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The Domar-Musgrave phenomenon and adverse selection

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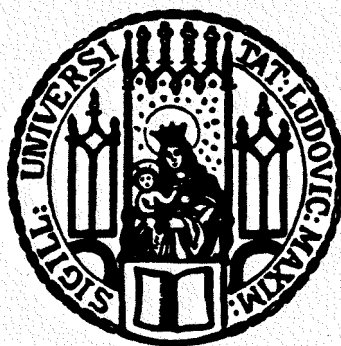
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March 1990

DISCUSSION PAPERS



HERAUSGEGEBEN VON DER
VOLKSWIRTSCHAFTLICHEN FAKULTÄT
DER
LUDWIG-MAXIMILIANS-UNIVERSITÄT
MÜNCHEN

**The Domar–Musgrave Phenomenon
and Adverse Selection**

by

Kai A. Konrad*

March 1990

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The Domar-Musgrave Phenomenon and Adverse Selection

Abstract

The taxation of risk-taking revenues induces an investor, who allocates a given amount of resources between a risky and a safe asset, to increase the risky share of his portfolio. This result was first derived by Domar and Musgrave in a partial analytic framework. In a more general framework it has been shown that, given that all tax revenues are redistributed in a lump-sum fashion, this Domar-Musgrave phenomenon can be expected to show up only if a substantial diversification of risks takes place within the tax proceeds. Given perfect capital markets, however, such "insurance"-properties of risk-taking-revenue taxes cannot be expected to exist. Is the Domar-Musgrave phenomenon only a partial analytic peculiarity without much importance? In this paper it is tried to reevaluate the Domar-Musgrave phenomenon. Asymmetric information, in particular with regard to the valuation of entrepreneurial firms that are considering going public, may induce a process of adverse selection. Some entrepreneurs decide not to go public or they sell only some part of their firms. In equilibrium, therefore, some unsystematic risk remains unconsolidated. In this case "compulsory insurance" such as a risk-taking-revenue tax is not ineffective. An additional risk consolidation takes place within the collected tax proceeds. However, the impact of such taxes on welfare is quite diverse, depending on the abilities of the owners of taxed entrepreneurial firms to react to the taxes. If owners of entrepreneurial firms cannot react via a change of ownership structure, then they will react like the investor in the portfolio model of the Domar-Musgrave framework, reproducing the effects derived in this literature. Good examples of this type of firm may be small business firms and craftsmen. However, if the original owner-managers of firms go public with a strictly positive percentage of their firms, while keeping a fraction of their own firms in their portfolios, the risk-taking-revenue tax induces them to increase this fraction. In this case the tax reinforces the process of adverse selection. The impact of a risk-taking-revenue tax on welfare depends on the type of firm.

1 Introduction

The taxation of risky profits, or, more specifically, *risk-taking revenues* induces an investor to increase his demand for the risky investment opportunity. This phenomenon was first studied by Domar and Musgrave (1944) and its robustness has been shown under quite general conditions¹. The intuitive explanation for this incentive to increase the risky fraction of the investor's portfolio has often been described: via risk-taking-revenue taxes government participates not only in the revenues of risk-taking, but also assumes some burden of risk. The risk to the investor is reduced by the amount the government assumes and he receives a proportionately lower reward. Given the same expected revenue per unit of risk borne by the investor, this investor has an incentive to adjust his supply of risk-taking towards the before-tax level. In a general equilibrium framework the risk-taking incentive of a risk-taking-revenue tax is less obvious. A risk-taking incentive cannot be taken for granted. It may prevail if tax proceeds are used in a "neutral" way, so that the investor's behavior is not affected by the riskiness of the expenditures financed by these tax proceeds², or substantial risk diversification might take place within the collected tax revenues, so that a redistributed risk-taking-revenue tax is equivalent to a compulsory mutual insurance³.

Assume, however, that no risk-diversification takes place within the collected tax proceeds, and tax-proceeds are redistributed in a lump-sum fashion. Except for income effects of redistribution, the incentive effect of a risk-taking-revenue tax, in this case, does not prevail. Given risky lump-sum redistributions and given that investors do not change their portfolios, the amount of risk they assume is not

¹ Cf. Ahsan (1974), (1975), (1988), (1989), Allingham (1972), Atkinson and Stiglitz (1980), Buchholz (1987), Mintz (1981), Mossin (1968), Sandmo (1969), (1977), (1985), Stiglitz (1969), (1972) and Tobin (1958) for some central contributions and further references.

² For the important case of financing public goods, the incentive effect depends on how public goods enter the utility function. For a deeper analysis of this widely used assumption see Stiglitz (1972).

³ This reasoning often is applied when redistributed income taxation is considered. See, e.g., Eaton and Rosen (1980a), (1980b), Hamilton (1987), Sinn (1985) and Varian (1980).

reduced by the tax and therefore the incentive effect does not appear. This neutrality result has been shown to hold for different conditions⁴ by Atkinson and Stiglitz (1980) and Konrad (1989b). If perfect capital markets exist, there is little reason why diversifiable risks of entrepreneurial firms should not be diversified in private capital markets. This has been pointed out, e.g., by Bulow and Summers (1984), and Sandmo (1985, p. 306) concludes that "there is little reason that the result [the incentive for additional risk taking] will continue to hold in general equilibrium or that similar results can be derived for a more general class of tax systems".

However, existing capital markets are far from perfect. Information in capital markets is asymmetric. Entrepreneurs who found and manage a firm generally know the true profitability of their firm better than potential buyers. As Myers and Mayluf (1984) pointed out, the capital market might be a "lemon" market (cf. Akerlof (1970)). If buyers cannot discriminate between different firms, their willingness to pay will be the same for each. At best, the market price of a firm will be a weighted average of the true values of firms in the case of perfect information. Assume that entrepreneurs who own exceptionally profitable firms are unable to signal the exceptionally high quality of their firms⁵. They choose to accept the market price for their firm below the true market value (related to perfect information), or they keep the firm for themselves. In the latter case they gain the higher expected revenues, but also they bear some nonsystematic risks involved with this firm that could be diversified in the capital market.

This paper considers the effect of a risk-taking-revenue tax in a capital market equilibrium with adverse selection. Entrepreneurs decide endogenously about the size of their firm and about the fraction of their firm that they are willing to sell in the capital market. The latter assumption, i.e. that the entrepreneur endogenously chooses the fraction of retention of his firm, turns out to be decisive for determining

⁴ Cf. also Gordon (1985).

⁵ See footnote 6 for a discussion of this assumption.

the incentive effect of a risk-taking-revenue tax .

In the capital market equilibrium with adverse selection, some unsystematic risk that could be diversified via the capital market is deliberately borne by some entrepreneurial investors. A risk-taking-revenue tax leads to some additional risk diversification within the tax proceeds. However, to judge the welfare impact of this tax, the question of whether or not a risk-taking-revenue tax leads to some adverse behavioral effects that might even overcompensate the welfare gains from this risk diversification will be considered here.

It will be shown that a risk-taking-revenue tax induces exactly the investor reactions that are known from the Domar-Musgrave literature if the entrepreneur cannot react via a change in the fraction of retention. This is the case if the entrepreneur wanted to keep the whole firm for himself even before the tax rate was increased. The well-known results that describe how an investor reacts to an increase in a risk-taking-revenue tax in the case of constant or decreasing returns to scale (cf., e.g., Mossin (1968) and Mintz (1981)) are thereby considerably generalized. It will be shown, however, that the reaction of entrepreneurs who bear diversifiable risks is quite different if they can react to the tax-rate change by changing the fraction of retention of their firms. In this case, a risk-taking-revenue tax turns out to reinforce the adverse-selection problem of asymmetric information, driving an even larger proportion of high quality firms from the capital market and, thereby, increasing the amount of deliberately not diversified risks that could be diversified via the capital market.

These results will be discussed under welfare perspectives, considering explicitly the problem of taking the use of tax proceeds into account. Finally, the results will be summarized.

2 A Capital Market Model

2.1 Model Assumptions

Consider the following two period, one good economy. There is a number of firms, $f = 1, \dots$ and each firm founded and managed by a single entrepreneur. Firms are described by their twice continuously differentiable stochastic production functions $f(a^f, s)$ that describe the relation between capital input $a^f \geq 0$ in period 0 and the output that is stochastic, depending on $s \in \{1, \dots, S\}$, the state of nature that exists in period 1 and is unknown in period 0 (cf., e.g., Diamond (1967)). Assume that it is possible not to produce, i.e., $f(0, s) = 0$ for all firms. Each entrepreneur chooses the activity level a^f of his firm. Given a complete set of contingent goods markets in period 0 and an equilibrium price vector π of these contingent goods, the decision problem of an entrepreneur is quite simple. He computes the profit of various production plans and chooses the plan that maximizes this profit. A similar situation prevails, if, instead of contingent commodity markets, shares of firms are traded and these shares generate the whole commodity space (see, e.g., Krouse (1986)). However, for this equivalence to hold, the buyers of shares of firms obviously must be able to know $f(a^f, s)$ of each single specific firm.

Here it is assumed [*information assumption*] that buyers of shares of a firm do know what different types of firms are traded and in what relative proportions. However, they are not able to observe the risk or return properties of the true production plan of a specific firm. The only firm-specific information they have is the amount of capital a^f that is used in this firm. Only the entrepreneur who founded and manages a firm knows the stochastic properties of "his" firm.

A second assumption concerns the stochastic properties of the market portfolio, i.e., the sum of all shares of all firms that are traded at the capital market. It is assumed that a share of this market portfolio is perfectly risk free [*perfect diversifiability assumption*]. A share of the market portfolio yields the same revenue (firms' output) in all states of nature s . At first glance, this assumption looks extraordinarily strong. It is, however, made only for convenience. It is intended to describe and isolate the specific effect of a risk-taking-revenue tax on the behavior

of an entrepreneur who bears diversifiable risks for reasons of asymmetric information. From this perspective, whether there is only a partial or a complete risk consolidation in the capital market is not important. In the adverse-selection literature on capital markets and insurance this assumption is quite usual. Prescott and Townsend (1984, p. 24), e.g., consider the case of an insurance market, assuming that total damage is perfectly known. More implicitly, this assumption is also made in models that assume that only the supply side or the demand side is risk averse, the other side of the market being risk neutral (cf., e.g., Rothschild and Stiglitz (1976) for the case of insurance markets and DeMeza and Webb (1988) for the case of credit markets). Consider the case of entrepreneurs who demand credit and private households which demand insurance because they are risk averse and risk neutral banks that supply risky loans or insurance companies that are risk neutral. Insurance companies and banks are firms and, usually, they must ultimately be owned by private households, probably the same ones which demand insurance or loans. The risk-neutral behavior of such firms is consistent with the risk aversion of households only if within the business activities of insurance companies and banks or in additional risk markets (reinsurance, capital markets) risks are consolidated to such an amount that single investors receive a practically risk-free income from their investment in assets supplied at the capital market.

The asymmetric information has some implications for the capital market equilibrium. Using the assumption of a safe market portfolio, the capital market equilibrium can be described as follows. Potential buyers of shares of two firms that produce with the same capital input a^f cannot discriminate between them with regard to profitability. Investors must be indifferent between shares of such firms and their willingness to pay must be the same for both firms. The market value v^f of a firm can only depend on the only observable factor, i.e. the amount of capital a^f used in this firm.

$$(1) \quad v^f = v(a^f).$$

Notice that potential buyers of shares even cannot observe the amount of a firm that the entrepreneurial investor keeps for himself. The amount of retention therefore cannot act as a signal of the true firm quality⁶. Assume that a capital market equilibrium with (1) exists and that v^f is a twice continuously differentiable function of a^f . Further assume that a capital market and a credit market exist, both with positive turnovers.

An entrepreneur who wants to found a firm in the first period $t = 0$ has to make three decisions. First, he has to determine a , the activity level of his firm⁷, second, he has to choose the fraction $(1-q) \in [0,1]$ of his firm that he wants to sell at the capital market, and, third, he has to decide how to invest his possibly remaining resources (including the proceeds from selling a fraction of his firm) in the credit market and the capital market. As both credit contracts at the credit market and shares of the market portfolio at the capital market are equally safe and, by assumption, both markets have positive turnovers in the equilibrium, investment yields the same, certain revenue in both markets, described by a safe rate of interest i . This makes the investment problem (3) simple. The investor is indifferent between investing in the credit market or buying shares of the market portfolio in the capital market. The decision problem of the entrepreneur reduces to the choices of a and q .

Assume that a proportional tax with tax rate t_r is levied on additional profits an entrepreneur receives by deciding not to sell his entire firm but to retain a substantial fraction q . Let $v(a)$ be the market price of a firm that uses the amount a of capital as given by (1). $f(a,s)$ is the output of the specific firm under consideration, depending on a and the state of nature s . Let retention $q \in [0,1]$ be the fraction of his firm that the entrepreneur wants to keep, i.e. does not sell in the

⁶ Leland and Pyle (1977), e.g., assume that the fraction of the firm that an entrepreneurial investor keeps for himself acts as a costly signal of true firm qualities, defusing the problem of adverse selection to some extent. The assumption that such retentions are not observable will be made here, however, because the adverse selection problem is most serious in this case and, therefore, the potential gain of additional risk diversification via a risk-taking-revenue tax is maximal. Moreover, it is not clear why incorporating costly signalling should change results qualitatively.

⁷ For simplicity, in what follows the superscript f of the entrepreneur or firm under consideration is omitted.

capital market, and let t_r be the proportional risk-taking-revenue-tax rate. Then the risk-taking-revenue tax⁸ is given by

$$(2) \quad T_r = t_r q [f(a,s) - (1+i)v(a)].$$

An entrepreneur who sells his entire firm yields $v(a)$ in period $t = 0$. Investing $v(a)$ at the credit (or capital) market yields $(1+i)v(a)$ in period $t = 1$. In contrast, an entrepreneur who decides to retain the fraction q of his firm gets $(1-q)v(a)$ from selling the fraction $(1-q)$ in period $t = 0$ and revenues $qf(a,s)$ from his retained fraction in period $t = 1$. The income difference between a complete buyout and retaining $q > 0$ is the revenue he receives for not selling the fraction q . However, to receive this uncertain additional revenue he has to bear the fraction q of the risks that are implied by the production activity of his firm. The uncertain additional revenue therefore is called risk-taking revenue.

The final wealth V_1 of the entrepreneur, dependent on a , q and t_r is therefore determined by

$$(3) \quad V_1 = qf(a,s) + (1+i)[(1-q)v(a) - a + V_0] - t_r q [f(a,s) - (1+i)v(a)].$$

This final wealth consists of the fraction q of output of his firm, in conformity with the retained fraction q , plus the returns of his investment in the credit and/or capital market, minus the risk-taking-revenue tax T_r . The investment in the credit and/or capital market equals his initial wealth V_0 minus his investment a in his firm plus proceeds from selling the fraction $(1-q)$ of his firm in the capital market.

An expected-utility maximizing entrepreneur solves the problem

⁸ For simplicity it is assumed here that tax proceeds are used in a way that does not affect the decisions of entrepreneurs. Different to the situation with perfect capital markets (cf., e.g., the cited neutrality results by Atkinson and Stiglitz (1980) and Konrad (1989b)), a redistribution of tax proceeds would not make the risk-taking-revenue tax ineffective. In contrary, given plausible assumptions about risk aversion, a redistribution of tax proceeds would confirm or even enlarge the reactions of entrepreneurs here. For a discussion see section 3.

$$(4a) \quad \max_{\{q,a\}} EW(V_1)$$

subject to (1), (3) and

$$(4b) \quad q \geq 0,$$

$$(4c) \quad 1-q \geq 0,$$

$$(4d) \quad a \geq 0.$$

$W(V_1)$ is the Neumann-Morgenstern utility function of the entrepreneur's final wealth V_1 . The intertemporal decision concerning his consumption and saving in the first period is not considered here. The initial wealth V_0 of the entrepreneur is assumed to be exogenously determined⁹. The Lagrangean of this non-linear programming problem (4) is

$$L(q,a,\lambda) = EW(V_1) + \lambda(1-q),$$

denoting the Lagrange multiplier here by λ . Maximization of L yields the following Kuhn-Tucker conditions

$$(5a) \quad \partial L / \partial a \leq 0, \quad a \geq 0 \quad \text{and} \quad a \partial L / \partial a = 0,$$

$$(5b) \quad \partial L / \partial q \leq 0, \quad q \geq 0 \quad \text{and} \quad q \partial L / \partial q = 0,$$

$$(5c) \quad \partial L / \partial \lambda \geq 0, \quad \lambda \geq 0 \quad \text{and} \quad \lambda \partial L / \partial \lambda = 0.$$

These Kuhn-Tucker conditions are necessary conditions for a maximum of problem (4), because the non-negativity constraints (4b), (4c) and (4d) are linear (cf. Chiang (1974), pp. 713n.).

The taxation problem is uninteresting if $a = 0$, because $f(0,s) = 0$ in this case. If a firm does not exist or if it is optimal not to produce, then the choice of q is

⁹This assumption usually is made in the Domar-Musgrave literature. For exceptions see Ahsan (1977), (1988), (1989) and Konrad (1989a).

irrelevant, and also the taxation problem disappears with the source of taxes, i.e. $da/dt_r = dq/dt_r = 0$ for this type of entrepreneur.

Similar reasoning applies to the case $q = 0$. Equation (2) reveals that $T_r \equiv 0$ holds in this case. An entrepreneur has no incentive to react to a tax change that does not affect him. An entrepreneurial investor who sells his entire firm in the capital market has no incentive to change his decisions on investment or retention, i.e., he chooses

$$da/dt_r = dq/dt_r = 0.$$

Notice that entrepreneurial investors with $q = 0$ are founders of firms with expected revenue that is at best as high as the revenue of the market portfolio, i.e. they are initial owners of "relatively bad" firms¹⁰. The result here shows that the supply of relatively bad firms will not be altered by a change of the risk-taking-revenue-tax rate. This property will turn out to be important when judging the welfare implications from a risk-taking-revenue tax.

Consider now the cases with $a \geq 0$, $q \geq 0$ not binding. From (5a) one gets

$$(6) \quad E \left[W'(V_1) \left[q(1-t_r)f_a + (1+i)[(1-q(1-t_r))v' - 1] \right] \right] = 0.$$

Condition (6) describes the situation where, in the optimum the entrepreneurial investor cannot increase his expected utility by a small change of the capital input a . Now the optimality condition that determines q has to be considered. This happens in two different subsections, because, depending on whether $(1-q) \geq 0$ is binding or not, the tax has very different implications for the entrepreneur's decisions.

¹⁰ Only if $Ef(a,s) \leq (1+i)v(a)$ does it not pay the initial owner of this firm to retain at least a possibly very small but positive fraction of his firm.

2.2 *Partially going public* [$0 < q < 1$]

Consider first the case of an entrepreneur that prefers to sell a strictly positive fraction of his firm $0 < (1-q) < 1$. Using this property one gets from (5b) and (5c)

$$(7) \quad E[W'(V_1)[f - (1+i)v]] = 0.$$

This condition (7) describes the situation where, in the optimum, given the activity level a of the firm, the expected utility of the entrepreneur cannot be increased by a small variation of q .

The case of an entrepreneur who retains a positive fraction of his firm ($q > 0$), but also sells some fraction of it ($q < 1$), seems to be very plausible for many entrepreneurial firms. Consider an entrepreneur who has a firm that produces the stochastic output $f(a,s)$. If he sells his whole firm ($q = 0$) he forgoes an expected revenue

$$(8) \quad \Delta = Ef(a,s) - (1+i)v(a).$$

By selling his firm he receives $v(a)$ in period 0, which, invested in the credit or capital market, yields the safe return $(1+i)v(a)$ in period 1. If, instead, he keeps his firm, he gets the risky return $f(a,s)$ in period 1. Given a capital market equilibrium with asymmetric information described by equation (1), among firms of a given activity level this difference in equation (8) is positive for the more profitable firms. Entrepreneurs who initially own these "high-quality" firms do not sell their whole firm, but retain a certain fraction q . Increasing the fraction q of retention they increase their expected final wealth by $\partial V_1 / \partial q = f - (1+i)v$, but they also have to bear the additional risk implied by their increased retention. Households are risk neutral with regard to small risks (cf. Merton (1982), p. 609). Therefore, an entrepreneurial investor who owns a firm that produces an above average expected revenue prefers to retain at least some positive fraction of his firm.

Positive retentions $q > 0$ imply a welfare loss in comparison to the case of a

perfect capital market. The entrepreneur who deliberately decides to retain $q > 0$ bears the burden of risks that could be reduced via the capital market. In the case of perfect diversifiability, the welfare loss can be calculated (for the case $t_r = 0$) to equal

$$Z = q \text{Ef}(a,s) - \mathcal{S}$$

with \mathcal{S} being the solution of the equation

$$E \left[W(qf + (1+i)((1-q)v - a + V_0)) \right] = W(\mathcal{S} + (1+i)((1-q)v - a + V_0)).$$

\mathcal{S} is the certainty equivalent of that part of the uncertain returns of the entrepreneurial firm that the entrepreneurial investor does not sell in the capital market. The difference between these expected returns and their certainty equivalent measures the additional risk costs that are borne by the entrepreneur. But no one has to bear these risk costs if the risk is diversified via the capital market.

Moreover, the activity level a that is described by (6) deviates from the activity level that is chosen under conditions of perfect information. Given perfect capital markets and complete risk diversification, the marginal condition for the optimal amount of investment is determined by

$$(9) \quad \text{Ef}_a = 1 + i,$$

that generally might differ from (6). Considering, e.g., the special case $v(a) = a$ and $t_r = 0$, (6) becomes

$$(10) \quad qE[W'(V_1)(f_a - (1+i))] = 0.$$

Analogously to Sandmo's (1971) analysis of entrepreneurial behavior in the case of

price uncertainty, for the special case $f(a,s) = \vartheta(s) h(a)$ with $h'(a) > 0$, $h''(a) \leq 0$, and with $\vartheta(s)$ a stochastic variable, condition (10) implies a smaller stock of capital than the stock that is implied by (9). This is shown in appendix 1.

If the entrepreneur could be forced to sell his whole firm, the optimal capital stock would differ from the capital stock that is implicitly determined by (6). Given that the entrepreneur retains some fraction $q > 0$ of his firm and bears all the risk that results from this fraction of the firm's activity it would not however be welfare enhancing to force the entrepreneur to deviate from his optimal choice regarding capital input a .

Consider now the reaction of an entrepreneur to a change of the tax rate t_r , given the case $1 > q > 0$ and $a > 0$. Differentiating (6) and (7) with regard to t_r yields the system of equations

$$(11) \quad \begin{bmatrix} x_{qq} & x_{qa} \\ x_{aq} & x_{aa} \end{bmatrix} \begin{bmatrix} dq \\ da \end{bmatrix} = \begin{bmatrix} y_{qt_r} \\ y_{at_r} \end{bmatrix} dt_r$$

$$\Leftrightarrow \quad X \begin{bmatrix} dq \\ da \end{bmatrix} = Y dt_r$$

with

$$\begin{aligned} x_{qq} &\equiv E \left[W''(V_1) [f - (1+i)v]^2 (1-t_r) \right], \\ x_{qa} &\equiv E \left[W'(V_1) [f_a - (1+i)v'] \right] + E \left[W''(V_1) [f - (1+i)v] [J] \right], \\ x_{aq} &\equiv E \left[W'(V_1) [f_a - (1+i)v'] (1-t_r) \right] + E \left[W''(V_1) [J] [f - (1+i)v] (1-t_r) \right], \\ x_{aa} &\equiv E \left[W'(V_1) [qf_{aa}(1-t_r) + (1+i)(1-q(1-t_r))v''] \right] + E \left[W''(V_1) [J]^2 \right], \\ y_{qt_r} &\equiv E \left[W''(V_1) [f - (1+i)v] [f - (1+i)v] q \right], \\ y_{at_r} &\equiv E \left[W'(V_1) [f_a - (1+i)v'] q \right] + E \left[W''(V_1) [J] [f - (1+i)v] q \right], \end{aligned}$$

and

$$J \equiv qf_a(1-t_r) + (1+i)[(1-q(1-t_r))v' - 1].$$

Using Cramer's rule yields

$$(12) \quad da/dt_r = [y_{at_r}x_{qq} - y_{qt_r}x_{aq}]/\det X = 0$$

and

$$(13) \quad dq/dt_r = [y_{qt_r}x_{aa} - y_{at_r}x_{qa}]/\det X = q/(1-t_r).$$

This result can be summarized by a proposition.

Proposition 1: Given a capital market equilibrium that is described by (1) consider an entrepreneur who sells some fraction $0 < q < 1$ of his firm in the capital market. In response to an increased risk-taking-revenue tax he reacts not by changing the activity level of his firm, but by increasing the fraction of his firm that he retains, so that $dq/dt_r = q/(1-t_r)$.

Intuitively this result can be interpreted as follows. An entrepreneurial investor has two ways of reacting to a change of risk-taking-revenue taxes. He can change the investment activity a or the amount of retention q . Consider an entrepreneur who reacts in the way described by (12) and (13). In this case, he receives exactly the same final wealth net of taxes as before the tax increase. Let q_0 be the fraction of retention for $t_r = 0$ and consider the introduction of a risk-taking-revenue tax. The fraction of retention changes to $q_0/(1-t_r)$. The final net wealth of the entrepreneur does not change in this case.

$$\begin{aligned} V_1(t_r > 0) &= q(t_r)f + (1+i)[(1-q(t_r))v - a + V_0] - t_r q(t_r) (f - (1+i)v) \\ &= [q_0/(1-t_r)]f + (1+i)[[1 - q_0/(1-t_r)]v - a + V_0] - t_r [q_0/(1-t_r)] (f - (1+i)v) \\ &= (1-t_r)q_0f / (1-t_r) + (1+i)[v - a + V_0] - (1-t_r)q_0(1+i)v / (1-t_r) \\ &= q_0f + (1+i)[(1 - q_0)v - a + V_0] \\ &= V_1(t_r = 0). \end{aligned}$$

Similarly to the original Domar-Musgrave phenomenon, the entrepreneur's possibility set does not change in a small neighbourhood of the old optimum $V_1(t_r = 0)$. A reaction described by (6) and (7) allows the entrepreneur to obtain this old optimum even after the tax increase.

Some general equilibrium repercussions of this reaction might weaken this effect.

The increased retention q implies a reduction of supply of shares in the capital market. The demand for market portfolio and/or supply of credit is reduced by an equal amount, because entrepreneurs who increase their retention get fewer resources from selling shares of their firms and, therefore, they have fewer resources that can be invested in the capital or credit market.

As long as the capital and credit market are perfect substitutes, price reactions seemingly are not implied. There is, however, another effect that is more serious. Remember that initial owners of relatively bad firms sell their entire firms and they do not react to a tax rate change, i.e. they do not increase their retentions if the tax rate increases. Only those entrepreneurs increase their retentions who own "more profitable" or "better" firms, i.e. firms with a positive Δ in equation (8). As $dq/dt_r > 0$ is true only for these better firms, an even larger fraction of better firms' shares are no longer supplied in the capital market, while the supply of "lower-quality" firms remains unchanged. As an effect, the profitability of capital market investments falls tendentially if $v(a)$ does not change. To equate the profitability of the credit and capital market, market prices of firms must fall, and this again leads to a reduction of the supply of "high-quality" firms in the capital market, i.e. reinforces the process of adverse selection.

The risk-taking-revenue tax, therefore, might induce an equilibrium with a fraction of non-diversified diversifiable risks that is larger than without this tax. This effect, taken in isolation, certainly is welfare deteriorating. If entrepreneurs can react to an increase of a tax by increasing their retentions, a risk-taking-revenue tax might not reduce or offset, but even enlarge the inefficiency of the capital market equilibrium with adverse selection considered here. Nevertheless, the tax proceeds of a risk-taking-revenue tax have quite different stochastic properties than in the case of perfect capital markets considered by Gordon (1985). Asymmetric information leads to equilibria with some diversifiable, but deliberately not diversified risk. Taxing the revenue of this non-diversified risk allows some of this risk to be diversified within the tax proceeds. Tax proceeds from taxing risk-taking revenues therefore do not have a market value of zero as in the case of perfect

capital markets. On the other hand, however, they cannot be collected without any tax burden as in the case of perfect capital markets.

Fortunately, however, there is a situation which allows a risk-taking-revenue tax with a very small, if any, burden to be collected. This happens if the entrepreneurs *cannot* react as in the case just considered.

2.3 The Case of Complete Retention ($q = 1$)

Consider now an entrepreneur who finds it optimal not to sell any share of his firm in the capital market, i.e., the case $a > 0$, $q = 1$. A risk-taking-revenue tax cannot induce him to increase the fraction of retention of his firm. Instead, a closer look at the optimality conditions will reveal that his reaction will be similar to that of an investor in the Domar-Musgrave framework.

Given $a > 0$, $q = 1$, differentiation of (6) with regard to t_r yields

$$(14) \quad \frac{da}{dt_r} = \frac{E \left[W'(V_1) [f_a - (1+i)v'] \right] + E \left[W''(V_1) [f_a(1-t_r) - (1+i)(1-t_r v')] [f - (1+i)v] \right]}{x_{aa}}$$

The denominator x_{aa} is defined in equation (11) and $x_{aa} < 0$ if the second-order conditions for a maximum are fulfilled. The sign of the numerator determines whether an increase of a risk-taking-revenue tax may increase or decrease the activity level of the firm.

Consider first a special case with

$$(15) \quad v(a) = a.$$

If there is a large number of entrepreneurial investors who can found and operate a firm that simply gets a loan and uses this money as capital, lending it to other entrepreneurs, a credit-market equilibrium with asymmetric information and a positive turnover on both the credit and capital market, fulfills this assumption.

This is shown in appendix 2. Given property (15), condition (14) simplifies to

$$(16) \quad da/dt_r = \frac{E[W''(V_1)[f_a - (1+i)][f - (1+i)a]}{E[W'(V_1)f_{aa}] + E[W''(V_1)[f_a - (1+i)]^2(1-t_r)]} \left\{ \begin{array}{l} \geq \\ \leq \end{array} \right\} 0.$$

This expression is identical with the condition that has been derived in the Domar-Musgrave literature, where the reaction of an investor to a change in the risk-taking-revenue-tax rate is determined. In this literature an investor is considered who allocates an initial amount of wealth between a safe asset and a risky investment opportunity that has a stochastic return $f(a,s)$. The stochastic return depends on the amount of investment a and the state of nature s that prevails. Let a denote the amount of initial wealth an investor uses to invest in the risky investment opportunity. Then the maximization calculus of this investor yields condition (16) exactly.

In the case of constant returns to scale, i.e. if $f(a,s) \equiv \vartheta(s) \cdot a$, condition (16) reduces to the ordinary Domar-Musgrave phenomenon

$$(17) \quad da/dt_r = a/(1-t_r) > 0$$

that has been derived, e.g., by Mossin (1968).

In a stimulating article Mintz (1981) derived condition (16) within the Domar Musgrave portfolio model and argued that da/dt_r cannot be signed without further assumptions concerning $f(a,s)$. This result probably led to a reconciliation of the importance of the Domar-Musgrave phenomenon. In addition to this result, he derived a condition that implies an increase in risk-taking. If $f(a,s) - af_a(a,s) \equiv \text{constant}$ with regard to s for given a and the entrepreneur has constant or increasing absolute risk aversion, then $da/dt_r > 0$. The generality of this statement, however, should not be overestimated. First, it does not consider the possibility that tax proceeds are partly distributed back to the taxed entrepreneurs. The result of Mintz (1981) draws on the property of a risk-taking-revenue tax (in the case of

decreasing returns to scale) to make the taxed entrepreneur tendentially less wealthy. Taking the use of tax proceeds into account, this property is not assured. Moreover, constant or increasing absolute risk aversion is not empirically an attractive case.

Fortunately, a somewhat more general result can be obtained. Assume that the market equilibrium does not necessarily fulfill property (15), but that the more general condition

$$(18) \quad v'(a) \equiv dv/da \geq 1$$

holds. Condition (18) contains the case $v(a) = a$ as a special case. Notice that the property $v'(a) \geq 1$ is self evident in the case of an equilibrium with perfect information. Given the optimal choice of capital a , the marginal increase of capital stock should increase the value of the firm by this unit. If buyers of shares cannot observe the true properties of a specific firm, condition (18) is not self evident anymore. Nevertheless, it is not implausible to say that the equilibrium value of a firm increases by at least one unit if one more unit of capital is used in this firm¹¹.

Following generalization can be shown to hold:

Proposition 2: Given a capital market equilibrium with asymmetric information that fulfills (1) and (18), consider an increase of the risk-taking-revenue-tax rate $t_r \in [0,1)$. If there is an ordering of the states of nature such that $f(a,s)$ and $f_n(a,s)$ are monotonically increasing in s , and the owner of the firm has constant or decreasing absolute and constant or increasing relative risk aversion, then

$$da/dt_r \geq 0$$

holds for firms that are entirely retained by their entrepreneurial

¹¹ Let a_{\max}^f be the capital input in the largest firm that is traded at the capital market. The following property is sufficient for (18) to hold. In the capital market firms of all sizes $a^f \in (0, a_{\max}^f]$ are traded and for each size there is at least one entrepreneur who sells his whole firm in the capital market, i.e., he chooses $q = 0$. This can be proved by contradiction. Assume that $\partial v(a^*)/\partial a^* < 1$ for any a^* from this interval. Then the entrepreneur of a firm with $a = a^*$ would be better to choose $(a^* - da)$. The revenue from selling the whole firm would fall by less than his saved capital costs da .

investor ($q = 1$), i.e. the risk-taking-revenue tax increases the activity level of these firms.

The proof of this proposition follows lines drawn by Kihlstrøm and Laffont (1983, p. 166) in a somewhat different context and it is given in appendix 3. Notice that proposition 2 is quite general, covering many cases that have been treated in the literature. In particular, it covers the case

$$(19) \quad f(a,s) \equiv \vartheta(s) h(a) + g(a)$$

with ϑ being a random variable with finite moments and h and g being real valued differentiable functions with $h' > 0$. Reordering $\vartheta(s)$ such that $\vartheta(s)$ is monotonically increasing in s shows that the stochastic production function (19) fulfills the requirement made in proposition 2. The production function (19) is widely used¹². Proposition 2 also generalizes Sandmo's (1971) proposition concerning the effect of a profit tax without having to meet the criticism of Katz (1983), (1985), because here the measure of risk aversion in proposition 2 is concerned with the final wealth of the entrepreneur and not with profits of a firm.

3 Discussion

The partial analytic results of the early Domar-Musgrave literature show that risk-taking-revenue taxes induce investors to increase the risky fraction of their investment portfolio. General equilibrium approaches show that this property critically depends on the stochastic properties of tax proceeds, its use and how this use affects entrepreneurs' utility. If, e.g., capital markets are perfect, so that within the tax proceeds an additional risk diversification cannot take place and tax proceeds are simply paid back to tax payers in a lump-sum fashion, there is little

¹² Cf., e.g., Leland (1974), Leland and Pyle (1977), Gordon (1985) and Britto (1988), (1989). More special cases are even more widespread.

reason to believe that the incentive effect of risk-taking-revenue taxes which is to increase risk taking, prevails.

This result is rather important under welfare perspectives. Risk-taking-revenue taxes do not generally increase the fraction of wealth that an investor wants to invest more in risky assets. Instead, this incentive effect shows up if, and only to the extent that such taxes lower the wealth risks borne by investors, i.e., if via these taxes an additional risk-diversification can take place. One reason for this risk diversifying property of risk-taking-revenue taxes has been considered here. If there is asymmetric information in the capital market and entrepreneurs who founded and initially own a firm know the true quality of their own specific firm best, a process of adverse selection may take place. The capital market equilibrium may be characterized by entrepreneurs who prefer not to sell their entire firms or even not to sell any share of their firms in the capital market. The similarity of this phenomenon to the adverse selection problem in insurance markets leads to the suggestion that risk-taking-revenue taxes might have welfare enhancing properties similar to compulsory insurance in insurance markets with adverse selection.

For completely credit-financed firms with exogenous given project size, the welfare enhancing effect of a interest tax has been shown by DeMeza and Webb (1988). Here, instead, not only does a credit market exist, but the capital and credit market exist simultaneously and both the *project size* and the *fraction* of the firm *that is not sold in the capital market* but is retained by the entrepreneur are *endogenous*. For the impact of risk-taking-revenue taxes, the endogeneity of the percentage of retentions turns out to be decisive. Given that the fraction of retentions is endogenous, the entrepreneur is forced to react to a risk-taking-revenue tax in a way that amplifies the process of adverse selection. Given an increase of a risk-taking-revenue-tax rate, entrepreneurs do not increase their risky production activity, but they simply increase the fraction of retentions, perfectly offsetting the effect of the increase of the tax rate in their net-of-tax portfolio. No additional risk-diversification is reached. The additional risk diversification within tax proceeds is compensated by a reduced amount of risk diversification via the private

capital market. Additionally, the adverse selection problem can be expected to be reinforced by the general equilibrium repercussions of these reactions.

If an entrepreneur cannot react to a tax-rate change via increasing his retentions because he already retains 100 % of the shares of his firm ($q = 1$), then the impact of an increase of a risk-taking-revenue tax is very different. Similarly to the classical portfolio model of Domar and Musgrave, an increase of the rate of this tax has an impact on the activity level of the risky activity. Under fairly general conditions an increase in the rate of a risk-taking-revenue tax increases this activity level, and, probably more important, a considerable amount of additional risk-diversification might take place within the tax proceeds.

In this analysis it has been assumed that tax proceeds are used in a way that does not affect entrepreneurs' decisions. In a model with endogenously explained capital market incompleteness this assumption is much weaker than in a model with perfect capital markets. In the introduction it has been mentioned that the Domar Musgrave phenomenon does not appear in a model with perfect capital markets. A lump-sum redistributed risk-taking-revenue tax, in contrary, is perfectly ineffective in this case. The reason is that in the case of perfect capital markets the risk-taking-revenue-tax proceeds are risky. No risk consolidation takes place within the tax revenue. By the redistributions of tax proceeds therefore all risk is transferred back to the private sector. The only consistent way to use risky tax proceeds in a neutral way in the case of perfect capital markets is to spend it on public goods, assuming that these public goods affect the utility function of households in an additively separable way (Ahsan (1988), (1989), e.g., makes this assumption). The welfare impact of the use of tax proceeds in this case depends much on households' attitudes towards the risky supply of these public goods. For a detailed consideration of these issues see, e.g., Stiglitz (1972) and Allingham (1972).

In the model here a much easier way to use the tax proceeds in a neutral way is possible. Moreover, this way can be judged much more clearly from a welfare perspective. By the *perfect diversifiability assumption* all risks are non-systematic here. All risks could be perfectly consolidated via the capital market. The

consolidation does not happen because the information structure leads to adverse selection in the capital market. Within the tax revenues of a risk-taking-revenue tax in this case a substantial risk consolidation can take place and tax proceeds are expected to be practically riskless. If these proceeds are lump-sum redistributed, then practically no risk is given back to the private sector. Households which receive the redistributions are simply made richer. If all tax proceeds are redistributed to households which are not entrepreneurs and this does not affect the supply of these households in the credit and capital market, then this is a perfectly neutral and clearly beneficial way to spend the tax revenue.

If entrepreneurial households receive some redistributions, things are more complicated. If decreasing or constant absolute risk aversion is assumed, however, it can be expected that the reaction of the entrepreneurs to a tax increase remains qualitatively the same. As they are richer now, their willingness to bear risk increases or remains constant. In the case of incomplete retention ($q < 1$), therefore, the adverse selection problem can be expected to be reinforced and in the case of 100 % retention ($q = 1$) the reaction of their risky investment to a change in the tax rate can be expected to be reinforced.

4 Summary

The Domar-Musgrave phenomenon does not appear if capital markets are perfect and tax proceeds are redistributed to households, even if this distribution takes place in a lump-sum fashion. Nevertheless, the Domar-Musgrave Phenomenon turns out not to be a partial analytic peculiarity. Asymmetric information, in particular with regard to the valuation of entrepreneurial firms that are considering to go public, may induce a process of adverse selection. Some entrepreneurs decide not to go public or they sell only some part of their firms. In equilibrium, therefore, some unsystematic risk remains unconsolidated. In this case a risk-taking-revenue tax is not ineffective. An additional risk consolidation takes place within the collected tax

proceeds. However, the impact of such taxes on welfare is quite diverse, depending on the abilities of the owners of taxed entrepreneurial firms to react to the taxes. If owners of entrepreneurial firms cannot react via a change of ownership structure, then they will react like the investor in the portfolio model of the Domar-Musgrave framework, reproducing the effects derived in this literature. However, if the original owner-managers of firms go public with a strictly positive percentage of their firms while keeping a fraction of their own firms in their portfolios, the risk-taking-revenue tax induces them to increase this fraction. In this case the tax reinforces the process of adverse selection.

Appendix 1

The proof follows the lines of Sandmo (1971, S. 67): If $v(a) = a$ and $f(a,s) = \vartheta h(a)$, then for $q > 0$ equation (6) becomes

$$E[W'(V_1)(\vartheta h')] = E[W'(V_1)(1+i)]$$

or, equivalently,

$$E[W'(V_1)(\vartheta h' - h')] = E[W'(V_1)((1+i) - h')].$$

As

$$W'(V_1(\vartheta)) \leq (\geq) W'(EV_1) \quad \text{for all } \vartheta h \geq (\leq) h,$$

it follows that

$$W'(V_1)[\vartheta h' - h'] \leq W'(EV_1)[\vartheta h' - h'] \quad \text{for all } \vartheta,$$

and, taking expectations,

$$E[W'(V_1)[\vartheta h' - h']] = E[W'(V_1)[(1+i) - h']] \leq 0,$$

and, therefore, the capital stock a that is chosen under uncertainty fulfills

$$a \leq a^*,$$

a^* being the capital stock with $h'(a^*) = 1+i$, i.e. the capital stock that is chosen under certainty.

Appendix 2

Consider the case $v(a) > a$. In this case "banks" will be founded, taking up and lending the amount of a . The entrepreneur who founds this bank can sell it in the capital market, yielding a certain profit of $v(a) - a$.

Consider the case $v(a) < a$. It is not profitable to found such firms and to sell them wholly at the capital market. The founder would make a certain loss. However, it might be profitable to found a firm of this type and to retain some fraction q of it. For this endeavour to be profitable, the net revenues from the retained part of the firm, $q(f(a,s) - (1+i)a)$, must overcompensate the founder for his loss from selling the fraction $(1-q)$, i.e., $(1-q)(v(a)-a)(1+i)$. This only can be the case if $Ef(a,s) \geq (1+i)a$.

Appendix 3

Considering (6),

$$\begin{aligned} & da/dt_r \left\{ \begin{array}{l} \geq \\ \leq \end{array} \right\} 0 \\ \Leftrightarrow & E \left[W'(V_1)[f_a - (1+i)v'] \right] + E \left[W''(V_1)[f_a(1-t_r) - (1+i)(1-t_r v')] [f - (1+i)v] \right] \left\{ \begin{array}{l} \geq \\ \leq \end{array} \right\} 0. \end{aligned}$$

Notice the sign change that is implied by $x_{aa} < 0$. The non-positivity of both terms, i.e., the validity of (A.3.1) and (A.3.2) (below) is shown as follows.

$$\begin{aligned} \text{(A.3.1)} \quad & E \left[W'(V_1)[f_a - (1+i)v'] \right] \leq 0 \\ \Leftrightarrow \text{ (as } t_r < 1) \quad & E \left[W'(V_1)[f_a - (1+i)v'](1-t_r) \right] \leq 0 \\ \Leftrightarrow & E \left[W'(V_1)[f_a(1-t_r) + (1+i)t_r v' - (1+i)v'] \right] \leq 0 \\ \Leftrightarrow & E \left[W'(V_1)[f_a(1-t_r) + (1+i)(t_r v' - 1)] \right] + (1+i)(1-v')EW'(V_1) \leq 0. \end{aligned}$$

but

$$E \left[W'(V_1)[f_a(1-t_r) + (1+i)(t_r v' - 1)] \right] = 0$$

because of (6) and $q = 1$, and $(1+i)(1-v')EW'(V_1) \leq 0$ for $v' \geq 1$. This shows the validity of (A.3.1).

$$\begin{aligned} \text{(A.3.2)} \quad & E \left[W''(V_1)[f - (1+i)v][f_a(1-t_r) - (1+i)(1-t_r v')] \right] \leq 0 \\ \Leftrightarrow & - (1/(1-t_r)) E \left[W'(V_1)(-W''/W') [f(1-t_r) - (1+i)(1-t_r v)] [\dots] \right] \leq 0 \end{aligned}$$

with $[\dots] \equiv [f_a(1-t_r) - (1+i)(1-t_r v)]$. The factor $1/(1-t_r)$ does not change the sign and, therefore, can be neglected. Now add

$$(1+i)[-a + V_0] + t_r(1+i)v - (1+i)[-a + V_0] - t_r(1+i)v (= 0)$$

to $[f(1-t_r) - (1+i)(1-t_r v)]$ and reorder to get

(A.3.2)

$$\Leftrightarrow -E\left[W'(V_1)(-W''/W')\{f(1-t_r) + (1+i)[-a + V_0] + t_r(1+i)v\}[\dots]\right] \\ - E\left[W'(V_1)(-W''/W')\{- (1+i)[-a + V_0] - t_r(1+i)v - (1+i)(1-t_r)v\}[\dots]\right] \leq 0.$$

Note that $V_1 = f(1-t_r) + (1+i)[-a + V_0] + t_r(1+i)v$ and substitute in the first expectations term and sum up the terms in the second expected value term to get

(A.3.2)

$$\Leftrightarrow -E\left[W'(V_1)[\dots](-W''/W')V_1\right] + (1+i)(V_0 + v - a) E\left[W'(V_1)[\dots](-W''/W')\right] \leq 0.$$

By (6) $E\left[W'(V_1)[\dots]\right] = 0$. Therefore, the first term is negative under increasing relative risk aversion if f_a and f monotonically increase in s , as this implies that also V_1 and, therefore, $[\dots]$ and $(-W''/W')V_1$ monotonically increase in s . Given constant relative risk aversion $(-W''/W')V_1$ is constant, implying that, because of (6), the first term becomes zero. A similar reasoning applies to the second term. Given (6), $E\left[W'(V_1)[\dots]\right] = 0$. If f and f_a monotonically increase in s , then also $[\dots]$ and V_1 monotonically increase in s . Given decreasing absolute risk aversion in the second term negative values of $W'[\dots]$ are multiplied with large values of $(-W''/W')$ and positive values of $W'[\dots]$ are multiplied with small values of $(-W''/W')$, such that $E\left[W'(V_1)[\dots](-W''/W')\right]$ becomes negative. $(V_0 + v - a)$ is positive, $V_0 > 0$ is assumed, and, as $v(0) \geq 0$ and $v' \geq 1$, $v \geq a$. Given constant absolute risk aversion the second expected-value term

$$E\left[W'(V_1)[\dots](-W''/W')\right] = (-W''/W')E\left[W'(V_1)[\dots]\right] = 0.$$

This proves the validity of (A.3.2).

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