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Urban-rural migration and congestion costs revisited: is there a triple dividend for cities in developing countries?

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Urban-rural migration and congestion costs revisited: is there a triple dividend for cities in developing countries?

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Abstract

Many cities in developing countries suffer from bad health and environmental conditions due to urbanization. The paper shows that increasing urban manufacturing congestion costs do not necessarily imply a reduction of a city's health as well as of environmental quality as one could expect ex-ante. The model distills a range of the urban manufacturing sector size which generates a triple dividend: a situation in which the government can simultaneously improve health, reduce pollution, and increase the productivity of labour by investing in either green capital or urban infrastructure that reduces congestion costs.

Keywords: Congestion Costs, Migration, Public Health, Pollution, Economic Development

JEL Classification Number: O18, H8, R12, R13, R23, Q52, I15, I18
1 Introduction

Urbanization can be generally understood as a dynamic event with a tremendous impact on the living conditions and the health status of urban population (Chang (1994)). So-called pull factors, such as the desire for better living conditions, profound education, the demand for cultural plurality and for a well developed communication infrastructure and, above all, improved access to health care attracts people from urban regions, resulting in overcrowded mega cities and extensive urbanized regions all over the world\(^1\). Although urbanization may improve the living conditions of many people as they benefit from increasing returns\(^2\). Hence, much of the growth process occurs in cities (Glaeser et al. (1992)). In a recent contribution, Das et al. (2014) show that urbanization is a highly significant factor explaining much of the variation of transitional growth in Indian’s states.

At the same time it implies challenges for the management of cities, especially in developing and emerging economies\(^3\). For instance, public infrastructure, such as roads, public refuse disposal or health care facilities are often in a poor condition or barely developed, and notably the socially deprived suffer most under these conditions (Bjorvatn (2000), Schwela et al. (2006), p.1.). Predominantly, weak institutions combined with economic mismanagement can be generally blamed for this situation. Further, as pointed out by Brakman and van Marrewijk (2010), urbanization also reflects the lumpiness of production factors, particularly in developing countries. Venables (2005) mentioned that "the presence of increasing returns to scale in [some developing country]cities leads to urban structures that are not optimally sized". Finally, as shown by Head and Ries (1995), foreign firms find it favorable to locate in cities with a sufficiently

\(^1\)Although predominantly relevant for developing and emerging economies, urbanization is a global phenomenon. For instance, in 2003, 73% of all European citizens live in urbanized regions. It is estimated that this number will increase towards 80% by 2030 (Tellnes et al. (2005)).

\(^2\)The NEG literature (Fujita et al. (2001), Krugman and Venables (1995)) argues that agglomeration reflects the existence of increasing returns together with factor movements towards cities.

\(^3\)Although Das et al. (2014) take into account some infrastructure related control variables, however, they do not explicitly focus on other important issues coming along with urbanization, such as health and environmental issues.
good infrastructure. It is further reasonable to assume that the majority of those firms produce cleaner compared to home firms. This can be used as an explanation, why so many cities in developing countries find themselves in a environmental development trap.

One of the challenging tasks primarily for cities belonging to the BRICS states is the dramatic increase of air pollution during the last ten years and its direct or indirect negative impact on health, not only for the urban-, but also for the rural population. For instance, it is estimated that indoor air pollution is responsible for 537,000 premature deaths annually in Asian urban regions alone. The WHO (2002) estimates that worldwide more than 1.4 billion urban residents are exposed to air pollution concentrations; this exceeds the determined WHO guidelines values dramatically by the factor equal or larger than three (WHO (2000, 2005)). In urban areas of developing countries, 2-5% of total deaths can be traced back to high fine particular matter (PM) levels, which are mainly caused by traffic, power plants and combustion industrial process (Haq and Schwela (2008)). Hence, urban growth causes significant consequences for the environment (Vennemo et al. (2009)), which in turn affects the cities attractiveness negatively.

There is compelling notably microeconomic evidence that infrastructure may have a significantly positive effect on health and worker’s productivity (Agénor (2008)). Behrman and Wolfe (1987), Lavy et al. (1996), Leipziger et al. (2003) and Wagstaff and Claeson (2004), pp. 170-174 note that sanitation and clean water strongly reduce infant mortality. Further, some survey find that slums, in which sanitation facilities are predominantly badly developed exhibit a twice as high infantile death rate (children

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4BRIC is an acronym for a group of countries at a similar stage of newly advanced economic development, consisting of Brazil, Russia, India, China and South Africa.

5Poor households often use most polluting fuels for cooking and heating. Together with the fact that more people, even the members of poor households spend more time at home due to a rising living-standard, indoor air pollution becomes a major challenge for health care (Haq and Schwela (2008)). The literature on urban air quality management systems are just beginning to focus on this fact.

6See WHO (2002).
under five) compared to non-slum urban regions. Irrespectively of any income effect, Wang (2003) however noted that access to electricity can significantly reduce child mortality. In many developing countries, urban and rural public health infrastructure is in poor condition\(^7\). The promise of higher income in urban areas compared to rural sectors create additional pressure on the urban health infrastructure, and, as a consequence, this will increase health risks due to insufficient treatment or medical malpractice because of the insufficient public health infrastructure. As a consequence of that health care services in developing countries are often delivered privately (Schlein (2013)) as many governments of developing countries often fall short providing sufficient public health care\(^8\). In the following, we explicitly introduce a medicare or health sector\(^9\) for two main reasons: first, new cohorts of urban population are richer and better educated and care more about health. Second, the inclusion of this modern sector catches the transition of many developing countries cities towards service sector areas\(^10\).

The majority of the relevant literature dealing with congestion effects has a pronounced supply-side focus but ignores demand-side congestion effects such as the induced congestion effect on public health mentioned above. One strand of literature embeds congestion effects into a AK neoclassical growth framework (among others, see Barro and Sala-i-Martin (1992), Eicher and Turnovsky (2000), Glomm and Ravikumar (1997) and Turnovsky (1997)). More recently, Grossmann (2013) discusses within a two-sector R&D based endogenous growth model the adverse productivity effects of a rising urban population. Literature established in the new economic geography sphere discusses congestion by mainly focusing on commuting and transporting costs. Generally, it is assumed that higher commuting and transporting costs in developing countries

\(^7\)The majority of African women living in the rural Sahara region blame bad infrastructure as the key obstacle for accessing health care (Collinson (2010)).

\(^8\)However it has to be noted that the size of the public sector varies by country and from region to region. For instance, in Uganda and Ghana the private sector demand is more than 60%, in Namibia it is less than 10% (BHIA (2008)).

\(^9\)In the following, we use the terms "health" and "medicare" synonymously.

\(^10\)Please refer to Zhen and Kahn (2013) who provide a superb overview of this issues for the case of China. Similar developments are likely to be generalized for other cities in other developing countries.
are the direct consequence of poor infrastructure quality\textsuperscript{11}. Prominent examples in this line of research are Krugman and Elizondo (2007), who focus on the centrifugal forces of commuting cost, whereas Tabuchi (1998) focuses on the interplay of transportation costs decrease and urban dispersion and concentration. Recently, within an evolutionary game-theoretic frame, Fujishima (2013) added housing congestion to the model developed by Fujita and Thisse (2003).

From that point of view it is however surprising that besides compelling empirical evidence\textsuperscript{12}, there are only a few attempts in the literature to study the relationship between health, environmental development and growing cities\textsuperscript{13}. Our economic model is mainly based on that of Bjorvatn (2000), who discusses supply-side congestion which constitutes an urban pull factor in a Todaro-style urban-rural setting. It also borrows elements from Acemoglu (2009), ch. 21, Kahn and Naqvi (1983) and Zenou (2011), Duranton (2008) and Combes et al. (2005). In particular, we follow Bjorvatn (2000), Duranton (2008) and Combes et al. (2005), because there is empirical support for this model structure, in particular for cities in developing countries.

However, our study contributes to the literature in several directions. (i) We expand his two-sector rural-urban economy towards a three sector economy, comprising a rural, a modern urban manufacturing and a modern urban, privately organized medicare sector. The latter two sectors define the city or urban region. The medicare good as well as the manufacturing good can only be produced in the urban region. The rural sector exclusively produces the agricultural good. Workers move to the sector which offers the higher wage. (ii) City growth causes congestion, not only on the supply side of the economy, but it also negatively affects the provision of public health services, which in turn negatively affects the personal health status. We follow Bjorvatn (2000) and assume that the traditional sector reacts less sensitive towards congestion, since this sector is less capital sensitive. This argument can be directly transferred to

\textsuperscript{11}In particular the higher transport cost structure of African countries compared to Asia and Latin America is generally regarded as the major bottleneck for economic development (JICA (2009) and Eifert (2005) et al.).

\textsuperscript{12}Please refer to Benneman and Kerf (2002) for a summary of conducted empirical studies.

\textsuperscript{13}There are some attempts, mainly with a macroeconomic perspective. For instance, see Agénor (2008), Agénor and Agénor (2009) and Agénor, Canuto, and Pereira da Silva (2012).
the medicare sector, which is service-orientated and, thus, more labor intensive. (iii) Consumption of goods causes environmental damage, which also affects the personal health status. (iv) Urban green capital positively affects urban labour productivity.

The paper contributes to one of the most pressing open research questions particularly for developing countries, namely, the effects of urban-rural migration on environmental and health when congestion effects are relevant.

In particular, the paper makes the following major points: (i) Based on a general equilibrium analysis, the paper disentangles intra- and intersectoral migration effects caused by supply-side congestion and shows that these are the driving forces which affect health and environmental development. These intra- and intersectoral migration effects can be associated with labour market spread-effects. Klarl (2013) in this context points out that labour market spread-effects affect city growth. (ii) The paper demonstrates that congestion costs as well as other externalities related to environmental and health issues make it difficult to implement a promising policy strategy to cope with the challenges which cities and urban areas in developing countries are faced with. Nevertheless, the paper shows that increasing urban manufacturing congestion costs do not necessarily imply a reduction of health as well as of environmental quality. (iii) It distills a range of the urban manufacturing sector’s size which allows for a triple dividend (increased health and environmental quality and higher urban manufacturing output) from a welfare point of view.

We note that there are papers who tackle some of this issues in isolation. Hence our model contributes to the literature in the way that we provide a basic model which includes to the best of our knowledge the first time the highly policy relevant causal

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14. Additionally, environmental damage can appear as a direct byproduct of the installed production technology. However, we neglect this additional source of pollution as this additional feature will not qualitatively change our overall results.

15. As mentioned above, the model assumes a city consisting of a medicare and urban manufacturing sector. Thus, with intersectoral migration we directly associate a labour force movement between the city and the rural area, whereas intrasectoral migration denotes migration within a city’s sectors.

16. For instance, recently Millard-Ball (2012) asks the question, whether city climate plans reduce emissions; Behrens and Robert-Nicoud (2014) tackles the issue of urbanization and inequality.
links between health, environment, migration in an general equilibrium model of city size for developing countries.

The model's structural form is heavily related to empirical evidence but even flexible enough for extensions in several directions. As our model is amenable to comparative static exercises and to numerical simulations, we use explicit functional forms where it is necessary. The plan of the paper is the following: Section 2 introduces the model, section 3 presents the general equilibrium, whereas section 4 is devoted to a characterization of the general equilibrium. Section 5 discusses the environmental and health quality equilibrium response if congestion costs are relevant, gives policy recommendations and links the model's result to the relevant empirical literature. Section 6 concludes.

2 Urban-rural migration and the green city: A basic framework for developing countries

As mentioned by Duranton (2008), an economic theory dealing with cities has a common underlying structure. In particular, three structural elements are essential: a spatial structure, some relevant assumptions regarding the mobility of factors, such as labour or capital and, finally, the production structure. Given these elements are embedded, the model is well specified. Following this argument, in this section we consider an economy consisting of three sectors, the urban manufacturing sector ($u$), the privately organized medicare sector ($m$), and the rural sector ($r$). We may assume that the urban manufacturing sector and the health or medicare sector form the urban area or city. The economy produces three goods, a manufactured good produced only in the manufacturing sector, a medicare good produced exclusively in the urban medicare sector and a agricultural good solely manufactured in the rural region. For simplicity, we neglect transportation costs. Hence, goods can be shipped without costs between the sectors. We assume further that labor is the only factor of production. Workers are mobile and there are no migration costs. Hence, workers base their location decision solely on sector-specific wage-differentials and, thus, move to that region which offers the higher wage. Although stylized, this model offers are very intuitive way of intro-
ducing a geographic dimension which is indeed needed in order to grasp the possible congestion effects on public health service induced by rapid city population growth and the induced feedback effect on workers health capital.

2.1 Household preferences

The utility function of each worker for the three goods is given by

\[ u(c_r, c_u, h) = c_r^\alpha c_u^\beta h^{1-\alpha-\beta}, \{\alpha, \beta\} \in (0, 1), 0 < \alpha + \beta < 1, \]  

where \( c_i, i = \{u, r\} \) denotes the consumption of the manufactured and agricultural good. \( h \) represents the worker’s health status, which is endogenized below. Let us further denote \( I \) the total income of the economy, which is the sum of the sector specific labor income \( w_j L_j, j = \{u, r, m\} \) and profits \( \pi \). Below we will see that \( h \) is endogenously explained by public health care spending as well as by the overall environmental damage, which are both exogenously given to the household. Hence, public health creates a positive externality, whereas environmental damage creates a negative externality to the representative household. Further, so called green capital adds positively to the household’s utility.

2.2 Production

1. Agricultural sector

The agricultural good production technique is linear. The total agricultural output is given by:

\[ y_r(L_r) = L_r. \]  

The market supply is perfectly competitive and everybody has free access to this technology.

2. Manufacturing sector

Manufacturing in the urban sector can only employ workers from the manufacturing sector in the urban area. As mentioned in the introduction, the manufactured good is produced with an advanced technology. For simplicity, we assume a monopoly structure in this market, where only one firm has access to
this technology due to financial restrictions\textsuperscript{17}. It is obvious to assume that the manufactured good sector’s firms productivity also depends on the available urban infrastructure, such as the urban transporting system. We follow Bjorvatn (2000) and assume that the production function therefore takes the form:

\[ y_u(L_u, c(L_u, \varsigma)) = \alpha_u(L_u - c(L_u, \varsigma)), \]

with \( \alpha_u > 1 \) as the urban manufacturing sector labor productivity and \( L_u \) as the labor input. \( c(L_u, \varsigma) \) represents the congestion cost function which increases with decreasing quality of urban infrastructure supply \( \varsigma \) for a given level of \( L_u \).

We further assume convex congestion costs, i.e. \( \frac{\partial c(L_u, \varsigma)}{L_u} > 0, \frac{\partial^2 c(L_u, \varsigma)}{L_u^2} > 0 \), \( \lim_{L_u \to \infty} \{ c(L_u, \varsigma) \} = \infty \), \( \lim_{\varsigma \to \infty} \{ c(L_u, \varsigma) \} = 0 \) and, finally, \( \lim_{\varsigma \to 0} \{ c(L_u, \varsigma) \} = \infty \). We follow Bjorvatn (2000) and express the congestion costs as:

\[ c(L_u, \varsigma) = \frac{L_u^2}{2\varsigma}. \]

Hence, public investment in urban manufacturing infrastructure \( \varsigma \) decrease congestion costs, whereas a growing urban manufacturing sector embodied by an increasing \( L_u \) decrease the supply of the manufacturing sector.

3. Medicare/health sector

The medicare sector produces health care services. We assume that the access to the health care market is limited due to accreditation issues or governmental regulation. In contrast to the manufacturing sector, the medicare sector reacts less sensitive towards congestion due to the service orientation of this sector. Hence, the production technology reads as:

\[ y_m(L_m) = \alpha_m L_m, \]

with \( \alpha_m > 1 \) as the medicare sector labor-productivity and \( L_m \) is the labor input for the medicare sector production\textsuperscript{18}.

\textsuperscript{17}It is worth mentioning that the monopoly power is limited as there is a positive probability of market entry, both from the urban and the medicare sector. We will explore this point in more detail later.

\textsuperscript{18}As for the urban manufacturing sector, monopoly power is limited as there is a positive probability of market entry both, from the manufacturing and the urban sector.
2.3 Green capital benefits

There is a large and well-established literature from several disciplines like medicine, economics, psychology, urban planning, and climatology indicating that encounters with nearby nature, such as parks and walkways, will positively affect mental and physical fitness and, hence, may contribute to improve productivity. For instance, Zhang (1999) points to the fact that green open space provides "green lungs" to a city by improving air quality. Zhang (1999) found that the total amount of green space in Beijing absorbs 4,240,000 tonnes of CO₂ per year and removes 30,516.56 tonnes of dust particles per year, which is enormous. Further, Finnigan et al. (1994) points out that green capital controls temperature extremes by improving climate conditions and reducing the "heat island effect", which is often associated with humid and hot summers (Shashu-Bar et al. 2010). Moreover, green capital reduces noise and provides sources for fresh water (Biao et al. 2010). Kuo et al. (1998) observed that the more green inner-city public spaces are, the more these spaces are used by residents. He further notes that people living closer to greenery have more social contacts and have stronger feelings of belongings and hence, increases quality of life.

Given the large body of evidence that, first, bad environmental conditions such as pollution are causally linked with poor health and reduced labour productivity and, second, as mentioned above, bad environmental conditions are causally linked with underinvestment into green capital, it is reasonable to conjecture that at least some of the green capital benefits - such as benefits stemming from public financed parks - can be interpreted as a positive externality for the city’s labour productivity.

Denoting $\alpha_i$ for $i = \{m,u\}$ the economy sector's specific labour productivity, it increases linearly with a known index $\Phi_i$ which measures the density of green capital of urban sector $i$:

$$\alpha_i(\Phi_i) = \bar{\alpha}_i I_{i\in\{u,m\}} + \chi_i \Phi_i,$$

(6)

with $\chi_i \in [0,1]$ as an exogenously given known constant.

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20 For the first, refer among others to Chay and Greenstone (2003). For the positive link between health and labour productivity refer to Graff Zivin and Neidell (2012).

21 One can argue that also brown capital improves labour productivity. To keep tractability of this
\( I_{i \in \{u,m\}} \) denotes an indicator function which is zero for the rural sector. \( \bar{\alpha}_i > 1 \) represents the so-called basic labour productivity which is unrelated to the greenness of the city. For simplicity, we assume that \( \bar{\alpha}_m = \bar{\alpha}_u = \bar{\alpha} \). It is worth mentioning that the assumption manifested in equation (6), although strongly justified by the literature, can be seen as a first approximation to a more detailed investigation of further factors which affect urban productivity\(^{22}\).

### 2.4 Public health service

The supply of public health service depends positively on the health infrastructure supply per worker. More specifically, let us assume that public health service can be written as a weighted average of each sector’s public health infrastructure \( \Omega_j, \forall j = \{u,r,m\} \):

\[
H_p(\Omega_j, L_j) = \prod_j \left[ \frac{\Omega_j}{L_j} \right]^{\nu_j},
\]

(7)

with \( \sum_j \nu_j = 1 \) which denotes the importance of the \( j \)-th sector’s public health infrastructure. The motivation underlying this function is that for a given health infrastructure level \( \Omega_j, j = \{u,r,m\} \), a growing population causes congestion which reduces \( H_p \)\(^{23}\). For instance, it would be a natural choice to assume that the individual health directly affects the city’s labour productivity (see Agénor (2008)). This relaxation in turn increases the complexity of the model to a certain extent. We found that for some specifications, a model’s solution cannot be guaranteed or we obtained solutions which hardly can be interpreted as additional non-linearities arise due to the endogeneity of health. Hence, little is gained but the model’s tractability is lost.

\(^{23}\)This implies that the government cannot react immediately by adjusting the public health infrastructure when population increases.
2.5 Health status

A representative worker’s health status increases with public health services $H_p$ and the privately offered medicare service. As mentioned above, urban as well as rural environmental damage, i.e., air pollution induced by consumption of the urban and rural good negatively affects the health status. Formally, we can represent these assumptions as

$$h(m,e,Ψ) = Γ^{1+ξ}m^{γ_1}e^{-γ_2}H_p^{γ_3} = m^{γ_1}e^{-γ_2}Ψ,$$  (8)

with $Ψ = Γ^{1+ξ}H_p^{γ_3}$. $Ψ$ contains the weighted public health services with corresponding weight $γ_3 \in (0,1)$ and the green capital $Γ$ weighted with $ξ = \prod_j Ψ^ω_j \geq 0$ with $\sum_j ω_j = 1$ which represents the strength of the positive externality of green capital for the society’s health. $Ψ$ is obviously independent from consumer choices. In contrast to $Ψ$, the level of both $m$ and $e$ are directly influenced by consumers activities. Now, $γ_1 - γ_2$ defines the scale elasticity, which can be positive, null or negative. As $\{γ_1,γ_2\} \in (0,1)$, it follows that $-1 < γ_1 - γ_2 < 1$.

2.6 Environmental damage

Product consumption causes environmental damage as a byproduct (Andreoni and Levinson (2001)), i.e. ambient air or water pollution which is a main challenge particularly for poor urban cities. We assume that the economy’s total environmental damage is the weighted average of each sector $i$’s pollutions. We make the reasonable assumption that only the rural and urban manufacturing sector contributes significantly to pollution. Drawing the arguments together, we propose the following functional form for the environmental damage:

$$e(c_i, A) = \left[ \phi \prod_i c_i^{δ_i} \right]^κ - A^{κ_ε},$$  (9)

with $\{φ,ψ_ε,κ\} > 0$ and $δ_i > 0 \forall i \in \{u,r\}$ 24. The rationale behind equation (9) is that environmental damage is a positive function of three sector consumption and is negatively related to public abatement effort $A$. For simplicity, we further assume that

24We depart from modeling the emissions as a byproduct of sectors’ firms production (see Kahn (2006), p. 505) as well as a byproduct of consumption. For all sectors, this would imply to relax
abatement effort $A$ is financed from a supranational authority or the federal state and, hence, is exogenous to the local $j$ sectors\textsuperscript{25} but is increasing with income $I$ as more developed countries pollutes more than least developed countries. Hence, $A \propto I^{26}$.

In the following we assume that $e(c_j, A) > 0$. With the restriction that $\psi_e > 1$, equation (9) can be also interpreted as the environmental Kuznets curve, as it exhibits an inverse U-shaped income-pollution relationship (Andreoni and Levinson equation (9) towards

$$e(c_j, y_j, A) = \left[ \phi_{c_j} \prod_j c_j^{\delta_{c_j}} \right]^{\kappa_e} \left[ \phi_{y_j} \prod_j y_j^{\delta_{y_j}} \right]^{\kappa_y} - A^{\psi_e}. \tag{10}$$

Obviously, this modification is more realistic since it assumes heterogeneity of pollution sources, however, it would not change the results qualitatively. Further, one can argue that since all production is instantly consumed, this distinction is somehow irrelevant. In other words, if we redefine the externality to depend on production this would obviously not change the models predictions. Following this arguments, we depart from the modification of equation (9).

\textsuperscript{25} Of course, this is a simplifying assumption but nevertheless realistic. For instance, World Bank’s Carbon Finance Unit (https://wbcarbonfinance.org/Router.cfm?Page=HomeItemID=24675) uses funds contributed by OECD governments and companies to finance abatement technologies in developing countries. In particular, the World Bank’s Carbon Finance Unit has a focus on cities and urban landscapes. Alternatively, it straightforward to include the local government into the model which imposes environmental taxes on income and profits. A further scenario comprises local firms which invest in an appropriate abatement technique. However, many developing countries charge pollution inputs or outputs with a tax as this is considered as a particular attractive policy because other policy instruments, such as regulations require experience in administrating, which many countries suffers from (Bruce and Ellis (1993)). Clearly, this would make the model more realistic but will not alter the overall results the paper make. Additionally, it is worth to mention that the paper’s aim is not devoted to a discussion regarding the efficiency of different environmental policy instruments.

\textsuperscript{26} For instance, the group of least developed countries (UN classification) were responsible for 0.3 metric tons of CO2 emissions in 2010, whereas South Asia emitted 1.4 metric tons in 2010 (see http://data.worldbank.org/topic/climate-change).
(2001) and Zheng and Kahn (2013)). Further (9) accounts for heterogeneity within emission sectors.

3 General Equilibrium

In our model, we have one labour market and three goods markets. The general equilibrium can be characterized as follows:

Definition 1. An general equilibrium is given by a known vector of sector specific prices \((p_r, p_m, p_u, w_u, w_m, w_r)\), sector specific quantities \((y_r, y_u, y_m, L_u, L_m, L_r, c_r, c_u, m)\) and economy wide vector of quantities \((e, H_p, h)\) such that

(i) the household maximizes her utility under the budget constraint, taking public health \(H_p\) and the environmental damage \(e\) as well as the public abatement activities \(A\) as given. Moreover, they are indifferent where to locate as the migration condition implies wage equalization across sectors: \(w_m = w_r = w_u\).

(ii) the sectors' firms maximize their profits.

(iii) labour markets clear: \(L = \sum_j L_j\) for \(j = \{u, r, m\}\).

(iv) consumption goods markets clear, i.e. \(c_j = y_j\), for \(j = \{u, r, m\}\).

In the following paragraphs we explain how to derive the general equilibrium in more detail. As we will see, we obtain the general equilibrium by solving the model backwards starting with the labour market equilibrium for the urban sector. For this purpose, let us first assume to set the price of the urban good as numeraire, \(p_u = 1\).

But what results for the wage rates of the urban as well as for the manufacturing sector given the sectors are not competitive as assumed in this paper? The point is that even if the e.g. urban sector is imperfectly competitive, there is a "natural" price ceiling, which induces the firm to set the price not above \(p_u = 1\) due to probability of market entry from the rural and medicare sector. Further, the urban manufacturer would never charge a price below unity. Theoretically, the manufacturer can set a infinitely high price in order to safe sector specific variable costs (Bjørvatn (2000)). But the unity price ceiling restricts him to set the price above \(p_u = 1\). Hence, his optimal strategy is to impose a price of \(p_u = 1\). The same line of arguments hold for the medicare
sector monopoly firm. We now further assume the existence of perfectly competitive organized informal counterparts for both, the urban manufacturing as well as for the medicare sector with production technique $\hat{y}_i = L_i$ for $i = \{m, u\}$. Those sectors are not confronted with congestion effects. Then those informal sectors would offer a wage rate of $\hat{w}_i$ for $i = \{m, u\}$, where $\hat{w}$ is the informal wage rate. Hence, the formal sector will set $w_i = \hat{w}_i$ for $i = \{m, u\}$. Further, from equations (3) and (5) we directly observe that profits increase with $L_i$ for $i = \{m, u\}$ as the congestion effects are given and $\alpha > 1$ which can be directly seen from equation (6). Hence, given the urban and medicare firms deliver their markets, they will employ the entire sector-specific labour force.

We are now prepared to determine the equilibrium labor market allocation for the three sectors. For this, we first derive the household’s demand structure for the sector specific goods of the economy. Employing (1), inserting equations (8) and (9) using and the budget restriction, the demand structure for the manufactured, the rural and the medicare good, respectively, is given as

$$[c_u, c_r, m] = \left[ \Theta_u \frac{I}{p_u}, \Theta_r \frac{I}{p_r}, \Theta_m \frac{I}{p_m} \right],$$

with $\Theta_u = \left[ \frac{\beta - (1-\delta)\gamma_2 \kappa(1-\alpha-\beta)}{\alpha + \beta + (\gamma_1 - \gamma_2 \kappa)(1-\alpha-\beta)} \right]$, $\Theta_r = \left[ \frac{\alpha - \delta \gamma_2 \kappa (1-\alpha-\beta)}{\alpha + \beta + (\gamma_1 - \gamma_2 \kappa)(1-\alpha-\beta)} \right]$ and $\Theta_m = \left[ \frac{\gamma_1 (1-\alpha-\beta)}{\alpha + \beta + (\gamma_1 - \gamma_2 \kappa)(1-\alpha-\beta)} \right]$ and $p_j$ as the product price for good $j = \{u, r, m\}$. Note that the commodities will be only consumed given $\beta - (1-\delta)\gamma_2 \kappa (1-\alpha-\beta) > 0$, $\alpha - \delta \gamma_2 \kappa (1-\alpha-\beta) > 0$ and $\alpha + \beta + (\gamma_1 - \gamma_2 \kappa)(1-\alpha-\beta) > 0$. The latter inequality holds given the environmental damage caused by the urban and rural commodities is smaller than the implied three-sector commodity utility.

From equation (11) we know that $c_j = \frac{\Theta_j I}{p_j}$ for $j = \{u, r, m\}$. As we have chosen the urban manufacturing price level acting as numeraire, it follows that $p_u = w_u = 1^{27}$. Employing further the equilibrium condition (iv) of Definition 1 together with the profit streams of the urban manufacturing and medicare sector, $\pi_i = p_i y_i - L_i w_i$ for $i = \{m, u\}$, we arrive at the following two-dimensional system of equations which explains the rural as well as the medicare wage as functional forms of different sector specific labour force allocations:

27This point can be justified as follows: Note that the informal urban sector maximize the profits $\hat{\pi}_u = \hat{y}_u - \hat{w}_u L_u$ which finally leads to $\hat{w}_u = p_u = 1$. But we argued that $\hat{w}_u = w_u$, which directly implies $w_u = p_u = 1$. 

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only one of them, namely the allocation linked with the superscript ∗.

Thus, for three different equilibrium labour market allocations denoted with the superscripts (12) and (13) we can solve for the equilibrium labour market allocations. For \( L > 0 \), we find three different equilibrium labour market allocations denoted with the superscripts \( \{*, \land, **\} \), which are delegated to the next Lemma. As we will see in the next section, only one of them, namely the allocation linked with the superscript * is stable.

**Lemma 1.** The possible sets of labour market allocations read as

\[
L_u^{*, \land, **} = \frac{A \gamma + \sqrt{\gamma} \left(2 \theta L (\bar{\alpha} + \chi_u \Omega_u) (\hat{\Omega}_1 \hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m) - 1) B + A^2 \gamma\right)}{\theta (\bar{\alpha} + \chi_u \Omega_u) B} + \frac{\hat{\Omega}_1 (\hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m) + 1) \left(\sqrt{\gamma} (2 \theta L (\bar{\alpha} + \chi_u \Omega_u) (\hat{\Omega}_1 \hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m) - 1) B + A^2 \gamma\right)}{\theta (\bar{\alpha} + \chi_u \Omega_u) (\hat{\Omega}_1 B + \hat{\Omega}_2)} + \frac{(\hat{\Omega}_1 + 1) \hat{\Omega}_2 (\theta L (\bar{\alpha} + \chi_u \Omega_u) B - A \gamma)}{\theta (\bar{\alpha} + \chi_u \Omega_u) B^2},
\]

\[
L_r^{*, \land, **} = \frac{\hat{\Omega}_1 (\hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m) + 1) \left(\sqrt{\gamma} (2 \theta L (\bar{\alpha} + \chi_u \Omega_u) (\hat{\Omega}_1 \hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m) - 1) B + A^2 \gamma\right)}{\theta (\bar{\alpha} + \chi_u \Omega_u) (\hat{\Omega}_1 B + \hat{\Omega}_2)} + \frac{(\hat{\Omega}_1 + 1) \hat{\Omega}_2 (\theta L (\bar{\alpha} + \chi_u \Omega_u) B - A \gamma)}{\theta (\bar{\alpha} + \chi_u \Omega_u) B^2},
\]

\[
L_m^{*, \land, **} = \frac{A \gamma + \sqrt{\gamma} \left(2 \theta L (\bar{\alpha} + \chi_u \Omega_u) (\hat{\Omega}_1 \hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m) - 1) B + A^2 \gamma\right)}{\theta (\bar{\alpha} + \chi_u \Omega_u) B} + \frac{\hat{\Omega}_1 (\hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m) + 1) \left(\sqrt{\gamma} (2 \theta L (\bar{\alpha} + \chi_u \Omega_u) (\hat{\Omega}_1 \hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m) - 1) B + A^2 \gamma\right)}{\theta (\bar{\alpha} + \chi_u \Omega_u) (\hat{\Omega}_1 B + \hat{\Omega}_2)} + \frac{(\hat{\Omega}_1 + 1) \hat{\Omega}_2 (\theta L (\bar{\alpha} + \chi_u \Omega_u) B - A \gamma)}{\theta (\bar{\alpha} + \chi_u \Omega_u) B^2},
\]

with \( A \equiv \hat{\Omega}_2 \left(\hat{\Omega}_1 (\bar{\alpha} + \chi_m \Omega_m) (\bar{\alpha} + \chi_u \Omega_u) - \chi_m \Omega_m + \chi_u \Omega_u\right) + \hat{\Omega}_1 (\bar{\alpha} + \chi_u \Omega_u) + 1 \) and \( B \equiv \hat{\Omega}_1 (\hat{\Omega}_2 (\bar{\alpha} + \chi_m \Omega_m + 1) + 1) + \hat{\Omega}_2 \).

It is important to mention that Lemma 0 is not sufficient to assess a valid labour market allocation, e.g. it does not exclude the possibility that all sectors are active in equilibrium, which implies that \([L_r^{*, \land, **} > 0, L_u^{*, \land, **} > 0, L_m^{*, \land, **} > 0]\). To see this last argument, note that for a sufficient level of \( L_u \), the congestion costs increases and this in turn cuts profits. Hence, the implied labour market pull effect of the congestion effects
may lead to a shutdown of the urban manufacturing sector in equilibrium. Fortunately, the model's set-up implies that determining a critical range of of $L_u$, which guarantees operating of the urban manufacturing firm, also directly induces the activity of the remaining sectors. However, $L_u$ in equilibrium is endogenously determined by a set of parameters (see equation (14)). As shown by the next Lemma, restricting solely the labour productivity $\bar{\alpha}$ ensures that all sectors operate in equilibrium by assuring that $[L_r^{*,\wedge,\alpha*>0}, L_u^{*,\wedge,\alpha*>0}, L_m^{*,\wedge,\alpha*>0}]$.

**Lemma 2.** (1) There exists a range for $L_u$ which guarantees that the urban manufacturing firm operates. This range is given by

$$L_u \in \left(0, \tilde{L}_u\right),$$

with

$$\tilde{L}_u = \left[\frac{2\kappa(\bar{\alpha}+\chi_u\Phi_u-1)}{\theta(\bar{\alpha}+\chi_u\Phi_u)}\right].$$

(2) The sector specific labour market equilibrium allocation $[L_r^{*,\wedge,\alpha*>0}, L_u^{*,\wedge,\alpha*>0}, L_m^{*,\wedge,\alpha*>0}]$ exists given $\bar{\alpha} \in (\bar{\bar{\alpha}}, \tilde{\bar{\alpha}})$ with

$$\bar{\bar{\alpha}} \equiv \left[\frac{\gamma_2\kappa(\alpha+\beta-1)\kappa\kappa(\alpha+\beta-1)(\chi_m\Omega_m-1)}{\gamma_1(1-\alpha-\beta)}\right]$$

and

$$\tilde{\bar{\alpha}} \equiv \left[\frac{-\gamma_2(\beta-1)\kappa(\alpha+\beta-1)\kappa\kappa(\alpha+\beta-1)(\chi_m\Omega_m-1)}{\gamma_1(1-\alpha-\beta)}\right].$$

(3) $\tilde{\Omega}_2$ is strictly positive for $\bar{\alpha} \in [\bar{\bar{\alpha}}, \tilde{\bar{\alpha}}]$. Further, given (2) holds, for $\bar{\alpha} \equiv \tilde{\Omega}_1 - \tilde{\Omega}_2$, we can deduce the following relationship:

$$\bar{\alpha} > \tilde{\bar{\alpha}} \Rightarrow \tilde{\Omega}_1 > \tilde{\Omega}_2 \text{ for } \bar{\alpha} \in [\bar{\bar{\alpha}}, \tilde{\bar{\alpha}}]$$

$$\bar{\alpha} = \tilde{\bar{\alpha}} \Rightarrow \tilde{\Omega}_1 \geq \tilde{\Omega}_2 \text{ for } \bar{\alpha} \in [\bar{\bar{\alpha}}, \tilde{\bar{\alpha}}]$$

$$\bar{\alpha} < \tilde{\bar{\alpha}} \Rightarrow \left\{\begin{array}{ll}
\tilde{\Omega}_2 - \tilde{\Omega}_1 \geq 0 & \text{for } \bar{\alpha} \in [\bar{\bar{\alpha}}, \tilde{\bar{\alpha}}] \\
\tilde{\Omega}_2 - \tilde{\Omega}_1 \leq 0 & \text{for } \bar{\alpha} \in [\bar{\bar{\alpha}}, \tilde{\bar{\alpha}}].
\end{array}\right.$$
manufacturing firm would not deliver the market. On contrary, if conditions (1) and (2) of Lemma 2 hold, a labour market equilibrium exists and, hence, it is straightforward to verify that we can recursively solve for the remaining quantities listed in Definition 1. Second, condition (3) although not relevant for the existence of the labour market equilibrium, it nevertheless offers some information regarding the proportion of \( \tilde{\Omega}_1 \) and \( \tilde{\Omega}_2 \) which has a main impact, not only e.g. on the labour market equilibrium outcome but also, as we will see, for the comparative statics.

It is worth to note that so far we only impose the existence of a labour market equilibrium. However, Lemma 2 is not sufficient to identify possible multiple equilibria and it is further not informative regarding the stability of existing equilibria. The next section is concerned with these issues.

4 Characterization of the Equilibrium

In this section we will directly focus on the characteristics of the obtained labour market equilibria \([L^*_r, \wedge, ** > 0, L^*_u, \wedge, ** > 0, L^*_m, \wedge, ** > 0]\) starting with the urban manufacturing sector. In particular, we want to answer the question whether multiple equilibria exists, and given they do exist, we want to explore whether they are stable or not.

In the preceding section, we have argued that the model can be solved recursively starting with the labour market equilibrium. Once again we make use of the recursive structure of the model and deduce that proving the stability of the labour market equilibrium directly proves the stability of the general equilibrium of the model.

In order to achieve a stable labour market equilibrium allocation in which all sectors
are active in business, we expect that (12) and (13) are increasing functions of the other sector’s labour force\footnote{This can be easily seen by using the fact that $L = \sum_j L_j$. Replacing $L_r$ in equation (12) by $L_r = L - L_m - L_u$ and $L_m$ in equation (13) by $L_m = L - L_r - L_u$ leads to this conclusion.}: E.g., an exogenous productivity shock which draws more people to the urban manufacturing sector, leads to a reduction of both, the rural and medicare sector employment, as we assume zero population growth. Consequently, this creates a direct upward pressure on the rural and medicare sectors’ price and wage levels. Hence, in this case, the labour market acts as the stabilizing force in the economy as it guarantees a stable long-run equilibrium. Otherwise, if the wages of one sector are increasing in the labour force of the same sector, this induces a continuous migration to the others sectors. Hence, a (short-run) equilibrium in this case cannot be stable in the long-run.

Using the fact that in equilibrium $w_u = 1$ and further evaluating $L_m$ and $L_r$ at their corresponding equilibrium values, we can represent the wage rates $w_m(L_{*,\wedge,*}, L_{*,\wedge,**})$ and $w_r(L_{*,\wedge,*}, L_{*,\wedge,**})$ with varying $L_u$, whereas the intersection of the wage equation $w_m(L_{*,\wedge,**}, L_{*,\wedge,**})$ and $w_r(L_{*,\wedge,**}, L_{*,\wedge,**})$ together with $w_u = 1$ determines $L_u^*$. Now, the specific parameter calibration of the model shifts $w_m(L_{*,\wedge,**}, L_{*,\wedge,**})$ and $w_r(L_{*,\wedge,**}, L_{*,\wedge,**})$ upwards or downwards for given $L_u$ and, hence, rules out or not the existence of multiple equilibria. As $w_m(L_{*,\wedge,**}, L_{*,\wedge,**})$ and $w_r(L_{*,\wedge,**}, L_{*,\wedge,**})$ are quadratic in $L_u$ we expect at most two equilibria. The next two Lemmas impose some restrictions which rule out or not the existence of multiple (urban) labour market equilibria.

**Lemma 3.** For $\bar{\alpha} \in [\bar{\alpha}; \hat{\alpha}]$ a multiple labour market equilibrium can be established.
Lemma 2 provides some further conditions which have to be fulfilled to establish another interesting result, the so-called knife-edge equilibrium.

**Lemma 4.** (1) There exists a so-called knife-edge equilibrium which leads \( w_m(\hat{L}_m, \hat{L}_r, \hat{L}_u) \) and \( w_r(\hat{L}_m, \hat{L}_r, \hat{L}_u) \) being tangent to \( w_u = 1 \). This is given if \( \hat{L}_u = \frac{\varsigma}{\theta} \). Given this, an equilibrium labour market allocation is defined by

\[
[\hat{L}_r, \hat{L}_u, \hat{L}_m] = \left[ \frac{\varsigma \Omega_1 \alpha_u (-\hat{\Omega}_2 \alpha_m - 1)}{2 \theta (\hat{\Omega}_1 \hat{\Omega}_2 \alpha_m - 1)}, \frac{\varsigma \hat{\Omega}_2 \alpha_u (-\hat{\Omega}_1 - 1)}{2 \theta (\hat{\Omega}_1 \hat{\Omega}_2 \alpha_m - 1)} \right].
\]

(18)

(2) The equilibrium allocation \([\hat{L}_r > 0, \hat{L}_u > 0, \hat{L}_m > 0] \) exists.

Now, for a sufficient \( \bar{\alpha} > 2 - \chi_u \Omega_u > 0 \) it follows that \( \hat{L}_u < \hat{L}_u \) and thus the urban manufacturing firm operates in the market (see Lemma 2) by employing \( \hat{L}_u = \frac{\varsigma}{\theta} \). However, a small shock moves the system away from the equilibrium. Although with a narrow interpretation, the knife-edge equilibrium is unstable itself, with a broader view, it separates stable from unstable equilibria as we will see below.

**Lemma 5.** (1) For given \( L_m > 0, L_r > 0 \), a labour market equilibrium is called stable, given there is no incentive to deviate from this equilibrium. This is the case if and only if

\[
L_u \in (0, \hat{L}_u),
\]

(19)

whereas \( \hat{L}_u \equiv \frac{\varsigma}{\theta} \) represents the urban labour force, knife-edge equilibrium.

In other words, for a given \( \theta \), the infrastructure quality of the urban manufacturing sector, \( \varsigma \), is critical for determining whether a labour market equilibrium is stable or not.

The arguments presented sofar will become more clear by introducing the following Example 1.
Example 1: Labour market equilibrium. Let us consider the following calibration:

$$\alpha = 0.4, \beta = 0.46, \kappa = 0.1, \gamma_1 = 0.15, \gamma_2 = 0.15, \gamma = 1.075, \theta = 1, \delta = 0.5, \chi_m = \chi_u = 1, \Omega_m = \Omega_u = 1$$ and $\bar{\alpha} = 5$. The entire labour supply is set to $L = 3.3$.

For simplicity and without loss of generality, the calibration exercise imposes a scale elasticity of $\gamma_1 - \gamma_2 = 0$, both urban sectors are endowed with the same level of green capital, i.e. $\Omega_m = \Omega_u = 1$ and labour productivity of both urban sectors can be improved in the same way by greening the sectors, i.e. $\chi_m = \chi_u = 1$, while urban manufacturing congestion costs are relevant, i.e. $\theta = 1$. Now, $\bar{\alpha} \in [\bar{\alpha}, \bar{\alpha}] = [2.72, 21.85]$ and further $\bar{\alpha} = 2.94 < \bar{\alpha}$. Hence we deduce that one equilibrium is stable, whereas the other does not exists in the sense as the urban manufacturing firm would realize negative profits.

Figure (1) provides a graphical representation of Example 1. It represents the knife-edge equilibrium as well as the stable and unstable urban labour market equilibria allocations denoted as $\hat{L}_u, L_u^*$ and $L_u^{**}$, respectively. As we can easily observe, Figure 1A and 1B represent the unstable and knife-edge equilibrium labour market allocations, whereas Figure 1C and 1D show the stable as well as the knife-edge equilibrium (see Lemma 3.). Note that the labour market allocations denoted with point $A_u$ fail to fulfill the employment condition $L = \sum_j L_j$ for $j = \{u, r, m\}$.

---

Ex ante, it seems straightforward to refer directly to equations (12) and (13) together with $L = \sum_j L_j$ for $j = \{u, r, m\}$ to represent the labour market equilibrium in $\mathbb{R}^3$. However, in this contribution we map the equilibria in $\mathbb{R}^2$ to guarantee an insightful representation of the model’s labour market equilibrium by evaluating $w_m(L_m, L_r)$ and $w_r(L_m, L_r)$ at their respective steady state values.
Having found a stable labour market equilibrium in the preceding section, in this section we want to explore the environmental and health response of a positive urban manufacturing congestion cost shock. This shocks manifests by increasing $\theta$. As we can directly deduce from Lemma 1, changing $\theta$ induces a reallocation of the sectoral labour market. Thus, by varying $\theta$ we can not only discuss the induced urban-rural migration effects (see Lemma 1) but it is also possible to highlight the induced health and the environmental changes of the entire urban-rural economy due to the recursive structure of the model.

The analysis rests upon comparative-statics based on the following system which
represents the equilibrium inter- and intrasectoral labour force migration:

\[
L^*_r = L^*_r(L^*_m, L^*_u, \theta, \mathbf{P})
\]

\[
L^*_m = L^*_m(L^*_r, L^*_u, \theta, \mathbf{P})
\]

\[
L^*_u = L - L^*_m - L^*_r,
\]

where the vector \( \mathbf{P} \) contains the set of remaining parameters which are not of particular interest for the next subsection’s analysis.

5.1 Urban manufacturing congestion costs: Comparative Statics

Let us now explore the effects of increasing \( \theta \). By total differentiating system (20) it is possible to highlight the complex migration dynamics between the sectors as a reflex to increasing urban manufacturing congestion costs.

**Result 1.** The intra- and intersectoral labour force migration due to an increasing importance of urban manufacturing congestion costs can be represented as:

\[
\frac{dL^*_u}{d\theta} = - \frac{(\epsilon_{m,\theta} + \epsilon_{m,\theta}) \frac{L^*_m}{\theta} + (\epsilon_{r,\theta} + \epsilon_{r,\theta}) \frac{L^*_r}{\theta}}{1 + (\epsilon_{m,\theta} \epsilon_{m,\theta} + \epsilon_{m,\theta}) \frac{L^*_m}{\theta} + (\epsilon_{r,\theta} \epsilon_{r,\theta} + \epsilon_{r,\theta}) \frac{L^*_r}{\theta} + (\epsilon_{m,\theta} \epsilon_{m,\theta})} \frac{L^*_m}{\theta} \frac{L^*_r}{\theta} > 0 \tag{21}
\]

\[
\frac{dL^*_m}{d\theta} = \frac{(\epsilon_{m,\theta} \epsilon_{m,\theta} + \epsilon_{m,\theta}) \frac{L^*_m}{\theta} + (\epsilon_{r,\theta} \epsilon_{r,\theta} - \epsilon_{m,\theta} \epsilon_{r,\theta}) \frac{L^*_m}{\theta} \frac{L^*_r}{\theta}}{1 + (\epsilon_{m,\theta} \epsilon_{m,\theta} + \epsilon_{m,\theta}) \frac{L^*_m}{\theta} + (\epsilon_{r,\theta} \epsilon_{r,\theta} + \epsilon_{r,\theta}) \frac{L^*_r}{\theta} + (\epsilon_{m,\theta} \epsilon_{m,\theta})} \frac{L^*_m}{\theta} \frac{L^*_r}{\theta} < 0 \tag{22}
\]

\[
\frac{dL^*_r}{d\theta} = \frac{(\epsilon_{m,\theta} \epsilon_{m,\theta} + \epsilon_{m,\theta}) \frac{L^*_m}{\theta} + (\epsilon_{r,\theta} \epsilon_{r,\theta} - \epsilon_{m,\theta} \epsilon_{r,\theta}) \frac{L^*_m}{\theta} \frac{L^*_r}{\theta}}{1 + (\epsilon_{m,\theta} \epsilon_{m,\theta} + \epsilon_{m,\theta}) \frac{L^*_m}{\theta} + (\epsilon_{r,\theta} \epsilon_{r,\theta} + \epsilon_{r,\theta}) \frac{L^*_r}{\theta} + (\epsilon_{m,\theta} \epsilon_{m,\theta})} \frac{L^*_m}{\theta} \frac{L^*_r}{\theta} < 0, \tag{23}
\]

whereas \( \epsilon_{i,j}, i \neq j \) represents the elasticity of migration of sector \( i \) with respect to sector

\[
\frac{dL^*_u}{d\theta} = - \alpha_u \frac{L^*_m}{\theta} \frac{L^*_r}{\theta} \left[ \Omega_1 \Omega_2 (1 + \alpha_m) + \Omega_1 + \Omega_2 \right] > 0 \tag{24}
\]

\[
\frac{dL^*_m}{d\theta} = - \alpha_u \frac{L^*_m}{\theta} \frac{L^*_r}{\theta} \left[ \Omega_1 \Omega_2 + \Omega_2 \right] < 0 \tag{25}
\]

\[
\frac{dL^*_r}{d\theta} = - \alpha_u \frac{L^*_m}{\theta} \frac{L^*_r}{\theta} \left[ \Omega_1 \Omega_2 \alpha_m + \Omega_1 \right] < 0, \tag{26}
\]
$j$, and $\epsilon_{i,\theta}$ shows the elasticity of sector $i$ due to changes in the importance of urban manufacturing congestion costs. With Lemma 2 it is straightforward to show that $\frac{dL_u}{d\theta} > 0$, $\frac{dL_r}{d\theta} < 0$ and $\frac{dL_m}{d\theta} < 0$ as $\mathbb{D} \equiv 1 + \alpha_u \left(1 - \frac{L_u}{\bar{\xi}}\right) \left[\tilde{\Omega}_1 \tilde{\Omega}_2 (1 + \alpha_m) + \tilde{\Omega}_1 + \tilde{\Omega}_2 \right] + \tilde{\Omega}_2 + \tilde{\Omega}_1 \alpha_m > 0$, with $\tilde{\Omega}_1 > 0$, $\tilde{\Omega}_2 > 0$.

The intuition behind Result 1 is as follows: increasing the relevance of urban manufacturing congestion costs decreases output and increases urban prices relative to the remaining sectors price levels. This in turn induces people migrating from both, the rural and medicare sector to the urban manufacturing sector. This again increases the congestion costs and reduces urban manufacturing profits. This shows that congestion costs act as a pull factor in this model, which triggers people to migrate to the urban manufacturing sector.

Mathematically, this is represented by the sum of the two terms of the enumerator of equation (21), which shows the inter- and intra-sectoral reallocation of the equilibrium rural as well as of the medicare sector labour force due to changes in $\theta$. As both, the enumerator as well as the denominator, turns out to be positive, urban manufacturing labour force increases due to changes in $\theta$. The last two terms of the enumerator of equations (23) and (25) show the convergence of prices and wages towards the new equilibrium, in which these last two terms are zero$^{30}$. The strength of migration obviously depends on the sector specific migration elasticities as well as on the sensitivity of a sector’s $j$ labour force due to changes in the urban manufacturing congestion costs.

$^{30}$It must be mentioned that we exclude the possibility that the urban manufacturing sector earns negative profits after transition and therefore cannot compete with the informal urban sector.
5.2 Urban manufacturing congestion costs: Environmental and health response

The aim of this sub-section is to distill the link between intra- and intersectoral labour force migration and the economy-wide response of environmental and health development due to changes in the urban manufacturing congestion cost parameter $\theta$. Given, the urban manufacturing firm is active (see Lemma 2), by differentiating equation (9), we obtain:

$$\frac{\partial h}{\partial \theta} = \frac{\partial h}{\partial e} \frac{\partial e}{\partial \theta} = \frac{\partial h}{\partial e} \left[ \sum_i \frac{\partial e}{\partial c_i} \frac{\partial c_i}{\partial I} \sum_j \frac{\partial I}{\partial L_j} \frac{\partial L_j}{\partial \theta} \right] \geq 0, \; j = \{u, r, m\} \text{ and } i = \{u, r\}.$$  \hspace{5cm} (27)

Indeed, the sign of equation (27) is not determined, i.e. it is ex ante not straightforward that increased congestion costs negatively or positively affect the environmental as well as the health level of the economy. However, it is possible to identify a critical urban manufacturing employment level $L_u$, which clearly determines the sign of equation (27). In other words, we have show for which critical level $L_u$ the term $\sum_j \frac{\partial I}{\partial L_j} \frac{\partial L_j}{\partial \theta}$ turns out be zero.

**Lemma 6.** A stable labour market equilibrium is constituted with Lemma 2. Let us further define the critical urban manufacturing employment level as

$$L_u \equiv -\frac{c(\alpha - \gamma_2 \delta_\kappa (\alpha + \beta - 1) + \alpha - \gamma_1 (\alpha + \beta - 1) \chi_m \Omega_m - \gamma_2 \delta_\kappa (\alpha + \beta - 1) \gamma_1 \Omega_u - \gamma_1 \Omega_u)}{\theta (\alpha + \chi_u \Omega_u) \left( \gamma_2 \delta_\kappa (\alpha + \beta - 1) - \gamma_1 (\alpha + \beta - 1) + \alpha \right)}.$$ \hspace{5cm} It holds that

(1) $\hat{L} > L_u$ as $\hat{L} - L_u = -\frac{c((2\tilde{\Omega}_1 + 1) \Omega_2 \alpha_m + \tilde{\Omega}_2)}{\theta \tilde{\Omega}_u (\Omega_1 \Omega_2 (\alpha_m + 1) + 1)} < 0$. Hence, $L_u$ is feasible with $\hat{\Omega}_1 > 0$ and $\hat{\Omega}_2 > 0$.

(2) The urban manufacturing employment level $\tilde{L}_u$, corresponding to urban manufac-
turing sector’s marginal costs (mc) equal to one, mc = 1, can be straightforwardly calculated as 
\[ \hat{L}_u \equiv \frac{\varsigma (\bar{\alpha} + \chi_u \Omega_u - 1)}{\theta (\bar{\alpha} + \chi_u \Omega_u)}, \]
which is greater than \( \mathcal{L}_u \) as 
\[ \hat{L}_u - \mathcal{L}_u = \frac{\varsigma \gamma_1 (1 - \alpha - \beta)(\bar{\alpha} + \chi_m \Omega_m - 1)}{\theta (\bar{\alpha} + \chi_u \Omega_u)(\gamma_2 \delta (\alpha + \beta - 1) - \gamma_1 (\alpha + \beta - 1) + \alpha)} > 0. \]

(3) Finally, we can show that
\[ \frac{\partial I}{\partial L_j} \frac{\partial L_j}{\partial \theta} \begin{cases} > 0, & \text{for } L_u < \mathcal{L}_u \hfill \text{(1)} \\
= 0, & \text{for } L_u = \mathcal{L}_u \hfill \text{(2)} \\
< 0, & \text{for } L_u > \mathcal{L}_u \end{cases} \]
Based on Lemma 6 we arrive at the following Result 2.

**Result 2.** Assuming that \( mc < 1 \). The respective response of the environment and health levels due to an increase of the urban manufacturing congestion costs is ambiguous. Whether the overall effect is positive or negative depends on the size of the urban manufacturing sector, measured by \( L_u \). Being more concrete, we have (1)
\[ \frac{\partial e}{\partial \theta} \begin{cases} > 0, & \text{for } \hat{L}_u > \mathcal{L}_u > L_u \hfill \text{(1)} \\
= 0, & \text{for } \hat{L}_u > L_u = \mathcal{L}_u \hfill \text{(2)} \\
< 0, & \text{for } \hat{L}_u > L_u > \mathcal{L}_u \end{cases} \]

(2) Further, we can directly deduce from equation (27) that
\[ \frac{\partial h}{\partial \theta} \begin{cases} < 0, & \text{for } \hat{L}_u > \mathcal{L}_u > L_u \hfill \text{(1)} \\
= 0, & \text{for } \hat{L}_u > L_u = \mathcal{L}_u \hfill \text{(2)} \\
> 0, & \text{for } \hat{L}_u > L_u > \mathcal{L}_u \end{cases} \]

In other words, increasing \( \theta \) induces intra- and intersectoral labour force migration and corresponding sector-specific income variations. This can be formally represented with \( \frac{\partial I}{\partial L_j} \frac{\partial L_j}{\partial \theta} \forall j \). Given \( L_u \) does not exceed \( \mathcal{L}_u \), it turns out that \( \sum_j \frac{\partial I}{\partial L_j} \frac{\partial L_j}{\partial \theta} > 0 \), i.e. more
people a drawn from the rural and medicare sector to the urban manufacturing sector (see Result 1) and additional urban manufacturing income gains over-compensate the sum of the remaining sector’s additional income reductions. Hence, for given preferences, consumption increases and, finally for given preferences, reduces environmental and health quality for $L_u < L_u$ (see equation (27)). Hence, the urban income effect dominates the urban manufacturing congestion effect.

On the other hand, given $L_u < L_u < \tilde{L}_u$, the marginal costs of an additional urban worker is lower than the market price but now $\frac{\partial I}{\partial L_u} < \sum_i \frac{\partial I}{\partial L_i}, \ i = \{r, m\}$ as urban manufacturing congestion costs increase. Hence, the urban manufacturing income gain due to congestion is over-compensated by the opportunity costs working in the urban sector. This in turn decreases the additional consumption possibilities and reduces ceteris paribus the pressure on health and environmental quality. However, it is important to note that $mc < 1$ still holds, i.e. the monopoly of the urban sector still dominates the congestion externality. Finally, if $L_u > \tilde{L}_u$, the congestion effects clearly dominates, i.e. the marginal cost of an additional urban worker in the manufacturing sector exceeds the urban sector’s market price, $mc > 1$.

The discussion above directly allows us to derive the following policy implications which is summarized in the Result 3.

**Result 3.** Given $L_u < L_u < \tilde{L}_u$, the negative externality induced by the urban manufacturing monopoly dominates the congestion effect. Hence, the policy maker should stimulate increased manufacturing towards $\tilde{L}_u$, since the monopoly generates to little output. At the same time, increasing $L_u$ from $L_u$ towards $\tilde{L}_u$ increases environmental as well as health quality. Hence, from a welfare point of view, the optimal urban
manufacturing size lies in the range \( L_u \in (\underline{L}_u, \hat{L}_u) \).

To illustrate the arguments above, let us consider Example 2.

**Example 2: Optimal urban manufacturing sector size.** Let us consider again the calibration from Example 1. We further assume for the environmental Kuznets curve that \( \delta_u = 0.4, \phi = 1 \) and \( \kappa = 2.0 \). The calibration based on Example 2 induces that the rural sector mainly contributes to environmental disaggregation\(^{31}\).

Figure (2) draws \( \frac{\partial h}{\partial \theta} \) and \( \frac{\partial \epsilon}{\partial \theta} \) and, further contains \( L_u, \hat{L}_u, \tilde{L}_u \) and \( \hat{L}_u \). The shaded area reflects the optimal urban manufacturing size.

It is worth to note that the findings above should be interpreted with caution, as Result 2 only establishes a ceteris paribus result. I.e. labour market fluctuations also induce changes in the public health level, which in turn also affect the health status (see equations (7) and (8)).

### 5.3 Policy experiments

As shown in the preceding section, increasing the urban manufacturing sector size towards \( \hat{L}_u \) may be justified in efficiency terms. Hence, an efficient solution is directly associated with a minimum distance \( |L_u, \hat{L}_u| \).

\(^{31}\)It is worth mentioning that we have tried to find reasonable calibrated values from the data to match the situation encountered in the real world. Unfortunately, disaggregated data for this purpose are only rarely available and only for some specific countries or small geographic areas. Nevertheless, at least we have tried to find some real world numbers for the relevant parameters based upon some case studies conducted in this area of interest. For instance, rural population density and the number of pollutant sources can often exceed that in cities (see Oguntoke et al. (2010) for the case of Nigerian villages.). From that point of view, setting \( \delta_r = 1 - \delta_u = 0.6 \) seems to be a reasonable choice.
In this section we examine how different government policy schemes affects the distance $|\mathcal{L}_u, \tilde{\mathcal{L}}_u|$. In the following, we first focus on a policy program which increases the importance of green capital in the city. Second, we discuss a policy which aims to increase the urban manufacturing sector’s infrastructure (for the latter see Bjorvand (2000)).

1. Greening the city

As mentioned above, there is rich empirical evidence that greening of urban areas positively affects (mental) health. In our model, $\chi_i, \ i = \{u, m\}$ measures the importance of green capital in an urban’s sector $i$. Differentiating of $\mathcal{L}_u$ with respect to $\chi_i, \ i = \{u, m\}$ results in

$$
\frac{\partial \mathcal{L}_u}{\partial \chi_m} = \frac{(\alpha + \beta - 1)\varsigma \gamma_1 \Omega_m}{\theta[\alpha + (1 - \alpha - \beta)](\gamma_1 - \delta \kappa \gamma_2)\alpha_u} < 0,
$$

$$
\frac{\partial \mathcal{L}_u}{\partial \chi_u} = \frac{[\alpha + (1 - \alpha - \beta)][(\gamma_1 \alpha_m - \delta \kappa \gamma_2)\Omega_u\varsigma]}{\theta[\alpha + (1 - \alpha - \beta)(\gamma_1 - \delta \kappa \gamma_2)]\alpha_u^2} > 0.
$$
Indeed, increasing $\chi_u$, ceteris paribus increases $\mathcal{L}$ more than $\tilde{\mathcal{L}}_u$ and, thus, reduces the distance $|\mathcal{L}_u, \tilde{\mathcal{L}}_u|$. In other words, the urban population associated with the highest profit, $\tilde{\mathcal{L}}$, increases with a lower rate than the urban population capacity, $\mathcal{L}_u$, which guarantees a constant health level, given congestion costs increase. This is due to the fact that, first, the remaining sectors become more attractive as $\alpha_u$ increases and, second, the existing pressure induced by the urban manufacturing costs on the urban manufacturing profits decreases. On the other hand, a higher $\chi_m$ draws more people into the urban manufacturing sector, increases congestion costs and, hence, decreases $\mathcal{L}_u$ but leaves $\tilde{\mathcal{L}}_u$ unaffected. Hence, the distance $|\mathcal{L}_u, \tilde{\mathcal{L}}_u|$ increases. In a nutshell, increasing $\chi_m$ increases the opportunity costs of migration to the city, whereas a higher $\chi_u$ decreases the costs. This is translated by an decreasing or increasing level of $\tilde{\mathcal{L}}_u$, respectively.

2. Increasing urban manufacturing infrastructure

Let us now study an improvement of the urban manufacturing infrastructure, which can be traced back to increasing $\varsigma$ in equation (4). If we differentiate $\mathcal{L}_u$ with respect to $\varsigma$ we obtain

$$\frac{\partial \mathcal{L}_u}{\partial \varsigma} = \frac{[\alpha - (1 - \alpha - \beta)\delta\gamma_2](\alpha_u - 1) + (1 - \alpha - \beta)(\chi_u\Omega_u - \chi_m\Omega_m)}{\theta[\alpha + (1 - \alpha - \beta)(\gamma_1 - \delta\gamma_2)]\alpha_u} \leq 0. \ (30)$$

Now, it is straightforward that for homogeneity in the city sector’s labour productivity, i.e. $\chi_u\Omega_u = \chi_m\Omega_m$, $\frac{\partial \mathcal{L}}{\partial \varsigma} > 0$. The same hold true, given the urban manufacturing labour productivity, $\alpha_u$, is higher compared to the medicare sector’s, i.e. $\chi_u\Omega_u > \chi_m\Omega_m$. Thus, the opportunity costs migrating to the city are low. However, for $\chi_m\Omega_m < \frac{(\alpha - 1)(-\gamma_2\delta\chi(\alpha + \beta - 1) - \alpha) - \chi_u\Omega_u(\gamma_2\delta\chi(\alpha + \beta - 1) - \gamma_1(\alpha + \beta - 1) + \alpha)}{\gamma_1(\alpha + \beta - 1)}$,
the opportunity costs migrating to the urban manufacturing sector are high. On the other hand, increasing $\zeta$ increases $\tilde{L}_u$, i.e. $\frac{\partial \tilde{L}_u}{\partial \zeta} > 0$. Now, for given sector specific productivities $\alpha_u$ and $\alpha_m$, one can show the following:

$$\frac{\partial L_u}{\partial \zeta} - \frac{\partial \tilde{L}_u}{\partial \zeta} = \gamma_1(\alpha + \beta - 1)(\tilde{\alpha} + \chi_{m}\Omega_m - 1) \frac{\theta}{[\tilde{\alpha} + \chi_{u}\Omega_u][\alpha - (1 - \alpha - \beta)\gamma_2+ \gamma_1(1 - \alpha - \beta)]} < 0.$$ (31)

Hence, irrespective of the sign of $\frac{\partial \tilde{L}_u}{\partial \zeta}$, the latter effect, $\tilde{L}_u$, dominates and hence, the distance $|L_u, \tilde{L}_u|$ decreases. In other words, increasing the urban manufacturing infrastructure increases the urban population size which is compatible with the highest urban manufacturing profits. This effect over-compensates the opportunity cost effect, which can be either positive or negative ($\frac{\partial C}{\partial \zeta} \leq 0$), and thus, accelerates or dampens the first effect.

**Result 4.** (1) If congestion costs are relevant, a policy, which aims to create incentives for investing in green capital increases the health and environmental quality of the economy if the policy is directed to the congested sector. (2) Investing in the infrastructure of a congested sector reduces the congestion costs pressure and tends to increase health and environmental quality.

### 5.4 Recent empirical findings

How can the the major findings summarized with Result 4 be linked to empirical findings? Our model predicts that a better quality of public infrastructure improves the economic and social conditions of cities. For instance, the improvement or installation of a public transport project, notably BRT systems in Bogotá and successfully emulated in Lagos, Ahmadabad and Guangzhou and Johannesburg significantly reduced
congestion costs (Un report 468). As mentioned by Suzuki et al. (2010), the synergetic interplay between an efficient bus system and urban form has significantly reduced the congestion in terms of a decreasing fuel waste (measured US dollar) in traffic jams in Curitiba (US dollar 930,000) compared to Rio de Janerio (US dollar 13.4 million). Hence, our model confirms the empirical findings that investments in infrastructure of a congested sector reduces the congestion costs.

Further, our model predicts that green capital positively affects labour productivity and health. This prediction is supported by a rich strand of literature (Kaplan (1993), Kaplan (1995), Shibata and Suzuki (2002), UN report () and Graff Zivin and Neidell (2012)). Although many cities in developing countries have made considerable progress in terms of improvements of their economic and social conditions, it seems that the majority of developing countries still focus on an economic growth strategy without or only with a limited acknowledgment of environmental and health issues\footnote{An issue not discussed in the model, but nevertheless relevant is the link between technology progress for health. Acemoglu et al. (2012) note that technological advances and their diffusion could moderate pollution.}. This kind of growth orientated policy might reinforce the congestion cost effect and, thus, reduces quality of life. Hence, from a policy maker’s point of view, our model suggest a green capital investment strategy to cope with this congestion cost effect. In a nutshell, our model is able to replicate several empirical findings.
6 Concluding comments

The majority of developing countries suffers from badly developed infrastructure. Further, a rapidly increasing population worsens the quality of an existing infrastructure even more. This radical urbanization undoubtedly has a tremendous impact on the quality of the environment and the health status of the population not only in cities but also in rural areas.

Although there is a strong empirical literature regarding the importance of urbanization and migration for cities in developing countries, the literature so far is silent regrading the health and environmental consequences of phenomenons like migration related congestion effects. This paper contributes to the this latter issue with a theoretical model.

The paper shows that increasing urban manufacturing congestion costs do not necessarily imply a reduction of health as well as of environmental quality as one could expected ex-ante. It distills a range of the urban manufacturing sector size which generates a triple dividend from a welfare point of view. To reach this, the paper recommends a policy which highlights the importance of investments in a city’s green capital and in the infrastructure of a congested sectors.

A key implicit assumption is that the economy is closed. If trade is allowed, the market price of the manufactured good would be bounded from above by the world price plus transportation costs. I suspect this would mitigate the pull factor of urban congestion by decoupling manufacturing wages from urban congestion. Hence, the potential effect of international trade seems a worth extension and defines an avenue
for further research. Insofar, the paper provides a starting point for a deeper exploration of the environmental and health challenges, cities, particularly in developing countries, are faced with.
7 Appendix

Proof of Lemma 2. (1) The profits of the urban sector turns out to be positive, given \( L_u < \frac{2(\alpha_u - 1)}{\alpha_u g} \). (2) Inserting \( L_u^* \) in \( \pi_u^*(\tilde{\alpha}) \) yields negative profits given \( \tilde{\alpha} > \tilde{\alpha} \). On the other hand, given \( \tilde{\alpha} < \tilde{\alpha} \) it directly follows that \( L_u^* \notin \mathbb{R}^+ \Rightarrow \{L_m^*, L_r^*\} \notin \mathbb{R}^+ \). Hence, \( \tilde{\alpha} \in (\tilde{\alpha}, \tilde{\alpha}) \) guarantees that \( L_u^* \in \mathbb{R}^+ \Rightarrow \{L_m^*, L_r^*\} \in \mathbb{R}^+ \).

Further, \( \tilde{\Omega}_2 \) is increasing in \( \tilde{\alpha} \) for \( \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}] \) with \( \tilde{\Omega}_2|_{\tilde{\alpha}} = -\frac{\gamma_1(\alpha + \beta - 1)}{\gamma_2 \delta \kappa (\alpha + \beta - 1) + \alpha} > 0 \) and \( \tilde{\Omega}_2|_{\tilde{\alpha}} = -\frac{\gamma_1(\alpha + \beta - 1)}{\gamma_2 \delta \kappa (\alpha + \beta - 1) + \alpha} > 0 \). Moreover, exhibits a pole at \( \tilde{\hat{\alpha}} = -\frac{\gamma_2 \delta \kappa (\alpha + \beta - 1) + \alpha}{\gamma_1(\alpha + \beta - 1)} > \tilde{\alpha} \) as \( \tilde{\alpha} = \tilde{\alpha} \). This can be directly seen as follows: first note that for \( \tilde{\hat{\Omega}}_1 - \tilde{\hat{\Omega}}_2 = \tilde{\alpha} \) we have \( \tilde{\alpha} < \tilde{\alpha} \) as \( \tilde{\alpha} - \tilde{\alpha} = -\frac{\gamma_2 \delta \kappa (\alpha + \beta - 1) + \alpha}{\gamma_1(\alpha + \beta - 1)} > 0 \). Hence, \( \tilde{\hat{\Omega}}_2 > 0 \) for \( \pi_u^*(\tilde{\alpha}) \) with \( \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}] \).

Finally, notice that \( \hat{\Omega}_1 \leq \hat{\Omega}_2 \) for \( \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}] \). This can be directly seen as follows: first note that for \( \hat{\Omega}_1 - \hat{\Omega}_2 = \tilde{\alpha} \) we have \( \tilde{\alpha} < \tilde{\alpha} \) as \( \tilde{\alpha} - \tilde{\alpha} = -\frac{\gamma_2 \delta \kappa (\alpha + \beta - 1) - \gamma_1(\alpha + \beta - 1)}{\gamma_2 \delta \kappa (\alpha + \beta - 1) + \alpha} > 0 \).

Hence, we can deduce the following relationship between \( \tilde{\Omega}_1 \) and \( \tilde{\Omega}_2 \) for \( \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}] \):

\[
\begin{align*}
\tilde{\tilde{\alpha}} > \tilde{\alpha} & \Rightarrow \tilde{\hat{\Omega}}_1 > \tilde{\hat{\Omega}}_2 \text{ for } \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}] \\
\tilde{\tilde{\alpha}} = \tilde{\alpha} & \Rightarrow \tilde{\hat{\Omega}}_1 \geq \tilde{\hat{\Omega}}_2 \text{ for } \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}] \\
\tilde{\tilde{\alpha}} < \tilde{\alpha} & \Rightarrow \\
& \quad \tilde{\hat{\Omega}}_2 - \tilde{\hat{\Omega}}_1 \geq 0 \text{ for } \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}] \\
& \quad \tilde{\hat{\Omega}}_2 - \tilde{\hat{\Omega}}_1 \leq 0 \text{ for } \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}].
\end{align*}
\]

Proof of Lemma 3. Suppose that \( \tilde{\alpha} > \tilde{\alpha} \), where \( \pi_{L_u}^*(\tilde{\alpha}) = 0 \), \( \pi_{L_u}^*(\tilde{\alpha}) > 0 \) and \( \pi_{L_u}^*(\tilde{\alpha}) < 0 \). It directly follows that for \( \tilde{\alpha} \in [\tilde{\alpha}, \tilde{\alpha}] \) a multiple labour market equilibrium exists.

Proof of Lemma 4. (1) The knife-edge equilibrium can be derived as follows: first, take the first derivatives of equation (12) and (13) with respect to \( L_u \), setting those expressions to zero and solving each equations for \( L_u \), we have \( \hat{L}_u = \frac{\chi}{g} \) for each sector.

Inserting \( \hat{L}_u \) in equations (12) and (13) we have two equations, namely \( w_m(L_m, L_r, \hat{L}_u) \)
and \( w_r(L_m, L_r, \hat{L}_u) \) with two unknowns, namely \( L_m \) and \( L_r \). Solving this system with the steady state condition that sector specific wages equals, i.e. \( w_j = 1 \) for \( j = \{u, r, m\} \), we finally arrive at equation (18).

(2) It is straightforward to show that \( \dot{L}_r > 0, \dot{L}_u > 0, \dot{L}_m > 0 \). As \( \dot{L}_u > 0 \), it should directly follow that \( \dot{L}_r > 0 \) and \( \dot{L}_m > 0 \). To show that \( \dot{L}_r > 0 \), notice that \( \left( \hat{\Theta}_1 \hat{\Theta}_2 \alpha_m \alpha_r - 1 \right) < 0 \) for \( \bar{\alpha} \in (\tilde{\alpha}, \hat{\alpha}) \). It is further easy to verify that \( \zeta \hat{\Theta}_2 \alpha_u \left( -\hat{\Theta}_1 \alpha_r - 1 \right) < 0 \) for \( \bar{\alpha} \in (\tilde{\alpha}, \hat{\alpha}) \). Hence, \( \dot{L}_r > 0 \) is strictly positive. Now turn to \( \dot{L}_m > 0 \). It is easy to verify that \( \zeta \hat{\Theta}_1 \alpha_u \left( -\hat{\Theta}_2 \alpha_m - 1 \right) < 0 \) for \( \bar{\alpha} \in (\tilde{\alpha}, \hat{\alpha}) \). Hence, \( \dot{L}_m > 0 \) is strictly positive. From Lemma 1 we know that \( \bar{\alpha} \in (\tilde{\alpha}, \hat{\alpha}) \) establishes that the urban manufacturing firm operates in equilibrium.

Proof of Lemma 5. By differentiating equation (12) with respect to \( L_j \), for \( j = \{u, r, m\} \) we find that (i) \( \frac{\partial w_m}{\partial L_u} > 0 \Leftrightarrow L_u < \frac{\zeta}{\theta} \), (ii) \( \frac{\partial w_m}{\partial L_r} > 0 \Leftrightarrow \frac{\alpha \hat{\Theta}_2}{L_m} > 0 \) for \( L_m > 0 \) and (iii) \( \frac{\partial w_m}{\partial L_m} < 0 \Leftrightarrow L_u \in [\hat{L}_{u1}; \hat{L}_{u2}] \), with \( \hat{L}_{u1} \equiv \frac{\alpha_u - \sqrt{\alpha_u (2L_r \alpha_r + \alpha_u)}}{\theta \alpha_u} = \hat{L}_u - \sqrt{\alpha_u (2L_r \alpha_r + \alpha_u)} \theta \alpha_u \) and \( \hat{L}_{u2} \equiv \frac{\alpha_u + \sqrt{\alpha_u (2L_r \alpha_r + \alpha_u)}}{\theta \alpha_u} = \hat{L}_u + \sqrt{\alpha_u (2L_r \alpha_r + \alpha_u)} \theta \alpha_u \). Now, we turn to equation (13). Once again, we take the first derivatives with respect to \( L_j \), for \( j = \{u, r, m\} \) and found (iv) \( \frac{\partial w_r}{\partial L_u} < 0 \Leftrightarrow L_u < \frac{\zeta}{\theta} \), (v) \( \frac{\partial w_r}{\partial L_m} > 0 \Leftrightarrow \frac{\alpha \hat{\Theta}_1}{L_r} > 0 \) for \( L_r > 0 \). Finally, we have (vi) \( \frac{\partial w_r}{\partial L_m} < 0 \Leftrightarrow L_u \in [\hat{L}_{u1}; \hat{L}_{u2}] \), with \( \hat{L}_{u3} \equiv \frac{\alpha_u - \sqrt{\alpha_u (2L_m \alpha_m + \alpha_u)}}{\theta \alpha_u} = \hat{L}_u - \sqrt{\alpha_u (2L_m \alpha_m + \alpha_u)} \theta \alpha_u \) and \( \hat{L}_{u4} \equiv \frac{\alpha_u + \sqrt{\alpha_u (2L_m \alpha_m + \alpha_u)}}{\theta \alpha_u} = \hat{L}_u + \sqrt{\alpha_u (2L_m \alpha_m + \alpha_u)} \theta \alpha_u \).

Note that conditions (i)-(vi) have to be fulfilled simultaneously to define a stable equilibrium. The critical conditions are (iii) and (vi). From (iii) and (vi) we can directly deduce that \( \hat{L}_{u1} < \hat{L}_u < \hat{L}_{u2} \) for \( L_r > 0 \) and \( \hat{L}_{u3} < \hat{L}_u < \hat{L}_{u4} \) for \( L_m > 0 \). From (i) together with (iii) and (iv) together with (vi) we further see that \( L_u \in [\hat{L}_{u1}, \hat{L}_{u2}] \) and \( L_u \in [\hat{L}_{u3}, \hat{L}_{u4}] \) are potential candidates for a stable labour market equilibria. Further
observe, first, \( \Theta_1(L_r) \equiv \sqrt{\frac{\alpha_u(2\theta L_r \alpha_r + \varsigma \alpha_u)}{\theta \alpha_u}} \) is increasing in \( L_r \) and, second, \( \Theta_1(0) = \hat{L}_u \). Third, \( \Theta_2(L_m) \equiv \sqrt{\frac{\alpha_u(2\theta L_m \alpha_m + \varsigma \alpha_u)}{\theta \alpha_u}} \) is increasing in \( L_m \) and, fourth, \( \Theta_2(0) = \hat{L}_u \). Hence, if \( L_r > 0 \Rightarrow \bar{L}_{u1} < 0 \) which is not a valid solution as \( \bar{L}_{u1} \notin L_u > 0 \) per assumption. Thus, the lower bound for a stable labour market equilibrium is given by \( L_u = 0 \). Further \( \bar{L}_{u2} \) is strictly increasing in \( L_r \) with \( L_r > 0 \). Hence for every \( L_r > 0 \), it follows \( \bar{L}_{u2} > \hat{L}_u \) and this contradicts condition (i) and (iii). Hence, the upper bound for a stable labour market equilibrium is given by \( L_u = \hat{L}_u \). A similar argument can be found for the medicare sector. \( L_m > 0 \Rightarrow \bar{L}_{u3} < 0 \). Hence, \( \bar{L}_{u3} \notin L_u > 0 \). \( \bar{L}_{u4} > 0 \) defines the unstable equilibrium: for every \( L_m > 0 \) it follows that \( \bar{L}_{u4} > \hat{L}_u \). From the latter arguments we can conclude that the lower bound for \( L_u \) must be \( L_u = 0 \), the upper bound for \( L_u \) must be \( \hat{L}_u \). Drawing the arguments together, we finally conclude that \( L_u \in (0, \hat{L}_u) \) and given \( L_m > 0 \) and \( L_r > 0 \) guarantees a stable labour market equilibrium by fulfilling conditions (i)-(vi) simultaneously.
References


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