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**Taxation and Risk Taking
in a Capital Market Equilibrium
with Self-Selection**

by

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January 1991

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**TAXATION AND RISK TAKING
IN A CAPITAL MARKET EQUILIBRIUM
WITH SELF-SELECTION**

by

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Abstract

A reactive competitive capital market equilibrium with self-selection is considered in a situation where entrepreneurs decide whether to go public and, if so, to what extent. It is shown that taxes on capital gains and a profit tax have strong general equilibrium effects on risk allocation, even if tax proceeds are lump-sum redistributed. This result is in contrast to the view that the Domar-Musgrave effect is only a partial equilibrium effect. Comparing the adverse effects of the tax in private risk markets with the additional risk diversification which is obtained within the tax proceeds reveals that the taxes can improve the risk allocation.

Keywords: risk taking, taxation, capital markets, adverse selection, signalling.

JEL classification numbers: 321, 313.

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1. Introduction

The taxation of risky profits induces an investor to increase his demand for the risky investment opportunity. This phenomenon was first studied by Domar and Musgrave (1944) and its robustness in partial equilibrium has been shown under very different tax regimes¹. In a general equilibrium framework the risk-taking incentive of such taxes cannot be taken for granted. If perfect capital markets exist, there is little reason why diversifiable risks of entrepreneurial firms should not be diversified in private capital markets. This has been pointed out, e.g., by Bulow and Summers (1984, p.24) and Gordon (1985).

However, existing capital markets are far from perfect. Information in capital markets is asymmetric. Entrepreneurs who found and manage a firm generally know the true profitability of their firm better than potential buyers. As Myers and Mayluf (1984) pointed out, the capital market might be a "lemon" market. Assume that entrepreneurs who own exceptionally profitable firms are unable to signal the exceptionally high quality of their firms. They can either accept the market price for their firm below the "true" market value or they keep the firm for themselves. In the latter case, they gain the higher expected revenues, but they also bear some nonsystematic risks involved with this firm that could be diversified in the capital market. In two recent papers, taxes on risky income in equilibria with capital market imperfections are considered. Konrad (1990) derives the partial equilibrium incentive effects of a risk-taking revenue tax in a capital market equilibrium with hidden knowledge, concentrating on the substitutability of portfolio and real investment choice. Konrad and Richter (1990) consider the general equilibrium reactions to taxes on risky income in equilibria where possible signals of firm quality are not observable. The equilibria are adverse selection equilibria where better quality firms partially or totally leave the capital market.

This paper is close in spirit to Konrad and Richter (1990), but, instead of an adverse selection equilibrium, a signalling equilibrium with self-selection is treated. The effects of a profit tax and two types of capital gains tax on private risk taking and aggregate risk

diversification in a general equilibrium model of capital markets with self-selection are analysed. The structure of the paper is as follows. Section 2 characterizes the basic elements of the model and outlines the properties of the separating equilibrium. Section 3 introduces taxes and derives their general equilibrium impacts. Taxes turn out to be effective. The private general equilibrium reaction is compared to the additional risk diversification within tax proceeds to derive the net impact of the tax on aggregate risk diversification. In section 4 the line of arguments and the main conclusions are set out in a less formal way.

2. The model

Consider an economy with only one physical good and two periods, $t = 0, 1$. There is a large set of entrepreneurs. All entrepreneurs are expected utility maximizers with the same strictly concave utility of end of period wealth functions $U(V_1)$ and are endowed with the same initial wealth $V_0 > 0$. Each founds his own firm f , investing an amount of a . For simplicity I assume $a = 0$. Given a neutral way of finance the results do not depend on this assumption and investment is not the concern of the paper. The firm yields a stochastic output in period 1, $z^f \equiv Ez^f + X^f$, with expected output Ez^f and some random fluctuation X^f . $EX^f = 0$ by definition and all X^f are assumed to be identically and independently distributed random variables. Superscripts (" f ") are used to designate variables related to firm f or the initial owner of this firm.

There are two types of firms, good ones (superscript " g "), and bad ones (superscript " b "), with $Ez^g > Ez^b$. Entrepreneurs know the true type of their firm, potential buyers of shares do not.

Entrepreneurs can sell all shares of their firm in the capital market, or may retain a certain fraction q^f and only sell the rest $(1-q^f)$. In choosing $q^f > 0$, an entrepreneur deliberately bears some of his firm's risk. By selling his whole firm, he would get rid of this risk and as there is no systematic risk no one would have to bear it. If all

entrepreneurs sell the whole of their firms and buyers cannot identify a firm's true type, then the market price of firms must be an average of the true expected revenues of all firms. Owners of better firms would probably be better off retaining some fraction of their firm, bearing the unsystematic risk of this retention, and, at the same time enjoying the exceptionally high expected revenue of their own firms. This makes retaining some positive fraction attractive for good types. In this case q^f , if observable, may serve as a quality signal, inducing buyers to offer a higher price for shares of firms whose entrepreneurs choose high retentions.

Leland and Pyle (1977) were probably the first to propose that, in capital markets, the fraction of retention might act as a signal of a firm's quality. For empirical evidence cf. Downes and Heinkel (1982) and Krinsky and Rotenberg (1989).

For credit financed firms with default risk, the question of the existence of a signalling capital market equilibrium with two types of firm has been considered by DeMeza and Webb (1990). Here, firms are self-financed or, at least, investment credits are safe. Risks are solely borne by shareholders. In the credit market, only safe loans are traded for a given rate of interest $(1 + r)$ which will be kept fixed during the analysis². Nevertheless, from a structural point of view, the model is rather similar to DeMeza and Webb (1990).

Given these assumptions, there might be more than one competitive equilibrium. The candidate for a reactive (Riley (1979)) separating equilibrium with self-selection is described as follows.

Buyers offer two different prices, p^g for firms whose owners retain some $q^g > 0$, and p^b for firms whose owners retain q^b with

$$(1) \quad q^b = 0,$$

$$(2) \quad p^g = E z^g / (1 + r),$$

$$(3) \quad p^b = E z^b / (1 + r),$$

and q_g implicitly determined by

$$(4) \quad EU(q_g z^b + [(1-q_g)p_g + V_0](1+r)) = U((p^b + V_0)(1+r)).$$

Good types choose the (p_g, q_g) offer, bad types the $(p^b, 0)$ offer.

For bad and good types the prices p^b and p_g paid by buyers "break even" in the following sense. By the law of large numbers, the risk involved in a portfolio of good or bad firms is negligible, given the assumptions about the distribution of the X^f . Buying a portfolio of all bad firms, buying a portfolio of all good firms, or investing in the safe credit market are three alternatives which yield a safe return. In equilibrium these returns must equate. This is what the pricing equations (2) and (3) ensure. (4) ensures that bad and good entrepreneurs maximize their expected utility on the set of offers $\{(p^b, 0), (p_g, q_g)\}$ if bad types choose $(p^b, 0)$ and good types choose (p_g, q_g) . If (4) holds, bad types are exactly indifferent between the offers. The left-hand side describes their expected utility if they choose (p_g, q_g) , the right-hand side describes the utility if they choose the offer made for them. It is without loss of generality to assume that they choose the $(p^b, 0)$ offer in this case. Good types get the right-hand side of (4) if they choose $(p^b, 0)$. If they choose (p_g, q_g) , they get a higher utility level than the left-hand side of (4), because they get $q_g z^g$ instead of $q_g z^b$. They strictly prefer the (p_g, q_g) offer. Therefore, the equilibrium is *informationally consistent* (Riley (1979)).

Among all informationally consistent separating equilibria with offers (p^b, \hat{q}^b) and (p_g, \hat{q}^g) , the offers $(p^b, 0)$, (p_g, q_g) are *pareto dominating*. All offers (p^b, \hat{q}^b) with p^b as in (3) break even if only bad firm's entrepreneurs choose it. Among these offers bad entrepreneurs strictly prefer $q^b = 0$. q_g now can be chosen such that it is not attractive for bad types to choose (p_g, q_g) , given that $(p^b, 0)$ is offered (*incentive compatibility constraint*), and such that, among all offers (p_g, \hat{q}^g) fulfilling this incentive compatibility

constraint, the offer is the most preferred one for good types. The incentive compatibility constraint is

$$(5) \quad EU(qgz^b + [(1-qg)pz + V_0](1+r)) \leq U((p^b + V_0)(1+r)).$$

Final wealth of a bad type who chooses the offer made for good types, (p^g, q^g) , is $qgz^b + [(1-qg)pz + V_0](1+r)$. If he chooses the offer made for bad types his final wealth is $(p^b + V_0)(1+r)$. Expected utility of a bad type pretending to be good (left hand side of (5)) must be no greater than the utility from accepting the offer $(p^b, 0)$ ³. Both the expected utility of pretending to be a good type (left hand side of (5)) and the expected utility of being good type and choosing (p^g, q^g) , i.e., $EU(qgz^g + [(1-qg)pz + V_0](1+r))$, are decreasing in q^g for risk averse entrepreneurs. The condition that, for good types, q^g must be optimal among all offers fulfilling (2), therefore makes (5) binding, i.e., makes it fulfilled with equality in equilibrium. This yields (4).

3. Taxes

Let us now take the self-selection equilibrium fulfilling (1), (2), (3) and (4) for granted⁴.

Consider the following taxes, a *pure insurance tax*,

$$(6) \quad T_i \equiv t_i q^f (z^f - pz(1+r)),$$

a *capital gains tax*,

$$(7) \quad T_c \equiv t_c q^f (z^f - p^b(1+r)),$$

and a *profit tax*

$$(8) \quad T_p = t_p z^f.$$

3.1 The pure insurance tax

$(z^f - Ez^f) \equiv X^f$ is the random deviation of actual output from expected output for a firm. q^f is the fraction of retention actually held in equilibrium by the initial owner of this firm, i.e., it is q^g if he chooses (p^g, q^g) , and $q^b = 0$, if he chooses the offer $(p^b, 0)$. In equilibrium only good firms pay taxes. Bad firms choose $(p^b, 0)$ so that their tax base is zero. Only if they pretended to be good, would they too have to pay taxes. The expected value of the tax levied on good firms is zero by (2). q^g times the deviations $X^g = z^g - Ez^g$ is the risk borne by the entrepreneur of this firm if he retains the fraction q^g . For given q^g , the fraction t_i of this risk is shifted to the government by taxation. The partial analytic effect of the tax is similar to a proportional insurance.

Tax proceeds are a fraction of aggregate output variations. By the no-systematic risk assumption, therefore, these proceeds are not risky. More precisely, redistributions of these proceeds are not risky. Moreover, proceeds have an expected value of $ET_i = t_i q^f EX^f = 0$. This makes it unnecessary to bother further about what happens with the proceeds of the insurance tax and with the risk it contains.

Equilibrium conditions (1)–(3) do not depend on the tax. Due to arbitrage considerations, the equilibrium market prices p^g of good types and p^b of bad types cannot change. Also $q^b = 0$ remains true. With the price p^b , bad types prefer to sell their whole firm, regardless of the tax rate $t_i \in [0, 1]$, which is in any case not paid by bad types who choose $(p^b, 0)$. The impact of a tax-rate increase on the equilibrium value of q^g can be calculated from the incentive compatibility constraint (4), which, including the tax defined in (6), is

$$(9) \quad EU(q^g[z^b - (1+r)p^g](1-t_i) + [V_0 + p^g](1+r)) = U((p^b + V_0)(1+r)).$$

The insurance tax slightly changes the self-selection choice problem of bad types compared to (4). If a bad type pretends to be good by choosing q^g , he has to pay some taxes $t_i q^g(z^b - Ez^g)$, which is a subsidy in expected value terms. This yields the left hand

side of (9). If he chooses the offer $(p^b, 0)$ designed for him, he does not pay any tax and gets the same final wealth as in the equilibrium without taxes. As (9) is binding in the equilibrium, the *general equilibrium* impact of the insurance tax can be calculated by differentiation of (9) to get

$$(10) \quad dqg/dt_i = qg/(1-t_i).$$

(10) is formally identical with the Domar–Musgrave effect (cf. Mossin (1968)) which was derived in the partial analytic portfolio model. However, (10) describes a general equilibrium effect here. This first result is summarized as

Theorem 1 Given a separating equilibrium characterized by (1), (2), (3) and (9), the increase in an insurance tax changes the general equilibrium amount of retentions by good type entrepreneurs by

$$dqg/dt_i = qg/(1-t_i).$$

Corollary 1 The tax rate change does not change the total amount of risk diversification.

Corollary 1 is proved as follows. In equilibrium all risks of bad firms are fully diversified, but the fraction

$$(11) \quad \sigma \equiv qg(1-t_i)$$

of good firms' risks is not diversified. $(1-qg)$ is diversified in the private capital market, $t_i qg$ via the tax. Increasing the tax does not change σ , as

$$d\sigma/dt_i = -qg + (1-t_i)(dqg/dt_i) = -qg + qg = 0.$$

3.2 A capital gains tax

$(z^f - E z^b)$ describes the excess output of a firm f over the expected output of a bad firm. An entrepreneur can always obtain $E z^b$ in the second period by selling his firm as a bad one. If he retains some fraction of his firm, he gets some additional return $q^f(z^f - E z^b)$ which might be positive or negative, depending on the actual realization of the random variable z^f . The expected value of $q^f(z^f - E z^b)$ is positive if f is a good firm and it is zero for bad firms. The additional return the entrepreneur receives is proportionally taxed by the capital gains tax. It is a subsidy for these cases where the actual realization $z^f < E z^b$ (perfect loss offset). Therefore, the expected value of these capital gains taxes is zero for bad firms which pretend to be good ones, but is positive for truly good ones.

Bad types in equilibrium again choose $q^b = 0$ and do not pay any taxes. However, their incentive for pretending to be good type is altered. The incentive compatibility constraint becomes

$$(12) \quad c_1 \equiv EU(q^g z^b + [(1 - q^g)p^g + V_0](1 + r) - T_c + S) = U((p^b + V_0)(1 + r) + S),$$

with the capital gains tax T_c defined in (7), and with S being safe lump-sum transfers. In contrast to an insurance tax, tax proceeds of the capital gains tax (7) have a positive expected value. In a general equilibrium analysis the tax proceeds should be redistributed. If they are given to entrepreneurs who signal that they are good ones, this is not lump sum, but it gives entrepreneurs an additional incentive to signal to be good ones. If they are given to entrepreneurs who signal that they are bad ones, they are made richer, but only if they are signalling to be bad ones. This would be an additional incentive to signal that they are bad ones, and would also not be lump sum. Therefore, let us assume that tax proceeds are equally distributed among all entrepreneurs. Let γ be the fraction of entrepreneurs of good type and $(1 - \gamma)$ the fraction of bad ones. Then, using also the assumption of no-systematic risk, the lump-sum redistributions to each

entrepreneur used in (12) are safe and are equal to $\gamma E T_c$, so that the government budget constraint becomes

$$(13) \quad c_2 \equiv S - \gamma t_c q g (E z g - E z^b) = 0.$$

Equations (1), (2) and (3) again are unaffected by the tax. We obtain the general equilibrium reactions by differentiating both the incentive compatibility constraint (12) and equation (13) for t_c and by solving the resulting system of equations

$$(14) \quad \begin{bmatrix} c_{1q} & c_{1s} \\ c_{2q} & c_{2s} \end{bmatrix} \begin{bmatrix} dqg \\ dS \end{bmatrix} = \begin{bmatrix} -c_{1t} \\ -c_{2t} \end{bmatrix} dt_c$$

with

$$c_{1q} \equiv E\{U'[z^b - p g(1+r) - t_c(z^b - E z^b)]\} = E\{U' X^b\}(1-t_c) - EU' \Delta$$

$$c_{1s} \equiv EU' - U'$$

$$c_{2q} \equiv -\gamma t_c \Delta$$

$$c_{2s} \equiv 1$$

$$-c_{1t} \equiv E\{U' X^b\} q g$$

$$-c_{2t} \equiv \gamma q g \Delta,$$

with

$$\Delta \equiv E z g - E z^b.$$

Using Cramer's rule,

$$dt_c/dqg = \frac{E\{U' X^b\}(1-t_c) - EU' \Delta + (EU' - U') \gamma t_c \Delta}{E\{U' X^b\} q g - (EU' - U') \gamma q g \Delta},$$

or, after some manipulations,

$$(15) \quad dt_c/dqg = (1-t_c)/qg + \frac{-EU' \Delta (1-\gamma) - U' \gamma \Delta}{E\{U' X^b\} q g - (EU' - U') \gamma q g \Delta}.$$

The second term of the right hand side is positive for nonincreasing absolute risk aversion. The numerator is negative as marginal utility and Δ are positive. $E\{U'X^b\} < 0$ by risk aversion, and $EU'-U'$ is shown to be non-negative for constant or decreasing absolute risk aversion in the lemma that is proved in the appendix. This makes the denominator also negative. Therefore, the inverse of (15),

$$(16) \quad dqg/dt_c < qg/(1-t_c).$$

This is summarized as

Theorem 2 A redistributed capital gains tax (7) increases the equilibrium amount of retentions by less than the pure Domar-Musgrave effect, i.e.,

$$dqg/dt_c < qg/(1-t_c),$$

if entrepreneurs exhibit constant or decreasing absolute risk aversion.

Corollary 2 A redistributed capital gains tax increases the overall amount of risk diversification in equilibrium if the entrepreneurs exhibit constant or decreasing absolute risk aversion.

Corollary 2 is proved as follows. Again, the fraction $\sigma \equiv qg(1-t_c)$ of good firms' risks is not diversified. $(1-qg)$ is diversified in the private capital market, t_cqg via the tax. Increasing the tax changes σ

$$d\sigma/dt_c = -qg + (1-t_c)(dqg/dt_c) < -qg + qg = 0.$$

The result might be surprising. It shows that a tax which is not paid by bad firms' entrepreneurs in the equilibrium makes them less effective in pretending to be a good firm.

3.3 A profit tax

Consider finally the profit tax T_p . Among the types of taxes analysed here, this is the most relevant one because it has the closest counterpart in real economies.

A tax is levied on entrepreneurial (net-)output. It has to be paid, no matter who owns the shares which entitle him to get the output. The profit tax changes the attractiveness of holding shares of firms, as a percentage of all profits goes to the government. To make investors indifferent between a safe portfolio of firms of type f and investment in the credit market, the net-of-tax return of buying firm shares of type f , $Ez^f(1-t_p)/p^f$ and the return of the credit market $(1+r)$ must be equal. The equations describing firm value in equilibrium become

$$(17) \quad Ez^g(1-t_p) = (1+r)p^g$$

and

$$(18) \quad Ez^b(1-t_p) = (1+r)p^b.$$

The final wealth for good types choosing the offer (p^g, q^g) and for bad types choosing the offer (p^g, q^g) or the offer $(p^b, 0)$ can be calculated to be

$$(19) \quad V_1^g(p^g, q^g) \equiv q^g z^g(1-t_p) + (1-q^g)Ez^g(1-t_p) + (1+r)V_0 + S,$$

$$(20) \quad V_1^b(p^g, q^g) \equiv q^g z^b(1-t_p) + (1-q^g)Ez^g(1-t_p) + (1+r)V_0 + S,$$

and

$$(21) \quad V_1^b(p^b, 0) \equiv Ez^b(1-t_p) + (1+r)V_0 + S,$$

respectively. Use of (17) and (18) is made to eliminate the price variables. Among all possible offers (p^b, \hat{q}^b) with p^b fulfilling (18), $(p^b, 0)$ is still the most preferred for bad types. $EV_1^b(p^b, \hat{q}^b)$ is independent of \hat{q}^b , but the risk is minimal for $\hat{q}^b = 0$.

The incentive compatibility constraint using (20) and (21) becomes

$$(22) \quad c_1 \equiv EU(V_1^b(p^g, q^g)) - U(V_1^b(p^b, 0)) = 0.$$

Again, if this incentive compatibility constraint holds for bad firms, good firms' entrepreneurs are always better off if they choose the (p^g, q^g) offer.

In equilibrium, with bad types choosing $(p^b, 0)$ and good types choosing (p^g, q^g) , both types pay taxes. Let γ again be the fraction of good types. The tax which is paid by any single good firm is stochastic. Aggregate tax proceeds, however, are practically *safe* by the no-systematic-risk assumption. All tax proceeds are used for safe lump-sum redistributions among all firms, i.e.,

$$(23) \quad c_2 \equiv S - [\gamma E z^g + (1-\gamma) E z^b] t_p = 0.$$

The equilibrium is determined by conditions (1), (22) and (23), making implicitly use of (17) and (18). Equation (1) is not affected by the tax. Differentiation of (22) and (23) for q^g , S and t_p with q^g and S endogenous and t_p exogenous leads to the system of equations

$$(24) \quad \begin{bmatrix} c_{1q} & c_{1s} \\ c_{2q} & c_{2s} \end{bmatrix} \begin{bmatrix} dq^g \\ dS \end{bmatrix} = \begin{bmatrix} -c_{1t} \\ -c_{2t} \end{bmatrix} dt_p$$

with

$$c_{1q} \equiv E\{U'(V_1^b(p^g, q^g))[z^b - E z^g]\}(1-t_p)$$

$$c_{1s} \equiv EU'(V_1^b(p^g, q^g)) - U'(V_1^b(p^b, 0))$$

$$c_{2q} \equiv 0$$

$$c_{2s} \equiv 1$$

$$-c_{1t} \equiv E\{U'(V_1^b(p^g, q^g))[q^g z^b + (1-q^g)E z^g]\} - U'(V_1^b(p^b, 0))E z^b$$

$$-c_{2t} \equiv \gamma E z^g + (1-\gamma)E z^b.$$

Using Cramer's rule,

$$dqg/dt_p = [(-c_{1t})c_{2s} - (-c_{2t})c_{1s}] / \det C$$

or,

$$(25) \quad dqg/dt_p = \frac{qg}{1-t_p} + \frac{[(1-\gamma)EU' + \gamma U'](Ez_g - Ez_b)}{(1-t_p)E\{U'(z^b - Ez_g)\}}$$

EU' , U' and $Ez_g - Ez_b$ are positive. Therefore, the numerator of the second term of the right hand side of (25) is positive. The denominator is clearly negative as $E\{U'(z^b - Ez_g)\} < E\{U'X^b\} < 0$. This can be summarized by

Theorem 3 A redistributed profit tax (8) increases the equilibrium amount of retentions by less than the pure Domar-Musgrave effect, i.e.,

$$dqg/dt_p < qg/(1-t_p).$$

Corollary 3 A redistributed profit tax increases the overall amount of risk diversification in equilibrium.

The proof of corollary 3 follows the lines of that of corollary 2.

4. Discussion

The risk-taking incentive of capital gains taxes and profit taxation with full loss offset was first put forward by many authors as an additional instrument of risk consolidation. The phenomenon was later criticized as being only partial analytic, without significance in more general equilibrium models with perfect risk markets.

This paper challenges the latter view. First I described a capital market equilibrium with self-selection, which is similar to the standard Spence equilibrium (cf., e.g., Cho and Kreps (1987)). Founders and entrepreneurs of firms with non-systematic revenue risks are interested to go public to diversify this non-systematic risk in the capital market. However, there is some information problem. Firms differ in quality.

Entrepreneurs know the true expected profitability of their firm, but potential buyers do not. Assume first that entrepreneurs are unable to signal the true firm quality: buyers cannot know the true profitability prospects of a particular firm. There would be a capital market equilibrium where firm prices reflect only the profitability of the average-quality firm. Entrepreneurs of more profitable firms choose to accept the market price for their firm below the "true" market value, or they retain (a fraction of) their firm for themselves. In this paper it was assumed that buyers cannot observe firm quality, but they can observe such retentions made by any entrepreneur. So, buyers may use the fraction of retentions as a signal of firm quality and therefore offer a higher price to firms with higher retentions. Given this relationship, bad firms' owners might find it profitable to imitate the behavior of good firms' owners, also retaining some fraction of their firms in order to receive the higher selling price offered to firms with high retentions. In this case, the retentions of good firms must be high enough to make it not attractive for bad firms to imitate good ones, otherwise the price paid by buyers again would have to reflect some average firm quality. The self-selection condition which determines this amount of retentions which just makes bad firms' owners not willing to imitate the behavior of good firms' owners, is called the *incentive compatibility constraint*.

Notice that the retentions made by the better firms' owners imply that these owners bear non-systematic risk. This risk could be diversified in the capital market, but, doing so, good firms' owners would suffer income losses (they would subsidize bad firm's owners).

This is basically the equilibrium situation where the paper started. Three different taxes were introduced and their general equilibrium impacts were derived. All these taxes are levied on the risky returns from retentions. Such taxes have two major effects. First, within the tax proceeds a perfect risk consolidation takes place as the tax is levied on returns which involve only non-systematic risks. This is the *insurance effect* of these taxes, and, taken in isolation, it is beneficial because it lowers the costs of risk bearing.

Second, taxation changes the incentive compatibility constraint. The risk burden for a given fraction of retentions is lowered by the tax. Generally, it can be expected that this effect increases the fraction of retentions which is necessary to prevent bad firms from imitating good firms (*incentive effect*). Taken in isolation, this means a reduction of risk sharing via private markets. Additionally, there are income effects of the tax and of the redistribution of tax proceeds. These effects complicate the derivation of private market reactions.

In the paper all these partially countervailing effects have been compared for three different taxes and clear conclusions could be drawn regarding the impact of taxes on total risk sharing in general equilibrium. A tax on risky revenues of retentions which acts as a fair insurance, i.e., which has zero expected tax proceeds in the equilibrium, is shown to have no effect on the total amount of risk sharing. A capital gains tax and a profit tax, instead, increase total risk sharing if all general equilibrium repercussions are taken into account. The intuition of this result is that these two taxes reduce the revenue differences between good and bad firms. This tendentially reduces the incentives for bad firms to imitate good ones and reduces the retentions necessary to deter them from imitation.

The result is considerably different from that of DeMeza and Webb (1990) who show that a 100 % profit tax is welfare improving. Their result is based on the fact that a 100 % tax removes the hidden-knowledge problem and allows a first best solution to be reached. However, due to political reasons, it might be not feasible to impose a 100 % tax on profits, whereas a small change in the tax rate might be feasible. The results obtained here hold for the case where there is still a reactive capital market equilibrium after the tax rate change. Capital gains taxes and profit taxes might improve the risk allocation, even if the basic features of the underlying equilibrium are not destroyed.

Compared with the results obtained in an adverse selection equilibrium (cf. Konrad and Richter (1990)), it is perhaps surprising that a tax is beneficial in the case of separating equilibria with self-selection when it performs badly in the case of an adverse

selection equilibrium. This shows that there is a severe caveat on drawing policy implications for tax reform. Results are rather tentative. It has to be scrutinized whether the equilibrium is a signalling equilibrium with self-selection or a simple adverse selection equilibrium.

Appendix

Lemma: Given the expected utility indifference curve $\overline{EU}(V_1)$ with

$$V_1 = m + \alpha X,$$

with m being expected final wealth, $\alpha > 0$ and X a random variable with mean $EX = 0$, the increase of expected utility of an additional amount of income is increasing (constant) in α for decreasing (constant) absolute risk aversion, i.e.,

$$(A.1) \quad \frac{d \frac{\partial EU}{\partial m}}{d\alpha} \Big|_{\overline{EU}} \{ \geq \} 0 \Leftrightarrow \frac{d(-U''/U')}{dV_1} \{ \leq \} 0.$$

Proof of the lemma.

$$(A.2) \quad \frac{d \frac{\partial EU}{\partial m}}{d\alpha} \Big|_{\overline{EU}} = \partial EU' / \partial \alpha + \partial EU' / \partial m (dm/d\alpha) \Big|_{\overline{EU}}$$

$$= E[U''X] + E\{U''[-(E[U'X]/EU')]\}$$

$$= -E[(-U''/U')U'X] + E[(-U''/U')U'](E[U'X]/EU').$$

Define $r \equiv (-U''/U')$ to be the measure of absolute risk aversion. Then, as $EU' > 0$, the sign of (A.2) equals

$$(A.3) \quad \text{sgn} \{ E[rU'] E[U'X] - E[rU'X] EU' \}.$$

Define x^* such that $x^*EU' = E[U'X]$ and transform (A.3) to

$$(A.4) \quad \text{sgn} \{ E[rU'] x^*EU' - E[rU'X] EU' \},$$

which equals

$$(A.5) \quad \text{sgn} \{ E[rU' (x^*-X)] EU' \}.$$

For $r = \text{constant}$, (A.5) is zero by the definition of x^* . For r decreasing, this sign is negative by standard reasoning. This completes the proof.

Footnotes

- * Helpful comments by Amihai Glazer, Tore Nilssen, Wolfram F. Richter and Hans-Werner Sinn are gratefully acknowledged.
1. For some early central contributions see Allingham (1972), Mossin (1968) and Stiglitz (1969), and Buchholz (1987) and Sandmo (1985) for further references.
 2. This is the case, e.g., if there is a non-stochastic constant returns to scale technology available to all households, yielding this safe rate of return.
 3. In principle, an incentive compatibility constraint for good types must also be considered. Good types must prefer the offer designed for them. If (4) is fulfilled, however, this is always the case. Their expected utility of choosing (p_g, q_g) is $EU(q_g z_g + (1-q_g)Ez_g + V_0(1+r)) > EU(q_g z^b + (1-q_g)Ez_g + V_0(1+r))$ for $q_g > 0$. By (4), the right hand side utility is equal to the utility of a good type who chooses the offer designed for bad types.
 4. The equilibrium described by (1)–(4) receives additional support from the fact that it passes the *Intuitive Criterion* of Cho and Kreps (1987), if the fraction of good types is small enough to rule out a pooling equilibrium. The "small enough" can be made more precise by the condition $\exists q_g < 1$ fulfilling (5) and

$$EU(q_g z_g + (1+r)(1-q_g)p_g + (1+r)V_0) > U((1+r)(p^* + V_0)),$$
 with p^* being the price that breaks even in a pooling equilibrium.

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