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**Risk Taking and Taxation
in Complete Capital Markets**

by

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May 1991

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**RISK TAKING AND TAXATION
IN COMPLETE CAPITAL MARKETS**

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*I would like to thank Harris Schlesinger for helpful comments.

ABSTRACT

In general equilibrium, with complete conventional securities markets and endogenous asset supply, taxes on risk remuneration are ineffective but harmless. They do not alter the real allocation of goods or the distribution of wealth, they impose no excess burden, and, in particular, have no impact on risk taking.

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1. INTRODUCTION

The taxation of risk remuneration, i.e. the deviations of risky returns from the safe rate of return, induces an investor to increase his demand for the risky investment opportunity.

There is an intuitive argument for this result in portfolio theory¹. By means of taxes, government becomes a silent partner. Collecting a fraction of risk remuneration government has a share in gains and losses. Hence, it assumes also a similar fraction of the risk burden. The private investor bears less risk and earns less risk premium. He has an incentive to increase risky investment to restore his initial portfolio.

This risk-taking incentive is less obvious in a general equilibrium framework. In the partial analytic portfolio model, the fraction of risk which is assumed by the government is dissipated. It never returns to the private sector. Stiglitz (1972) shows that the validity of the incentive effect depends on the stochastic properties of the tax proceeds, their use and how this use enters the utility of investors. Atkinson and Stiglitz (1980 p. 118) consider the case of an economy with identical individuals and with perfectly correlated risks, showing that a profit tax has no incentive effects on risk taking if the risky tax proceeds are lump-sum redistributed. Ahsan (1990) extends this result in a similar portfolio model with endogenized savings. Bulow and Summers (1984) provide some intuition of such neutrality results in a partial analytic framework. Gordon (1985) shows a similar result in a capital asset pricing model.

I analyse a tax on risk remuneration in a general equilibrium with heterogeneous agents and perfect and complete conventional security markets. Production plans and savings decisions are endogenous. All tax proceeds are redistributed, but not necessarily to the individuals who paid the tax. I show that the tax does not affect the production decisions of firms and does not alter the consumption plans of individuals. The tax does not alter the real

¹ The phenomenon was first studied by Domar and Musgrave (1944). Cf. Buchholz (1987) for a unified approach, Ahsan (1989a) for normative aspects, and Buchholz (1987) and Sandmo (1985) for further references.

allocation of goods. In particular, the tax does not change savings or increase private or aggregate risk taking.

The risk-taking incentive of taxes on risk remuneration raises also a normative question. The risk-taking incentive may be distorting or welfare improving. An equilibrium without taxes but with complete capital markets is Pareto optimal. The best outcome that could be expected after introducing a lump-sum redistributed tax is that the allocation does not change, except for income effects. Most taxes, however, cause an excess burden. This paper shows that taxes on pure risk remuneration in the case of perfect and complete capital markets cause no excess burden. They do not change the allocation at all. Hence, they are harmless.

2. THE SECURITY MARKET MODEL

Consider the following standard financial model. The economy consists of I individuals indexed by i , and F firms indexed by f . There are two periods, 0,1, and S possible states of nature in period 1, indexed by s . Also, there is only one commodity available for consuming or investing in each period.

A production decision by firm f is a vector $y^f \in Y^f \subset \mathbb{R}^{S+1}$, where $y^f \equiv (y_0^f, y_1^f) \in \mathbb{R}_- \times \mathbb{R}_+^S$. y_0^f is safe input (investment) in period 0. y_1^f is the vector of contingent output, which can be considered as a random variable. Similarly, a consumption decision by individual i is $x^i \equiv (x_0^i, x_1^i) \in X^i \subseteq \mathbb{R}_+^{S+1}$. Each individual has a concave differentiable utility function u^i from \mathbb{R}_+^{S+1} to \mathbb{R} , and consumption always has positive marginal utility, i.e., $u_0^i \equiv \partial u^i / \partial x_0^i > 0$ for all x_0^i and $u_s^i \equiv \partial u^i / \partial x_s^i > 0$ for all x_s^i . Investors also have an initial commodity endowment, $\underline{x}^i \equiv (\underline{x}_0^i, 0, 0, \dots, 0) \in \mathbb{R}_+^{S+1}$, i.e., for convenience, endowment is positive only in period 0.

Further, the investors have initial shareholdings $\underline{\alpha}^{if} \geq 0$ with $\sum_i \underline{\alpha}^{if} = 1$ for all f . Each investor has to pay the fractions $\underline{\alpha}^{if} y_0^f \in \mathbb{R}$ of the investment in firms, according to his initial ownership fractions. All firms are traded at the end of period 0, i.e., before production takes place. The market value of firm f is v^f . $\tilde{\alpha}^{if}$ describes the fraction of firm f that is owned by

investor i after trade. In period 1 production takes place. The true state of nature is revealed and investor i receives $\tilde{\alpha}^{if} y_{is}^f$, the fraction $\tilde{\alpha}^{if}$ of this firm's output in state s .

The security market is assumed to be complete. Therefore, a safe consumption plan is feasible. Without loss of generality, assume that firm 1 provides this safe security by its safe production plan: $y^1 = (y_0^1, y_1^1)$ with $y_{1s}^1 \equiv \bar{y}_1^1 \in \mathbb{R}_+$.

Consider now a tax on risk remuneration with full loss offset,

$$(1) \quad T^i = t_r \sum_f \tilde{\alpha}^{if} \left[y_1^f - [1] v^f (\bar{y}_1^1 / v^1) \right],$$

with $[1] \equiv (1, 1, \dots, 1)' \in \mathbb{R}^S$ the S -dimensional column vector of ones (throughout the paper). The ratio (\bar{y}_1^1 / v^1) is the ratio between output in period 1 and market price of the safe firm in period 0. It determines the safe rate of return. $y_1^f - [1] v^f (\bar{y}_1^1 / v^1)$ is the vector describing the risk remuneration for holding shares of f . It is the difference between y_1^f , the vector of actual random output in the second period, and the opportunity costs of an investor who buys this firm in the first period. Opportunity costs are determined by v^f , the market value of the firm, and the safe rate of return. Risk remuneration is the investor's reward for choosing a risky investment opportunity instead of the riskless one. The components of the vector $y_1^f - [1] v^f (\bar{y}_1^1 / v^1)$ can be understood as the possible realizations of a random variable and are positive in some states of nature and negative in others. The sum over all $y_1^f - [1] v^f (\bar{y}_1^1 / v^1)$ weighted by the shares of firms that investor i chooses to hold is risk remuneration $\sum_f \tilde{\alpha}^{if} [y_1^f - [1] v^f (\bar{y}_1^1 / v^1)] \in \mathbb{R}^S$ on investor i 's portfolio. The expected value of risk remuneration usually is positive. Risk remuneration is subject to a tax with strictly proportional tax rate t_r . This tax (which is negative in states of nature where risk remuneration is negative) is the straightforward generalization of Stiglitz's (1969), Ahsan's (1989b) and Sandmo's (1989) capital gains tax with full loss offset for the intertemporal case, with many assets and rather unspecific production technologies, in a general equilibrium framework with endogenous production.

All tax proceeds are lump-sum redistributed to tax payers. Tax payer i receives fractions $\gamma^{if} \geq 0$ of the risk remuneration of firms f for $f = 1, \dots, F$, i.e.,

$$(2) \quad L^i = \sum_f \gamma^{if} (y_1^f - [1]v^f(\bar{y}_1/v^1)) \in \mathbb{R}^S,$$

with

$$(3) \quad \sum_i \gamma^{if} = t_r \quad \text{for all } f = 1, \dots, F.$$

(3) is the government budget constraint: the fractions γ^{if} can be arbitrary, positive or negative, but government fulfills its budget constraint, distributing exactly all tax revenues.

The net absorption of the public sector is zero. Actual redistribution $L^i(s) = \sum_f \gamma^{if} (y_1^f - [1]v^f(\bar{y}_1/v^1))$ takes place after production in period 1.

Consider a general equilibrium. It is described by an allocation (x, y, α, v) with

$$(4) \quad x \equiv (x^1, \dots, x^I), \quad \text{with } x^i = (x_0^i, x_1^i) \in \mathbb{R} \times \mathbb{R}^S,$$

$$(5) \quad y \equiv (y^1, \dots, y^F), \quad \text{with } y^f = (y_0^f, y_1^f) \in \mathbb{R}_+ \times \mathbb{R}_+^S,$$

$$(6) \quad \alpha \equiv (\alpha^1, \dots, \alpha^I) \text{ with } \alpha^i \equiv (\alpha^{i1}, \dots, \alpha^{iF}), \text{ and}$$

$$(7) \quad v \equiv (v^1, \dots, v^F).$$

Completeness of the capital market implies that production plans y^1, \dots, y^F generate \mathbb{R}^{S+1} . The equilibrium fulfills the usual requirements for a general capital market equilibrium. x^i is an optimal plan for investor $i = 1, \dots, I$ given his utility function and budget constraints, i.e.,

$$(8) \quad x^i = \arg \max u(\bar{x}^i),$$

for $\bar{x}^i \in X^i$ fulfilling

$$(9) \quad \bar{x}_0^i + \sum_f \bar{\alpha}^{if} v^f \leq \underline{x}_0^i + \sum_f \underline{\alpha}^{if} (v^f - y_0^f)$$

and

$$(10) \quad \begin{aligned} \bar{x}_1^i &= \sum_f \bar{\alpha}^{if} y_1^f - T^i + L^i = \\ &= \sum_f \bar{\alpha}^{if} y_1^f - t_r \sum_f \bar{\alpha}^{if} \left[y_1^f - [1]v^f(\bar{y}_1/v^1) \right] + \sum_f \gamma^{if} (y_1^f - [1]v^f(\bar{y}_1/v^1)), \end{aligned}$$

using the equilibrium values of y and x in determining T^i and L^i . The present value of i 's present consumption \bar{x}_0^i , plus the market value of his portfolio after trade at the end of period 0, does not exceed the present value of his initial endowment. This endowment consists of his endowment in goods, \underline{x}_0^i , plus the market value of his endowment of firm shares, $\sum_f \alpha^{if} v^f$, minus his share of investment costs, $\sum_f \alpha^{if} y_0^f$. This is described by condition (9)². Equation (10) determines his second period consumption given his portfolio choice $\bar{\alpha}^i$ in period 0, taxes, and subsidies. Notice that $T^i = T^i(\bar{\alpha}^{if})$. The amount of taxes depends on i 's choices. Taxation is not lump-sum. Subsidies L^i are lump-sum.

Further, equilibrium requires that y^f is an optimal plan for the firm f (for all $f = 1, \dots, F$). y^f maximizes the market value net of period-0 investment:

$$(11) \quad y^f = \arg \max (v^f(\bar{y}^f) - \bar{y}_0^f)$$

for $\bar{y}^f \in Y^f$.

Additionally, in equilibrium the market clearing conditions³

$$(12) \quad \sum_i x^i = \sum_f y^f + \sum_i \underline{x}^i$$

and

$$(13) \quad \sum_i \alpha^i = \sum_i \alpha^i = (1, 1, \dots, 1) \in \mathbb{R}^F$$

must be satisfied.

Assume that the equilibrium is an interior equilibrium with $\alpha^{if} \neq 0$ for all i and f , $x^i > 0$

² In general (9) is very different from a condition $\bar{x}_0^i \leq \underline{x}_0^i - \sum_f \alpha^{if} y_0^f$. Consider an investor who is particularly well endowed with firm shares, but did not get any \underline{x}_0^i . This investor may liquidate some of his wealth initially invested in firm shares for present consumption, i.e., he can choose $\sum_f \bar{\alpha}^{if} v^f < \sum_f \alpha^{if} v^f$. His savings decision is endogenous.

³ As a consequence of Walras's law, condition (12) is always fulfilled if all individuals and government fulfill their budget restrictions (9) and (3) respectively.

for all i , and $y^f \neq 0$ for all f . The first-order conditions⁴ with regard to α^{if} yield

$$(14) \quad -u_0^i v^f + \sum_s u_{1s}^i \left[y_{1s}^f - t_r(y_{1s}^f - [1]v^f(\bar{y}_1/v^1)) \right] = 0$$

for all $f = 1, \dots, F$. Notice that this exhausts the number of first-order conditions. If α^i is determined, (x_j^i, x_1^i) also is determined by (9) and (10). Condition (14) shows that the marginal utility from selling an additional share of firm f and consuming the revenue today equals the marginal utility of keeping this share and enjoying the future benefits. Using $y_{1s}^1 = \bar{y}_1$, for $f = 1$ equation (14) yields

$$(15) \quad \bar{y}_1/v^1 = u_0^i / \sum_s u_{1s}^i.$$

(15) describes that in equilibrium the safe rate of return \bar{y}_1/v^1 equals the marginal rate of substitution between period 0 and period 1 consumption. (15) can be used to transform (14) to

$$(16) \quad v^f = \langle u_1^i, y_1^f \rangle / u_0^i,$$

$\langle a, b \rangle$ denoting the scalar product of vectors a and b .

Notice that $(1, u_{11}^i/u_0^i, \dots, u_{1s}^i/u_0^i, \dots) \equiv (1, p_1^i) \equiv p^i$ is the *sustaining state claim price vector* for investor i . Using the assumption that the system of conventional securities is complete, this vector is unique and is the same for all $i = 1, \dots, I$ (see, e.g. Krouse (1986), p. 118).

Therefore, the *implicit state price vector* can be defined to be $p \equiv (1, p_1) \equiv (1, p_1^i)$ omitting the superscript, where p is the equilibrium price vector of the pure securities market equilibrium that corresponds to the conventional securities market equilibrium being considered here.

Using the definition of p equation (16) simplifies to

⁴ The second-order conditions are fulfilled if $u(\bar{x}^i)$ is concave and the production possibility sets Y^f of firms are convex.

$$(17) \quad v^f = \langle y_1^f, p_1 \rangle ,$$

showing that the market value of the firm is the sum of state contingent outputs multiplied by their state claim prices.

3. THE NEUTRALITY OF A TAX-RATE CHANGE

Consider now the impact of a tax rate change, assuming that the additional tax proceeds are fully lump-sum redistributed.

Proposition 1

If (x, y, α, v) is an interior capital market equilibrium for a tax rate $t_r = 0$, then, for any given tax rate $t_r^* \in [0, 1)$, there is an equilibrium $(x^*, y^*, \alpha^*, v^*)$ with

$$(18) \quad \begin{aligned} x^* &= x \\ y^* &= y \\ v^* &= v \\ \alpha^{i*} &= \begin{bmatrix} \alpha^{i1} - \sum_{f=2}^F (t_r \alpha^{if} - \gamma^{if}) v^f / (v^i (1 - t_r)) \\ (\alpha^{i2} - \gamma^{i2}) / (1 - t_r) \\ \vdots \\ (\alpha^{iF} - \gamma^{iF}) / (1 - t_r) \end{bmatrix} . \end{aligned}$$

For a proof see the appendix. Proposition 1 states (i) that a redistributed tax on risk remuneration can be fully counteracted by market transactions described by (18), so that the allocation of goods is not affected by the tax. (ii) this counteraction is individually optimal for the investors. Their choices of savings and net risk taking are unchanged. (iii) firms have no incentive to change their production plans. Individuals may get lump-sum subsidies that

are very different from their individual tax payments⁵. Even in this case, by (18) individuals can perfectly counteract the effects of the tax and the subsidies. Individuals could also take action to avoid taxes in a way that leads to some excess tax burden. Proposition 1 reveals that a tax on risk remuneration imposes no such excess burden. The real allocation of goods, even for each single investor and firm, is not affected by this tax. The tax is not distorting and has no redistributive effects on wealth.

The assumption of a full loss offset is critical for the incentive for additional risk taking in the partial equilibrium portfolio analysis. Proposition 1 shows that this incentive disappears under the same full-loss-offset provision in complete capital markets. The neutrality result, however, also depends critically on the loss-offset provision.

4. THE SYMMETRIC CASE

It may be suspected that, similarly to the partial analytic decision of the investor in the Domar–Musgrave portfolio model, the net portfolio stays the same *because* the risk-taking incentive effect prevails. This is not the case. To see this, consider the perfectly symmetric case with I identical investors. Assume that each feels atomistic, and behaves competitively in the markets, taking as given the market prices of assets and the equilibrium values of risky lump-sum redistributions.

Proposition 2:

In a symmetric equilibrium with identical investors

$$(19) \quad d\alpha^{if}/dt_r \equiv 0$$

holds for all $i = 1, \dots, I$ and for all $f = 1, \dots, F$.

⁵ This is a major difference to the approaches by Atkinson and Stiglitz (1980, p. 118), Gordon (1985) and Ahsan (1990). It allows the tax on risk remuneration to be shown as neutral, not because each individual gets back his individual risky tax payments, but because all risk is returned to, and borne by, the private sector.

Proposition 2 is a corollary of proposition 1. A look to (18) reveals that, in the symmetric case with $\gamma^{if} = t_r \alpha^{if}$ equation (18) reduces to $\alpha^{if*} = \alpha^{if}$ for all f .

The Domar–Musgrave phenomenon known from the portfolio model is $d\alpha^{if}/dt_r = \alpha^{if}/(1-t_r) > 0$ (cf. Mossin (1968) and Sandmo (1989) for the many asset case). Feldstein (1969), among others, has described the intuition of the risk-taking incentive of taxation. By taxing the risk remuneration, the government not only collects a fraction of the reward for risk taking, but also assumes some fraction of the risk burden initially assumed by the investor. The investor now bears less risk than planned, given the price or revenue of risk taking. This makes him willing to extend his risky investment until the amount of risk he takes (net of taxes) is the same as in the equilibrium before the tax rate change occurred. If capital markets are complete and therefore no risk diversification takes place within the tax proceeds, taking the redistribution of taxes into account changes the result. If the proceeds of the tax on risk remuneration are lump-sum redistributed, not only are the rewards for risk-taking redistributed, but also the risk burden assumed by the government is shifted back to the investors. Investors, who choose the amount of risk they are willing to bear, take the risks associated with the lump-sum rebates into account. They have no incentive to increase their demand for risk taking.

The initial equilibrium without taxes but with complete capital markets is Pareto optimal. The best outcome that could be expected in a symmetric world after introducing a lump-sum redistributed tax is that the allocation does not change. Most taxes, however, have incentive effects, changing the allocation in an unfavorable way, regardless of whether tax proceeds are lump-sum redistributed or not. They have an excess burden. Proposition 1 shows that this is not the case for a tax on risk remuneration. This tax imposes no excess burden. The tax does not even redistribute wealth. No individual's net consumption plan is changed by the tax. The main insight of this paper may be the reason for this result. Consider the impact of the tax or the lump-sum subsidies on budgetary constraints. These constraints are not changed. The expected value of the tax proceeds is positive, but, valued by the equilibrium asset prices, the market value of the tax proceeds is zero (cf. (A.5) in the

appendix). The same holds for redistributions (cf. (A.6) in the appendix). By taxing risk remuneration, government acquires tax revenues with a zero market value. Therefore, the marginal burden of a tax on risk remuneration is zero. The tax is similar to other taxes with a marginal tax burden of zero. Such taxes do not induce a substitution effect that brings about an excess burden.

5. CONCLUSIONS

If there is no risk diversification within the proceeds of a tax on risk remuneration, such taxes provide no incentive to increase risk taking. This result is generalized here for the case of an economy with many heterogeneous agents and many firms with endogenous risky production and complete risk markets. If firms maximize their market value and only firms' shares are traded, a lump-sum redistributed tax on risk remuneration is perfectly ineffective. In particular, there is no incentive to change the portfolio structure by increasing the fraction of the more risky assets, or by shifting resources from safe to risky industries.

This result has some welfare theoretic impact. If the Domar–Musgrave risk-taking incentive of taxation were to show up in general equilibrium here, it would generally imply welfare losses (other market defects being absent), as the initial equilibrium is Pareto optimal. However, the incentive only shows up if the riskiness of tax proceeds is not returned to tax payers, either because within the tax proceeds some risk diversification takes place or because the tax proceeds are used for other purposes and are not returned at all. This shows that capital-gains taxes do not lead to "too much" risk taking in general. They do induce additional risk assumption in certain cases. These are where the government can bear the risk at lower cost than the private sector or where some of the risks can be transformed in such a way that they no longer affect the risk-taking decisions of the investors. However, the conditions for these cases to appear are restrictive. In the context of complete risk markets considered here, lump-sum redistributed taxes on risk remuneration are useless but harmless.

APPENDIX

The proof of proposition 1 has 5 steps. In step 1 it is shown that the opportunity sets X^i do not depend on $t_r \in [0,1)$ in an exchange equilibrium. In step 2 it is shown that $\alpha^* = \alpha$ is individually optimal for any tax rate in an exchange equilibrium with $y^* \equiv y$ and $v^* = v$. In step 3 it is shown that the allocation of firm shares α^* associated with $\alpha^* = \alpha$ is feasible on the aggregate level. In step 4 it is shown that $v^* = v$ holds given the altered choices of α^{if*} , such that $\alpha^* = \alpha$, provided that production plans are not changed. In step 5 it is shown that no firm has an incentive to alter its production plan.

Step 1. Let x^{i*} be the optimal choice of investor i for $t_r \in [0,1)$ described by (8), (9) and (10). If (y^1, y^2, \dots, y^F) generate \mathbb{R}^{S+1} , as is the case by the assumption of completeness, then, by a proper choice of $\tilde{\alpha}^{if}$, all $\tilde{x}^i \in \mathbb{R}_+^{S+1}$ are feasible, irrespective of $t_r \in [0,1)$. I show that, in particular, $x^{i*} = x^i$ is feasible. It has to be shown that for any $t_r \in [0,1)$ a proper choice of α^{if*} allows to obtain

$$(A.1) \quad x_j^i = \underline{x}_j^i - \sum_f \alpha^{if*} v^f + \sum_f \underline{\alpha}^{if*} (v^f - y_0^f)$$

$$(A.2) \quad x_1^i = \sum_f \alpha^{if*} y_1^f - t_r \left[\sum_f \alpha^{if*} [y_1^f - [1]v^f(\bar{y}_1^f/v^1)] \right] + \sum_f \gamma^{if} [y_1^f - [1]v^f(\bar{y}_1^f/v^1)].$$

Let α^i be the vector of shares which yields x^i for the case $t_r = 0$. For given t_r and γ^{if} , a solution of this problem is (18). This shows that the opportunity set X^i does not depend on t_r .

Step 2. The market value of a firm with output y_1^f is v^f . Therefore, the market value of the tax is

$$(A.5) \quad t_r \sum_f \tilde{\alpha}^{if} [v^f - v^f(v^1/v^1)] = 0,$$

and, similarly, the market value of subsidies is

$$(A.6) \quad \sum_f \gamma^{if} [v^f - v^f(v^1/v^1)] = 0.$$

These considerations show that the maximization calculus of an investor who takes $v^* = v$ and $y^* = y$ as given does not depend on the taxes and subsidies. The objective function $u(\bar{x}^i)$ and the constraints (9) and (10) are independent of t_r or γ^i .

Step 3. Government has zero net absorption, i.e., $\sum_i T^i = \sum_i L^i$. The budget constraint of the whole economy is (12). In an exchange equilibrium with $y^* = y$ and $\alpha^* = \alpha$ condition (12) is fulfilled, as it was fulfilled in (y, α, α, v) for $t_r = 0$. Condition (13) is fulfilled for α^{if*} given by (18): using the property $\sum_{i=1}^I \alpha^{if} = 1$ (for all $f = 1, \dots, F$) of the initial equilibrium and $\sum_{i=1}^I \gamma^{if} = t_r$ from equation (3), summing up equation (18) over all $i = 1, \dots, I$ yields

$$\sum_{i=1}^I \alpha^{i1*} = \sum_{i=1}^I \alpha^{i1} - \sum_{f=2}^F \left[\sum_{i=1}^I t_r \alpha^{if} - \sum_{i=1}^I \gamma^{if} \right] (v^f / ((1-t_r)v^1)) = 1 - 0 = 1$$

and

$$\sum_{i=1}^I \alpha^{if*} = [\sum_{i=1}^I \alpha^{if} - \sum_{i=1}^I \gamma^{if}] / (1-t_r) = 1 \quad \text{for all } f = 2, \dots, F.$$

Step 4. The implicit state price vector

$$(A.7) \quad p = (1, p_1) = (1, u_1^i(x^i)/u_0^i(x^i), \dots, u_S^i(x^i)/u_0^i(x^i))$$

only depends on x^i , but $\alpha^* = \alpha$. Using $y^* = y$ in (17), $v^f = \langle y_1^f, p_1 \rangle$ shows $v^{f*} = v^f$.

Step 5. Firms maximize their market value net of investment costs. Using equation (17) this is

$$(A.8) \quad \max \{ v^f - \bar{y}_0^f \} = \max \{ \langle p_1, \bar{y}_1^f \rangle - \bar{y}_0^f \}.$$

If firms anticipate that p does not change by the tax rate change, the production plan y^f that maximizes (A.8) is independent of t_r . Therefore firms choose $y^{f*} = y^f$ in the new equilibrium. This completes the proof.

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