Hasnas, Irina; Wey, Christian

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Full versus partial collusion among brands and private label producers

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Editor:

Prof. Dr. Hans-Theo Normann
Düsseldorf Institute for Competition Economics (DICE)
Phone: +49(0) 211-81-15125, e-mail: normann@dice.hhu.de

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Full Versus Partial Collusion Among Brands and Private Label Producers*

Irina Hasnas† Christian Wey‡

July 2015

Abstract

We analyze the incentives to collude when brand manufacturers compete with a private label producer of inferior quality. Full collusion is easier to sustain than partial collusion from the brands’ perspective when horizontal differentiation is large and vertical differentiation is small. The private label firm is better off under full collusion than under partial collusion if goods are sufficiently homogenous (horizontal and/or vertical). Partial collusion could be preferred by the private label exactly when full collusion is easier to sustain. Improving the private label’s quality makes full collusion more likely, either because it relaxes the brand producers’ incentive constraint or because it shifts the preference of the private label firm from partial collusion to full collusion. Fully collusive behavior reveals itself through a nonnegative price effect on the brands’ side caused by a quality increase of the private label good.

JEL-Classification: L11, L13

Keywords: Oligopoly, Product Differentiation, Private Label, Collusion

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†Düsseldorf Institute for Competition Economics (DICE), Heinrich-Heine University of Düsseldorf, Universitätsstrasse 1, 40225 Düsseldorf, Germany; E-mail: hasnas@dice.hhu.de.

‡Düsseldorf Institute for Competition Economics (DICE), Heinrich-Heine University of Düsseldorf, Universitätsstrasse 1, 40225 Düsseldorf, Germany; E-mail: wey@dice.hhu.de.
1 Introduction

We present a Salop-circle model which captures market competition between branded products and a private label substitute.\footnote{Another application of our analysis are pharmaceutical products where brands and their generic equivalents compete against each other after the end of patent protection (see Frank and Salkever, 1997).} All products are differentiated in the horizontal dimension. In addition, the private label good is assumed to be inferior in the vertical dimension. We use an infinitely repeated game approach to examine under which circumstances full collusion of heterogeneous firms is easier or harder to sustain than partial collusion. In the former case all firms (the brands and their private label substitute) collude, while in the latter case only the brands form a self-enforcing cartel.

Private label products (also called store brands) encompass all merchandise sold under a retailer’s brand. Their market share has risen significantly and today private labels are an integral part of almost all retail markets. For instance, based on 2011 sales data of Nielsen, private labels accounted for 42 per cent in the United Kingdom.\footnote{Market shares are calculated based on the turnover of fast-moving consumer goods, excluding fresh food. The market shares differ significantly across countries (for instance, Spain: 39\%, Germany and Portugal 32\%, while Greece has the lowest with 10\%). Market shares have been increasing steadily. According to European Commission (2011), from 2003 to 2009 their share increased by 2-7 percentage points in Western and Southern Europe (except Spain) and by 10-26 percentage points in Spain and Central Europe. Inderst (2013) provides a survey of these developments.} Private labels were initially part of a low-price, low-quality strategy allowing retailers to compete for price-sensitive consumers (Hassan and Monier-Dilhan 2006). These budget private labels were often designed as me too products and were positioned at the lower end of the quality and price spectrum. Private labels were especially successful in markets where no strong national brands were present (European Commission, 2011). However, over recent years they have grown in the segment of added-value and premium products.\footnote{For instance, the German supermarket chain Real offers its own premium labels in many product categories.} Based on GfK German retailing data, Inderst (2013, Figure 3, p. 14) reports that over the period 2007-2012 budget private labels’ market share stayed put at around 25 per cent, while the market share of premium private labels increased from 9 per cent to 12.9 per cent over the same period.

The rise of private label products has been explained by cost savings, buyer power reasons
and retailer differentiation.\textsuperscript{4} The role private labels play within the context of collusion has been largely neglected so far.\textsuperscript{5} This is surprising because collusion is an ongoing issue among food manufacturers. In Germany, for instance, three cartel cases with food manufacturers involved were decided recently. The coffee roasters’ cartel was a partial cartel in which only brand coffee makers were found guilty of forming a cartel, while there was no evidence found that private label producers (mainly store brands of retailers Aldi and Lidl) have participated in the cartel (Bundeskartellamt, 2009). The coffee roasters’ cartel is remarkable, because it lasted for many years even though it did not include the private labels which have a market share of 17 per cent in the German coffee market in 2011 (Bundeskartellamt, 2014a, p. 208). In the sausage cartel, to the opposite, private label producers participated in the cartel which included almost all branded sausage producers (see Bundeskartellamt, 2014b). The confectionery manufacturers’ cartel only included six branded products (see Bundeskartellamt, 2013).\textsuperscript{6}

Behind this background our main research questions are the following: First, how do horizontal product differentiation and the private label’s (vertical) quality affect the stability of full and partial collusion (the former being an all encompassing cartel, while the latter only includes the branded goods)? And relatedly: When is a more homogenous cartel among branded manufacturers more likely to form than a heterogeneous cartel which also includes private label substitutes? Second, how is the private label producer’s incentive to close the quality gap towards the branded goods affected by market conduct which can be competitive, partially collusive or

\textsuperscript{4} Hoch and Banerji (1993) have shown that cost savings can be so large that private labels may generate even higher profit margins than the respective national brands. Private labels can substantially enhance a retailer’s bargaining position vis-à-vis brand manufacturers because it enhances their outside option (Mills, 1998, Bon- tems, Monier-Dilhan, and Requillart, 1999, and Steiner, 2004). Moreover, private labels can increase retailer differentiation as retailers would otherwise carry the same assortment of branded goods (Gabrielsen and Sörgard, 2007).

\textsuperscript{5} An exception is Steiner (2004) who warns explicitly that the issue of collusion between private label goods and national brands may become more of an issue in the future. Interestingly, the issue of collusion between private labels and branded goods does not play a major role in recent retailing sector inquires by competition authorities (see, e.g., Bundeskartellamt, 2014a, Competition Commission, 2008, European Commission, 1999).

\textsuperscript{6} It should be noted that cartel cases are decided on explicit evidence of cartel formation. The question, therefore, whether or not private label producers participated in the cartel via tacit collusion was not decided in those cases where only brand manufacturers found guilty.
fully collusive?

We analyze a Salop circle model with three firms that differ in their (vertical) quality parameter. Two out of three firms are high-quality brand producers, while the third firm is a private label producer that has an inferior quality. We are interested to analyze whether collusion among three heterogeneous firms is easier to sustain than partial collusion among the two brand producers. We use an infinitely repeated game approach to examine the stability of collusion. We first show that the brand producers’ incentive constraint is critical to obtain full collusion over partial collusion, whenever nonparticipation of one firm leads to noncooperative market conduct. In those instances, the private label firm always joins the brand producers for a full collusion outcome, given that full collusion is incentive compatible for the brand manufacturers. If, however, nonparticipation of the private label firm induces the brand manufacturers to form a partial cartel (which is always better than noncooperative conduct), then the private label may prefer partial collusion over full collusion. This is, ceteris paribus, more likely to be the case, the lower the intensity of competition (i.e., the higher the horizontal product differentiation) and the larger the (vertical) quality gap between the private label and the branded goods. Thus, a private label firm is more likely to join the branded goods producers to form an all encompassing cartel the higher the quality of the private label good and the more intense competition are.

We also show that the incentives to increase the private label’s quality is largest under full collusion with partial collusion and noncooperative behavior following in that order. The incentives are further enhanced by the prospect of making full collusion feasible in the first place. There are two reasons why a quality upgrade of the private label good can trigger full collusion. First, it relaxes the incentive constraint for the brand producers, and second, it makes it more likely that the private label firm prefers full collusion over partial collusion.

Our paper contributes to the collusion literature that deals with cartel stability when firms’ are heterogenous. Häckner (1994) shows that an all-inclusive cartel is harder to sustain when

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7Selten (1973) analyzes cartel stability as a coalition formation process (i.e., without referring to an infinitely repeated game context). He assumes homogeneous products and Cournot competition. Full cartelization is only possible when there are few firms. See Prokop (1999) for a related approach within a model of price competition, where it is also shown that the chance of full cartelization is very much limited. For a survey, see Bos and Harrington (2010).
products become more vertically differentiated. Based on a spatial model of horizontal product differentiation Ross (1992) argues that increased product differentiation could enhance cartel stability (see also Chang, 1991). Given those results, opposing forces are present in a model that combines variety-differentiated products with (vertical) quality differentiation. In addition, partial collusion has not been addressed in those works. Closely related is Bos and Harrington (2010) who analyze the sustainability of collusion (full and partial) within an infinitely repeated game framework. Their focus is on capacity asymmetries among firms, while firms’ products are homogenous. Overall, they show that full collusion is harder to sustain, when firms become more asymmetric (with regard to their capacity). Moreover, smaller firms are more likely to stay out of the cartel giving rise to partial collusion among the largest firms in the market. We apply the same stability analysis as they do; namely, we suppose that nonparticipation of a firm in the all encompassing cartel will keep collusion among the remaining firms if it is profitable for them.

We also contribute to the economic analysis of private labels (for a survey, see Berges-Sennou, Bontems, and Requillart, 2004). Price effects and product positioning incentives were analyzed in Mills (1995) and Bontems, Monier-Dilhan, Requillart (1999), and Gabrielsen and Sörgard (2007). Those works focused on the strategic effects within a vertical relations setting without considering the collusion problem. Empirical works have shown ambiguous price effects of private labels on branded substitutes. Quite interestingly, Putsis (1997) and Cotterill and Putsis (2000) have provided evidence that brands’ prices decreased after the introduction of private label substitutes. In contrast, Ward et al. (2002) show a positive association of private

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8 A related result is obtained in Rothschild (1992).

9 Thomadsen and Rhee (2007) show that collusion is always harder to sustain the more differentiated the products if costs of forming the cartel are sufficiently large.

10 Our model builds on Economides (1989, 1993) which are early models (of the Hotelling and Salop type, respectively) with both horizontal and vertical product differentiation.

11 The debate about how to formalize a cartel’s stability in case of heterogeneous firms is ongoing (see Bos and Harrington, 2010, for a survey). The impact of cost asymmetries in association with an indivisible cost of collusion is analyzed in Ganslandt, Persson, and Vasconcelos (2012).

12 Choi and Coughlan (2006) show (disregarding the vertical relation problem) that a private label should position close to a strong (weak) national brand when its quality is high (low).
labels’ market shares and branded products’ prices.\textsuperscript{13} A similar relationship is uncovered in Frank and Salkever (1997) who investigated price responses of branded pharmaceuticals after patent protection expired and generic substitutes entered the market.

The paper proceeds as follows. In Section 2 we present the model setup and Section 3 provides the equilibrium analysis. In Section 4 we analyze the private label producer’s quality incentives and we show how collusion can be detected from market data. Finally, Section 5 concludes.

2 The Model

We specify a variant of the Salop circle model (Salop, 1979) which combines horizontal product differentiation (as a measure of overall competition intensity) and vertical product differentiation (which mirrors the inferior quality of the private label vis-à-vis the branded goods). Let there be three firms \((j = 0, 1, 2)\) located equidistantly on the unit circle. Firms 1 and 2 produce two brands with (vertical) quality index \(s_i\), where \(i = 1, 2\).\textsuperscript{14} Both firms are horizontally differentiated and they are located at \(x_1 = 1/3\) and \(x_2 = 2/3\), respectively. We refer to these goods as brands 1 and 2, respectively. Firm 0 is located at \(x_0 = 0\) on the unit circle and produces a private label product which is a horizontally differentiated variant, but of a lower (vertical) quality \(s_0 \leq s_i\) for \(i = 1, 2\). We set production costs equal to zero.\textsuperscript{15}

Consumers are distributed uniformly along the unit circle with mass of one. Each consumer buys at most one unit of the good. A consumer’s position \(x\) on the unit circle represents her most preferred product variant in the horizontal dimension. The utility of a consumer with address \(x \in [0, 1]\) buying from firm \(i = 1, 2\) is given by

\[
U_i^x = s_i - t |x_i - x| - p_i, \tag{1}
\]

where \(p_i\) is firm \(i\)’s price. According to (1), we consider a linear transportation cost function.

\textsuperscript{13}Bontemps, Orozco, and Requillart (2008) provide related evidence for France.

\textsuperscript{14}We use the index \(i\) to refer only to the brands \(i = 1, 2\), whereas the index \(j\) is used to refer to all firms \(j = 0, 1, 2\).

\textsuperscript{15}A three-firms Salop model is also used in Rasch and Wambach (2009) to analyze the effect of a two-firm merger and internal-decision making rules on cartel stability. Yet, in their model, all products have the same vertical quality.
where $t > 0$ is the exogenously given transportation cost parameter and $s_i$ is the (vertical) quality index.\(^{16}\) Correspondingly, the utility of a consumer with address $x$ buying the private label product is given by

$$U^x_0 = s_0 - \min\{tx; t(1-x)\} - p_0,$$

where $s_0 \leq 1$ and $p_0$ is the price charged by firm 0. We set $s_1 = s_2 = 1$ and define $s := s_0$ with $s \in [0, 1]$. Thus both brands are assumed to be of the same quality and their quality is higher than the private label’s quality. The quality gap between the brands and the private label is given by $1 - s \geq 0$. Consumers only buy a product if their utility is not negative.

We consider an infinitely repeated price competition game to study firms’ collusion incentives. In the stage game, all firms set their prices simultaneously. All firms have the same discount factor $\delta \in [0, 1]$. In the infinitely repeated game, we focus on trigger strategies with Nash reversal in the punishment phase.\(^{17}\)

We consider two types of collusion: \(i)\) full collusion (FC), where all three firms collude, and \(ii)\) partial collusion (PC), where only firms 1 and 2 collude, while firm 0 behaves noncooperatively. In addition, we denote by $N$ the case that all firms behave always noncooperatively.

We analyze the stability of collusion under full and partial collusion. We denote by $\pi^N_j$ the noncooperative (stage game) profit of firm $j$, by $\pi^C_j$ the collusive (stage game) profit of firm $j$, and by $\pi^{D,C}_j$ the deviation profit of a colluding firm $j$, where the superscript $C$ refers either to the partial collusion case or the full collusion case; i.e., $C \in \{FC, PC\}$. Given trigger strategies with Nash-reversal, firm $j$ has no incentive to deviate from the collusive behavior if and only if the discount factor is large enough; i.e., if

$$\delta \geq \delta^C_j := \frac{\pi^{D,C}_j - \pi^C_j}{\pi^{D,C}_j - \pi^N_j} \quad (3)$$

holds. We impose the following parameter restrictions which ensure that the market is always

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\(^{16}\)See Economides (1989) for a similar approach to combine both horizontal and vertical product differentiation within a Salop model.

\(^{17}\)We use a grim strategy as in the seminal paper of Friedman (1971) to derive firms’ collusion incentives. While this is standard practice in the tacit collusion literature (see Mas-Colell, Whinston, and Green, 1995, Chap. 12D), optimal punishments (so-called stick-and-carrot strategies) can be more effective in sustaining collusion (see Abreu, 1986, 1988, and Abreu, Pearce, and Stacchetti 1986). Both approaches can be expected to lead to the same qualitative results (see Häckner, 1996, and Chang, 1991).
covered in equilibrium.

**Assumption 1.** *We restrict the analysis to all parameter pairs* \((t, s)\) *which fulfill the conditions* \(1 \geq s \geq \max\{1 - 5t/6, t/2, 13t/6 - 2\}.* *The minimal possible values of* \(t\) *and* \(s\) *are* \(t_{\min} = 6/11\) *and* \(s_{\min} = 3/8,\) *while the maximal possible values are* \(t_{\max} = 18/13\) *and* \(s_{\max} = 1.\)

Assumption 1 specifies the feasible set of parameters we are considering throughout the analysis (all conditions are derived in the Appendix). Specifically, restriction \(s \geq 1 - 5t/6\) ensures that the equilibrium market share and price of the private label good are positive under noncooperative behavior. Conditions \(s \geq t/2\) and \(s \geq 13t/6 - 2\) ensure that the market is covered under full and partial collusion, respectively. Specifically, condition \(s \geq t/2\) implies that the deviation price of the private label firm under full collusion is lower than its collusive price. Finally, \(t \geq t_{\min} = 6/11\) makes sure that the deviating firm under (both partial and full) collusion realizes a market share which is less than 100 per cent.\(^{18}\)

**Nash Equilibrium of the Stage Game.** Before we analyze the infinitely repeated game, we solve the stage game to derive \(\pi_j^N,\) for \(j = 0, 1, 2.\) We first derive the demand functions. Note that firms are located equidistantly on the unit circle. Denote the indifferent consumer between firms 0 and 1 by \(x_0.\) Given prices \(p_0\) and \(p_1\) the indifferent consumer between firms 0 and 1 is given by

\[
s - p_0 - tx_0 = 1 - p_1 - t(1/3 - x_0)
\]

which gives her location on the segment \(x \in (0, 1/3):\)

\[
x_0 = (s + t/3 - p_0 + p_1 - 1)/(2t)\) for \(x_0 \in (0, 1/3).
\]

Similarly, the indifferent consumer between firms 1 and 2 (denoted by \(x_1\)) is obtained from

\[
1 - p_1 - t(x_1 - 1/3) = 1 - p_2 - t(2/3 - x_1),
\]

which gives her location on the segment \(x \in (1/3, 2/3):\)

\[
x_1 = (t - p_1 + p_2)/(2t)\) for \(x_1 \in (1/3, 2/3).
\]

Finally, the indifferent consumer between firms 2 and 0 (denoted by \(x_2\)) is given by

\[
1 - p_2 - t(x_2 - 2/3) = s - p_0 - t(1 - x_2)
\]

\(^{18}\)The remaining maximal and minimal values of \(t\) and \(s\) follow from the restrictions that constrain \(s\) as stated in the first sentence of Assumption 1.
which gives her location on the segment \( x \in (2/3, 1) \):

\[
x_2 = (1 - s + 5t/3 + p_0 - p_2)/(2t) \text{ for } x_2 \in (2/3, 1).
\] (6)

Using (4), (5), and (6) we can write firm \( j \)'s demand \( D_j \) as

\[
D_j = \begin{cases} 
  x_0 + 1 - x_2, & \text{if } j = 0 \\
  x_j - x_{j-1}, & \text{if } j = 1, 2.
\end{cases}
\] (7)

Using (7) we can solve the firms' maximization problems \( \max_{p_j \geq 0} \pi_j = p_j D_j \) simultaneously to obtain the equilibrium prices and firms' equilibrium profits under noncooperative behavior.

**Proposition 1.** Suppose that all firms behave noncooperatively. We obtain the following equilibrium values:

i) Prices: \( p_i^N = (3(1 - s) + 5t)/15 \), for \( i = 1, 2 \), and \( p_0^N = (6(s - 1) + 5t)/15 \). Moreover, \( p_1^N = p_2^N \geq p_0^N \) if \( s \leq 1 \) (with equality holding for \( s = 1 \)).

ii) Profits: \( \pi_i^N = (5t + 3(1 - s))^2/(225t) \), for \( i = 1, 2 \), and \( \pi_0^N = (6(s - 1) + 5t)^2/(225t) \); moreover, \( \pi_1^N = \pi_2^N \geq \pi_0^N \) (with equality holding for \( s = 1 \)).

iii) Locations of the indifferent consumers: \( x_0^N = 1/6 - (1 - s)/(5t) \), \( x_1^N = 1/2 \), and \( x_2^N = 5/6 + (1 - s)/(5t) \).

**Proof.** See Appendix.

Parts i) and ii) of Proposition 1 state firms' prices and profits, respectively. The prices and profits of the brand producers decrease when the quality of the private label increases, while the opposite holds for the private label producer. As long as the private label good is of a strictly lower quality than the branded goods (\( s < 1 \)), the brand producers realize higher profits than the private label producer. Part iii) of Proposition 1 shows that the private label producer 0 serves the consumers with addresses \((0, x_0^N) \cup (x_2^N, 1)\), while the branded manufacturers 1 and 2 serve the consumers on the intervals \((x_0^N, 1/2)\) and \((1/2, x_2^N)\), respectively. If the quality of the private label good is inferior, \( s < 1 \), then firm 0 serves less consumers than firms 1 and 2, despite the fact that it charges the lowest price. The relatively low quality of the private label good reduces its equilibrium demand. This benefits the brands, because they can sell their products at a higher price and also enjoy a larger equilibrium demand. However, if firm 0's quality increases, firms 1 and 2 face stronger competition and reduce their prices. If \( s = 1 \), then all three firms are homogeneous in the vertical dimension and they share the market equally.
3 Equilibrium Analysis

We next analyze the infinitely repeated game for the cases of full collusion and partial collusion. We then compare our results and we relate them to the case where firms always behave noncooperatively. Finally, we compare the stability of both types of collusion.

3.1 Full Collusion

Assume that all firms in the market collude (case FC). Then all firms maximize their joint profit \( \pi^{FC} := \sum_{j=0}^{2} \pi_j \) and charge collusive prices.\(^{19}\) The maximization problem is given by

\[
\max_{p_0, p_1, p_2 \geq 0} \pi^{FC} = p_0D_0 + p_1D_1 + p_2D_2.
\]

We impose the following constraints: First, the market is always covered and each firm obtains a strictly positive market share. Second, all consumers realize a nonnegative utility when buying one of the offered products. In the Appendix we show that these constraints pin down the equilibrium under full collusion. The branded firms set the same price (they are both symmetric) such that the indifferent consumer located at \( x = 1/2 \) gets a utility of zero. The private label firm then sets a price such that the indifferent consumers located at \( x = 1/6 \) and \( x = 5/6 \) obtain also a utility of zero and are therefore indifferent between buying the private label good or the next branded good. In addition, we also derive the optimal deviation prices where we impose that the maximal market share of the deviating firm is less than 100 per cent. The following proposition states the fully collusive prices and profits as well as the deviation prices and deviation profits.

**Proposition 2.** Consider collusion by all three firms. We obtain the following equilibrium values:

i) Prices: \( p_1^{FC} = 1 - t/6 \), for \( i = 1, 2 \), and \( p_0^{FC} = s - t/6 \), so that \( p_0^{FC} \leq p_1^{FC} \) holds (with equality holding at \( s = 1 \)).

ii) Profits: \( \pi_i^{FC} = 1/3 - t/18 \), for \( i = 1, 2 \) and \( \pi_0^{FC} = s/3 - t/18 \).

\(^{19}\)We follow Donsimoni (1985) and Athey and Bagwell (2001) who select the collusive outcome which maximizes joint profits (see Bos and Harrington, 2010, and Thomadsen and Rhee, 2007, for discussions of this issue and for related literature).
iii) Demands: Firm 0 serves consumers located at \([0, 1/6) \cup (5/6, 1]\). Firms 1 and 2 serve consumers located at \((1/6, 1/2)\) and at \((1/2, 5/6, 1)\), respectively.

iv) Deviation by firm \(i\), \(i = 1, 2\): The deviation prices and profits are \(p_{i}^{D, FC} = t/12 + 1/2\) and \(\pi_{i}^{D, FC} = (t + 6)^{2} / (144t)\), respectively.

v) Deviation by firm 0: The deviation price and profits are \(p_{0}^{D, FC} = t/12 + s/2\) and \(\pi_{0}^{D, FC} = \frac{(6s + t)^{2}}{(144t)}\), respectively.

Proof. See Appendix.

Parts i) and ii) of Proposition 2 state the collusive prices and profits when all three firms collude. The firms charge higher prices than in the noncooperative case. According to part iii) of Proposition 2, each firm’s market share is \(1/3\). It also implies that the demand for the private label good increases compared to the noncooperative case, while the market share of the brands is reduced accordingly.

Parts iv) and v) give the deviation prices and profits of the firms. The deviating firm undercuts its rivals by setting a lower price; it then obtains a higher market share and a higher profit. The number of consumers served depends on the transportation cost parameter. We assume that the transportation cost is large enough, so that the deviating firm obtains a market share of less than 100 per cent (which is ensured by \(t \geq 6/11\); see Assumption 1).

3.2 Partial Collusion

In the case of partial collusion (PC), firms 1 and 2 collude, while firm 0 behaves noncooperatively. Let \(p_{i}^{PC}\) denote the price of firm \(i\), for \(i = 1, 2\), and let \(p_{0}^{PC}\) be the noncooperative price set by firm 0 under partial collusion. The colluding brands maximize their joint profit. Their maximization problem is given by

\[
\max_{p_{1}, p_{2} \geq 0} \pi^{PC} = p_{1}D_{1} + p_{2}D_{2},
\]

while the maximization problem of firm 0 is

\[
\max_{p_{0} \geq 0} \pi_{0} = p_{0}D_{0}.
\]

Solving the maximization problems gives rise to a set of first-order conditions which determine the equilibrium outcome.
Proposition 3. Consider partial collusion between firms 1 and 2, while firm 0 behaves noncooperatively. We obtain the following equilibrium values:

i) Prices: \( p_i^{PC} = 5t/9 + (1-s)/3 \), for \( i = 1, 2 \), and \( p_0^{PC} = 4t/9 - (1-s)/3 \), so that \( p_0^{PC} < p_i^{PC} \) holds always.

ii) Profits: \( \pi_0^{PC} = (4t - 3(1-s))^2 / (81t) \) and \( \pi_i^{PC} = (5t + 3(1-s))^2 / (162t) \), for \( i = 1, 2 \).

iii) Demands: Firm 0 serves consumers located at \([0, x_0^{PC}] \cup (x_2^{PC}, 1] \), where \( x_0^{PC} > x_0^N \) and \( x_2^{PC} < x_2^N \). Firm 1 serves consumers located at \((x_0^{PC}, 1/2) \) and firm 2 at \((1/2, x_2^{PC}) \), respectively. Moreover, \( x_0^{PC} = (4t - 3(1-s)) / (18t) \), \( x_1^{PC} = 1/2 \), and \( x_2^{PC} = (14t + 3(1-s)) / (18t) \).

iv) If firm \( i, i = 1, 2 \), deviates from partial collusion, then its price and profits are \( p_i^{D,PC} = (1-s)/4 + 5t/12 \) and \( \pi_i^{D,PC} = ((1-s)/4 + 5t/12)^2 / t \), respectively.

Proof. See Appendix.

Part iii) of Proposition 3 says when the two brands collude, they reduce their market shares and serve less consumers compared to the noncooperative case. Part iv) of Proposition 3 states that a brand could deviate from the collusive agreement by charging a lower price than those set by the rival firms. Such a deviation increases its profits.

3.3 Comparison of Results

By comparing the results derived so far, we can order firms’ prices, demands and profits under the three different types of conduct (noncooperative, partially collusive, and fully collusive).

Corollary 1. By comparing the equilibrium prices under noncooperation, full and partial collusion, we get: \( p_j^{FC} > p_j^{PC} > p_j^N \), for \( j = 0, 1, 2 \).

Proof. Follows directly from comparing the equilibrium prices as stated in Propositions 1-3.

Corollary 1 states that prices are increasing when firms’ conduct becomes more collusive. Prices are maximal when there is full collusion. They remain higher under partial collusion than under noncooperative behavior. Combining the latter observation with the fact that the private label firm sets a lower price than the branded goods producers in case of a partial cartel (see Proposition 3), we get that the brands’ prices serve as an umbrella such that the private label’s price increases above the fully noncooperative price.\(^{20}\)

\(^{20}\)In the EU, umbrella effects are potentially becoming more important for the assessment of the harm created...
Corollary 2. By comparing the equilibrium demands under noncooperation, full and partial collusion, we get the following orderings:

i) $D_0^{FC} > D_0^N$, $D_0^{PC} > D_0^N$ and $D_0^{PC} > D_0^{FC}$ for $s > 1 - t/3$.

ii) $D_i^N > D_i^{FC}$, $D_i^N > D_i^{PC}$ and $D_i^{FC} > D_i^{PC}$, for $s > 1 - t/3$ with $i = 1, 2$.

iii) $D_0^{FC} = D_1^{FC} = D_2^{FC} = 1/3$.

Proof. Follows directly from calculating firms’ demands (7) by using the locations of the indifferent consumers as stated in Propositions 1-3.

By comparing the equilibrium demands, we notice that the brands serve the highest share of the market in the noncooperative case. Each brand serves more than one third of the market. Under full collusion all firms share the demand equally. Under partial collusion, brands charge a higher price than under full collusion and serve less consumers; thus, both firms’ market shares become smaller than one third.

Corollary 3. By comparing the equilibrium profits under noncooperation, full and partial collusion, we get the following orderings:

i) $\pi_i^{FC} > \pi_i^{PC} > \pi_i^N$, for $i = 1, 2$.

ii) $\pi_0^{FC} > \pi_0^N$ and $\pi_0^{PC} > \pi_0^N$ hold always.

iii) $\pi_0^{FC} > \pi_0^{PC}$ if $s > s^*(t) := \left[ t - 3\sqrt{3}\sqrt{t(4-3t)} \right]/6 + 1$ and $\pi_0^{FC} < \pi_0^{PC}$ if $s < s^*(t)$ (with equality holding at $s = s^*(t)$). Moreover, $\partial s^*(t)/\partial t > 0$.

Proof. See Appendix.

Corollary 3 states that firms’ profits are always higher under collusion (both partial and full collusion) compared to the profits under noncooperative behavior. Full collusion always leads to higher profits than partial collusion for the brand producers, $i = 1, 2$, which is not the case for the private label firm. In fact, the private label firm can realize higher profits under partial collusion than under full collusion. This observation is important for the stability of full and partial collusion, respectively.\footnote{This result is related to Donsimoni (1985) who analyzed cartel stability when firms differ in costs (but produce}

\footnote{by a cartel in private law suits. According to the new EU Damages Directive (see EU, 2014) members of a cartel can be held responsible for higher prices independently charged by firms competing with cartel members. Umbrella pricing then refers to a market outcome, where independent firms increased their prices in response to the cartel’s price increases.}
to join the brand manufacturers for a full collusion outcome, given that the brands keep their collusive conduct (i.e., partial collusion is realized).

The reason is that the market share of the private label firm always increases under partial collusion when compared with its market share under full collusion. In the former case, the private label’s market share on the segment \( x \in [0, 1/3] \) is \( x_{0}^{PC} = 1/6 \) and in the latter case the private label’s market share is \( x_{0}^{FC} = (4t - 3(1 - s))/(18t) \). We then get \( x_{0}^{PC} > x_{0}^{FC} \) if \( s > 1 - t/3 \) (see Corollary 2). Note also that \( \partial(x_{0}^{PC} - x_{0}^{FC})/\partial t = (1 - s)/(6t^{2}) > 0 \) holds, so that the difference of the market shares is increasing in \( t \). Accordingly, from part \( iii) \) of Corollary 3, we can infer that \( \pi_{0}^{PC} > \pi_{0}^{FC} \) only becomes feasible when \( t > 6/5 \) holds. It means that for the profit of the private label being larger under partial collusion than under full collusion, the increase in the market share must be large enough to compensate for the price decrease. In line with this observation, part \( iii) \) of Corollary 3 also states that the critical value \( s^{*}(t) \) is increasing in \( t \). This means that the range of the quality parameter \( s \) for which partial collusion is preferred by the private label firm increases in \( t \). Thus, everything else equal, a reduced competitive intensity (high value of \( t \)) makes it more likely that the private label firm prefers partial collusion over full collusion.

**Corollary 4.** Comparing the optimal deviation prices and profits under full and partial collusion, we get the following orderings:

1. \( \pi_{j}^{D,PC} > \pi_{j}^{FC} \) and \( p_{j}^{D,PC} < p_{j}^{FC} \) with \( j = 0, 1, 2 \).
2. \( \pi_{j}^{D,FC} > \pi_{j}^{PC} \) and \( p_{j}^{D,FC} < p_{j}^{PC} \) with \( j = 0, 1, 2 \).

**Proof.** Follows directly from comparing the respective values as stated in Propositions 2-3.

Corollary 4 states that firms always deviate by charging a lower price to earn a higher profit. This result also implies that firms’ critical discount factor (3) is always in the range between zero and one.

### 3.4 Stability Analysis

Collusion is sustainable if the discount factor (3) is large enough. Full collusion is stable if \( \delta \geq \delta_{j}^{FC} \) holds for all \( j = 0, 1, 2 \). Partial collusion is stable, whenever \( \delta \geq \delta_{i}^{FC} \) holds for all \( a \) homogenous good). He showed that the most efficient firms always join the cartel while the less efficient firms could stay outside.
The critical discount factor of the brand producers \(i = 1, 2\) under full collusion is given by

\[
\delta_i^{FC} = \frac{\pi_i^{D,FC} - \pi_i^{FC}}{\pi_i^{D,FC} - \pi_i^N} = \frac{75 (2 - t)^2}{-48s^2 + 160st + 96s - 125t^2 - 60t + 252},
\]

where \(\partial \delta_i^{FC} / \partial s < 0\) and \(\partial \delta_i^{FC} / \partial t < 0\). Thus, the critical discount factor \(\delta_i^{FC}\) is reduced (impling that full collusion is easier to sustain) when the transportation cost parameter and/or the quality parameter of the private label are increasing. Intuitively, when \(s\) increases the profit in the noncooperative stage game (see part \(ii\) of Proposition 1) is reduced which lowers the expected profit of deviation. The noncooperative stage game profit is realized in the punishment phase after the deviation period. At the same time, both the fully collusive profit and the profit in the deviation stage are independent of private label’s quality. Thus a higher quality of the private label makes full collusion easier to sustain (lower value of \(\delta_i^{FC}\)). Increasing horizontal product differentiation increases the likelihood of full collusion as in Ross (1992).

The critical discount factor of firm the private label firm under full collusion is given by

\[
\delta_0^{FC} = \frac{\pi_0^{D,FC} - \pi_0^{FC}}{\pi_0^{D,FC} - \pi_0^N} = \frac{75 (2s - t)^2}{108s^2 - 220st + 384s - 125t^2 + 320t - 192},
\]

where it can be shown that \(\partial \delta_0^{FC} / \partial s > 0\) and \(\partial \delta_0^{FC} / \partial t < 0\). The former derivative says that the private label firm’s incentive to collude is reduced when \(s\) increases. This is due to the fact that in the punishment phase the profit of the private label good increases the smaller the quality gap becomes, so that the expected profit in the punishment phase increases in \(s\). As for the brands, the incentive constraint of the private label firm is more likely to be fulfilled when the transportation cost parameter increases (i.e., horizontal product differentiation is high).

The critical discount factor of the brand producers \(i = 1, 2\) under partial collusion is given by

\[
\delta_i^{PC} = \frac{\pi_i^{D,PC} - \pi_i^{PC}}{\pi_i^{D,PC} - \pi_i^N} = \frac{25}{81},
\]

so that the stability of partial collusion among the brand producers neither depends on the intensity of competition \(t\) nor on the private label’s quality \(s\). Thus partial collusion between the branded products is immune against changes of the quality of the private label good and changing intensities of competition.\(^{22}\) We summarize the comparison and properties of the

\(^{22}\) This may explain the relative stability of recently detected partial cartels among branded manufacturers in Germany as mentioned in the Introduction.
critical discount factors as follows.

**Proposition 4.** The orderings of the critical discount factors are as follows:

i) \( \delta^F_0 < \delta^F_i \) holds always (equality holding at \( s = 1 \)).

ii) \( \delta^F_i < \delta^P_i \) (\( \delta^F_i > \delta^P_i \)) holds if \( s > \bar{s}(t) \) (\( s < \bar{s}(t) \)), with equality holding at \( s = \bar{s}(t) \), where \( \bar{s}(t) := 5t/3 - (\sqrt{201t - 44t^2 - 126})/3 + 1 \). Moreover, \( \partial \bar{s}(t)/\partial t < 0 \) and \( \partial^2 \bar{s}(t)/\partial t^2 > 0 \).

Furthermore, \( \partial \delta^F_i / \partial s < 0 \), \( \partial \delta^F_i / \partial t < 0 \), \( \partial \delta^F_0 / \partial s > 0 \), and \( \partial \delta^F_0 / \partial t < 0 \) hold always.

Finally, \( \partial \delta^P_i / \partial s = \partial \delta^P_0 / \partial t = 0 \).

**Proof.** See Appendix.

Part i) of Proposition 4 states that the brand producers are critical for sustaining full collusion.23 If the discount factor is high enough such that the brand producers collude, then the private label firm’s incentive constraint is also fulfilled (in the special case \( s = 1 \), all firms have the same quality levels, so that their incentive conditions are the same as well). Part ii) shows that either full collusion or partial collusion is easier to sustain. There exists a critical value \( \bar{s}(t) \) such that for \( s > \bar{s}(t) \) full collusion is easier to sustain than partial collusion. If this condition is reversed, then the opposite holds with partial collusion being easier to sustain than full collusion.

The function \( \bar{s}(t) \) consists of all pairs \((t, s)\) at which the critical discount factors under full and partial collusion are the same. This function is convex and negatively sloped in a \((t, s)\) diagram over the feasible set. This means that relatively large values of \( s \) and \( t \) make it more likely that full collusion is easier to sustain than partial collusion. Or put differently, partial collusion is easier to sustain than full collusion when the values of \( s \) and \( t \) are relatively low.24

Figure 1 illustrates the critical discount factors as functions of \( s \) when we set the parameter value \( t = 1 \). The upward sloping curve represents the critical discount factor of the private label firm (thin line) which is never binding for \( s < 1 \). If the incentive constraint for full collusion is fulfilled for the branded firms, then it is also fulfilled for the private label firm. Comparison of

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23 This result is also obtained in Häckner (1994) who shows that it is always the high quality firm which has the largest deviation incentives.

24 If we allow for prices under full collusion which take care of the incentive constraint of the brand producers, then the critical discount factor can be reduced for the brands (see Bos and Harrington, 2010). This would mean that the private label had to increase its price to shift revenues to the branded firms. However, such an optimization problem is also constrained by the private label firm’s participation constrained as given by part iii) of Corollary 3.
the critical discount factors of the brand producers under full collusion (dashed line) and under partial collusion (bold line) shows that full collusion is easier to sustain than partial collusion when \( s \) is larger than the threshold value \( \bar{s} \), (which is reached at the intersection of both curves), while the opposite holds for lower values of \( s \). Thus, full collusion is, ceteris paribus, easier to sustain than partial collusion if the quality gap of the private label good is not too large.

From Corollary 3 (which states firms’ profit levels under the three types of conduct) we know that the brand producers always prefer full collusion over partial collusion, while both types of collusion are preferred over noncooperative behavior. The private label firm also realizes the lowest profit level under noncooperative behavior. In contrast to the brand producers, the private label firm, however, may prefer partial collusion over full collusion. Assuming that the firms select the type of conduct which maximizes their profits and taking into account the feasibility of collusion, we get the following market conduct depending on firms’ discount factors (the same stability criterion is applied in Bos and Harrington, 2010).

**Proposition 5.** Depending on the discount factor \( \delta \), firms market conduct is as follows:

i) If \( \delta > \max\{\delta_i^{FC}, \delta_i^{PC}\} \), then market conduct is FC if \( s > s^* \), whereas it is PC if \( s < s^* \).

ii) If \( \delta_i^{FC} > \delta > \delta_i^{PC} \), then market conduct is PC.

iii) If \( \delta_i^{PC} > \delta > \delta_i^{FC} \), then market conduct is FC.
iv) If $\delta < \min\{\delta^FC_i, s^{PC}_i\}$, then market conduct is $N$.

**Proof.** Follows from combining the results of Proposition 5 with part iii) of Corollary 3.

Part i) of Proposition 5 refers to the case, where firms’ discount factor is sufficiently large to make both full collusion and partial collusion stable. In this area, the private label firm’s preference for either type of collusion determines the market conduct. From Corollary 3, part iii), we know that the private label firm prefers partial collusion if $s > s^*$, whereas the opposite holds for $s < s^*$. The type of conduct then follows immediately from the private label firm’s preference. Interestingly, we notice that the private label firm prefers partial collusion over full collusion in the parameter area, where full collusion is easier to sustain than partial collusion from the brand producers’ perspective. A prediction of the likely market conduct only based on the critical discount factors would be misleading in those instances. One has to consider the incentives of the private label firm to join the brand manufacturers in their collusive conduct or to behave noncooperatively. Inspection of the critical value $s^*$ yields that full collusion is always the outcome for low values of $t$ (precisely, $t < 6/5$), while in the remaining parameter area partial collusion is preferred by the private label firm only if $s < s^*(t)$. In that area it holds that intense competition (low value of $t$) makes, *ceteris paribus*, full collusion more likely (given that both types of collusion are feasible). In the same way, it follows from the shape of $s^*$, that low values of $s$ tend to make partial collusion more likely, while for large values of $s$ it becomes less likely.

Part ii) deals with cases where partial collusion is easier to sustain than full collusion. If firm’s discount factor allows only for partial collusion, it will also be the market conduct chosen by the firms. Part iii) refers to the opposite case, where full collusion is easier to sustain than partial collusion, while only the former is feasible. Clearly, in this case full collusion is the type of conduct selected by the firms.

Finally, part iv) gives the case where neither type of collusion is incentive compatible, so that noncooperative behavior is the market conduct. A noncooperative outcome is more likely the larger the quality gap and/or the more intense competition. This follows directly from $\partial \delta^FC_i / \partial s < 0$ and $\partial \delta^FC_i / \partial t < 0$, so that an increasing quality of the private label good and reduced competition make full collusion easier to sustain. The former relation is in line with Steiner’s (2004) observation that private labels’ quality has been increasing, while signs of col-
lusion between brands and private label goods have also emerged more recently.

Figure 2 illustrates our results presented in Proposition 5. The thin lines represent the constraints of the feasible set as specified in Assumption 1. The upward sloping dashed curve is the locus of all pairs \((t, s)\) such that the private label firm is indifferent between partial and full collusion. It represents the critical value \(s^*(t)\) as specified in part \(iii)\) of Corollary 3. The private label firm gets a higher (lower) profit under full collusion than under partial collusion northeast (southwest) of the dashed curve. Inspection of the dashed curve \(s^*(t)\) yields that \(t\) must pass a minimal value (namely, \(\tilde{t} = 6/5\) where \(s(\tilde{t}) = s^*(\tilde{t})\) holds), so that partial collusion can be more attractive than full collusion for the private label firm. Only if the intensity of competition is sufficiently low (\(t > 6/5\)) partial collusion can be preferred by the private label firm. In that range, however, the private label’s quality must not surpass the critical value \(s^*(t)\) (dashed curve), to get partial collusion instead of full collusion (while assuming that both are feasible).

![Figure 2: colored lines: feasible set; bold curve: \(s(t)\); dashed curve: \(s^*(t)\)](image)

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25The colored lines describe the feasible set as specified in Assumption 1. The red line represents \(s \leq 1\), the blue line \(s \geq 1 - 5t/6\), the green line \(s \geq t/2\), and the yellow line \(s \geq 13t/6 - 2\). Note also that \(t = t_{\min} = 6/11\) holds at the origin of the graph. Of course, the following discussion of Figure 2 only refers to the range of parameters within the feasible set.
The downward sloping bold curve in Figure 2 depicts the critical value \( \bar{s}(t) \) which is the locus of all \((t, s)\) pairs where the critical discount factors of the brand producers are equal under full and partial collusion. Full collusion is easier (harder) to sustain than partial collusion for all \((t, s)\) pairs northeast (southwest) of the bold curve. Interestingly, exactly when full collusion is easier to sustain than partial collusion (i.e., we are in the area northeast of the bold curve), then it can happen that the private label firm prefers partial collusion over full collusion.

The intensity of competition is measured by the parameter \( t \). If the intensity of competition is large (low value of \( t \)), then full collusion becomes harder to sustain, so that partial collusion is more likely. In contrast to this observation (which relies on the brand producers’ incentives to engage in full or partial collusion), the private label producer tends to prefer partial collusion over full collusion when the intensity of competition is reduced (high value of \( t \)). With regard to vertical quality differentiation (parameter \( s \)), both the incentives of the brand manufacturers and the private label producer are more in line. An increase of the quality of the private label makes it more likely that the incentive constraint of the brand producers is fulfilled and that the private label producer prefers full collusion over partial collusion.

4 Extensions and Discussion

In this section we analyze the private label firm’s incentives to improve its quality. We also show how our results can be used to detect collusive conduct from market data. This argument is based on the markedly different market responses to an improvement of the private label’s quality under the three types of market conduct (noncooperative, partially or fully collusive).

**Strategic choice of private label quality.** The incentive of the private label producer to close the quality gap, \( 1 - s \), between the private label’s and the brands’ qualities critically depends on the type of conduct. Assume an initial decision knot, where the private label’s quality is set before the infinitely repeated market game starts. Suppose that the type of market conduct is fixed being either \( N \), \( PC \) or \( FC \). Taking the total derivative of the private label producer’s expected flow of profits (around the equilibrium values \( p_0, p_1, \) and \( p_2 \) under the three different conduct regimes) with respect to the quality parameter \( s \) yields

\[
\frac{d\pi_0}{ds} = \frac{\partial \pi_0}{\partial s} + \frac{\partial \pi_0}{\partial p_0} \frac{dp_0}{ds} + \sum_{i=1}^{2} \frac{\partial \pi_0}{\partial p_i} \frac{dp_i}{ds},
\]  

(11)
where we canceled out the factor $1/(1 - \delta)$. The first term of the right-hand side of (11) is the direct effect of a change in $s$ on the private label firm’s profit which is positive but different depending on the type of market conduct. The second term on the right-hand side is zero because of the envelope theorem. The third term on the right-hand side represents the strategic effect of the quality investment. It is noteworthy that it is negative under $N$ and $PC$ (because both $dp_i^N/ds$ and $dp_i^{PC}/ds$ are strictly negative), but disappears under $FC$ (because of $dp_i^{FC}/ds = 0$). 

In the parlance of Fudenberg and Tirole (1984), there exists an investment reducing strategic effect under $N$ and $PC$ \textit{(puppy dog ploy)}, while there is no such effect present under $FC$. Not surprisingly, marginal investment incentives are largest under $FC$, what can be derived from calculating the total derivatives of (11) under the three different types of market conduct.

**Proposition 6.** The private label firm’s marginal incentives to close the quality gap $1 - s$ are largest under $FC$ with $PC$ and $N$ following in that order; i.e., $d\pi_0^{FC}/ds > d\pi_0^{PC}/ds > d\pi_0^N/ds > 0$ hold always.

**Proof.** See Appendix.

From Proposition 4 and Corollary 3, we know that an increase of the quality of the private label makes full collusion more likely because of the following two reasons. First, a higher value of $s$ makes it easier for brand producers to sustain full collusion; i.e., $\partial \delta_i^{FC}/\partial s < 0$. As $\partial \delta_i^{PC}/\partial s = 0$ holds, full collusion becomes relatively easier to sustain than partial collusion the larger $s$ becomes (see Proposition 4). Second, a higher value of $s$ makes full collusion more attractive than partial collusion for the private label firm (part iii) of Corollary 3).

Suppose now a generic change in $s$, say from $s_1$ to $s_2$, with $s_1 < s_2$ such that this increase in the private label’s quality changes market conduct from noncooperative ($N$) or partially collusive ($PC$) to fully collusive ($FC$). Investment incentives, which are given by the profit differential $\pi_0^{FC}(s_2) - \pi_0^{C}(s_1)$ (with $C = FC, PC, N$) are then driven only by the replacement effect and the following result follows immediately.\(^\text{26}\)

**Proposition 7.** Suppose a generic increase of the private label’s quality from $s_1$ to $s_2$ with $s_1 < s_2$ which changes market conduct from $N$ or $PC$ to $FC$. The private label firm’s incentives

\(^{26}\text{Note that a change from } N \text{ to } PC \text{ is not possible through an increase of } s \text{ as } \delta_i^{PC} \text{ does not depend on } s.\)
to close the quality gap \(1 - s\) are then ordered as follows:

\[
\pi_0^{FC}(s_2) - \pi_0^N(s_1) > \pi_0^{FC}(s_2) - \pi_0^{PC}(s_1) > \pi_0^{FC}(s_2) - \pi_0^{FC}(s_1) > 0.
\]

**Proof.** Follows directly from \(\pi_0^{FC} > \pi_0^{PC} > \pi_0^N\).

Proposition 7 follows from the ordering of the private label firm’s profit under the three different types of market conduct. The private label producer has larger investment incentives the lower the degree of collusion, because the net gain from a quality increase is higher when competition is more intense initially.\(^{27}\)

These observations have two implications: *First*, an increase in the quality of the private label may be strategic so as to obtain a full collusion outcome as the type of market conduct chosen by the firms. *Second*, an increase in the quality of the private label may have significant adverse effects for consumers, whenever it is used to trigger a full collusion outcome. In fact, incentives to close the quality gap between the private label good and the brands are maximal whenever noncooperative behavior would prevail without any investment. Such a constellation is obtained if \(\delta_i^{FC}(s_1) > \delta > \delta_i^{FC}(s_2)\) holds.

The pro-collusive effect of a higher quality of the private label good sheds new light on the possible market effects of so-called premium private label goods. While the trend of an increasing quality of private label goods has been generally interpreted as pro-competitive, our investigation highlights their role in stabilizing full collusion between private label and brand producers.

**Identifying collusive conduct.** The following Table 1 describes the change in the prices of the brands and the private label depending on an improvement of the private label’s quality \(s\). Rows 2-4 relate to cases where the type of conduct is fixed as being noncooperative, partially collusive or fully collusive, respectively. The fifth row refers to those instances where initially conduct is either noncooperative or partially collusive, while a quality improvement of the private label induces a fully collusive outcome.

\(^{27}\)A qualitatively similar result is stated in Aubert, Rey, and Kovacic (2006) where a drastic innovation is considered.
<table>
<thead>
<tr>
<th>Conduct</th>
<th>Brands’ prices</th>
<th>Private label’s price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>↓ (by $-\frac{1}{5}$)</td>
<td>↑ (by $\frac{2}{5}$)</td>
</tr>
<tr>
<td>$PC$</td>
<td>↓ (by $-\frac{1}{3}$)</td>
<td>↑ (by $\frac{1}{3}$)</td>
</tr>
<tr>
<td>$FC$</td>
<td>no change</td>
<td>↑</td>
</tr>
<tr>
<td>switch to $FC$</td>
<td>↑</td>
<td>↑</td>
</tr>
</tbody>
</table>

Table 1: Effects of an increase in $s$ on brands’ and private label’s prices

Two observations are noteworthy: *First*, given that the type of conduct is fixed at $N$, $PC$ or $FC$, the private label’s price is the more quality-sensitive, the more collusive the market conduct becomes (Table 1 states in the third column the sign of the own-price effect and the marginal effect is given in brackets). If the type of conduct changes through an increase in $s$ to full collusion (see last row of Table 1), then the change in the private label’s price is a discrete jump upward. *Second*, full collusion can be inferred from the absence of a negative sensitivity of the brands’ prices with respect to the private label’s quality. According to Table 1, the brands’ prices stay put if full collusion prevails. The price effect is even positive for the branded goods if a higher private label quality leads to fully collusive conduct.\(^{28}\)

Incidentally, Ward et al. (2002) showed in their empirical analysis of scanner data (obtained at cash registers) from US grocery stores that an increasing market share of private labels tends to increase the brands’ prices. While this can be explained by same static theories of product differentiation and vertical relations, our model suggests that such an outcome can also result from a combination of an increasing private label quality and collusive conduct.

The relations stated in Table 1 suggest two empirical strategies to identify collusion in markets where brands and private labels compete. From the first observation it follows that the private label’s own price response to a quality improvement is the larger the more collusive industry conduct becomes. The second observation suggests that fully collusive behavior can be inferred from a nonnegative price effect of the brands resulting from an increase of the private label’s quality.

\(^{28}\)See Gabrielsen and Sörgard (2007) for a vertical restraint theory which also shows that branded suppliers could increase their prices in the presence of a private label (particularly of poor perceived quality). See also Gilo (2008) for a survey of vertical restraints leading to a cartel outcome between retailer controlled private labels and branded goods.
5 Conclusion

We have analyzed collusion between brands and a private label substitute. We focused on heterogeneity of firms due to product differentiation; both horizontal and vertical. We assumed that nonparticipation of the private label producer in a cartel does not necessarily lead to fully noncooperative behavior, but may induce the brand manufacturers to form a partial cartel. In fact, forming such a partial cartel is always optimal for the brand producers when being incentive compatible. Given that both partial and full collusion are feasible, the private label firm is more likely to revert to noncooperative behavior (with a partial cartel following), if horizontal and/or vertical differentiation is sufficiently large. Thus, focusing on the private label’s incentive to participate in a full cartel, we get that this is more likely whenever product differentiation (vertical and/or horizontal) is low enough.

Interestingly, this picture is different when considering the brand producers’ incentive conditions. Then a higher degree of horizontal product differentiation works in favor of full collusion, while the quality gap of the private label must not be too large. Thus, comparing the brand producers’ incentive constraints gives the result, that full collusion is more likely when horizontal product differentiation is large but vertical differentiation is low. Taking both results together, we find that exactly in the parameter range, where full collusion is easier to sustain than partial collusion (from brands’ perspective), it could happen that the private label producer opts for a partial collusion outcome by behaving noncooperatively. For this to happen, horizontal and vertical differentiation must be sufficiently large.

The partial collusion case has two characteristic features which merit mentioning. First, the stability conditions of the brand manufacturers are independent of the degree of product differentiation (both horizontal and vertical). Second, the private label firm can increase its price under partial collusion above the equilibrium price under fully noncooperative conduct. The first observation can be used to explain the remarkable stability of cartels among brand manufacturers even in an environment in which the degree of competition and the private label’s quality change over time. The second result is potentially important for the assessment of the harm created by a cartel that only involves explicit collusion among the brand manufacturers. As private law suits also allow for damages created by a cartel’s umbrella effects on outsiders’ prices the question emerges whether or not the outsiders did join the explicit cartel by colluding tacitly (so
that in fact a full cartel was in operation) or whether the outsiders behaved noncooperatively (in which case the cartel was partial). In both instances, the outsiders increase their prices, however, by different amounts.

We have also analyzed the private label’s incentive to increase its quality. We showed that it is maximal under full collusion with partial collusion and noncooperative behavior following in that order. In addition, a quality increase makes full collusion more likely because of two reasons: First, it relaxes the brand producers’ incentive constraint for full collusion. Second, it makes it more likely that the private label firm prefers full collusion over partial collusion. The latter observation has also implications for the competitive assessment of private label goods. As long as private label goods were of the budget type, private label producers had only little incentives to join into a full cartel, while branded firms found it also easier to sustain a partial cartel. This may have changed as private labels’ quality increased over time. As the quality gap becomes smaller, private label firms and branded producers should have found it more attractive to form an all encompassing cartel.

We have also shown that the price responses associated with a quality improvement of the private label good can give important information for detecting cartelization. First, if a quality increase does not induce a negative effect on brand producers’ prices (everything else equal), then either full collusion is present or the market is triggered into fully collusive conduct. Second, the larger the private label’s own-price effect of a quality increase, the more likely it is that collusive conduct exists in the market.

Appendix

In this Appendix we present the missing proofs.

Proof of Proposition 1. Using firms’ demand functions (7), we can write the firms’ profits as

$$
\pi_0 = D_0 p_0 = (x_0 + 1 - x_2)p_0, \quad (12)
$$

$$
\pi_1 = D_1 p_1 = (x_1 - x_0)p_1, \quad \text{and} \quad (13)
$$

$$
\pi_2 = D_2 p_2 = (x_2 - x_1)p_2. \quad (14)
$$

Substituting the values of the indifferent consumers (4), (5), and (6) into these expressions, we
$$\pi_0 = p_0 \left( 6s + 2t - 6p_0 + 3p_1 + 3p_2 - 6 \right) / (6t),$$
$$\pi_1 = p_1 \left( 2t - 3s + 3p_0 - 6p_1 + 3p_2 + 3 \right) / (6t),$$
$$\pi_2 = p_2 \left( 2t - 3s + 3p_0 + 3p_1 - 6p_2 + 3 \right) / (6t).$$

Maximization of each firm’s profit gives the following system of first-order conditions:

$$(6s + 2t - 12p_0 + 3p_1 + 3p_2 - 6) / (6t) = 0,$$
$$(2t - 3s + 3p_0 - 12p_1 + 3p_2 + 3) / (6t) = 0,$$
$$(2t - 3s + 3p_0 + 3p_1 - 12p_2 + 3) / (6t) = 0.$$

All profit functions are strictly concave, so that this system of first-order conditions determines the unique equilibrium outcome. Solving for the prices we get the following equilibrium values as stated in part i) of the proposition:

$$p_0^N = (6(s - 1) + 5t) / 15 \text{ and }$$
$$p_i^N = (3(1 - s) + 5t) / 15 \text{ for } i = 1, 2.$$

Substituting the equilibrium prices into (4), (5), and (6), we get the equilibrium locations of the indifferent consumers (see part iii) of the proposition)

$$x_0^N = 1/6 - (1 - s) / (5t),$$
$$x_1^N = 1/2, \text{ and }$$
$$x_2^N = 5/6 + (1 - s) / (5t).$$

Note that $$x_0^N \in (0; 1/3)$$ and $$x_2^N \in (2/3; 1)$$ must hold in an interior solution. This is true for $$s > 1 - 5t/6$$ which is implied by Assumption 1. Substituting the equilibrium prices and locations of the indifferent consumers into the profit functions (12), (13), and (14) yields

$$\pi_0^N = (6(s - 1) + 5t)^2 / (225t) \text{ and }$$
$$\pi_1^N = \pi_2^N = (5t + 3(1 - s))^2 / (225t),$$

which are stated in part ii) of the proposition. We finally check whether the utilities of the
indifferent consumers (15), (16), and (17) are nonnegative. We obtain

\[ U_{x_0} = U_{x_2} = (2s + 3)/5 - t/2 \]
\[ U_{x_1} = (s + 4)/5 - t/2. \]

Setting \( U_{x_0}, U_{x_1}, U_{x_2} > 0 \), we get the condition \( s > 5t/4 - 3/2 \), which holds by Assumption 1.

**Proof of Proposition 2.** Consider collusion by all three firms. The three firms maximize their joint profit

\[ \max_{p_0^{FC}, p_1^{FC}, p_2^{FC} \geq 0} \pi^{FC} = p_0 D_0 + p_1 D_1 + p_2 D_2 \]

subject to consumers’ reservation utilities (which must be nonnegative). Substituting the demand functions into \( \pi^{FC} \) and differentiating with respect to the prices, we get the following system of first-order conditions

\[ (3s + t - 6p_0 + 3p_1 + 3p_2 - 3)/(3t) = 0 \text{ and } \]
\[ (2t - 3s + 6p_0 - 12p_i + 6p_{i'} + 3)/(6t) = 0, \text{ for } i, i' = 1, 2, i \neq i'. \]

This system of first-order conditions is only fulfilled at \( t = 0 \). Hence, there cannot exist an interior solution to the maximization problem. We next show that Assumption 1 ensures that the joint profit is maximized in the corner solution where all indifferent consumers get a utility of zero, while the branded goods prices \( p_1 \) and \( p_2 \) are the same and the market is fully covered.

Suppose that all indifferent consumers get a utility of zero; i.e., \( U_{x_0} = U_{x_1} = U_{x_2} = 0 \) holds.

Firms 1 and 2 are symmetric, hence, the location of the indifferent consumer is \( x_1 = 1/2 \) with \( p_1 = p_2 \). Substituting these values into \( U_{x_1=1/2} = 0 \), we get the following equilibrium prices of the brands under full collusion

\[ p_1^{FC} = p_2^{FC} = 1 - t/6. \]  \hspace{1cm} (18)

We assume that the market is always covered. Therefore, the utility of the indifferent consumer on the segments \( (0, 1/3) \) and \( (2/3, 1) \) must be zero, \( U_{x_0} = U_{x_2} = 0 \). By substituting (4) and the collusive prices of the brands (18) into the utility functions we get the price for the private label good from the indifferent consumers:

\[ p_0^{FC} = s - t/6. \]  \hspace{1cm} (19)
Given the prices of the brands (18), we still have to check whether it is indeed optimal to set the price $p^\text{FC}_0 = s - t/6$ which ensures that the market is fully covered. Note first that the joint profit can never increase with a lower price for the private label, because $p^\text{FC}_0 < p^\text{FC}_i$ for $i = 1, 2$. However, we still have to ensure that there is no incentive to set a higher price for the private label than $p^\text{FC}_0$. If this were optimal, the market would not be covered. In other words, under a higher collusive price charged by the private label there is a new indifferent consumer whose address is $x_0 = (s - p_0)/t < 1/6$ on the segment $(0, 1/3)$ (correspondingly, the new location on the segment $(2/3, 1)$ is then $x_0 = 1 - (s - p_0)/t < 5/6$). Hence, given the prices of the brands (18), the joint profit cannot be increased by a price of the private label which is higher than $p^\text{FC}_0$ if

$$p^\text{FC}_0 \geq \hat{p}_0 \text{ with } \hat{p}_0 := \arg \max_{p_0} D_0(p_0) p_0.$$ 

Solving the maximization problem

$$\max_{p_0} D_0(p_0) p_0 = \max_{p_0} \frac{1}{t} 2p_0 (s - p_0) \quad (20)$$

gives

$$\hat{p}_0 = \frac{1}{2} s$$

which implies

$$\hat{p}_0 \leq p^\text{FC}_0 \text{ if and only if } s \geq \frac{1}{3} t.$$ 

The latter inequality is assumed in Assumption 1.29 Again, it ensures that the market is fully covered under full collusion. We have, therefore, proven part i) and part iii) of the proposition. The indifferent consumer between the private label and brand 1 (2) is located at $x_0 = 1/6$ ($x_2 = 5/6$). Substituting the equilibrium prices and locations of the indifferent consumers into the profit functions (12), (13), and (14) yields the profits under full collusion (as stated in part ii) of the proposition)

$$\pi^\text{FC}_0 = \frac{s}{3} - t/18 \text{ and } \quad \pi^\text{FC}_1 = \pi^\text{FC}_2 = 1/3 - t/18.$$

29 Applying the same deviation analysis to the brands, we get that joint profits cannot increased with a higher brand price if $t \leq 3$. 

28
The sum of all firms’ profits under full collusion is then given by
\[ \pi^{FC} = (s + 2)/3 - t/6. \]

**Derivation of the deviation profits.** We first solve the deviation problem of one of the brand producers which are symmetric. Next we solve the deviation problem of the private label firm.

**Case 1 (deviation by firm 1):** If firm 1 deviates, it charges a deviation price \( p_{D,FC}^{1} \). Substituting the collusive prices (18)-(19) into (4) and (5), we get the locations of the indifferent consumers depending on firm 1’s deviation price:

\[ x_{0}^{D,FC} = \frac{(t + 2p_{D,FC}^{1} - 2)}{4t} \quad \text{and} \quad x_{1}^{D,FC} = \frac{(5t - 6p_{D,FC}^{1} + 6)}{12t}. \]  

(22) 

(23)

Note that we must ensure that \( x_{1}^{D,FC} - x_{0}^{D,FC} < 1 \). Firm 1 maximizes its deviation profit

\[ \max_{p_{1}^{D,FC} \geq 0} \pi_{1}^{D,FC} = p_{1}^{D,FC} (x_{1}^{D,FC} - x_{0}^{D,FC}). \]

By substituting the locations of the indifferent consumers (22) and (23) into the profit function and solving the maximization problem, we find the optimal deviation price of firm 1:

\[ p_{1}^{D,FC} = t/12 + 1/2. \]

Note that the optimal deviation price of the brand \( p_{1}^{D,FC} \) is smaller than the collusive price \( p_{i}^{FC} \) for all \( t < 2 \) which holds by Assumption 1. Substituting the optimal deviation price into (22) and (23), we get the locations of the indifferent consumers

\[ x_{0}^{D,FC} = \frac{(7t - 6)}{(24t)} \quad \text{and} \quad x_{1}^{D,FC} = \frac{(3t + 2)}{(8t)}. \]

We must check whether the demand of the deviating firm is smaller than one, \( x_{1}^{D,FC} - x_{0}^{D,FC} < 1 \). This is true if \( t \geq 6/11 \) which is assumed in Assumption 1. The deviation profit is then given by

\[ \pi_{1}^{D,FC} = (t + 6)^2/(144t). \]

This proves part iv) of the proposition.
Case 2 (deviation by firm 0): Substituting the collusive prices into (4) and (6), we get the locations of the indifferent consumers depending on the private label firm’s deviation price:

\[ x_0^{D,FC} = \left( 6s + t - 6p_0^{D,FC} \right) / (12t) \quad \text{and} \]
\[ x_2^{D,FC} = \left( 11t/6 - s + p_0^{D,FC} \right) / (2t). \quad (24) \]

Again, we must ensure that \( x_0^{D,FC} + 1 - x_2^{D,FC} < 1 \). Firm 0 maximizes its deviation profit

\[ \max_{p_0^{D,FC} \geq 0} \pi_0^{D,FC} = p_0^{D,FC} (x_0^{D,FC} + 1 - x_2^{D,FC}). \]

This problem is only well defined for \( p_0^{D,FC} \leq p_0^{FC} \) because in this case the market is fully covered. For \( p_0^{D,FC} > p_0^{FC} \) the maximization problem (20) applies where we already showed that \( p_0^{FC} \) is optimal for \( s \geq t/3 \). By substituting the locations of the indifferent consumers (24) and (25) into the profit function and solving the maximization problem, we find the optimal deviation price of firm 0:

\[ p_0^{D,FC} = t/12 + s/2. \quad (26) \]

The optimal deviation price (26) is smaller than the collusive price of the private label (19) if \( p_0^{FC} - p_0^{D,FC} \geq 0 \) holds, which gives the condition

\[ s \geq \frac{t}{2}. \quad (27) \]

Condition (27) is part of Assumption 1. Note that a deviation price of the private label larger than the collusive price \( p_0^{FC} \) cannot be optimal for \( s \geq t/3 \) as we have shown by solving the maximization problem (20). Thus, the optimal deviation price of the private label would be the collusive price \( p_0^{FC} \) for \( t/2 > s \geq t/3 \). By Assumption 1 this case is ruled out, so that the private label firm finds it optimal to deviate with a price which is smaller than the collusive price.

By substituting the optimal deviation price of the private label good into the locations of the indifferent consumers (24) and (25), we get

\[ x_0^{D,FC} = (6s + t) / (24t), \quad \text{and} \]
\[ x_2^{D,FC} = (23t - 6s) / (24t). \]

We must check that under deviation the demand of the private label does not exceed one, \( x_0^{D,FC} + 1 - x_2^{D,FC} < 1 \). This holds for \( t > 6s/11 \) which is implied by assuming \( t > 6/11 \) (see
Assumption 1. The deviation profit of firm 0 is then given by
\[ \pi_0^{D, FC} = (6s + t)^2 / (144t). \]

This proves part \( v) \) of the proposition.

Proof of Proposition 3. Consider collusion by firms 1 and 2, while firm 0 behaves noncooperatively. Firms 1 and 2 maximize their joint profit
\[ \max_{p_1, p_2 \geq 0} \pi^{PC} = p_1 D_1 + p_2 D_2, \]
while the maximization problem of firm 0 is
\[ \max_{p_0 \geq 0} \pi_0 = p_0 D_0. \]

Substituting the demand functions into both problems and maximizing over the respective prices, we get the following system of first-order conditions:
\[
(2t - 3s + 3p_0 - 12p_i + 6p_{i'} + 3) / (6t) = 0, \text{ for } i, i' = 1, 2, i \neq i', \text{ and} \]
\[
(6s + 2t - 12p_0 + 3p_1 + 3p_2 - 6) / (6t) = 0. \]

All maximization problems are strictly concave, so that the solution of this system of first-order conditions gives the equilibrium prices as stated in part \( i) \) of the proposition; namely,
\[
p_0^{PC} = 4t/9 - (1 - s)/3, \\
p_1^{PC} = p_2^{PC} = 5t/9 + (1 - s)/3. \]

All prices are strictly positive under Assumption 1. Substituting the collusive prices into (4), (5), and (6), we get the equilibrium locations of the indifferent consumers under partial collusion (see part \( iii) \) of the proposition)
\[
x_0^{PC} = (4t - 3(1 - s)) / (18t), \\
x_1^{PC} = 1/2, \text{ and} \\
x_2^{PC} = (14t + 3(1 - s)) / (18t). \]

Assumption 1 ensures that \( x_0^{PC} \in (0, 1/3) \) and \( x_2^{PC} \in (2/3, 1) \). Substituting the equilibrium prices and locations of the indifferent consumers into the profit functions (12), (13), and (14)
yields (part ii) of the proposition)

\[
\begin{align*}
\pi_0^{PC} &= (4t - 3(1 - s))^2 / (81t), \\
\pi_1^{PC} &= \pi_2^{PC} = (5t + 3(1 - s))^2 / (162t).
\end{align*}
\] (28)

We finally check whether the utilities of the indifferent consumers (15), (16), and (17) are nonnegative. We obtain

\[
\begin{align*}
U_{x_0}^{pC} &= U_{x_2}^{pC} = (1 + s) / 2 - 2t / 3, \text{ and} \\
U_{x_1}^{pC} &= (s + 2) / 3 - 13t / 18.
\end{align*}
\]

These utility levels are nonnegative if \( s > \max\{13t / 6 - 2, 4t / 3 - 1\} \) which is assumed in Assumption 1.

We next derive the deviation price and profit of one of the brand producers (both are symmetric) as stated in part iv) of the proposition. Consider deviation by firm 1. Substituting the collusive prices \( p_0^{PC} \) and \( p_2^{PC} \) into (4) and (5), we get the locations of the indifferent consumers depending on the deviation price of firm 1:

\[
\begin{align*}
x_0^{D,PC} &= \left(6s - t + 9p_1^{D,PC} - 6\right) / (18t) \text{ and} \\
x_1^{D,PC} &= \left(14t - 9p_1^{D,PC} + 3 - 3s\right) / (18t).
\end{align*}
\] (29) (30)

Firm 1 maximizes its deviation profit

\[
\max_{\pi_1^{D,PC}, \geq 0} \pi_1^{D,PC} = p_1^{D,PC} (x_1^{D,PC} - x_0^{D,PC}).
\]

By substituting the locations of the indifferent consumers (29) and (30) into the profit function and solving the maximization problem, we find the optimal deviation price of firm 1:

\[
p_1^{D,PC} = (1 - s) / 4 + 5t / 12.
\]

Substituting the optimal deviation price into (29) and (30), we get the locations of the indifferent consumers under deviation

\[
\begin{align*}
x_0^{D,PC} &= (15s + 11t - 15) / (72t) \text{ and} \\
x_1^{D,PC} &= (41t - 3s + 3) / (72t).
\end{align*}
\]
We can show that $x_1^{D,PC} - x_0^{D,PC} < 1$ if $s > 1 - 7t/3$ which holds by Assumption 1. The deviation profit of firm 1 is then given by

$$\pi_1^{D,PC} = (5t/12 + (1 - s)/4)^2/t.$$ 

Proof of Corollary 3. The proof of parts i) and ii) of the corollary follows immediately from comparing the respective profit levels under noncooperative behavior, full collusion and partial collusion. Part iii) compares the profit levels of the private label firm under full collusion (21), and under partial collusion (28). This comparison gives rise to the condition

$$\pi_0^{FC} - \pi_0^{PC} = \frac{-18s^2 + 6st + 36s - 41t^2 + 48t - 18}{162t} \geq 0$$

which holds if and only if

$$s \geq s^*(t) := \frac{1}{6}t - \frac{1}{2}\sqrt{3}\sqrt{t(4 - 3t) + 1}.$$

Moreover,

$$\frac{\partial s^*(t)}{\partial(t)} = \frac{\sqrt{t(4 - 3t)}\left(\sqrt{t(4 - 3t)} + 9\sqrt{3}t - 6\sqrt{3}\right)}{6t(4 - 3t)}$$

which obtains three zeros at $t \in \{0, (2/3) - (\sqrt{2}\sqrt{41})/123, 4/3\}$. It is easily checked that $\partial s^*(t)/\partial(t) > 0$ for all $t \in [(2/3) - (\sqrt{2}\sqrt{41})/123, 4/3]$. As $s^*(t)$ cuts through the feasible set (as specified in Assumption 1) over the interval $t \in [6/5, 54/41]$, it then follows that $\partial s^*(t)/\partial(t) > 0$ holds always.

Proof of Proposition 4. Part i) Inspecting the critical discount factors $\delta_0^{FC}$ and $\delta_i^{FC}$ (see (9) and (8), respectively), we get that $\delta_0^{FC} = \delta_i^{FC}$ holds at $s = 1$. To prove that $\delta_i^{FC} > \delta_0^{FC}$ holds for $s < 1$, we first show that $\delta_i^{FC}$ is monotonically decreasing in $s$ over the relevant parameter range. Taking the derivative with respect to the parameter $s$, we get

$$\frac{\partial\delta_i^{FC}}{\partial s} = 75(2 - t)^2 \frac{-160t + 96s - 96}{(48s^2 - 160st - 96s + 125t^2 + 60t - 252)^2},$$

so that the sign of the derivative depends on the sign of the term $-160t + 96s - 96$. This term is negative for all $s < 5t/3 + 1$ which is implied by Assumption 1. Thus, $\partial\delta_i^{FC}/\partial s < 0$ holds everywhere. We next show that $\delta_0^{FC}$ is monotonically increasing in $s$ over the relevant parameter
range. Taking the respective derivative, we get
\[
\frac{\partial \delta_0^{FC}}{\partial s} = -28s^2t + 96s^2 - 76st^2 + 160st - 96s + 45t^3 - 104t^2 + 48t
\]
\[
\frac{1}{(108s^2 - 220st + 384s - 125t^2 + 320t - 192)^2} /1200 ,
\]
so that the sign of the derivative depends on the sign of the numerator. The numerator has two potentially relevant roots at
\[
s'(t) = \frac{1}{2}t \quad \text{and} \quad s''(t) = \frac{-104t + 45t^2 + 48}{14t - 48} \quad \text{for} \quad t \neq \frac{24}{7}.
\]
Further inspection yields that \( s > \max\{s'(t), s''(t)\} \) holds in the feasible area as specified in Assumption 1. This implies that the numerator of the right-hand side of (31) and thus \( \partial \delta_0^{FC}/\partial s \) is strictly positive in the relevant parameter range. Combining these results concerning the slopes of both critical values with the fact that both values are equal at \( s = 1 \) gives the ordering stated in the proposition.

Part ii) Setting \( \delta_i^{FC} = \delta_i^{PC} \) we can calculate the unique threshold value \( \pi(t) := 5t/3 - (\sqrt{201t - 44t^2 - 126})/3 + 1 \) which cuts through the feasible set (the expression below the square root sign is always positive). The orderings stated in the proposition are then easily verified. Calculating the first and second derivative with respect to \( t \) we get
\[
\frac{\partial \pi(t)}{\partial t} = \frac{188t + 10\sqrt{-44t^2 + 201t - 126} - 201}{6} < 0 \quad \text{and}
\]
\[
\frac{\partial^2 \pi(t)}{\partial t^2} = \frac{6075}{4(-44t^2 + 201t - 126)^{\frac{3}{2}}} > 0,
\]
where the signs hold within the considered parameter range.

**Proof of Proposition 6.** We have to calculate the marginal profit changes of the private label producer under the three types of market conduct. This yields
\[
\frac{d\pi_0^N}{ds} = \frac{72s + 60t - 72}{225t}, \quad \frac{d\pi_0^{PC}}{ds} = \frac{18s + 24t - 18}{81t}, \quad \text{and} \quad \frac{d\pi_0^N}{ds} = \frac{1}{3}.
\]
Comparison of those values gives the ordering stated in the proposition. It remains to show that
\[
\frac{\partial \delta_i^{FC}}{\partial t} = 1200(t - 2) \frac{32s - 35t + 10st - 6s^2 + 24}{(48s^2 - 160st - 96s + 125t^2 + 60t - 252)^2} < 0
\]
which holds because the numerator of the right-hand side is positive if \( s > \frac{5}{6}t - \frac{5}{6}\sqrt{t^2 - 2t + 16} + \frac{8}{3} \)
which is implied by Assumption 1. Finally,

\[
\frac{\partial \delta_{FC}}{\partial t} = \frac{28s^3 + 76s^2t - 176s^2 - 45st^2 + 48st + 48s + 20t^2 - 24t}{(108s^2 - 220st + 384s - 125t^2 + 320t - 192)^2/1200} < 0
\]

which holds because the numerator of the right-hand side is negative if

\[ s > \frac{22}{7} - \frac{5}{28}\sqrt{81t^2 - 272t + 256} - \frac{45}{28}t \]

which is implied by Assumption 1, again.

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