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Personality traits, subjective learning, and entrepreneurial decision making

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We present a dynamic occupational choice model with a learning algorithm simultaneously capable to explain entrepreneurial entry, exit, and survival. According to our model, those individuals decide to become entrepreneurs who expect their productivity to be highest when managed by themselves. As we further assume that individuals have incomplete information about their own non-cognitive skills, which are relevant for entrepreneurial processes, entrepreneurial entry in our model is driven by overconfidence in the own skills—in line with earlier empirical findings. After entry, entrepreneurs receive noisy feedback from the market. Depending on a set of traits different from those driving the entry process into the market, entrepreneurs decide to either stay or leave the market. Our learning-based model generates survival rates decreasing at decreasing rates and captures findings on the earnings puzzle according to which median entrepreneurs do not earn more than median wage workers.

JEL classification
C53, D83, L22, L26

Keywords
Occupational choice, non-cognitive traits, subjective learning
1 Introduction

There is puzzling evidence on entrepreneurship, entrepreneurial processes, and decision making. Individuals appear to systematically overestimate their own entrepreneurial abilities before establishing a business. Therefore, a substantial number of individuals end their entrepreneurial adventure before it really started, while others remain in entrepreneurship although they could earn more by returning to wage employment. Hence, the income of entrepreneurs is sometimes lower and sometimes higher than those of their counterparts in wage employment.

Confronted with this empirical evidence, research points to entrepreneurial personality as one potential explanation of this puzzling contradiction. However, empirical results show that although personality seems to play an important role, it is not the main determinant of entrepreneurial entry and success. Critically, the influence of personality traits on entrepreneurial decision making is non-linear, such that it makes no sense to try defining the average or representative personality of an entrepreneur.

There is no formal economic model integrating personality traits into entrepreneurial decision processes,\(^1\) while capturing all empirical observations on entrepreneurial entry, income, and survival using a unified set of modeling assumptions. In this paper, we contribute to this discussion by proposing an occupational choice model that explains the existing evidence by adding one important assumption to present approaches on entrepreneurial choices. We assume that individuals have incomplete information about their own relevant non-cognitive skills\(^2\) but that they start learning about the characteristics of these specific skills once they start entrepreneurial activities.

The dynamic model we construct extends the static approach of Roessler and Koellinger (2012) on entrepreneurial choices and combines it with a learning algorithm provided by Camerer and Ho (1999). This algorithm, tested with experimental data, is altered in two ways so that traits are allowed to interact with the learning process: First, traits are one determinant of the initial decision, which we define as an occupational choice under a restricted set of information—when individuals have no personal experience with dependent employment or entrepreneurship, thus at the beginning of their professional career. Second, if there is personal experience in a certain occupational status, traits also

\(^1\) This is not to say that personality was ignored in economic theory. Non-standard preferences and bounded rationality are two ways of integrating personality traits (see Almlund et al., 2011). Moreover, Borghans et al. (2008) depict an array of ways of how to integrate cognitive and non-cognitive traits into economic theory.

\(^2\) We are aware that economists and psychologists pursue different objectives in applying the notion of non-cognitive skills (as used by economists) and the construct of personality traits (as used by psychologists). There is certainly a general problem of mixing both notions, as economists are more interested in the influence of personality on economic behavior, while psychologists have a stronger focus on understanding personality (see Thiel and Thomsen, 2013). Our rationale behind using these terms interchangeably is rather pragmatic: In the theoretical setting of our approach, the difference between both notions is minimal.
determine information processing and consequently affect occupational-choice decisions based on internalized information. The decision of each individual can either be to keep the current status (entrepreneur or worker) or to change the status (start or abandon an own business).

Our main question is: How do certain sets of personality traits influence the probability of entry and survival of entrepreneurs? Abstracting from all non-personality factors, we argue in our model that a certain set of traits, accountable for the subjective estimation of individual entrepreneurial fitness, affects the selection into entrepreneurship. After entry, entrepreneurs receive noisy positive or negative feedback from the market. Depending on sets of traits different from the set driving the entry process into the market, entrepreneurs decide to either stay or leave the market. In particular, we claim that it might be helpful to be reluctant to fundamentally revise previous decisions after a negative market feedback in order to survive or even succeed as an entrepreneur—thus, some degree of “grit” could be crucial. Furthermore, we also incorporate in our model that traits leading to good entrepreneurial performance is always of importance for entrepreneurial survival.

The remainder of the paper is organized as follows. Section 2 reviews the related occupational choice models research. Section 3 presents a basic outline of our model’s static components. In Section 4, we introduce dynamics. Moreover, we discuss conditions of a transition between entrepreneurship and wage employment. In Section 5, we analyze the outcomes of the model by providing numerical simulations that allow us to evaluate properties of the whole system, given some plausible local conditions. Section 6 concludes. The Appendix provides the analytical background of our approach and some additional results.

2 Previous research

Currently, there are several threads of research on entrepreneurial processes. An entry into entrepreneurship is, first of all, an occupational choice. Earlier models analyze variables that drive this choice. Lucas (1978) emphasizes the entrepreneurial talent of individuals, Kihlstrom and Laffont (1979) highlight that entrepreneurs need to be more risk tolerant than workers, while Holmes and Schmitz (1990) point to the opportunity-seeking character of entrepreneurship. Lazear (2005) argues that as a consequence of personal preferences and talent, some people choose to be jacks of all trades, where others specialize, with jacks of all trades being more entrepreneurship prone. Roessler and Koellinger (2012) provide a refined explanation for the emergence of entrepreneurs by simultaneously determining occupational choice and job matching. They argue that those who are relatively unmanageable, while possibly being excellent managers (of themselves and others), become entrepreneurs—a first conclusion that will be incorporated in our model.
Reviewing these occupational choice models raises the question whether the personality of individuals influences variables driving occupational choices towards entrepreneurship. This leads to the second line of research that we are interested in. Psychological research claims that personality is, in general, an essential determinant of occupational choices (see Holland, 1997). Moreover, personality theory asserts that the influence of personality variables on entrepreneurial decisions is mediated by the strategies and goals of the decision maker (see, e.g., Baum and Locke, 2004).

There is also ongoing discussion with respect to the influence of personality traits on performance in various areas and on different levels. Including self-productivity, Cunha et al. (2005), for instance, develop a model depicting the development of abilities over the life cycle, from ability level at birth until later stages. Since Cunha et al. (2005) build the model with the goal of interpreting empirical evidence on skill formation, their approach is one way how personality traits may affect entrepreneurial performance.

Empirical evidence reveals that personality traits are important in describing entrepreneurship. The personality structure of entrepreneurs is distinct compared to that of managers, either when measured by the psychological toolbox of the Big Five personality construct (see Zhao and Seibert, 2006) or when measured by a specific set of personality characteristics (see Rauch and Frese, 2007). Research makes clear, however, that observing differences in personality characteristics between entrepreneurs and other population groups does not indicate that these traits influence entrepreneurial entry or success and that separating the influence of personality traits on different entrepreneurial decisions is relevant (see Caliendo et al., 2014). One key insight of the analysis of Caliendo et al. (2014) is that different traits affect entrepreneurial entry and survival—entry is influenced by traits like Openness to Experience, Extraversion, and Emotional Stability, while survival is affected by traits like Agreeableness or Conscientiousness (see, also, Ciavarella et al., 2004).

Since personality matters, the explanatory power of the personality approach is important. In this line, Rauch and Frese (2007) match traits to tasks of entrepreneurs. They report an effect size of about 25%. In the same vein, Caliendo et al. (2014) report that the explanatory power of all observed personality constructs amounts to 30% among all observable variables. This clarifies, and will be used in our model, that traits are important but not the main determinant of entrepreneurial entry and success, and that the human personality has some influence on entrepreneurial decisions, just not linearly.

The assumption that personality traits related to entrepreneurial performance are unknown to oneself—the central assumption of our model—deserves further attention and is the third line of research we would like to touch here. Unknown traits can be explained by a gap between self-perception and reality, i.e. a gap between self-perceived and real personality traits. Morin (2006) points out that some studies find a large gap between self-perception of traits and peer-reports on the same traits. For instance, Fiedler et al. (2004) let 1,080 military recruits self-report on their own traits and compared the results to peer-
reports. Correlations between self-reports and peer-reports were low and ranged between 0.13 and 0.24, allowing for the assumption that traits are (partially) unknown. There also exist empirical results showing that those who enter into entrepreneurship seem to make self-assessment errors of a certain type, which can best described as overconfidence (see Koellinger et al., 2007). The main result of Koellinger et al. (2007) is that individuals tend to rely on subjective perceptions (estimates) rather than on objective success probabilities and that if we relate perceptions to objective probabilities, we see a positive bias: Nascent entrepreneurs tend to overestimate their own success probability. Camerer and Lovallo (1999) establish a similar result in an experimental setting (a game of entry into a competitive market). They reveal that if subjects know that outcomes will depend on their own skills, excess entry is more pronounced than in other settings. Put differently, overconfidence is mostly a property of self-selected entry. In a survey-based approach, Busenitz and Barney (1997) show that entrepreneurs tend to have a stronger confidence bias than managers in large organizations, thereby providing additional support for the phenomenon of generally overconfident entrepreneurs. All these observations are consistent with a distorted self-perception detected by Fiedler et al. (2004). Yet, the direction of the bias of entrepreneurs (too optimistic self-assessment) has to be explained and is the third point of our model.

Not knowing about own entrepreneurship-related traits leads to the question of whether individuals learn about this influence over time and is the fourth line of research, addressed here. There are occupational choice models with a learning component but no model accounts for personality traits. MacDonald (1988) was the first to assert that there are two kinds of entrepreneurs with either low or high abilities. According to his model, individuals with high abilities remain in entrepreneurship, while low-ability individuals return to wage employment after some time. Astebro et al. (2011) explain entrepreneurship by the existence of frictions in the assignment of individuals to tasks in firms. They argue that inefficient assignments lead to entrepreneurship, while in case of frictionless assignments there is no need for entrepreneurship at all.

Another related theoretical approach to our model is Braguinsky et al. (2012). They assume a three-period learning framework. In the first period, individuals draw an entrepreneurial idea of a certain quality (from a distribution) but do not gain any additional information. In the second period, individuals receive a noisy signal and update the idea’s quality prior according to Bayesian rules. In the third period, individuals know the exact quality of

3 Overconfidence has positive and negative aspects. One of the most important positive aspects of overconfidence is that overconfident individuals generate information that would not be available without them (see Bernardo and Welch, 2001).

4 There are explanations for the direction of the bias in the literature. For instance, Gervais and Odean (2001) show that overconfidence can result from taking too much credit for successes, while underweighting failures. Such a mechanism relies on different treatments of different types of information. In our model, all types of information are treated the same way, while overconfidence is generated because some information is missing.
the entrepreneurial idea. Assessing the realism of their learning algorithm, Braguinsky et al. (2012, p. 873) state:

In reality, feedback about the value of entrepreneurial ideas typically comes in the form of noisy realizations, such as earned profits. Consequently, entrepreneurs never truly learn the value of their ideas but become better informed about them over time as feedback accumulates.

The main difference between our model—as we will show in the next sections—and the model of Braguinsky et al. (2012) is that we do not abstract from these two real-world conditions. Even more so, the assumption that feedback comes in the form of noisy earnings and that entrepreneurs never truly learn the value of their entrepreneurial idea but can accumulate feedback are fundamental to our model.

Our model is, thus, a synthesis of the above discussed four research lines allowing us to overcome difficulties associated with each line. We combine the occupational choice model of Roessler and Koellinger (2012) with a learning algorithm. For the learning part, we rely on Camerer and Ho (1999) and Ho et al. (2007), who model learning as the development of the attractiveness of a particular option over time. A slight modification of the Camerer-Ho learning algorithm allows us to include traits into the learning process. Moreover, we will discriminate between different sets of traits. The model structure we impose features traits in the strict sense and models basic components as influenced by traits.

The studies briefly reviewed in this section discuss several variables and constructs that may explain individuals’ occupational choice decisions to enter and to exit entrepreneurship. Among them, we observe that personality traits play a crucial, but not unique, role, with a non-linear influence on entry and exit processes. To capture these observations, we will construct a model where personality influences and interferes with occupational choice decisions through a set of intermediary variables.

A rigorous test of our model will be to what extent it is able to capture two further empirical observations: Firstly, focusing on exits from entrepreneurship, there is empirical evidence that entrepreneurial survival rates are only about 50% after five years (see, e.g., Helmers and Rogers, 2010). The highest exit rates are in the first year after entry into self-employment showing a relatively high rate of “revolving-door entrepreneurs,” while exit rates in the following years become lower (see Knaup, 2005; Knaup and Piazza, 2007; Georgarakos and Tatsiramos, 2009; or Evans and Leighton, 1989). Thus, firm survival is mostly threatened immediately after entry.

Parallel research analyzes the incomes of entrepreneurs. Several empirical studies find that median entrepreneurs earn less than median wage-employed (see Astebro and Chen, 2014, for an overview), repeatedly leading to the question, why do individuals remain in entrepreneurship if they could earn more in dependent employment? Thus, a model needs to explain the seemingly
absent positive return to entrepreneurial activities for a substantial share of entrepreneurs.\(^5\)

3 The model’s core

This section presents the assumptions of our endogenous occupational choice model, the earnings functions, and the model’s static core explaining how firms are organized. Additionally, the section analyzes the preferences of entrepreneurs with regard to the construction of their firms.

3.1 Basic assumptions

We assume that individuals cannot fully anticipate to what extent they will succeed as entrepreneurs, but they can make an informed guess and learn. Each newly experienced period will alter the attractiveness of their former choice and may present a previously discarded option in a new and more attractive light.

Formally, consider that there are \(1, \ldots, I\) individuals who can decide between entrepreneurship and employment (working under someone else’s supervision). The set of all individuals involved in the model is \(N\). Moreover, we assume that there is a second set of individuals \(N^c\) not involved in the model. Think of \(N^c\) as accessible historical data, which can be used as a reference group by each \(i \in N\). Every individual will decide whether she will start or abandon entrepreneurship in every period \(t > 0\) where \(t = 0, \ldots, T\). An action decided upon in one period will be executed in the next period.

Decisions on occupational status are assumed to be influenced by four forces: personality traits, skills, liability restrictions, and labor market interactions. All individuals are affected by the same restrictions. Personality traits are different for each individual, but we assume that traits and skills are interrelated. If we conceptualize entrepreneurship by arguing that individual \(i\) is a potential entrepreneur, this stands for a certain combination of personality traits and skills.\(^6\) The importance of labor market interactions is obvious. Some individuals may quit their job to start an own business, thereby causing a change in their former employer’s income. Some individuals may abandon

\(^{5}\) An option value argument, as developed by Vereshchagina and Hopenhayn (2009), might explain the absence of a risk premium for entrepreneurs. However, Vereshchagina and Hopenhayn (2009) assume that entrepreneurs are self-financed, which, in combination with the availability of an outside option different from entrepreneurship (wage work), creates an incentive for poor entrepreneurs to invest in risky projects. The two main differences between our model and an option value argument are: (i) We use a richer concept of personality and empirically tested learning heuristics. (ii) Borrowing constraints are endogenous in our model—entrepreneurs are financed by a financial institution unless their behavior forces the financial institution to withdraw.

\(^{6}\) See also Gifford (2010) for some interesting examples clarifying why certain traits beneficial for entrepreneurial activities are not sufficient to be a successful entrepreneur if they are not complemented by an appropriate set of skills.
entrepreneurship and be absorbed into the structure of existing firms having consequences for all individuals working in those firms.

In our model, we will assume that we are able to differentiate between different sets of personality traits. The first set consists of traits that are responsible for entrepreneurial performance, thus performance-related traits denoted by $\tau^p_i \in \Theta$ for all $i$. Second, we assume that there is a set of so-called adaptive-responsiveness traits. Responsiveness traits are given by $\tau^r_i \in \Theta$ for all $i$. The responsiveness trait’s main responsibility is to alter the intensity of new compared to old stimuli, for instance by processing new information based on the feedback from the market in differing ways. The third set of traits is related to self-confidence. This set of traits can rationalize the self-confidence level of an individual if she is faced with ambiguous signals. We denote them by $\tau^c_i \in \Theta$ for all $i$.

The crucial point is that entrepreneurs making this occupational choice for the first time are in an entirely new situation. They cannot anticipate the feedback of the market and their own reactions on that. To learn whether an individual is a successful entrepreneur, this individual has to become an entrepreneur. Hereby, traits play a major role. In this context, we assume that their own performance traits may be unknown to individuals. Unknown performance traits capture all non-cognitive characteristics determining entrepreneurial proficiency. Information on these performance traits is provided by the market. In other words: The market evaluates personal characteristics and reports them to the individual with some noise. We will refer to the noisy market signal as market feedback. Responsiveness and confidence traits are assumed to influence behavioral heuristics that cannot be modified by the individuals.

Performance, responsiveness, and confidence traits belong in one category of personality traits. The second category is composed of skills formalized by $s_i \in \Psi$ for all $i$ where $\Psi$ is some appropriate vector space. We assume that skills are common knowledge. Although skills can be approximated by formal degrees, etc., skills and traits are not necessarily two independent categories. We assume that skills correlate with some skill-related personality traits $\tau^s_i \in \Theta$ (the fourth set of personality traits) such that $s_i = s(\tau^s_i, x_i)$ for all $i$ where $x_i$ is a vector (from a suitable vector space) of non-trait-related determinants of skills.

We assume that traits are stable or time-invariant such that $\tau_{i,t} = \tau_i$ for each of the four trait sets. This assumption is consistent with empirical findings. For example, Cobb-Clark and Schurer (2012) argue that the Big Five traits of working-age adults are stable enough to include personality as a time-invariant input into economic models.

With respect to the “development” of skills, one empirically verified explanation, exemplifying how traits $\tau^s_i$ and skills $x_i$ interact to produce $s_i$, is the following. Given $x_i$, there exists a potential skill level $s^*_i$. Whether this potential skill level is actually realized in a given environment depends on $\tau^s_i$ in the sense that individuals may not realize their full potential because they can

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7 We assume that each set of traits can be measured through the same questionnaire $\Theta$.
8 See also Baum and Locke (2004) on the relationship between entrepreneurial traits and skill.
be inhibited by their personality. Our model’s incentive structure is implicit but the same for each individual. Two individuals \( i \) and \( j \) with \( x_i = x_j \) will have different skill levels if they react to the model’s incentives in different ways implying \( \tau^s_i \neq \tau^s_j \).

We should note that trait constructs within each set of traits may be the same, they may overlap, or could be distinct—this is an empirical question. If, for example, sets determining self-confidence and responsiveness have common elements, \( \tau^c \) and \( \tau^r \) will follow a joint distribution producing a correlation between the two sets of traits. If self-confidence and responsiveness have neither common nor related elements, \( \tau^c \) and \( \tau^r \) can be drawn from two independent distributions.

The variables \( \tau^p, \tau^r, \tau^c, \) and \( \tau^s \) cannot be used in the model in their given form. To operationalize them, we will define functions (rationalizing functions) with certain properties that assign each variable a certain space of action. Although we do not define explicit rationalizing functions, it should be possible to either find functions with required properties or to find proxies for the variables to conduct an econometric analysis. We assume, furthermore, that relations between traits carry over to model variables influenced by traits. For instance, if two trait variables are correlated, we expect a correlation between the outcomes of rationalizing functions associated with them.

Decision criteria are attractivenesses of options. The attractiveness of the entrepreneur option is given by \( A_i,t \) for all \( i \) and \( t \). By construction, \( A_i,0 \) will be different from \( A_i,t \) with \( t > 0 \) since \( i \) will gain experience once she is exposed to the market. The attractiveness of the best non-entrepreneurial option is \( \bar{A}_{i,t} \) for all \( i \) and \( t \). There is a difference, resulting from experience formation, between \( t = 0 \) and \( t > 0 \) again. We will treat \( A \) and \( \bar{A} \) as utility functions. \( i \) will choose entrepreneurship in \( t + 1 \) if and only if \( A_i,t \geq \bar{A}_{i,t-1} \). Otherwise, in case of \( A_i,t < \bar{A}_{i,t-1} \), \( i \) will choose to be a worker in \( t + 1 \). We denote the set of all entrepreneurs in period \( t \) by \( E(t) = \{i | A_i,t-1 \geq \bar{A}_{i,t-1}\} \) and the set of all workers in the same period by \( W(t) = \{i | A_i,t-1 < \bar{A}_{i,t-1}\} \).

Entrepreneurs expect that they can sell any amount of their good or service at a price normalized to 1. Actual revenue can vary from period to period but is assumed to provide some useful information. Firms created by entrepreneurs can consist of many individuals but solo entrepreneurs (the only employee of a firm is the entrepreneur) are allowed as well.

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9 This postulated mechanism is consistent with experimental findings of, for instance, Borghans et al. (2006), who reveal that it depends on personality traits how students—taking a cognitive test—react to a change in rewards. Faced with larger rewards, students with personality traits that lead to a high motivation did not increase their performance as much as students with a different set of traits.

10 If profit levels would not provide any useful information on entrepreneurial fitness, it becomes difficult to construct an argument pointing out why they should not be ignored altogether in attractivity development.
The basic components of the model are conditional productivities. In accordance with assumptions of Roessler and Koellinger (2012), we understand a conditional productivity as the productivity of an individual (say \( j \)) if she is supervised by another individual (say \( i \)). Formally, the productivity of \( j \) under \( i \)’s management is given by \( v_{ij} \) for all \( i \) and \( j \) where \( i \neq j \). \( v_{ij} \) for \( i \neq j \) is assumed to depend on some interaction of skills \( s_i \) and \( s_j \).\(^{11}\) Because \( s_i \) is common knowledge, \( v_{ij} \) for all \( i \) and \( j \) where \( i \neq j \) is known to all individuals too.

\( v_{ij} \) for all \( i \) and \( j \) where \( i \neq j \) collects all productivities based on a collaboration of two individuals. Non-collaborative productivities, referred to as self-management productivities, are accounted for by \( v_{ii} \) for all \( i \). \( v_{ii} \) depends on skills \( s_i \) and on performance traits \( \tau_i^p \). Notice that, in line with the observation that personality does not play the main role, self-management productivities are only codetermined by trait set \( \tau_i^p \). We assume that two-person collaborations are the maximal degree of immediate mutual work interactions—individual \( i \) can collaborate with \( j \) and \( k \), but then \( j \) and \( k \) do not directly collaborate. Hence, the whole productivity system is described by \( v_{ij}(s_i, s_j) \) for all \( i \) and \( j \) where \( i \neq j \) and \( v_{ii}(s_i, \tau_i^p) \) for all \( i \). We can write the system of inter-dependent and self-dependent productivities as a matrix

\[
\Gamma = \begin{bmatrix}
v_{11}(s_1, \tau_1^p) & v_{12}(s_1, s_2) & v_{13}(s_1, s_3) & \cdots & v_{1I}(s_1, s_I) \\
v_{21}(s_2, s_1) & v_{22}(s_2, \tau_2^p) & v_{23}(s_2, s_3) & \cdots & v_{2I}(s_2, s_I) \\
v_{31}(s_3, s_1) & v_{32}(s_3, s_2) & v_{33}(s_3, \tau_3^p) & \cdots & v_{3I}(s_3, s_I) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
v_{I1}(s_I, s_1) & v_{I2}(s_I, s_2) & v_{I3}(s_I, s_3) & \cdots & v_{II}(s_I, \tau_I^p)
\end{bmatrix}
\] (1)

\( \Gamma \) can be referred to as the true productivity matrix.

Following our main assumption, we argue that \( \Gamma \) is partially not known.\(^{12}\) By definition, individuals lack knowledge of performance traits \( \tau_i^p \). As a consequence, they will be forced to construct an estimate for \( v_{ii} \) or \( \tau_i^p \). A reasonable assumption for these estimates is that each \( i \in N \) uses historical data \( N^c \). Assume that \( i \in N \) can find some historical entrepreneurs—only entrepreneurs can reliably estimate their self-management productivity having been exposed to the market—having a similar skill level, \( i.e. \ i \in N \) finds a reference group \( R_i = \{v_{jj}|d(s_i, s_j) \leq \varepsilon \text{ for all } j \in E^c\} \) where \( \varepsilon \) is a similarity measure and

\(^{11}\) As suggested by Roessler and Koellinger (2012), one example could be the professional distance between both individuals \( d(s_i, s_j) \) where \( d(\cdot) \) is some measure of distance. The cooperation of two engineers \( i \) and \( j \) is more productive than the cooperation of an engineer \( i \) and a philologist \( k \), which is reflected by \( d(s_i, s_j) < d(s_i, s_k) \) and results in \( v_{ij}(s_i, s_j) > v_{ik}(s_i, s_k) \).

\(^{12}\) In our model, we make the simplifying assumption that individuals have complete information about \( v_{ij} \) (in contrast to \( v_{ii} \)). This assumption can be justified by arguing that the productivity of employees can be approximated by the level of their formal education level—information that is common knowledge.
$E^c \subseteq N^c$ are historical entrepreneurs.\footnote{See Bosma et al. (2012) and Chlosta et al. (2012) for the impact of reference groups (role models) on entrepreneurial decision making.} Then, $i \in N$ can estimate her fitness with $\hat{v}_{ii} = (\sum_{j \in R_i} \hat{v}_{jj}) / |R_i|$.

Thus, we assume that individuals use reference groups to construct

$$\hat{\Gamma} = \begin{bmatrix}
\hat{v}_{11} & v_{12} & v_{13} & \cdots & v_{1I} \\
v_{21} & \hat{v}_{22} & v_{23} & \cdots & v_{2I} \\
v_{31} & v_{32} & \hat{v}_{33} & \cdots & v_{3I} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
v_{I1} & v_{I2} & v_{I3} & \cdots & \hat{v}_{II}
\end{bmatrix}$$

which is the matrix of estimated self-management and known conditional productivities. Thus, individuals assume a certain value for their self-management productivity and make decisions based on this assumption. Estimates $\hat{v}_{ii}$ depend on skill-related traits because these traits influence skills and skills are used to construct reference groups. Due to the fact that data from $E^c$ are freely accessible and the distance constituting a reference group is common, a reported $\hat{v}_{ii}$ can be validated by each $j \neq i$. Effectively, $\hat{\Gamma}$ is common knowledge.

Similar to Roessler and Koellinger (2012), we require $v_{ij} \in \mathbb{R}^+$ for all $i$ and $j$ where $i \neq j$, i.e. any cooperation of two individuals will produce a positive amount of goods. In contrast to Roessler and Koellinger (2012), we do not require $v_{ii} \in \mathbb{R}^+$ for all $i$ but only $\hat{v}_{ii} \in \mathbb{R}$ for all $i$. The reason is that there is an important difference between the two tasks associated with $v_{ij}$ and $v_{ii}$, namely being in supervised employment or in entrepreneurship with the entrepreneur being their own boss without supervision.

We further define that an individual $i$ with $v_{ii} \in \mathbb{R}^-$ is completely unfit for entrepreneurship. Let $U = \{i|v_{ij} < 0\}$ be the set of such individuals. In contrast, individuals in $\hat{U} = \{i|\hat{v}_{ii} < 0\}$ consider themselves as completely unfit, which does not mean that they are unfit, since $v_{ij} > \hat{v}_{jj}$ for all $i \in \hat{U}$ and a fixed $j \neq i$. 

Rewrite $\hat{\Gamma}$ as

$$\hat{\Gamma} = [ \begin{array}{cccc}
v_1 & v_2 & \cdots & v_{I-1} \\
v & \end{array} ]$$

where $v$ is a productivity column vector of individual $i$ under different managers. Assume that each vector $v_i$ allows us to create a distinct ranking $v_{(1)i} > v_{(2)i} > \cdots > v_{(I)i}$ where $\{1\}i$ is the best manager for $i$, $\{2\}i$ is the second-best manager for $i$, and so on. (Note that this ranking refers to $\hat{\Gamma}$ and not $\Gamma$.)

It is necessary to impose the following restriction on rankings:

**Static non-circularity assumption (SNCA).** Suppose that for a subset of $N$ we can assign ranks from 1 to $n$ such that $v_{12} > \hat{v}_{22}, v_{23} > \hat{v}_{33}, v_{34} > \hat{v}_{44}, \ldots, v_{[n-1]n} > \hat{v}_{nn}$. Given the previous ranking, $v_{n1} < \hat{v}_{11}$ must hold. (Since our model’s players act according to $\hat{\Gamma}$, SNCA does not need to hold for true conditional productivities represented by $\Gamma$.)
SNCA makes management transitive.\textsuperscript{14} We construct equilibrium firm organization in an intuitive way.\textsuperscript{15} First, assume that individual utility functions are increasing in income. Second, assume that managerial assignments are hierarchical such that each individual will be managed by one another individual unless she is an entrepreneur. An entrepreneur is managed by herself per definition. Firms consist of firm units of optimally assigned individuals.\textsuperscript{16} A unit $\mathcal{U}_i^{(0)} = \{ j \mid v_{ij} > v_{kj} \text{ for all } j \text{ and } j, k \neq i \}$ will collect all individuals whose best manager is $i$ (unless $i$ is an entrepreneur). Note that if $i$ is not the best manager of anyone, we have $\mathcal{U}_i^{(0)} = \emptyset$. Units are unique, $\mathcal{U}_i^{(0)} \cap \mathcal{U}_j^{(0)} = \emptyset$ for $i \neq j$, since there is only one best manager for each individual. A potential entrepreneur is not part of any firm unit and believes to be her own best manager. We assume that such an individual exists, i.e. $\exists i \text{ s.t. } \hat{v}_{ii} > v_{ji}$ for all $j \neq i$.

Entrepreneurs are indifferent between receiving their income in wages and profit.\textsuperscript{17} We can equate entrepreneurial income with the sum of profits and wages entrepreneurs pay to themselves. Let $F_i^{(0)}$ denote a set containing all individuals in the firm of $i$ (firms are named after entrepreneurs). Thus, if we ignore dynamics, the estimated entrepreneurial income of $i$ is given by

\begin{equation}
\hat{\pi}_{i,0} = v_{(1)i} + \sum_{j \in F_i^{(0)} \setminus i} v_{(1)j} - \sum_{j \in F_i^{(0)} \setminus i} w_j
\end{equation}

where $v_{(1)i} = \hat{v}_{ii}$ and $w_j \ (j \neq i)$ is a worker’s wage. Eq. (3) is formalized as an estimated income, as we only have estimates for $v_{ij}$. If $i$ acts according to her estimated income, her deterministic income (if it is noise-free) is

\begin{equation}
\pi_{i,0} = v_{ii} + \sum_{j \in F_i^{(0)} \setminus i} v_{(1)j} - \sum_{j \in F_i^{(0)} \setminus i} w_j
\end{equation}

where we replace the estimated value $\hat{v}_{ii}$ by its actual value $v_{ii}$, which is unknown to $i$. The output that can be unequivocally attributed to entrepreneur $i$ is $i$’s self-management productivity and $i$’s managerial advantage over the second-

\textsuperscript{14} For instance, if 1 is the best manager of 2, 2 is the best manager of 3, and 3 is the best manager of 1, we will not be able to construct a proper firm. To do this, we, for example, can rule out the last relation “3 is the best manager of 1,” which is done by SNCA for arbitrary long chains.

\textsuperscript{15} A formal proof is provided by Roessler and Koellinger (2012). The only difference is that instead of $\Gamma$ Roessler and Koellinger (2012) work with $\Gamma$. Since our assumptions on $\Gamma$ are equivalent to Roessler and Koellinger (2012) assumptions on $\Gamma$, there is no need to reproduce the proof in detail. The game is a normal-form game (actions are simultaneous) with public information on productivities and ultimatum wage offers where the solution is a Nash-equilibrium satisfying hierarchical assignments.

\textsuperscript{16} Roessler and Koellinger (2012) suggest the following algorithm to solve for equilibrium firms: Define a function $f^{(0)} : N \rightarrow N$ mapping an individual to her best manager ($f^{(0)}(i) = j$ if $v_{ij} > v_{kj}$ for all $k \neq i$). Define $f^{(1)}(i) = f^{(0)}(f^{(0)}(i))$ and iterate. $f^{(0)}(i)$ will assign the best manager to $i$, $f^{(1)}(i)$ will assign the best manager to $i$’s best manager, etc. Stop if $f^{(l+1)}(i) = f^{(l)}(i) \equiv f^{(\infty)}(i)$. Since $l < \infty$ and SNCA holds, the algorithm will stop after a finite number of iterations. $f^{(\infty)}(i)$ is an entrepreneur since $f^{(l+1)}(i) = f^{(l)}(i) = j$ only if $j$ is her own best manager.

\textsuperscript{17} Thus, we abstract in our model from non-monetary utilities, also associated with entrepreneurial choices (see, e.g., Benz and Frey, 2008).
best managers of individuals in the unit of $i$. Roessler and Koellinger (2012) demonstrate that this is indeed the unique Nash-equilibrium outcome given hierarchical assignments. It follows that Eq. (3) can be written as

$$\hat{\pi}_{i,0} = v_{\{1\}i} + \sum_{j \in \mathcal{U}_i^{(0)}} v_{\{1\}j} - v_{\{2\}j}$$

(5)

Again, $\hat{v}_{ii}$ can be replaced by $v_{ii}$ to get true earnings.

There is no unique outcome for wages since entrepreneurs are indifferent between different schemes that do not change profits (see Roessler and Koellinger, 2012). The indirect contribution of a worker $i$ to a firm $j$ consists of her managerial advantage over the second-best manager of individuals in her unit. However, units can be moved and it is not enough to reward $i$ for her managerial advantage only. If $i$ is in an optimally constructed firm, her direct contribution to output is $v_{\{1\}i}$. For a firm without $i$’s first-best manager, the direct contribution is $v_{\{n\neq1\}i} < v_{\{1\}i}$, while the managerial advantage remains the same if we move $i$ with her unit $\mathcal{U}_i^{(0)}$.

Consider the following wage offer:

$$w_{i,0} = v_{\{2\}i} + \sum_{j \in \mathcal{U}_i^{(0)}} v_{\{1\}j} - v_{\{2\}j}$$

(6)

The intuition behind Eq. (6) is that $i$’s net-contribution (the sum of direct and indirect contributions minus $i$’s wage given by Eq. [6]) to a firm with her first-best manager is $v_{\{1\}i} - v_{\{2\}i} > 0$, while her net-contribution to a firm without her first-best manager is $v_{\{n\neq1\}i} - v_{\{2\}i} \leq 0$. Only the firm with $i$’s second-best manager will be willing to match the offer in Eq. (6) since $v_{\{n\neq1\}i} - v_{\{2\}i} < 0$ for $n \neq 1, 2$ (and profit from employing $i$ is negative). All firms $j$ where $\{1\}i, \{2\}i \notin \mathcal{E}_j^{(0)}$ will offer less than $w_{i,0}$. Hence, $i$ gets the same highest offer twice—one from the firm with her first-best manager and one from the firm with her second-best manager—and chooses the firm with her first-best manager by assumption. Roessler and Koellinger (2012) demonstrate that Eq. (6) constitutes one, but not a unique, Nash-equilibrium, which will be used in further calculations.

Entrepreneurs might be indifferent between different firm structures if restructuring does not change entrepreneurial incomes. It can be shown that entrepreneurs might be indifferent toward the employment status of lower hierarchy employees.

**Indifference toward employment status of lower hierarchy workers.** Entrepreneurs are indifferent toward the existence of workers who are not in their own unit. Or: An entrepreneur is indifferent between different firm structures as long as her own unit is left untouched since only the unit of entrepreneurs has an effect on entrepreneurial incomes. (See Appendix A for a demonstration.)

In the dynamic setting, we will allow for firm restructurings resulting from the fact that some workers can become entrepreneurs or some entrepreneurs become workers (they abandon entrepreneurship) such that units can change.
Restructurings can affect units low in firm hierarchy. According to the property above, entrepreneurs do not necessarily have an incentive to fully compensate the manager of a unit low in firm hierarchy for a change in unit structure if her unit is affected by restructurings. This will result in a wage regime where wages of managers low in firm hierarchy are not increased although there is an increase in management tasks because more individuals are to be managed.

This section introduced the core of our model. Based on conditional productivities (the productivity of an individual managed by another individual), we derived static wages and (expected) entrepreneurial incomes. In addition, we emphasized a crucial result: For an entrepreneur, lower-hierarchy workers do not play a significant role. This result will be important for wage dynamics.

4 Entry, survival, and exit decisions

This section introduces the dynamic analysis of our model by describing the decision process of workers and entrepreneurs in time. The decision process we implement in the model accounts for the fact that personality traits are a component of entrepreneurial decision making, but are not the main determinant of the process. Therefore, we further incorporate a number of other variables not related to traits, such as costs and a budget constraint.

The section is structured as follows. We begin by stating two assumptions that close model gaps and insert liability constraints. Then, we model firm restructuring capturing the entry decision into and the exit decision from entrepreneurship. After explaining firm restructuring, we discuss possible wage regimes and decision dynamics.

4.1 Assumptions in the dynamic setting

4.1.1 Wage structure and dealing with missing information

Before we are able to start analyzing entry and exit (survival) decisions in the setting of firm restructuring processes, we make two assumptions telling us how to compute wages per period and how to handle missing information.

**Binding-wage-structure assumption (BWSA).** *Wages can be renegotiated in every period if the entrepreneur chooses to do so. However, if wages in \( t = 0 \) are structured according to a certain rule determining how to allocate firm income between workers and the entrepreneur, this structure must not be altered in \( t > 0 \).*

BWSA allows us to use wage equations already defined without the need to determine wage equilibria in every period. One way to explain BWSA is to argue that wage renegotiations are costly and the entrepreneur may choose to keep the wage of her workers constant if this does not hurt the entrepreneur’s income. For reasons of simplicity, we assume that if an individual applies for a job in period \( t > 0 \), she gets an updated wage.

Next, we need to stipulate how to handle missing information.
Ignorance-of-hidden-obstacles principle (IHOP). *An individual who decides on entrepreneurship will ignore future obstacles if these obstacles are not fully transparent to her. Obstacles occurring in the course of entrepreneurship will be dealt with when individuals are facing them.*

IHOP is a weak form of Hirschman’s Hiding-hand principle. According to Hirschman (1967, p. 13):

We would not consciously engage upon tasks whose success clearly requires that creativity be forthcoming. Hence, the only way in which we can bring our creative resources fully into play is by misjudging the nature of the task, by presenting it to ourselves as more routine, simple, undemanding of genuine creativity than it will turn out to be.

Thus, IHOP is a weaker approach than the Hiding-hand principle, as it does not assume any kind of hidden rationality (ignorance as a necessary condition for success). According to IHOP, we just ignore the non-obvious.

4.1.2 Liability constraints

So far, model individuals who start their firms could accumulate losses without any restrictions. To replace this unrealistic assumption, we assume that individuals face liability constraints. Therefore, if an individual is willing to start a firm, the financial institution in the system can provide capital, which is used as a safety buffer. Thus, to exclude unrestricted losses, we assume that according to regulatory rules an entrepreneur cannot operate without capital. There exists an entity that regulates entrepreneurial entry in a minimalist manner. It permits entry if the financial institution reports a capital safety buffer and denies entry if not.

Given IHOP, also applying to the financial institution, the institution computes the safety buffer according to

$$B_{i,t_A} = \frac{\beta(\beta_{i,t_A} - \beta^T)}{1 - \beta} \times \frac{c\hat{\pi}_{i,t_A}}{1 + m}$$  \hspace{1cm} (7)$$

where $t_{i,t_A} \in [0, T]$ is the period in which $i$ applies for capital ($i$ becomes entrepreneur in period $t_{i,t_A} + 1$). $m > 0$ is the financial institution’s mark-up to diversify risk and establish profit (computed according to the institution’s criteria), $\beta \in (0, 1)$ is a discount factor\(^{18}\), and $c \in (0, 1)$ is the share of income that the entrepreneur agrees to transfer to the financial institution per period. The safety buffer is positive in expected entrepreneur’s income of $i$. $i$ has no incentive to misreport $\hat{\pi}_{i,t_A}$ since the financial institution can verify it by combining information on matrix $\hat{\Gamma}$ and data on past restructurings.\(^{19}\)

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\(^{18}\) The discount factor may be linked to interest. It also can represent the average probability of entrepreneurial survival over one period, which is derived from some set of information by the financial institution. In the second case, financial institution and individual should not use the same discount factor.

\(^{19}\) Assume that the financial institution verifies the information with probability 1 and that verification costs are included in $c$.  

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The safety buffer is negative in the mark-up, positive in the income share, positive in the discount factor, and negative in the application period since the financial institution receives payments for $T-t_i$ periods only. The financial institution fixes income shares and mark-up.

The rules of the financial institution are:

#1 Entrepreneurs are committed to transfer a constant income share of $c$ to the bank in every period of entrepreneurship.

#2 Capital must not be consumed.

#3 If an entrepreneur exits, capital is returned to the financial institution.

#4 If losses occur, a share $c$ is covered by the financial institution but the financial institution immediately subtracts it from the safety buffer. The rest $1-c$ has to be covered by the entrepreneur.

#5 If capital becomes non-positive, the entrepreneur is forced by the financial institution to exit.

#6 In case of a forced exit of an entrepreneur, the financial institution will not grant another safety buffer.

#7 If an entrepreneur exits voluntarily, the financial institution will grant another safety buffer, which is computed according to Eq. (7).

To exclude different fraud schemes, we assume that the financial institution is able to enforce all rules #1–7.

4.2 Firm restructuring

A restructuring takes place if some individual leaves a firm to start her own company or some individual exits entrepreneurship and integrates into the firm of another individual. Call an individual who leaves a ‘nascent entrepreneur’ and an individual who integrates into another individual’s firm an ‘applicant.’

Leaving from and applying to a firm are possible in periods $t > 1$. Before departure, a nascent entrepreneur $j$ is part of a firm and correspondingly part of the unit of some $i \neq j$. Thus, we can define a set of nascent entrepreneurs depending on the unit they are leaving from and period of departure as $D_i(t)$. If $j$ leaves in period $t$ from the unit of $i$, we state that $j \in D_i(t)$. An equivalent holds for applicants. An applicant applies to a specific unit in the firm. We

Since we allow for firm restructuring (firm structure in period $t > 1$ can be different from the structure in $t = 1$), we have to make sure that restructuring does not have unintended side effects. First, we assume that there exists one firm managed by $i^* \in N$ that always survives. Second, we ensure that an alternative to entrepreneurship always exists by assuming that the second-best manager of entrepreneur $j$ is never part of $j$’s company, i.e., $v_{(2)}j \notin F_{j}^{(t)}$ for all $j$ and $t$. Note that the last assumption is just a dynamic extension of SNCA.
assume that she is immediately integrated into the unit if she chooses to apply. The set of applicants for the unit of \( i \) in period \( t \) is given by \( A_i(t) \).

The structure in period \( t = 1 \) is set in period \( t = 0 \) according to matrix \( \hat{\Gamma} \) and other variables like costs of entrepreneurship. Thus, we have a starting value \( U_i(0) \) for all \( i \). For the next periods, we have \( U_i(1) = U_i(0) \) for all \( i \) because decisions in \( t \) determine actions in \( t + 1 \). Furthermore, we have

\[
U_i(t) = \{ U_i(t-1) \cup A_i(t) \} \setminus D_i(t) \quad (8)
\]

for all \( i \) and all \( t > 1 \). Firms in period \( t \), \( F_i(t) \) for \( i \in N \), can be constructed using restructured units. An example is provided in Fig. 1 where we see two consecutive periods \( T \) and \( T + 1 \). In period \( T \), only one firm exists (the firm of individual 1). In period \( T + 1 \), individual 6 decides to become entrepreneur. Because 6 is the first-best manager of 7, she will take 7 with her. 3’s unit is affected by individual 6 who is excluded from \( U_3(T) \). The case of applicants works vice versa (by interpreting Fig. 1 from right to left).

Restructuring will have effects for workers and entrepreneurs. If \( i \) is a worker, her actual wage in period \( t \) is given by

\[
w_{i,t} = v_{\{2\}i} + \sum_{j \in U_i(t)} \left[ (v_{\{1\}j} - v_{\{2\}j}) \mathbb{I}_{j,1}^w + (v_{\{2\}j} - v_{\{3\}j})(1 - \mathbb{I}_{j,1}^w) \right] \quad (9)
\]

where \( \mathbb{I}_{j,1}^w = 1 \) if \( j \in W(1) \) and \( \mathbb{I}_{j,1}^w = 0 \) else. The indicator function is essential since a necessary condition for \( j \in E(1) \) is \( v_{\{1\}j} = \hat{v}_{jj} \). If \( j \) entered entrepreneurship in \( t = 1 \), she considers herself as her first-best manager. If \( j \) abandons entrepreneurship, she does not consider herself as her first-best manager anymore.

![Figure 1. Restructuring example](image-url)
4.3 Wage regimes

When describing the model’s core, we mentioned that entrepreneurs are indifferent between different firm structures as long as their own unit is left untouched. This result will be used now to construct a wage regime where some employees are denied opportunities and might decide to become entrepreneur as a consequence.

Notice that wages can be written as

\[ w_{i,t} = w_{i,0} + \Delta w_i(t) \]  

where \( \Delta w_i(t) = \sum_{k=1}^{t} \Delta w_{i,k} \) and \( \Delta w_{i,k} = w_{i,k} - w_{i,k-1} \) is the wage difference between two consecutive periods. \( \Delta w_{i,t} \) is determined by the behavior of \( i \)'s unit. Potential entrepreneurial incomes can be written in a similar way such that

\[ \hat{\pi}_{i,t} = \hat{\pi}_{i,0} + \Delta \pi_i(t) \]

where \( \Delta \pi_i(t) = \sum_{k=1}^{t} \Delta \pi_{i,k} \) and \( \Delta \pi_{i,k} = \pi_{i,k} - \pi_{i,k-1} \) is the income difference between two consecutive periods.

Given the equilibrium outcome, income changes of \( i \) depend on the behavior of \( i \)'s unit. Assume that a former entrepreneur is integrated into the firm of \( i \)'s entrepreneur such that \( i \) is the new manager of the former entrepreneur, \textit{i.e.} \( i \) has to perform additional work. Given such a situation, \( i \)'s entrepreneur might renegotiate wages but there are also reasons not to renegotiate.

If \( i \)'s entrepreneur compensates \( i \) for additional work, \( i \) is not denied any opportunities. Moreover, \( i \) would always compensate herself for additional work if she was an entrepreneur. Hence if wages are always renegotiated, we have \( \Delta w_{i,k} = \Delta \pi_{i,k} \), \textit{i.e.} \( i \)'s entrepreneur is willing to pay \( i \) the income change that \( i \) would pay to herself if \( i \) would be an entrepreneur. Yet, \( i \)'s entrepreneur might select the following wage regime:

**Wages bounded from above.** Entrepreneurs fully renegotiate wages with their own units such that wages are cut if there is less managerial work and increased if there is more. Entrepreneurs cut wages in all other units in their companies if necessary. However, entrepreneurs do not raise wages in units not managed by themselves.

The main consequence of a regime with wages bounded from above is that a worker might see an opportunity to increase her income by becoming entrepreneur. It is clear that an entrepreneur wants to keep the employees in her own unit since entrepreneurial income would be reduced if someone leaves this unit. Thus, entrepreneurs always fully adjust the wages of their units such that workers in these units will have no reason to start an own business. If someone from a non-entrepreneurial unit leaves the company, then firm output is reduced. Thus, entrepreneurs will have to reduce the wage bill. Application (entry into some unit) increases output. If an applicant applies to the entrepreneur's unit, the entrepreneur automatically increases her income. Entry into a non-entrepreneurial unit increases output too. Yet, entrepreneurial income does not increase if the entrepreneur fully rewards additional managerial effort by the
individual who manages the applicant. This hints at a strong incentive to not raise wages contingent upon entry into a non-entrepreneurial unit.

Let $i$ be a manager who is not managed by the entrepreneur and whose wage is not increased although there is an additional employee to be managed (due to entry into $i$’s unit). $i$ has two options: She can either stay or start an own firm. However, for $i$’s entrepreneur it does not matter whether $i$ stays or leaves. What is more, $i$’s entrepreneur can generate a surplus before $i$ departs because $i$’s wage was not increased. Hence, there is an opportunity\textsuperscript{21} opening up to $i$ that she can only exploit as an entrepreneur since $i$ would always compensate herself for additional work if she would be an entrepreneur.\textsuperscript{22} The withholding of opportunities can, for instance, be rationalized by wage adjustment costs the entrepreneur wants to avoid.

4.4 Initial period decision

In this section, we apply our framework to the initial period and corresponding choices. Individuals in the initial period suffer from a lack of information. Given IHOP, we can ignore everything that is not obvious and are given two sources of information: a wage offer and an estimated entrepreneurial income based on an estimate of self-management productivities.

Individuals will discount future incomes by some factor. Assume that they use the same factor $\beta$ as the financial institution. To decide which occupation to select, individuals are assumed to project income streams for each occupation. Projected income streams determine the attractiveness of an option in period 0. The income stream of wage work is

$$A_{i,0} = \sum_{t=1}^{T} \beta^t w_{i,0} = \tilde{\beta} w_{i,0}$$

(12)

where $\tilde{\beta} \equiv \beta^{(1-\beta^T)/(1-\beta)} \approx \beta/(1-\beta)$ for a large $T$. Similarly, the income stream of the entrepreneur status is

$$A_{i,0} = \sum_{t=1}^{T} \beta^t [1-c] \pi_{i,0} = \tilde{\beta}[1-c] \pi_{i,0}$$

(13)

\textsuperscript{21} In the empirical literature on entrepreneurship (see, for instance, Maritz, 2004; Harding et al., 2006; or Caliendo and Kritikos, 2009) there is usually a differentiation between entrepreneurs who enter because they see an opportunity or who enter because of necessity (a lack of other opportunities than entrepreneurship). The bounded wage regime produces opportunity entrepreneurs. Necessity entrepreneurs are a subgroup of the model’s set of entrepreneurs who enter because they perceive their self-management productivity as the productivity generating the best outcome.

\textsuperscript{22} Formally, we have $\Delta w_{i,k} \neq \Delta \pi_{i,k}$ since $\Delta w_{i,k} \leq 0$. More importantly, we have $\Delta \pi_{i,k} \geq \Delta w_{i,k}$, i.e. $i$’s entrepreneur is not willing to pay $i$ the income change which $i$ would pay to herself if $i$ would be an entrepreneur.
If the discount factor is zero, individuals would be indifferent toward the amount of future incomes and, consequently, toward the source of it.

Conditional on a positive discount factor, individuals will select entrepreneurship if and only if

\[ \hat{\pi}_{i,0} \geq \frac{w_{i,0}}{1 - c} \]  

(14)

Those who consider themselves as unfit for entrepreneurship, \( i \in \hat{U} \), will never select entering entrepreneurship because their wage is always larger than the estimated entrepreneurial income. Eq. (14) implies that there must be a critical value for estimated self-management productivities given by

\[ \hat{v}_{ii}^* = \frac{1}{1 - c} v_{(1)} - v_{(2)} + c \sum_{j \in \mathcal{U}_i^{(0)}} [v_{(1)}j - v_{(2)}j] \]  

(15)

where Eq. (14) will hold if \( \hat{v}_{ij} \geq \hat{v}_{ii}^* \). Critical self-productivities are positive in all three variables: costs \( c \), the productivity under the second-best manager \( v_{(2)i} \), and cumulated managerial advantage of \( i \) over the second-best managers of her unit \( \sum_{j \in \mathcal{U}_i^{(0)}} [v_{(1)}j - v_{(2)}j] \).

Thus, a necessary condition for entrepreneurship is \( v_{(1)i} = \hat{v}_{ii} \); the estimated self-management productivity has to be the largest conditional productivity. If the estimated self-management productivity is large enough to offset costs, the sufficient condition for starting up a firm is fulfilled. Or, if \( v_{(1)i} = \hat{v}_{ii}, \) \( i \) is a potential entrepreneur. Whether she becomes an entrepreneur depends on the constraints of the financial institution, i.e. costs \( c \). Personality sets the necessary condition, while costs determine when the sufficient condition is fulfilled.

The decision to become entrepreneur can be the result of at least two combinations of levels of productivity under the second-best manager and managerial advantage. (i) For example, individual \( i \) could assume (estimate) that her self-management productivity is very high. Then, \( i \) would become entrepreneur even if her productivity under the second-best manager and managerial advantage are also very high. In other words, individual \( i \) may assume that she is a natural-born entrepreneur. (ii) Individual \( i \) may have a very low productivity under her second-best manager and a very low managerial advantage. Then, \( i \) can become entrepreneur even if her estimated self-management productivity is very low. In case of a natural-born entrepreneur, \( i \) sees an opportunity to generate a high income in entrepreneurship. Given the second case, \( i \) selects entrepreneurship because the alternative of wage work is not very promising. We interpret the first case (natural-born entrepreneurs) as opportunity entrepreneurship and the second case as necessity entrepreneurship. Note that entrepreneurs starting a business in period 1 have neither experience with wage work nor with entrepreneurship. They are entrepreneurs without any work-experience.
Decisions influenced by experience

So far, decisions were based on deterministic information or on estimated self-management productivities while no learning took place. When introducing learning, we argue that information is provided by the market in form of stochastic feedback where individuals are assumed to internalize this kind of feedback to some extent. Dynamic decisions are driven by attractivities of options. Attractivities develop according to the scheme laid out by Camerer and Ho (1999) and are self-enforcing such that the previous attractivity of an option influences today’s attractivity of the same option. First, we will introduce feedback. Second, we characterize attractivity dynamics.

4.5.1 Market feedback

We assume that the market is correct in expectations but reports with noise. Denote market feedback in period $t$ of individual $i$ by $f_{i,t}$. Under our assumption, $E[f_{i,t}] = \pi_{i,t}$. One way to satisfy this assumption is $f_{i,t} = \pi_{i,t} + u_{i,t}$ where i.i.d. $u_{i,t} \sim (0,\sigma^2_{u,t})$ and $\sigma^2_{u,t} = \hat{u}|v_{ii}|$ such that the feedback varies by some fraction $\hat{u} > 0$ of $i$’s true productivity.

Individuals know that the entrepreneurial income they receive is influenced by some random factors that are not based on their performance. To this extent, they will not fully internalize market feedback but they should allow the market feedback to correct their initial estimate of their own performance. Internalized entrepreneur’s income of $i$, denoted by $\vartheta_{i,t}$, is given by

$$\vartheta_{i,t} = \omega(\tau^c_i)\tilde{\pi}_{i,t} + \left[1 - \omega(\tau^c_i)\right]\left[\pi_{i,t} + u_{i,t}\right]$$  \hspace{1cm} (16)

where $\omega : \Theta \rightarrow (0,1)$. Given individual confidence traits, only the feedback portion $1 - \omega_i$, where $\omega(\tau^c_i) = \omega_i \in (0,1)$, is internalized depending on how confident $i$ is about her initial estimate. Taking expectations yields

$$E[\vartheta_{i,t}] = \pi_{i,0} + t\tilde{\pi}_{i} + \omega_i[\tilde{\pi}_{i,0} - \pi_{i,0}] = \pi_{i,0} + t\tilde{\pi}_{i} + \omega_i[\hat{v}_{ii} - v_{ii}]$$  \hspace{1cm} (17)

where $\hat{v}_{ii} - v_{ii}$ is the initial estimation error. The size of the correction depends on $\omega_i$. $\omega_i = 0$ (no self-confidence) results in a full correction in expectations.

Introducing variance in the willingness to correct estimates needs a short discussion. From our point of view, the most plausible argument, for not fully correcting the initial estimate, is that the self-confidence parameter $\omega$ could reflect some construct related to grit. Grit is a personality trait defined by Duckworth et al. (2007, p. 1087) as the “passion for long-term goals.” The long-term goal of an individual $i$ could be to become an entrepreneur (given a corresponding initial estimate $\tilde{\pi}_{i,0}$). A gritty individual will stick to this objective as long as possible. This type of behavior will occur if the individual in question has a rather large self-confidence parameter $\omega_i$. If we interpret $\omega_i$ as directly related to grit (i.e. the trait $\tau^c_i$ is grit), we have a direct correspondence. The
statement “individual $i$ is grittier than individual $j$” can be directly translated into $\omega_i > \omega_j$.\(^{23}\)

Notice, first, that by including the stochastic variable $u_{i,t}$, we introduce failure by chance. Even an entrepreneur with a high true performance $\pi_{i,t}$ could decide to abandon entrepreneurship because feedback is below 0 by chance (because of $u_{i,t}$). Second, ignoring losses (because of grit, or something else correlated with $\omega_i$) is not a general formula for an entrepreneur to survive since there is still the financial institution that can enforce an exit if the entrepreneur accumulates too many losses.

To conclude, individuals faced with market feedback experience a trade-off between correcting their estimates and sticking to their decision. Each individual will counterbalance the trade-off in her own way by having recourse to her self-confidence traits.

4.5.2 Attractivity formation

Last but not least, we have to discuss in this section how attractivity formation evolves over time. We argue that this process is governed by experience, responsiveness traits, and the former attraction of an option.

Each individual starts with an initial horizon of experience $N_{i,0}$. Experience of $i$ is updated according to

$$N_{i,t} = \rho_i N_{i,t-1} + 1$$

(18)

where $\rho_i \in [0, 1]$ describes what fraction of a former period is integrated into $i$’s horizon of experience. Since the horizon of experience is the same for each option (employee and entrepreneur) we can, without adverse consequences, set $\rho_i = \rho = 1$ and $N_{i,0} = N_0 = 1$ to get

$$N_t = t + 1$$

(19)

According to Eq. (19), the horizon of experience increases by 1 in each period.

The attractivity of being a worker is then assumed to be given by:

**Attractivity of being a worker.**

$$\tilde{A}_{i,t} = \frac{N_{t-1}}{N_t} \phi(\tau_i^r) \tilde{A}_{i,t-1} + \frac{1}{N_t} w_{i,t}$$

(20)

where the wage $w_{i,t}$ depends on the regime applied by $i$’s employer.

The structure of attractivity formation follows the one suggested by Camerer and Ho (1999). (a) The attractivity of an option depends on the former attraction of this option. (b) Attractivity changes as time elapses.

\(^{23}\) Grit is one way to justify a non-full correction in expectations. A similar phenomenon is described as identity protection. Murtagh et al. (2012) consider identity protection as one important reason to resist changes. Tesser and Cornell (1991) found that maintaining self-evaluation may be more important than maximizing it. In our model, the instability of self-evaluation is captured by $\text{Var}[\theta_{i,t}]$, which is negative in $\omega_i$.\(^{21}\)
Attractivity dynamics are assumed to be directly influenced by the remaining trait set we specified as responsiveness trait. \( \phi : \Theta \to (0, 1) \) maps an individual responsiveness traits vector onto a parameter \( \phi_i \). \( \phi_i \in (0, 1) \) describes how attractivity from the previous period is integrated into recent-period attractivity. Consider the (ruled out) limiting cases. If \( \phi_i = 0 \), decisions are based only on recent events. If \( \phi_i = 1 \), previous attractivity is fully integrated into today’s attraction. In total, \( \phi_i \) describes how much information individual \( i \) extracts from her own working history in a certain occupation.\(^{24}\)

To formalize the attractivity of entrepreneurship, which follows the same structure as attractivity of being a worker, we consider two cases.

**Attractivity of being an entrepreneur.** First, if \( i \) is an entrepreneur in period \( t \), we have

\[
A_{i,t|E(t)} = \frac{N_{i-1}}{N_t} \phi(\tau_i^t) A_{i,t-1} + \frac{1-c}{N_t} \left[ \omega(\tau_i^t) \hat{r}_{i,t} + \left[ 1 - \omega(\tau_i^t) \right] f_{i,t} \right] \tag{21}
\]

Second, if \( i \) is a worker in period \( t \) who was never before forced by the financial institution to exit entrepreneurship:

\[
A_{i,t|\bar{W}(t)} = \frac{N_{i-1}}{N_t} \phi(\tau_i^t) A_{i,t-1} + \ldots + \frac{1-c}{N_t} \left[ \omega(\tau_i^t) \hat{r}_{i,t} + \left[ 1 - \omega(\tau_i^t) \right] \left[ f_{i,t} + \Delta \pi_i(t) - \Delta \pi_i(t^*) \right] \right] \Pi_{i,f<\tau}^r + \ldots + \frac{1-c}{N_t} \hat{r}_{i,t} \left[ 1 - \Pi_{i,f<\tau}^r \right] \tag{22}
\]

where \( \Pi_{i,f<\tau}^r = 1 \) if \( \exists \bar{t} < t \) such that \( i \in E(\bar{t}) \). In words, \( \Pi_{i,f<\tau}^r = 1 \) if \( i \) was an entrepreneur in a previous period and \( \Pi_{i,f<\tau}^r = 0 \) else. When \( i \) was an entrepreneur in periods \( t_1, \ldots, t_n \) where \( n \) is the number of periods in which \( i \) was an entrepreneur, \( t^* = \max\{1, \ldots, t_n\} \) is the last period of \( i \)’s entrepreneurship. \( i \) will take the feedback of this period \( f_{i,t^*} \) and adjust it to the development of her unit between period \( t^* \) and the actual period \( t \). If \( i \) was never an entrepreneur, the only information she has is her estimate.

In this section, we introduced all dynamic components of our model. Given some assumptions and liability constraints set by a financial institution, we described effects resulting from changes in firm structures if individuals decide to start or to abandon an own business. Employees might decide to start an own firm because they are denied opportunities by their employers. Dynamics start with an initial period where individuals without any work-experience decide if they want to be workers or entrepreneurs. After the initial period, entrepreneurs are confronted with feedback from the market. Conditional on their self-confidence traits entrepreneurs will internalize some part of feedback, while some parts can be ignored. Internalized feedback is then used to construct dynamic attractiveness where we assume that entrepreneurship is selected if it

\[^{24}\text{The parameter } \phi_i \text{ also influences learning efficiency. For instance, if } \phi_i \text{ is high, new and old information receive similar weights. All in all, learning efficiency decreases as } \phi_i \text{ increases.}\]
is at least as attractive as wage work. In addition, attractivity development is influenced by responsiveness traits.

5 Impact of personality on decisions

This section demonstrates, with the help of numerical simulations, our model’s basic mechanism. Simulations will allow us to analyze more deeply the decisions of heterogeneous individuals with mixed career histories (with individuals switching from entrepreneurship to wage work and vice versa without restrictions).

In the first subsection, we state all assumptions necessary to simulate the model. The second subsection examines the self-selection mechanism into and out of entrepreneurship. In the third subsection, we depict earning dynamics before we conclude with an overview.

5.1 Simulation framework

The model fully determines all interactions such that we can simulate the behavior of individuals under some global assumptions, which are as follows.

Assumption 1. The population mean of non-self-management productivities $\mu_{N\text{SP}}$ is larger than the population mean of self-management productivities $\mu_{SP}$. Or, we assume that the individual with average traits is better in managing others than herself.

Assumption 2. The variance of (true and expected) self-management productivities is larger than the variance of non-self-management productivities $\sigma_{N\text{SP}}^2$. Or, entrepreneurial abilities are more heterogeneous than management abilities.

Assumption 3. The variance of estimated self-management productivities $\sigma_{\text{ESP}}^2$ is larger than the variance of true self-management productivities $\sigma_{T\text{SP}}^2$ such that more individuals consider themselves entrepreneurs than really are.

Assumption 4. Non-self-management productivities are assumed to be independent from true or estimated self-management productivities. Hence, we abstract from the question of how interactions between two individuals are related to self-management performance. Furthermore, conditional productivities are assumed to be independent from each other, i.e. individuals who are good at managing a particular individual are not necessarily doing well as universal managers.

Assumption 5. Estimated self-management productivities are correlated with true self-management productivities where the correlation is $\gamma$. $\gamma$ determines the accuracy of self-assessments. $\gamma = 0$ is zero accuracy and $\gamma = 1$ is “good” self-assessment.

25 In Appendix B, we present analytical results based on more restrictive assumptions. Analytical results and results based on simulations of the model are consistent, which is shown in Appendix B. However, simulations are substantially less demanding with respect to assumptions and allow to derive some results that cannot be established with a more restrictive set of assumptions.

26 The corresponding code is provided on request.
**Assumption 6.** Responsiveness $\phi_i$ and confidence $\omega_i$ are random. In principle, responsiveness and confidence can be correlated. However, we assume that they are uncorrelated when describing the selection mechanism.  

**Table 1.** Distributional assumptions and fixed parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range or distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>300</td>
</tr>
<tr>
<td>$T$</td>
<td>60</td>
</tr>
<tr>
<td>$c$</td>
<td>0.01</td>
</tr>
<tr>
<td>$m$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>$u_{i,t}$</td>
<td>Normal with zero mean and standard deviation $\tilde{u}_{i}$</td>
</tr>
<tr>
<td>$\tilde{u}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\phi_i$, $\omega_i$</td>
<td>Jointly truncated normal with mean $[\mu_{\phi}, \mu_{\omega}]$, covariance matrix $\begin{bmatrix} \sigma_{\phi}^2 &amp; \gamma_{RC} \sigma_{\phi} \sigma_{\omega} \ \gamma_{RC} \sigma_{\phi} \sigma_{\omega} &amp; \sigma_{\omega}^2 \end{bmatrix}$, and truncation $\phi_i, \omega_i \in [0.001, 0.999]$</td>
</tr>
<tr>
<td>$\gamma_{RC}$</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_{\phi}, \sigma_{\omega}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\mu_{\phi}, \mu_{\omega}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$v_{ij}$ (for $i \neq j$)</td>
<td>Truncated normal with mean $\mu_{NSP}$, standard deviation $\sigma_{NSP}$, and left truncation such that $v_{ij} \in [0, 0.001, \infty)$</td>
</tr>
<tr>
<td>$\mu_{NSP}$</td>
<td>1.4</td>
</tr>
<tr>
<td>$\sigma_{NSP}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$v_{ii}$, $\tilde{v}_{ii}$</td>
<td>Jointly normal with mean $[\mu_{TSP}, \mu_{ESP}]$ and covariance matrix $\begin{bmatrix} \sigma_{TSP}^2 &amp; \gamma \sigma_{TSP} \sigma_{ESP} \ \gamma \sigma_{TSP} \sigma_{ESP} &amp; \sigma_{ESP}^2 \end{bmatrix}$, $\sigma_{TSP}$, $\sigma_{ESP}$, $\gamma$</td>
</tr>
<tr>
<td>$\mu_{TSP}, \mu_{ESP}$</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table 1 collects all global, distributional, and parameter assumptions. Costs are small to encourage entrepreneurship.

Simulations are mainly based on Eq. (6), (9), (7), (12), (13), (20), (21), (22), SNCA (with a dynamic extension), restructuring rules, and rules #1–7. Simulations proceed in three steps. The first step is to fix the variables of interest or to define a common grid for these variables. The second step consists in testing if $\Gamma(\gamma)$ and $\tilde{\Gamma}(\gamma)$ have a tolerable level of circularity. The third step is simulation and evaluation. Note that one draw of all parameters necessary to determine the behavior of $I$ individuals in periods $1, \ldots, T$ is counted as one simulation run.

Simulations revealed that the correlation between $\phi$ and $\omega$ does not play a large role for the selection mechanism.
By drawing $\Gamma$ and $\hat{\Gamma}$ from corresponding distributions, it is almost unavoidable that issues of circularity arise in the matrices such that at some stage the second-best manager of an entrepreneur may be employed in her own company. Since non-circularity cannot be realized in full form, we use soft criteria. A tolerable level of circularity is given if (a) SNCA holds for $\hat{\Gamma}$ and (b) $\hat{\Gamma}$ survives chain-restructurings with 5 chain links.\(^{28}\) Also, the simulation function includes circularity diagnostics. We stipulate that circularity diagnostics are bad if there is at least one relevant circularity problem. In this form, simulation results are not severely contaminated by issues related to circularity.\(^{29}\)

It is common to examine self-selection into and out of entrepreneurship by inspecting group differences between employees, nascent entrepreneurs, and surviving entrepreneurs to reveal which traits have an effect. We will do the same but with a slight modification: In addition to conditioning on type (worker, nascent entrepreneur, \textit{etc.}), we condition on the degree of self-assessment errors. This allows us to see why the group-differences approach might be insufficient as group differences can change with a change in the degree of self-assessment error.

5.2 Self-selection into and out of entrepreneurship

When discussing the results of our model, we will focus on two different scenarios. In Scenario A, there is no correlation between estimated and true self-management productivities, \textit{i.e.} we set $\gamma = 0$. In Scenario B, estimated and true self-productivity are highly, but not perfectly, correlated, \textit{i.e.} we set $\gamma = 0.9$. By comparing Scenario A and B, we can examine the difference between systems with different degrees of self-assessment errors. We conduct 500 simulations for each scenario.\(^{30}\) Both scenarios share the parameters depicted in Table 1. The selection mechanism of the model is fully described by Fig. 2, 3, 4, and 5, which are generated with simulated data.

Fig. 2 presents results on entry into entrepreneurship without any work-experience (entrepreneurs entering in period 1 denoted by EwWE in the figure) by depicting distributions of estimated self-management productivities (ESP) and true self-management productivities (TSP) of entrants without experience. These productivities are compared to distributions of the overall population

\(^{28}\) Chain restructuring refers to the following concept: Assume an entrepreneur (according to $\hat{\Gamma}$) $i$ abandons her firm and goes to her best alternative manager $j$. This is one chain link. $j$ is said to survive the restructuring if she has a real alternative if she decides to abandon entrepreneurship, \textit{i.e.} the second-best manager of $j$ is not part of $i$'s firm. $\hat{\Gamma}$ is said to survive a chain-restructuring with one chain link if the above holds for every entrepreneur and her best alternative manager in $\hat{\Gamma}$. A $n$ chain-link restructuring is defined in a similar way. $i$ abandons her firm and goes to $j$, $j$ abandons her firm and goes to $k$, and so on.

\(^{29}\) Note that never having circularity problems is a rather unrealistic assumption. The real-world counterpart of $\hat{\Gamma}$ and $\Gamma$ will not be perfectly non-circular. But, being not perfectly non-circular is usually not an obstacle if the number of individuals involved in the system is large. Two individuals without well-defined interactions do not pose an issue if they never interact. We follow the same strategy by letting $I$ be large enough.

\(^{30}\) Results are qualitatively stable to an increase in simulation numbers.
The upper figure depicts Scenario A and the lower figure Scenario B. We can observe that self-selection in the initial period (the period without any first-hand knowledge) into entrepreneurship is based solely on estimated self-management productivities. If estimated self-management productivities are not correlated with true productivities (see upper subfigure of Fig. 2), entrepreneurs entering in period 1 have a higher average estimated self-management productivity than the overall population. If estimated and true productivities are positively correlated (see lower subfigure of Fig. 2), the group of entrepreneurs entering in the initial period will have a higher true productivity on average because of this correlation but not because of any performance-related aspect of the self-selection mechanism: The mechanism cannot determine if performance traits are appropriate at this stage since entrepreneurs without any work-experience had no chance to accumulate feedback.

Fig. 3 present results on nascent entrepreneurs without entrepreneurial experience but with experience as employees. Fig. 3 compares the group means of nascent entrepreneurs (denoted by NE in the figure) to population means with respect to confidence, responsiveness, estimated self-management productivities (ESP), and true self-management productivities (TSP). We see that the statements for entrepreneurs without any work-experience also hold for nascent entrepreneurs (with employee-experience). Although opportunity entrepreneurs have more or less an average level of confidence and responsiveness near the population levels, they differ from the overall population in one regard. Their estimated self-management productivities are approximately twice as large as the overall population analogue. If the self-perception gap is large (Scenario A), true entrepreneurial productivities are at population levels. If the self-perception gap is partially closed (Scenario B), true entrepreneurial productivities are above average. The result is the same as for entrepreneurs without any work-experience.

Given the assumption that individuals in our model are rather good managers than entrepreneurs, entrants into entrepreneurship will come from the right tail of the distribution of estimated self-management productivities. As a consequence, entrants systematically overestimate their entrepreneurial abilities if a self-perception gap exists. This can explain the direction of the bias (too positive assessment), also found in field and experimental data.

The size of the bias decreases if the size of the self-perception gap is reduced. This can be seen by comparing Scenario A and B in Fig. 2 (upper subfigure is A and lower is B) and 3: If we partially close the self-perception gap, the distance between estimated and true self-management productivities decreases. This means that an overestimation bias still exists even when the perception gap is partially closed.

Thus, according to our model, individuals self-select into entrepreneurship based on estimated self-management productivities. This holds for entrepreneurs without any work-experience (entrepreneurs entering in the first period) and for nascent entrepreneurs with employee-experience. The result is demonstrated by a comparison between attributes of entrants and the overall population;
and shows that if only a partial self-assessment error exists, entrants tend to overestimate their entrepreneurial performance.

Fig. 4 presents survival dynamics by following a cohort of entrepreneurs over 61 periods (plotting the number of individuals still in entrepreneurship) where the number of entrepreneurs in each period is normalized by the number of entrants in period 1 such that we can examine the share of the original cohort still in business in each period $t = 2, \ldots, 61$. A decrease in the share can be interpreted as a decrease in probabilities to survive.

As one can see in this figure, the probability of survival strongly decreases in the first periods after entry (call it ‘the turbulent phase’) and stabilizes in later periods (‘the consolidation phase’). This corresponds to survival rates in reality. Given the attractivity dynamics of our model, the shape of the survival curve is influenced by three weighted components. First, there is the initially assumed difference in incomes (entrepreneurial and wage income), which determines the decision to become entrepreneur. If an individual is an entrepreneur, the initial difference is a factor encouraging entrepreneurship. Second, individuals learn about their true productivities, which can encourage or discourage them. Last, there are income changes due to restructurings of units. The initially assumed difference in incomes loses most of its influence right in the first periods of entrepreneurship if attractivity dynamics are as assumed in our model. Consequently, a transition is most likely in periods after entry. If responsiveness is not too low, the weights of the three components above will stabilize in time, thus generating the consolidation phase where the number of transitions from entrepreneurship to wage work is minimal.

Our model has another crucial feature: Better knowing true entrepreneurial fitness results only in small increases of survival probabilities (compare Scenario A and B in Fig. 4). This points to the relevance of factors not associated with entrepreneurial performance in the strict sense.

Trait sets important for survival are analyzed with quantiles. We compute 1%-quantiles for survivors’ traits given that entrepreneurs survived over $t$ periods. The interpretation is straightforward. If, for example, the 1%-quantile of confidence is $q$, this implies that 99% of survivors have a confidence level above $q$. Then, we can, for instance, observe the 1%-quantile of the self-confidence distribution of the survivors group in period $t$ and compare it to the same measure in the previous period $t - 1$. If the confidence quantile of survivors in $t$ is above the quantile in $t - 1$, confidence levels increased.

Why survival probabilities do not increase more when individuals make less self-assessment errors is explained in Fig. 5. Fig. 5 plots growth rates of the 1%-quantile for each attribute’s distribution of the survivor group as a function

31 Simulated data also reveals failure by chance. We define a failure by chance as a failure given that $\hat{a}_{i,0} \leq a_i,0$, i.e., the corresponding individual estimated her entrepreneurial abilities correctly or even underestimated abilities and still failed. Failure by chance is an interesting aspect for further research.

32 We develop an intuitive argument showing why learning can generate a certain survival curve shape in Appendix C.
of time. Attributes are confidence, responsiveness, estimated entrepreneurial performance (ESP), and true entrepreneurial performance (TSP). This approach allows for the determination of how attributes, respectively the distributions of attributes, of survivors change over time, i.e., we can determine the attribute most responsible for survival. If, as shown in the first row of Fig. 5, there is no correlation between estimated and true self-management productivities—interpretable as a high degree of unsystematic self-assessment errors—the largest difference between those entrepreneurs who enter in the first period and those who survive the turbulent and the consolidation phase is given by true self-management productivities or, put differently, by performance traits (notice the huge peak of TSP in early periods in the first row of Fig. 5). All other factors are less important than performance traits.

Allowing for fewer self-assessment errors, as in the second row of Fig. 5, changes the picture because the survival mechanism does not need to sort out as many potential but not true entrepreneurs as in the scenario with many self-assessment errors. Automatically, performance traits lose their outstanding group-defining function—though, they are still an important factor—and confidence becomes more important.

Thus, we can establish two conditional statements. Given no correlation between estimated and true self-management productivities, entrepreneurial performance (performance traits) determines who survives, while all other traits of survivors are not much different from the traits of the entrants group. Conditional on a partially closed perception gap, performance traits lose relevance—however, they are still important—and confidence traits become important in addition.

The two conditional statements clarify why personality might have a non-linear impact on entrepreneurial decisions and why it is hard to construct a representative successful entrepreneur (two empirical observation mentioned in the Introduction). Furthermore, neither performance nor confidence traits play a prominent role with respect to self-selection into entrepreneurship. This captures another observation: Trait sets important for entry are not necessarily significant for entrepreneurial survival.

5.3 Earnings

An observation that attracted considerable attention in the empirical literature is that many entrepreneurs do not exit despite the option of earning more as

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33 The loss in importance of true entrepreneurial performance can be illustrated with two simple numbers. Given a full self-perception gap (Scenario A), entrepreneurs who survived until the last period have a 1%-quantile of true entrepreneurial performance that is by approx. 85% above the same performance quantile of entrants. If the self-perception gap is partially closed (Scenario B), the 1%-quantile of true entrepreneurial performance of entrepreneurs who survived until the last period is above the 1%-quantile of entrants by ca. 22% only.

34 The relative importance of self-confidence is underpinned by the fact that, compared to responsiveness and the degree of self-assessment errors, increasing confidence results in the largest increase in average survival probabilities. This result is demonstrated in Appendix D.
Entrepreneurs without work—experience EwWE estimate their self—management productivities ESP above the population value POP while their true productivities TSP are at population level due to complete self—assessment error.

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Entrepreneurs without work—experience EwWE estimate their self—management productivities ESP above the population value POP while their true productivities TSP are also above population levels but below estimated values due to partial self—assessment error.

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**Figure 2.** Population and entrepreneurs without work—experience in Scenario A (upper figure) and B (lower figure)

This observation is often based on a cross-sectional comparison of entrepreneurial earnings with the contrafactual earnings (the wage an entrepreneur could earn). The simulation data generated by our model not only allows for a direct comparison of these two groups without the need to adjust for selection bias, but also reveals a thorough explanation for the so called entrepreneurial income puzzle.

Fig. 6 addresses the observation by depicting distributions of wage income and net entrepreneurial earnings in three different periods. Distributions for $T = 2$ are most similar to empirical distributions presented by Hamilton (2000). $T \in \{1, 2\}$ belongs to the turbulent phase. For $T \in \{1, 2\}$, we see that the

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Data is generated by Scenario A.
Estimated self-management performance ESP sets nascent entrepreneurs NE without entrepreneurship-experience most apart from the population (red line). In case of complete assessment error, all non-ESP-attributes (including true performance TSP) are at population levels.

Figure 3. Attributes of nascent entrepreneurs without entrepreneurial experience

individual receiving the median entrepreneurial income (who is an entrepreneur) would be better off as employee. In $T = 50$ we are in the consolidation phase. In this phase of entrepreneurship, most, though not all, entrepreneurs who could get a higher wage income made an exit from entrepreneurship as a consequence of learning. In $T = 50$, the median income of an entrepreneur is strictly better than the median wage income.

Fig. 7 shows the share of individuals who remain in entrepreneurship even though they could earn more in dependent employment. The share is substantial but decreases with time to nearly zero. Entrepreneurs who do not exit although they would be better off as employees are most present in the turbulent phase. This again is a feature of learning. An entrepreneur needs some time to realize that she is better off as employee and—as our model clarifies—the transition can be inhibited by individual self-confidence.

Strictly speaking, entrepreneurs who do not exit even though they would be better off as employees can only be observed in the turbulent phase. In reality, there will be smooth transitions between the turbulent phase and the consolidation phase: Some entrepreneurs will be in their consolidation phase and some in their turbulent phase due to overlapping cohorts of entrants. The number of entrepreneurs in the turbulent phase is always larger than in the consolidation phase (this is a general prediction of the model). Hence, lower earnings of entrepreneurs compared to workers may be observed because the characteristics of entrepreneurs are mostly driven by turbulent-phase entrepreneurs. Furthermore, real-world learning can be substantially less efficient than model learning such that the turbulent phase might be extended.
6 Summary of main results and conclusions

In this paper, we develop an occupational choice model that allows us to discuss how personality traits influence self-selection into and out of entrepreneurship, which is also able to explain known features of the two processes at the same time. From previous empirical research, we know that although personality matters, it is not the main determinant of entrepreneurial processes and that personality traits affect entry and survival in different ways. We also know from previous research that entrepreneurs tend to overestimate their abilities. Therefore, more individuals enter entrepreneurial activities than are able to survive, while survival rates tend to decrease at decreasing rates. Empirical evidence also reveals that some individuals stay in entrepreneurship even though they could earn more in wage employment.

We show that this evidence can be explained in one coherent model if individuals are not fully aware of their traits characteristics and, thus, of the differing influence of personality variables on entrepreneurial decision making. Our theory is motivated by three stylized facts, namely that personality matters for occupational choices, that individuals are unaware of their specific characteristics, and that they are able to learn about their own characteristics.

We claim that entrepreneurship-related personality traits can be sorted in four different sets of traits. Skill-related traits (trait set 1) influence the skill level of an individual. Specifically, they influence individual non-entrepreneurial and subjectively estimated entrepreneurial productivities. Performance traits (trait set 2) have an impact on the objectively true individual entrepreneurial productivity. Confidence traits (trait set 3) determine the share of stochastic market feedback (entrepreneurial income) an individual internalizes in every period. More confidence results in a smaller internalized share. Responsiveness traits (trait set 4) determine how quickly an individual reacts if faced with a new...
With complete self-assessment error, true self-management performance TSP is group defining attribute of survivors. Estimated self-management productivities ESP, responsiveness, and confidence are substantially less important.

\[ \text{Confidence ESP Responsiveness TSP} \]

\[ \begin{array}{cccc}
0 & 0 & 0 & 0 \\
10 & 10 & 10 & 10 \\
20 & 20 & 20 & 20 \\
30 & 30 & 30 & 30 \\
40 & 40 & 40 & 40 \\
\end{array} \]

Time

Growth of 1% quantile of survivors’ attributes

With only partial self-assessment error, distance between survivors’ attributes drastically decreases. Confidence of survivors is second most important attribute.

\[ \text{Confidence ESP Responsiveness TSP} \]

\[ \begin{array}{cccc}
-0.05 & -0.05 & -0.05 & -0.05 \\
0.00 & 0.00 & 0.00 & 0.00 \\
0.05 & 0.05 & 0.05 & 0.05 \\
0.10 & 0.10 & 0.10 & 0.10 \\
0.15 & 0.15 & 0.15 & 0.15 \\
\end{array} \]

Time

Growth of 1% quantile of survivors’ attributes

Figure 5. Change in group attributes of survivors over time given Scenario A (upper) and Scenario B (lower).

stimulus. One important consequence of having differentiated between these sets of traits is that individuals with different personalities learn in different ways and make different assumptions on potential outcomes.

Our model proposes a simple selection mechanism to separate entrepreneurs from workers. The main results of our model are:

Individuals become entrepreneurs because they assume that their self-management productivity is the largest conditional productivity. If the estimated self-management productivity is higher than all other conditional productivities and if (entry) costs are low enough, individuals directly start an own business. If the estimated self-management productivity is not higher than all other productivities but still high (higher than the population average) or if (entry) costs are not low enough, individuals start as workers but eventually will become entrepreneurs once they see an opportunity not exploitable as...
a worker. Only skill-related traits matter for entry since the estimate of self-management productivities is determined by skills. Neither performance, nor confidence, nor responsiveness traits are involved in the self-selection process into entrepreneurship.

After entry, other traits are important. If estimated self-management productivities are not correlated with true self-management productivities (thus, if self-assessment of entrepreneurial performance is bad), the group of survivors is mostly defined by performance traits. If estimated and true self-management productivities are highly correlated (if self-assessment is good), performance traits do not define the group of survivors anymore because having appropriate performance traits is likely to be ensured by having appropriate skill-related traits. In case of a high correlation, in particular confidence traits gain relevance, with confidence traits, which can be approximated by grit, unfolding a strong influence on exit decisions.

Thus, traits affecting entry and survival can be different because skill-related traits (responsible for entry) as well as performance and confidence traits (responsible for survival) are not necessarily correlated. Furthermore, we can only derive conditional statements on the importance of each trait.
(conditional on the degree of self-assessment errors) for survival and cannot rely on unconditional population differences.

Our mechanism produces plausibly shaped entrepreneur survival curves—survival probabilities decreasing at a decreasing rate. As certain individuals (depending on their skill-related traits) self select into entrepreneurship, the market sorts out fresh entrepreneurs who do not possess the “right” traits to survive (mostly performance and confidence traits in our model). The sorting mechanism does most of its work in initial periods following entry such that the probability to fail is largest in this turbulent phase. In later periods, the probability to survive stabilizes.

Some entrepreneurs could earn more in wage employment but may not realize it. However, the number of individuals who stay in entrepreneurship, even if their contrafactual wage income is larger, decreases over time. The reason is that entrepreneurial income is an imperfect signal that comes in form of noisy market feedback. Therefore, individuals need some time to learn if they are better off in wage work or in entrepreneurship. Since this holds for each new cohort of entrepreneurs, there will always be a number of entrepreneurs who should switch to wage work but stay in entrepreneurship if cohorts overlap.

The model also provides a simple explanation for overconfidence of nascent entrepreneurs detected by Koellinger et al. (2007), Camerer and Lovallo (1999), and Busenitz and Barney (1997). Since individuals self select into entrepreneurship because of a relatively high perceived entrepreneurial productivity, the self-selection mechanism can lead to overconfident nascent entrepreneurs if a large enough self-perception gap exists.

Thus, all results of our model are consistent with observations presented in the Introduction. A main insight of our analysis is that the driving force behind most of the results are effects of potential self-assessment errors due to a difference between perceived abilities (estimated self-management productivities) and true abilities (true self-management productivities). Moreover, the relation of traits to entrepreneurial survival is rather complex since no particular trait matters unconditionally and the relation of traits to each other is crucial too. Consequently, the prototype successful entrepreneur, if it is aimed to build such
a construct, cannot be described by a static definition. The prototype has the right traits in the right situation, which demands a dynamic concept.

In contrast to existing occupational choice models with a learning component, our model includes personality and relies on differing assumptions on why entrepreneurship emerges. For instance, Astebro et al. (2011) use the assumption that entrepreneurs result from non-optimal assignments of workers to firms and there would be no entrepreneurship if assignments were frictionless. We assume that entrepreneurs are individuals who are best managed by themselves, which is the main assumption of our model tracing back to the approach of the Roessler and Koellinger (2012) model.

The model provides a dynamic learning algorithm predominantly operating with observable variables and trait rationalizing variables. Traits can be measured. Hence, future research should be directed at the estimation of model parameters with representative data.
References


A Indifference between firm structures

In our model, we emphasized one particular feature of the payment structure. Entrepreneurs are indifferent toward the existence of workers who are not in their own unit. Consider the following example. Let $s_1$ be an optimally constructed firm with $M_1 > 0$ employees (the entrepreneur is not counted as employee). $m_1$ individuals are in the entrepreneur’s unit where $m_1 > 0$. $M_1 - m_1 = m_1'$ individuals are in other units where $m_1' > 0$. Let $s_2$ be the same firm structure but with no individuals in other units other than the entrepreneur’s unit (individuals in the entrepreneur’s unit are the same in $s_1$ and $s_2$), i.e. we have $m_2 = M_2 = m_1$ and $m_2' = 0$. Worker’s wage bills $W$ are

$$W_{s_1} = \sum_{i=1}^{m_1} [v_{i}^{(2)}] + \sum_{i=m_1+1}^{m_1'} [v_{i}^{(2)}]$$

$$W_{s_2} = \sum_{i=1}^{m_1} v_{i}^{(2)}$$

where $\Delta v_i^{(0)}$ is the managerial advantage of $i$ over second-best managers of her unit. Entrepreneurial income in structure 1 is

$$\pi_{s_1} = v_{i}^{(1)} + \sum_{i=1}^{M_1} v_{i}^{(1)} - W_{s_1}$$

or

$$\pi_{s_1} = v_{i}^{(1)} + \sum_{i=1}^{m_1} [v_{i}^{(1)} - v_{i}^{(2)}] - \Delta v_i^{(0)} + \sum_{i=m_1+1}^{m_1'} [v_{i}^{(1)} - v_{i}^{(2)}] - \Delta v_i^{(0)}$$

However, since the aggregate managerial advantage of a worker is fully rewarded to this worker, we must have

$$\sum_{i=1}^{m_1} \Delta v_i^{(0)} + \sum_{i=m_1+1}^{m_1'} \Delta v_i^{(0)} = \sum_{i=m_1+1}^{m_1'} [v_{i}^{(1)} - v_{i}^{(2)}]$$

and

$$\pi_{s_1} = v_{i}^{(1)} + \sum_{i=1}^{m_1} [v_{i}^{(1)} - v_{i}^{(2)}]$$

We, thus, can verify that $\pi_{s_2} = \pi_{s_1}$, i.e. entrepreneurial income is the same across structures $s_1$ and $s_2$. The example provides an illustration of an important consequence of Eq. (5) and (6) regarding dynamics.
In Appendix B, we demonstrate some properties of “behavior in expectations.” In the first subsection, we state necessary assumptions. Then we examine what factors influence entrepreneurial entry and survival. The last subsection analyzes how specific variables affect the time of a transition from entrepreneurship to wage work or vice versa.

### B.1 Assumptions

We model interactions between individuals in the following simplified form. If individual \( j \neq i \) interacts with individual \( i \), \( j \) can either leave \( i \)'s unit or firm, apply to \( i \)'s firm, or enter \( i \)'s unit. All these actions might be reflected by some change in \( i \)'s income (viz. entrepreneurial income, potential entrepreneurial income, or wage income). From \( i \)'s perspective, the actions of all individuals \( N \setminus i \) will create certain distributions of income changes. Though we cannot say how these distributions will exactly look like, we assume that we are confronted with i.i.d. distributions with an existing expected value and variance.

We assume i.i.d. \( \Delta w_{i,t} \sim (\Delta w_i, \sigma^2_{w,i}) \). The expected wage of \( i \) in period \( t \) is

\[
E[w_{i,t}] = w_{i,0} + \overline{\Delta w_i}t
\]  

(Nascent entrepreneurs and applicants affect wages by affecting units. Entrepreneurs will be affected if someone leaves their unit or applies to the unit they manage. The effect on entrepreneurs is numerically the same as for workers (the same person once treated as a worker and once as an entrepreneur where either entrepreneurship or wage work is a contrafactual). Accordingly, we can define

\[
\Delta \pi_{i,t} \equiv \pi_{i,t} - \pi_{i,t-1} = \check{\pi}_{i,t} - \check{\pi}_{i,t-1} = \Delta w_{i,t}
\]  

Since the effect is channeled through the unit, actual and estimated income (i.e. \( \pi_{i,t} \) and \( \check{\pi}_{i,t} \)) are affected in equal measure because changes in units are obvious and cannot be ignored by IHOP. To avoid misleading notation, we define i.i.d. \( \Delta \pi_{i,t} \sim (\Delta \pi_i, \sigma^2_{\pi,i}) \). Expected estimated entrepreneurial earnings are given by

\[
E[\check{\pi}_{i,t}] = \check{\pi}_{i,0} + \overline{\Delta \pi_i}t
\]  

To get expected true entrepreneurial earnings, just replace \( \check{\pi}_{i,0} \) by \( \pi_{i,0} \) such that

\[
E[\pi_{i,t}] = \pi_{i,0} + \overline{\Delta \pi_i}t
\]  

Given our two assumptions on distributions of income changes represented by Eq. (23), (25), and (26), the bounded-above wage regime can be formalized in a simple manner. If \( j \) starts to work in period \( t^* > 0 \) in the firm of \( i \) but is not part of \( i \)'s unit, \( j \notin U_{i}^{(t^*)} \), \( j \)'s wage is \( w_{j,t} \leq w_{j,t} \) for every period \( t > t^* \) she stays in \( i \)'s firm. By the effect of the regime, the expected wage change is
\( \Delta w_i \leq 0 \) instead of \( \Delta w_i \in \mathbb{R} \). In addition to \( \Delta \pi_i \) an entrepreneur \( i \) may generate a surplus for not adjusting wages in every period. Denote the expected value of this surplus by \( \Delta \pi_i' \geq 0 \) such that \( \mathbb{E}[\Delta \pi_{i,t}] = \Delta \pi_i + \Delta \pi_i' = \Delta \pi_i \), given the bounded regime.

To simplify analysis, we will look only at two types of individuals. Type \( w \) individuals were workers up to period \( t: i \in W(1), \ldots, i \in W(t) \). Type \( e \) individuals are individuals who were entrepreneurs up to period \( t: i \in E(1), \ldots, i \in E(t) \).

b.2 Incentives to enter and to exit

Conditional on the work history \( w \) or \( e \), we get four equations (attractivity of entrepreneurship and wage work for both a worker and entrepreneur), where we apply expectations (thus, we consider “behavior in expectations”) and use successive iteration to get each equation.

**Expected attractivity of wage work for an employee.** The bounded-above regime implies non-positive periodical changes in income \( \Delta w_i \leq 0 \) for a worker such that

\[
\mathbb{E}[\tilde{A}_{i,t|w}] = \frac{1}{t+1} \left[ (\phi_i' \tilde{\beta} + \varphi_i(t)) w_{i,0|w} + \Phi_i(t) \Delta w_i \right]
\]

(27)

where

\[
\Phi_i(t) = \frac{(1 - \phi_i) t + \phi_i^{t+1} - \phi_i}{(\phi_i - 1)^2} > 0, \quad \partial \Phi_i(t) / \partial \phi_i, \quad \partial \Phi_i(t) / \partial t > 0
\]

\[
\varphi_i(t) = \frac{1 - \phi_i'}{1 - \phi_i}
\]

We assume that \( i \) is not part of the entrepreneur’s unit.

**Expected attractivity of wage work for an entrepreneur.** Expected attractivity is given by

\[
\mathbb{E}[\tilde{A}_{i,t|e}] = \frac{1}{t+1} \left[ (\phi_i' \tilde{\beta} + \varphi_i(t)) w_{i,0|e} + \Phi_i(t) \Delta w_i \right]
\]

(28)

We assume that an applicant receives an updated wage such that the wage difference \( \Delta w_i \) can be larger than \( \Delta w_i \).

**Expected attractivity of entrepreneurship for an employee.** An employee has no information on entrepreneurship besides her estimated entrepreneurial performance \( \hat{\pi}_{i,0|w} \) such that

\[
\mathbb{E}[\hat{A}_{i,t|w}] = \frac{1}{t+1} \left[ (\phi_i' \tilde{\beta} + \varphi_i(t)(1 - c) \hat{\pi}_{i,0|w}) + \ldots \right.
\]

\[
\ldots + \frac{1}{t+1} \left[ [1 - c] \Phi_i(t) \Delta \pi_i \right]
\]

(29)
Expected attractivity of entrepreneurship for an entrepreneur. An entrepreneur has an estimate of entrepreneurial performance and feedback based on true entrepreneurial performance such that

\[ E[A_{t,i|e}] = \frac{1}{t+1} \phi_i^t \beta [1 - c] \tilde{\pi}_{i,0|e} + \ldots \]

\[ \cdots + \frac{1}{t+1} \left[ \varphi_i(t) \left( [1 - c] (\omega_i \pi_{i,0|e} + [1 - \omega_i] \pi_{i,0}) + [1 - c] \Phi_i(t) \Delta \pi_i \right) \right] \]

(30)

where \( \Delta \pi_i \) is the change in entrepreneurial income given the surplus, which is generated by denying upward wage adjustments to some workers. Furthermore, changes in entrepreneurial income can be larger than changes in wage income due to surplus such that \( \Delta \pi_i \geq \Delta w_i \).

We will consider two time windows: short and long run.

**Short run.** A firm starts with a set of hierarchically assigned workers and neither expands nor shrinks for some periods. And, a newly hired worker’s area of responsibility stays the same for a number of periods. Hence:

\[ \Delta \pi_{i,t} = \Delta w_{i,t} = 0 \]  

(31)

**Long run.** In the long run, we can have non-zero income changes. A unit can be dominated by nascent entrepreneurs, \( \Delta w_i = \Delta \pi_i < 0 \), it can be dominated by applicants, \( \Delta w_i = \Delta \pi_i > 0 \), or applicants and nascent entrepreneurs can perfectly balance each other in the long run, \( \Delta w_i = \Delta \pi_i = 0 \).

There are two possible transitions. An individual can transition from entrepreneur to worker or from worker to entrepreneur. Transitions are determined by differences in attractivity where we can examine expected differences in attractivity.

**Expected difference in attractivity: transition from entrepreneurship to wage work.** Let us turn to the first kind of transitions. Solely type e individuals can abandon entrepreneurship and transform into applicants. Denote relative attractivity of entrepreneurship by

\[ \Delta_{i,t}^{E\rightarrow W} \equiv E[A_{i,t|e}] - E[\tilde{A}_{i,t|e}] \]

A transition to wage work will take place if \( \Delta_{i,t}^{E\rightarrow W} < 0 \). The significant difference is

\[ \Delta_{i,t}^{E\rightarrow W} = \frac{1}{t+1} \phi_i^t \beta \left( [1 - c] \pi_{i,0|e} - w_{i,0|e} \right) + \ldots \]

\[ \cdots + \frac{1}{t+1} \varphi_i(t) \left( [1 - c] (\omega_i \pi_{i,0|e} + [1 - \omega_i] \pi_{i,0}) - w_{i,0|e} \right) + \ldots \]

\[ \cdots + \frac{1}{t+1} \Phi_i(t) \left[ -c \Delta w_i + [1 - c] \Delta \pi_i \right] \]

(32)
where $\Delta \pi_i > 0$ is entrepreneurial surplus. Notice that $[1 - c] \hat{\pi}_{i,0|e} - w_{i,0|e} \geq 0$ because $i$ started as entrepreneur.

**Expected difference in attractivity: transition from wage work to entrepreneurship.** The relevant difference for a transition from worker to entrepreneur, the second kind of transitions, is given by

$$\Delta_{i,t}^{W \rightarrow E} = E[A_{i,t|w}] - E[\hat{A}_{i,t|w}]$$

A transition will take place if $\Delta_{i,t}^{W \rightarrow E} = 0$. The difference can be written as

$$\Delta_{i,t}^{W \rightarrow E} = \frac{\phi_i^t}{t+1} \tilde{\beta} [(1 - c) \hat{\pi}_{i,0|w} - w_{i,0|w}] + \ldots$$

$$\ldots + \frac{1}{t+1} \Phi_i(t) [(1 - c) \bar{\Delta} \pi_i - \bar{\Delta} w_i]$$

(33)

**Push and pull factors.** To organize the analysis, we will disaggregate differences in attractivity into factors pulling towards the present status and factors pushing into the second non-present status. For clarification, a push factor for an entrepreneur is a reason to abandon entrepreneurship. A push factor for a worker is a reason to start an own firm. A pull factor for an entrepreneur has a positive impact on entrepreneurial survival. A pull factor for a worker has a negative impact on entrepreneurial entry.

**Table 2. Transitions from entrepreneur to worker**

<table>
<thead>
<tr>
<th>Factor</th>
<th>Push factor (exit)</th>
<th>Pull factor (do not exit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average periodical change in entrepreneurial income*</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Average periodical change in wage income</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Initial estimated difference in incomes</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>True entrepreneurial income</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Self-confidence</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Surplus</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

* Without surplus

Table 2 shows an overview of pull and push factors influencing entrepreneurial survival. To compile Table 2, we examined the factors in Eq. (32) and (33). Factors that are always positive are constant pull factors; factors that are always negative are constant push factors; and factors that can be negative or positive can be pull or push factors. As can be seen in Table 2, there are as many factors advancing exit as factors advancing survival (non-exit) and most factors advancing survival can also stimulate exit. The initial estimated difference in incomes and entrepreneurial surplus are the only factors constantly supporting entrepreneurship, while all other factors (can) support the exit from entrepreneurship.
Table 3. Transitions from worker to entrepreneur

<table>
<thead>
<tr>
<th>Factor</th>
<th>Push factor (enter)</th>
<th>Pull factor (do not enter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential average periodical change in entrepreneurial income</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Average periodical change in wage income</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Initial estimated difference in incomes</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

Note that average periodical changes in wage income are weakly below 0 in the bounded wage regime.

Table 3 presents factors that influence transitions into entrepreneurship, however only after the first period of the model. The only factor unambiguously encouraging entry into entrepreneurship is the average periodical change in wage income. Since entrepreneurs bound wages from above, changes in wage incomes are restricted to negative values only. Changing the status from wage worker to entrepreneur removes the restriction on income changes. Hence, changes in wages represent opportunities withheld by the wage regime.

b.3 Impact of different factors on the timing of transition

Assume that a transition takes place. We are interested in how each factor affects the timing of transitions or put differently: Does a factor rather delay or accelerate a transition? First, we examine transitions in the short run. Second, transitions are depicted in the long run. Last, insights are summarized in one table, which is possible as long and short run results are consistent.

b.3.1 Short-run transitions

First, consider the short run without income changes.

Worker-to-entrepreneur transitions. In the short run, workers do not transition to entrepreneurship, as in the short run there are no changes in incomes. This also means that there are no denied opportunities. Put differently, if we ignore income changes, the only remaining factor influencing transitions is the initially estimated difference in incomes. But, the initially estimated difference in incomes is always a factor supporting wage work (see Table 3 where the estimated difference is always a pull factor).

Entrepreneur-to-worker transitions. Entrepreneurs may decide to abandon their business even in the short run. Given an arbitrary variable $z$, the marginal effect of $z$ on the pre-transition period $T_{i}^{E \rightarrow W}$ (a transition takes place in $T_{i}^{E \rightarrow W} + 1$) is

$$
\frac{\partial T_{i}^{E \rightarrow W}}{\partial z} = \frac{\partial \log(\phi_i)}{\partial z} \log(-\lambda_{i|e} / [\tilde{\beta}(1 - \phi_i) - \lambda_{i|e}]) + \ldots
$$

$$
\ldots + \frac{1}{\log(\phi_i)(-\lambda_{i|e} / [\tilde{\beta}(1 - \phi_i) - \lambda_{i|e}])} \frac{\partial (-\lambda_{i|e} / [\tilde{\beta}(1 - \phi_i) - \lambda_{i|e}])}{\partial z}
$$
\[
\lambda_{i|e} = \frac{[1 - c] \{\omega_i \hat{\pi}_{i,0} + [1 - \omega_i] \pi_{i,0}\} - w_{i,0}}{\hat{\pi}_{i,0} - w_{i,0}}
\]

where \(\log(\phi_i) < 0\). Note that if \(\lambda_{i|e} < 0\), internalized income is smaller than wage income in expectations. If \(\lambda_{i|e} > 0\), internalized income is larger than wage income in expectations and \(T_i^{E-W}\) is not defined, i.e. a transition will not occur because all factors are pull factors. We can establish the following:

**Marginal effects on the period of transition from entrepreneur to worker in the short run (EEWS).** **Marginal effects are as follows:**

\[
\frac{\partial}{\partial \phi} T_i^{E-W} > 0, \frac{\partial}{\partial \beta} T_i^{E-W} > 0, \frac{\partial}{\partial \pi_{i,0}} T_i^{E-W} > 0
\]

\[
\frac{\partial}{\partial c} T_i^{E-W} = \begin{cases} 
0 & \text{if entrepreneurial performance was overestimated} \\
> 0 & \text{if performance estimated correctly} \\
< 0 & \text{if entrepreneurial performance was underestimated}
\end{cases}
\]

\[
\frac{\partial}{\partial \omega_i} T_i^{E-W} = \begin{cases} 
0 & \text{if entrepreneurial performance was overestimated} \\
> 0 & \text{if performance estimated correctly} \\
< 0 & \text{if entrepreneurial performance was underestimated}
\end{cases}
\]

\[
\frac{\partial}{\partial \chi} T_i^{E-W} > 0
\]

**Proof.** To see, note:

i. \(\partial \log(\phi_i) / \partial \phi_i > 0\) and \(\partial ( -\lambda_{i|e}/[\bar{\beta}(1 - \phi_i) - \lambda_{i|e}] ) / \partial \phi_i < 0\). Hence: \(\partial T_i^{E-W} / \partial \phi_i > 0\).

ii. \(\partial \log(\phi_i) / \partial \beta = 0, \partial \bar{\beta} / \partial \beta > 0, \text{and } \partial ( -\lambda_{i|e}/[\bar{\beta}(1 - \phi_i) - \lambda_{i|e}] ) / \partial \beta < 0\). Hence: \(\partial T_i^{E-W} / \partial \beta > 0\).

iii. \(\partial \log(\phi_i) / \partial \chi = 0\) and

\[
\frac{\partial \lambda_{i|e}}{\partial \chi} = \frac{[1 - \omega_i][\hat{\pi}_{i,0|e} - \pi_{i,0|e}]w_{i,0|e}}{w_{i,0|e}(2[1 - c]\hat{\pi}_{i,0|e} - w_{i,0|e}) - (c - 1)^2\hat{\pi}_{i,0|e}^2}
\]

where \(\partial \lambda_{i|e} / \partial \chi = 0\) if \(\hat{\pi}_{i,0|e} = \pi_{i,0}\), \(\partial \lambda_{i|e} / \partial \chi > 0\) if \(\hat{\pi}_{i,0|e} > \pi_{i,0}\), and \(\partial \lambda_{i|e} / \partial \chi < 0\) if \(\hat{\pi}_{i,0|e} < \pi_{i,0}\). Hence: If entrepreneurial performance was overestimated, \(\partial T_i^{E-W} / \partial \chi > 0\) and if underestimated, \(\partial T_i^{E-W} / \partial \chi < 0\).

iv. \(\partial \log(\phi_i) / \partial \pi_{i,0} = 0\) and

\[
\frac{\partial \lambda_{i|e}}{\partial \pi_{i,0}} = \frac{(1 - c)(1 - \omega_i)}{(1 - c)\hat{\pi}_{i,0|e} - w_{i,0|e}} > 0
\]

Hence: \(\partial T_i^{E-W} / \partial \pi_{i,0} > 0\).
v. \[ \frac{\partial \log(\phi_i)}{\partial \omega_i} = 0 \] and

\[ \frac{\partial \lambda_{i,e}}{\partial \omega_i} = \frac{(1 - c)(\hat{\pi}_{i,0}e - \pi_{i,0})}{(1 - c)\hat{\pi}_{i,0}e - w_{i,0}e} \]

Hence: If entrepreneurial performance was overestimated,

\[ \frac{\partial T_{i}^{E \rightarrow W}}{\partial \omega_i} > 0 \]

and if underestimated,

\[ \frac{\partial T_{i}^{E \rightarrow W}}{\partial \omega_i} < 0 \]

diagram

vi. The effect of the initial income difference on the exit period is obviously positive.

\[ \square \]

b.3.2 Long-run transitions

Second, we want to examine the long run.

Notice that every pre-transition period (a transition takes place in the next period) can be written as

\[ T_{i|\Delta_{i},t=0} = r - \frac{1}{\log(\phi_i)} W_0 \left( -\frac{\log(\phi_i) \exp\{\log(\phi_i)r\}}{\bar{r}} \right) \]

where \( W(\cdot) \) denotes Lambert’s W function, i.e. a function such that

\[ t = W(t) \exp\{W(t)\} \]

for \( t \in \mathbb{R} \). \[ ^{36} \] We will only need the upper branch \( W_0 \).

Given an arbitrary variable \( z \neq \phi_i \), we get

\[ \frac{\partial T_{i|\Delta_{i},t=0}}{\partial z} = \frac{\partial r}{\partial z} - \frac{1}{\log(\phi_i)} \frac{W_0 \left( -\frac{\log(\phi_i) \exp\{\log(\phi_i)r\}}{\bar{r}} \right)}{1 + W_0 \left( -\frac{\log(\phi_i) \exp\{\log(\phi_i)r\}}{\bar{r}} \right)} \times \ldots \]

\[ \ldots \times \frac{\partial \left\{ -\frac{\log(\phi_i) \exp\{\log(\phi_i)r\}}{\bar{r}} \right\}}{\partial z} \]

where

\[ \frac{\partial \left\{ -\frac{\log(\phi_i) \exp\{\log(\phi_i)r\}}{\bar{r}} \right\}}{\partial z} = \frac{\log(\phi_i) \exp\{\log(\phi_i)r\} \frac{\partial r}{\partial z} - \log(\phi_i)^2 \exp\{\log(\phi_i)r\} \bar{r} \frac{\partial r}{\partial z}}{\bar{r}^2} \]

\[ ^{36} \] Thus we need to solve an equation of the form \( \exp(-qt) = q_0(t - q_1) \) for \( t \) where \( q, q_0, \) and \( q_1 \) are arbitrary real constants.
If \( z = \phi_i \), we have

\[
\frac{\partial T_i}{\partial \phi_i} \bigg|_{\Delta_i=t=0} = \frac{\partial r}{\partial \phi_i} + \frac{1}{\phi_i \log(\phi_i)} W_0 \left( - \log(\phi_i) \exp\{\log(\phi_i)r\} \right) - \ldots
\]

\[
\ldots - \frac{1}{\log(\phi_i)} \frac{W_0 \left( - \log(\phi_i) \exp\{\log(\phi_i)r\} \right)}{\log(\phi_i) \exp\{\log(\phi_i)r\}} \left[ 1 + W_0 \left( - \log(\phi_i) \exp\{\log(\phi_i)r\} \right) \right] \times \ldots
\]

\[
\ldots \times \frac{\partial \left\{ - \frac{\log(\phi_i) \exp\{\log(\phi_i)r\}}{\bar{r}} \right\}}{\partial \phi_i}
\]

where

\[
\frac{\partial \left\{ - \frac{\log(\phi_i) \exp\{\log(\phi_i)r\}}{\bar{r}} \right\}}{\partial \phi_i} = \frac{\log(\phi_i) \exp\{\log(\phi_i)r\} \frac{\partial \bar{r}}{\partial \phi_i}}{\bar{r}^2} - \ldots
\]

\[
\ldots = \frac{\log(\phi_i) \exp\{\log(\phi_i)r\} \left[ \log(\phi_i) \frac{\partial r}{\partial \phi_i} + \frac{\bar{r}}{\phi_i} \right]}{\bar{r}} - \ldots
\]

\[
\ldots - \exp\{\log(\phi_i)r\} \frac{1}{\phi_i \bar{r}}
\]

To get a specific transition type, we have to replace \( r \) and \( \bar{r} \) by appropriate values.

**Worker-to-entrepreneur transitions.** We have to replace \( r \) by \( r_1 \) and \( \bar{r} \) by \( \bar{r}_1 \):

\[
r_1 = \frac{(\phi_i - 1)^2 \kappa_i |w| - (1 - \phi_i) \phi_i \kappa_i}{(1 - \phi_i) \kappa_i}
\]

\[
\bar{r}_1 = \frac{(1 - \phi_i)^2(1 - \beta) \bar{\kappa}_i}{(\phi_i - 1)^2(1 - \beta) \kappa_i |w| - (\phi_i - 1)^2(1 - \phi_i) \beta \kappa_i |w| - (1 - \phi_i)(1 - \beta) \bar{\kappa}_i \phi_i}
\]

\[\kappa_i |w| = [1 - c] \pi_{i,0} |w| - w_{i,0} |w|\]

\[\bar{\kappa}_i = [1 - c] \Delta \pi_i - \Delta w_i \]

We restrict: \( \bar{r}_1 > 0 \) and \( \Delta w_i \leq 0 \). The next restriction is that \( i \) started as a worker: \( \kappa_i |w| < 0 \). There is one unique push factor. A push can come from \( \kappa_i |w| > 0 \).

**Entrepreneur-to-worker transitions.** \( r \) has to be replaced by \( r_2 \) and \( \bar{r} \) by \( \bar{r}_2 \) where

\[
r_2 = \frac{(1 - \phi_i) \phi_i \Delta I - (\phi_i - 1)^2 \kappa_i |e|}{(1 - \phi_i)^2 \Delta I}
\]

\[
\bar{r}_2 = \frac{(1 - \phi_i)^2 \kappa_i |e| - \beta(1 - \phi_i)(\phi_i - 1)^2 ((1 - c) \pi_{i,0} |e| - w_{i,0} |e|) - (1 - \phi_i) \phi_i \Delta I}{(1 - \phi_i)^2 \Delta I}
\]

\[\kappa_i |e| = [1 - c] \omega_i \pi_{i,0} |e| + [1 - c] \omega_i \pi_{i,0} |e| - w_{i,0} |e|\]
\[ \Delta I = -c \Delta w_i + [1 - c] \Delta \pi_i^s \]

Assume \( \tilde{r}_2 > 0 \). By definition, we have \( \Delta \pi_i^s \geq 0 \). Since \( i \) started as an entrepreneur, \( \hat{\pi}_{i,0|e} > 0 \) and \( [1 - c] \hat{\pi}_{i,0|e} - w_{i,0|e} > 0 \). A push can come from \( \kappa_{il} < 0 \) or from \( \Delta I < 0 \) (or from both).

**Evaluation of marginal effects.** Derivatives are too complex to deliver simple analytical results but we can derive the most common effect with the aid of a numerical evaluation. We are interested in the impact of \( \phi_i, \omega_i, \beta, c, w_{i,0}, \hat{\pi}_{i,0}, \pi_{i,0}, \Delta w_i, \Delta \pi_i, \) and \( \Delta \pi_i^s \). Only effects that present some general interest are taken into account.

We can exploit the fact that some variables have natural boundaries, that some explicit conditions must hold, and that only relative differences matter for a numerical evaluation of the long run. \( \phi_i, \omega_i, \beta, \) and \( c \) (group-1 variables) are located in the interval \((0, 1)\). Assume furthermore that \( w_{i,0} \in (0, 8] \), \( \hat{\pi}_{i,0}, \pi_{i,0} \in [-8, 8], \Delta w_i, \Delta \pi_i \in [-3, 3], \) and \( \Delta \pi_i^s \in [0, 3] \) (group-2 variables). The restriction on group-2 variables is unproblematic since for a transition only differences matter.

For transitions from worker to entrepreneur, we use a grid with step size 0.2 for group-1 variables and step size 1 for group-2 variables. The resulting grid is a \( 12,529 \times 7 \) matrix (7 variables and 12,529 combinations of these variables). For transitions from entrepreneurship to wage work, we use a grid with step size 0.2 for group-1 variables and a step size of 1.5 for group-2 variables. The resulting overall grid is a matrix with dimensions \( 641,602 \times 9 \).

An overview over dominating marginal effects (most common effects) is depicted in Table 4. A positive value shows that the transition period is delayed while a negative value stands for an acceleration given a marginal increase in the variable examined.

**Table 4. Dominating marginal effects conditional on defined grid**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Entry into entrepreneurship</th>
<th>Exit from entrepreneurship</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_i )</td>
<td>( &gt; 0^* )</td>
<td>( &gt; 0 )</td>
</tr>
<tr>
<td>( \omega_i )</td>
<td>( &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>( &gt; 0^* )</td>
<td>( &gt; 0 )</td>
</tr>
<tr>
<td>( \hat{\pi}_{i,0} )</td>
<td>( &lt; 0^* )</td>
<td>( &gt; 0^* )</td>
</tr>
<tr>
<td>( w_{i,0} )</td>
<td>( &gt; 0^* )</td>
<td>( &lt; 0^* )</td>
</tr>
<tr>
<td>( \pi_{i,0} )</td>
<td></td>
<td>( &gt; 0 )</td>
</tr>
<tr>
<td>( c )</td>
<td>( &gt; 0 )</td>
<td>( &lt; 0 )</td>
</tr>
<tr>
<td>( \Delta \pi_i )</td>
<td>( &lt; 0^* )</td>
<td></td>
</tr>
<tr>
<td>( \Delta w_i )</td>
<td></td>
<td>( &gt; 0^* )</td>
</tr>
<tr>
<td>( \Delta \pi_i^s )</td>
<td></td>
<td>( &lt; 0 )</td>
</tr>
<tr>
<td>( \Delta w_i )</td>
<td></td>
<td>( &gt; 0 )</td>
</tr>
</tbody>
</table>

*Note: \( * \) signifies that the effect is not only dominant but also the only measurable effect.*

37 Since the number of variables interacting is high, there will be some special cases depending on circumstances of no general interest, which we will ignore.
### Summary on marginal effects on transition period

#### Table 5. General impact of an increase in model variables on a transition

<table>
<thead>
<tr>
<th>Variable</th>
<th>Entry into entrepreneurship</th>
<th>Exit from entrepreneurship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average periodical change in (potential) entrepreneurial income</td>
<td>acceleration</td>
<td>acceleration</td>
</tr>
<tr>
<td>Average periodical change in (potential) wage income</td>
<td>delay</td>
<td>delay</td>
</tr>
<tr>
<td>Average periodical surplus</td>
<td>delay</td>
<td>delay</td>
</tr>
<tr>
<td>True entrepreneurial income</td>
<td>delay</td>
<td>delay</td>
</tr>
<tr>
<td>Responsiveness</td>
<td>delay</td>
<td>delay</td>
</tr>
<tr>
<td>Costs of entrepreneurship</td>
<td>delay</td>
<td>acceleration</td>
</tr>
<tr>
<td>Initial estimated difference in incomes</td>
<td>delay</td>
<td>delay</td>
</tr>
<tr>
<td>Self-confidence when faced with market feedback</td>
<td>delay</td>
<td>delay</td>
</tr>
<tr>
<td>Discount factor</td>
<td>delay</td>
<td>delay</td>
</tr>
</tbody>
</table>

*Note: We depict only most common effects, i.e. we ignore effects relying on a too specific parameter constellation.*

If we combine our insights gathered in the analysis of short and long run, we can compile Table 5. Long run effects do not contradict short run effects summarized by EEWS such that we can combine all results on marginal effects in one table.

Table 5 gathers all variables’ effects on the entry time of nascent entrepreneurs into entrepreneurship and entrepreneurial exit. Every variable works in the long run but the first three variables (changes in entrepreneurial income, changes in wage income, and surplus) are purely long-term variables that do not influence decisions in the short run.

With respect to marginal effects of trait-related variables, we can establish the following results. True entrepreneurial income influenced by performance traits delays a transition from entrepreneur to worker. Responsiveness determined by responsiveness traits delays a transition from worker to entrepreneur and from entrepreneur to worker. Hence, increasing responsiveness will results in later transitions to entrepreneurship but those who become entrepreneur will survive longer. Confidence determined by self-confidence traits has a positive effect on entrepreneurial survival as well, in the sense that more confident individuals survive longer. An increase in estimated self-management productivity affected by skill-related traits results in an increase of initial estimated difference in incomes of an entrepreneur. Consequently, increasing estimated entrepreneurial income will prolong entrepreneurship. Note that the analytical
approach cannot take into account relations (such as correlations) between trait sets.
This section explains the shape of survival curves in our approach. We can summarize our findings in the following manner.

**Factors influencing the shape of survival curves (FICS).** There are three factors affecting survival dynamics: (a) the initial estimated difference in incomes; (b) new information; and (c) periodical changes in entrepreneurial income. The initially assumed difference in incomes is always a factor supporting entrepreneurship. The other two factors (new information and periodical changes) can either support entrepreneurship or encourage a transition to wage work. The pull from the initial difference will lose its influence during the first periods such that a transition to wage work is most likely right after beginning. This generates the turbulent phase. If responsiveness is not too low, relative weights will stabilize in time. This generates what we called the consolidation phase where the number of transitions from entrepreneurship to wage work is minimal.

**Demonstration of FICS.** To show that FICS holds, we will demonstrate the following. First, the initial assumed difference in incomes is assigned a decreasing weight, which decreases at a decreasing rate. Weight decreases are more pronounced in early periods. Second, individuals with very low responsiveness will generally assign an increasing weight to new income level information but weights will increase at a decreasing rate. Individuals with very high responsiveness will assign a decreasing weight with a decreasing rate to new information. Individuals who are not very low and not very high in responsiveness can assign an increasing weight to new information in the first periods but then switch to a decreasing weight in later periods. Third, periodical changes are assigned an increasing weight, which increases at a decreasing rate.

Consider the following equation:

\[ \Delta E_{i,t}^{W} = w_a(t)\nu_a + w_b(t)\nu_b + w_c(t)\nu_c \]  

(34)

where

\[ w_a(t) = \frac{\phi_i^t}{t+1}, \quad w_b(t) = \frac{1 - \phi_i^t}{(1 - \phi_i)(t+1)}, \quad w_c(t) = \frac{(1 - \phi_i)t + \phi_i^{t+1} - \phi_i}{(\phi_i - 1)^2(t+1)} \]

\[ \nu_a = \tilde{\beta} (\{1 - c\} \hat{\pi}_{i,0}\nu_e - w_{i,0}\nu_e) \],  

\[ \nu_b = (\{1 - c\} \{\omega_i \hat{\pi}_{i,0}\nu_e + \{1 - \omega_i\} \pi_i,0\} - w_{i,0}\nu_e) \]

\[ \nu_c = ([1 - c] - 1) \Delta w_i + [1 - c] \Delta \pi_i \]

Note that Eq. (34) is just a reproduction of incentives in expectations to abandon entrepreneurship we already presented. \( \nu_a \) is the initial estimated relative attractiveness of entrepreneurship; \( \nu_b \) is the attractiveness of entrepreneurship given that individual \( i \) experiences her true productivity \( \pi_{i,0} \) and compares it with her assumed productivity \( \hat{\pi}_{i,0} \nu_e \), given \( i \)'s confidence parameter \( \omega_i \); \( \nu_c \) represents
changes in entrepreneurial income. \( w_a, w_b, \) and \( w_c \) are time-dependent weights assigned to components of incentives \( u_a, u_b, \) and \( u_c. \)

\[ u_c \geq 0 \text{ if } c \Delta \pi_i / (1-c) \geq \Delta w_i, \text{ which is possible, and } u_c < 0 \text{ if } c \Delta \pi_i / (1-c) < \Delta w_i, \text{ which is also possible. We assume } u_a > 0 \text{ such that the decision for entrepreneurship was not a decision made on a razor’s edge. Depending on } \pi_i, u_i, \text{ and } \omega, u_b \text{ can be negative, positive, or zero.} \]

Note that \( w_a(t) \) and \( w_b(t) \) converge to 0 as \( t \to \infty \), while \( w_c(t) \) converges to \( (1-\phi_1)/ (\phi_1 - 1)^2 > 0 \). If entrepreneurs believe that income changes of a worker and an entrepreneur are exactly the same and positive in expectations, all entrepreneurs are “asymptotic workers” since, then, \( u_c = -c \Delta w_i < 0. \)

The reason is that initial attractivity \( u_a \) and internalized income \( u_b \) will fade completely in time, while costs are external and never fade completely. If entrepreneurs believe that income changes of entrepreneurs are larger than income changes of workers, entrepreneurs can be “asymptotic entrepreneurs” if \( u_c \geq 0 \).

The initial difference \( u_a \) is always a pull factor. \( u_b \) and \( u_c \) can be pull or push factors. Furthermore, \( w_a(t), w_b(t), w_c(t) > 0 \) for all \( t = 1, 2, \ldots \). \( w_a(t), w_b(t), \) and \( w_c(t) \) are augmenting pushes and pulls as time passes. They can support a push, support a pull, or reduce pushes and pulls. Consequently, we have to check the dynamics of \( w_a(t), w_b(t), \) and \( w_c(t) \). Let \( \dot{w} = dw(t)/dt \) and \( \ddot{w} = d^2w(t)/dt^2 \).

**Dynamics of \( w_a \).** Although \( t \) is discrete, consider

\[
\dot{w}_a(t) = \frac{\phi_i^t \log(\phi_i)}{t+1} - \frac{\phi_i^t}{(t+1)^2}
\]

\[
\ddot{w}_b(t) = -\frac{\phi_i^t \log(\phi_i)^2}{t+1} - \frac{2\phi_i^t \log(\phi_i)}{(t+1)^2}
\]

Given that \( \phi_i \in (0, 1) \), it is easy to see that \( \dot{w}_a < 0 \) and \( \ddot{w}_b > 0 \). Hence, most of initial attractivity \( u_a \) is lost in the first periods.

**Dynamics of \( w_b \).** Consider

\[
\dot{w}_b(t) = \frac{\psi_1(\phi_i,t)}{(1-\phi_i)(t+1)^2}
\]

\[
\psi_1(\phi_i,t) = \phi_i^t (1-(t+1) \log(\phi_i)) - 1
\]

\( w_b(t) \) is not necessarily a reduction rate because \( \psi_1(\phi_i,t) \) can be positive, negative, or zero. \( (1-\phi_i)(t+1)^2 \) is always positive. Since \( \phi_i \) has natural boundaries and we are only interested in the difference between first periods and later periods (but not the latest periods), we can evaluate \( \psi_1(\phi_i,t) \) numerically for \( t = 1, \ldots, T \). Examine Fig. 8a where we plot if either \( \psi_1(\phi_i,t) > 0 \) (coded with 1) or \( \psi_1(\phi_i,t) < 0 \) (coded with -1) for \( T = 20 \). If \( \psi_1(\phi_i,t) > 0 \), we have \( \dot{w}_b(t) > 0 \) and \emph{vice versa}. There are no instances where \( \psi_1(\phi_i,t) \) is exactly 0. The effect

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38 An interpretation is that entrepreneurs “retire” one day, \emph{i.e.} they start to prefer a fixed income without additional costs.
shown in Fig. 8a is realistic. $v_b$ provides new information. If individuals are low in responsiveness (i.e. have a high $\phi_i$), new information is boosted over a number of periods and then starts to be forgotten. An individual who is high in responsiveness (i.e. has a low $\phi_i$) tends to forget new information faster.

Now, consider

$$\tilde{w}_b(t) = \frac{\psi_2(\phi_i, t)}{(1 - \phi_i)(t + 1)^3}$$

$$\psi_2(\phi_i, t) = -\phi_i^j \log(\phi_i)^2(t + 1)^2 + 2\phi_i^j \log(\phi_i)(t + 1) + 2(1 - \phi_i^j)$$

Fig. 8b describes the behavior of $\tilde{w}_b$ for $T = 20$ (the coding is the same as in Fig. 8a).

We can combine Fig. 8a and 8b to arrive at the following conditional statement: (i) If individuals have low responsiveness, the subjective value of new information, where new information is represented by $v_b$, can first increase
but the increase is weakened. (ii) If responsiveness is high, the value of new information can decrease and the decrease can be weakened or accelerated. (iii) If we start with a value for responsiveness that is not too high and not too low, we will most likely have \( \dot{w}_b > 0 \) and \( \ddot{w}_b < 0 \) in the first periods, and \( \dot{w}_b < 0 \) and \( \ddot{w}_b > 0 \) in later periods. Note that dynamics of \( w_b(t) \) are the exact opposite of dynamics of \( w_a(t) \) in the first periods, and become similar in later periods. 

**Dynamics of \( w_c \).** Consider 

\[
\dot{w}_c(t) = \frac{\psi_3(\phi_i, t)}{(\phi_i^2 - 2\phi_i + 1)t^2 + (2\phi_i^2 - 4\phi_i + 2)t + (\phi_i^2 - 2\phi_i + 1)}
\]

\[
\psi_3(\phi_i, t) = \phi_i^{t+1} \log(\phi_i) + \phi_i^{t+1} (\phi_i \log(\phi_i) - \phi_i) + 1
\]

It is easy to verify that \((\phi_i^2 - 2\phi_i + 1)t^2 + (2\phi_i^2 - 4\phi_i + 2)t + (\phi_i^2 - 2\phi_i + 1) > 0\). It is also obvious that \( \psi_3(\phi_i, t) > 0 \) for our area of interest. A simple condition for \( \psi_3(\phi_i, t) > 0 \) is

\[
\phi_i^{t+1} [\log(\phi_i)(t + \phi_i) - \phi_i] > -1
\]

which can be shown to hold for a sufficiently large \( t \). Hence: \( \dot{w}_c > 0 \).

Now, consider 

\[
\dot{w}_c(t) = \frac{\psi_4(\phi_i, t)}{(\phi_i^2 - 2\phi_i + 1)t^3 + (3\phi_i^2 - 6\phi_i + 3)t^2 + (3\phi_i^2 - 6\phi_i + 3)t + (\phi_i^2 - 2\phi_i + 1)}
\]

\[
\psi_4(\phi_i, t) = \phi_i^{t+1} \log(\phi_i)^2 t^2 + \phi_i^{t+1} (2\phi_i \log(\phi_i) - 2\phi_i \log(\phi_i)) t + \ldots
\]

\[
\ldots + \phi_i^{t+1} (\phi_i \log(\phi_i)^2 - 2\phi_i \log(\phi_i) + 2\phi_i) - 2
\]

It is easy to see that the denominator of \( \dot{w}_c \) is positive by checking every expression in brackets not depending on time. In case of the nominator, \( \psi_4(\phi_i, t) \), we just have to verify that \( \psi_4(\phi_i, t) + 2 > 0 \), that \( \psi_4(\phi_i, 1) < 0 \), that \( \psi_4(\phi_i, t) + 2 \) is decreasing in \( t \), and that \( \lim_{t \to \infty} (\psi_4(\phi_i, t) + 2) = 0 \) to get \( \psi_4(\phi_i, t) < 0 \) and consequently \( \ddot{w}_c < 0 \).

Thus, the weight of the periodical change in income \( v_c \) increases with time at a decreasing rate. This is different from the effect on \( v_a \) and \( v_b \) because periodical changes are observed in every period such that pressure accumulates instead of being reduced. 

**Combining all arguments.** The argument for survival curves to be decreasing at a decreasing rate can be revealed if we combine all our insights. We plot weight dynamics for \( T = 20 \) and \( \phi_i \in \{0.1, 0.5, 0.9\} \) in Fig. 9. If responsiveness is high \((i.e. \phi_i = 0.1)\), the new information weight \( w_b \) is decreasing. If responsiveness is middle \((i.e. \phi_i = 0.5)\), the new information weight \( w_b \) first increases and then decreases. The same holds if responsiveness is low \((i.e. \phi_i = 0.9)\) but the period where the new information weight \( w_b \) increases is longer compared to a middle responsiveness level.

The deciding factor is the difference in weights. As we can observe in Fig. 9, the difference in weights mostly increases in the first periods for \( \phi_i \in \{0.1, 0.5\} \) and stabilizes in later periods. For \( \phi_i = 0.9 \), the difference between the weights
Figure 9. Weight dynamics

of initial attractivity $w_a$ and new information $w_b$ also mostly increases in first periods and stabilizes in later periods but the distance to the weight of income changes $w_c$ does not stabilize in sufficient time.

Initial attractivity is a pull factor (a factor supporting survival). Assume that new information results in a push (a factor supporting exit). When does a transition take place? It will take place in the first periods because these are the time periods where pushes get the largest relative weight—relative to pulls.

Assume that income changes $u_c$ are a push factor. If responsiveness is not low, the push from $u_c$ increases its relative weight (relative to the pull from $u_a$ and a potential pull from $u_b$) mostly in the first periods. Hence, we have the same argument pointing at a high exit probability in the first but not in later periods.
If responsiveness is very low, the probability to exit is not necessarily highest in the first periods. This is demonstrated by $\phi_i = 0.9$ in Fig. 9. A low responsiveness tends to produce a large increase in the weight of income changes that does not stabilize in the short term. This can be explained as follow: An (expected) income change is seen in every period. We established that a push from income changes accumulates up to a certain point of stability near $(1 - \phi_i)/(\phi_i - 1)^2$, which is increasing in $\phi_i$. So, if individuals fully remember almost all income changes ($\phi_i$ is near 1), the push from $\nu_c$ will take a large number of periods to stabilize. If $\nu_c$ is a pull factor, it is weakest in the first periods—as can be seen in Fig. 9, income changes $\nu_c$ will get the smallest weight in earlier periods. As a consequence, the counterweight from $\nu_c$ will be smallest in first periods resulting in an increased probability to exit in these periods compared to later time periods.

To summarize: Given a level of responsiveness that is not too low—a $\phi_i$ that is not too high—the probabilities to survive will mostly decrease in first periods and stabilize in later periods.
D Impact of personality on overall survival

This section investigates how strongly different personality traits affect the aggregate survival outcome. The main result is as follows.

Relative importance of self-confidence traits (RISC). Compared to responsiveness and the degree of self-assessment errors, confidence has a larger positive effect (given an one unit increase) on survival.

Demonstration of RISC. We will now demonstrate RISC. We are interested in the impact of the block of variables that rationalizes population mean confidence traits, population mean responsiveness traits, and the degree of self-assessment errors (viz. the levels of average confidence $\mu_\omega$ and average responsiveness $\mu_\phi$, and the correlation between real and expected self-management productivities $\gamma$).

We vary $\mu_\phi$ and $\mu_\omega$ in steps of 0.2 such that $\mu_\phi, \mu_\omega \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, i.e. we avoid too extreme average parameter characteristics in personality, while intermediate personalities are all present. Moreover, we vary $\gamma$ in steps of 0.9, i.e. $\gamma \in \{0, 0.9\}$, such that there are exactly two states with respect to self-assessment errors: There can be a full self-perception gap ($\gamma = 0$) or the self-perception gap can be almost closed ($\gamma = 0.9$). For the other parameters we assume the same values as in the main text. Parameter combinations result in a parameter grid we can iterate over. We compute 20 simulations for each combination of parameters.

We concentrate on entrepreneurs who started their business in period 1 (entrepreneurs without any work-experience) and follow cohorts from period 1 until the last period $T + 1$. Conditional on a certain parameter constellation, every survivorship curve can be characterized by one central outcome: the share of entrepreneurs who started in the first period and survived until the last period. By measuring the impact of changes in the variables $\mu_\omega, \mu_\phi$ and $\gamma$ on the share of last-period survivors, we can measure the effect of average confidence, average responsiveness, and the degree of self-assessment errors on overall survival outcomes.

Simulation results are summarized in Fig. 10. A given point in Fig. 10 fixes the value of one parameter, while other parameters can vary. In other words, we do not use a full ceteris paribus assumption. There is one distinct pattern in Fig. 10. No matter what values the other two parameters take, more confidence is associated with a higher lower (minimal share of last-period survivors) and upper boundary (maximal share of last-period survivors) of survival. The boundaries can be interpreted as the best-possible outcome (upper boundary) and the worst-possible outcome (lower boundary) given a certain value of one parameter.

For a comparative analysis, consider Fig. 11 where we plot the change in the average survival outcome (average share of survivors) in the last period per

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39 Remember that occupational status in period $t$ is determined by relative attractivity in $t − 1$ such that we have status data for $T + 1$. 

Impact of confidence, responsiveness, and self-assessment errors on survival

- Confidence
- Correlation between estimated and true productivities
- Responsiveness

Note: Points are jittered to avoid overplotting.

Figure 10. Impact of personality population averages on entrepreneurial survival

Figure 11. Confidence, responsiveness, self-assessment errors, and average survival outcomes

one-step parameter increase. Note that we compute effects induced by changes in one parameter given that the other two parameters vary. More confidence, less self-assessment errors, and less responsiveness all increase the probability to be an entrepreneur in the last period for individuals who were entrepreneurs in the first period. However, confidence has the largest impact of all variables and so it is the most important variable of the three. Independent from the other personality trait responsiveness and the degree of self-assessment errors, a more confident group of individuals will have a notably higher probability to survive than a less confident group.