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## The advantage of being poor: private provision of public goods, strategic incentives and the role of public provision

Münchener Wirtschaftswissenschaftliche Beiträge, No. 92-18

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*Suggested Citation:* Konrad, Kai A. (1992) : The advantage of being poor: private provision of public goods, strategic incentives and the role of public provision, Münchener Wirtschaftswissenschaftliche Beiträge, No. 92-18, Volkswirtschaftliche Fakultät der Ludwig-Maximilians-Universität München, München

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**The Advantage of Being Poor: Private Provision  
of Public Goods, Strategic Incentives, and the Role of Public Provision**

von

**Kai A. Konrad**

September 1992

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**THE ADVANTAGE OF BEING POOR:  
PRIVATE PROVISION OF PUBLIC GOODS, STRATEGIC INCENTIVES,  
AND THE ROLE OF PUBLIC PROVISION**

by

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May 9, 1992

\* I thank Ray Rees and Kjell Erik Lommerud for helpful discussion. Financial support by NAVF and the hospitality of the Department of Economics at the University of Bergen are gratefully acknowledged.

**Abstract**

When people know a public good is to be privately provided in the future, economic behavior may change in periods before the one with the private provision game. Individuals have an incentive to reduce their disposable income so as to shift the burden of provision to others. The incentive is reduced if individuals know that the government will provide some of the public good, as long as this provision is large enough. In a second-best world, private provision of public goods has been claimed to be a reasonably good alternative to public provision. This claim needs to be reconsidered in the light of these results.

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## 1. Introduction

People react in different ways when a down-and-out asks for a few coins. Some, unwilling to appear mean, will say something like "sorry, but I am very short of money myself", or, "sorry, I am poor too". Being poor is seen as a good reason for not giving handouts. Charitable giving is a standard example for a public good. The standard model of private provision of public goods explains why in a Nash equilibrium the rich especially can be expected to contribute (see, e.g., Bergstrom *et al.* 1986), particularly if the group of potential contributors is large (see, e.g., Andreoni 1988, and Fries *et al.* 1991). In general, however, this does not make being poor advantageous. In the worst case, where being rich is really a nuisance, it is usually easy enough to get rid of potential excess wealth. The advantage of being poor is a relative advantage: the poor are more likely to get a free ride in a Nash game of private provision of a public good. Therefore, if they know that such a game will take place some time in the future, individuals may have an incentive today to behave in a way that lets them have a free ride tomorrow.

The idea that the anticipation of future interactions may change agents' behavior today has become common in economics. Examples are entry deterrence in oligopoly theory, preemptive behavior in upcoming contests (Dixit 1987), principal-agent theory, or the public choice literature on political decision making in an intertemporal context (see, e.g., Glazer 1989, Tabellini and Alesina 1990).

This paper considers the incentive effects of an upcoming private-provision-of-a-public-good game on individuals' choice of future disposable income in the period before the private-provision game takes place. Individuals first make their labor-leisure choice. Then, when their monetary income is determined, they decide how to spend this income on a private good and on contributions to a public good. However, individuals may have an incentive to cut back their income generating activities at the first stage. If they have less disposable income, they have to contribute less to the public good. It turns out that, given the nature of the Nash

equilibrium of private provision of a public good, for each dollar an individual reduces her labor income, her imputed income is reduced by only a small fraction of this dollar.

Since Samuelson's (1954, 1955) contributions on efficient public good provision and individual free-riding incentives, it has been claimed in Public Finance that public goods should be publicly provided. The Nash equilibrium of private provision of the public good as described by Chamberlin (1974) and McGuire (1974) is typically one with underprovision of the public good. The equilibrium is only efficient in some degenerate cases, e.g., in a 1-person economy (see, e.g., Bergstrom *et al.*, 1986).

Bergstrom *et al.* (1986, p. 43) also point out that it is not clear, a priori, whether private or public provision of the public good is preferable from an efficiency point of view. Certainly there are inefficiencies of administration, informational imperfections inherent in democratic decision making, and an excess burden involved in collecting taxes. These costs have to be compared with the social costs of underprovision in the Nash equilibrium. Moreover, government contributions to the public good may crowd out private contributions. In the extreme case, where the revenue used for governmental provision is taken only from individuals who contribute to the public good, this crowding out is 'one to one' (see, e.g., Warr 1982, Roberts 1984). Suppose that one dollar of tax revenue costs  $c > 1$  dollars. The first units of governmental supply only crowd out private contributions; so, if the government wants to improve the allocation it can only do so by supplying over and above what was provided in the private provision equilibrium. However, in this case, all governmental provision including the units that just crowd out private provision must be financed by taxes. Given that government has already crowded out private contributions completely, a further increase of the public good supply may be welfare enhancing, but overall, it is possible that the best allocation with government provision may be worse than the Nash equilibrium of private provision.

In section 3 the paper contributes to the question of whether public goods should



be supplied publicly, or whether private provision is a reasonably good alternative. It considers the effect of governmental provision of the public good on the strategic incentives to keep disposable income low in the period when the private provision game takes place. It turns out that governmental provision of the public good can alleviate not only the underprovision problem that arises in the static Nash equilibrium of private provision, but also reduces the welfare losses that result from the strategic incentives, provided that the amount of public provision is sufficiently high.

## 2. The model

There are  $n$  individuals in an economy. Let  $x^i$  denote  $i$ 's purchase of a private good,  $g^i$  his contributions to the privately provided public good, and  $G = \sum_{i=1}^n g^i$  aggregate contributions to the public good. The individuals have identical twice continuously differentiable utility

$$(1) \quad U^i = v(\bar{L} - L^i) + u(x^i, G),$$

where  $\bar{L} - L^i$  is individual  $i$ 's leisure. Additive separability between leisure and the other consumption goods is only for analytical convenience. For the same reason assume that  $v$  has positive but decreasing marginal utility, and  $u$  is strictly concave and, to avoid degenerate corner solutions where individuals prefer, e.g., not to work at all, suppose that  $v'(\bar{L}) = 0$ , and  $\lim_{L^i \rightarrow \bar{L}} v'(\bar{L} - L^i) = \infty$ .

There are two stages, 1 and 2. At stage 1 all individuals simultaneously choose their labor supply  $L^i \in [0, \bar{L}]$ . The wage rate  $w$  is exogenous, and without loss of generality,  $w = 1$ . Their income therefore is  $y^i = y_0 + L^i$ , where  $y_0 \geq 0$  is some exogenous income. At stage two a public good is privately provided.

Consider first the Nash equilibrium of private provision of the public good at the second stage when incomes are given. It is the equilibrium of the standard model of

private provision of public goods. Suppose that  $y = (y^1, \dots, y^n)$  is the vector of incomes. Individual  $i$ 's choice problem in this framework can be described as follows. She chooses  $x^i$ , her private good consumption, and  $G$ , the *aggregate* provision of the public good, to maximize her utility

$$(2) \quad u(x^i, G)$$

s.t.

$$(3) \quad x^i + G \leq y^i + G_{-i},$$

$$(4) \quad x^i \geq 0,$$

$$(5) \quad G - G_{-i} \geq 0.$$

The units of the public good and of the private good are chosen here such that prices are normalized to one without loss of generality. The budget restriction is  $L^i + y_0 = y^i \geq x^i + g^i$ : private good consumption  $x^i$  and individual  $i$ 's contribution to the public good cannot exceed the given income  $L^i + y_0$ . Add on both sides  $G_{-i} \equiv \sum_{j \neq i} g^j$ , the quantity of the public good that is provided by others, to obtain condition (3). Constraints (4) and (5) express that consumption  $x^i$  and individual contributions to the public good,  $g^i = G - G_{-i}$ , cannot be negative. Note that this decision problem could equivalently be formulated as a choice of  $x^i$  and individual contribution  $g^i$ : each individual takes the contributions  $G_{-i}$  of others as given. Therefore, her own contribution adds to these contributions and uniquely determines aggregate provision.

Define  $\omega^i \equiv L^i + y_0 + G_{-i}$  the *imputed income* of individual  $i$ . Define  $f_i = f_i(\omega^i)$  to be the demand function of  $i$ , as being the solution of maximizing (2) subject to (3) only, but not subject to (4) and (5). Assume that this demand function is single-valued and a differentiable function of imputed income. The marginal propensity to consume for the public good is  $\partial f_i / \partial \omega^i \in (0, 1)$ , i.e., both goods,  $x^i$  and  $G$  are supposed to be strictly normal goods for the individual. This *normality property* implies that the Nash equilibrium of private provision exists and is unique (see Bergstrom *et al.*, 1986, p. 33n.). They also show that the equilibrium has a

particularly simple characterization for the case of identical utility functions. For a given vector of incomes there is a level  $y^* \in \mathbb{R}^+$  of income with

$$(6) \quad g^i = \max\{y^i - y^*, 0\}.$$

Everyone contributes the part of her income that exceeds  $y^*$ , and consumes  $x^i = y^*$  units of the private good, where  $y^*$  is implicitly determined by

$$u_x(y^*, \Sigma_{j=1}^n \max\{(y^j - y^*), 0\}) = u_G(y^*, \Sigma_{j=1}^n \max\{(y^j - y^*), 0\})$$

for contributors, and

$$u_x(y^i, \Sigma_{j=1}^n \max\{(y^j - y^*), 0\}) \geq u_G(y^i, \Sigma_{j=1}^n \max\{(y^j - y^*), 0\})$$

for individuals  $i$  that do not contribute. If endowments are also symmetric, then all individuals contribute the same amount (see Bergstrom *et al.*, 1986, p. 37).

Consider next the effect of an income reduction of a single individual  $i$  on the equilibrium values of  $G$ ,  $y^*$ ,  $x^i$ , and  $g^i$ , for given amounts of income of all other individuals. The exposition concentrates on a symmetric interior equilibrium with  $y^i > y^*$ . Suppose that individual  $i$  reduces her income by a marginal unit,  $dL^i = dy^i$ . In the initial equilibrium,  $g^i = y^i - y^* = G/n$ ,  $x^i = y^*$  for all  $i = 1, \dots, n$ , such that

$$(7) \quad u_x(y^*, \Sigma_{j=1}^n (y^j - y^*)) = u_G(y^*, \Sigma_{j=1}^n (y^j - y^*)).$$

Differentiation of (7) with respect to  $y^i$  yields

$$(8) \quad u_{xx} dy^* + u_{xG} dy^i - n u_{xG} dy^* = u_{Gx} dy^* + u_{GG} dy^i - n u_{GG} dy^*,$$

or,

$$(9) \quad \frac{dy^*}{dy^i} = \frac{u_{GG} - u_{xG}}{u_{xx} - (n+1)u_{xG} + nu_{GG}} = \frac{1}{\frac{u_{xx} - u_{xG}}{u_{GG} - u_{xG}} + n}$$

Recall that  $y^* = x^i$ . Therefore, individual  $i$  expects that, if she changes her income by a marginal unit, (9) is also the change of the equilibrium level of her consumption of the private good.

When an individual chooses her labor input at stage 1, she knows that a reduction of her labor input by one marginal unit changes aggregate public good supply at stage 2 by

$$(10) \quad \begin{aligned} \frac{dG}{dy^i} &= -n \frac{dy^*}{dy^i} + 1 \\ &= -n \frac{u_{GG} - u_{xG}}{u_{xx} - (n+1)u_{xG} + nu_{GG}} + 1 \\ &= \frac{1}{n \frac{u_{GG} - u_{xG}}{u_{xx} - u_{xG}} + 1} > 0, \end{aligned}$$

and her private good consumption by

$$(11) \quad \frac{dx^i}{dy^i} = \frac{dy^*}{dy^i} = \frac{1}{\frac{u_{xx} - u_{xG}}{u_{GG} - u_{xG}} + n} > 0.$$

The inequalities in (10) and (11) follow from concavity of  $u$ .<sup>1</sup> Equation (10) shows

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<sup>1</sup> Strict quasi concavity is actually sufficient here. It implies  $u_{GG} - u_{xG}(u_G/u_x) > 0$ , and  $u_{xx}(u_G/u_x) - u_{xG} < 0$ . Using the fact that in the equilibrium, the marginal rate of substitution equals the price ratio of  $x$  and  $G$ , i.e.,  $u_G/u_x = 1$ , yields  $u_{GG} - u_{xx} < 0$  and  $u_{xx} - u_{xG} < 0$ .

that the increase of individual  $i$ 's income by \$1 has two effects. First, as individual  $i$  contributes any dollar of income that exceeds  $y^*$  to the public good,  $G$  is increased by this dollar. However, this additional income also changes the equilibrium. In particular, the  $y^*$ , which determines private consumption ( $x^i = y^*$ ) and contributions ( $g^i = y^i - y^*$ ), is changed, as described by (9), and all individuals change their contributions and consumption of the private good accordingly.

An individual who reduces her income by a marginal dollar changes her consumption vector  $(x^i, G)$ . Both private good consumption and consumption of the public good are reduced. However, the sum of these reductions weighted with their prices (that are equal to one) is

$$(12) \frac{dx^i}{dy^i} + \frac{dG}{dy^i} \stackrel{(9),(10),(11)}{=} \frac{u_{GG} + u_{xx} - 2u_{xG}}{u_{xx} - u_{xG} + n(u_{GG} - u_{xG})} < 1 \quad \text{for } n \geq 2.$$

The inequality sign in (12) for  $n \geq 2$  follows from strict concavity of  $u$ ; but again strict quasi concavity would be sufficient (see fn. 1). Equation (12) shows that individual  $i$ 's consumption value (valued with producer prices) is reduced by less than her income reduction. If she reduces her income by a unit, she does not have to reduce her consumption of  $x$  and  $G$  by a unit. The general equilibrium repercussions make the individual bear only a fraction of her true income reduction. This is the main result of the paper and central to the following pages.

The intuition for this result is as follows. In the initial equilibrium the individual contributes  $y^i - y^*$  to the public good. Now the individual  $i$  reduces her income by  $\Delta$  and would only contribute  $y^i - \Delta - y^*$ . This reduces the aggregate amount of contributions and the imputed income of all others and leads to the following general equilibrium repercussions. In the initial equilibrium each individual chose  $G$  such that, for this  $G$ , her marginal utility of consumption of the private good equalled her marginal utility of the public good (cf. equation (7)). Now  $G$  is reduced by  $\Delta$ . This implies  $u_x^j < u_G^j$  for all  $j = 1, \dots, n$ . In the new equilibrium, all individuals use less of their imputed income for private good consumption and more for contributions to

the public good.

The effect is stronger the larger the set of contributors. For very large groups of contributors,

$$(13) \quad \lim_{n \rightarrow \infty} \frac{dG}{dy^i} = \lim_{n \rightarrow \infty} -n \frac{u_{GG} - u_{xG}}{u_{xx} - (n+1)u_{xG} + nu_{GG}} + 1 = 0,$$

and, similarly,

$$(14) \quad \lim_{n \rightarrow \infty} \frac{dx^i}{dy^i} = \lim_{n \rightarrow \infty} \frac{1}{\frac{u_{xx} - u_{xG}}{u_{GG} - u_{xG}} + n} = 0.$$

With large groups of contributors, the individual cost of reducing one's own income by a unit becomes infinitesimal.

Now the equilibrium choice of the labor input can be determined. Suppose that each individual chooses her labor input  $L^i$  to maximize her utility given the labor input choices of others, knowing that the equilibrium values of  $G$  and  $x^i$  are monotonic functions of her labor supply as described by (10) and (11). The condition determining a symmetric equilibrium is

$$(15) \quad v'(\bar{L} - L^i) = u_x(x^i(L^i), G(L^i)) \frac{dx^i}{dL^i} + u_G(x^i(L^i), G(L^i)) \frac{dG}{dL^i}$$

$$(7) \quad = u_x(x^i(L^i), G(L^i)) \left[ \frac{dx^i}{dy^i} + \frac{dG}{dy^i} \right].$$

Individual  $i$  chooses her labor supply to equate her individual opportunity cost of a marginal unit of income, which is the foregone marginal utility of leisure, to her benefits of this additional unit of income in terms of additional marginal utility of private good consumption and additional marginal utility of the public good. Equation (15) makes use of  $dy^i = dL^i$ , and the last equality makes use of the Nash equilibrium condition (7). Labor choices  $L^i = L$  for  $i = 1, \dots, n$  characterize a symmetric equilibrium if, given the choice  $L_j = L$  by all others, for any individual  $i$

her choice  $L^i = L$  fulfills (15).<sup>2</sup> The fact that  $\frac{dx^i}{dy^i}$  and  $\frac{dG}{dy^i}$  do not sum up to 1 (see (12)), but, for sufficiently large  $n$ , become rather small, indicates that, for  $n \geq 2$ ,

$$(16) \quad v' < u_x = u_G.$$

The labor choice in (16) is too small. The individual only internalizes a small fraction of the true opportunity costs of leisure. This can be verified by showing that a simultaneous increase of the labor supply by all individuals yields a strict Pareto improvement.

Consider the utility change of a representative individual  $i$  if all individuals simultaneously increase their labor supply by  $dL^1 = dL^2 = \dots = dL^n = dL$ .

$$(17) \quad \frac{dU^i}{dL} = -v' + [u_x(x^i(L^i), G(L^i)) \sum_{j=1}^n \frac{dx^j}{dL} + u_G(x^i(L^i), G(L^i)) \sum_{j=1}^n \frac{dG}{dL^j}]$$

$$\stackrel{(9)}{=} -v' + u_x(x^i(L^i), G(L^i)) \frac{dy^*}{dy^i} + u_G(x^i(L^i), G(L^i)) \frac{dG}{dy^i} +$$

$$(n-1) \left[ u_x(x^i(L^i), G(L^i)) \frac{dy^*}{dy^i} + u_G(x^i(L^i), G(L^i)) \frac{dG}{dy^i} \right]$$

$$\stackrel{(15)}{=} (n-1) \left[ u_x(x^i(L^i), G(L^i)) \frac{dy^*}{dy^i} + u_G(x^i(L^i), G(L^i)) \frac{dG}{dy^i} \right] > 0.$$

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<sup>2</sup> This first-order condition only characterizes a local optimum. The condition is sufficient if the individual's objective function is strictly concave in  $L^i$  over the whole range  $[0, \bar{L}]$ . The limiting properties of  $v$  guarantee an interior global maximum in this case. However, concavity of  $u$  does not seem to be sufficient to make  $U^i(L^i)$  globally concave for given labor supply of all others. A sufficient condition is, e.g.,  $u$  to be strictly concave and additively separable.

This makes use of  $x^i = y^*$ , and  $dy^i = dL^i$ . This result is summarized as

**Proposition 1**      In the equilibrium that is characterized by (15), a simultaneous increase of labor supply by all individuals is welfare increasing.

Proposition 1 indicates that, in the symmetric two-stage equilibrium with private provision of a public good at stage 2, individuals have a strategic incentive to reduce their income generating activity at stage 1. They gain the full benefits of their labor reduction, i.e., the additional leisure utility. The effective income reduction perceived by an individual who reduces her labor supply is much smaller than the reduction of aggregate income in the society. The perceived individual income reduction is about the order of magnitude of  $1/n$ -th of the true income reduction. The general equilibrium repercussions in the Nash equilibrium at the second stage make all other individuals provide more public good so that, for large  $n$ , the equilibrium provision of the public good stays almost constant, and  $x^i$  of the representative individual goes down only by about  $1/n$ -th of her individual income reduction  $dL^i$ .

The incentive to cut back in income generating activities in situations where there is to be a game of private provision of a public good in the future has been derived here for the choice of labor supply in a period before the one in which contributions to the public good are determined. The result can be taken as more general and applies in particular to commitment with respect to future expenditure, human capital investment or other ways of shifting freely disposable resources away from the period in which the private provision game takes place.



### 3. The role of government

Private provision of public goods may not be so desirable, given the ex-ante incentives just derived. The mere existence of the private provision game in the future may lead to massive behavioral changes today and may cause welfare losses that exceed the losses caused by the simple underprovision in the Nash equilibrium. Is there a role for government?

**Proposition 2** Consider a symmetric equilibrium that is characterized by  $L^i = L^*$ ,  $x^i = y^*$  for all  $i = 1, \dots, n$ , and  $G = G^* > 0$ . If government levies a lump-sum tax and uses the tax proceeds  $T = nT^i < G^*$  for public provision of the public good, then  $(L^*, y^*, G^*)$  stays an equilibrium.

**Proof.** The maximization problem of a representative individual for a given labor supply is

$$\begin{aligned} & \max u(x^i, G) \\ \text{s.t.} \quad & x^i + G \leq y_0 + L^i - T^i + \sum_{j \neq i} g^j + g_g, \\ & x^i \geq 0, \\ & G - g_g - \sum_{j \neq i} g^j \geq 0, \end{aligned}$$

where  $g_g$  is government's provision, and  $T^i = g_g/n$  is the lump-sum tax paid by individual  $i$ . The Nash equilibrium of private provision to the public good is unique for given labor supply. For a lump-sum financed change of  $g \in [0, G^*)$ , the perfect crowding out result of Warr (1982) or Bergstrom *et al.* (1986) applies: suppose that all individuals contribute a positive amount  $g^i$  in the laissez-faire equilibrium such that, for all  $i$ ,  $T^i < g^i$ . For this case, the stage-2-Nash equilibrium is invariant to the level of  $g_g \in [0, G^*)$ .

The equilibrium labor supply is determined by

$$v' = u_x \frac{dy^*}{dy^i} + u_G \frac{dG}{dy^i} .$$

Given perfect crowding out,  $T^i$  and  $g_g$  do not enter this condition (cf. (10) and (11)) and, therefore, the same labor supply stays optimal. ■

Proposition 2 shows that a policy of public provision of the public good is ineffective in the 2-stage game considered here if the government does not provide more than is contributed in the laissez-faire equilibrium with private provision. A social planner would like to maximize social welfare by a choice of the vectors of labor input, consumption of the private good and the amount of the public good. In a symmetric world with concave utility, this problem reduces to maximizing the utility of a representative consumer by a choice of a general level of labor input  $L^i = L^P$  for  $i = 1, \dots, n$ , consumption of the private good  $x^i = x^P$  for  $i = 1, \dots, n$ , and the amount of the public good,  $G^P$ . The welfare optimum  $(L^P, x^P, G^P)$  fulfills the marginal conditions

$$(18) \quad v' = u_x = nu_G.$$

The first equation in (18) shows that the marginal utility of income used for private good consumption has to be equal to its opportunity costs in terms of foregone leisure utility. The second equation in (18) describes Samuelson's rule that, for a public good, marginal utilities of users should sum up to the marginal cost of providing it, i.e., here, the opportunity cost of foregone consumption of private good. Clearly,  $x^P$  is determined once  $L^P$  and  $G^P$  are determined:

$$x^P = y_0 + L^P - G^P/n.$$

**Proposition 3**     Suppose that government provides  $G^P$  and finances it with a lump-sum tax. Then  $(L^P, x^P, G^P)$  is an equilibrium.

*Proof.* Given that  $G^p$  (or more) of the public good is provided by others (including government), no individual will voluntarily provide public good at stage 2 if it has an income smaller or equal  $\hat{y}$  with  $\hat{y}$  determined by

$$u_x(\hat{y}, G^p) = u_G(\hat{y}, G^p).$$

Now,  $\hat{y} > x^p$  for  $n \geq 2$ , as

$$(19) \quad u_x(x^p, G^p) = n u_G(x^p, G^p).$$

Suppose that individuals think that  $G_{-i} = G^p$  and  $G^p$  is provided by the government. Then the objective function of individual  $i$  is

$$\max [v(\bar{L} - L^i) + u(L^i + y_0 - G^p/n - g^i, G^p + g^i)],$$

where  $G^p/n$  is the lump-sum tax to finance  $G^p$ . Recall that  $L^i \geq 0$  is never binding. The first-order conditions are

$$(20) \quad v' = u_x \frac{dx^i}{dL^i} + u_G \frac{dG}{dL^i}$$

and

$$(21) \quad u_x \geq u_G \quad \text{with} \quad g^i(u_x - u_G) = 0.$$

Suppose that others do not contribute and government provides  $G^p$ . For  $L^i = L^p$  net income is  $y^i = y_0 + L^i - G^p/n < \hat{y}$ . This implies that  $g^i = 0$ ,  $dg^i/dL^i = 0$ , and therefore  $dx^i/dL^i = 1$ , and  $dG/dL^i = 0$ . Equation (20) reduces to

$$(22) \quad v' = u_x.$$

Therefore,  $L^i = L^p$ ,  $x^i = x^p$ , and  $g^i = 0$  solve (21) and (20). Moreover, the choices

$L^p$ ,  $x^p$ , and  $g^i = 0$  are feasible. The choice of  $L^p$  and  $g^i = 0$  just generates income  $x^p = y^i = L^p + y_0 - G^p/n$ . This also justifies the assumption that, given governmental provision of  $G^p$ , other individuals do not provide positive quantities to the public good, in addition to  $G^p$ , i.e.,  $G_{-i} = G^p$ . ■

The analysis of public provision of the public good has so far considered public provision in the range  $g_g \in [0, G^*]$ . In this range perfect crowding out resulted and public provision was shown to be ineffective. In particular, such provision has no impact on the incentives in the periods before the private provision game takes place. The paper also considered the case  $g_g = G^p$ . Once government provides the Samuelson quantity, not only the amount of public good provided is optimal, but also the incentive problem with respect to labor supply disappears. No private individual contributes to the public good in any case. So, additional income is fully appropriated by that individual who generates this income, i.e.,  $dG/dy^i = 0$ , and  $dx^i/dy^i = 1$ . Income has no external effect.

Between  $G^*$  and  $G^p$ , there is a whole range of possible quantities government could choose to provide. The impact of such provision on the equilibrium choices of labor and private provision are now examined. Suppose  $g_g \in (G^*, G^p)$ . A representative individual has to determine her optimal labor supply. Recall that, for given labor choices, the resulting Nash equilibrium of private provision of the public good is unique.

Consider first a *Gedankenexperiment*. For given  $g_g \in (G^*, G^p)$ , the general level of labor supply is continuously increased from zero to  $\bar{L}$ . For small  $L^i$ , equilibrium contributions to the public good are zero. Therefore,  $G = g_g$ . More formally, recall that the Nash equilibrium is determined by some level  $y^*$ , such that everybody contributes the part of his net income  $y^i$  that exceeds  $y^*$ ,

$$(23) \quad g^i = \max \{0, y^i - y^*\} = \max \{0, y_0 - g_g/n + L^i - y^*\},$$

where  $y^*$  is determined by

$$(24) \quad u_x(y^*, G) = u_G(y^*, G)$$

with

$$G = g_g + \sum_{j \neq i} g^j + \max\{0, y^i - y^*\},$$

or, by symmetry,

$$(25) \quad G = g_g + n \max\{0, y^i - y^*\}.$$

For small  $L^i$ ,  $y^*$  that solves (24) is larger than individuals' net income  $y_0 - g_g/n + L^i$ , and therefore no individual contributes to the public good. For such  $L^i$  a marginal increase of individual  $i$ 's income fully increases her consumption of the private good and has no external effect. For given  $g_g$ , there is a  $\hat{L}(g_g)$  with  $y^* = y_0 - g_g/n + \hat{L}(g_g)$ . For all labor inputs  $L^i > \hat{L}(g_g)$ , the marginal unit of income is not simply used to buy additional units of the private consumption good. There is private provision of the public good. Everyone will contribute part of her income to the public good. In this range of incomes an increase of income has an effect on an individual's consumption as described by (10) and (11). Therefore, for  $L^i > \hat{L}(g_g)$ , an individual does not get the full purchasing power of an additional unit of income she generates. Most of the additional income is appropriated by others in the general equilibrium.

Now it is possible to derive the optimal labor supply decision in the symmetric equilibrium. Symmetry is used in the following sense. In the equilibrium all individuals choose the same labor supply. However, they maximize under the Nash conjecture. Each individual  $i$  thinks that her marginal deviation from the equilibrium level does not change the labor choice of other individuals. The equilibrium is determined by a general level of labor input  $L^1 = L^2 = \dots = L^n = L^*$  such that it does not pay for a single individual to deviate from this  $L^*$ .

Individual  $i$  considers whether to deviate from this level of labor input and thinks that this does not affect others' labor input decision:

$$(26) \quad \left. \frac{dU^i}{dL^i} \right|_{L^*} = \begin{cases} -v'(\bar{L} - L^*) + u_x(y_0 + L^* - \frac{g_g}{n}, g_g) & \text{for } L^* < \hat{L}(g_g) \\ -v'(\bar{L} - L^*) + u_x(y^*, g_g + n[y_0 + L^* - \frac{g_g}{n} - y^*]) \left[ \frac{dy^*}{dy^i} + \frac{dG}{dy^i} \right] & \text{for } L^* > \hat{L}(g_g) \end{cases},$$

where  $\frac{dy^*}{dy^i} + \frac{dG}{dy^i}$  is derived in (12). For the case  $L^* > \hat{L}(g_g)$  use of (24) is made.

Equation (26) summarizes the above discussion. The additional unit of labor always reduces leisure utility. If  $L^* < \hat{L}(g_g)$ , the individual uses the additional income fully to purchase additional  $x$  and receives the additional utility of private good consumption. If  $L^* > \hat{L}(g_g)$ , then part of the additional income is used for contributions to the public good. These contributions crowd out some of the contributions of others. For each  $g_g$ , marginal utility of labor input is kinked at some  $\hat{L}(g_g)$  where the private-provision-of-public-goods-game incentive becomes active. At  $\hat{L}(g_g)$ ,  $U^i$  is continuous but not differentiable, with

$$\lim_{L^* \uparrow \hat{L}(g_g)} \left. \frac{dU^i}{dL^i} \right|_{L^*} > \lim_{L^* \downarrow \hat{L}(g_g)} \left. \frac{dU^i}{dL^i} \right|_{L^*}.$$

As  $\hat{L}(g_g)$  is determined by the condition

$$(27) \quad u_x(\hat{L}(g_g) + y_0 - \frac{g_g}{n}, g_g) = u_G(\hat{L}(g_g) + y_0 - \frac{g_g}{n}, g_g),$$

$\hat{L}(g_g)$  is an increasing function of  $g_g$ . The higher public provision the higher the range of labor choices for which the strategic incentives of the private provision game are not at work. An interior equilibrium labor supply  $L^*$  is determined by the condition

$$\left. \frac{dU^i}{dL^i} \right|_{L^*} = 0 \quad \text{for } L^* \neq \hat{L}(g_g),$$

with  $\frac{dU^i}{dL^i}|_{L^*}$  defined as in (26), and

$$\lim_{L^* \uparrow \hat{L}(g_g)} \frac{dU^i}{dL^i}|_{L^*} > 0 > \lim_{L^* \downarrow \hat{L}(g_g)} \frac{dU^i}{dL^i}|_{L^*} \quad \text{for } L^* = \hat{L}(g_g).$$

Again, this condition is not sufficient without further assumptions about the curvature of  $U^i(L^i)$  (see fn. 2).

#### 4. Generalizations

The paper assumed that at stage 2 a Nash game of private provision of a pure public good occurs. The implications about crowding out and income redistribution of the equilibrium in the standard model of private provision of a pure public good are known to be at odds with empirical facts. Abrams and Schmitz (1978), e.g., report that an additional dollar of public provision crowds out only 28 cents of private provision, where the standard model predicts that, if this public provision is financed by lump-sum taxes paid by contributors, public provision crowds out private provision with a one-to-one ratio (see, e.g., Warr 1982 and Bergstrom *et al.* 1986). A generalization that is flexible enough to fit empirical facts is the *warm-glow approach* proposed by Andreoni (1989, 1990) and Roberts (1984). In this approach, the individual's contribution to the public good generates some private extra utility for the donor. Utility (2) in the private provision game is substituted by  $u(x^i, G, g^i)$ . The contributor has some benefit from contributing *himself*, in addition to the effect of increasing  $G$  with his contribution. The private-good property of contributions to the public good reduces somewhat the effect derived in this paper. However, the standard model results are at least suggestive for this "extended" model.<sup>3</sup>

This paper considered commitment at stage 1 with regard to the choice of labor

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<sup>3</sup> For a similar argument see Boadway *et al.* (1989, p. 160).

supply. The results can be interpreted as being more general. The private provision game that takes place at stage 2 has incentive effects on many decisions in earlier periods. The amount to be provided by an individual in the private provision game can be influenced by all kinds of commitments. Buying a durable consumption good and financing it with a loan or mortgage that binds future earnings may be a way to reduce one's contributions to public goods in future periods. Generally, commitment to spending, reduced savings, or rigorous savings plans that do not allow for flexible adjustment, and other measures that reduce freely disposable income in periods when private provision of public goods occurs may be a means to shift the burden of provision to other individuals.

The result of this paper is also applicable in the theory of international transfers. Voluntary contributions of countries to development aid, investment aid to eastern European countries etc. are essentially public goods. The procedure that determines development aid contributions basically follows the structure of the standard model of private provision of a public good (Kemp, 1984). Therefore, applying the basic idea in this paper, a country has an incentive to precommit in earlier periods and thereby changes its economic situation in the period when contributions are to be made, in a way that makes its Nash contribution low. For example, high budget deficits today narrow the scope for participation in development aid programs tomorrow.



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