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Fiscal policy coordination in currency unions (at the zero lower bound)

Thomas Hettig and Gernot J. Müller*

February 28, 2015

– *Preliminary* –

Abstract

Within currency unions, according to the pre-crisis consensus, countries can rely on fiscal policy to stabilize economic activity locally. Monetary policy's role, in turn, is to stabilize economic activity at the union level. Against this background, we reassess the optimal degree of fiscal stabilization within currency union provided that monetary policy is constrained by the zero lower bound on nominal interest rates. Specifically, we contrast the optimal level of government consumption from an individual country's perspective with the optimal level from the union's perspective and explore the need for coordinating expansionary fiscal policies.

Keywords: Currency union, fiscal policy, zero lower bound, coordination, EMU

JEL-Codes: E61, E62, F41

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1 Introduction

According to a pre-crisis consensus, fiscal policy may have an important role to play in stabilizing economic activity within a currency union. Yet this role is limited to economic activity within individual member states of a currency union, because monetary policy is in a better position to stabilize economic activity at the union level (Beetsma and Jensen, 2005; Galí and Monacelli, 2008). Yet the recent economic and financial crisis has exposed a shortcoming of this paradigm: in a severe economic downturn monetary policy may be constrained in stabilizing economic activity by the zero lower bound on nominal interest rates. As a result, economic activity may stall and fiscal policy becomes very effective in stimulating economic activity (see, e.g., Christiano et al. 2011, Woodford 2011 and Eggertsson and Krugman 2012).

What are the implications for coordinating fiscal stabilization policies in a currency union? For the extent of fiscal stimulus which is optimal from the perspective of the union as a whole may generally be different from what is optimal from the perspective of an individual member state. In fact, during the course of 2008–09 many observers have called for stronger policy coordination, urging European governments to engineer larger fiscal expansions (e.g. Krugman 2008).

And indeed, a casual inspection of the data supports the notion that fiscal stimulus has been muted in the euro area, at least if compared to the US. Figure 1 displays two rough measures of the discretionary fiscal stance, both for the US and the euro area. The left panel shows the change in the cyclically adjusted government budget deficit, measured in percentage-point changes relative to the pre-crisis year 2007. It illustrates that fiscal stimulus in the euro area fell short of that in the US. The same picture emerges in the right panel. It shows the level of government consumption relative to trend output. While the average level is considerably lower in the US, the increase relative to the pre-crisis level is only 7 percent in the EA, but 9 percent in the US.¹

In this paper, we approach the issue from a theoretical perspective. Specifically, we draw on Galí and Monacelli (2008) who put forward a model of a currency union consisting of a continuum of small open countries, each negligible in terms of aggregate outcomes. This framework abstracts from complications which give rise to strategic interactions across the

¹To some extent, European fiscal stabilization policies have been coordinated, namely through the *European Economic Recovery Plan*, discussed and legislated in late 2008–09. According to Cwik and Wieland (2011) the measures foreseen by the plan amounted to 1.04 and 0.86 percent of 2009 and 2010 GDP, respectively. They were thus considerably smaller than legislation in the United States under the *American Recovery and Reinvestment Act* which amounted to roughly 5 percent of GDP. Still, Cwik and Wieland (2011) stress that, in contrast to the measures in the US, the European package was concentrated on two years and front loaded in 2009.

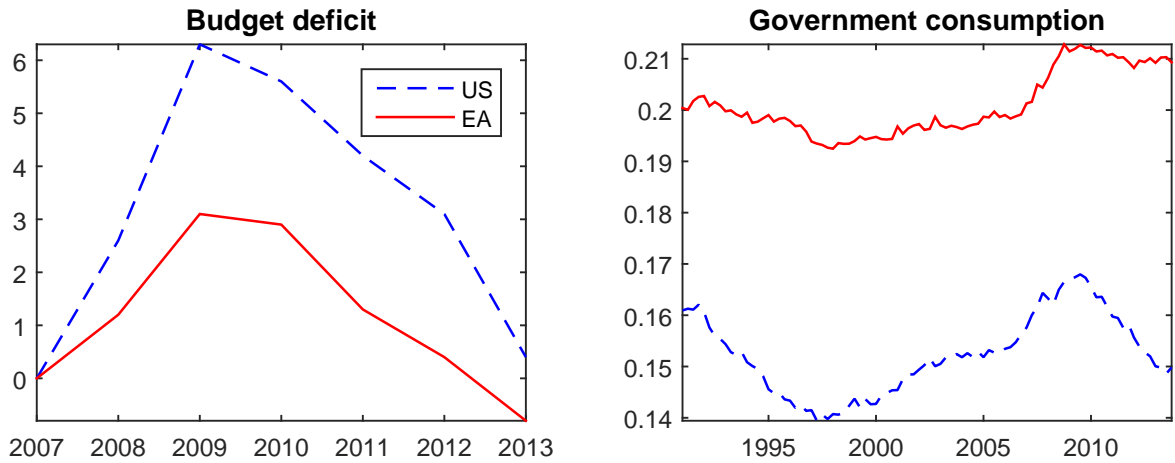


Figure 1: Left: cyclically adjusted deficit, annual observations, change relative to 2007, measured in percentage points of potential output; right: government consumption of general government in units of trend output, quarterly observations, source: OECD Economic Outlook.

member states of a currency union. A representative household within each country consumes, supplies labor and trades a complete set of state-contingent assets across countries. Each country is a monopolistic supplier of a particular set of goods which enter consumption baskets in all member states. As a result, member states have the power to influence their terms of trade. Goods, in turn, are produced in a monopolistic competitive environment and firms are restricted in their ability to adjust prices.

This gives a role for monetary policy to stabilize the economy. In our setting, however, it is constrained by the zero lower bound so that it cannot fulfill its role giving rise to the second policy instrument exhaustive government spending, which is dominated by monetary policy to offset aggregate shocks in normal times. Government expenditure is determined and financed through lump-sum taxes within each country and falls only on domestically produced goods. In order to solve the model analytically we consider a first-order approximation to the private-sector equilibrium conditions and a second-order approximation to the welfare criterion of the policy maker. The welfare criterion of policy makers may differ. Under coordination fiscal policies are set to maximize union-wide welfare. In the absence of coordination fiscal policy makers maximize country-specific welfare. We focus on discretionary policies.

Within our framework we recoup two results which have already been established in the literature, but are crucial to put our main result into perspective. First, we compute fiscal multipliers assuming that government spending is exogenous and the economy is stuck at

the zero lower bound. We contrast the case where all members of the currency union raise government spending and the case where government spending is raised in one country only. We find that the multiplier is larger than unity in the first case, but small than unity in the second case (Fahri and Werning, 2012).² Second, we confirm an earlier finding of Devereux (1991): if countries coordinate the level of government spending in steady state it will generally take a lower share of output than in case each country sets it non-cooperatively (Nash). In the absence of cooperation, governments seek—in vain—to improve on their country's terms of trade by raising purchases of domestically produced goods.³

Our main result establishes the optimal discretionary response of government spending to a shock which pushed the economy at the zero lower bound. Specifically, we consider a scenario where a shock pushes the economy to the zero lower bound and assume that monetary policy is unable to stabilize the economy through non-conventional policies.⁴ Further, once the shock ceases to persist the economy immediately returns to a zero-inflation steady state. As a result we find that—in line with the conjecture above—countries provide too little stimulus in the absence of cooperation.

This result obtains because in each instant fiscal policy makers face different constraints as far as the conduct of monetary policy is concerned. Cooperating fiscal policy makers face a monetary policy which de facto targets union-wide inflation. An inflationary impulse due to fiscal policy is thus not offset by monetary policy and reduces—at the zero lower bound—long-term real interest rates. From the perspective of a government within an individual member state there is, however, a de facto price level target, because purchasing parity holds in the long run. Any inflationary impulse due to fiscal policy thus triggers an offsetting deflationary tendency and causes an increase in the long term real rate on impact (Corsetti et al., 2013b). Hence, these opposite effects on real rates lead to a smaller fiscal stimulus in the non-cooperative environment.

Related to our analysis, Cook and Devereux (2011) study optimal fiscal policy in two-country model when monetary policy is stuck at the zero lower bound. However, they focus on the

²Erceg and Lindé (2012) compare results for a small open economy which may be stuck at the zero lower bound or operate a fixed exchange rate. In this case, spending multipliers at the zero lower bound will exceed those obtained for fixed exchange rates only if prices are sufficiently flexible. Nakamura and Steinsson (2014), in turn, show that multipliers may be high within a currency union when compared to the multiplier at the union level in the absence of a zero-lower-bound constraint. Acconcia et al. (2014) find for Italian data that variations in local government government spending, given an unchanged tax burden of local residents as well as constant national monetary and fiscal policy, have fairly strong output effects.

³For the terms of trade externality see also Corsetti and Pesenti (2001), Benigno and Benigno (2003) and De Paoli (2009). The last two papers focus only on monetary policy.

⁴Such policies may include forward guidance as in Gaulti B. Eggertsson and Michael Woodford (2003) or, depending on the nature of shock which pushes the economy at the zero lower bound, credit policies by the central bank, see, e.g., Cúrdia and Woodford (2011).

case of coordination. Blanchard et al. (2014) calibrate a two-country model to capture key features of the euro area, notably of its core and periphery. They share our focus on the gains from cooperation relative to self-interested policies once the zero lower bound binds. However, since their model is more complex than ours, their analysis is limited to numerical evaluations and based on an ad-hoc welfare criterium.

We stress that our result holds, even though trade is assumed to be balanced. Of course, trade spillover from fiscal stimulus are a first-order concern for policy makers and an important dimension when it comes to coordinating fiscal policies across countries. However, evidence on the empirical importance of trade spillovers is mixed and, more importantly, trade spillovers are not confined to countries within monetary unions.⁵ For these reasons, we abstract from trade spillovers and focus on the aspect of fiscal coordination which is specific to currency unions.

We also disregard complications due to sovereign risk which may impact the effects of fiscal stabilization policies in currency unions (Corsetti et al., 2014). In fact, as Ricardian equivalence obtains in our model, we do not consider public debt and deficits at all. Clearly, at least within the European Union coordinating budget and deficit policies have featured highly on the actual policy agenda. Also theory provides a rationale for coordination in the presence of a deficit bias coupled with an international externality (Beetsma and Uhlig, 1999; Krogstrup and Wyplosz, 2010).

The remainder of the paper is structured as follows: In section 2 we describe the basic setup of the model. Section 2.3 compares government spending multipliers at the zero lower bound at the union and at the country level. In section 3 we analyze the need for coordination by computing optimal government spending with and without coordination.

2 Model

The currency union, we model, consists of a continuum of small open economies each denoted by $i \in [0, 1]$. Each of these countries consists of a representative household, a firm and a government sector. As we build essentially on Galí and Monacelli (2008) we will not present all derivations of the model but rather present only the objectives and constraints of the three sectors. Further we will present the linearized equilibrium conditions at the country and the union level. We refer to their paper for a more thorough description of the model.

⁵In an empirical analysis spillovers from the US to the euro area and the UK, Corsetti and Müller (2014) find the role of the trade channel quite limited.

2.1 Objectives

Households

The representative household in country i gains utility from private consumption, C_t^i , public consumption, G_t^i , and suffers disutility of labor, N_t^i , according to:

$$U(C_t^i, N_t^i, G_t^i) = (1 - \chi) \log C_t^i + \chi \log G_t^i - \frac{(N_t^i)^{1+\varphi}}{1 + \varphi}$$

The parameter $\chi \in [0, 1)$ describes the weight that the household puts on public relative to private consumption. The inverse of the Frisch elasticity of labor supply is denoted by $\varphi > 0$.⁶ Private consumption is an composition of a domestic good ($C_{H,t}^i$) and an import good ($C_{F,t}^i$):

$$C_t^i \equiv \frac{(C_{H,t}^i)^{1-\alpha} (C_{F,t}^i)^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha}$$

Parameter $\alpha \in [0, 1]$ characterizes the openness of the economy. When $\alpha < 1$ there is home bias in consumption. The household seeks to minimize expenditures while reaching a given level of C_t^i . This implies the demand function

$$C_{H,t}^i = \left(\frac{P_{H,t}^i}{P_{c,t}^i} \right)^{-1} C_t^i, \quad C_{F,t}^i = \left(\frac{P_{F,t}^i}{P_{c,t}^i} \right)^{-1} C_t^i$$

where $P_{c,t}^i = (P_{H,t}^i)^{1-\alpha} (P_{F,t}^i)^\alpha$ is the consumption price index; $P_{H,t}^i$ and $P_{F,t}^i$ denote the price of the domestic good and the import good, respectively.

Households seek to maximize their expected discounted lifetime utility subject to their budget constraint:

$$\max_{\{C_t^i, N_t^i, A_t^i\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} (e_t \beta^t) U(C_t^i, N_t^i, G_t^i) \quad (1)$$

$$\text{s.t.} \quad P_{c,t}^i C_t^i + E_t \{ Q_{t,t+1} A_{t+1}^i \} \leq A_t^i + W_t^i N_t^i - T_t^i + D_t \quad (2)$$

where e_t is a shock to the subjective discount factor $\beta \in (0, 1)$. Households have access to a complete set of state contingent securities that allow them to insure against idiosyncratic risk.⁷ A_t^i denotes their portfolio of nominal assets and $Q_{t,t+1}$ is the asset pricing kernel

⁶This is a plausible assumption as otherwise labor supply would (always) decline in response to an increase in the real wage.

⁷This risk could e.g. stem from idiosyncratic technology shocks as in Galí and Monacelli (2008). As we analyze optimal policy in response to an aggregate shock that pushes the currency union at the zero lower bound we abstain from idiosyncratic risk in our exposition for simplicity.

more commonly referred to as the unique stochastic discount factor (common across countries). A riskless one-period discount bond that pays one unit of the common currency for costs $E_t Q_{t,t+1}$. We denote by $R_t^* = \frac{1}{E_t Q_{t,t+1}}$ the gross nominal interest rate. We assume that monetary policy has control over this rate and uses it to implement its policy (when not constrained by the zero lower bound). The consumer price index (CPI) is given by $P_{c,t}^i$ and T_t^i are lump-sum taxes.⁸ The union-wide profits of firms are distributed to households in a lump-sum fashion (D_t). Households are wage takers and wages are perfectly flexible.

Government

Opposed to the private sector the government consumes no foreign goods. It allocates its expenditure in a cost minimizing manner with the resulting demand function for a generic good j :

$$G_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\varepsilon} G_t^i \quad (3)$$

The value of aggregate expenditure, G_t^i , remains to be determined below.

Final good firms

There are two perfectly competitive sectors which assemble the domestic good and the import good from a variety of domestically and imported goods, respectively. In each instance, the final good is CES aggregator of the product varieties. For the domestic bundle we have

$$C_{H,t}^i = \left(\int_0^1 C_{H,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Parameter $\varepsilon > 1$ denotes the elasticity of substitution between different varieties. Expenditure minimization implies that domestic demand for the generic domestic good j is given by

$$C_{H,t}^i(j) = \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\varepsilon} C_{H,t}^i.$$

Here $P_t^i(j)$ is the price of good j and $P_t^i \equiv \left(\int_0^1 P_t^i(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is the domestic (producer) price index.

Importers, in turn, operate an aggregation technology as in Galí and Monacelli (2008)

$$C_{F,t}^i = \exp \int_0^1 c_{f,t}^i df$$

⁸In the following we will focus on the producer price index, P_t^i , which is related to the CPI according to $P_t^i = P_{c,t}^i (S_t^i)^\alpha$ where S_t^i denotes the terms of trade which are specified below.

where $c_{f,t}^i = \log C_{f,t}^i$, in turn, is the basket of goods imported from country f

$$C_{f,t}^i = \left(\int_0^1 C_{f,t}^i(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Expenditure minimization implies that demand from country i for good j produced in country f is given by

$$C_{f,t}^i(j) = \left(\frac{P_t^f(j)}{P_t^f} \right)^{-\epsilon} C_{f,t}^i,$$

where $P_t^f(j)$ is the price of good j in country f and $P_t^f \equiv \left(\int_0^1 P_t^f(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$.

Intermediate good firms

There is a continuum of firms in every country i of which each firm $j \in [0, 1]$ produces a differentiated good $Y_t^i(j)$ in monopolistic competition. These goods can be traded across countries and the law of one price is assumed to hold. Firms are subject to nominal rigidities and in general cannot adjust their price $P_t^i(j)$ every period. Following Calvo (1983) firms can reset their price in a given period with probability $1 - \theta$ while their current price remains with probability θ . The probability of resetting the price is independent of a firm's last adjustment. Firms hire labor $N_t^i(j)$ and produce with a linear technology $Y_t^i(j) = N_t^i(j)$. Their objective is to maximize expected nominal payoffs taking the demand for their product into account:

$$\begin{aligned} \max_{P_t^i(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k}^i(j) (\bar{P}_t^i(j) - (1 - \tau^i) W_{t+k}^i) \right\} \\ \text{s.t. } Y_{t+k}^i(j) = \left(\frac{\bar{P}_t^i(j)}{P_{t+k}^i} \right)^{-\epsilon} Y_{t+k}^i \end{aligned}$$

where $\bar{P}_t^i(j)$ is the newly set optimal price in period t , and W_t^i is the nominal wage. Further, firms receive a labor subsidy τ^i of the domestic government. When chosen efficiently it is used to offset the negative effects of monopolistic competition in steady state.

2.2 Equilibrium conditions for approximate model

We solve for equilibrium taking linear approximations to optimality and market clearing conditions. We denote by \hat{x}_t the log deviation of a generic variable $x_t \equiv \log X_t$ from its steady state value x . Union wide variables are obtained by integrating over all countries in the union: $\hat{x}_t^* = \int_0^1 \hat{x}_t^i di$.

First we derive the market clearing condition in country i by inserting the optimal demand functions for domestic, foreign and government consumption in the market clearing condition for a generic good j in country i . Integrating over all goods j and linearizing gives:

$$\hat{y}_t^i = (1 - \gamma)(\hat{c}_t^i + s_t^i) + \gamma\hat{g}_t^i \quad (4)$$

Parameter γ denotes the steady state ratio of government consumption to output. The above equation links domestic output \hat{y}_t^i to domestic consumption \hat{c}_t^i , the terms of trade $s_t^i = p_t^* - p_t^i$ and domestic government spending \hat{g}_t^i . Further, the assumption of complete markets implies the following risk sharing condition:

$$\hat{c}_t^i = \hat{c}_t^* + (1 - \alpha)s_t^i$$

Combining it with (4) gives

$$\hat{y}_t^i = \gamma\hat{g}_t^i + (1 - \gamma)\hat{c}_t^* + (1 - \gamma)s_t^i \quad (5)$$

In our currency union setup equation (5) replaces an otherwise standard dynamic IS-equation as an equilibrium condition. In equilibrium a standard dynamic IS-curve holds, too, however prices are not determined via the IS-curve on the country level. That is, even if the central bank follows the Taylor principle it will not react to inflation in country i because of its negligible share in union wide inflation. Instead prices are determined via the exchange rate peg i.e. by targeting domestic to union wide prices (which are determined by the Taylor principle). Below we show the price determinacy on the country level formally by providing a stationary solution for the terms of trade.

Integrating equation (5) over all economies $i \in [0, 1]$, noting that $\int_0^1 s_t^i di = 0$, leads to the aggregate market clearing condition

$$\hat{y}_t^* = \gamma\hat{g}_t^* + (1 - \gamma)\hat{c}_t^* \quad (6)$$

Finally, for convenience we combine the goods market equilibrium in country i (5) with the union wide equilibrium (6) and express the resulting equation in terms of inflation instead of the terms of trade:

$$\Delta\hat{y}_t^i = \Delta\hat{y}_t^* + \gamma(\Delta\hat{g}_t^i - \Delta\hat{g}_t^*) - (1 - \gamma)(\pi_t - \pi_t^*) \quad (7)$$

where $\pi_t^i = p_t^i - p_{t-1}^i$ denotes the inflation rate.

The second equilibrium condition on the country level can be computed by combining the optimal price setting behavior of the firm sector with the law of motion for aggregate prices. This yields a usual Phillips curve:

$$\pi_t^i = \beta E_t\{\pi_{t+1}^i\} + \lambda \left(\frac{1}{1 - \gamma} + \varphi \right) \hat{y}_t^i - \frac{\lambda\gamma}{1 - \gamma} \hat{g}_t^i \quad (8)$$

with $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. Equations (7) and (8) characterize the equilibrium in the small open economy given a process for government spending. For later purposes note that as $\theta \rightarrow 1$ (complete price stickiness), $\lambda \rightarrow 0$. One can show that the terms of trade evolve according to⁹

$$s_t^i = \omega s_{t-1}^i + \omega \beta E_t \{s_{t+1}^i\} - \omega \lambda \varphi \gamma (\hat{g}_t^i - \hat{g}_t^*) \quad (9)$$

where $\omega \equiv \frac{1}{1+\beta+\lambda[1+\varphi(1-\gamma)]} \in [0, \frac{1}{1+\beta})$. This second order stochastic difference equation has a unique stable solution:

$$s_t^i = \delta s_{t-1}^i + \delta \lambda \varphi \gamma \sum_{k=0}^{\infty} (\beta \delta)^k E_t \{ \hat{g}_t^* - \hat{g}_t^i \} \quad (10)$$

with $\delta \equiv \frac{1-\sqrt{1-4\beta\omega^2}}{2\omega\beta} \in (0, 1)$. Thus prices are uniquely determined at the country level via the exchange rate peg. This representation of the Phillips curve is convenient to show that the government spending multiplier is smaller than 1 at the country level (see section 2.3).

The union wide equilibrium conditions are given by an aggregate Phillips curve

$$\pi_t^* = \beta E_t \{ \pi_{t+1}^* \} + \lambda \left(\frac{1}{1-\gamma} + \varphi \right) \hat{y}_t^* - \frac{\lambda \gamma}{1-\gamma} \hat{g}_t^* \quad (11)$$

where $\pi_t^* \equiv \int_0^1 \pi_t^i di$, $\hat{y}_t^* \equiv \int_0^1 \hat{y}_t^i di$, $\hat{g}_t^* \equiv \int_0^1 \hat{g}_t^i di$ and by a dynamic IS curve which can be obtained by integrating the country specific IS curves (not shown)

$$\hat{y}_t^* = E_t \{ \hat{y}_{t+1}^* \} - (1-\gamma)(r_t^* - E_t \{ \pi_{t+1}^* \} - E_t \{ r_{t+1}^{net} \}) - \gamma E_t \{ \Delta \hat{g}_{t+1}^* \} \quad (12)$$

with $r_{t+1}^{net} = -\log \beta - \log e_{t+1} + \log e_t$ and where r_t^* denotes the union wide riskless nominal interest rate set by the monetary authority.

Definition of equilibrium An equilibrium is a collection of

1. country specific stochastic processes $\{ \hat{y}_t^i, \pi_t^i \}$ for all $i \in [0, 1]$
2. union wide stochastic processes $\{ \hat{y}_t^*, \pi_t^*, r_t^*, e_t \}$ with $\hat{y}_t^* = \int_0^1 \hat{y}_t^i di$, $\pi_t^* = \int_0^1 \pi_t^i di$,

such that for given $\{ \hat{g}_t^i \}$ with $\hat{g}_t^* = \int_0^1 \hat{g}_t^i di$

3. equilibrium conditions (7) and (8) are satisfied on the country level
4. and equilibrium conditions (11) and (12) are satisfied on the union level.

To close the model a rule for the nominal interest rate, r_t^* , has to be specified. We only consider two possibilities: $r_t^* = 0$ as long as the zero lower bound binds or $r_t^* = (1-\beta)/\beta$ otherwise (steady state).

⁹See the working paper version of Galí and Monacelli (2008).

2.3 Multipliers from country and union wide perspectives

In this section we will collect findings on the size of the multipliers in a closed economy when the zero lower bound is binding and in a small open economy with an exchange rate peg. The disparity of the multipliers gives incentives to choose different levels of government spending under optimal policy from a union wide and a country specific perspective. In order to compute the multiplier at the zero lower bound on the union level, we model this scenario by a shock to the discount factor such that e_t temporarily falls to a value $0 < e_t < 1$. That is, we assume the shock to be large enough such that the central bank is unable to lower r_t^* enough to offset the effects of the drop in $E_t r_{t+1}^{net}$. Thus the zero lower bound becomes binding. It is assumed that the shock remains with probability μ and that the economy returns with the reverse probability to the zero inflation steady state in which $e_t = 1$. Once this happened the economy stays in steady state forever. We assume that μ is unaffected by fiscal and monetary policy.¹⁰ As long as the shock state remains variables take on a value x_L . With these assumptions the Phillips curve and the IS equation become:

$$\pi_L^* = \frac{1}{1 - \beta\mu} \kappa (\hat{y}_L^* - \psi_{yg} \hat{g}_L^*) \quad (13)$$

$$(1 - \mu)(\hat{y}_L^* - \gamma \hat{g}_L^*) = (1 - \gamma)\mu \pi_L^* + (1 - \gamma)r_L \quad (14)$$

with $\bar{\sigma} \equiv \frac{1}{1-\gamma}$, $\psi_{yg} \equiv \frac{\bar{\sigma}\gamma}{\bar{\sigma}+\varphi}$ and $\kappa \equiv \lambda \left(\frac{1}{1-\gamma} + \varphi \right)$. This system can be solved for \hat{y}_L^* in dependence of \hat{g}_L^* in order to compute the government spending multiplier. As we are interested in the effects of a 1% point increase of government spending in terms of GDP we have to divide the derivative of \hat{y}_L^* with respect to \hat{g}_L^* by γ :

$$\frac{1}{\gamma} \frac{\partial \hat{y}_L^*}{\partial \hat{g}_L^*} = \frac{(1 - \mu)(1 - \beta\mu) - (1 - \gamma)\mu\kappa \frac{\bar{\sigma}}{\bar{\sigma}+\varphi}}{(1 - \mu)(1 - \beta\mu) - (1 - \gamma)\mu\kappa} \geq 1. \quad (15)$$

The inequality follows from the condition for determinacy of the model, $(1 - \mu)(1 - \beta\mu) > (1 - \gamma)\mu\kappa$.

Following the working paper version of Galí and Monacelli (2008) we can use equations (4) and (10) to compute the value of the government spending multiplier at the country level. For simplicity and without loss of generality we set union wide variables to zero and assume that government spending follows a two state Markov switching process: with probability μ government spending takes on a value $\hat{g}_L^i > 0$ and with probability $1 - \mu$ government spending returns to steady state. With these assumptions the two equations become in the

¹⁰Monetary policy could for instance affect this probability by influencing expectations of agents with policy commitments once the zero lower bound ceases to bind.

first period of the increase in government spending

$$\hat{y}_{1L}^i = \gamma \hat{g}_L^i - (1 - \gamma) p_{1L}^i \quad (16)$$

$$p_{1L}^i = \frac{\delta \lambda \varphi \gamma}{1 - \beta \delta \mu} \hat{g}_L^i \quad (17)$$

Combining both the government spending multiplier on impact is given by:

$$\frac{1}{\gamma} \frac{\partial \hat{y}_{1L}^i}{\partial \hat{g}_L^i} = 1 - (1 - \gamma) \underbrace{\frac{\delta \lambda \varphi}{1 - \beta \delta \mu}}_{\geq 0} \leq 1 \quad (18)$$

The upper bound is reached when prices are completely sticky. Galí and Monacelli (2008) show in their working paper that under full price stickiness, that is, when $\lambda \rightarrow 0$, it follows that $\delta \rightarrow 1$ such that the multiplier becomes 1. With less than full price stickiness prices respond to the increase in government spending such that consumption is crowded out. Thus the multiplier is smaller than 1 as long as prices are not completely sticky.¹¹ Corsetti et al. (2013b) provide an intuition: According to the Markov switching process for government spending the increase in spending is expected to be reverted in the future. As the exchange rate cannot adjust because of the peg, long run purchasing power parity requires prices to decline in the future. Households thus expect future deflation which has dampening effects on the economy already today.

Thus we recover the following result from Fahri and Werning (2012) for the government spending multipliers:

$$\frac{1}{\gamma} \frac{d\hat{y}_L^*}{d\hat{g}_L^*} \geq 1 \geq \frac{1}{\gamma} \frac{d\hat{y}_{1L}^i}{d\hat{g}_L^i}$$

3 Optimal policy

In this section we compute optimal fiscal policy under coordination and without coordination (Nash) at the zero lower bound by maximizing a quadratic approximation to welfare subject to the equilibrium conditions. We approximate the respective welfare functions around a zero inflation steady state that is consistent with the social planner solutions under the corresponding approaches. The respective steady states can be decentralized under flexible prices by choosing a specific value for the labor subsidy τ and a certain rule for government spending.¹² A derivation of union wide welfare under coordination can be

¹¹One can further show that the multiplier is positive for all degrees of price stickiness, that is, for all $\lambda \in [0, 1]$.

¹²See Galí and Monacelli (2008) for the decentralization of the steady state under coordination and appendix A for the case without coordination.

found in Galí and Monacelli (2008). In appendix B we derive the welfare function without coordination.¹³

3.1 Steady states under coordination and Nash

We start our analysis by computing the mentioned steady states and analyze their properties. Under coordination the social planner (of the union) maximizes union wide welfare subject to the production function and the goods market clearing condition:

$$\begin{aligned} \max \int_0^1 U(C_t^i, N_t^i, G_t^i) di &= \int_0^1 \left[(1 - \chi) \log C_t^i + \chi \log G_t^i - \frac{(N_t^i)^{1+\varphi}}{1 + \varphi} \right] di \\ \text{s.t.} \quad Y_t^i &= N_t^i \\ Y_t^i &= C_{i,t}^i + \int_0^1 C_{i,t}^i df + G_t^i \end{aligned}$$

In steady state we have:

$$\left(\frac{G}{Y} \right)^{Coord} = \chi; \quad Y^{Coord} = 1$$

Under Nash the social planner (of a given country i) takes only domestic welfare into account which is maximized subject to the production function, the risk sharing condition and the goods market clearing condition:

$$\begin{aligned} \max U(C_t^i, N_t^i, G_t^i) &= (1 - \chi) \log C_t^i + \chi \log G_t^i - \frac{(N_t^i)^{1+\varphi}}{1 + \varphi} \\ \text{s.t.} \quad Y_t^i &= N_t^i \\ C_t^i &= C_t^*(S_t)^{1-\alpha} \\ Y_t^i &= C_t^i(S_t)^\alpha + G_t^i \end{aligned}$$

In steady state we have:

$$\begin{aligned} \gamma^{Nash} &= \left(\frac{G}{Y} \right)^{Nash} = \frac{\chi}{1 - \chi} \left((1 - \alpha) + \frac{\chi}{1 - \chi} \right)^{-1} \\ Y^{Nash} &= ((1 - \alpha)(1 - \chi) + \chi)^{\frac{1}{1+\varphi}} \end{aligned}$$

It holds that $\chi < \gamma^{Nash}$ while $Y^{Coord} > Y^{Nash}$. We thus confirm the earlier finding by Devereux (1991) that government spending takes an excessive share of output without coordination. Furthermore output is inefficiently low. The intuition is that the government tries to

¹³Without coordination there remain linear terms in a second order approximation to utility. With these a correct evaluation of welfare is not possible as the linear terms contain errors of second order. We follow Benigno and Woodford (2006) and substitute for the linear terms by using a second order approximation to the goods market clearing condition.

improve the terms of trade by increasing domestic demand as the loss of fewer exports to the rest of the union is—*ceteris paribus*—outweighed by reduced disutility of labor of the domestic households. In equilibrium, however, all countries behave the same such that no country gains from increased government spending. It can be shown that utility is higher in the cooperative steady state as long as $\alpha \neq 0$. Thus the Nash steady state is inefficient.¹⁴

3.2 Optimal policy at the zero lower bound: coordination and Nash

In the following we will compute optimal discretionary fiscal policy under coordination and Nash. We maximize the welfare functions subject to the equilibrium conditions. Under the assumed process for r_t^{net} we will have a two state solution in equilibrium. When maximizing welfare, however, we do not apply this solution to the constraints because otherwise the policymaker could influence expectations even under discretion.

We start with the case of coordination where we assume a symmetric solution i.e. $x_t^i = x_t^*$ for all $i \in [0, 1]$ as there is only an aggregate shock and no idiosyncratic ones. Therefore there are no integrals in the optimization problem. The optimization problem under discretion is given by:

$$\begin{aligned} \max_{\pi_t^*, \hat{y}_t^*, \hat{g}_t^*} \mathbb{W}_t^* &\simeq -\frac{1}{2} \left(\frac{\varepsilon}{\lambda} (\pi_t^*)^2 + (1 + \varphi) (\hat{y}_t^*)^2 + \frac{\chi}{1 - \chi} (\hat{g}_t^* - \hat{y}_t^*)^2 \right) \\ \text{s.t.} \quad \hat{y}_t^* &= E_t \{ \hat{y}_{t+1}^* \} - (1 - \chi) (r_t^* - E_t \{ \pi_{t+1}^* \} - r_t^{net}) - \chi E_t \{ \Delta \hat{g}_{t+1}^* \} \\ \pi_t^* &= \beta E_t \{ \pi_{t+1}^* \} + \lambda \left(\frac{1}{1 - \chi} + \varphi \right) \hat{y}_t^* - \frac{\lambda \chi}{1 - \chi} \hat{g}_t^* \end{aligned}$$

Taking first order conditions of the corresponding Lagrangian and substituting for the Lagrange multipliers yields (see appendix C.1):

$$\pi_t^{*, Coord} + \frac{1}{\varepsilon} \hat{y}_t^{*, Coord} = -\psi_g^{Coord} \hat{g}_t^{*, Coord} \quad (19)$$

with $\psi_g^{Coord} \equiv \frac{1}{\varepsilon \varphi}$. When $\varphi > 0$ it is optimal to increase government spending at the zero lower bound given a drop in output and inflation.

Under Nash the optimization problem under discretion looks as follows:

$$\begin{aligned} \max_{\pi_t^i, \hat{y}_t^i, \hat{g}_t^i} \mathbb{W}_t^i &\simeq -\frac{1}{2} \frac{1 - \alpha}{1 - \gamma^{Nash}} \left(\frac{\varepsilon}{\lambda} (\pi_t^i)^2 + (1 + \varphi) (\hat{y}_t^i)^2 + \frac{\gamma^{Nash}}{1 - \gamma^{Nash}} (\hat{g}_t^i - \hat{y}_t^i)^2 \right) \\ \text{s.t.} \quad \Delta \hat{y}_t^i &= \Delta \hat{y}_t^* + \gamma^{Nash} (\Delta \hat{g}_t^i - \Delta \hat{g}_t^*) - (1 - \gamma^{Nash}) (\pi_t - \pi_t^*) \\ \pi_t^i &= \beta E_t \{ \pi_{t+1}^i \} + \lambda \left(\frac{1}{1 - \gamma^{Nash}} + \varphi \right) \hat{y}_t^i - \lambda \frac{\gamma^{Nash}}{1 - \gamma^{Nash}} \hat{g}_t^i \end{aligned}$$

¹⁴Our setup differs from Devereux (1991) in some respects. For instance, we have linear production opposed to decreasing returns to scale but do not have log disutility of labor supply. Under his calibration government spending exceeds the one under coordination even in levels (see his Table 3), which is never the case in our setup.

Taking first order conditions of the corresponding Lagrangian and substituting for the Lagrange multipliers yields (see appendix C.2):

$$\pi_t^{i,Nash} + \frac{1}{\varepsilon} \hat{y}_t^{i,Nash} = -\psi_g^{Nash} \hat{g}_t^{i,Nash} \quad (20)$$

with $\psi_g^{Nash} \equiv \frac{1}{\varepsilon\varphi} (\lambda\varphi + (1 + \lambda))$. In equilibrium all countries will behave the same such that $\hat{g}_t^{i,Nash} = \hat{g}_t^{*,Nash}$. Comparing equations (19) and (20) leads to the following proposition:

Proposition 1

As $\varphi, \lambda > 0$ a drop in inflation and output ($\pi_t + \frac{1}{\varepsilon} \hat{y}_t$) at the zero lower bound implies

$$0 < \hat{g}_t^{*,Nash} < \hat{g}_t^{*,Coord}.$$

Thus, at the zero lower bound there is underprovision of public goods without coordination of fiscal policy.

3.3 Quantitative analysis

In order to compute the optimal level of government consumption at the zero lower bound, we assign values to the model parameters. In doing so, we target observations for the euro area and use the US as a benchmark for a currency union in which government spending is set cooperatively. For the euro area we assume that government expenditures are set no-cooperatively (baseline), as there is basically no exhaustive government spending administered at the area-wide level.¹⁵ Here and in what follows we consider average values for the period 1999–2006.

We set $\chi = 0.148$ in order to match the average share of exhaustive government spending relative to GDP in the US (see Figure 1 above). We set $\alpha = 0.168$ to match the average ratio of imports in EA countries from within the EA relative to GDP (Source: OECD, Monthly Foreign Trade Statistics). This implies $\gamma^{Nash} = 0.172$. The actual level of government consumption in the euro area is 19.6 percent. Hence, our model accounts for approximately half the gap in the average level of government consumption between the US and the EA.

Further we set $\beta = 0.99$, $\theta = 0.925$ implying that prices last on average 13 quarters. Such a high degree is difficult to reconcile with microevidence, but may be justified in light of evidence suggesting very sluggish price adjustments, notably at the zero lower bound (see Corsetti et al. (2013a) for further discussion. We will illustrate to what extent results vary

¹⁵This is an idealization, as according to NIPA data only 36.3% of all exhaustive government expenditures in the US are determined at the federal level. In the EU there is a common budget. However, it is very small and contains mostly transfers.

Table 1: Model parameters

Steady-state ratios		
χ	0.148	Public consumption weight: US data
α	0.168	Openness: EA data
γ_{Nash}	0.172	Compare to EA data 0.196
β	0.99	Discount factor
θ	0.925	Calvo parameter
ε	6	Elasticity of substitution
φ	4	Inverse of Frisch elasticity of labor supply
r_L^n	-0.01	Natural rate
μ	0.80	Expected duration of ZLB: 5 quarters

with θ . $\varepsilon = 6$ for the elasticity of substitution, $\varphi = 4$ for the inverse of the Frisch elasticity of labor supply and $r_L = -0.01$ implying real policy rate required to maintain a constant path for private expenditure of 4%. Finally, we assume that $\mu = 0.8$ (baseline). This implies an expected duration of the zero lower bound episode of 5 quarters. This is a relatively short period, but necessary in order to compute optimal policies for lower degrees of price stickiness without running into indeterminacy problems (see Woodford (2011)). Table 1 summarizes the parameter values.

Given these parameter values, we compute the optimal level of government consumption with and without coordination. Figure 2 shows the results. The vertical axis measures the optimal level of government consumption in percentage deviations from steady state. In the upper panel, we vary the expected duration of the zero lower bound episode along the horizontal axis. The dashed line represents the case of coordination, the solid line corresponds to Nash. We make two observations. First, the optimal level of government spending increases, the longer the zero lower bound episode. This reflects the fact that output and inflation decline more strongly, as the expected duration of the zero lower bound episode increases. It is then optimal to raise government consumption both under coordination and without, see (19) and (20). Second, we throughout the proportional increase in government consumption is always larger under coordination, see Proposition 1.

The lower panel of Figure 2 shows the optimal policy response as a function of the degree of price stickiness. The lower the degree of price stickiness, the larger the gap between the optimal policy under coordination and Nash. To understand this result, note that inflation responds more strongly to higher government spending if prices are more flexible. Higher

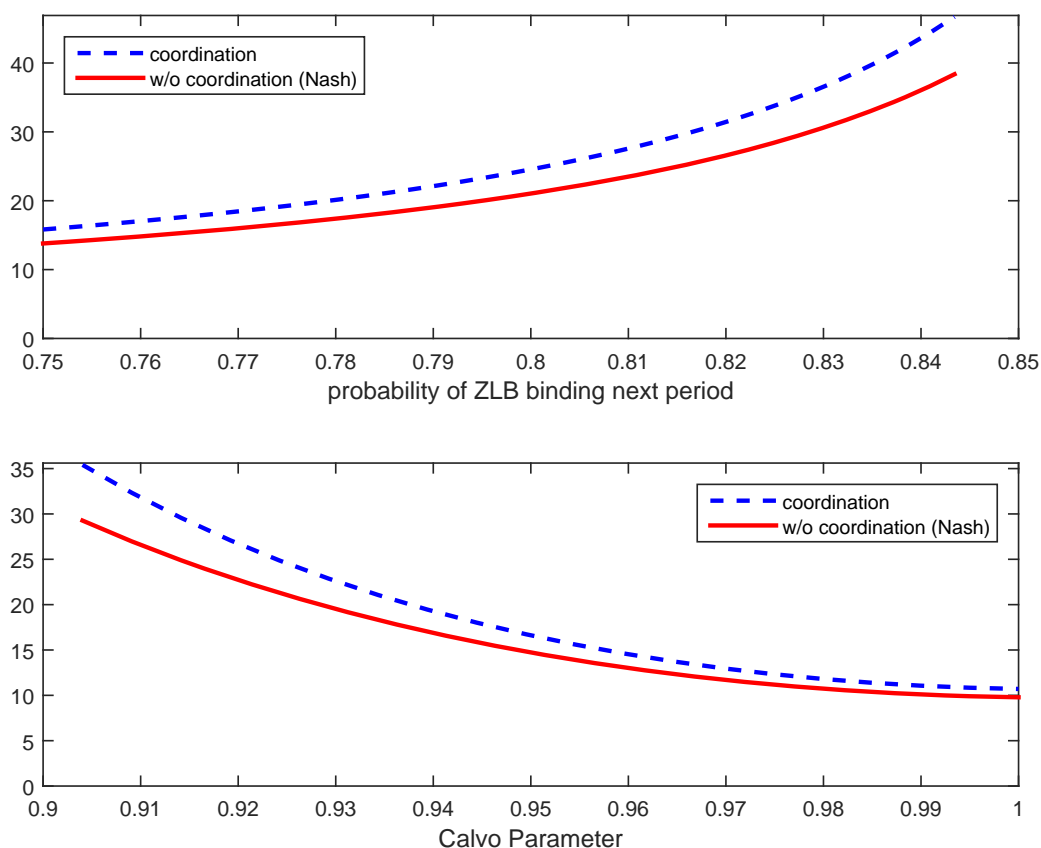


Figure 2: Optimal government consumption under coordination (dashed) and Nash (solid) in percentage deviation from steady state. Note: horizontal axis measures probability μ that the zero lower bound remains binding (top panel) and the probability θ that a price remains unadjusted (lower panel). We only compute results for combinations of parameters which give rise to a unique and stable equilibrium.

inflation at the union level reduces real interest rates and thus stimulates aggregate demand at the union level. At the country level, instead, higher inflation appreciates the terms of trade and thus reduces the demand for domestically produced goods. Hence, the more flexible prices, the larger is the need for coordination.

In the absence of coordination government consumption is too high in steady state, because government try to improve their terms of trade. Off the steady state, as the zero lower bound binds, government consumption is raised too little as an individual government's consumption has no effect on area-wide demand and inflation. It is instructive to compute the actual level of government spending in units of the steady-state output under coordination in order to assess how the two opposing effects play out. Figure 3 shows the result. The vertical axis measures the optimal level of government consumption with (dashed lines) and with-

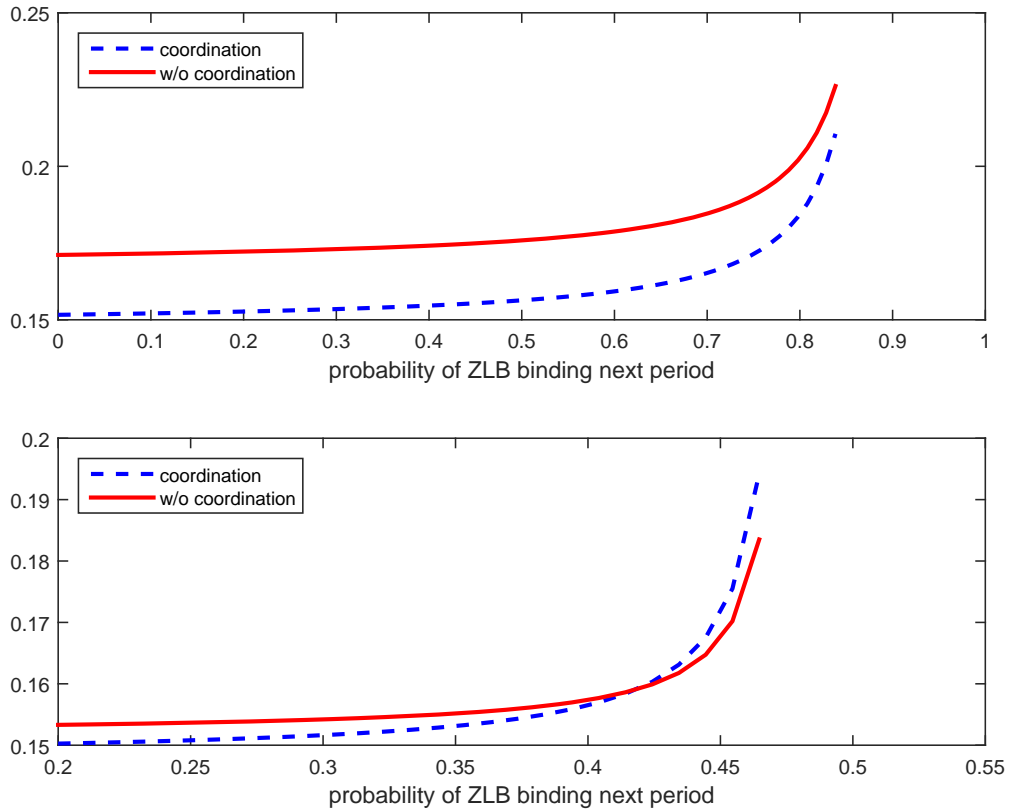


Figure 3: Optimal government spending under coordination (dashed) and Nash (solid) relative to steady state output under coordination. Notes: upper panel shows results for baseline parameterization (see Table 1), lower panel assumes a lower Frisch elasticity ($\varphi = 0.2$) and a lower Calvo parameter ($\theta = 0.5$).

out (solid lines) coordination. Along the horizontal axis we measure the expected duration of the zero lower bound episode. The upper panel assumes otherwise the parameter values listed in Table ?? above. In this case, we find that the steady-state effect dominates: the level of spending without coordination exceeds the optimal level with coordination for all value of μ .

In order to to illustrate that it is possible for the optimal spending level under coordination to exceed the level under Nash, we consider alternative parameter values. First, we assume a lower value for Frisch elasticity ($\varphi = 0.2$) and a lower Calvo parameter ($\theta = 0.5$). Results are shown in the lower panel of Figure 3. In this case we find that if the expected duration of the zero lower bond episode is sufficiently long, the optimal level of public consumption under coordination is higher than under Nash.

4 Conclusion

In the context of the global financial crisis fiscal stabilization policy made a comeback of sorts. A central consideration in this regard is that monetary policy may become constrained by the zero lower bound on nominal interest rate if the need for stabilization is particularly large. In this case, it not only seems natural to turn to fiscal policy for additional support, it has also been documented that fiscal policy is likely to be particularly effective under such circumstances.

Given these insights, we consider a scenario of a currency union where a common monetary policy operates jointly with many fiscal policies. Provided that the common monetary policy is unable to stabilize area-wide inflation and output, we ask whether there is a need to coordinate fiscal policies to achieve the socially optimal outcome. This question arises, because fiscal multipliers tend to be small in countries with fixed exchange rate regimes relative to an economy which operates at the zero lower bound (Corsetti et al., 2013b; Fahri and Werning, 2012).

We take up the question within a New Keynesian model of a monetary union due to Galí and Monacelli (2008) and derive the optimal level of government spending, contrasting the results under cooperation and without cooperation. In the absence of cooperation each country seeks to improve its terms of trade. In a symmetric equilibrium, however, the terms of trade are unchanged and economic activity is reduced relative to the optimal steady state because government spending takes an inefficient share of output (see also Devereux, 1991). Instead, we find that the optimal fiscal response to a union-wide shock which the common central bank is unable to neutralize implies too little fiscal stimulus in the absence of cooperation. In this case governments do not internalize the aggregate effects of their additional spending and hence contribute less than what is optimal from a union wide perspective.

References

- Acconcia, A., Corsetti, G., and Simonelli, S. (2014). Mafia and public spending: Evidence on the fiscal multiplier from a quasi-experiment. *American Economic Review*, 104(7):2185–2209.
- Beetsma, R. and Jensen, H. (2005). Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *Journal of International Economics*, (67):320–352.
- Beetsma, R. and Uhlig, H. (1999). An analysis of the stability and growth pact. *Economic Journal*, (109):546–571.
- Benigno, G. and Benigno, P. (2003). Price Stability in Open Economies. *Review of Economic Studies*, 70(4):743–764.
- Benigno, P. and Woodford, M. (2006). Optimal taxation in an RBC model: A linear-quadratic approach. *Journal of Economic Dynamics and Control*, 30(9-10):1445–1489.
- Blanchard, O., Erceg, C. J., and Linde, J. (2014). The euro area recovery: Should the core expand spending to help the periphery? mimeo.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383 – 398.
- Christiano, L., Eichenbaum, M., and Rebelo, S. (2011). When Is the Government Spending Multiplier Large? *Journal of Political Economy*, 119(1):78 – 121.
- Cook, D. and Devereux, M. B. (2011). Optimal fiscal policy in a world liquidity trap. *European Economic Review*, 55(4):443–462.
- Corsetti, G., Kuester, K., Meier, A., and Müller, G. J. (2013a). Sovereign risk, fiscal policy, and macroeconomic stability. *The Economic Journal*, 123(566):F99–F132.
- Corsetti, G., Kuester, K., and Müller, G. (2013b). *Floats, pegs and the transmission of fiscal policy*, volume 17 of *Fiscal Policy and Macroeconomic Performance, Central Banking, Analysis, and Economic Policies Book Series*, chapter 7, pages 235–281. in Céspedes, L.F. and J. Galí (eds.), Santiago, Chile, Central Bank of Chile.
- Corsetti, G., Meier, A., Kuester, K., and Müller, G. J. (2014). Sovereign risk and belief-driven fluctuations in the euro area. *Journal of Monetary Economics*, (61):53–73.

- Corsetti, G. and Müller, G. J. (2014). Multilateral economic cooperation and the international transmission of fiscal policy. In Feenstra, R. and Taylor, A., editors, *Globalization in an Age of Crisis: Multilateral Economic Cooperation in the Twenty-First Century*, chapter 7, pages 257–299. University of Chicago Press.
- Corsetti, G. and Pesenti, P. (2001). Welfare And Macroeconomic Interdependence. *The Quarterly Journal of Economics*, 116(2):421–445.
- Cúrdia, V. and Woodford, M. (2011). The Central-Bank Balance Sheet as an Instrument of Monetary Policy. *Journal of Monetary Economics*, (58):54–79.
- Cwik, T. and Wieland, V. (2011). Keynesian Government Spending Multipliers and Spillovers in the Euro Area. *Economic Policy*, (26):493–549.
- De Paoli, B. (2009). Monetary policy and welfare in a small open economy. *Journal of International Economics*, 77(1):11–22.
- Devereux, M. B. (1991). The terms of trade and the international coordination of fiscal policy. *Economic Inquiry*, (102(3)):720–736.
- Eggertsson, G. B. and Krugman, P. (2012). Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach. *The Quarterly Journal of Economics*, 127(3):1469–1513.
- Erceg, C. J. and Lindé, J. (2012). Fiscal Consolidation in an Open Economy. *American Economic Review*, (102(3)):186–191.
- Fahri, E. and Werning, I. (2012). Fiscal multipliers: Liquidity traps and currency unions. mimeo.
- Galí, J. (2008). *Monetary policy, inflation, and the business cycle*. Princeton Univ. Press.
- Galí, J. and Monacelli, T. (2008). Optimal monetary and fiscal policy in a currency union. *Journal of International Economics*, 76(1):116 – 132.
- Gauti B. Eggertsson and Michael Woodford (2003). The Zero Interest-Rate Bound and Optimal Monetary Policy. *Brookings Papers on Economic Activity*, 1:139–211.
- Krogstrup, S. and Wyplosz, C. (2010). A common pool theory of supranational deficit ceilings. *European Economic Review*, 54:273–282.
- Krugman, P. (2008). The economic consequences of Herr Steinbrueck. New York Times.

Nakamura, E. and Steinsson, J. (2014). Fiscal stimulus in a monetary union: Evidence from us regions. *American Economic Review*, 104(3):753–92.

Woodford, M. (2011). Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1 – 35.

A Social planner solution without coordination

Without coordination the social planner in a small open economy maximizes domestic utility given production, the risk-sharing condition and the aggregate goods market clearing condition:

$$\begin{aligned} \max U(C_t^i, N_t^i, G_t^i) &= (1 - \chi) \log C_t^i + \chi \log G_t^i - \frac{(N_t^i)^{1+\varphi}}{1 + \varphi} \\ \text{s.t. } Y_t^i &= N_t^i \\ C_t^i &= C_t^*(S_t)^{1-\alpha} \\ Y_t^i &= C_t^i(S_t)^\alpha + G_t^i \end{aligned}$$

Note that the risk sharing condition and the market clearing condition imply $C_t = (Y_t - G_t)^{1-\alpha} (C_t^*)^\alpha$. The Lagrangian can be written as:

$$\mathcal{L}_t = (1 - \chi) \log C_t^i + \chi \log G_t^i - \frac{(N_t^i)^{1+\varphi}}{1 + \varphi} + \lambda (C_t - (N_t - G_t)^{1-\alpha} (C_t^*)^\alpha)$$

First order conditions are given by:

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = (1 - \chi) \frac{1}{C_t} + \lambda = 0 \quad (21)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial N_t} &= -(N_t)^\varphi - \lambda(1 - \alpha)(N_t - G_t)^{-\alpha} (C_t^*)^\alpha \\ &= -(N_t)^\varphi - \lambda(1 - \alpha) \frac{C_t}{Y_t - G_t} = 0 \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial G_t} &= \chi \frac{1}{G_t} + \lambda(1 - \alpha)(N_t - G_t)^{-\alpha} (C_t^*)^\alpha \\ &= \chi \frac{1}{G_t} + \lambda(1 - \alpha) \frac{C_t}{Y_t - G_t} = 0 \end{aligned} \quad (23)$$

Combine (22) and (23) to get:

$$(N_t)^\varphi = \chi \frac{1}{G_t} \quad (24)$$

Combine (21) and (23):

$$\begin{aligned}
\frac{\chi}{1-\chi} \frac{C_t}{G_t} &= (1-\alpha) \frac{C_t}{Y_t - G_t} \\
\frac{\chi}{1-\chi} (Y_t - G_t) &= (1-\alpha) G_t \\
\left((1-\alpha) + \frac{\chi}{1-\chi} \right) G_t &= \frac{\chi}{1-\chi} Y_t \\
G_t &= \frac{\chi}{1-\chi} \left((1-\alpha) + \frac{\chi}{1-\chi} \right)^{-1} Y_t
\end{aligned} \tag{25}$$

Thus the steady state under Nash is given by:

$$\frac{\bar{G}^{Nash}}{\bar{Y}} = \frac{\chi}{1-\chi} \left((1-\alpha) + \frac{\chi}{1-\chi} \right)^{-1}$$

Combining (24) and (25):¹⁶

$$\begin{aligned}
N^{Nash} &= \left[\left((1-\alpha) + \frac{\chi}{1-\chi} \right) (1-\chi) \right]^{\frac{1}{1+\varphi}} \\
(N^{Nash})^{1+\varphi} &= (1-\alpha)(1-\chi) + \chi
\end{aligned} \tag{26}$$

Decentralization

In the following we show, how this allocation can be decentralized under flexible prices. In a flexible price equilibrium it holds that firms choose a constant mark-up over marginal costs MC_t^i :

$$\overline{MC}_t^i = 1 - \frac{1}{\varepsilon} \tag{27}$$

where the bar is used to denote the flexible price allocation. The marginal costs can be expressed as:

$$\overline{MC}_t^i = \frac{1-\tau^i}{1-\chi} (\bar{N}_t^i)^{1+\varphi} \left(1 - \frac{\bar{G}_t}{\bar{Y}_t} \right)$$

Inserte this into (27) and let government spending \bar{G}_t^i follow a rule according to (25). Solving the resulting expression for \bar{N}_t^i , gives:

$$(\bar{N}_t^i)^{1+\varphi} = \frac{1-\chi}{1-\tau^i} \left(1 - \frac{\chi}{1-\chi} \left((1-\alpha) + \frac{\chi}{1-\chi} \right)^{-1} \right)^{-1}$$

¹⁶Note that N_t^{Nash} would also be constant when we had technology shocks A_t such that $Y_t = A_t N_t$. The same holds true for $(G_t/Y_t)^{Nash}$, while G_t and Y_t would fluctuate with technology.

The subsidy that decentralizes the social planner solution without coordination under flexible prices can now be computed by inserting for \bar{N}_t according to (26) and solving for τ^i . The solution is given by:

$$(1 - \tau^i)(1 - \alpha) = 1 - \frac{1}{\varepsilon}$$

B Deriving the welfare function without coordination

B.1 Second order approximation to utility

When deriving the welfare function from the perspective of a small open economy within the currency union there remain linear terms. With these no correct evaluation of welfare is possible as linear terms contain errors of second order. We follow the approach described by Benigno and Woodford (2006) and substitute for the linear terms using second order approximations to equilibrium conditions. In the following we show how to apply this method in our case.

Utility $U_t = U(C_t, G_t, N_t)$ is additively separable in its arguments. A second order approximation to utility around a generic steady state C, G, N therefore gives:

$$\begin{aligned} U_t - U &\approx U_C C \left(\frac{C_t - C}{C} \right) + U_G G \left(\frac{G_t - G}{G} \right) + U_N N \left(\frac{N_t - N}{N} \right) \\ &\quad + \frac{1}{2} U_{CC} C^2 \left(\frac{C_t - C}{C} \right)^2 + \frac{1}{2} U_{GG} G^2 \left(\frac{G_t - G}{G} \right)^2 \\ &\quad + \frac{1}{2} U_{NN} N^2 \left(\frac{N_t - N}{N} \right)^2 \end{aligned}$$

Rewriting the expression in terms of log deviations, that is, using for a generic variable X_t

$$\frac{X_t - X}{X} \approx \hat{x}_t + \frac{1}{2} \hat{x}_t^2$$

where $\hat{x}_t = x_t - x$ and $x_t = \log X_t$, the above approximation becomes:

$$\begin{aligned} U_t - U &\approx U_C C \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + U_G G \left(\hat{g}_t + \frac{1}{2} \hat{g}_t^2 \right) + U_N N \left(\hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) \\ &\quad + \frac{1}{2} U_{CC} C^2 \hat{c}_t^2 + \frac{1}{2} U_{GG} G^2 \hat{g}_t^2 + \frac{1}{2} U_{NN} N^2 \hat{n}_t^2 \end{aligned}$$

Rearranging:

$$\begin{aligned} U_t - U &\approx U_C C \left(\hat{c}_t + \frac{1}{2} \left(1 + \frac{U_{CC} C}{U_C} \right) \hat{c}_t^2 \right) + U_G G \left(\hat{g}_t + \frac{1}{2} \left(1 + \frac{U_{GG} G}{U_G} \right) \hat{g}_t^2 \right) \\ &\quad + U_N N \left(\hat{n}_t + \frac{1}{2} \left(1 + \frac{U_{NN} N}{U_N} \right) \hat{n}_t^2 \right) \end{aligned}$$

Define further: $\sigma \equiv -\frac{U_{CC}C}{U_C}$, $\sigma_g \equiv -\frac{U_{GG}G}{U_G}$ and $\sigma_n \equiv \frac{U_{NN}N}{U_N}$. Thus

$$U_t - U \approx U_C C \left(\hat{c}_t + \frac{1}{2} (1 - \sigma) \hat{c}_t^2 \right) + U_G G \left(\hat{g}_t + \frac{1}{2} (1 - \sigma_g) \hat{g}_t^2 \right) + U_N N \left(\hat{n}_t + \frac{1}{2} (1 + \sigma_n) \hat{n}_t^2 \right).$$

As utility is given by

$$U_t = (1 - \chi) \log C_t + \chi \log G_t - \frac{N_t^{1+\varphi}}{1 + \varphi}$$

the above defined parameters become: $\sigma = \sigma_g = 1$ while $\sigma_n = \varphi$ such that we get:

$$U_t - U \approx U_C C \hat{c}_t + U_G G \hat{g}_t + U_N N \left(\hat{n}_t + \frac{1}{2} (1 + \varphi) \hat{n}_t^2 \right)$$

Deviding by $U_C C$:

$$\frac{U_t - U}{U_C C} \approx \hat{c}_t + \frac{U_G G}{U_C C} \hat{g}_t + \frac{U_N N}{U_C C} \left(\hat{n}_t + \frac{1}{2} (1 + \varphi) \hat{n}_t^2 \right)$$

Because of monopolistic competition firms set a markup on marginal costs. If not offset by a certain value for for the labor subsidy there will be a wedge Φ between the marginal rate of substitution and the marginal product of labor (MPN) in steady state (see, e.g., Galí (2008), p.106):

$$-\frac{U_N}{U_C} = \text{MPN}(1 - \Phi)$$

In our setup we have $\text{MPN} = Y/N$. Therefore

$$\begin{aligned} \frac{U_N N}{U_C C} &= -\frac{Y}{N} \frac{N}{C} (1 - \Phi) \\ \frac{U_N N}{U_C C} &= -\frac{1}{1 - \gamma} (1 - \Phi) \end{aligned}$$

with $\gamma \equiv G/Y$ and $1 - \gamma = C/Y$.¹⁷ Making use of this relation and the one for $\frac{U_G G}{U_C C}$ under the assumed utility function, we can rewrite the approximation to utility as:

$$\frac{U_t - U}{U_C C} \approx \hat{c}_t + \frac{\chi}{1 - \chi} \hat{g}_t - \frac{1 - \Phi}{1 - \gamma} \left(\hat{n}_t + \frac{1}{2} (1 + \varphi) \hat{n}_t^2 \right)$$

In order to substitute for \hat{c}_t , we approximate the goods market clearing condition $Y_t = C_t S_t^\alpha + G_t$ second order. Taking logs and rearranging:

$$\log C_t = \log(Y_t - G_t) - \alpha \log S_t$$

¹⁷The relation for C/Y follows from the goods market clearing condition and the assumption of a symmetric steady state where the terms of trade S are equal to 1.

Using the approximation for $\log(Y_t - G_t)$ as in the appendix of Galí and Monacelli (2008) and noting that up to third order $\log C_t$ and $\log S_t$ are linear we get:

$$\hat{c}_t \approx \frac{1}{1-\gamma} (\hat{y}_t - \gamma \hat{g}_t) - \frac{1}{2} \frac{\gamma}{(1-\gamma)^2} (\hat{g}_t - \hat{y}_t)^2 - \alpha s_t \quad (28)$$

Therefore

$$\begin{aligned} \frac{U_t - U}{U_C C} &\approx \frac{1}{1-\gamma} (\hat{y}_t - \gamma \hat{g}_t) - \frac{1}{2} \frac{\gamma}{(1-\gamma)^2} (\hat{g}_t - \hat{y}_t)^2 - \alpha s_t \\ &\quad + \frac{\chi}{1-\chi} \hat{g}_t - \frac{1-\Phi}{1-\gamma} \left(\hat{n}_t + \frac{1}{2} (1+\varphi) \hat{n}_t^2 \right) \end{aligned}$$

In order to substitute for \hat{n}_t it can be shown that $N_t = Y_t \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj$. Define $z_t \equiv \log \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj$. It thus holds around a symmetric zero inflation steady state that:

$$\hat{n}_t = \hat{y}_t + z_t$$

Further it can be shown that z_t is of second order and proportional to $\frac{1}{2} \frac{\varepsilon}{\lambda} \pi_t^2$ (see again the appendix of Galí and Monacelli (2008)). Finally, the approximation to utility can be expressed as:

$$\begin{aligned} \frac{U_t - U}{U_C C} &\approx \frac{1}{1-\gamma} (\hat{y}_t - \gamma \hat{g}_t) - \frac{1}{2} \frac{\gamma}{(1-\gamma)^2} (\hat{g}_t - \hat{y}_t)^2 - \alpha s_t \\ &\quad + \frac{\chi}{1-\chi} \hat{g}_t - \frac{1-\Phi}{1-\gamma} \left(\hat{y}_t + z_t + \frac{1}{2} (1+\varphi) \hat{y}_t^2 \right) \\ &\approx \frac{\Phi}{1-\gamma} \hat{y}_t + \left(\frac{\chi}{1-\chi} - \frac{\gamma}{1-\gamma} \right) \hat{g}_t - \alpha s_t \\ &\quad - \frac{1}{2} \frac{\gamma}{(1-\gamma)^2} (\hat{g}_t - \hat{y}_t)^2 - \frac{1}{2} \frac{1-\Phi}{1-\gamma} (1+\varphi) \hat{y}_t^2 - \frac{1}{2} \frac{1-\Phi}{1-\gamma} \pi_t^2 \quad (29) \end{aligned}$$

Define: $A_y = \frac{\Phi}{1-\gamma}$; $A_g = \frac{\chi}{1-\chi} - \frac{\gamma}{1-\gamma}$; $A_s = -\alpha$.

In the coordination case, the linear terms drop out of the union wide welfare function if one approximates around the efficient steady state from the union wide perspective. That is, in that steady state the labor subsidy is chosen such that $\Phi = 0$ and steady state government spending γ equals χ . The terms of trade drop out because from a union wide perspective it is recognized that it is futile to manipulate them.¹⁸

B.2 Second order approximation to the market clearing condition

We know from (28) that

$$\hat{c}_t \approx \frac{1}{1-\gamma} (\hat{y}_t - \gamma \hat{g}_t) - \frac{1}{2} \frac{\gamma}{(1-\gamma)^2} (\hat{g}_t - \hat{y}_t)^2 - \alpha s_t$$

¹⁸Technically they drop out by integrating over them as one considers union wide welfare. The terms of trade are defined as $s_t^i = p_t^* - p_t^i$ from which it follows that $\int_0^1 s_t^i di = 0$.

Further the risk sharing condition implies:

$$\hat{c}_t = \hat{c}_t^* + (1 - \alpha)s_t$$

Combining both, we get:

$$0 \approx \frac{1}{1 - \gamma} (\hat{y}_t - \gamma \hat{g}_t) - \frac{1}{2} \frac{\gamma}{(1 - \gamma)^2} (\hat{g}_t - \hat{y}_t)^2 - s_t + t.i.p. \quad (30)$$

where \hat{c}_t^* is captured by terms independent of policy (*t.i.p.*) as the average evolves exogenously for a given member of the currency union and thus will not be taken into account when maximizing welfare.

Define: $B_y = \frac{1}{1 - \gamma}$; $B_g = -\frac{\gamma}{1 - \gamma}$; $B_s = -1$.

B.3 Substituting for the linear terms

We approximate the welfare function without coordination around the steady state that is optimal from the viewpoint of a small open economy. From the social planner problem of the perspective of a small open economy we have that

$$\frac{\bar{G}^{Nash}}{\bar{Y}} = \gamma^{Nash} = \frac{\chi}{1 - \chi} \left((1 - \alpha) + \frac{\chi}{1 - \chi} \right)^{-1}$$

This steady state can be implemented by a certain rule for government spending and by choosing the following subsidy:

$$1 - \tau = \frac{\varepsilon}{\varepsilon - 1} \frac{1}{1 - \alpha}.$$

From (Galí, 2008, e.g. p.73) we know further that

$$\Phi = 1 - \frac{\varepsilon - 1}{\varepsilon} \frac{1}{1 - \tau}.$$

such that with the above mentioned subsidy

$$\Phi = \alpha.$$

Inserting for γ^{Nash} and Φ in the coefficients A and B we get:

$$A_y = \frac{\alpha[(1 - \chi)(1 - \alpha) + \chi]}{(1 - \chi)(1 - \alpha)}; \quad A_g = -\frac{\alpha\chi}{(1 - \chi)(1 - \alpha)}; \quad A_s = -\alpha$$

$$B_y = \frac{(1 - \chi)(1 - \alpha) + \chi}{(1 - \chi)(1 - \alpha)}; \quad B_g = -\frac{\chi}{(1 - \chi)(1 - \alpha)}; \quad B_s = -1$$

Thus, it is easily seen that subtracting α times condition (30) from (29) –both evaluated at the Nash steady state– removes the linear terms from the welfare approximation.

Making use of this the welfare function approximated around the Nash steady state becomes:

$$W_0^{Nash} \approx -\frac{1}{2} \frac{1-\alpha}{1-\gamma^{Nash}} \sum_{t=0}^{\infty} e_t \beta^t \left(\frac{\varepsilon}{\lambda} (\pi_t^i)^2 + (1+\varphi)(\hat{y}_t^i)^2 + \frac{\gamma^{Nash}}{1-\gamma^{Nash}} (\hat{g}_t^i - \hat{y}_t^i)^2 \right) + t.i.p.$$

C Optimal policy with and without coordination

C.1 Coordination

Our strategy is as follows: we derive optimal policy under discretion without applying the two state equilibrium right away because otherwise the policymaker could influence expectations. Then we evaluate the solution at the zero lower bound with the two state solution. Further, we assume a symmetric solution i.e. $x_t^i = x_t^*$ for all $i \in [0, 1]$ as we only have an aggregate shock. Therefore there are no integrals in the optimization problem. The optimization problem under discretion looks as follows:

$$\begin{aligned} \max_{\pi_t^*, \hat{y}_t^*, \hat{g}_t^*} U_t^* &\simeq -e_t \frac{1}{2} \left(\frac{\varepsilon}{\lambda} (\pi_t^*)^2 + (1+\varphi)(\hat{y}_t^*)^2 + \frac{\chi}{1-\chi} (\hat{g}_t^* - \hat{y}_t^*)^2 \right) \\ \text{s.t.} \quad \hat{y}_t^* &= E_t\{\hat{y}_{t+1}^*\} - (1-\chi)(r_t^* - E_t\{\pi_{t+1}^*\} - r_t^{net}) - \chi E_t\{\Delta \hat{g}_{t+1}^*\} \end{aligned} \quad (31)$$

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda \left(\frac{1}{1-\chi} + \varphi \right) \hat{y}_t^* - \frac{\lambda \chi}{1-\chi} \hat{g}_t^* \quad (32)$$

The corresponding Lagrangian is given by:

$$\begin{aligned} \mathcal{L}_t &= -e_t \frac{1}{2} \left(\frac{\varepsilon}{\lambda} (\pi_t^*)^2 + (1+\varphi)(\hat{y}_t^*)^2 + \frac{\chi}{1-\chi} (\hat{g}_t^* - \hat{y}_t^*)^2 \right) \\ &+ e_t \psi_y^* [\hat{y}_t^* - E_t\{\hat{y}_{t+1}^*\} + (1-\chi)(r_t^* - E_t\{\pi_{t+1}^*\} - r_t^{net}) + \chi E_t\{\Delta \hat{g}_{t+1}^*\}] \\ &+ e_t \psi_\pi^* \left[\pi_t^* - \beta E_t\{\pi_{t+1}^*\} - \lambda \left(\frac{1}{1-\chi} + \varphi \right) \hat{y}_t^* + \frac{\lambda \chi}{1-\chi} \hat{g}_t^* \right] \end{aligned}$$

First order conditions read as follows

$$\frac{\partial \mathcal{L}_t}{\partial \pi_t^*} = -\frac{\varepsilon}{\lambda} \pi_t^* + \psi_\pi^* = 0 \quad (33)$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{y}_t^*} = -(1+\varphi)\hat{y}_t^* + \frac{\chi}{1-\chi} (\hat{g}_t^* - \hat{y}_t^*) + \psi_y^* - \lambda \left(\frac{1}{1-\chi} + \varphi \right) \psi_\pi^* = 0 \quad (34)$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{g}_t^*} = -\frac{\chi}{1-\chi} (\hat{g}_t^* - \hat{y}_t^*) - \chi \psi_y^* + \frac{\lambda \chi}{1-\chi} \psi_\pi^* = 0 \quad (35)$$

Rearrange (35):

$$\psi_y^* = -\frac{1}{1-\chi} (\hat{g}_t^* - \hat{y}_t^*) + \frac{\lambda}{1-\chi} \psi_\pi^*$$

Combining it with (34):

$$\begin{aligned}
0 &= -(1 + \varphi)\hat{y}_t^* + \frac{\chi}{1 - \chi}(\hat{g}_t^* - \hat{y}_t^*) - \frac{1}{1 - \chi}(\hat{g}_t^* - \hat{y}_t^*) + \frac{\lambda}{1 - \chi}\psi_\pi^* - \lambda \left(\frac{1}{1 - \chi} + \varphi \right) \psi_\pi^* \\
0 &= -(1 + \varphi)\hat{y}_t^* - (\hat{g}_t^* - \hat{y}_t^*) - \lambda\varphi\psi_\pi^* \\
\psi_\pi^* &= -\frac{1}{\lambda}\hat{y}_t^* - \frac{1}{\lambda\varphi}\hat{g}_t^*
\end{aligned} \tag{36}$$

Put (36) in (33):

$$\pi_t^* = -\frac{1}{\varepsilon}\hat{y}_t^* - \alpha_g^{coord}\hat{g}_t^* \tag{37}$$

with $\alpha_g^{coord} \equiv \frac{1}{\varepsilon\varphi}$. Optimal policy is characterized by equation (19). In order to compute the equilibrium allocation at the zero lower bound one has to combine it with the dynamic IS equation and the Phillips curve.

C.2 No coordination

As under coordination we derive optimal policy under discretion and then evaluate the solution at the zero lower bound with the two state solution. The optimization problem under discretion looks as follows:

$$\begin{aligned}
\max_{\pi_t^i, \hat{y}_t^i, \hat{g}_t^i} U_t^i &\simeq -e_t \frac{1 - \alpha}{2(1 - \gamma)} \left(\frac{\varepsilon}{\lambda} (\pi_t^i)^2 + (1 + \varphi) (\hat{y}_t^i)^2 + \frac{\gamma}{1 - \gamma} (\hat{g}_t^i - \hat{y}_t^i)^2 \right) \\
\text{s.t.} \quad \Delta \hat{y}_t^i &= \Delta \hat{y}_t^* + \gamma (\Delta \hat{g}_t^i - \Delta \hat{g}_t^*) - (1 - \gamma) (\pi_t - \pi_t^*)
\end{aligned} \tag{38}$$

$$\pi_t^i = \beta E_t \{ \pi_{t+1}^i \} + \lambda \left(\frac{1}{1 - \gamma} + \varphi \right) \hat{y}_t^i - \lambda \frac{\gamma}{1 - \gamma} \hat{g}_t^i \tag{39}$$

With the Lagrangian given by

$$\begin{aligned}
\mathcal{L}_t &= -e_t \frac{1 - \alpha}{2(1 - \gamma)} \left(\frac{\varepsilon}{\lambda} (\pi_t^i)^2 + (1 + \varphi) (\hat{y}_t^i)^2 + \frac{\gamma}{1 - \gamma} (\hat{g}_t^i - \hat{y}_t^i)^2 \right) \\
&\quad + e_t \psi_y^i \left[\Delta \hat{y}_t^i - \Delta \hat{y}_t^* - \gamma (\Delta \hat{g}_t^i - \Delta \hat{g}_t^*) + (1 - \gamma) (\pi_t - \pi_t^*) \right] \\
&\quad + e_t \psi_\pi^i \left[\pi_t^i - \beta E_t \{ \pi_{t+1}^i \} - \lambda \left(\frac{1}{1 - \gamma} + \varphi \right) \hat{y}_t^i + \lambda \frac{\gamma}{1 - \gamma} \hat{g}_t^i \right]
\end{aligned}$$

First order conditions read as follows

$$\frac{\partial \mathcal{L}_t}{\partial \pi_t^i} = -\frac{1 - \alpha}{1 - \gamma} \frac{\varepsilon}{\lambda} \pi_t^i + (1 - \gamma) \psi_y^i + \psi_\pi^i = 0 \tag{40}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{y}_t^i} = -\frac{1 - \alpha}{1 - \gamma} (1 + \varphi) \hat{y}_t^i + \frac{1 - \alpha}{1 - \gamma} \frac{\gamma}{1 - \gamma} (\hat{g}_t^i - \hat{y}_t^i) + \psi_y^i - \lambda \left(\frac{1}{1 - \gamma} + \varphi \right) \psi_\pi^i = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{g}_t^i} = -\frac{1 - \alpha}{1 - \gamma} \frac{\gamma}{1 - \gamma} (\hat{g}_t^i - \hat{y}_t^i) - \gamma \psi_y^i + \frac{\lambda \gamma}{1 - \gamma} \psi_\pi^i = 0 \tag{42}$$

Rearrange (42):

$$\psi_y^i = -\frac{1-\alpha}{1-\gamma} \frac{1}{1-\gamma} (\hat{g}_t^i - \hat{y}_t^i) + \frac{\lambda}{1-\gamma} \psi_\pi^i \quad (43)$$

Combining it with (41) and solving for ψ_π^i :

$$\psi_\pi^i = -\frac{1}{\lambda} \frac{1-\alpha}{1-\gamma} \hat{y}_t^i - \frac{1}{\lambda\varphi} \frac{1-\alpha}{1-\gamma} \hat{g}_t^i \quad (44)$$

Inserting for ψ_y^i and ψ_π^i in (40) and rearranging gives:

$$\pi_t^i = -\frac{1}{\varepsilon} \hat{y}_t^i - \alpha_g^{Nash} \hat{g}_t^i \quad (45)$$

with $\alpha_g^{Nash} \equiv \frac{1}{\varepsilon\varphi} (\lambda\varphi + (1+\lambda))$.

Thus we have that:

$$\alpha_g^{coord} = \frac{1}{\varepsilon\varphi} < \frac{1}{\varepsilon\varphi} (\lambda\varphi + (1+\lambda)) = \alpha_g^{Nash} \quad \text{as } \lambda, \varphi > 0. \quad (46)$$