

Konrad, Kai A.

Working Paper — Digitized Version

Private provision of public goods by firms

Münchener Wirtschaftswissenschaftliche Beiträge, No. 92-20

Provided in Cooperation with:

WZB Berlin Social Science Center

Suggested Citation: Konrad, Kai A. (1992) : Private provision of public goods by firms, Münchener Wirtschaftswissenschaftliche Beiträge, No. 92-20, Volkswirtschaftliche Fakultät der Ludwig-Maximilians-Universität München, München

This Version is available at:

<https://hdl.handle.net/10419/112684>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



WZB-Open Access Digitalisate

WZB-Open Access digital copies

Das nachfolgende Dokument wurde zum Zweck der kostenfreien Onlinebereitstellung digitalisiert am Wissenschaftszentrum Berlin für Sozialforschung gGmbH (WZB).

Das WZB verfügt über die entsprechenden Nutzungsrechte. Sollten Sie sich durch die Onlineveröffentlichung des Dokuments wider Erwarten dennoch in Ihren Rechten verletzt sehen, kontaktieren Sie bitte das WZB postalisch oder per E-Mail:

Wissenschaftszentrum Berlin für Sozialforschung gGmbH

Bibliothek und wissenschaftliche Information

Reichpietschufer 50

D-10785 Berlin

E-Mail: bibliothek@wzb.eu

The following document was digitized at the Berlin Social Science Center (WZB) in order to make it publicly available online.

The WZB has the corresponding rights of use. If, against all possibility, you consider your rights to be violated by the online publication of this document, please contact the WZB by sending a letter or an e-mail to:

Berlin Social Science Center (WZB)

Library and Scientific Information

Reichpietschufer 50

D-10785 Berlin

e-mail: bibliothek@wzb.eu

Digitalisierung und Bereitstellung dieser Publikation erfolgten im Rahmen des Retrodigitalisierungsprojektes **OA 1000+**. Weitere Informationen zum Projekt und eine Liste der ca. 1 500 digitalisierten Texte sind unter <http://www.wzb.eu/de/bibliothek/serviceangebote/open-access/oa-1000> verfügbar.

This text was digitizing and published online as part of the digitizing-project **OA 1000+**.

More about the project as well as a list of all the digitized documents (ca. 1 500) can be found at <http://www.wzb.eu/en/library/services/open-access/oa-1000>.

Private Provision of Public Goods by Firms

von

Kai A. Konrad

September 1992

Correspondence Address:

Kai A. Konrad
Department of Economics
University of Munich
Ludwigstraße 33
D-8000 München 22
Germany

Die MÜNCHENER WIRTSCHAFTSWISSENSCHAFTLICHEN BEITRÄGE werden von der Volkswirtschaftlichen Fakultät der Ludwig-Maximilians Universität München herausgegeben. Die inhaltliche Verantwortung für die Beiträge liegt bei den Autoren und nicht bei der Fakultät.

Volkswirtschaftliche Fakultät
Ludwig-Maximilians-Universität München
Ludwigstraße 28
D-8000 München 22

PRIVATE PROVISION OF PUBLIC GOODS BY FIRMS*

by

Kai A. Konrad
University of Munich, Germany

May 1992

* Helpful comments by Ray Rees are gratefully acknowledged. The usual disclaimer applies.

ABSTRACT

In the standard Nash equilibrium of private provision of public goods only the firm with the highest demand contributes. The existence of the contribution game may affect firms' ex-ante decisions on technology, factor inputs and capacity in a way that leads to a decrease of the quantity of the public good that is finally provided. Further, the sequential equilibrium of private provision of a public good by firms is considered. The quantity provided is typically smaller than in the Nash equilibrium, although some firms may contribute more than they would in autarchy.

Kai A. Konrad
Department of Economics
University of Munich
Ludwigstrasse 33
D-8000 Munich 22
Germany

1. Introduction

Some inputs in firms' activities are public goods. Some of them, like the legal system, are publicly provided. Some others are not. There are numerous examples for public goods that are privately provided by firms.

Arrow (1962) and Nelson (1959) pointed out that technological knowledge has the characteristics of a pure public good. Mansfield *et al.* (1981) and Jaffe (1986) show that R&D investment has spillovers. The major fraction of patent bearing innovations is successfully imitated within 4 years (avoiding patent infringements). Investment in R&D therefore may be considered as provision of a public good. Similarly, some firms provide infrastructure and firms invest in non-firm specific human capital. Advertisement may increase the market turnover and has spillovers that constitute a positive externality. Entry deterrence which is provided by one firm in an oligopoly benefits all incumbents (McLean and Riordan, 1989). Buchholz (1991) shows that firms' decisions in duopoly can be understood as a game of private provision of public goods. For example, if one firm reduces its supply, both firms benefit in similar ways from the resulting increase of the equilibrium price. Finally, many rent-seeking activities aim in obtaining favors by government that benefit all firms in an industry. An import ban for particular goods or a subsidy for a particular industry benefits all firms that produce this good or get this subsidy. It has been recognized, e.g., by Magee *et al.* (1989), Katz *et al.* (1990) and Ursprung (1990) that lobbying is often aimed towards a public good.

The standard model of private provision of public goods (see, e.g., Bergstrom *et al.*, 1986) seems to be a good candidate to explain and describe properties of voluntary provision of public goods by firms. In particular, it seems to be plausible that profit maximizing firms are mainly driven by their individual advantage of using the public good they provide. The additional motivations for voluntary provision of public goods by individuals that are discussed in the literature are less compelling for profit maximizing firms. We do not expect a firm to enjoy a "warm

glow" of giving in Andreoni's (1989, 1990) sense if other firms also benefit from a firm's lobbying expenditure.¹ Instead, competition is widely accepted to be the more natural description of a firm's environment.

Section 2 applies the standard Nash-equilibrium concept to firms' provision of a pure public good. Differently to the case of private individuals contributing to a public good, for the symmetric case, there is a multiplicity of equilibria. In the asymmetric case, the equilibrium is generically unique and only one firm provides the whole amount of public good, independently of the number of firms and the intensity of their interest. Moreover, the firm with the highest demand for the public good may benefit least from the equilibrium outcome. Section 3 analyses how the private provision game influences firm's choices of technology, of capacity constraints and their input choices that are made in earlier periods. Firms have a strategic incentive to avoid to be the one firm that provides the public good. This incentive affects firm's choices of their inputs, technology and capacity and the equilibrium outcome of private provision of the public good. Section 4 returns to the 1-stage game of private provision of the public good but considers the case of Stackelberg leadership. If firms commit themselves sequentially on the amount of the public good they provide, this may reduce the aggregate equilibrium amount of public good. However, differently from the asymmetric Nash equilibrium, more than one firm may make positive contributions to the public good. Some firm may contribute more than it would in the Nash equilibrium, and some firm may contribute even more than it would if it were the only firm existing. Section 5 summarizes the results.

¹ Such motivations may become relevant in a principal-agent framework of firms. The manager of a firm may have some private fringe benefits or simply a "warm glow of giving" if this firm makes donations, and his decisions on these donations may be not perfectly observable by the owners of the firm.

2. The Nash equilibrium of voluntary provision

Suppose there are firms $i = 1, \dots, n$. Each firm maximizes its profit

$$(1) \quad \Pi^i = R^i(x^i, G) - x^i - g^i,$$

where g^i is i 's contribution to the aggregate amount G of public good, with $G = \sum_i g^i$. The variable x^i describes cost of factor inputs other than the public good. In particular, in the simplest case, x^i is a single factor of production and its quantity is standardized: its price in units of the public good equals one. Equation (1) describes that profit of firm i is a function of its private input (or input cost), its contribution to the public good, and aggregate provision of the public good. Suppose that for the first and second derivatives of revenue R^i with regard to x^i and G standard convexity assumptions hold: $R^i_x \geq 0$, $R^i_G \geq 0$, $R^i_{xx} \leq 0$, $R^i_{GG} \leq 0$, $R^i_{xG} = R^i_{Gx} \geq 0$. Moreover some standard limiting properties are useful to exclude case studies for corner solutions: $R^i_G(x^i, 0) > 1$ for $x^i > 0$, $R^i_x(0, G) > 1$ for $G \geq 0$, and $R^i(x^i, G)$ is strictly bounded from above.

The problem of the single firm can be reformulated as to maximize profit

$$(2) \quad \Pi^i = R^i(x^i, G) - x^i - (G - G_{-i}),$$

by a choice of x^i and G , s.t.

$$(3) \quad G - G_{-i} \geq 0,$$

$$(4) \quad x^i \geq 0.$$

If the firm considers $G_{-i} = \sum_{j \neq i} g^j$, the contributions of all others, as exogenously given, this problem has an unconstrained maximum that determines the demand of

firm i for the public good, say G^{i*} , neglecting constraint (3). This demand will be called firm i 's *unconstrained demand* for G . Of course, the firm cannot provide a negative amount. Therefore, aggregate provision is equal to G^{i*} if $G_{-i} < G^{i*}$, and it is equal to G_{-i} otherwise.

Example 1.

Firms produce output with a single input, e.g., labor. Their production functions are $y^i = f^i(L^i)$, with $f^i_L > 0$, $f^i_{LL} < 0$, $f^i_L(0) > w$, and $\lim_{L^i \rightarrow \infty} f^i_L < w$. The price of the output is exogenous and equal to one. The wage rate w is also exogenous. There is a proportional profit tax on the difference between revenue and cost with a tax rate $\tau(G)$. This tax rate depends on the aggregate expenditure G on lobbying for a reduced tax rate, with $d\tau/dG < 0$, $d^2\tau/(dG)^2 > 0$. Each firm can contribute to lobbying. The contribution of firm i is $g^i \geq 0$, and can be deducted from the tax base. The net profit of firm i is

$$(5) \quad \Pi^i = (1 - \tau(G)) [f^i(L^i) - g^i - wL^i].$$

This is a special case of (2).

Example 2.

Firms produce output using two inputs, a private input x^i , and a public good G . Their production functions are

$$(6) \quad f^i(x^i, G), \quad i = 1, 2, \dots, n,$$

with standard assumptions about decreasing marginal returns and concavity, i.e., $f^i_x > 0$, $f^i_G > 0$, $f^i_{xx} \leq 0$, $f^i_{GG} \leq 0$, $f^i_{Gx} = f^i_{xG} \geq 0$. Further, f^i exhibits decreasing returns to scale and is bounded from above. The price of private input, of the public good, and of the output good are equal to 1 (by appropriate

standardization of units of goods), and are exogenous. In this case the profit of firm i is determined as

$$(7) \quad \Pi^i = f^i(x^i, G) - x^i - \max\{G - G_{-i}, 0\}.$$

Profit equals revenue $f^i(x^i, G)$ that depends on aggregate public good provision and input x^i , minus input cost x^i , minus cost of contributing to the public good.

Consider the Nash equilibrium for the *general* case:² if firm i maximizes (2) subject to the constraints $x^i \geq 0$ and $G \geq G_{-i}$, then the resulting Kuhn–Tucker problem leads to the first-order conditions³

$$(8) \quad \frac{\partial \Pi^i}{\partial x^i} = 0,$$

and

$$(9) \quad \frac{\partial \Pi^i}{\partial G} \leq 0 \text{ with } (G - G_{-i}) \frac{\partial \Pi^i}{\partial G} = 0.$$

Suppose that firms $1, \dots, n$ are reordered as a_1, a_2, \dots, a_n such that

$$(10) \quad G^{a_1*} \geq G^{a_2*} \geq G^{a_3*} \geq \dots \geq G^{a_n*},$$

² To keep the analysis simple, it is assumed throughout that firm's decisions affect other firms via the private provision of public goods problem only. In particular, firms are not direct competitors: their output decisions do not influence the market demand or prices other firms face.

³ The objective function is strictly concave and the non-negativity constraints are linear. Therefore, the first-order conditions (8) and (9) are necessary and sufficient. Equation (8) uses that $x^i > 0$ in the optimum.

where (x^{a_i*}, G^{a_i*}) is the unconstrained solution of (2) for firm a_i with $i = 1, \dots, n$.

Proposition 1 $G = G^{a_1*}$ is the only Nash-equilibrium outcome.

Proof. Firms' contributions in a Nash equilibrium are $g^{a_i} = \max\{0, G^{a_i*} - G_{-a_i}\}$. First, $G \equiv G^{a_1*}$ is a Nash-equilibrium. Suppose that firm a_1 provides $g^{a_1} = G^{a_1*}$. This makes $G_{-a_i} \geq G^{a_1*}$ for all a_i for $i \geq 2$. Their contribution is

$$g^{a_i} = \max\{G^{a_i*} - G_{-a_i}, 0\} \leq \max\{G^{a_i*} - G^{a_1*}, 0\} = 0. \quad (10)$$

This makes $G_{-a_1} = 0$ and makes firm a_1 provide $G = G^{a_1*}$. Second, $G \equiv G^{a_1*}$ is the only Nash equilibrium. Any $\hat{G} < G^{a_1*}$ cannot be a Nash equilibrium. Firm 1 would provide at least the difference between G^{a_1*} and \hat{G} . Any $\hat{G} > G^{a_1*}$ is not an equilibrium. There would be at least one j with $g^{a_j} > 0$. For this, $g^{a_j} = \hat{G} - G_{-a_j} > G^{a_j*} - G_{-a_j}$ which violates the definition of G^{a_j*} . ■

Proposition 1 says that the aggregate amount of public good in the equilibrium equals the unconstrained demand of the firm with the highest demand. The amount of aggregate public good that is provided by a group of firms in the Nash equilibrium is independent of the number of firms and of the unconstrained demand of any firm except the firm with the highest unconstrained demand for the public good. The intuitive reason for this outcome is that firms have no income effect. The marginal opportunity costs of lobbying for a firm are constant. Marginal costs are independent of the level of their own lobbying expenditure. For the case

$$G^{a_1*} = \dots = G^{a_j*} > G^{a_{j+1}*} \geq \dots \geq G^{a_n*},$$

the fractions $\gamma^{a_i} G^{a_i*} \geq 0$ that are provided by firms $a_i \in \{a_1, \dots, a_j\}$ are indeterminate. Any $(\gamma^{a_1}, \dots, \gamma^{a_j})$ with $\sum_{i=1}^j \gamma^{a_i} = 1$ is a Nash equilibrium. For a more

asymmetric situation, however, the equilibrium is generically unique.

Corollary 1 If $G^{a_1*} > G^{a_2*}$, then $g^{a_1} = G = G^{a_1*}$, $g^{a_2} = \dots = g^{a_n} = 0$ is the only Nash equilibrium.

Proof. Suppose $g^{a_j} > 0$ with $g^{a_j} + G_{-a_j} = G$ for $a_j \neq a_1$. By definition, $g^{a_j} = \max \{0, G^{a_j*} - G_{-a_j}\}$. Hence, by $g^{a_j} > 0$, $G = G^{a_j*}$. But $G^{a_j*} < G^{a_1*}$, and, by Proposition 1, $G = G^{a_1*}$. This is a contradiction. ■

Corollary 1 shows that, except for the very symmetric case, the standard model of private provision of public goods predicts that an industry leader (someone with the highest demand for the public good) will play a major role in the provision of public goods.

3. Strategic commitment

Firms make some of their choices on production technology, inputs, capacity, marketing strategy, etc. before the decisions on public good provision are made. These choices determine the individual firm's demand function for the public good, because they determine the shape of the marginal revenue curve R^i_G . Graphically, each firm i 's unconstrained demand for the public good can be described by the point of intersection of the R^i_G -curve with the cost of providing an additional unit of the public good. This is illustrated in Figure 1 for the case with two firms, 1 and 2.

Consider the marginal revenue curves R^1_G and R^2_G in Figure 1. Given the nature of the Nash equilibrium of private provision of a public good, firm 1 provides $G = G^{a_1*}$, firm 2 provides zero. Firm 1 may have an incentive to choose a different technology, input mix, or capacity, that leads, e.g., to the dashed marginal revenue curve in Figure 1. In this case firm 2 would provide all public good in the Nash equilibrium, i.e., G^{a_2*} . Of course, anticipating this, firm 2 would possibly like to avoid this outcome by also making a different technology choice.

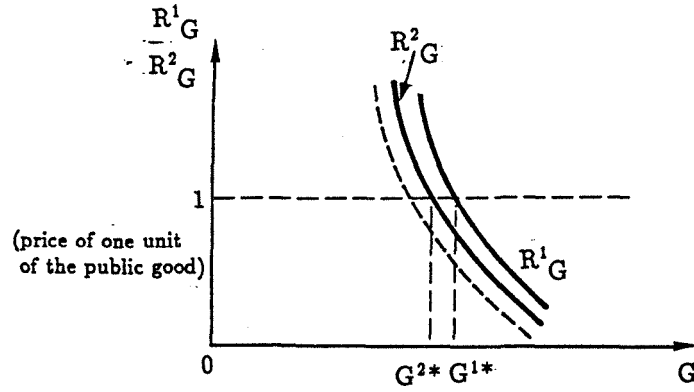


Figure 1

This section concentrates on such strategic behavior. It uses the particular framework of example 2 with two firms, but for the case where the firms first commit themselves with regard to the input factor x^i . When x^i is chosen for $i = 1, 2$, decisions on contributions g^i are made. The production functions are

$$(11) \quad f^i(x^i, G), \quad \text{for } i = 1, 2,$$

with the properties outlined in example 2 in section 2.

Simultaneous decisions on private inputs

First consider the case where firms in stage 1 simultaneously decide on their input x^1 and x^2 , and then, once x^1 and x^2 are determined, in stage 2, they play a Nash game to determine their contributions g^1 and g^2 to the public good. Given their choices x^1 and x^2 , the unconstrained demand of firms is now a function of input choices on x^1 and x^2 , written as $G^{1*} = G^{1*}(x^1)$ and $G^{2*} = G^{2*}(x^2)$. This *unconstrained demand* is implicitly determined by the conditions $\frac{\partial f^i(x^i, G)}{\partial G} \Big|_{G = G^{i*}} = 1$ for $i = 1, 2$.

The profit of firm 1 is

$$(12) \quad \Pi^1 = f^1(x^1, G) - x^1 - \max\{G - G_{-1}, 0\}.$$

Profit equals revenue $f^1(x^1, G)$ that depends on aggregate public good provision and input x^1 , minus input cost x^1 , minus actual cost of firm 1's contribution to the public good. Figure 2 depicts the profit of firm 1 as a function of factor input x^1 for a given input choice x_2^0 of firm 2. If $x^1 = 0$, then $f^1(0, G) = 0$, and firm 1 will not provide any public good. Similarly, if the factor input x^1 is small enough, given the factor input choice of firm 2, the unconstrained demand for public good of firm 2 will be higher than that of firm 1. Therefore, firm 2 will provide all public good in the equilibrium in stage 2, $G = G^{2*}(x_2^0)$. For small x^1 and given x_2^0 , profit of firm 1 is

$$(13) \quad f^1(x^1, G^{2*}(x_2^0)) - x^1.$$

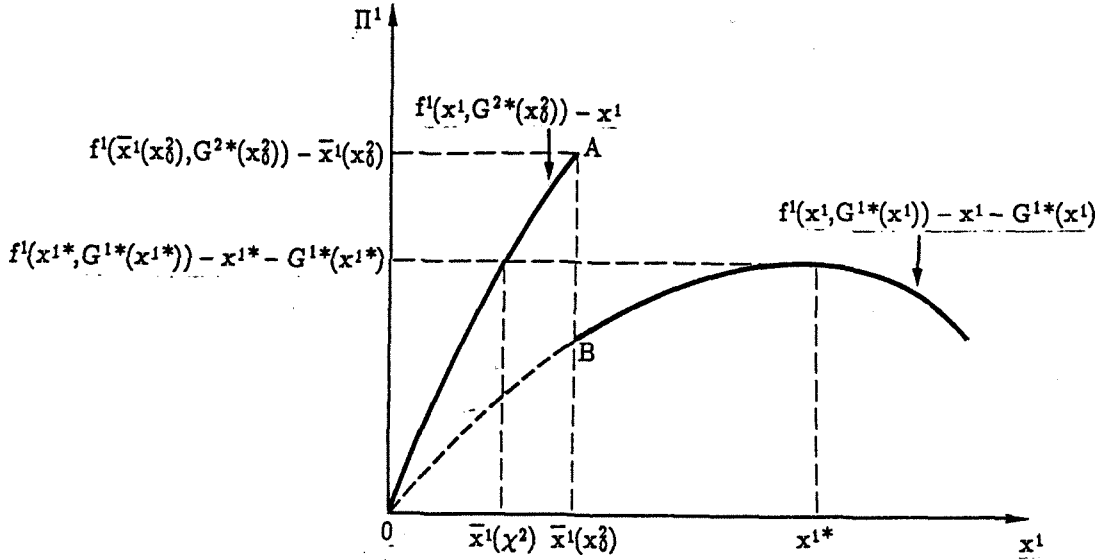


Figure 2

However, there is a critical $\bar{x}^1(x_2^0)$, such that, for $x^1 > \bar{x}^1(x_2^0)$, the unconstrained demand of firm 1 for the public good is larger than $G^{2*}(x_2^0)$, and, therefore, only firm

1 will provide the whole amount of public good in the stage-2-Nash equilibrium of private provision of the public good, i.e., $G = G^1(x^1)$. Profit becomes

$$(14) \quad f^1(x^1, G^1(x^1)) - x^1 - G^1(x^1)$$

for $x^1 > \bar{x}^1(x_2^*)$. The x^1 that maximizes (14) on $x^1 \in (\bar{x}^1(x_2^*), \infty)$ for firm 1 is labelled x^{1*} .

For $x^1 = \bar{x}^1(x_2^*)$, finally, $G^1(x^1) = G^2(x_2^*)$. In this symmetric case there is a continuum of Nash equilibria of private provision. In any of these equilibria the aggregate provision equals $G = G^1(x^1) = G^2(x_2^*)$, where a fraction $\gamma \in [0, 1]$ is provided by firm 1, and $1-\gamma$ is provided by firm 2. Therefore, depending on γ , for $\bar{x}^1(x_2^*)$, the profit of firm 1 is somewhere between A and B in Figure 2. Firm 1 chooses x^{1*} or $\bar{x}^1(x_2^*) - \epsilon$ with $\epsilon > 0$ infinitesimal for given x_2^* , depending on which of the two yields the higher profit.⁴

There is at most one $x_2^* = \chi^2$ for which these two profit levels coincide, i.e.,

$$(15) \quad f^1(\bar{x}^1(\chi^2), G^2(\chi^2)) - \bar{x}^1(\chi^2) = f^1(x^{1*}, G^1(x^{1*})) - x^{1*} - G^1(x^{1*}).$$

For $x^2 < \chi^2$, the autarchy input choice x^{1*} yields higher profit for firm 1, for $x^2 > \chi^2$, the choice $\bar{x}^1(\chi^2) - \epsilon$ for infinitesimally small ϵ yields higher profit for firm 1. This shows that a firm may have a strategic incentive to choose its inputs such that it has not to provide the public good in stage 2. The pay-off function is inherently discontinuous in the input choices (x^1, x^2) . Such discontinuities are well

⁴ I do not consider the uninteresting case with (13) having an interior global optimum on $[0, \bar{x}^1(x_2^*)]$ for relevant parameters of x_2^* . The two firms are very different with regard to their demand for G in this case. The strategic considerations this section concentrates on may not occur in such cases (see the example below in this section).

known to cause problems of existence of an equilibrium.

The problem does not necessarily occur if firms are sufficiently different. The following example illustrates this point. Firm i 's unconstrained or *autarchic* choice is again denoted $(x^i, G^i(x^i))$. Define $\hat{x}^1 = \arg \max \{f^1(x^1, G^2(x^2)) - x^1\}$. Suppose that, for all $G \leq G^1(\hat{x}^1)$,

$$(16) \quad \max_{x^2} f^2(x^2, G) - x^2 < f^2(x^2, G^2) - x^2 - G^2,$$

(which also implies $G^2(x^2) > G^1(\hat{x}^1)$). In this case, firm 2 can never hope to be in a situation where it pays to choose x^2 small enough to free ride in stage 2. Note that \hat{x}^1 is the highest input firm 1 could possibly choose in stage 1. But still, even for this high level, there is no choice x^2 that makes firm 2's profit higher than its autarchic profit (the right-hand side of (16)). The conditions in the example are much tighter than needed. The main property that is needed is that the marginal productivity of the public good in both firms is sufficiently different so that it never pays for the firm which is in more serious need for G to make the other firm pay. Generally, this property is less likely to hold if firms become more similar. In fact:

Proposition 2 Let there be two firms $i = 1, 2$, with identical production functions with

$$(17) \quad f^i(x^i, G^i(x^i)) - x^i - G^i(x^i) > 0.$$

There is no Nash equilibrium in pure strategies.

Proof. Assume that (x^{1n}, x^{2n}) is a Nash equilibrium. Suppose first that $x^{1n} = x^{2n}$. In this case, at least one of the firms would provide some positive fraction γ of the public good, say it is firm 1. Given x^{2n} , if firm 1 reduces its demand for x^1 by some infinitesimal $\varepsilon > 0$, this has only a marginal direct impact on its revenue, but reduces firm 1's equilibrium provision of the public good to zero without reducing the aggregate amount that is provided in the equilibrium. This holds for any positive

$x^{1n} = x^{2n}$. But, by (17), $x^1 = x^2 = 0$ is also not a Nash equilibrium as each firm i can do better by choosing $(x^{i*}, G^{i*}(x^{i*}))$. Suppose instead that $x^{1n} < x^{2n}$. In this case only firm 2 provides the public good, $G = G^{2*}(x^{2n})$. If firm 1 takes x^{2n} as exogenously given, its optimal choice therefore is $x^1 = x^{2n} - \epsilon$. If, however, firm 2 takes $x^{1n} = x^{2n} - \epsilon$ as exogenously given, then its optimal choice is $x^2 = x^{1n} - \epsilon = x^{2n} - 2\epsilon < x^{2n}$. This contradiction shows that (x^{1n}, x^{2n}) is not an equilibrium. ■

Proposition 3 Under the premises of Proposition 2, there is a Nash equilibrium in mixed strategies, if, for perfect symmetry on stage 2 ($x^1 = x^2$), both firms contribute some positive amount of the public good.

Proof. The choice sets of firms 1 and 2 can be chosen as closed convex intervals $A^1 \equiv [0, x^1_{\max}]$ and $A^2 \equiv [0, x^2_{\max}]$ for sufficiently large x^i_{\max} since revenue is bounded from above. The pay-off functions are

$$(18) \quad \Pi^1 = \begin{cases} f^1(x^1, G^{2*}(x^2)) - x^1 & \text{for } x^1 < x^2 \\ f^1(x^1, G^{1*}(x^1)) - x^1 - G^{1*}(x^1) & \text{for } x^1 > x^2 \\ f^1(x^1, G^{1*}(x^1)) - x^1 - \gamma G^{1*}(x^1) & \text{for } x^1 = x^2 \text{ with } \gamma \in (0,1), \end{cases}$$

and similarly for Π^2 with all "1" substituted by "2" and vice versa, and γ substituted by $(1-\gamma)$.

These pay-off functions are continuous in (x^1, x^2) for all $x^1 \neq x^2$. Further,

$$(i) \quad \lim_{x^1 \rightarrow x, x^2 \rightarrow x} \Pi^1(x^1, x^2) \geq \Pi^1(x, x) \geq \lim_{x^1 \rightarrow x, x^2 \rightarrow x} \Pi^1(x^1, x^2)$$

and

$$(ii) \quad \lim_{x^1 \rightarrow x, x^2 \rightarrow x} \Pi^2(x^1, x^2) \leq \Pi^2(x, x) \leq \lim_{x^1 \rightarrow x, x^2 \rightarrow x} \Pi^2(x^1, x^2),$$

where, for $\gamma \in (0,1)$, all inequalities are strictly fulfilled. This makes the assumptions fulfilled to apply Theorem 5b of Dasgupta and Maskin (1986, p. 16) and Proposition 3 follows. ■

Sequential private input choice

Consider now a slightly modified 2-stage game. Suppose that the two firms make their input choices sequentially. Firm 2 decides first on x^2 . Then firm 1 decides on x^1 , taking x^2 as given.⁵ Then, once x^2 and x^1 are determined, firms contribute their Nash-equilibrium amounts g^1 and g^2 of the public good. The decision problem of firm 2 becomes

$$(19) \quad \max_{x^2} \{f^2(x^2, \max\{G^2^*(x^2), G^1^*(x^1(x^2))\}) - x^2 - g^2\}$$

$$\text{with } g^2 = \begin{cases} 0 & \text{for } G^1^*(x^1(x^2)) > G^2^*(x^2) \\ G^2^*(x^2) & \text{for } G^1^*(x^1(x^2)) < G^2^*(x^2) \\ \gamma G^2^*(x^2) & \text{for } G^1^*(x^1(x^2)) = G^2^*(x^2), \text{ with } \gamma \in [0,1]. \end{cases}$$

For its decision on x^2 , firm 2 knows that the choice of x^1 depends on x^2 : $x^1 = x^1(x^2)$, and, equilibrium contributions of the public good depend on x^2 and x^1 .

Figure 3 illustrates the solution of this problem. Recall from (15) that χ^2 is the level of private good chosen by firm 2 that makes firm 1 just indifferent between choosing $(x^1^*, G^1^*(x^1^*))$ and choosing $(\bar{x}^1(\chi^2) - \varepsilon, 0)$, where $\bar{x}^1(x^2)$ for some x^2 is defined as the quantity of x^1 such that $G^1^*(\bar{x}^1(x^2)) = G^2^*(x^2)$.

For $x^2 < \chi^2$, firm 1 is better-off choosing its optimal unconstrained level of its private input, x^1^* , knowing that it will also provide all public good, i.e., $G^1^*(x^1^*)$. This combination $(x^1^*, G^1^*(x^1^*))$ is the maximand of the unconstrained problem to

⁵ It is just a convention that the higher numbered firm contributes first. This convention is convenient in some later proofs that use induction. The numbering of firms does not imply anything about the relative size of firms' unconstrained demands for the public good.

maximize $f^1(x^1, G^1) - x^1 - G^1$. The profit of firm 2 in this case is

$$(20) \quad f^2(x^2, G^{1*}(x^{1*})) - x^2.$$

For all $x^2 > \chi^2$, firm 1 will not contribute to the public good. Only firm 2 will contribute its optimal amount, given x^2 . Therefore, in this range, the profit of firm 2 is

$$(21) \quad f^2(x^2, G^{2*}(x^2)) - x^2 - G^{2*}(x^2).$$

For $x^2 = \chi^2$, firm 1 is just indifferent between choosing $(x^{1*}, G^{1*}(x^{1*}))$ and choosing $\bar{x}^1(\chi^2) - \varepsilon$, in which case assume that it chooses $(x^{1*}, G^{1*}(x^{1*}))$. Firm 2 chooses $x^2 \leq \chi^2$ or x^{2*} , depending on whether

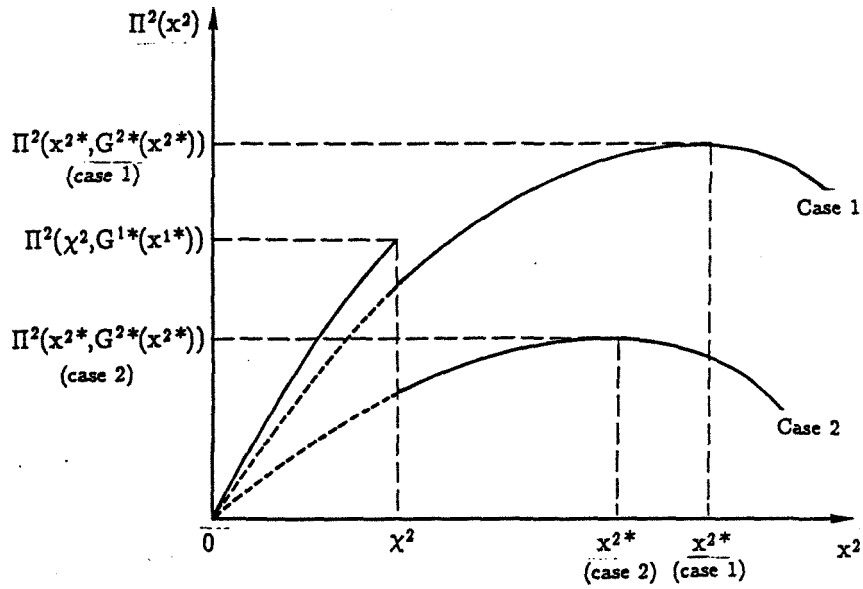
$$(22) \quad \max_{x^2 \leq \chi^2} [f^2(x^2, G^{1*}(x^{1*})) - x^2] \left\{ \begin{array}{l} \geq \\ \leq \end{array} \right\} \max_{x^2 > \chi^2} [f^2(x^2, G^{2*}(x^2)) - x^2 - G^{2*}(x^2)].$$

The decision problem of firm 2 is depicted in Figure 3 for two different shapes of (21). In case 1 firm 2 prefers to choose its unconstrained input vector $(x^{2*}, G^{2*}(x^{2*}))$, which yields profit $\Pi^2(x^{2*}, G^{2*}(x^{2*}))$. In case 2, firm 2 prefers to restrict its input demand x^2 strategically to χ^2 (or $\chi^2 - \varepsilon$). For this choice, firm 1 provides $G^{1*}(x^{1*})$ of the public good and firm 2 free-rides. The profit of firm 2 is $\Pi^2(\chi^2, G^{1*}(x^{1*}))$ and is higher than the profit of choosing $(x^{2*}, G^{2*}(x^{2*}))$.

These considerations show that cases are possible where the firm adjusts its technology, factor demand, or capacity choice (in the example given simply the demand for the other production factor) strategically in stage 1 in order not to become the sole provider of the public good in the Nash equilibrium.

For given choices of the private input, the amount of public good that is provided

in the equilibrium is such that the marginal willingness to pay of the firm with the highest marginal willingness to pay equals marginal cost for an additional unit of the public good. Samuelson's rule would require that the sum of marginal willingnesses of firms equals marginal cost. This is the standard underprovision result of private provision of public goods. In addition to this "Samuelsonian" underprovision, in an ex-ante situation, firms may try to avoid to be a contributor to the public good by strategic choices of their inputs, their technology, or their capacity constraints. Condition (22) shows that this strategic behavior has the tendency of aggravating the problem of underprovision.



$$\text{Case 1: } \max_{x^2 \leq x^2} [f^2(x^2, G^{1*}(x^{1*})) - x^2] < \max_{x^2 > x^2} [f^2(x^2, G^{2*}(x^{2*})) - x^2 - G^{2*}(x^2)]$$

$$\text{Case 2: } \max_{x^2 \leq x^2} [f^2(x^2, G^{1*}(x^{1*})) - x^2] > \max_{x^2 > x^2} [f^2(x^2, G^{2*}(x^{2*})) - x^2 - G^{2*}(x^2)]$$

Figure 3

4. Sequential provision of the public good

If there are few firms that provide and use a public good, the Nash assumption may be less reasonable. An alternative hypothesis is that firms provide the public good sequentially. Such Stackelberg games for two players in the context of private provision of a public good by individuals have been described by Varian (1990). The purpose of this section is to consider the sequential equilibrium for the case of more than two firms.

There are n firms. They are numbered according to their order of play: firm n plays *first*, then $n-1$, etc. Each firm $m \in \{1, \dots, n\}$ maximizes an objective function

$$(23) \quad R^m(G) - g^m,$$

which is a particularly simple case of (2) for exogenous x^m . Suppose that $R^m(G)$ is twice differentiable with $R^m_G(0) > 1$, $R^m_{GG} < 0$ for $G < G^m_{\max}$, and $R^m_G = 0$ for all $G \geq G^m_{\max}$ for some firm specific G^m_{\max} . Define $G_{\max} = \max \{G^m_{\max} \mid m \in \{1, \dots, n\}\}$. Again, G is the aggregate amount of the public good that is provided by all firms, and g^m is firm m 's contribution. Firm m takes the contributions of firms $n, n-1, \dots, m+1$ as exogenous. These firms already chose their contributions. The contributions of all other firms $m-1, \dots, 1$ depend on m 's own contributions, i.e., firm m takes into account that

$$(24) \quad G = g^n + \dots + g^{m+1} + g^m + g^{m-1}(\cdot) + \dots + g^1(\cdot),$$

where the argument (\cdot) that determines firm $(m-k)$'s contribution g^{m-k} is the amount of contributions made by firms $n, \dots, m-k+1$, i.e., $\sum_{j=m-k+1}^n g^j$.

The sequential game with two players is particularly simple. Firm 1's unconstrained demand for the public good is G^{1*} . If firm 2 provides more than this quantity, then firm 1 will not provide any public good. If firm 2 provides less than

G^1^* , then firm 1 provides the difference $G^1^* - g^2$. Therefore, it never pays for firm 2 to provide a quantity between zero and G^1^* . It may provide nothing, in which case its profit is

$$(25) \quad R^2(G^1^*),$$

or it provides more than G^1^* , in which case firm 1 does provide zero such that firm 2 provides all. In this case firm 2 provides its unconstrained demand G^2^* , and its profit is

$$(26) \quad R^2(G^2^*) - G^2^*.$$

Firm 2's choice depends on which of these profits is higher.

Proposition 4 Suppose that, whenever a firm is indifferent between different amounts of contribution, it chooses the highest one. In this case, a sequential contribution equilibrium exists for any finite number of firms and is unique.

Proof by induction. Consider the case with only one firm. Suppose that this firm finds an amount of ${}_1G \geq 0$ of the public good already provided. It decides on its optimal amount of contributions and the choice on $[0, G_{\max}]$ is unique. This is the Stackelberg game solution for one firm. Suppose now that there are m firms. Firm m is the first to decide on its provision, and, an amount of ${}_mG \geq 0$ of the public good is already provided exogenously. Suppose that, for any ${}_mG \geq 0$, the Stackelberg game has a unique solution, i.e., a vector of contributions $(g^m, g^{m-1}, \dots, g^1)$. Now add an additional firm $m+1$ that finds an initial amount of contributions ${}_{-(m+1)}G$ and decides on its contributions $g^{m+1} = {}_{-(m+1)}G - {}_mG$ and makes its contributions

before firm m starts deciding. By its decision, firm $m+1$ determines g_m . This firm $m+1$ simply chooses its preferred amount of contribution, given the equilibria resulting from possible choices. Provided that, for indifference, it chooses the larger contribution, this choice is unique⁶. Therefore, if a unique Stackelberg equilibrium for m firms exists, it also does for $m+1$ firms, and, therefore, for an arbitrary number of firms. ■

Now some properties of the sequential equilibrium are derived.

Proposition 5 If $G^{1*} > G^{m*}$ for all $m \in \{2, \dots, n\}$, then the Nash equilibrium and the sequential equilibrium are identical.

Proof: Firm 1 is the last firm that possibly contributes. Its contributions are equal to $\max\{0, G^{1*} - g_1\}$. Therefore, firm 2 will not contribute, independently of how much firms 3, ..., n contributed, as $G^{2*} < G^{1*}$. Suppose that firms 2 to j do not contribute, but firm 1 finally contributes $\max\{0, G^{1*} - g_1\}$. Of course, in this case also firm $j+1$ does not contribute, and so for all $j = 2, \dots, n$ for arbitrary n . ■

Varian (1990) shows in a similar model for two consumers that two properties hold in the sequential equilibrium.

- a) Aggregate contributions in the sequential equilibrium are never larger than in the Nash equilibrium.
- b) The amount of the public good contributed by the Stackelberg leader, i.e., by the agent who provides first in the sequential equilibrium is never larger than

⁶ As is standard in principal-agent theory, I disregard here the theoretical problem that the optimal contribution may be infinitesimally smaller or larger than a particular g_0 .

the amount provided by this agent in the Nash equilibrium (Varian, 1990, Theorem 2, p. 17).

Varian's first result a) is robust for the case with n agents. Aggregate contributions in the sequential equilibrium are not higher than in the Nash equilibrium. The second result b) does not generalize. It turns out that, differently to the Nash equilibrium with non-identical firms, in the sequential equilibrium, more than one firm may contribute a positive amount, and, e.g., in the 3-firm case, the firm that contributes first may contribute more than it would contribute in the Nash equilibrium, and may even contribute more than its unconstrained demand for the public good. These properties are established by the following propositions.

Proposition 6 (Varian 1990) Aggregate provision in the Nash equilibrium is not smaller than in the sequential equilibrium.

Proof. In the Nash equilibrium, $G^{\text{nash}} = \max \{G^i^* \mid i \in \{1, \dots, n\}\}$. In the sequential equilibrium the following property holds. No firm m that finds an amount $_{-m}G$ already provided has an incentive to provide $g^m > 0$ such that $_{-m}G + g^m > G^{\text{nash}}$. A proof of this property is by induction. The property is true for $m = 1$. Suppose it is true for the last m firms, i.e., for $_{-i}G < G^{\text{nash}}$, $_{-i}G + g^i \leq G^{\text{nash}}$ for all $i = 1, \dots, m$, and, for $_{-i}G \geq G^{\text{nash}}$, $g^i = 0$. This implies that $g^m = g^{m-1} = \dots = g^1 = 0$, if $_{-m}G \geq G^{\text{nash}}$. Consider now firm $m+1$. If it finds $_{-(m+1)}G \geq G^{\text{nash}}$, then this firm chooses $g^{m+1} = 0$. Suppose it finds $_{-(m+1)}G < G^{\text{nash}}$. It will provide $g^{m+1} \leq G^{\text{nash}} - _{-(m+1)}G$. If it provides $g^{m+1} > G^{\text{nash}} - _{-(m+1)}G$, then $g^m = g^{m-1} = \dots = g^1 = 0$. In this case g^{m+1} is suboptimal. E.g., $\hat{g}^{m+1} = G^{\text{nash}} - _{-(m+1)}G$ is a better choice: still, $g^m = g^{m-1} = \dots = g^1 = 0$, but for $G > G^{\text{nash}} \geq G^m^*$, $R^{m+1}_G(G) < 1$. This shows that the property holds for any m . Proposition 6 follows immediately. ■

Proposition 7 In a sequential equilibrium, $g^m > G^{m*}$ is possible for some m .

Proof (by an example). Consider $n = 3$. Suppose that

$$(26) \quad R^2(G^{2*}) - G^{2*} = R^2(G^{1*}) - \varepsilon$$

with $\varepsilon > 0$ but small, and $G^{2*} > 2G^{1*}$. These conditions are fulfilled for appropriate shapes of $R^2(G)$ and $R^1(G)$. If firm 3 chooses $g^3 = {}_{-2}G = 0$, then the equilibrium value of the sequential game of firms 2 and 1 is G^{1*} . Firm 2 does not provide. It prefers to leave it to firm 1 to provide the public good, although this implies that only a relatively small amount of G is provided in the equilibrium. If $g^3 = {}_{-2}G > \varepsilon$, then firm 2 will provide $g^2 = G^{2*} - {}_{-2}G$. Suppose that $G^{3*} = \varepsilon/2$, but that $1 > R^3_G \geq 1/2$ for $G \in (\varepsilon/2, G^{2*}]$. For sufficiently small ε , firm 3 prefers to contribute $g^3 = \varepsilon + \delta$ with some infinitesimal $\delta > 0$, because, in this case, it gets $G = G^{2*}$ instead of $G = G^{1*}$. Its benefit is

$$(27) \quad \int_{G^{1*}}^{G^{2*}} R^3_G(G) dG > G^{1*}/2,$$

his cost is only infinitesimally larger than ε . Therefore, $g^3 = \varepsilon + \delta$ is preferred to any $g^3 \in [0, \varepsilon/2]$ for sufficiently small ε . In this example firm 3 provides more than its unconstrained demand G^{3*} in the equilibrium. ■

The intuition behind the example that proves Proposition 7 is that, in a sequential provision game, a small change in the contribution of a firm that provides early can influence the aggregate provision in the equilibrium in a discontinuous way. Again, similar to section 3, input choices are made strategically. The resulting equilibrium choices therefore may have seemingly strange properties, like in the

example, where a firm contributes units that yield marginal revenue that is below marginal costs.

5. Conclusions

This paper considered private provision of public goods by firms. The standard Nash-equilibrium of private provision of public goods, in which firms' only decision variable is the amount of public good they provide, is very simple because income effects are absent. The equilibrium is generically unique. The firm with the highest demand for the public good provides the amount it would provide if other firms were non-existent. There is a tendency for underprovision (from firms' point of view) that is well-known from the standard Nash equilibrium model of private provision of public goods. The marginal unit that is provided in the equilibrium is determined by the marginal revenue of the firm with the highest demand for the public good, not of by the sum of marginal revenues of all firms.

The underprovision problem is aggravated once the choice set of firms is enlarged. If they can influence their marginal benefits of using the public good before the Nash game of private provision of the public good takes place, then they may have a strategic incentive not to be the firm with the highest demand for the public good. If a firm undercuts the demand of a competitor slightly, this firm may save the whole amount of provision, because, in equilibrium, only the firm with the highest demand provides. The paper considers the ex-ante choice of a complementary production factor, showing that this choice may be inefficiently small for the strategic reasons outlined. But the result can be taken as more general. The expectation of an upcoming Nash game of private provision of a public good may influence firms' choices of their capacity, technology, fixed factor inputs, or the degree of product differentiation in a way to rely less on the public good.

Many examples are conceivable. Firms may try to become less dependent on the

outcome of an industry's lobbying activity. If lobbying aims in an export subsidy, a firm may find it useful to concentrate its marketing effort more in the domestic market. If lobbying aims in a factor input subsidy, a firm may find it useful to invest in the development of a substitute for this factor in its own production. Or consider an industry with spillovers of advertising. A new entrant's capacity choice may be influenced by strategic considerations about his prospects to get a free-ride on advertising. Finally, if a firm is known to be a very efficient imitator, its competitors have less reason to expect this firm to carry out major R&D effort.

The paper also considered the case where the private provision game is sequential with a well-defined order in which firms contribute to the public good. This strategic game also aggravates the underprovision problem. The maximum amount possibly provided in this game is equal to the Nash equilibrium amount, and this equilibrium outcome can always be generated by making the firm with the highest demand contribute last. However, the equilibrium amount of aggregate provision can be lower than the Nash equilibrium amount. The equilibrium has also the property that, for strategic reasons, some firm may provide an amount that is higher than its unconstrained demand, i.e., an amount such that the marginal unit of its contribution increases this firm's revenue by less than the cost of this marginal unit.

References

- Andreoni, James, 1989, Giving with impure altruism: applications to charity and Ricardian equivalence, *Journal of Political Economy*, 97, 1447–1458.
- Andreoni, James, 1990, Impure altruism and donations to public goods: a theory of warm-glow giving, *Economic Journal*, 100, 464–477.
- Arrow, Kenneth J., 1962, Economic welfare and the allocation of resources for invention, in Richard R. Nelson (ed.), *The Rate and Direction of Inventive Activity: Economic and Social Factors*, Princeton University Press, Princeton, 609–625.
- Bergstrom, Theodore, Lawrence Blume, and Hal Varian, 1986, On the private provision of public goods, *Journal of Public Economics*, 29, 25–49.
- Buchholz, Wolfgang, 1991, Isoliertes und koordiniertes Verhalten im Duopol und bei öffentlichen Gütern – eine einheitliche grafische Darstellung, *Jahrbuch für Sozialwissenschaft*, 42, 336–357.
- Dasgupta, Partha, and Eric Maskin, 1986, The existence of equilibrium in discontinuous economic games, I: theory, *Review of Economic Studies*, 53, 1–26.
- Jaffe, Adam B., 1986, Technological opportunity and spillovers of R&D: evidence from firm's patents, profits, and market value, *American Economic Review*, 76, 984–1001.
- Katz, Eliakim, Shmuel Nitzan, and Jacob Rosenberg, 1990, Rent-seeking for pure public goods, *Public Choice*, 65, 49–60.
- Magee, Stephen P., William A. Brock, and Leslie Young, 1989, *Black Hole Tariffs and Endogenous Policy Theory*, Cambridge University Press, Cambridge.
- Mansfield, Edwin, Mark Schwarz, and Samuel Wagner, 1981, Imitation costs and patents: an empirical study, *Economic Journal*, 91, 907–918.
- McLean, Richard P. and Michael H. Riordan, 1989, Industry structure with sequential technology choice, *Journal of Economic Theory*, 47, 1–21.

- Nelson, Richard R., 1959, The simple economics of basic scientific research, *Journal of Political Economy*, 67, 297–306.
- Tullock, Gordon, 1988, Future directions for rent-seeking research, in Ch.K. Rowley, R.D. Tollison, and G. Tullock (eds), *The Political Economy of Rent-Seeking*, Boston, Kluwer, 465–480.
- Ursprung, Heinrich W., 1990, Public goods, rent dissipation, and candidate competition, *Economics and Politics*, 2, 115–132.
- Varian, Hal R., 1990, Sequential Provision of Public Goods, CREST working paper, University of Michigan.