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Working Paper

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Tinbergen Institute Discussion Paper, No. 15-062/VIII

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Robot Cars and Dynamic Bottleneck Congestion: The Effects on Capacity, Value of Time and Preference Heterogeneity

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Robot Cars and Dynamic Bottleneck Congestion: 
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version of 9 June 2015

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Abstract

‘Robot cars’ are cars that allow for automated driving. By allowing cars to safely drive closer together than human driven ‘normal cars’ do, robot cars raise road capacity. By allowing drivers to perform other activities in the vehicle, they lower the value of travel time delays (VOT). We investigate the social welfare effect of robot cars using a dynamic equilibrium model of congestion that captures the following mechanisms: the resulting increase in capacity, the decrease in VOT and the implications for the heterogeneity in the VOT. We do so for a number of market organizations: private monopoly, perfect competition and public supply. Increasing the share of robot cars raises average capacity, but may hurt existing robot car users as the switchers, through their altered departure time behaviour, will impose higher bottleneck-congestion externalities. Depending on which effect dominates, buying a robot imposes a net negative or positive externality. Numerical analysis suggests that a net positive externality is more likely; nevertheless, for a small, but still plausible, capacity effect a net negative externality results. With a positive (negative) externality, marginal cost provision under perfect competition tends to lead to an undersupply (oversupply) of robot cars, and a public supplier needs to subsidise (tax) robot car purchase in order to maximise welfare. A monopolist supplier ignores the externality and tends to add a mark-up to its price. This almost always leads to a substantial undersupply.

Keywords: robot cars, heterogeneity, bottleneck model, autonomous cars, self-driving cars, market structure
JEL codes: D42, D62, H23, L12, L51, R41, R48

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1. Introduction

‘Robot cars’—also referred to as self-driving or autonomous cars—are cars that drive themselves. Robot cars have automated speed choice, based on the own speed and the speed of and distance to surrounding vehicles, and can thus drive closer together and at a more uniform speed than human driven ‘normal cars’. All else equal, a group of robot cars can move at a greater density for any given speed than normal cars, and thus raise the flow (speed times density) capacity of roads (Chang and Lai, 1997). Besides this capacity effect, people using a robot car instead of a ‘normal car’ may gain a decrease in their value of travel time losses (VOT), as the time in the car can be spent on other activities besides driving and this makes travel time more useful and lowers its costliness. As a result, the VOT may become (more) heterogeneous for a mix of drivers using normal and robot cars. Such heterogeneity may strongly affect the overall welfare effects of policies such as congestion pricing and capacity expansion, and enforces the relevance of distributional effects of policies (see, e.g., Arnott et al. (1993, 1994), Lindsey (2004) and Van den Berg and Verhoef (2011ab) all of whom only consider ‘normal cars’).

Furthermore, robot cars may lead to fewer accidents, to fewer cars and parking spots if robot cars could drive around and pick-up other (paying) passengers, to higher (safe) speed limits, and to reduced fuel use. If only robot cars use a road, it may become possible to redesign it: e.g., to reduce the lane widths and turn a 2-lane motorway into a 3-lane one. There are also possible problems, including the question of liability if something goes wrong, software and hardware reliability, loss of privacy, and hacking of and virus intrusion into robot cars.

Clearly, therefore, robot cars can be expected to have a large influence on urban transport and the layout of cities, explaining the strong interest of policymakers, media and general public in robot cars as a solution to transport problems. This paper focusses on the effects of robot cars on (the social cost of) congestion, focussing on the following mechanisms: the resulting increase in capacity, the decrease in VOT, and the implications for the heterogeneity in the VOT. We are the first to consider the effects via the VOT and heterogeneity therein, as well as to consider robot cars while making departure time choices endogenous following a dynamic user equilibrium condition.

We do not consider sources of heterogeneity other than from the decisions to obtain a robot car or not, so that the VOT is homogeneous if everyone has the same car.3 We use the Vickrey (1969) bottleneck model to capture the effects in a stylised setting that permits analysis using closed-form solutions, while also capturing the important behavioural aspects sketched above.

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3 Pre-existing heterogeneity seems a natural extension of our paper. Vickrey (1969) already included heterogeneity in the preferred arrival time. Arnott et al. (1988, 1994) studied three cases: heterogeneity in the preferred arrival time, heterogeneity between the value of time and values of schedule delay, and heterogeneity between values of schedule delay early and late. Vickrey (1973) introduced proportional heterogeneity in values of time and schedule delay where all three values vary in fixed proportions. This heterogeneity was also used by Xiao et al. (2011) and Van den Berg (2014). Arnott and Kraus (1995) and Van den Berg and Verhoef (2011ab) combined proportional heterogeneity with heterogeneity between the value of time and the values of schedule delay. Newell (1987), Lindsey (2004), Börjesson and Kristoffersson (2014) and Liu et al. (2014) looked at general heterogeneity.
With a heterogeneous VOT, as Arnott et al. (1988, 1993) show, users self-select over time. In an equilibrium with queuing, the users with the highest VOTs travel in the edges of the peak when queuing delays are short; users with the lowest VOTs travel in the centre peak thus accepting long travel times in exchange for a convenient arrival moment. If a user obtains a lower VOT after buying a robot car, this increases the congestion cost that she imposes on all users who arrive closer to preferred arrival time than she does. The reason is that she now cares less about travel times and hence needs a steeper travel time development over time to remain in user equilibrium (Lindsey, 2004). Consequently, the marginal external cost (MEC) of a user decreases with her VOT (Van den Berg and Verhoef, 2011a) and when her VOT decreases due to getting a robot car this raises her MEC.

Total social cost includes travel costs but also automobile costs due to depreciation and fuel use. Robot cars are likely to be more expensive than normal cars and may become obsolete quicker, which implies higher depreciation costs. At the same time, robot cars may use less fuel as they drive at more uniform speeds. The ‘heterogeneity effect’ via the MEC means that, ignoring the capacity effect, an increase in the share of robot cars raises travel times and costs for existing robot car users. If the capacity effect is strong enough, total social cost is minimised when everyone has a robot car; but if not, it may be socially best when not everyone has a robot car. Gubins and Verhoef (2011) find a similar result when considering the impact of teleworking technology on the social cost and benefits of commuting.

With public interest in congestion being high, and the development of robot cars in private hands, it is an interesting question if the supply of robot cars should be left to the market or if it should be publicly guided or even controlled. We therefore examine three provision schemes of robot cars: socially-optimal ‘public provision’, the benchmark perfectly competitive case of ‘marginal cost provision’, and provision by a profit-maximising ‘monopolist’. The effects of provision regimes of robot cars have not been studied before. Public and marginal cost provision can be expected to differ as using a robot car instead of a normal car imposes a different externality. When the capacity effect is stronger than the heterogeneity effect, there is a net positive externality and thus robot cars need to be priced below marginal production cost when roads are not priced optimally. If the heterogeneity effect dominates, which the numerical analysis suggests may occur, marginal cost provision leads to overconsumption of robot cars. Private supply would approach the marginal cost case as the provision market becomes more competitive. Provision by a monopolist private firm typically leads to a substantially lower supply of robot cars than marginal cost provision, since it uses its market power to maximise profits. This also leads to a large welfare loss.

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4 We only consider settings with unpriced congestion. With an optimal congestion charge, marginal production cost pricing of the cars is optimal. Still, the congestion charge would need to be different for normal and robot cars, as robot cars impose lower schedule delays due to their higher capacity. If this differentiation is impossible, one may still need to subsidise robot cars.
For our results, a critical parameter is the size of the capacity effect. A literature overview in Section 2 shows large variations in predictions from engineering studies. The major determinant seems to be how well robot cars communicate and cooperate. The average prediction is that capacity would increase by around 100% when we go from all normal cars to all robot cars. VanderWerf et al. (2002) find that capacity may be only 7% higher with only ‘uncooperative robot cars’ than with only normal cars, while it may be 200% higher with efficient cooperation. Another important parameter is the effect on the VOT, and its size is an even more open question than the size of the capacity effect. A final question is how much higher (or lower) other automobile cost (i.e. depreciation and fuel cost) of robot cars will be compared to normal cars. The effects of the provision regimes and the question of whether second-best social optimum needs a corrective subsidy or tax are sensitive to these three parameters. Hence, it is vital to do extensive sensitivity analyses, as we will do in Section 5. Finally, Section 6 studies modelling extensions such as pre-existing heterogeneity and a non-linear capacity effect. For ease of reference, the below Nomenclature box summarises the notation as also explained in text.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Value of time (VOT) for normal car users: the cost of an hour of travel time.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Value of schedule delay early: the cost of an hour earlier arrival than the preferred arrival time $t^*$.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Value of schedule delay late: the cost of an hour later arrival than the preferred arrival time $t^*$.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Compound preference parameter: $\delta = \beta / (\beta + \gamma)$.</td>
</tr>
<tr>
<td>$\theta$</td>
<td>VOT reduction parameter for a robot car: the VOT for a robot car is $\theta \alpha$ with $\beta / \alpha &lt; \theta &lt; 1$.</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>Change in travel cost when switching from a normal car to a robot car: $\Delta C = C_r - C_n$.</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Travel cost for car type $i={r, n}$ with $r$ indicating a robot car and $n$ the normal car. Travel cost is the sum of free-flow travel time cost and bottleneck cost where the bottleneck cost equals queuing time cost plus schedule delay cost.</td>
</tr>
<tr>
<td>$CTT$</td>
<td>Travel time cost.</td>
</tr>
<tr>
<td>$CSD$</td>
<td>Schedule delay cost, i.e. the cost of arriving at a different time than the preferred arrival time.</td>
</tr>
<tr>
<td>$F$</td>
<td>Fraction of users that has a robot car.</td>
</tr>
<tr>
<td>$MC_r$</td>
<td>Marginal automobile cost of the robot car. The corresponding cost for a normal car is normalised to zero, and hence $MC_r$ gives how much higher or lower for the robot car the sum is of fuel cost and per trip marginal production cost.</td>
</tr>
<tr>
<td>$MEB$</td>
<td>Marginal external benefit of marginally increasing number of users with a robot car, it equals the change in travel cost minus change in average social cost: $MEB = \Delta C - (\partial TTC/\partial F)/N$.</td>
</tr>
<tr>
<td>$MU_r$</td>
<td>Per trip mark-up on the robot car. The mark-up on the normal car is (normalised to) zero.</td>
</tr>
<tr>
<td>$N$</td>
<td>Total number of users.</td>
</tr>
<tr>
<td>$P$</td>
<td>(Generalised) price equals travel cost plus per trip marginal automobile cost and mark-up.</td>
</tr>
<tr>
<td>$r$</td>
<td>Capacity parameter for the robot car: this car faces a bottleneck capacity of $r\cdot s$.</td>
</tr>
<tr>
<td>$s$</td>
<td>Bottleneck capacity for a normal car.</td>
</tr>
<tr>
<td>$t$</td>
<td>Arrival time.</td>
</tr>
<tr>
<td>$t^*$</td>
<td>Preferred arrival time.</td>
</tr>
<tr>
<td>$t_e$</td>
<td>Moment that the last car arrives and thus the peak ends.</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Moment that the first car arrives and thus the peak starts.</td>
</tr>
<tr>
<td>$TT$</td>
<td>Travel time which equals free-flow travel time plus queueing time.</td>
</tr>
<tr>
<td>$TT_{ff}$</td>
<td>Free-flow travel time.</td>
</tr>
<tr>
<td>$TTC$</td>
<td>Total travel cost which equals $N(F \cdot C_r + (1-F \cdot C_n))$.</td>
</tr>
<tr>
<td>$TC$</td>
<td>Total cost which equals $TTC + F \cdot N \cdot MC_r$.</td>
</tr>
</tbody>
</table>
2. Overview of the research on the effect of robot cars on road capacity

It difficult to determine the possible effect of robot cars on the capacity of a road bottleneck. It not only depends on how good the robot technology is, but also on how well the cars communicate and cooperate, on the choice of headway between robot cars, on current traffic conditions (without initial congestion, the capacity effect would bring no benefit) and on how efficiently normal car users drive.

There is a large engineering literature on this question, but it remains an open and debated question. Zwaneveld and Van Arem (1997) review the early literature and argue that a doubling or tripling of capacity seems likely. However, in more recent literature, the increase in effective capacity can be as low as 7% for robot cars that do not communicate (VanderWerf et al., 2002) and as high as 270% for very efficiently communicating/cooperating cars (Tientrakool et al. 2011). The predictions thus vary from almost no effect to a quadrupling of capacity. Table 1 gives an overview of the predictions in the literature. All results refer to switching from 100% normal cars to 100% robot cars, which seems most relevant for our study as in our setting normal and robot car users travel fully separated over time. If car types would travel mixed, the effects would be much less beneficial as, for an increasing level of penetration, most gains occur when going from a majority of robot cars (e.g. 80-90%) to only robot cars (e.g. Tientrakool et al. 2011). Van Arem et al. (1996, 2006) find that introducing uncooperative robot cars may even lessen capacity especially when the fraction with a robot car is low.

Table 1: Percentage increase in capacity from going from only normal cars to only robot cars

<table>
<thead>
<tr>
<th>Study</th>
<th>Uncooperative robot cars</th>
<th>Cooperative robot cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chang and Lai (1997)</td>
<td>33%</td>
<td>x</td>
</tr>
<tr>
<td>Shladover et al. (2001) &amp; VanderWerf et al. (2002)</td>
<td>7%</td>
<td>About 220%</td>
</tr>
<tr>
<td>Shladover (2011)</td>
<td>x</td>
<td>80% for cars (much smaller effect for trucks)</td>
</tr>
<tr>
<td>Ni et al. (2010)</td>
<td>x</td>
<td>20%-50%</td>
</tr>
<tr>
<td>Tientrakool et al. (2011)</td>
<td>90% at 50 km/h, 40% at 100km/h</td>
<td>200% at 50 km/h, 270% at 100km/h</td>
</tr>
</tbody>
</table>

3. The model

3.1. Set-up

We model the effects of robot cars using Vickrey’s (1969) point-queue bottleneck model. We focus on how introducing robot cars changes the outcome, and will hence be brief in our description of the standard bottleneck model. For a complete overview see, for instance, Arnott et

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5 Yet, even if cooperation is technically possible, it may not occur in practice because users and/or producers choose not to. Just as in the prisoners’ dilemma, all actors cooperating may make everyone better off, but from this cooperative outcome each actor could increase its pay-off by unilaterally playing an uncooperative strategy.
al. (1988, 1990, 1993, 1994) or Small and Verhoef (2007). We assume that demand is fixed and thus ignore that if robot cars lower costs this increases demand and thereby increase congestion. We also ignore other transport modes such as public transport and taxis.

Travel time, $TT$, is the sum of free-flow travel time, $TT_{ff}$, and delays from queueing at the bottleneck. When there is no queue and the arrival rate at the bottleneck is below its capacity, travel time equals $TT_{ff}$. The queuing delay equals the number of cars in the queue at the instance of joining it divided by the capacity of the bottleneck during the queuing time. When normal cars are passing the bottleneck, the capacity is $s$. When robot cars are passing the bottleneck, capacity equals $r\cdot s$ where the capacity-increase parameter, $r$, is at least one: $r \geq 1$. The review in Section 2 suggests that $r$ may be close to 1 or as high as 4. The value of travel time losses (VOT) with a normal car is $a$, with a robot car the VOT is $\theta \cdot a$ with $\theta < 1$ being the VOT decrease parameter. Travel time cost, $C_{TT}$, of a user equals her travel time, $TT$, multiplied by her value of time.

A person’s bottleneck cost equals queuing time cost plus schedule delay cost, $C_{SD}$. Schedule delay cost is the cost due to arriving on a different time than the most preferred arrival time, $t^*$. We assume that $t^*$ is the same for all, and normalize it to 0 for convenience. We follow Small (1982) and most of the literature and use schedule delay costs that are linear in the time difference between $t^*$ and the actual arrival time, $t$. The cost per hour earlier arrival than $t^*$, also referred to as an hour of early schedule, is $\beta$; per hour late arrival the cost is $\gamma$. Travel cost equals bottleneck cost plus free-flow travel time cost:

$$C_n[t] = C_{TT}[t] + C_{SD}[t] = \alpha \cdot TT[t] + \text{Max}(-\beta \cdot t, \gamma \cdot t),$$
with a normal car,  

$$C_r[t] = C_{TT}[t] + C_{SD}[t] = \alpha \cdot \theta \cdot TT[t] + \text{Max}(-\beta \cdot t, \gamma \cdot t),$$
with a robot car.

where subscript $n$ indicates a normal car and $r$, a robot car. As Arnott et al. (1988) show, users with a lower VOT self-select to arrival times in the centre peak where travel times are long and schedule delays are small: these users care little about travel times losses and can thus enjoy the small schedule delays in the centre peak. Drivers with a larger VOT self-select to the early and late edges of the peak. This means that normal and robot cars will travel separated over time, and do not share the bottleneck. Therefore, the bottleneck capacity is either $s$ or $r \cdot s$. If traffic would be mixed, the capacity at each moment would depend on the shares of the car types passing the bottleneck then.

The generalised price per trip (or price for brevity) is the sum of travel cost, automobile cost and mark-up per trip on the car. Generalised cost (or cost for brevity) is price minus the mark-up per trip. Automobile cost includes fuel cost (which are assumed a fixed amount per trip) and depreciation per trip due to the (long-run) marginal production cost of the car. The mark-up on

\[^6\] It seems more accurate that the depreciation also considers the rest value when selling the car. For brevity, we will use marginal production cost.
the car is also reworked to an amount per trip. Total profit on a car type equals the number of this type times the mark-up. Robot cars will probably be more expensive to produce; robot cars may also become obsolete more quickly, which raises the depreciation cost. This will be partly compensated by robot cars having a lower fuel cost. Nevertheless, it seems likely that robot cars will have a higher per-trip automobile cost. Still, we do not impose this, and our numerical analyses also study robot cars having lower automobile costs.

For normal cars, we normalise automobile cost and mark-up to zero. Accordingly, the (extra) marginal automobile cost for a robot car, $MC_r$, is how much higher (or lower) this cost is for robot cars. The $MU_r$ is the (extra) mark-up per trip on robot cars, where this mark-up may be negative.

For equilibrium, travel cost for each type of car needs to be constant over the arrival times used by its users and no lower on other times: otherwise someone could lower her travel cost by changing arrival time and thus the system would not be in equilibrium. Therefore, we can leave out the arrival time indicator and equilibrium prices are

\[
P_n = C_n, \quad P_r = C_r + MC_r + MU_r.
\]

### 3.2. Standard bottleneck model without robot cars

We now briefly reintroduce the bottleneck model without robot cars, and thus with homogeneous users. The three subsections hereafter look at when robot cars only affect capacity, only the VOT, and finally at the full model when they affect both. This order of presentation helps in identifying the different effects of robot cars.

In equilibrium, travel cost as in (1) should be constant over time as longs as arrivals occur. This implies $\alpha \cdot \partial TT / \partial t + \partial CSD / \partial t = 0$. From this, we derive that travel time must grow linearly over arrival time by $a/\beta$ before $t^*$, and shrink by $-a/\gamma$ after $t^*$. In equilibrium, the bottleneck is used at capacity throughout the peak. This means that the total number of users, $N$, can just pass the bottleneck during the peak and the peak lasts $N/s$. The first driver arrives at the starting time, $t_s$, of the peak and faces a zero queuing time as there is no queue yet. The last driver to arrive at $t_e$ also incurs no queuing, otherwise he could lower his cost by departing slightly later. It is simple to show that a fraction $\gamma/(\beta+\gamma)$ of users arrives early and the remainder late, so that equilibrium travel cost is:

\[
C = \delta \cdot \frac{N}{s} + a \cdot TT, \quad \text{with} \quad \delta = \frac{\beta \cdot \gamma}{\beta + \gamma}.
\]

Here, $N$ is the fixed number of users and $\delta$ is a compound preference parameter.

Fig. 1 illustrates the equilibrium. It shows the travel-time pattern over arrival times which leads to constant costs. The curve can be interpreted as an iso-cost function as costs are constant along
it; shifting it upwards (downwards) would mean a higher (lower) cost level. This iso-cost interpretation will prove useful later on in this paper.

*Fig. 1: Equilibrium queuing times without robot cars*

\[ C_r = C_n = \delta \cdot \frac{N}{s} \left( 1 - F \right) + \frac{1}{r} F + \alpha \cdot TT_{y}. \] (6)

This is the same cost function as for the standard bottleneck model in (5) except for that a fraction \( F \) of users a higher capacity of \( r \cdot s \) applies. Travel cost does not vary over car type, as all users have the same preferences. The higher capacity for robot cars is equally beneficial for both types.

Fig. 2 illustrates all this. The thin dashed line shows the equilibrium travel times without robot cars; the thick black line represents a positive fraction, \( 1 \geq F > 0 \), with a robot car. The increased capacity shifts all users to the lower thick iso-cost curve, and hence everyone gains a lower cost. The slope of the equilibrium travel times remains the same for both types of users as \( \alpha, \beta \) and \( \gamma \) are equal across users. All users therefore have the same iso-cost curve and thus the same cost.

---

7 We assume that also in this setting normal and robot cars travel separated over time, and thus that capacity varies discretely over time. If car types have different VOTs, types will automatically travel separated over time. With a homogeneous VOT, it is equally possible that types travel mixed, but in that case the capacity at a certain moment would depend on the fraction with a robot car at the bottleneck then. This would complicate the model and make this subsection incomparable with the later ones.
Total travel cost, $TTC$, equals:

$$TTC = N \cdot (C_n \cdot (1 - F) + C_r \cdot F) =$$
$$= \delta \cdot \frac{N^2}{s} \left( (1 - F) + \frac{1}{r} \right) + a \cdot TTC \cdot N. \quad (7)$$

The higher the fraction with a robot car, the higher total capacity and the lower travel cost. Since total travel cost in (7) decreases linearly in $F$, we have a corner solution for the optimum where everyone has a robot car if the extra automobile cost of robot cars is not too large. Otherwise, total social cost is minimised when nobody has a robot car.

Because normal and robot cars have the same travel costs, a marginal user switching car type faces a zero change in travel cost, $\Delta C$:

$$\Delta C = C_r - C_n = 0. \quad (8)$$

So that nobody would get a robot car if they are more expensive to buy than a normal car. Yet, total travel cost decreases with $F$. Buying a robot car instead of a normal car imposes a positive externality, and there is a positive ‘marginal external benefit’ ($MEB$):

$$MEB = \Delta C - \frac{\partial TTC}{\partial F} = \delta \cdot \frac{N}{s} \left( 1 - \frac{1}{r} \right) > 0. \quad (9)$$

To conclude, when robot cars only affect capacity, all users have the same cost and everyone gains by the same amount if the share with a robot car increases. Therefore, getting a robot car causes a positive externality, because it lessens the negative congestion externality. If cars are priced at marginal production costs, users will ignore the positive externality and there is
underconsumption of robot cars (unless robot cars are cheaper to buy than normal cars and thus everyone still gets a robot car even though they ignore the externality).

3.4. Special case 2: having a robot car only lowers the value of time

Now we turn to robot cars only affecting the VOT while capacity remains unaffected. Users of a normal car have a VOT of $\alpha$, and in a robot car this becomes $\theta \cdot \alpha$. The following constraint must hold: $\beta \alpha < \theta < 1$. When some users have a robot car and others not, this means that the VOT is heterogeneous. As Arnott et al. (1998) and Van den Berg (2011ab) show, more VOT heterogeneity lowers total bottleneck cost (ceteris paribus). In our setting, the degree of heterogeneity is maximised when the robot car share is 50%. Bottleneck cost is queuing time cost plus schedule delay cost, and travel cost is bottleneck cost plus free-flow travel time cost. A user who switches from a normal car to a robot car not only gains a lower free-flow travel-time cost due the lowered VOT, but often also gains a lower bottleneck congestion cost.

**Fig. 3: Equilibrium queuing times when robot cars only affect the value of time**

<table>
<thead>
<tr>
<th>Case 1: no robot cars</th>
<th>Case 2: 50% has a robot car</th>
<th>Case 3: everyone has a robot car</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1: no robot cars</td>
<td>Case 2: 50% has a robot car</td>
</tr>
</tbody>
</table>

Note: For the 50% robot car case, $t_w$ is the moment robot cars start to pass the bottleneck and thus arrive at the destination; $t_e$ is the moment the last robot car arrives. The $t_1$ and $t_2$ are the moments the iso-cost function of the robot car intersects the Y-axis (note that, at this moment, they do not arrive, but it is the moment they would have the same cost as in equilibrium if the travel time were zero then which it is not). Therefore, the thin dashed lines are out-of-equilibrium continuations of the iso-cost function for robot cars in the 50% robot car setting.

Fig. 3 illustrates this subsection by depicting three outcomes: when nobody has a robot car (solid lines), when 50% does (dashed lines), and when everybody does (dotted lines). For the 50% robot car case, the thin dashed lines are an out-of-equilibrium continuation of the iso-cost function

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8 The $\theta$ has to be above $\beta / \alpha$ as in the dynamic congestion models the VOT must be above the value of schedule delay early ($\beta$). If the VOT equals the $\beta$, then the departure rate would be infinite and a mass departure results (Arnott et al., 1990). The constraint is also intuitive: a VOT below $\beta$ would imply that people prefer sitting in the car over getting out and reaching the destination (to work, see a movie, visit family, etc.). It would also imply that, in the early morning, people get a higher utility from being in the car than being at home. This would mean that a rational person, in the morning, would wake up, get out of the car to go to the toilet, wash and make breakfast, and then return to the car. The empirical evidence for such behaviour seems scant.

Similarly, we assume that using a robot car will decrease the VOT and thus $\theta < 1$. A $\theta$ equal to 1 leads to the uninteresting case with no effect on the VOT. We ignore $\theta > 1$: firstly, because an increased VOT would be strange if only because you could still switch off the robot driving and, secondly, because this would complicate the modelling.
for robot cars. Robot cars arrive between \( t_{re} \) and \( t_{re}^* \) and the thin dashed lines outside this period indicate what the travel times would need to be, but are not in equilibrium, for them to have the same cost. Consistent with the literature, when we have both car types, robot car users (with the lower VOT) self-select to the centre peak, and normal car users to the edges of the peak. The decreased VOT of the robot car users means that they can enjoy the lower schedule delays in the centre peak, as they care less about the longer travel times there than normal car users.\(^9\)

As we show below, when both car types are present, normal car users impose lower externalities than robot car users. The lower VOT in the robot cars means that its users need a steeper travel time development over time to be in user equilibrium, and hence impose longer travel times on those who travel closer to the central peak (Lindsey, 2004). Consequently, the marginal external cost (MEC) of a user decreases with his VOT (Van den Berg and Verhoef, 2011a); and when a driver’s VOT decreases because he acquires a robot car, this raises his MEC. Increasing the share of robot cars thus hurts existing robot car users, as there are fewer users who impose a lower queuing externality on them. Without a capacity effect, there is no effect on normal car users who always travel closer to shoulders of the peak than robot car users and therefore will not face an upward pressure of equilibrium queuing times. Phrased differently, robot cars only impose schedule delays on them, and this external effect is independent of the VOT. Normal car users always have the same travel cost, and thus iso-cost curve, as with homogeneity, and this cost is independent of the share of robot car users.

During the early arrival window of normal car users, the queuing time increases at a rate of \( \beta/\alpha \) by arrival time; it decreases by \( -\gamma/\alpha \) for late arrivals. These arrival rates are needed to keep the travel cost constant over time. For robot car users to be in equilibrium, these slopes need to be steeper, and should be \( \beta/(\theta\cdot\alpha) \) before \( t^* \) and \( -\gamma/(\theta\cdot\alpha) \) thereafter. In Fig 3, with 50% robot cars, the equilibrium bottleneck cost of robot car users is \( \beta(t^*-t_1) = \gamma(t_2-t^*) \); it is \( \beta(t^*-t_s) = \gamma(t_e-t^*) \) for normal cars, which is the same as without robot cars.

Normal car users impose less steep travel time changes over arrival time than robot cars users, and hence normal car users impose lower queuing time costs on them than fellow robot car users do. This, and their lowered VOT, shifts robot car users to a lower iso-cost curve. This also means that normal car users cause a lower congestion externality with 50% robot cars than with 0%.

The lower \( \theta \), and thus the more a robot car lowers a driver’s VOT, the larger the difference in external cost imposed by a normal and a robot car. When the share of robot car users, \( F \), increases, there are fewer normal car users who impose relatively low queuing time costs on robot car users. Consequently, the queuing times increase for robot car users and they end up on a higher iso-cost curve. When we go from the 50% to 100% robot-car case in the diagram (but this argument holds

\(^9\) Both normal and robot car users would be worse off when traveling with the other group. Hence, temporal separation of users is sustained as equilibrium.
for any two positive percentages), users who already had a robot car in the 50% case are hurt, and are shifted to a higher iso-cost curve. With 100% robot cars, the bottleneck cost is back at the level without robot cars: the length of the queue at any moment has then increased by a proportion \(1/\theta\), just compensating the decrease in the VOT. In both cases, the VOT is homogeneous, and then the bottleneck cost is independent of the VOT (Arnott et al., 1988). Of course, travel cost is lower in the 100% case, as the free-flow travel time cost is lower.

Following Van den Berg and Verhoef (2011ab), travel costs for respectively normal and robot cars can be shown to be

\[
C_n = \delta \cdot \frac{N}{s} + \alpha \cdot TT_{ff},
\]

\[ (10) \]

\[
C_r = \delta \cdot \frac{N}{s} \left( \theta (1-F) + F \right) + \alpha \cdot \theta \cdot TT_{ff}.
\]

\[ (11) \]

Increasing the share, \( F \), has no effect on the travel cost of a normal car, but it increases the travel cost of a robot car. For a user who switches from a normal car to a robot car, travel cost decreases by

\[
\Delta C = C_r - C_n = -\delta \frac{N}{s} \left( (1-F)(1-\theta) \right) - \alpha \cdot TT_{ff} \left( 1 - \theta \right).
\]

where, in the rightmost equation, the first term is the decrease in bottleneck cost and the second the decrease in free-flow travel-time cost.

Total travel cost is

\[
TTC = N(C_n (1-F) + C_r \cdot F)
\]

\[
= \delta \frac{N^2}{s} \left( 1 + F (1-\theta) \right) + \alpha \cdot TT_{ff} \cdot N (1 - F (1-\theta)).
\]

\[ (13) \]

The \( F \) affects total travel cost because it changes the heterogeneity in the VOT. Therefore, we will also refer to this as the heterogeneity effect. The derivative of total travel cost with respect to the share of robot car users is

\[
\frac{\partial TTC}{\partial F} = (1-\theta)N \left[ \delta \frac{N}{s} \left( 2 \cdot F - 1 \right) - \alpha \cdot TT_{ff} \right].
\]

\[ (14) \]

For \( F>1/2 \), the derivative is larger (i.e. total cost decreases less or increases more with \( F \)) when \( \delta \cdot N/s \) is larger as then increasing \( F \) hurts current robot car users more. A larger \( \delta \cdot N/s \) strengthens the heterogeneity effect, and this increases the difference between the external costs imposed by a normal car and a robot car. A higher \( F \) also increases \( \partial TTC/\partial F \): firstly, because there are more robot car users to hurt by turning a normal car user into a robot car user and, secondly, because the
marginal user who switches car type sees a smaller decrease, $\Delta C$, in own travel cost. When $\alpha \cdot TTff$ increases, $\partial TTC / \partial F$ decreases: as free-flow travel time becomes more costly, it becomes more beneficial to lower the VOT by getting a robot car. Bottleneck and free-flow cost are both scaled by $1-\theta$, and thus $\theta$ has no effect on the sign of the derivate.

Switching to a robot car lowers one’s own cost, but imposes a negative externality by increasing the travel cost of other robot car users. The implied $MEB<0$ is

$$MEB = \Delta C - \frac{\partial TTC}{\partial F} = -\delta \frac{N}{s} (F (1-\theta)) < 0.$$

(15)

If cars are priced at marginal production costs, users will ignore this negative externality and there will be overconsumption of robot cars. Conversely, with only a capacity effect, we found a positive externality and marginal cost pricing resulted in underconsumption. There is no a priori reason why either effect would dominate.

### 3.5. General case: robot cars affect capacity and value of time

Now we turn to the full setting, where robot car ownership brings changes in capacity and VOT. The effects will prove to be a combination of the effects in the previous two subsections. Increasing the share with a robot car hurts existing robot car users due to the heterogeneity effect, as the new robot car users will now cause a steeper travel time pattern. Yet, increasing $F$ also increases total capacity and this lowers travel cost for everyone. If the heterogeneity effect dominates, there is a net negative externality. In the reverse case, there is a net positive externality.

Travel costs are a combination of those in the previous two subsections:

$$C_n = \delta \cdot \frac{N}{s} \left( (1-F) + F/l + r \right) + \alpha \cdot TTff,$$

(16)

$$C_r = \delta \cdot \frac{N}{s} \left( \theta (1-F) + F/l + r \right) + \alpha \cdot \theta \cdot TTff.$$

(17)

Setting $\theta=1$ would make (16) and (17) equal cost function (6) that only had a capacity effect; setting $r=1$ would result in eqs. (10)-(11) that reflect the case with an effect on the VOT only.

The iso-cost curves of Fig. 4 are also combinations of those in Figs. 2 and 3. The left panel of Fig. 4 indicates the equilibrium with a relatively small effect on capacity for three penetration levels: 0% robot cars, 50% robot cars, 100% robot cars. The right panel does this for a relatively large capacity effect. The only difference between the two panels is that, in the left panel, going from 50% to 100% robot cars hurts those that already had a robot car by shifting them to a higher iso-cost curve while, in the right panel, existing robot car users then gain a cost decrease because the capacity effect dominates.
When a user switches from a normal to a robot car, her travel cost decreases by

$$\Delta C = C_r - C_n = -\delta \frac{N}{s} \left( (1 - F) (1 - \theta) \right) - \alpha \cdot TT_{ff} \cdot (1 - \theta).$$

This cost decrease falls with $F$: a higher $F$ means that there is less to gain in lowered bottleneck congestion cost since there are fewer normal car users who impose the lower externality.

Total travel cost is:

$$TTC = N(C_n(1 - F) + C_r \cdot F)$$

$$= \delta \frac{N^2}{s} \left( 1 + F^2 (1 - \theta) - F \left( 2 - \frac{1}{r} - \theta \right) \right) + \alpha \cdot TT_{ff} \cdot N \cdot (1 - F (1 - \theta)).$$

In the lower equation, the first part is the total bottleneck cost and the second is the cost due to free-flow travel time. The derivative of $TTC$ w.r.t. $F$ is

$$\frac{\partial TTC}{\partial F} = \delta \frac{N^2}{s} \left( 2 \cdot F(1 - \theta) - \left( 2 - \frac{1}{r} - \theta \right) \right) - (1 - \theta) \alpha \cdot TT_{ff} \cdot N.$$  

The effects of the parameters are as in the previous two subsections with the exception of $\theta$. A larger $\theta$ (i.e. a smaller VOT reduction due to getting a robot car) has two opposing effects on derivative (20), and the net effect is ambiguous. The first effect is that a higher $\theta$ makes switching to a robot car reduce own cost less and thus $\frac{\partial TTC}{\partial F}$ becomes less negative or more positive. The second effect is via the bottleneck externality. A normal car user imposes a smaller cost on robot car users than a robot car user, but, with a larger $\theta$, this difference is smaller and thus a person switching to a robot car imposes smaller cost increases. When $F < 1/2$, there are few existing robot car users that are harmed by increasing $F$, and the first effect via own cost always dominates and $\frac{\partial TTC}{\partial F}$ increases with $\theta$. When $F > 1/2$, there are relatively many robot car users that are
harmed and the effect of $\theta$ on $\partial TTC / \partial F$ becomes ambiguous. As we will see in the next section, this means that the effect of $\theta$ on equilibrium shares is also ambiguous.

The marginal external benefit due to switching to a robot car can be negative or positive:

$$MEB = \Delta C - \frac{\partial TTC / \partial F}{N} = \frac{\partial N}{s} \left( -F(1-\theta) + \left( 1 - \frac{1}{r} \right) \right).$$

(21)

It is negative when the heterogeneity effect dominates the capacity effect which occurs when $F(1-\theta) > 1 - 1/r$. The $MEB$ is more likely to be negative when $F$ is larger as then there are more robot car users that are hurt by increasing the share. A smaller $\theta$ means getting a robot car decreases the VOT more and this strengthens the heterogeneity effect and thereby lowers the marginal external benefit. When $r$ is larger, switching to a robot car brings a larger capacity gain and thus the $MEB$ is higher. Eq. (21) hence shows how these different forces jointly determine the overall $MEB$.

4. Provision regimes for the full model

Now we turn to determining the equilibrium outcome under three provision regimes: 1) marginal cost pricing, 2) socially-optimal public supply, and 3) profit-maximising monopoly provision. The provision of robot cars at (long-run) marginal production cost leads to overconsumption of robot cars if there is a negative externality from robot car purchase ($MEB<0$), and to underconsumption under a positive externality ($MEB>0$). Under private provision, as the market becomes more competitive the outcome would approach marginal cost provision. The other extreme of private provision is our monopoly.

Total cost, $TC$, equals total travel cost, $TTC$, plus total automobile cost due to buying robot cars instead of normal cars, $MC_r F N$, as the automobile cost of normal cars is normalised to zero:

$$TC = TTC + MC_r F N.$$  

(22)

With an interior solution of the car market, the travel cost reduction, $\Delta C$, due to a robot car is exactly offset by the sum of per trip extra marginal cost and mark-up of a robot car:

$$\Delta C + MC_r + MU_r = 0,$$

(23)

such that no user would like to switch car type. For a corner solution with no robot cars, the generalised price of robot cars should be above that of normal cars and thus $\Delta C + MC_r + MU_r > 0$, and vice versa when there are no normal cars. As we will see in our numerical analysis, corner outcomes are likely to occur, which is partly due to our assumption of homogeneous individuals and a homogeneous effect of robot cars on VOTs.
4.1 Marginal cost provision

Perfect competition leads to marginal cost provision of robot cars at marginal production cost, and thus a zero mark-up. Assuming an interior solution, (23) then leads to an equilibrium share of robot car users, $F^{MC}$, of

$$F^{MC} = 1 - s \frac{MC_r}{\delta N} \left( \frac{MC_r}{1 - \theta} - \alpha \cdot TT_{ff} \right)$$

$$= 1 - s \frac{1}{\delta N \cdot (1 - \theta)} \left( MC_r - \alpha (1 - \theta) \cdot TT_{ff} \right).$$

(24)

However, often there is a corner solution. It can be that nobody has a robot car, $F^{MC} = 0$, because the extra marginal cost of a robot car, $MC_r$, is excessive. It can also be that everyone has a robot car because $MC_r$ is low, or because free-flow travel time is very costly (i.e. a large $\alpha \cdot TT_{ff}$), or because $\theta$ is very small making a robot car very effective at lowering travel time costs. The term $s/(\delta \cdot N)$ is the inverse of the term $\delta \cdot N/s$ that scales the bottleneck cost. Assuming $F^{MC} < 1$ and thus $MC_r / (1 - \theta) - \alpha \cdot TT_{ff} > 0$, a higher $\delta \cdot N/s$ increases bottleneck costs and not surprisingly a larger share of users then wants a robot car in order to lower these cost.

4.2 Public provision

Public provision minimises total cost under the second-best limitation that there is no direct congestion pricing. In optimum and ignoring corner solutions, the derivative of total cost w.r.t. $F$ should be zero:

$$\frac{\partial TC}{\partial F} = \frac{\partial TTC}{\partial F} + MC_r \cdot N = 0.$$  

(25)

Combining f.o.c. (25) with condition (23) for the auto-market equilibrium implies a mark-up of

$$MU_r = \frac{\frac{\partial TTC}{\partial F}}{N} - \Delta C = -MEB.$$  

(26)

Therefore, the public operator imposes a negative or positive mark-up to correct for the externality from getting a robot car, where naturally a subsidy applies when $MEB$ is positive.

For an interior solution, the second order condition for minimization is fulfilled and we find

$$F^{pub} = \frac{s}{2N\delta} \left( \alpha \cdot TT_{ff} - \frac{MC_r}{1 - \theta} \right) + \left( \frac{2 - \frac{1}{r} - \theta}{2(1 - \theta)} \right)$$

$$= \frac{1}{2(1 - \theta)} \left( \frac{s}{N\delta} \left( \alpha (1 - \theta) TT_{ff} - MC_r \right) + 2 - \frac{1}{r} - \theta \right).$$

(27)
Just as with marginal cost pricing, the cost-minimizing $F^{pub}$ increases with $\alpha \cdot TT_{ff}$ and decreases with $MC_r$. The $F^{pub}$ also increases with the capacity-effect parameter $r$. The effect of $\theta$ is ambiguous since, as discussed, there are two counteracting effects: the first via the own travel cost and the second via the externality.

One may often expect $F^{pub}$ to exceed $F^{mc}$, because of a dominating positive externality. Yet, this is not always true, as we will also see in our numerical analysis, if $r$ is small, the heterogeneity effect may dominate and there is a negative externality due to robot car purchase. Alternatively, we may be in a corner solution with 0% or 100% robot car in both regimes. Assuming interior solutions, the difference in equilibrium shares of marginal cost and public provision is

$$F^{pub} - F^{MC} = \frac{1}{2(1-\theta)} \left( s \left( MC_r - \alpha (1-\theta) TT_{ff} \right) + \theta \frac{1}{r} \right),$$

if $0<F^{pub}<1$ and $0<F^{MC}<1$. (28)

The public supplier has a larger share compared to the marginal cost supplier when $r$ is higher as the public supplier wants to take advantage of the capacity effect, whereas under marginal cost provision car buyers ignore the capacity effect. When $MC_r$ is smaller or $\alpha (1-\theta) TT_{ff}$ larger, the robot car reduces own cost more and the equilibrium share in both regimes is larger, but this effect is smaller with a public supplier as it also considers the car purchase externality. The effects of $\theta$ is ambiguous.

4.3. Provision by a monopolist

The monopolist maximizes its profit, which (under the assumption of constant marginal production cost) equals the mark-up per trip multiplied by the number of robot cars:

$$\Pi = MU_r \cdot F \cdot N = (-\Delta C - MC_r) \cdot F \cdot N,$$ (29)

where $\Delta C$ gives how much lower the travel cost of a robot car is than of a normal one, and thus $-\Delta C$ measures the willingness to pay for a robot car. Maximisation of profit gives:

$$\frac{\partial \Pi}{\partial F} = 0 = -(-\Delta C + MC_r)N - \frac{\partial \Delta C}{\partial F} \cdot F \cdot N$$

$$= -\left( \frac{\Delta N}{s}(1-F)(1-\theta) - \alpha \cdot TT_{ff}(1-\theta) + MC_r \right) N - \left( \frac{\Delta N}{s}(1-\theta) \right) \cdot F \cdot N$$

$$= \left( \frac{\Delta N}{s}(1-2F)(1-\theta) + \alpha (1-\theta) TT_{ff} - MC_r \right) N.$$ (30)

Consistent with the aviation literature (e.g. Brueckner, 2002; Brueckner and Van Dender, 2008) and private road literature (e.g. Edelson, 1971; Verhoef, 2007; Wu et al. 2011), the monopolist internalises the externality that robot cars impose on each other. In (30), it considers that increasing $F$ raises the travel cost of existing robot car users, which lowers their willingness to pay.
the mark-up. Still, the monopolist ignores the capacity effect as this affects normal and robot car users equally and does not affect the willingness to pay for the robot car.

For interior solutions, the second order condition holds and the optimal share for a monopolist is half of that under marginal cost provision, where the later already often leads to undersupply:

\[
F_{\text{mon}}^{\text{opt}} = \frac{1}{2} \left( 1 - \frac{s}{\delta N (1-\theta)} \left( MC_r - \alpha \cdot TT_r (1-\theta) \right) \right)
\]

The corner solution \( F_{\text{mon}} = F_{\text{MC}} = 0 \) occurs in both regimes when the travel cost reduction due to a robot car is so small that nobody wants one (unless it is subsidised): \( \Delta C[F = 0] + MC_r \geq 0 \).

The monopolist supplies fewer robot cars than the public supplier does (unless we are in a corner outcome \( F_{\text{mon}} = F_{\text{pub}} = 0 \) or \( F_{\text{mon}} = F_{\text{pub}} = 1 \)):

\[
F_{\text{Mon}} - F_{\text{Pub}} = - \frac{r - 1}{2r (1 - \theta)}, \quad \text{if } 0 < F_{\text{Pub}} < 1 \text{ and } 0 < F_{\text{Mon}} < 1.
\]

As the numerical analysis will also show, both corner outcomes are unlikely. \( F_{\text{mon}} = 1 \) needs a rather negative \( MC_r \) and, as fuel savings can only be so large, this may require that robot cars must be cheaper than normal cars, which seems unlikely. \( F_{\text{pub}} = 0 \) needs a small \( r \) and an unrealistically large \( MC_r \). Ignoring the implausible corner outcomes, the monopolist supplies too few robot cars and there thus is an undersupply. For interior solutions, the degree of undersupply of a monopoly compared to the public case decreases with \( \theta \) and increases with \( r \).

5. Numerical example

This section presents results using two base-calibrations: one for the USA and one for the Netherlands, where the latter may be viewed as being representative for European countries. After discussing the calibration and the results for these base cases, this section finally turns to sensitivity analyses.

Compared with the USA, The Netherlands has a lower VOT and fewer driven kilometres before a car is sold second-hand. The Netherlands also has a much higher fuel price, which is partly offset by a higher fuel efficiency. These differences in country specific parameterisation will turn out to be important: we will find that for the USA calibration, marginal cost and public provision lead to the same outcome \( F_{\text{MC}} = F_{\text{Pub}} = 1 \), while for Dutch case marginal pricing supply leads to an underconsumption of robot cars.

The model outcome is sensitive to the parameterisation, and hence it is important to do sensitivity analyses. We focus on the effects of the \( MC_r, \theta, \) and \( r \). These parameters are specific to
the trade-off between a robot vs a normal car. Moreover, for the other parameters, we have more guidance for their values from the theoretical or empirical literature.

5.1. Calibration of the numerical models
We focus on petrol passenger cars. For both countries, the schedule delay parameters will be based on the ratios $\beta/\alpha$ and $\gamma/\alpha$ from Small (1982) as is common in the literature. As in Van den Berg and Verhoef (2011ab), we use $N=9000$ and $s=3600$. We consider a trip of 20km, with a free-flow travel time of 20 minutes. The VOT with a robot car must be larger than the value of schedule delay early: $\theta > \beta/\alpha = 39/64 \approx 0.61$. Therefore, the sensitivity analyses will vary $\theta$ between 39/64 and 1. Our base value is $\theta=0.8$, which means that, when switching to a robot car, the VOT is decreased by about half of what it can be decreased (before the regular equilibrium is replaced by one with a mass departure). Our base calibration assumes that robot cars lead to a doubling of capacity: $r=2$; the sensitivity analyses will make $r$ vary from 1 to 5. This seems reasonable, as our literature review found an average effect of about $r=2$ and predictions ranging from 1.07 to 3.7.

For the USA our value of time is $18.82/h$, which is the recommended VOT (US department of transportation, 2011) updated using 2013 income data (U.S. Census Bureau, 2014). Our base-case USA value of $MC_1$ is 1.13. For the USA in 2013, the fuel expenditure per mile was\(^{10}$ $0.13$, implying a fuel efficiency of 26 miles-per-gallon\(^{11}$ or 11km-per-litre. Let us assume that a robot car has a 20% higher fuel efficiency. This implies a fuel cost saving of a robot car of $0.31$ per trip. We further assume that the extra cost of a robot car is $7500$. We use that in 2013 a car was on average driven 13476 miles per year\(^{12}$ and assume that a new car is sold after 4.5 years. Then, the extra depreciation cost for a robot car becomes $1.54$ per trip. This results in the extra automobile cost per trip of $1.13$.

For The Netherlands, the recommended VOT is €10 (Kouwenhoven et al., 2014) and, following a similar calculation as for the USA, a $MC_1$ of €1.51 seems reasonable.\(^{13,14}$


\(^{13}\)The average daily km for people with a car is 41km (CBS, 2015a). We assume an extra purchase cost minus rest value of €5000 and people selling the car after 3.5 years (which is lower than for the USA as a large part of new Dutch cars are lease cars (leased by employers for their employees due to tax advantages) and are leased for a few years only). This results in a depreciation cost per trip of €1.91. Fuel savings would be €0.40 per trip, using an average fuel price for 2014 of €1.695/litre (CBS, 2015b) and 14km/litre fuel efficiency (http://gemiddeldgezien.nl/meer-gemiddelden/68-gemiddeld-verbruik-auto accessed on 14 February 2015) and again a 20% higher fuel efficiency for a robot car.

\(^{14}\)In our numerical welfare evaluation we ignore that a part of the fuel price is tax. Including this would complicate the modelling but would not change the individual’s choice. It may affect the choice of the public supplier, but in our bases cases it does not as the fuel part in the automobile cost is small and in both bases cases public optimum will turn out to far into the corner solution of 100% robot cars anyway. Ignoring the tax part does affect the cost savings but even there the effect is small. For instance, in The Netherlands, taxes form about 64% of the fuel price (www.nl.wikipedia.org/wiki/Benzineaccijns accessed on 15 April 2015), removing the reduction in tax payments from the cost saving would reduce these cost savings in the public case by only 3.7%.
5.2. Base-case

Table 2 tabulates the outcomes for the base calibrations for the USA and The Netherlands. It shows four cases: no robot cars, perfectly competitive provision at marginal production cost, second-best public provision that minimises total cost, and monopoly provision that maximises the profit of the robot car manufacturer. TTC is the total travel cost and TC is the total cost including automobile cost; %ΔTTC and %ΔTC are the corresponding percentage changes in totals from the case without robot cars. Relative efficiency is the total cost saving of a policy from the case without robot cars divided by the corresponding cost reduction due to public provision.

| Table 2: Outcomes under the base calibrations for the USA and The Netherlands |
|-----------------------------------------------|-------------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                               | USA               |                               | The Netherlands |
|                               | No robot cars     | Marginal cost | Public | Monopoly | No robot cars | Marginal cost | Public | Monopoly |
| Share with a robot car        | 0                 | 1                | 1    | 0.51     | 0               | 0.65           | 1    | 0.33 |
| Total travel cost (TTC)       | 261839            | 147857           | 147857 | 193030   | 139128         | 94675          | 78563.9 | 114580 |
| %ΔTTC                         | -                 | -44%             | -44%  | -26%     | -               | -32%           | -44%  | -18%  |
| Total cost (TC)               | 261839            | 158027           | 158027 | 198254   | 139128         | 103539         | 92153.9 | 119012 |
| %ΔTC                          | -                 | -40%             | -40%  | -24%     | -               | -26%           | -34%  | -14%  |
| Relative efficiency#          | 0                 | 1                | 1    | 0.61     | 0               | 0.76           | 1    | 0.43 |
| Price per trip for a robot car| -                 | 17.56            | 10.71 | 24.37    | -               | 11.50          | 6.60  | 14.01 |
| Price per trip for a normal car| 29.09            | 17.68            | 17.68 | 24.37    | 15.46           | 11.50          | 9.40  | 14.01 |
| Mark-up                       | -                 | 0                | -6.85*| 2.34     | -               | 0             | -3.64*| 0.79  |
| Marginal external benefit     | -                 | 6.85             | 6.85  | 9.07     | -               | 4.48           | 3.64  | 5.27  |
| Difference in cost between robot car and normal car | - | -0.12 | -0.12 | -2.34 | - | 0 | 0.84 | -0.79 |
| Corner solution?              | -                 | yes              | yes   | no       | -               | no            | yes   | no   |

Note: *As there is a corner solution, the public agency could ask any mark-up that is not higher than the difference in cost between the normal and robot car. The table assumes that the mark-up equals the negative of the Marginal External Benefit.

*Relative efficiency is the welfare gain of a policy from the case without robot cars divided by the gain from public provision.

The introduction of robot cars substantially lowers costs and even with a monopolist users are better off than without robot cars (as they can always choose to continue to use normal cars). This is even though the monopolist asks a substantial mark-up per trip of $2.34, which implies a 60% profit rate (i.e. profit over revenue). Costs are higher for the USA than for the Netherlands as the USA has a higher value of time, which also increases the values of schedule delay in our calibration.

For the USA, marginal cost provision leads to the same corner solution of 100% robot cars as under public provision. The robot car has a lower cost (i.e. travel cost plus automobile cost) than the normal car, and hence everybody wants one. For the Netherlands, marginal cost provision
results in too few robot cars as users ignore the positive externality. This MC regime attains only
76% of the cost savings due to public provision (relative to the case without robot cars).\footnote{The table assumes that the public supplier always adds a mark-up equal to \(\text{MEB} \). Yet, if there is a corner solution, it could add any mark-up below the difference in cost between car types. For the USA, the public agency could even ask a positive mark-up of \$0.12 and make a profit; for the Netherlands, the mark-up must be below \(-0.84\) and could hence be much less negative than \(-3.64\).}

For the USA, a realistic lower bound for \(MC_r\) seems \(-\$0.75\) (using a fuel efficiency gain of
50%, an extra purchase cost of \$1000, and a usage of 5.5 years). A reasonable upper bound seems
\(MC_r=\$4\) (using a 5% higher fuel efficiency, an extra purchase cost of \$15000, and 3 years of
usage). Under the lower bound case for the USA of \(MC_r=\$0.75\), the outcome is similar as in the
main case: both marginal cost pricing and public provision lead to 100% robot cars. Monopoly
supply has a similar relative efficiency as in Table 2, although the fraction with a robot car is
larger. For the upper-bound case of \(MC_r=\$4\), public supply still leads to 100% robot cars but this
requires a very large subsidy. Accordingly, robot cars need to be very costly for it not to be
socially worthwhile for everyone to get one if the capacity effect is as large as \(r=2\). As we will see
later on, when \(r\) is lower, it will be optimal that only some, or even no, users get a robot car. In the
upper-bound case, marginal cost and monopoly provision lead to a small fraction with a robot car
and these regimes have low relative efficiencies.

5.3 Sensitivity with respect to \(\theta\) and \(MC_r\).

The previous subsection looked at two base calibrations. Now, we will turn to the outcome over
wider ranges of the parameters. The focus will be on the effect of the parameters on the share with
a robot car under the different provision regimes. This subsection varies the VOT reduction
parameter, \(\theta\), and it varies the extra marginal automobile cost of a robot car, \(MC_r\), from \(-2\) to \(8\). A
\(MC_r\) below \(-1\) will tend to imply that robot cars are cheaper to produce than normal cars, as fuel
savings can only be so large. Subsection 5.4 will vary \(r\) and \(\theta\). The effects of the other parameters
were in-line with the theoretical discussion (see the appendix for the numerical sensitivity
analysis).

Fig. 5 shows the equilibrium share with a robot car for marginal cost (left panel), public
(middle panel) and monopoly provision (right panel). The graph is for the Dutch VOT of €10,
where the other parameters are the same for the USA and The Netherlands. Using the VOT for the
USA leads to very similar pictures but that for public supply the \(MC_r\) needs to be above 7 for the
share to be below one; therefore, to get an interesting picture instead of only just white, we would
need to consider very unrealistic parameter values. The darker the area of a contourplot is, the
lower the share with a robot car; a white area is for the corner solution \(F=1\), and a black area for
\(F=0\).
Fig. 5 shows that, when the capacity effect is large enough with \( r=2 \), public supply leads to 100% robot cars for reasonable values of \( \theta \) and \( MC_r \). Marginal cost supply tends to lead to a large share with a robot car as well. If \( MC_r \) and \( \theta \) are low enough, marginal cost pricing and public provision both lead to 100% robot cars. Still, for higher, but plausible, values of \( MC_r \), marginal cost provision leads to a below-optimal share with a robot car. Monopoly provision of robot cars leads to an even larger undersupply, and hence even larger welfare loss. Only when \( MC_r<0 \) (which means that the robot car has a lower automobile cost than a normal car) and when \( \theta \) is large enough (which means that the effect on the VOT of a robot car is limited) does the monopoly not lead to too few robot cars, as then all regimes lead to 100% robot cars. For interior solutions, the monopoly leads to half as few robot cars as marginal cost pricing. Marginal cost and monopoly provision lead to 0% robot cars for the exactly same black parameter range, as in this range it always raises own cost to get a robot car.

As argued, the effect of \( \theta \) on the equilibrium share is ambiguous as there are two opposing effects: a higher \( \theta \) means that a robot car reduces free-flow travel time costs less and this lowers the equilibrium \( F \), but it also lessens the heterogeneity effect and this may raise the equilibrium \( F \).

**Fig. 5: Effect on share of robot cars of \( MC \), and \( \theta \): the left panel is for marginal cost provision, the middle panel for public provision and the right panel for monopoly provision**

![Three panels showing the effect of \( MC \) and \( \theta \) on the share of robot cars](image)

Note: The darker the area of a contourplot is, the lower the share with a robot car; a white area is for \( F=1 \), and a black area for \( F=0 \).

Figs. 6 and 7 compare the welfare effects of the regimes. Monopoly supply can clearly lead to a large welfare loss compared to public supply. But also marginal cost supply, when it does not result in the same fraction as public supply, can lead to substantial welfare losses even when the difference in equilibrium fractions is not that large. In this sensitivity analysis, marginal cost and public provision only lead to the same outcome when there are corner solutions, otherwise marginal cost provision leads to an undersupply as the positive capacity externality dominates.
Fig. 6: Effects of $MC_r$ and $\theta$ on percentage change in total cost (from with only normal cars) of marginal cost provision (left panel), public provision (middle panel) and monopoly provision (right panel)

<table>
<thead>
<tr>
<th>Left: $%\Delta TC$ for marginal cost provision</th>
<th>Middle: $%\Delta TC$ for public provision</th>
<th>Right: $%\Delta TC$ for monopoly provision</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
</tr>
</tbody>
</table>

Fig. 7: Effects of $MC_r$ and $\theta$ on the relative efficiency, $\Omega$, of marginal cost provision (left panel) and the monopoly provision (right panel)

<table>
<thead>
<tr>
<th>Left: relative efficiency of marginal cost provision</th>
<th>Right: relative efficiency of monopoly provision</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4" alt="Graph" /></td>
<td><img src="image5" alt="Graph" /></td>
</tr>
</tbody>
</table>

Note: Relative efficiency is the cost reduction of a policy from the case without robot cars divided by the reduction from public provision. The relative efficiency of the public regime is by definition 1.

5.4. Sensitivity analysis with respect to $\theta$ and $r$.

Next, we look at the impact of the capacity-increase parameter, $r$. We range $r$ from 1 to 5 while varying $\theta$ over all possible values. Fig. 8 depicts the shares with a robot car for the three regimes. Consistent with the analytics, $r$ has no effect on the share of robot cars under marginal cost provision or monopoly provision, as the share is then solely determined by $MC_r$ and the cost gain, $\Delta C$, that a user gets when switching car type and these are independent of the capacity effect. Therefore, users ignore the external capacity effect. The value of $r$ does affect $P^{pub}$. For most of the range of $r$, the public supplier ensures that everyone gets a robot car. Only when, the capacity effect is relatively small (e.g. a 50% increase in capacity), is it optimal for not everyone to be equipped. It should, however, be noted that $r=1.5$ is within the plausible range for this parameter.

Fig. 9 (left panel) compares the share under marginal cost provision and public provision. It does this for a smaller range of $r$ than Fig. 8 as the pattern for $r>2$ looks the same as for $r=2$, while for lower values of $r$ there are interesting effects. The difference in shares is largest when $\theta$ is large and the $r$ is not too small. Then, marginal cost provision leads to 0% robot cars as the individual
gains little from getting one; conversely, the public supplier will ensure that the share is large as there is a net positive externality. In the bottom left corner of the figure (for instance when $\theta=0.7$ and $r=1.25$), there is a net negative externality and marginal cost provision leads to overconsumption of robot cars. In this area, $\theta$ is small and getting a robot car thus leads to a large reduction in own cost, whereas $r$ is small and hence society hardly gains in terms of increased capacity. As the right panel of Fig. 9 shows, monopoly provision never leads to overconsumption and only leads to the similar share as public provision when $r$ is very small.

**Fig. 8: Effect of $r$ and $\theta$ on equilibrium shares of robot cars: the left panel is for marginal cost provision, the middle panel for public provision and the right panel for monopoly provision**

Note: The darker the area of a contourplot is, the lower the share with a robot car; a white area is for $F=1$, and a black area for $F=0$.

**Fig. 9: Effect of $r$ and $\theta$ on differences between equilibrium shares of robot cars: left panel is marginal cost vs public provision and the right panel monopoly vs public provision**

We also repeated this analysis first with a lowered $MC_r$ of $-2$ and then with an increased $MC_r$ of 7 (see the appendix), whereas here $MC_r$ was 1.51. When $MC_r=-2$, it always reduces own cost to get a robot car, as this always lowers automobile cost while it never increases travel cost. Consequently, marginal cost provision always leads to 100% robot cars. Conversely, when $MC_r=7$, getting a robot car always increases own cost, and both marginal cost pricing and monopoly
supply lead to 0\% robot cars. Still, if the capacity effect is not too small, there is a positive externality from getting a robot car and accordingly the socially optimal share may still be positive and needs a relatively large subsidy. Besides these changes, the effects of the parameters are qualitatively the same as in the main sensitivity analysis here.

6. Model extensions

We used the simplest possible dynamic model to investigate the effects of robot cars via increased capacity, a lower VOT, and the resulting VOT heterogeneity. There are number of logical extensions and it seems worthwhile to discuss how these might affect our results.

First, it seems plausible that the extent to which the VOT is reduced by getting a robot car will differ over people. Some office workers may spend the freed-up time preparing documents or checking email becoming nearly as productive as in the office. Conversely, manual workers probably face some difficulty in using the freed-up time working, but may spent the travel time on leisure activities. A businessperson may already be using the time in a normal car in a productive way, and hence buying a robot car may bring only modest benefits. In any case, the parameter \( \theta \) is likely to vary over individuals. This would then mean that users self-select: the users for whom a robot car reduces travel cost more (i.e. with a lower \( \theta \)) are more likely to get one. This also means that increasing the share with a robot car may be less beneficial. For a high enough share, the marginal user who switches has a relatively small reduction in VOT and thus a small cost saving. Heterogeneity in \( \theta \) may thus strengthen the VOT heterogeneity effect, as there will be more heterogeneity. Finally, a heterogeneous \( \theta \) gives marginal cost and monopoly supply an extra disadvantage: these suppliers only consider the marginal user in deciding their \( F \), whereas the public supplier also considers the average user who has a lower \( \theta \) and thus lower VOT than the marginal user. Such an extra difference between public and private supply is common with heterogeneous users.

There may also be a pre-existing heterogeneity in the values of time and/or schedule delay. Then, users arrive ordered by the heterogeneous ratios \( \beta_i / VOT_i \) and \( \gamma_i / VOT_i \), which are now heterogeneous for two reasons: the pre-existing heterogeneity and the differences in car type. Pre-existing heterogeneity may lessen the heterogeneity effect of robot cars, as robot cars may add relatively little in heterogeneity and heterogeneity always remains. It may also strengthen the heterogeneity effect, as the last users to switch to a robot car will often have the lowest own cost reduction. Pre-existing heterogeneity also means that some normal and robot car users may have the same ratio \( \beta_i / VOT_i \) or \( \gamma_i / VOT_i \), which would mean that they travel mixed, reach the bottleneck at the same time, and are on the same iso-cost curve.

Many studies have found that the capacity effect increases with the share that has a robot car. Going from no robot cars to a few has little capacity benefit, as there are few robot cars to
cooperate with and they have to travel surrounded by normal cars. Going from mostly robot cars to 100% robot cars bring large gains (Tientrakool et al., 2011). This consideration could be included by making \( r \) an increasing function of the share of robot cars in total or at the time of reaching the bottleneck. This may have limited effects on the marginal cost and monopoly regimes as these providers ignore the capacity effect anyway. It would affect the public optimum, but it may depend on the parameterization whether the share is higher or lower with the increasing \( r \) than with the fixed \( r \). However, our model implies that robot cars may cluster in time (as their preferences are more similar), which would raise the capacity effect during their travel moments.

7. Conclusion

We investigated the effects of introducing robots cars (also referred to as autonomous or automated cars) on costs of travel via a number of distinct channels. These include the increase in capacity, the decrease in values of time (VOTs), and the resulting heterogeneity in VOTs. The share of users with a robot car was endogenous as we incorporated the equilibrium of the car purchase market. We considered three provision regimes: perfect competition leading to marginal cost pricing, second-best public provision and monopoly supply.

Buying a robot car instead of a normal one raises road capacity and thus imposes a positive externality; but it also lowers the user’s VOT and, as the bottleneck congestion externality (under heterogeneity) decreases with the user’s VOT (see also Lindsey, 2004), buying a robot car thus also imposes a negative externality. The net externality may be positive or negative. The numerical analyses for the USA and The Netherlands suggest that a net positive externality is most likely and occurs unless the capacity effect is small with say a 25% increase in capacity from robot cars. In the engineering literature, the average predicted effect is about a 100% increase in capacity, still predictions around 25% or lower do occur.

If there is a positive external benefit due to buying a robot car, this means that marginal cost pricing tends to lead to underconsumption of robot cars. To prevent this, and attain to the second-best optimum, the public supplier needs to give a subsidy. However, it may also occur that there is a negative externality and then a corrective tax is needed to prevent overconsumption. The private monopolist is likely to lead to a large undersupply and welfare loss, unless robot car have lower automobile cost than normal cost, which seems unlikely.

The results are sensitive to the model’s crucial parameters. For instance, in the USA base case, public and marginal cost supply both lead to a corner solution of 100% robot cars, whereas, for the Dutch base case, marginal cost supply leads to only 76% robot cars and public supply to a 100%. There is also great uncertainty about these parameter values, and this makes investigating them an important future research topic.
We already discussed interesting modelling extensions in the penultimate section. But there are also important and interesting policy and market structure extensions. We only analysed unpriced congestion. First-best pricing removes the negative heterogeneity externality, while the positive capacity externality remains unless we can differentiate the toll between normal and robot cars. However, what happens under step tolling? We only consider one supplier, but, in reality, there will be multiple suppliers of robot cars, which may also supply normal cars. Will these suppliers face incentives to let their robot cars cooperate or would it be more profitable to keep them incompatible? And what about supporting infrastructure for robot cars along the road? Will each car supplier need to build its own? Finally, the introduction of robot cars will have great effects on public transport and taxi transport, and in the long-run it may even affect the structure of the city. These questions deserve attention in future research efforts.

Acknowledgements
We thank the participants of the OPTION conference (2015) and Eureka seminar (2015) in Amsterdam (2015). We thank Sylvia Bleker for valuable comments. Financial support from the ERC (AdG Grant #246969 OPTION) is gratefully acknowledged. All mistakes are ours.

Appendix. Further sensitivity analyses
A.1. Effect of $\theta$ and $r$ for different automobile costs
This section of the appendix repeats the analysis of subsection 5.4 of changing $\theta$ and $r$ but for a lower $MC_r$ of $-1$ and for a higher $MC_r$ of $7$. The main text discusses the effects and the changes from the main sensitivity analysis.

Fig. A.1: For $MC_r=-1$, the shares of robot cars over $r$ and $\theta$: the left panel is for public provision and the right one for monopoly provision (marginal cost provision always has a share of one)

Note: We omit the figure for marginal cost provision. When $MC_r<0$, marginal cost provision means that all users always get a robot car.
A.2. Effects of changing $\delta \cdot N/s$

Sections A.2 and A.3 look at changing the other parameters besides those specific to the robot car setting by respectively changing $\delta \cdot N/s$ and $\alpha \cdot TT_f$. For the USA base case, $\alpha \cdot TT_f$ was about 6.23 and $\delta \cdot N/s$ about 22.82; for the Dutch case, these respectively were approximately 3.33 and 12.13. It does not matter how we change $\delta \cdot N/s$, what matters is the combined size of the term. Doubling $\delta$ and halving $N$ keeps $\delta \cdot N/s$ constant and means that nothing really changes. Increasing $\delta \cdot N/s$ raises bottleneck cost, making it more attractive to get a robot car and thus, for an interior solution, $F$ increases. If $\theta$ is high, however, there is still little to gain in reduction in own cost from getting a robot car and thus then marginal cost and monopoly provision lead to 0% robot cars, regardless of the size of $\delta \cdot N/s$.

Fig. A.3: Effect of $\delta \cdot N/s$ and $\theta$ on the shares of robot cars: the left panel is for marginal cost provision, the middle panel for public provision and the right panel for monopoly provision.
A.3. Effects of changing $\alpha \cdot TT_{ff}$

When $\alpha$ or $TT_{ff}$ increases, getting a robot car lowers free-flow travel cost more and thus the share with a robot car becomes higher. Public supply leads to 100% robot cars for almost the entire parameter range, only for when $\theta$ and $\alpha \cdot TT_{ff}$ are both very low does it not.

Fig. A.4: Effect of $\alpha \cdot TT_{ff}$ and $\theta$ on the shares of robot cars: the left panel is for marginal cost provision, the middle panel for public provision and the right panel for monopoly provision

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