Lechthaler, Wolfgang; Tesfaselassie, Mewael

**Working Paper**

**Trend growth, unemployment and optimal monetary policy**


**Provided in Cooperation with:**
Kiel Institute for the World Economy (IfW)

Suggested Citation: Lechthaler, Wolfgang; Tesfaselassie, Mewael (2015) : Trend growth, unemployment and optimal monetary policy, Kiel Working Paper, No. 2003, Kiel Institute for the World Economy (IfW), Kiel

This Version is available at:
http://hdl.handle.net/10419/112796

**Standard-Nutzungsbedingungen:**
Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.
Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.
Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Trend Growth, Unemployment and Optimal Monetary Policy
by Wolfgang Lechthaler and Mewael Tesfasellassie

No. 2003 | August 2015
Trend Growth, Unemployment and Optimal Monetary Policy

by Wolfgang Lechthaler and Mewael Tesfaselassie

Abstract:

We analyze the implications of changes in the trend growth rate for optimal monetary policy in the presence of search and matching unemployment. We show that trend growth in itself does not generate a trade-off for the monetary authority, but that it interacts importantly with the inefficiencies stemming from the labor market. Higher trend growth exacerbates the inefficiencies of the labor market and therefore calls for larger deviations from price stability.

Keywords: E12; E24; E52

JEL classification: trend growth, trend inflation, unemployment

Wolfgang Lechthaler
Kiel Institute for the World Economy
24106 Kiel, Germany
E-mail: wolfgang.lechthaler@ifw-kiel.de

Mewael Tesfaselassie
Kiel Institute for the World Economy
24106 Kiel, Germany
E-mail: mewael.tesfaselassie@ifw-kiel.de

The responsibility for the contents of the working papers rests with the author, not the Institute. Since working papers are of a preliminary nature, it may be useful to contact the author of a particular working paper about results or caveats before referring to, or quoting, a paper. Any comments on working papers should be sent directly to the author.

Coverphoto: uni_com on photocase.com
Trend Growth, Unemployment and Optimal Monetary Policy

Wolfgang Lechthaler* and Mewael F. Tesfaselassie**

July 15, 2015

Abstract

We analyze the implications of changes in the trend growth rate for optimal monetary policy in the presence of search and matching unemployment. We show that trend growth in itself does not generate a trade-off for the monetary authority, but that it interacts importantly with the inefficiencies stemming from the labor market. Higher trend growth exacerbates the inefficiencies of the labor market and therefore calls for larger deviations from price stability.

JEL Classification: E12; E24; E52

Keywords: trend growth, trend inflation, unemployment.

---

*Kiel Institute for the World Economy, Kiellinie 66, 24105 Kiel, Germany. E-mail: wolfgang.lechthaler@ifw-kiel.de.
**Kiel Institute for the World Economy, Kiellinie 66, 24105 Kiel, Germany. E-mail: mewael.tesfaselassie@ifw-kiel.de.
1 Introduction

The slow recovery after the Great Recession experienced in many countries sparked a new debate about whether GDP growth has permanently slowed down. Most prominently, Larry Summers publicised the notion that the developed world has entered a period of secular stagnation, low growth caused by weak demand (see Summers (2014)). Some researchers even argue that the slowdown in GDP growth is not a recent phenomenon but a trend that has started long before that (see, e.g., Fernald (2014) and Gordon (2014)). In this paper we show that these profound changes have an impact on business cycle dynamics and thereby optimal monetary policy. We demonstrate our results using a model with search and matching unemployment and trend productivity growth.

Our paper lies in the tradition of a recent literature that analyzes optimal monetary policy in the presence of labor market frictions (see Faia (2009), Thomas (2008), Faia, Lechthaler, and Merkl (2014), or Lechthaler and Snower (2013)). The main finding in this literature is that in the presence of real shocks labor market frictions may lead to inefficient fluctuations in output and employment that call for optimal monetary policy to deviate from price stability. We contribute to this literature by analyzing the effects of trend productivity growth, and find that higher trend growth exacerbates the distortions stemming from the labor market, thus calling for larger deviations from price stability.

We augment the standard search and matching model by disembodied technological progress (see, e.g., Pissarides (2000)), and by Calvo-type nominal price rigidity, (see, e.g., Walsh (2005)). In the presence of disembodied technological progress trend productivity of workers increases each period at an exogenous rate, and all workers operate with the same, most recent technology. Unlike Pissarides (2000), who assumes an exogenous and constant real interest rate, we consider an endogenous real interest rate as a result of intertemporally optimizing consumers. We consider business cycle fluctuations that are induced by temporary shocks to productivity and to government spending.

We first show analytically that trend growth is not a source of inefficiency on its own. In the model at hand, labor market outcomes are efficient if unemployment benefits are zero and the so-called Hosios condition is fulfilled. In that case and in the presence of real shocks, maintaining zero inflation is still optimal, irrespective of the growth rate. However, if labor market outcomes are inefficient, then trend growth interacts with these inefficien-

---

1 This is in contrast with embodied technological progress, under which only newly hired workers can use the latest technology.
2 According to Hosios (1990) the labor market works efficiently if the bargaining power of firms equals the elasticity of the matching function.
cies and thereby influences the optimal deviation from price stability. We show that, under plausible assumptions about household consumption smoothing (see, e.g., Eriksson (1997)), higher productivity growth lowers the effective discount factor and thereby amplifies the inefficiencies due to labor market distortions (arising from the presence of unemployment benefits and violation of the Hosios condition).

To demonstrate this result numerically we use the calibration approach suggested by Hagedorn and Manovskii (2008). As is well known, (see, e.g., Shimer (2005) and Costain and Reiter (2008)) the standard calibration of the search and matching model is not able to generate the large fluctuations in employment observed in the data. Hagedorn and Manovskii suggest an alternative calibration strategy that is based on very high unemployment benefits and a very high bargaining power of the firm. This shrinks the surplus of a match and makes the wage rate relatively rigid over the business cycle, which in turn implies larger fluctuations in employment. However, as already demonstrated by Arsenau and Chugh (2012), these fluctuations are largely inefficient, calling for business cycle stabilization. More specifically, monetary policy can use inflation to dampen markup fluctuations, and thus make employment more stable (see Faia (2009)).

In the present paper higher trend growth leads, along a balanced growth path, to a steeper consumption profile, implying lower future marginal utility from consumption (due to decreasing marginal utility). This makes consumers less patient, or put differently, they require a higher interest rate in order to support the rise in consumption growth. In this setting, higher trend growth leads to even more rigid wages and even more volatile employment, thus exacerbating the inefficiency stemming from the labor market. However, by reinforcing the markup distortion and the relative price distortion of changes in the real interest rate, higher trend growth makes fighting the labor market distortion through monetary policy more costly. The first (second) effect calls for larger (smaller) deviations from price stability when trend growth is higher. In the calibrated version of the model, we find that the first effect dominates and thus optimal inflation volatility increases with trend growth. In our benchmark scenario, the optimal volatility of inflation rises from below 0.2% for zero growth to 0.26% for four percent growth. The optimal reduction in output volatility rises from 4.5% for zero growth to almost 6% for four percent growth.

Apart from the literature on optimal monetary policy in the presence of labor market frictions, our paper is related to a recent literature that analyzes the consequences of trend growth in a business cycle setting. For instance, Amano, Moran, Murchison, and Rennison (2009) examine the effect productivity growth on the optimal steady state inflation rate in the presence of Calvo-type nominal wage and price staggering. In a similar
New-Keynesian model with non-separable utility Tesfasellassie (2013) studies the effect of productivity growth on the government spending multiplier, while Snower and Tesfasellassie (forthcoming) analyze the joint effect of trend growth and job turnover on the steady state optimal inflation rate. In contrast to these papers, we analyze optimal monetary policy over the business cycle, i.e., in response to temporary shocks.

The rest of the paper is structured as follows. Section 2 describes the model, including the optimization problem of households and firms. Section 3 compares the equilibrium outcomes under the decentralized economy with flexible prices and under the social planner. Section 4 discusses the Ramsey optimal monetary policy and presents the main results of the paper under our benchmark calibration. Section 5 contains our sensitivity analysis. Section 6 provides concluding remarks and suggestions for future work.

2 The model

We incorporate trend growth by assuming a deterministic growth in labor productivity $A_t$, where $\Gamma$ denotes gross productivity growth and $\gamma$ is the growth rate (both in quarterly terms). Along a balanced growth path, consumption, output and the real wage grow at the rate of $\gamma$. We assume that productivity growth is reflected in all existing and new jobs (i.e., technology is disembodied).

Production takes place in two sectors. Firms in the intermediate goods sector hire workers subject to search and matching frictions and produce their output using a linear production technology with labor being the sole input. They sell their products under perfect competition to the final goods sector. Firms in the final goods sector transform the intermediate good into slightly differentiated consumption goods and sells them under monopolistic competition to the households. They face nominal rigidities in price-setting as in Walsh (2005).

2.1 Households

There is a representative household with a continuum of members over the unit interval and period utility function $C_t^{1-\sigma}/(1-\sigma)$, $\sigma > 0$. $C_t$ is a Dixit-Stiglitz composite of a continuum of differentiated goods $C_t = \left(\int_0^1 C_{k,t}^{1/\mu_p} dk\right)^{\mu_p}$ where each good is indexed by $k$, $\mu_p = \frac{\epsilon}{1-\epsilon}$ and $\epsilon$ is the elasticity of substitution between goods. Optimal consumption allocation across goods gives the demand equation $C_{k,t} = \left(\frac{p_{k,t}}{p_t}\right)^{-\epsilon} C_t$ where
\( P_t = \left( \int_0^1 P_{k,t}^{1-\epsilon} \, dk \right)^{\frac{1}{1-\epsilon}} \) is the price index.

In a given period a fraction \( N_t \) of household members are employed by firms and earn a nominal wage \( W_t \). The rest earn nominal unemployment benefits of \( P_t u_b A_t, u_b > 0 \).

As common in the literature, we assume that the income is pooled within the household so that unemployed workers do not face lower consumption than employed workers (see, e.g., Andolfatto (1996)). The household maximizes the lifetime utility

\[
E_t \sum_{i=0}^{\infty} \beta^i \frac{C_{t+i}}{1-\sigma},
\]

subject to the budget constraint

\[
P_tC_t + B_t = W_t N_t + P_t u_b A_t (1 - N_t) + R_{t-1} B_{t-1} + D_t - T_t. \tag{2}
\]

Here \( \beta \) is the subjective discount factor, \( R_t \) is the nominal interest rate on bond holdings \( B_t \), \( D_t \) is aggregate nominal profit income from the monopolistically competitive firms, and \( T_t \) represents lump-sum taxes.

It is straightforward to derive the familiar Euler equation

\[
1 = E_t \left( \frac{Q_{t,t+1} R_t}{\Pi_{t+1}} \right), \tag{3}
\]

where \( \Pi_t \equiv P_t/P_{t-1} \) is gross inflation rate and \( Q_{t,t+1} \equiv \beta (C_{t+1}/C_t)^{-\sigma} \) is the household’s stochastic discount factor, which is used to discount future real payoffs. It can be rewritten as

\[
Q_{t,t+1} = \beta \Gamma^{-\sigma} \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma}, \tag{4}
\]

where \( c_t = C_t/A_t \). From Eq. (3) a steady state growth path with higher trend growth implies a higher gross real rate \( R/\Pi \). A higher real rate implies stronger discounting of future payoffs.

## 2.2 Firms

### 2.2.1 Intermediate goods sector

Firms in the intermediate goods sector face search and matching frictions a’la Diamond, Mortensen and Pissarides (see, e.g., Pissarides (2000)). There is an unlimited number of

\(^3\)The presence of \( A_t \) ensures that along a balanced growth path real unemployment benefits grow at the same rate as the real wage.
potential entrants that need to post a vacancy $V_t$ at cost $A_t \kappa$ to have the chance to find a worker and enter the market. The vacancy posting cost is measured in terms of the CES bundle of final goods. The presence of $A_t$ ensures that along the balanced growth path the cost of vacancy posting increases at the same rate as final goods output (otherwise vacancies would converge towards infinity and unemployment towards zero). Each firm can employ only one worker and produces with technology $a_t A_t$, where $a_t$ is a transitory but persistent productivity shock.

**Hiring.** Aggregate employment evolves according to the dynamic equation

$$N_t = (1 - \delta)N_{t-1} + M_t,$$

(5)

where $M_t$ is the number of newly formed matches in period $t$, which become productive immediately.

The size of the labor force is normalized to 1. At the beginning of each period a fraction $\delta$ of previously employed workers are separated from their jobs. They immediately engage in job search. Thus the number of searching workers is given by

$$U_t = 1 - (1 - \delta)N_{t-1},$$

(6)

and the unemployment rate after hiring takes place is $u_t = 1 - N_t$.

The number of newly created matches, $M_t$, is determined by a constant returns-to-scale matching function, with the number of searching workers, and the number of posted vacancies as its arguments

$$M_t = \mu U_t^\alpha V_t^{1-\alpha},$$

(7)

where $\mu > 0$ is a scale parameter describing the efficiency of the labor market and $\alpha > 0$ is the elasticity of the matching function. Dividing this equation by $V_t$ and defining labor market tightness as $\theta_t \equiv V_t/U_t$ we can write the vacancy filling rate as

$$q(\theta_t) \equiv \frac{M_t}{V_t} = \mu \theta_t^{-\alpha}.$$  

(8)

The value of a vacancy is then given by $-A_t \kappa + q(\theta_t) J_t$ where $J_t$ is the value of an existing match. Free entry of firms drives down the value of a vacancy to zero so that

$$A_t \kappa = q(\theta_t) J_t$$

(9)
which is the standard vacancy creation condition. The cost of posting a vacancy equals
the benefit of posting a vacancy, the potential profits that can be earned in case the search
for a worker was successful. If the cost of posting a vacancy were lower than the expected
profit of posting a vacancy, new vacancies would be posted, lowering the vacancy filling
rate and thereby expected profits until the incentive to post further vacancies vanishes.

Active firms in this sector face a perfectly competitive output market. Let \( P_t^I \) denote the
nominal market price and \( p_t^I \equiv P_t^I / P_t \) the real market price. Then the value of an existing
match can be defined as

\[
J_t = a_t A_t p_t^I - w_t + \beta (1 - \delta) E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} J_{t+1}
\]  

(10)

where \( w_t = W_t / P_t \) is real wage. The value of a firm consists of contemporaneous profits
plus the expected future value of the match discounted by the appropriate discount factor.
Combining equations (9) and (10) and dividing by \( A_t \), the vacancy creation condition can
be written as

\[
\frac{\kappa}{q(\theta_t)} = a_t p_t^I - \frac{w_t}{A_t} + \beta (1 - \delta) E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \Gamma^{-\sigma} E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \Gamma \frac{\kappa}{q(\theta_{t+1})} \right\}
\]  

(11)

where \( \kappa / q(\theta_t) \) has now the interpretation of expected hiring costs, the cost of posting a
vacancy, \( \kappa \), multiplied with the expected time until the vacancy is filled, \( 1 / q(\theta_t) \). Equation
(11) says that in equilibrium the cost of a hiring worker must equal the contemporaneous
profits generated by a worker plus the saved hiring costs of the next period.

From the right hand side of equation (11) we see that there are two counteracting effects of
higher trend growth on the firm’s hiring policy. On the one hand it implies larger savings
in future hiring costs from current hiring (this effect has been labelled “capitalization
effect” by Aghion and Howitt (1994)). On the other hand higher trend growth implies
higher consumption growth, which lowers the stochastic discounting factor or raises the
real interest rate (i.e., a stronger discounting of future hiring costs). We call this the
discounting effect of growth. The discounting effect dominates the hiring cost effect if
and only if \( \sigma > 1 \). In this case higher trend growth reduces the returns to hiring by
lowering the discounted savings in future hiring costs.

Wage setting. Wages are set by Nash-bargaining. For this we need to define the value
functions of workers and firms. The real value to the household of an employed worker is
given by

\[
V_t^e = w_t + \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[ (1 - \delta (1 - \theta_{t+1} q(\theta_{t+1})))V_{t+1}^e + \delta (1 - \theta_{t+1} q(\theta_{t+1}))V_{t+1}^u \right] \right\}
\]  

(12)
where $\theta_{t+1} q(\theta_{t+1}) = M_{t+1}/U_{t+1}$ is an unemployed worker’s job finding rate. The corresponding real value of an unemployed worker is given by

$$V^u_t = u_b A_t + \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} [\theta_{t+1} q(\theta_{t+1}) V^c_{t+1} + (1 - \theta_{t+1} q(\theta_{t+1})) V^u_{t+1}] \right\} \quad (13)$$

Thus the household surplus from an employment relationship is given by

$$S^h_t = (\equiv V^e_t - V^u_t) = w_t - u_b A_t + \beta (1 - \delta) E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (1 - \theta_{t+1} q(\theta_{t+1})) S^h_{t+1} \right\} \quad (14)$$

which in detrended form becomes

$$s^h_t = w^d_t - u_b + \beta (1 - \delta) E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (1 - \theta_{t+1} q(\theta_{t+1})) \Gamma s^h_{t+1} \right\} \quad (15)$$

where $s^h_t \equiv S^h_t / A_t$ and $w^d_t \equiv w_t / A_t$. The surplus is smaller the lower is the stochastic discount factor.

The firm’s surplus in detrended form is

$$s^f_t = \frac{J_t}{A_t} = a_t p^f_t - w^d_t + \beta (1 - \delta) \Gamma^{1-\sigma} E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \theta_{t+1} \right\} s^f_{t+1} \quad (16)$$

Under the standard assumption of Nash bargaining the optimal surplus sharing rule is given by $s^h_t = (1 - \nu) s^f_t = (1 - \nu) (\kappa / q(\theta_t))$, where $\nu > 0$ is the bargaining power of the firm and the second equality is implied by equations (11) and (16). Using the surplus sharing rule to substitute out $s^h_t$ in equation (15) and in turn using equation (11) to substitute out $\kappa / q(\theta_t)$ gives, after rearranging, the wage setting equation

$$w^d_t = \nu u_b + (1 - \nu) \left\{ a_t p^f_t + \beta (1 - \delta) \kappa \Gamma^{1-\sigma} E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \theta_{t+1} \right\} \right\} \quad (17)$$

Given $\sigma > 1$, higher growth has similar effects on the wage as does a lower discount factor or a higher job separation rate.

### 2.2.2 Final goods sector

Each firm $k$ produces a differentiated final good using a linear technology $Y_{k,t} = Y^f_{k,t}$ and receives a government subsidy of $\tau$ percent of its total input costs, both of which imply that the firm’s real marginal cost, $m_{ck,t}$, is given by $(1 - \tau)p^f_t$. Price setting is subject to Calvo-type price staggering, where $\omega$ is the fraction of firms whose prices are fixed in any
given period. Let $P_{k,t}$ denote firm $k$’s output price. Each firm $k$ maximizes lifetime profit

$$E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} \left( P_{k,t}/P_{t+i} - (1 - \tau)p^*_t \right) Y_{k,t+i}$$

subject to the total demand for good $k$, $Y_{k,t+i} = (P_{k,t}/P_{t+i})^{-\epsilon} Y_{t+i}$, where $Y_{t+i} = C_{t+i} + G_{t+i} + A_{t+i} + V_{t+i}$ is total aggregate demand including government consumption, $G$, and the vacancy posting costs. The resulting optimal price is

$$p^*_t = \mu_p \frac{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} (1 - \tau)p^*_t Y_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^\epsilon}{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} Y_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon-1}}, \quad (18)$$

where $p^*_t \equiv P^*_t / P_t$ and $\mu_p$ is the price markup in the absence of price staggering. Eq. (18) can be rewritten as

$$p^*_t = \frac{F_{n,t}}{F_{d,t}}, \quad (19)$$

where $F_{n,t}$ and $F_{d,t}$ are auxiliary variables given by

$$F_{n,t} = (1 - \epsilon) p^*_t y_t c_t^{-\sigma} + \beta \omega \Gamma^{1-\sigma} \Pi^{\sigma-1}_{t+1} F_{n,t+1}, \quad (20)$$

and

$$F_{d,t} = y_t c_t^{-\sigma} + \beta \omega \Gamma^{1-\sigma} \Pi^{\sigma-1}_{t+1} F_{d,t+1}. \quad (21)$$

Under Calvo-type price staggering the aggregate price index can be rewritten as

$$1 = (1 - \omega) p_t^{*(1-\epsilon)} + \omega \Pi_t^{\sigma-1}. \quad (22)$$

Aggregating both sides of the market clearing condition for the intermediate good and using the demand equation for the final good $k$ leads to a relationship between aggregate final output $y_t$ and intermediate good output $y^I_t$,

$$y^I_t = \Delta_t y_t, \quad (23)$$

where $\Delta_t \equiv \int_0^1 (P_{k,t}/P_t)^{-\epsilon} df$ is a measure of price dispersion, which can be rewritten as

$$\Delta_t = (1 - \omega) p_t^{* - \epsilon} + \omega \Pi_t \Delta_{t-1}. \quad (24)$$

As aggregate output in the intermediate good sector is equal to aggregate employment, Eq. (23) can be rewritten as

$$a_t N_t = \Delta_t y_t. \quad (25)$$
Finally, the government sector budget constraint is \( t_t = P_t u_b u_t + \tau p^t_i a_t N_t + g_t \) and the aggregate resource constraint is

\[
y_t = c_t + g_t + \kappa V_t.
\]

To summarize, the dynamic system is given by equations (3), (4), (5), (6), (8), (17), (19), (20), (21), (22), (24), (25), (26), the definition \( \theta_t \equiv V_t / U_t \) and exogenous processes for government spending \( g_t \) and the transitory productivity shock \( a_t \).

### 3 Efficiency of flex-price equilibrium

This section compares the solution of the social planner economy with the equilibrium allocation under the decentralized economy with flexible prices and where subsidies are used to eliminate the monopolistic distortion in the final goods sector (the optimal level of the subsidy rate \( \tau \) is set equal to \( 1/\epsilon \)). The latter assumption allows us to concentrate on the potential distortions stemming from the labor market. The purpose of this exercise is to show under which conditions the first best outcome is feasible, and to identify the role trend growth for potential inefficiencies.

#### 3.1 Social planner’s problem

To simplify the social planner’s problem take the law of motion for employment (5) and substitute out \( M_t \) using the matching function to get

\[
N_t = (1 - \delta)N_{t-1} + \mu U_t^\alpha V_t^{1-\alpha}.
\]

Further substitute out \( U_t \), using equation (6), to get

\[
N_t = (1 - \delta)N_{t-1} + \mu (1 - (1 - \delta)N_{t-1})^\alpha V_t^{1-\alpha}. \tag{27}
\]

Now the social planner’s problem is to maximize household utility (1) subject to two constraints—the employment dynamics equation (27) and the aggregate resource constraint

\[
C_t = a_t N_t A_t - A_t \kappa V_t. \tag{28}
\]
Defining the multiplier on the resource constraint as $\lambda^c_t$ and the multiplier on the employment constraint as $\lambda^e_t$, the first order conditions of the Lagrangian $L$ are

\[
\begin{align*}
\frac{\partial L}{\partial C_t} &= C_t - \lambda^c_t = 0 \\
\frac{\partial L}{\partial N_t} &= -\lambda^e_t + \beta (1 - \delta) E_t \left\{ \lambda^n_{t+1} \left( 1 - \alpha \frac{M_{t+1}}{U_{t+1}} \right) \right\} + \lambda^e_t a_t A_t = 0 \\
\frac{\partial L}{\partial V_t} &= \lambda^n_t (1 - \alpha) \frac{M_t}{V_t} - \lambda^e_t A_t \kappa = 0.
\end{align*}
\]

Combining the first order conditions leads to

\[
\frac{\kappa}{q(\theta_t)} = a_t (1 - \alpha) + \beta (1 - \delta) \Gamma^{1-\sigma} E_t \left\{ \frac{\kappa}{q(\theta_{t+1})} \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \left( 1 - \alpha \frac{M_{t+1}}{U_{t+1}} \right) \right\}, \tag{29}
\]

where from equation (8) $M_t/V_t = q(\theta_t)$.

### 3.2 Decentralized economy

The optimality condition for vacancy posting in the decentralized economy is

\[
\frac{\kappa}{q(\theta_t)} = a_t - \frac{w_t}{A_t} + \beta (1 - \delta) \Gamma^{1-\sigma} E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{\kappa}{q(\theta_{t+1})} \right\}, \tag{30}
\]

while the wage setting equation is

\[
\frac{w_t}{A_t} = \nu u_b + (1 - \nu) \left( a_t + \beta (1 - \delta) \Gamma^{1-\sigma} E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{\kappa}{q(\theta_{t+1})} \right\} \left( 1 - \alpha \frac{M_{t+1}}{U_{t+1}} \right) \right). \tag{31}
\]

Using equation (31) in equation (30) and simplifying gives

\[
\frac{\kappa}{q(\theta_t)} = \nu (a_t - u_b) + \beta (1 - \delta) \Gamma^{1-\sigma} E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{\kappa}{q(\theta_{t+1})} \left( 1 - \alpha \frac{M_{t+1}}{U_{t+1}} \right) \right\}. \tag{32}
\]

Comparing equations (29) and (32) it is clear that the decentralized economy yields the same outcome as the social planner economy if $\nu = 1 - \alpha$ (the so-called Hosios condition is fulfilled) and $u_b = 0$. Under these assumptions the economy under flexible prices is efficient, and the best that the Ramsey planner can do is to keep inflation at zero at any time to avoid the price distortions that would follow from non-zero inflation. This is so irrespective of the trend growth rate.
Table 1: Parameter configuration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>discount factor</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>coefficient of relative risk aversion</td>
</tr>
<tr>
<td>( \omega )</td>
<td>fraction of non-optimizing firms</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>elasticity of substitution between final goods</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>matching efficiency</td>
</tr>
<tr>
<td>( \delta )</td>
<td>job separation rate</td>
</tr>
<tr>
<td>( \nu )</td>
<td>firm’s share of surplus</td>
</tr>
</tbody>
</table>

However, whenever unemployment benefits are positive and/or the Hosios condition is violated the labor market does not function efficiently. This implies that the Ramsey planner will have an incentive to deviate from price stability in response to temporary shocks. Put differently, the Ramsey planner will trade off higher price distortions against lower labor market distortions by following activist monetary policy. Interestingly, trend growth interacts with the labor market distortions in non-trivial way, thus affecting the trade-off faced by the Ramsey planner. In the following we will explore this trade-off numerically.

4 Ramsey optimal monetary policy

This section looks at the Ramsey optimal monetary policy, whereby the Ramsey planner maximizes household utility subject to the competitive equilibrium under nominal price rigidity and labor market frictions, i.e., the Ramsey planner takes the distortions on the labor market as given. As is standard, we assume that in steady state the monopolistic distortion is eliminated by the use of an appropriate subsidy. The idea of this approach is to isolate the distortion of the labor market and to make sure that deviations from price stability are not driven by the monopolistic distortion. Note, however, that the subsidy is not time-varying, and therefore the monopolistic distortion reappears in response to business cycle shocks.

Table 1 shows the calibration of some of the model parameters. The parameter values are somewhat standard in the business cycle literature.

The other parameters are calibrated as follows. In line with Hagedorn and Manovskii (2008) and Arsenau and Chugh (2012) we allow worker’s share of surplus \( (1 - \nu) \) to differ from the matching efficiency parameter \( \alpha \) (i.e., we do not impose the Hosios condition).
The scale parameter in the matching function \( \mu \) and steady state labor market tightness \( \theta \) are set such that the job-finding rate is 0.7 (see, e.g., Blanchard and Gal (2010)) and the job-filling rate is 0.9 (see, e.g., Andolfatto (1996), Arsenau and Chugh (2012)). Assuming a steady state unemployment rate \( U \) of 12% (see, e.g., Krause and Lubik (2007)), the steady state mass of vacancies \( V \) is pinned down by the definition of \( \theta \). \( u_b \) is set such that the the steady state return to non-market activity represents 95% of real wage in the steady state with 2 percent (annualized) trend growth rate (henceforth denoted by \( \gamma_a \)).

The vacancy posting cost parameter \( \kappa \) is set such that the steady state solution of the model under 2 percent trend growth rate is consistent with the above target values for \( \theta \), \( q(\theta) \), \( U \) and \( V \).

Regardless of the presence of trend growth under the optimal plan the steady state inflation rate is zero. The reason is that the presence of trend growth does not introduce another distortion. Given zero steady state inflation, higher trend growth implies lower steady state employment, as higher trend growth raises the effective discount rate (see, e.g., Tesfaselassie (2014) and the references therein).

The results are shown in terms of optimal impulse responses to a temporary but persistent shock to the economy and under alternative trend growth rates. As in related studies two types of shocks are considered: a shock to transitory productivity \( a_t \) and a shock to government spending, both of which are assumed to follow an AR(1) process (i.e., an autoregressive process of order 1). To be specific \( g_t / g = (g_{t-1} / g)^{\rho_g} u_{gt} \), \( 0 < \rho_g < 1 \), and \( a_t = a_{t-1}^{\rho_a} u_{at} \), \( 0 < \rho_a < 1 \). In line with previous studies the autocorrelation coefficients \( \rho_g \) and \( \rho_a \) are set equal to 0.9 while the standard deviation of the innovations \( u_{gt} \) and \( u_{at} \) are set equal to 0.01. Moreover, steady state government spending \( g \) represents 20 percent of aggregate output \( y \).

Figure 1 shows baseline (i.e., assuming 2 percent annualized trend growth rate) impulse responses of output, unemployment, inflation rate, the nominal rate of interest, labor market tightness and the wage rate to a positive shock to productivity under the Ramsey optimal policy (solid line), the zero-inflation policy (dashed line) and the efficient economy, i.e., under \( u_b = 0 \) and \( \nu = 1 - \alpha \) (dot-dashed line). In the absence of labor market distortions all three lines would coincide with each other, because then zero inflation would be optimal, but this is not the case in the presence of labor market distortions.

Let us first focus on the efficient economy and the economy under zero inflation. Obviously, the efficient economy exhibits much smaller fluctuations in all quantity allocations, especially the ones related to the labor market, the employment rate and the tightness of the labor market. In contrast, the wage rate is much more volatile in the efficient econ-
The calibration approach of Hagedorn and Manovskii (2008) is designed to generate relatively stable wages by shrinking the surplus of a firm-worker match through a very high threat point of the worker (high unemployment benefits, $u_b$). The comparison with the efficient economy reveals that this implies inefficiently large fluctuations in employment. This is in line with Arsenau and Chugh (2012) who also use a search and matching model under the Hagedorn-Manovskii calibration but analyze optimal fiscal policy.

Thus, the decentralized economy features excess volatility of quantity allocations in response to productivity shocks, which gives the Ramsey planner a motivation to reduce the volatility of the economy. Indeed, in reaction to the rise in labor productivity the Ramsey planner deviates from price stability and allows a reduction in the inflation rate (by inducing a rise the real interest rate) so as to smooth output and employment fluc-
tuations. That is, in the presence of nominal price rigidity the Ramsey planner uses the aggregate demand channel to reduce demand for the final good and in turn demand for the intermediate good. The resulting reduction in the relative price of the intermediate good $p^I_t$ implies that, given $a_t$ and $w_t$, the marginal revenue product of labor, and in turn the match surplus, also decline. At the same time, a reduction in $p^I_t$ implies a reduction in the real marginal cost of final good producers and therefore a corresponding reduction in the optimal relative price $p_t^*$ (see equation (19)). Finally, a reduction in $p_t^*$ implies, by definition, lower inflation (see equation (22)).

The deviation from price stability is due to the presence of labor market distortions associated with the violation of the Hosios condition (in particular, $\nu > 1 - \alpha$—bargaining power of firms is larger than the elasticity of matches with respect to vacancies) and the presence of unemployment benefits ($u_b > 0$). Both sources of distortions imply that the wage rate is relatively insensitive to the rise in productivity and therefore job creation and employment are relatively excessive. In order to partially offset the resulting distortions the Ramsey planner deviates from price stability and induces a smaller rise in match surplus than is the case under price stability. This in turn implies that the wage rate is more stable under the Ramsey planner than under price stability.

Turning next to the effect of trend growth on the optimal plan, Figure 2 shows the gap between the Ramsey optimal plan and the zero inflation policy under alternative values for the annualized trend growth rate—namely, 0 percent, 2 percent (baseline) and 4 percent. We see that the Ramsey planner induces the largest fall in inflation, and correspondingly the largest fall in the output and employment volatilities relative to the case of price stability, under the 4 percent growth rate (dashed line).

This is the outcome of two opposing effects of higher trend growth. On the one hand, higher trend growth reinforces the excessive job creation, in response to positive productivity shocks, stemming from firms having excessive bargaining power. As discussed above, this excessive bargaining power of firms implies that the wage rate is relatively insensitive to a given rise in future labor market tightness associated with the persistent rise in productivity (see equation (31)). By raising the effective discount rate, higher trend growth strengthens the insensitivity of the wage rate to a given productivity shock, which results in an increase in the output wedge between the efficient economy and the zero-inflation economy. This effect is illustrated by the left panel of Figure 3, which shows that higher trend growth increases the wedge between output under the efficient economy

---

4 Its inverse, the average price markup, is a measure of market power.
5 Note that a larger negative gap in figure 2 implies larger negative deviation from the zero-inflation economy and thus lower volatility.
and the corresponding one under zero-inflation policy. This increase in the output wedge reflects the strengthening of labor market distortions due to higher trend growth.

On the other hand, higher trend growth reinforces the markup distortion and relative price distortion of a given rise in the real interest rate (a given fall in aggregate demand). As discussed above, the Ramsey planner dampens the response of employment to higher productivity by raising the nominal interest rate so as to reduce demand for the final good and in turn demand for the intermediate good. The latter effect lowers $p_w$ (which is the current real marginal cost) and thus lowers the optimal relative price $p^*_t$. By raising the effective discount rate, higher growth raises the sensitivity of $p^*_t$ (and in turn inflation) to a given reduction in $p^*_t$, and therefore raises the cost (in terms of markup distortion) of
mitigating a given degree of labor market distortion.\footnote{Note that a stronger reduction in \( p_t^* \) due to higher trend growth also implies higher price dispersion (equation (24)). This effect alone, which increases relative price distortion and thereby lowers output, partly mitigates the excessive rise in output implied by the rise in the markup distortion.}

In order to illustrate the reinforcement of the markup distortion by trend growth the right panel of Figure 3 shows the gap in output between the Ramsey economy and the zero-inflation economy in a counterfactual scenario where the labor market effect of trend growth is suppressed—i.e., the degree of labor market distortion is unchanged as trend growth rises. This is achieved by assuming that the trend growth rate does not show up in the wage equation and the value of a filled job. We see that the gap in output is now smaller at higher levels of trend growth, while it is the opposite in the baseline model. In line with our discussion above, the higher markup distortion associated with higher trend growth, given an unchanged degree of labor market distortion, moves the Ramsey plan towards price stability—thus the smaller the deviation of output relative to the zero-inflation policy.

Figure 4 shows the optimal inflation volatility and the optimal reduction in output volatility as a function of the annualized trend growth rate over the interval \([0\%, 4\%]\). The dashed lines illustrate what happens under the counterfactual scenario, where trend growth does not affect the labor market distortions, while the solid lines illustrate our baseline case, taking into account the strengthening of labor market distortions due to higher trend growth. We see that in the counterfactual scenario there is a negative relationship between trend growth and inflation volatility. Consistent with this outcome there is a negative relationship between trend growth and the magnitude of the reduction in output volatility relative to the zero-inflation economy.

The solid lines in Figure 4 show contrasting outcomes to those in dashed lines—i.e., in the baseline model there is a positive relationship between trend growth and inflation volatility and between trend growth and the magnitude of the reduction in output volatility. This result indicates that the strengthening, due to higher trend growth, of labor market distortions dominates the corresponding strengthening of markup distortions. The figure also illustrates that the effect of trend growth is quantitatively important. Under zero growth the reduction in output volatility induced by the Ramsey planner is 4.5\%, while it is about 6\% under a growth rate of 4\%, an increase of one third.
5 Sensitivity analysis

Our benchmark result regarding the optimal deviation from price stability is not specific to the presence of a productivity shock. Other types of shocks, such as a government spending shock, that take the economy off steady state also imply deviations from price stability. To illustrate this, the left panel of Figure 5 shows the effect of trend growth on the optimal inflation volatility under a positive productivity shock (solid line, which replicates the benchmark case shown in the left panel of Figure 4) and a positive government spending shock (dashed line). As with the productivity shock, a government spending shock induces the Ramsey planner to deviate from price stability, and the deviation from price stability is larger the higher is trend growth. Moreover, in line with previous studies (see, e.g., Faia (2009), Faia, Lechthaler, and Merkl (2014) or Lechthaler and Snower (2013)) the optimal inflation volatility is smaller under a government spending shock than under a productivity shock. The reason is that higher government spending crowds out household consumption so that aggregate output, and in turn employment, rises less strongly than under a positive productivity shock, which implies that the output wedge moves by less.

The right panel of Figure 5 shows sensitivity of our benchmark result under alternative
calibrations of some model parameters—the Calvo parameter \( \omega \), the risk aversion parameter \( \sigma \) and the elasticity of substitution parameter \( \epsilon \). The alternative values are such that under a zero trend growth rate they imply lower inflation volatility.

First, relative to the benchmark calibration \( (\omega = 0.75) \) the alternative calibration \( \omega = 0.8 \) represents a stronger degree of nominal price rigidity, because the firm has a lower probability to reset its price. Considering the effect on the optimal volatility of inflation, a rise in \( \omega \) has two opposing effects. On the one hand, the co-movement between the optimal price of a firm and marginal cost is strengthened. On the other hand, the co-movement between the optimal price of a firm and inflation is weakened.

More rigid prices imply that an optimizing firm worries more about having a markup that is relatively high, in the face of a declining price level induced by lower aggregate demand. In this case the optimal relative price is less sensitive to a reduction in the current real marginal cost, which implies lower markup distortions. Put differently, a given movement in the marginal cost is associated with a larger movement in the optimal relative price. This effect alone calls for more inflation volatility, but is weaker the higher is the trend growth rate (see equation (19)). However, by equation (22), the larger is \( \omega \) the smaller is the implied (mechanical) drop in inflation of a given fall in the optimal relative
Figure 5: The effect of trend growth on the optimal inflation volatility. Sensitivity.

This effect calls for lower inflation volatility, and dominates the first effect, the markup effect. That explains why the line under $\omega = 0.8$ lies below the line under the benchmark case in Figure 5. Furthermore, the second effect is independent of trend growth and, thus, the line under $\omega = 0.8$ is flatter than in the benchmark case.

Analogous effects are at play in the case of changes in the value of $\epsilon$. The alternative calibration $\epsilon = 8$ represents a weaker degree of substitution between final goods than in the benchmark $\epsilon = 11$. On the one hand, it implies that relative demand is less price elastic so that an optimizing firm worries less about having a markup that is relatively high, in the face of a declining price level induced by lower aggregate demand, in case the firm does not get a chance to reset its price. In this case, the optimal relative price is more sensitive to a reduction in real marginal cost. It implies larger markup distortions and calls for lower inflation volatility. As before, this effect is weaker at higher trend growth rates (see equation (19)). On the other hand, by equation (22) the smaller is $\epsilon$ the larger is the implied drop in inflation of a given fall in the optimal relative price around the initial steady state. This effect calls for larger inflation volatility and is independent of trend growth. Therefore, the line under $\epsilon = 8$ is steeper than the corresponding one under the benchmark case. For lower (higher) growth rates, the first (second) effect dominates and, thus, the Ramsey planner induces a smaller (larger) deviation from price stability,
relative to the benchmark.

Finally, the alternative calibration $\sigma = 4$ represents a lower degree of consumption smoothing than the benchmark case with $\sigma = 5$ and thereby implies higher sensitivity of consumption and aggregate demand to changes in the real interest rate. In this case the Ramsey planner is able to stabilize the economy with a smaller deviation from price stability than implied by the benchmark calibration. At the same time, a lower degree of consumption smoothing weakens both the labor market and markup distortions. This implies less inflation volatility because a lower degree of consumption smoothing has effects similar to lower trend growth.

6 Concluding remarks

We study the implications of trend productivity growth for Ramsey optimal monetary policy in the presence of nominal rigidity and search and matching frictions in the labor market. We build on insights from two strands of the recent literature, one of which incorporates trend growth in the standard New-Keynesian model but abstracts from Ramsey optimal monetary policy or labor market frictions, and the other studies Ramsey optimal monetary policy in the presence of labor market frictions but abstracts from considerations of trend growth.

We show that higher productivity growth lowers the effective discount factor and thereby amplifies the inefficiencies due to labor market distortions (arising from the presence of unemployment benefits and higher wage bargaining power of firms). In this environment the wage rate is shown to be less responsive, and job creation more excessive, to aggregate shocks (e.g., productivity and government spending shocks), the higher is trend growth. As a result the Ramsey planner deviates further from price stability so as to stabilize job creation and employment. The deviation from price stability is somewhat weakened by a second effect of trend growth, which amplifies the inefficiencies due to markup distortions (arising from monopolistic competition and nominal price staggering). By calibrating the model to the US economy, we show that, under zero growth the reduction in output volatility induced by the Ramsey planner is 4.5%, while it is about 6% under a growth rate of 4%, an increase of one third.

In our analysis productivity growth arises from disembodied technological progress. Aghion and Howitt (1994) analyze the steady state effect of growth on unemployment and identify a creative destruction effect brought about by embodied technological progress. They
show that by reducing the duration of an existing job match faster growth leads to higher job destruction and therefore higher long-run unemployment. A possible extension of our analysis is thus to allow for embodied technological progress and thus endogenous job destruction. We consider this a potential topic for future research.

References


23