Schinkel, Maarten Pieter; Tóth, Lukás; Tuinstra, Jan

Working Paper
Discretionary Authority and Prioritizing in Government Agencies

Tinbergen Institute Discussion Paper, No. 15-058/VII

Provided in Cooperation with:
Tinbergen Institute, Amsterdam and Rotterdam

Suggested Citation: Schinkel, Maarten Pieter; Tóth, Lukás; Tuinstra, Jan (2015) : Discretionary Authority and Prioritizing in Government Agencies, Tinbergen Institute Discussion Paper, No. 15-058/VII, Tinbergen Institute, Amsterdam and Rotterdam

This Version is available at:
http://hdl.handle.net/10419/111731
Discretionary Authority and Prioritizing in Government Agencies

Maarten Pieter Schinkel
Lukas Toth
Jan Tuinstra
Tinbergen Institute is the graduate school and research institute in economics of Erasmus University Rotterdam, the University of Amsterdam and VU University Amsterdam.

More TI discussion papers can be downloaded at http://www.tinbergen.nl

Tinbergen Institute has two locations:

Tinbergen Institute Amsterdam
Gustav Mahlerplein 117
1082 MS Amsterdam
The Netherlands
Tel.: +31(0)20 525 1600

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

Duisenberg school of finance is a collaboration of the Dutch financial sector and universities, with the ambition to support innovative research and offer top quality academic education in core areas of finance.

DSF research papers can be downloaded at: http://www.dsf.nl/

Duisenberg school of finance
Gustav Mahlerplein 117
1082 MS Amsterdam
The Netherlands
Tel.: +31(0)20 525 8579
Discretionary Authority and Prioritizing in Government Agencies

Maarten Pieter Schinkel, Lukáš Tóth and Jan Tuinstra

May 2015

Abstract

Government agencies typically have a certain freedom to choose among different possible courses of action. This paper studies agency decision-making on priorities in a principal-agent framework with multi-tasking. The agency head (the principal) has discretion over part of the agency’s budget to incentivize his staff (agents) in the pick-up of cases. The head is concerned with society’s benefits from the agency’s overall performance, but also with the organization’s public image as formed from its case record and various non-case specific activities. Based on their talent and the contracts offered by the head, staff officials choose which type of task to pursue: complex major, yet difficult to complete cases with an uncertain outcome, or basic minor and simple cases with a much higher probability of success. The size of the agency’s discretionary budget influences not only the scale, but also the type of tasks it will engage in. Social welfare is non-monotonic and discontinuous in the agency’s budget. Small changes in the budget may cause extensive restructuring from major to minor tasks, or vice versa. A budget cut can improve welfare more than extra budget would, even if resources are below the welfare-maximizing level. For lower binding budgets, the head continues to suboptimally incentivize work on complex tasks, when the agency should have shifted down to simpler tasks. Yet a reluctant head may need to be nudged with more resources to pursue productive cases. In determining the discretionary space of the agency head, government can limit the extraction of resources, but thereby also benefits less from the head’s expertise. Antitrust authorities serve as one illustration of policy implications for institutional design.

JEL-codes: D02, H11, L44, M52

Keywords: Government agency, discretion, budget, enforcement priorities, antitrust

*We thank Alberto Alesina, Jonathan Baker, Gergely Csorba, Franklin Fisher, Germain Gaudin, Joseph Harrington, Morten Hviid, William Kovacic, John Kwoka, Giorgio Monti, Enrico Perotti, Daniel Sokol, João Vareda, Bauke Visser, Wouter Wils and conference participants at CLEEN 2013, CRESSE 2013, EARIE 2013, “The Reform of Competition and Regulatory Authorities in Europe,” University of Barcelona, 2013, the Haifa-Loyola joint workshop 2014 at Haifa University and IIOC 2015 for comments to earlier versions of this paper. All remaining errors are ours.

†Amsterdam School of Economics, ACLE and Tinbergen Institute, University of Amsterdam. Correspondence at: Plantage Muidergracht 12, 1018TV Amsterdam, The Netherlands. E-mail: m.p.schinkel@uva.nl.

‡Amsterdam School of Economics and ACLE, University of Amsterdam.

§Amsterdam School of Economics, CeNDEF and Tinbergen Institute, University of Amsterdam.
1 Introduction

Government agencies are organizations in the machinery of government with a certain amount of autonomy and independence from political influence in the execution of their functions in oversight and administration. Examples are central banks, intelligence agencies, internal revenue services, antitrust authorities, public prosecutors, energy regulators and gambling control boards. The laws these institutions enforce typically leave them considerable freedom to choose among different possible courses of action according to their own judgment. Principally tasked with decision making in specific cases, government agencies have varying levels of discretion over how to prioritize potential matters to pursue, how to conduct investigations, and what remedies to impose upon a finding of a breach of law.

The agencies also have other, non-case specific concerns of impression management, that is, self-presentation through professional communication and public relations directed at forming the organization’s public image. To a government agency, image is particularly important, as their tasks, be it controlling inflation, terrorism, money laundering, cartels or gambling schemes, by their nature often are enigmatic to the general public. Impression management helps secure public support for the agency’s stately goals and tasks - and so indirectly also its future budget.

Like all organizations, government agencies are networks of principal-agent relationships, and therefore riddled with agency issues that influence the allocation of resources over various tasks. Economic theory has long studied goal mismatches and incentive-provision schemes in principal-agent relationships within firms.\(^1\) While there have been calls for the introduction of incentive contracts into public organizations as well, the canonical principal-agent model is not directly transferable to non-profit organizations.\(^2\) Government agencies differ from profit-maximizing firms in important aspects, including external (political) resource assignment and difficult measurement of output. These characteristics allow for other civil servant incentives than just serving social welfare to take hold, among which mission-motivation, empire-building and conformity are known to induce suboptimal spending, bureaucratic slack and promotion of third-party interest.\(^3\)

In this paper we study in an agency-context how variations in its available resources can qualitatively change the range of activities the government agency will engage in, and how this will affect welfare. In a principal-agent model with multi-tasking, the agency has a two-level organizational structure. The head of the agency (principal) is concerned with society’s benefit from the agency’s overall performance, but also with impression management. He may seek high-profile cases for the exposure these will generate, but instead may also be lobbied or captured to avoid certain matters. The agency’s employed staff of officials (agents) can choose, individually or as a team with some autonomy, from various types of activities. The tasks vary in their expertise requirements and yield of social welfare gains upon completion, the probability of which is a function of staff effort and complexity. The

---


2 See Barzelay (2001).

officials are all permanently employed at a fixed wage, but the head has discretion over part of the agency’s budget to offer additional rewards to further incentivize his staff. These variable contracts can contain explicit incentive pay, but typically also the value of future career perspectives, in- and outside the agency, schooling opportunities, or tertiary benefits officials enjoy, such as participation in the agency’s international network, research projects, summer courses or conferences abroad. Depending on the contract offered, the agency officials either pursue the more demanding and complex high-profile cases, or opt to do simpler basic tasks. The head spends the budget residue that ends up not being awarded to the staff on other purposes, which typically do not have comparable social benefits. The expected residue decreases both in the size of the rewards and in the probability of the tasks being completed successfully.

We identify how the size of the head’s discretionary budget qualitatively affects the type of activities a government agency will perform. Small changes in the budget can have drastic consequences for society’s benefits from agency enforcement. Depending on the institution’s status, discretionary budget changes over certain thresholds can cause extensive restructuring, both away from and towards activities that require more expertise and yield a major outcome. Social welfare is non-monotonic and discontinuous in the budget, as a result, so that at a jump discontinuity an infinitesimal budget cut may lead to a substantial increase in welfare, beyond the direct fund savings. At other budget levels, however, more discretionary spending increases welfare. For lower binding budgets, for example, the head sub-optimally incentivizes its staff to work on complex tasks for too long, while the agency should already have shifted down to simpler tasks that fit the limited budget. These insights underline the importance of socially optimal budgeting for government agencies. They also reveal how institutional design and budget assignments can become a control tool in the arsenal of a government pursuing political goals or promoting third-party interests against the agency’s public tasks. The latter constitutes another principal-agent setting, in which the budget-setter is the principal and the agency head is the agent, for the analysis of which this paper lays some foundations.

Our approach adds to the literature on contracting in government agencies by considering both optimal and feasible agency choice of task and performance with a given budget constraint and a multi-level hierarchy. The economic literature on governmental organization has predominantly focused on political control of government agencies, as in Macey (1992), incentives created by sharing regulatory rights between several regulatory bodies, as in Martin (1996), and regulatory competition or collusion with separation of powers, as in Tirole (1986) and Laffont & Martin (1999).

We set up the objective of the agency head as a combination of social benefits and self-interest. Niskanen (1968) points at the incentives of bureaucrats for self-interested budget maximization and empire-building. Stigler (1971) and later Laffont & Tirole (1991) identified how agencies created to

4 Throughout the paper we use “task” and “case” interchangeably. While working on a case will typically entail multiple tasks that need to be prioritized over, that but moves our analysis to a lower level of the decision-making process.

5 See also Niskanen (2007).
act in the public interest, may be captured instead to advance the commercial or special concerns of interest groups that dominate the industry. Leaver (2009) reveals how a minimal squawk theory, in which officials try to minimize their mistakes for fear of being publicly marked as incompetent, rather than maximize the social welfare can explain institutional behavior. Kreps (1997), Murdock (2002) and Benabou & Tirole (2003) endogenize intrinsic motivation, particularly in public services and nonprofit organizations. Francois (2000, 2003) stresses the importance in these institutions of mission-motivation (a desire to promote the agency’s goals) and warm-glow utility (a desire for positive appraisal).

Dewatripont et al. (1999) derive implications for the optimal incentive provisions when civil servants are largely driven by career concerns and mission-motivation. Alesina & Tabellini (2007) and Alesina & Tabellini (2008) study consequences for the type of tasks a politician should delegate to career-concerned bureaucrats, if the agency head has either bureaucratic or political concerns. Makris (2009) shows, in an analysis of the effect of budget changes by a principal on a single mission-motivated agent, that the application of standard incentive contracts to government agencies may lead to a suboptimal provision of public services.

Our model provides a formal context to arguments put forward in an emerging legal and public administration literature on institutional design. Competition authorities, such as the U.S. Department of Justice’s Antitrust Division, the Federal Trade Commission and the European Commission’s Directorate General Competition fit our model particularly well. They are visible and relatively transparent agencies. The U.S. and European competition law principles are generally formulated, whereas cases are specific, various and regularly novel. As a result, competition authorities have considerable discretion in enforcement, which is reflected in vast case law. Some secondary and tertiary incentive pay for officials, on top of fixed wages, is common in these agencies. Both across and within types of anti-competitive behavior, there is variation in expertise requirements and yield of social welfare gains upon completion of tasks. Monopolization or abuse of dominance are often harder, take longer and are more complex to prosecute successfully than collusion, which is per se illegal by object. Within the class of cartel investigations, there is a choice of resource allocation between cases that were brought to the attention of the authorities by a leniency application of a remorseful cartel member for amnesty, and actively detecting the better organized cartels that manage to keep their ranks closed. The extent to which antitrust agencies should rely on the less burdensome leniency cases for the public good depends on staff talent, budget and the difficulty of independent discovery.

Antitrust agencies furthermore display a number of non-case specific activities. With large interests at stake, competition cases are extensively debated, both professionally and in the popular press. Kovacic et al. (2011) document that competition authority heads have concerns other than social welfare alone, including appearing "being busy", with an eye to the media and political superiors. Indeed do competition authority heads appear to value attention, giving interviews and contributing

---

6See, for example, Muris (2005), Crane (2011) and Kovacic & Hyman (2012).
7See Wils (2011).
regularly in conferences on landmark decisions or developments in enforcement tools. In addition, there are numerous competition policy outreach products, including enacted movies, online games and manga comic books, that agencies spend resources on. The authors express a worry that competition authorities may be wasting effort on big good-looking cases and image, while under-performing in other tasks like suggesting legislation refinements and staff preparation for advanced enforcement. Our model of multitasking in the principal-agent setting support some of these and other concerns.

Competition cases against known companies are sure to attract media attention right from the opening of the investigation - often even more so than from its conclusion. There are examples of zealously pursued high-profile cases that eventually ended without a forceful application of the law. Throughout the 1970's, for example, the US DoJ unsuccessfully prosecuted IBM for over ten years for violation of Section 2 of the Sherman Act, to ultimately conclude in 1982 that the case was without merit. In hindsight, some of the senior officials involved in the decision to pursue this case admitted other motivations than just expected social gain. A more recent ambitious case that ultimately inflated was around Google's alleged "search bias". After an extensive investigation, the FTC concluded that the evidence in the case was not enough to support a challenge of Google for monopolization under American law. The chairman of the FTC at the time, Jon Leibowitz, explained so in a lengthy press conference that was broadcasted live on national television. The European Commission's parallel investigations for abuse of dominance by Google continued, underlining the importance of agency governance in the pursuit of cases.

Studying antitrust authorities, Hyman & Kovacic (2013) stress a need for "engineering" strategies for the organization of government. They point at resource allocation as one key element of government agency functioning and observe that:

"[...] to do multiple things well requires both sufficient capacity and continuous fine-tuning of the agency’s allocation of resources [...] Some areas will flourish while other will languish – even if budgets keep pace with new responsibilities." Hyman & Kovacic (2013), p. 20.

Our analysis can shed light on how such fine-tuning may be done when the choices over agency’s priorities are driven by the motivations and constraints of the actors involved. In addition, we show that the transmission from budget to priorities pivots around the head.

The remainder of this paper is organized as follows. The basic model is introduced in Section 2. In Section 3, the main results under a budget restriction are presented. Welfare implications are discussed in Section 4. In Section 5, the model is extended to multiple tasks and officials. Section 6 analyzes

---

8Some of these and other examples of marketing products can be found on the ICN’s web-blog on "outreach". Such outreach spending can be substantial. Monti (2014) states that agencies in countries where the switch to competitive markets has been recent can spend up to 50% of resources on advocacy.

9See Fisher et al. (1985).

10See the contribution by Fredric M. Scherer in Slottje (1999).
the problem of the budget-setting body in government to determine the discretionary space it wants to give the head. In Section 7 several implications for institutional design are discussed, including an optimal level of autonomy from government in spending, returning to competition authorities as an illustration. Section 8 concludes. The proofs are given in an appendix.

2 A Model of Government Agency

Consider a government agency, which has a head and officials, that can undertake several classes of activities. Each of these $n$ tasks $i \in I$, with $I = \{1, \ldots, n\}$, is characterized by a double $(\psi_i, d_i)$, where $\psi_i \geq 0$ represents the difficulty of task $i$ and $d_i \geq 0$ are the social benefits that the task yields upon successful completion. A complex high-profile case, with the potential of becoming a landmark case, has a high value of $d_i$, while a a low $d_i$ is associated with a simple basic task. Agency officials differ in their skills, knowledge and talent level, captured by parameter $\theta \in [0, 1]$, which is known both to them and to the head.$^{11}$ The agency is assigned a budget, which consist of two components. A generic part of the budget pays for such costs as employing the head and the officials on fixed wage contracts for regular work, support staff, overhead for facilities, and other expenses. An additional part $D$ of the budget is at the head’s sole discretion. He can use it for motivational rewards for his officials to take on additional tasks, or for impression management of his agency’s public image.

The agency’s organizational structure is as follows. The agency head (principal) offers an upfront take-it-of-leave-it contract to each official, or case-handling team of officials (agent), who subsequently undertake the actual tasks. In the following, we refer to the decision making agent as "the official". A contract is a list of rewards for completing the tasks. After both the head and the official learn whether the task that the official has picked up has been completed successfully, the head pays the official according to the contract terms. The residual budget part, the head has available for non-case specific activities.

In this section, we study how the restrictions on these incentive contracts affect the agency’s way of prioritizing activities. Throughout this section we assume that $D$ is sufficient to finance any contract the agency head might wish to offer. In Section 3 we study the implications of a binding (discretionary) budget. We begin by analyzing a representative official, who decides among the performance of $n$ tasks. We then turn to the head’s strategies.

2.1 Effort chosen by the Agency’s Official

Within the context of his contract with the agency, the official uses his professional expertise on the task set to choose which task he will try to complete in addition to his basic work load. The official cannot do more than one task at the time. In taking on a task, the official exerts effort $a \geq 0$ at a

$^{11}$The implications of introducing asymmetric information between the head and his officials about the talent levels of the latter are discussed in Section 8.
personal cost \( c(a) \). His reward cannot be made conditional on his effort level directly, which is either unobservable or in-contractible by the head. Instead, the outcome is contracted. If task \( i \) is successfully completed, the official receives the reward \( R_i \geq 0 \) specified in his contract with the agency. If no task is completed, the official’s additional pay is zero. The offered contract is thus \( R \in \mathbb{R}_{++}^n \) where \( R_i \geq 0 \) for every \( i \). The official can always exert no additional effort \( (a = 0) \) and ensure basic utility from regular work for himself. On top of his fixed income, we normalize the official’s reservation utility level to zero.

Effort translates into probabilities that the task worked on will be completed, depending on talent and difficulty. If the official decides to pursue task \( i \), the probability of completing that task is modeled to be
\[
p_i(a, \theta) = a \times \theta^{\psi_i},
\]
while the probability of completing any other task is zero.\(^{12}\)

The official is risk neutral. When he decides to pursue task \( i \), his expected utility is
\[
E[U^O(a, \theta)] = p_i(a, \theta)R_i - c(a).
\]
The costs of his effort are set as \( c(a) = \frac{1}{2} \gamma a^2 \), where \( \gamma \) is a scaling parameter. The effort costs are increasing and convex in \( a \), which amounts to diminishing returns of the probability of completing a task from exerting higher effort. Note that assuming \( \gamma > D \) implies that optimal effort will remain in the interval \( [0, 1] \), since even if the budget is fully spent on one reward, for instance \( R_i = D \), under \( \gamma > D \) for activity \( a \geq 1 \) marginal costs of the official’s effort, \( \frac{\partial c}{\partial a} \geq \gamma \), will be higher than marginal benefits, \( \frac{\partial p_i(a, \theta)}{\partial a} R_i < D \). In other words, if \( \gamma > D \), the agency never has enough money to pay the official to exert effort \( a = 1 \) or more.\(^{13}\)

For a given talent level \( \theta \), the official chooses both which task to undertake and the effort he will make towards its completion, leaving the probability of obtaining a reward for completing the other tasks at zero. The official’s payoff maximization problem can thus be written as
\[
max_{i \in I} \{ max_a (\theta^{\psi_i} aR_i - \frac{1}{2} \gamma a^2) \},
\]
which returns a simple rule for the official’s choice of task to take up.

\(^{12}\)Note that the probabilities of tasks’ completion are not additively separable in \( \theta \) and \( \psi_i \), so that the budget values at which the agency switches among tasks to perform under binding resource constraints in the latter chapters depend also on the talent level of the official. Results are not qualitatively different with different specifications of the probability of task completion, as long as it is increasing in \( \theta \) and decreasing in \( \psi_i \).

\(^{13}\)In most situations, only lower values of \( \gamma \) are needed to keep the official’s effort and the probability of task completion in \( [0, 1] \). The assumption \( \gamma > D \) ensures this for any values of the remaining parameters and is unnecessarily strong, for instance in cases when the agency head does not want to offer the full budget as a reward for one task. The assumption will be relaxed in some numerical examples in the text.
Lemma 1. Given a contract $R \in \mathbb{R}_{+}^{n}$, an official with talent level $\theta$ will undertake the task with the highest value of

$$\theta^{\psi} R_i.$$  \hfill (1)

Moreover, under the official’s optimal choices of effort, the probabilities of task completion will be:

$$p_i = \frac{\theta^{2 \psi} R_i}{\gamma} \quad \text{and} \quad \forall j \neq i : p_j = 0,$$

if the official chooses to work on task $i$.

**Proof.** See Appendix. ■

### 2.2 Contracts offered by the Agency’s Head

The agency head adds value to the governance of his institution by using the full information he has about the official’s talent $\theta$ to set tailored rewards to make the official take up suitable tasks. The head does so by considering the expected benefits that the agency’s activities bring to society directly, but also the value of self-presentation through impression management. We formalize the latter in two different ways.

First, the head may have an interest in just opening cases, or rather just not opening certain cases, independent of the question whether these cases will ultimately be successful from a legal point of view or not. In particular this is so for high-profile cases that will generate a lot of exposure. This instantaneous incentive captures that exposure reflects on the agency’s public image, as well as the head personally. Driven by political pressures or career concerns to enforce, the head’s discounting of the probability of success may come from the fact that a big case opened would possibly run longer than the head’s term in office, so that, while the head gets credit for taking action, it will be his successor, not him, who will have to see these cases through.

Alternatively, the head may instead be under pressure not to draw public attention to certain matters, and ignore a potentially successful landmark case. Disutility from action may result from the head personally having been lobbied or captured not to touch a certain matter, with longer term career concerns attached. Yet, the head may just as well have his agency’s best interests in mind, where the agency may truly be harmed by investigating certain cases, despite those cases being legally promising. After all, other social concerns than competition alone may make a case politically nonviable. In addition may a company with extensive media overreach create political support for its position and publicly paint a pursuing agency as incompetent or unjust. The possibility of such detrimental effects to the agency’s good name should enter a head’s preferences for performance management, as the final outcome of these cases remains uncertain, the agency always runs a risk of making mistakes, and
together such considerations will affect future budgets.\footnote{Leaver (2009) shows that a “minimal squeak” theory, where officials try to minimize their mistakes for fear of being publicly marked as incompetent, rather than maximize the social welfare, explains behavior of the US State Public Utility Commissions better than the capture hypothesis.}

Second, the head values any residual budget that is not spent on paying rewards. He can use this residual budget for non-case specific activities that will put him and his agency in the professional and general public eye, such as giving informal opinions or speeches, appearing in the media, and marketing. The residual budget is assumed to be fully spent on such non-case specific activities. We assume that the social welfare gains from handling cases is different from comparable gains generated by impression management spending.

The head is risk neutral. His expected utility when employing the representative official performing task $i$ is

$$E[U^H] = p_i d_i + \phi d_i + V(D - p_i R_i).$$

The product $p_i d_i$ is society’s expected benefit from the agency’s overall activity. The parameter $\phi \in \mathbb{R}$ captures the head’s instantaneous incentive to open a high-profile case. If $\phi > 0$, the agency’s head gains positive utility from opening cases with potentially high impact on social welfare, while the head is punished for opening big cases when $\phi < 0$. In addition, the head values spending the residual budget, being the difference between the budget $D > 0$ and the expected payment to the official $p_i R_i$, as compared to the agency’s performance by parameter $V \in [0, +\infty)$.

If $V = 0$, all budget is offered as a reward for successfully completing cases, yet in expectation there is still a nonzero budget residue available for impression management spending. The higher $V$, the higher the head’s tendency to channel funds away from rewarding tasks.\footnote{In our formal analysis, $V$ is assumed to be strictly positive to assure a solution exists. $V = 0$ is discussed in later sections as a limit case that does not bring new fundamental insights.} Note that $V$ can capture a number of concerns the head may have, including his intrinsic motivation and personal preferences, the way in which he is evaluated by his superiors, in terms of monetary rewards, future (political) career prospects, and budget appropriation, combined with the workplace culture and the institutional system of controls in which he operates. The extent to which the head is monitored depends in large part on whether there exist informative measures of the agency’s performance. Often, such measures will be noisy. In addition, the value of $V$ may change during the head’s term.

Given his incentives, and knowing the agency official’s optimal response, the head solves

$$\max_{i \in I} \max_{R_i} \left\{ \max_{R_i} \left( \frac{\theta^2 \psi_i R_i}{\gamma} d_i + \phi d_i + V \left( D - \frac{\theta^2 \psi_i R_i^2}{\gamma} \right) \right) \right\} ; VD$$

In the following, we assume $D \neq 0$ to avoid degenerate cases. When $D = 0$, the agency performs none of its discretionary tasks, and the utilities of both the head and the official, as well as social welfare are zero.
\[ s.t. \, R_i \leq D \, \forall i \in I. \]

Under the assumption that any contract is affordable in the agency budget, this leads to the following simple optimal rule for the head.

**Lemma 2.** If the agency’s budget is non-binding, by setting the appropriate rewards, the agency head will make the official with talent level \( \theta \) pick up the task with the highest value of

\[ Q_i = \frac{(\theta \psi_i d_i)^2}{4V\gamma} + \phi d_i, \]  

as long as \( Q_i > 0 \) for at least one task \( i \in I \).\(^{17}\) Accordingly, the head will offer the official a contract (superscript ‘u’ for unconstrained):

\[ R_{i}^u = \frac{d_i}{2V}; \forall j \neq i : \, R_j = ICC_j, \]

where \( i \) is the task for which (2) is highest and \( ICC_j \) is any value that satisfies the official’s incentive constraint (1).\(^{18}\) These contracts lead to effort levels generating probabilities of task completion

\[ p_{i}^u = \frac{\theta^2 \psi_i d_i}{2\gamma V}; \forall j \neq i : \, p_j = 0. \]

If \( Q_i \leq 0 \) for all tasks \( i \in I \), the agency’s head keeps all resources for financing the performance management and the contract offered becomes

\[ R_{i}^u = 0; \forall i \in I, \]

amounting to probabilities of task completion

\[ p_{i}^u = 0; \forall i \in I. \]

**Proof.** See Appendix. ■

For every task, \( R_{i}^u \) is independent of the agency’s budget. It is the reward for which the head’s marginal benefit of spending an additional unit of the budget on incentivizing the official is equal to

\(^{17}\)In case \( Q_i = Q_j \) for some tasks \( \{i,j\} \subseteq I \), we assume that the agency’s head opts for the task that brings higher social benefits upon completion. Similarly, we assume that if \( Q_i = 0 \), task \( i \) is preferred by the agency’s head over performing no task at all. These assumptions are natural and inconsequential for the results.

\(^{18}\)Note that the official’s incentive constraint (1) is always satisfied by putting \( ICC_j \) equal to zero.
his marginal benefit of impression management spending. Note therefore that even when resources are unlimited, there is a maximum reward, that the agency head is willing to offer to the official, as he will prefer to extract any remaining resources above that reward level, due to diminishing returns to rewards offered. This maximum reward does not depend on the talent level, but only on the head’s preference for extraction $V$, since once the task choice has been made, $\theta$ influences only the probability of task completion. While more talented officials might not be offered higher rewards, in expectation they will still earn more than officials with lower $\theta$, as they can produce a higher probability of task completion with the same effort.

If the head does not value the mere opening of high-profile cases ($\phi = 0$), and also values the budget residue just as society values its resources ($V = 1$), the agency head would incentivize pickup of those cases that maximize the expected returns from the agency’s activities minus their expected costs, $p_i d_i - p_i R_i$. That is, he would choose to reward the task with the highest $\theta^\psi d_i$. The contracts offered would then be $R_i^\psi = \frac{d_i}{2}$, which represent an optimal balance between the benefits of completing the tasks and the costs of making the official pursue them properly in terms of effort level. Do note, however, that the head still pockets the budget residue. Only if $V$ was allowed to attain its lower bound, $V = 0$ (and for $\phi = 0$), does the head allocate the budget entirely on pursuing cases. His choice of rewards would then always be $R_i = D$\(^{19}\). An agency head who derives disutility from the opening of cases ($\phi < 0$) leans towards pursuing cases with a small impact on social welfare upon successful completion. Furthermore, if the head’s disutility of action is too high, so that $Q_i \leq 0$ for all tasks, the head rewards no tasks and spends all available budget on impression management, giving him a utility of $U^H = V D$.

The agency’s head limits his choices to only a subset of the agency’s task portfolio $I$, based alone on the tasks’ characteristics: difficulty $\psi_i$ and social welfare yield upon completion $d_i$. Lemma 3 establishes that the head will never incentivize - and so the staff never pick up - tasks that are both more difficult to complete and yield lower returns upon completion than another task in the agency’s tasks set.

**Lemma 3.** Take two tasks $\{A,B\} \in I$, such that $d_A > d_B$. If $\psi_B \geq \psi_A$, the agency’s head will prefer either task $A$ and no task at all over the pickup of task $B$.

*Proof. See Appendix. ■*

Note that Lemma 3 and the task ordering it establishes holds for any budget levels, not just for non-binding values of $D$. While the head could prefer a task that is both more difficult and brings lower benefits upon completion if he gained substantial disutility from opening tasks ($\phi d_i < 0$), which is in absolute value necessarily higher for tasks with higher benefits upon completion, the proof to

\(^{19}\)It is for this reason that we have bounded $V$ away from zero in this section for analytical convenience, since the budget is assumed to be unlimited, and the head’s problem would otherwise have no solution.
Lemma 3 shows that the agency’s head then will always prefer incentivizing no task at all. We can thus focus only on those tasks that are not dominated in the sense that there is no task that would be both easier to complete and yield higher benefits upon completion. The remaining tasks we can order as \((d_1, \psi_1) < (d_2, \psi_2) < \ldots < (d_m, \psi_m)\), so that the most difficult task is also the most beneficial to society.\(^{20}\)

To obtain some further insight into the head’s strategy when the budget is non-binding, first consider the case when \(\phi = 0\), that is, the head does not value opening cases for exposure only. The head’s objective then reduces to maximizing \(\theta^\psi d_i\), so that task \(A\) is picked over task \(B\) if \(\theta^\psi_A - \psi_B > \frac{d_B}{d_A}\). Suppose that task \(A\) is more difficult, i.e. \((d_B, \psi_B) < (d_A, \psi_A)\). The agency head then compares the ratio of \(\theta^\psi_A\) and \(\theta^\psi_B\), which are the terms by which the probability of the task’s completion is scaled, with the factor by which completing task \(B\) is less worthwhile to society than completing task \(A\). When the official’s talent level \(\theta\) increases, the more difficult task \(A\) becomes more attractive for the agency’s head.\(^{21}\) In other words, if the agency officials are skilled enough, the agency will perform more difficult tasks. More difficult tasks will also be performed when the social benefit from completing task \(A\) is sufficiently much higher than from completing the low-yielding task \(B\).\(^{22}\)

The head’s incentive to open high-profile cases for generating positive exposure (\(\phi > 0\)) makes the more complex tasks with high impact on social welfare generally more attractive to him. It is still true that a higher level of staff talent makes the more complex cases more attractive to the agency’s head, but if impression management is important enough to him - i.e., either \(\phi\) or \(V\) is very high - it becomes obsolete as a determinant of the agency’s behavior. While without the incentive to flash high-profile cases the head’s extraction plays no role in determining which task will be performed, with \(\phi > 0\) there is an interaction between the two motivations. Once \(\phi\) or \(V\) become high enough, only complex tasks will be performed by the agency, irrespective of the official’s talent, since the agency head expects to be able to extract a higher budget residue thanks to a lower probability of the official successfully completing complex tasks and the higher fixed reward from exposure through the high-profile cases.

3 Binding Discretionary Budget and the Agency’s Task Focus

We now turn to the role of the size of the budget that is at the agency head’s discretion. Note that for any value of \(D\) above the head’s optimal reward offer for his most preferred task, changes in \(D\) only influence the head’s utility through a different budget residue, not the optimal contract. However, once \(D\) falls below the head’s optimal reward offer for his most preferred task, the agency may come to

\(^{20}\)Here \(m \leq n\) since \(n - m\) task were excluded from the set via the above mechanism.

\(^{21}\)As long as \((d_B, \psi_B) < (d_A, \psi_A)\), the agency head will prefer \(A\) when the official’s talent level gets closer to 1, and \(B\) when \(\theta\) goes to zero with no further restrictions on the parametrization. This will no longer be the case when \(\phi > 0\). With high enough \(\phi\) the head might prefer task \(A\) for any official’s talent level.

\(^{22}\)These results hold when the budget does not constrain. As will be derived later, with restrictions on the level of the rewards, it might be the head’s optimal choice that the agency performs task \(B\) even if the official is very skilled. See Section 3.
perform different tasks under different budget constraints, leading to discontinuities in social welfare. To clarify how, we further focus on changes in the agency’s performance when there are only two possible tasks, A and B ordered as \((d_B, \psi_B) < (d_A, \psi_A)\), to choose from by one representative staff official employed by the agency’s head. The main insights obtained carry over to a more general setup with multiple tasks and officials, as discussed in Section 5.

First, we specify how the budget constraint determines which tasks the agency will perform under what conditions.

**Proposition 1.** For a large range of parameter values \(d_A, d_B, \psi_A, \psi_B, \phi, \theta, \gamma, V\), there exist critical values of \(D\), at which the agency abruptly changes its focus in the following way:

(i) Whenever task \(A\) is the head’s most preferred task when the budget is not binding, i.e. \(Q_A = \max\{0, Q_A, Q_B\}\), there may exist a critical budget value \(D_A^*\), so that the agency focuses on the complex task \(A\) when \(D \geq D_A^*\), and task \(B\) for budget values directly below \(D_A^*\). If such a \(D_A^*\) exists, there will always exist one critical budget value of the type \(D_0^*\) or \(D_1^*\).

(ii) Whenever \(\phi < 0\) and the agency does perform some task without a binding budget constraint, i.e. \(0 \neq \max\{0, Q_A, Q_B\}\), then and only then exists a critical budget value \(D_0^*\), so that the agency does not perform any task when \(D < D_0^*\) and performs task \(A\) or \(B\) for budget values directly above \(D_0^*\). The agency may also switch once between \(A\) and \(B\) for some critical budget value \(D_2^* \geq D_0^*\).

(iii) Whenever \(\phi > 0\) and the agency does not perform task \(A\) for all budget values, then and only then exists a critical budget value \(D_1^*\), so that the agency does perform task \(A\) when \(D \leq D_1^*\) and performs task \(B\) for some budget interval above \(D_1^*\). The agency may also switch once between \(A\) and \(B\) for some critical budget value \(D_2^* \geq D_1^*\).

Proof. See Appendix. ■

The intuition for the agency’s change in focus from the more difficult to the simpler task when the discretionary budget falls below \(D_2^*\) is as follows. With enough budget, the head prefers to incentivize the more complex task. Since \(R_{A}^{u} > R_{B}^{u}\), as \(D\) goes down from non-binding high values, this first starts influencing the probability of completing the more complex task and the utility it generates for the head. Note that as soon as the budget is binding, the head will put it all towards incentivizing his designated task, rather than spread it between the ex ante reward and impression management activities directly. As the budget is reduced further, the head suffers two types of utility loss. One is from his inability to still sufficiently incentivize his staff to exert effort on completing the complex task to society’s benefit. The other is from reduced expected budget for impression management. At some point, the head may then switch from stimulating the take up of the complex task to rewarding the simple task, which requires a lower reward to complete. However, when the budget is decreased further, below the head’s most preferred reward for the simple task, the official’s effort decreases further, and the probability of completion of the simple task with it. When the number of officials and/or the tasks
they can choose among increases, there may be more critical budget values of the type $D_3^*$, while there always exists at most one of the jumps of the type $D_0^*$ or $D_1^*$, leading to discontinuities in the agency’s performance.

As the head’s fixed incentive to open a case, $\phi d_i$, is constant, for low enough budget values it becomes the most important determinant of the agency’s performance, as is the case in (ii) and (iii). When $\phi > 0$, for those low budget levels the head will simply prefer the tasks with the highest fixed reward just for opening them. In addition, the probability of the head actually paying the reward for completing the complex task will be low, thus increasing his expected budget residue. This latter combination of effects also provides the intuition behind existence of the critical budget values of the type $D_1^*$. Moreover, the switch to the simpler task for intermediate budget values in (iii) only happens when there is a moderate difference between the tasks’ benefits, or a high difference between the tasks’ difficulties, while the head is concerned primarily with social welfare. If these conditions are violated, the head cares too little about society’s benefits to mind the sure ineffectiveness of trying to incentivize the complex task with too little budget. In other words, the head keeps pushing his staff to open complex high-profile cases, knowing they will most likely fail to complete them successfully, just to enjoy the exposure that such cases generate, while pocketing the unclaimed rewards for impression management purposes as well.

The intuition behind the critical budget budget values of the type $D_0^*$ is that when $\phi < 0$, the head shies away from opening cases, especially the complex ones that yield a high instantaneous disutility. The agency will therefore perform no discretionary tasks at all for these lowest budget values, which are not sufficient to incentivize the official enough to exert the kind of effort that would overcome the head’s dislike for opening the case.

Whereas generally variations in the agency’s discretionary budget will affect its focus of attention, there are three scenarios in which there is no shift in the agency’s priorities for any budget values. First, when $\phi$ is negative and large enough, the agency will never perform discretionary tasks for any budget, as even successfully incentivized and completed complex tasks yield the head insufficient gain to overcome his disutility from opening the case. Second, when $\phi > 0$ and $Q_A >> Q_B$, the task $A$ is just rewarding enough for the head to keep the agency at it no matter the budget. Third, when $B$ is the head’s most preferred task without a binding budget constraint and $\phi = 0$, the agency will perform task $B$ for all budget values above zero, since there is no fixed reward for opening a case.

Figure 1 (left) illustrates the effect of budget changes on the agency’s case focus for one particular parametrization with $\phi > 0$. For non-binding budget values, the agency head prefers the more complex task $A$ and rewards it with a constant reward $R_A^*$ upon successful completion. Once the budget falls below $R_A^*$, the reward offered for task $A$ becomes $D$. A linear decrease in $D$ then causes a linear decrease in the probability of completion of task $A$ via the official’s behavior described in

---

23 In the example in Figure 1 (left), parameter values are: $\theta = 0.5; (\psi_A, \psi_B) = (1.8, 1.1);$ $(d_A, d_B) = (400, 250);$ $\gamma = 6; V = 10; \phi = 0.07.$
Lemma 1. As the available resources decrease, so does the head’s opportunity to extract them for impression management purposes. The lower probability of completing task A - that is, the higher probability of extracting the whole budget - is no longer as attractive for the agency’s head, as the total amount of money to be had is little. Instead, the agency’s performance gains relatively higher importance in determining the head’s overall utility, so that he changes the contract with the official to incentivizing task B below point $D^*_2$. Moreover, with decreasing $D$ on the interval $(R^u_B, R^u_A)$, the head’s utility generated by rewarding task A decreases at a higher rate than the utility from rewarding task B - which itself decreases with the slope $V$, the head’s marginal loss of resources to extract - up until the point $R^u_B$.

This second intuition may cause $D^*_2$ to occur above $R^u_B$. Figure 1 (right) serves as an illustration with $\phi < 0$.\(^{24}\) Between $D^*_2$ and $D^*_1$ the head’s utility is the highest for rewarding task B. As the budget decreases further, the fixed reward for opening a task becomes relatively more important in determining head’s utility, and the agency’s head decides to reward task A at point $D^*_1$ and below. In addition, Figure 1 (right) illustrates how the agency performs no discretionary tasks for budget values below $D^*_0$, where the head derives negative utility from opening a case.

A lower probability of successful completion of the complex task increases the expected left-over budget that the head can put towards non-case specific activities. The less the head values impression management, the bigger will be the intervals $(D^*_1, D^*_2)$ and $(D^*_0, D^*_2)$. In Figure 1 (right), the lower accent on impression management makes the head less likely to shy away from opening cases. In Figure 1 (left), opening complex tasks despite having insufficient funds to induce the effort to complete them successfully with high enough probability still generates the exposure desired by the head.

If the head cares only about real cases, i.e. $V = 0$ and $\phi = 0$, the agency will perform the more

---

\(^{24}\)In the example in Figure 1 (right), parameter values are: $\theta = 0.7; (\psi_A, \psi_B) = (2, 1); (d_A, d_B) = (400, 200); \gamma = 10; V = 10; \phi = -0.2$. 

---
complex task only when its officials have high enough talent levels \( \theta \) as compared to the difference between the tasks’ benefits and difficulties, and a high enough discretionary budget \( D \). Performing complicated tasks requires talent, combined with sufficient resources to motivate those officials skillful enough to perform them. In determining whether there will be a nonzero probability of completing task \( A \), talent levels and the budget act as substitutes. However, the extent of this substitutability is limited. Higher (lower) \( D \) always means that lower (higher) talent levels are needed in order for the agency to perform task \( A \). The opposite is not always the case. Some talent values can make the budget constraint irrelevant as a determinant of the \textit{type} of task performed - however not the \textit{extent} by which it is performed, that is, the probability of completion. If \( \theta^{\psi_B} d_B > \theta^{\psi_A} d_A \), for instance if the official’s talent is close to zero, only task \( B \) will be performed for any budget value. Similarly, if the official’s talent level is close to one, only task \( A \) is performed and there is no budget interval in which the agency shifts to the simpler task \( B \). In this sense, talent availability is more important in determining the type of task the agency performs than the budget assigned.

4 Welfare Implications of the Agency’s Focus Shifts

The head’s switches between incentivizing high-profile and basic cases do not generally serve society’s interest. While the social welfare gains from handling cases and those from impression management spending will both be hard to quantify with much precision in practice, the latter even more so than the former, there is no reason to think they would generally be the same. In general, the net benefits to social welfare of impression management spending are ambiguous. Self-presentation towards building the organization’s public image is a delicate exercise. On the one hand may public appearances create a wider public awareness of the agency, its interventions and the rules it enforces. A public image of a strong agency certainly is likely to help compliance with the rules it oversees. On the other hand, public signaling can just as well have negative effects. It could fuel suspicion of the agency being politically bound, poorly informed or myopically focused, for example, making the agency lose grip on its regulatees. A published sector study, criminal profile, or a code red warning, while possibly impressing the general public, can also give away crucial information about the agency’s thinking to the initiated. Intended to come across authoritative and well-informed, such communications may reveal what the agency’s blank spots are as well.\(^{25}\) In many instances, a strong reputation would be best built through successfully pursuing meaningful cases. At the same time may zealously visible agency activity lead to over-deterrence when perfectly fine activities are curbed for fear of being mistaken for a violation that would trigger an intervention and possibly sanctions.

In this section, we study how social welfare is influenced by the agency’s focus shifts resulting from changes in \( D \). For simplicity, we analyze the normalized case in which impression management on

\(^{25}\)See Schinkel (2011) on the interaction between market overseers and their overseen being a strategic game of cat-and-mouse, in which it may not be clear who outsmarts who in the end.
balance does not benefit society. The qualitative results carry through generally when relaxing this assumption, as discussed at the end of this section. Under the assumption that budget not spent on cases generates no welfare, expected social welfare net from spending $D$ is:

$$E[W] = \sum_{i \in I} p_i d_i - D,$$

All tasks that the agency has not picked up have a probability of completion equal to zero. Note that this particular formulation of the social welfare function reflects the assumption made earlier that the entire budget is spent by the agency, that is, no residue is returned to society. In addition, for analytical convenience we imply that the head’s and the official’s private utilities are negligible in total welfare, which amounts to assuming that as individuals they are atomistic in society.

In principle, given perfect information about the functional forms and the values of $\theta, \forall i: (d_i, \psi_i), \phi, V, \gamma$ - plus the gains from impression management when there are any - it is possible to determine the social welfare maximizing budget level in any given case. It seems unrealistic, however, to assume that the budget-setter would have all of this knowledge. It is therefore more interesting to study the welfare implications of a range of possible budgets - including those that would be socially optimal. Doing so reveals, amongst other things, that a change in the budget often affects welfare by more than just the resources allocated.\(^\text{26}\)

While the agency head’s utility is continuous in the assigned budget, social welfare is not. The head switches between rewarding one task and the other only at point(s) $D^*$, where his utility from the two reward schedules is equal. Welfare changes discretely then, with those task switches. Consider Figure 2, which continues the two-task example set up in Figure 1, for which social welfare reduces to:

$$E[W] = p_i d_i - D,$$

where $i$ is the task rewarded by the agency head (if any) and pursued by the official.\(^\text{27}\) When the budget is non-binding, social welfare declines at a rate proportional to resources spent, since any additional resources provided to the agency are extracted by its head.

Social welfare is linear in the probabilities of task completion, so that it is a linear function of the budget whenever the two probabilities are linear in the budget, as illustrated in Figure 1. Note that society might be better off by not rewarding any of the tasks and simply keeping the discretionary budget instead. This is the case when either the head incentivizes no task given the budget constraint, or the official’s costs of effort are very high compared to the tasks’ benefits and difficulty, so that the expected social benefits of the agency’s discretionary activities do not justify the “investment” by society in additional rewards. Incentivizing an official with high $\gamma$ comes at a high price to society, yet

\(^{26}\)We return to the problem of budget-setting under incomplete information in Section 6.

\(^{27}\)In the example in Figure 2 (left), parameter values are the same as in Figure 1 (left). In the example in Figure 2 (right), parameter values are the same as in Figure 1 (right).
not to the agency’s head, who wants to offer nonzero rewards for task completion as long as \( \phi \geq 0 \), irrespective of the official’s costs, since lower effort caused by higher \( \gamma \) translates into higher expected residual budget for the head to spend. In addition, the head prefers the opening of high-profile cases for show.

In the wide range of circumstances under which society does want the agency’s head to use additional resources to promote effort by his staff, welfare develops as illustrated by a typical example in Figure 2 (left). There are several local maxima, with \( D^*_2 \) being the global welfare maximum.\(^{28}\) In particular, note that for budget levels just over \( D^*_2 \) marginal welfare is positive in \( D \). While this may suggest that it is optimal to further increase the budget, in fact a discrete welfare increase can be had by slightly decreasing the budget below \( D^*_2 \). While moving away from the global maximum, this budget cut increases welfare through a decrease in the head’s incentive to extract resources. On the other hand, had \( R^*_uA \) been a global maximum - which is attainable e.g. by a slight increase in \( d_A \) or the official’s talent level - better would be a large budget increase far towards, or best at, maximum welfare at \( R^*_3A \), but when that is not attainable, society may gain more from a budget cut than too small an increase. Finally, note that a discretionary budget squeezed below \( D^*_1 \) gives the head an incentive to just open high-profile cases, from which he derives positive instantaneous utility.

Proposition 2 generalizes the results, establishing that the jumps in welfare go in a predetermined direction.

**Proposition 2.** For a small budget change that passes any critical budget value \( D^* \), so that the agency’s focus of attention shifts:

(i) Whenever \( \phi \geq 0 \), a budget cut that makes the agency switch from a complex task to a simpler one increases social welfare discontinuously. Likewise, a budget increase that makes the agency switch

\(^{28}\)Note however that \( D^*_2 \) is not an attainable maximum, since task \( A \) is performed when \( D = D^*_2 \), as a result of the technical assumptions of our model. We could, however, similarly assume the opposite.
from a simpler to a complex case reduces welfare - even if \( D \) is still below its welfare-maximizing level.\(^{29}\) When \( \phi < 0 \), the effects of these switches on welfare are ambiguous.

(ii) A budget cut that makes the agency switch from a simple task to a more complex one decreases social welfare discontinuously. Likewise, a budget increase bringing \( D \) above such \( D^\ast \) improves welfare.\(^{30}\)

(iii) A budget cut that makes the agency switch from performing any task to performing no tasks decreases social welfare discontinuously, and vice versa.\(^{31}\)

Proof. See Appendix. ■

A head who derives positive utility from opening a case, is more willing to open high profile cases than is good for society, essentially for two reasons: the instantaneous utility in them, plus the higher expected left-over budget for impression management spending. When the head derives disutility from opening cases, the two incentives work in opposing directions, leading to ambiguous welfare effects in (i). The result in (ii) is independent of \( \phi \), since \( \phi > 0 \) is a necessary condition for the critical budget value of the type \( D^\ast \) to occur, as shown in Proposition 1. In other words, when \( \phi < 0 \), the agency’s focus shift brought about by a budget cut is always from a task with higher social benefits to a task yielding lower social benefits upon completion. Finally, (iii) follows from the fact that society always prefers at least some agency performance, however small, to using all resources for impression management activities. The reason for this is that while social welfare can in principle be negative for all budget values - for example when the tasks’ benefits are very low or the official’s cost parameter \( \gamma \) is very high - it will always be negative when the agency performs no task at all. The agency’s operations will then cost the society its full budget \( D \) and produce nothing in return.

Note that as long as \( \phi \geq 0 \), the agency head will design contracts leading to higher (lower) probability of completion of the complex (simple) tasks than would be society’s preference, since these tasks have lower (higher) probability of being successfully completed upon an official’s attempt, and the head receives higher fixed rewards from them. The wedge between the head’s preferred and the socially optimal contract is due to the head’s taste for impression management. Social welfare is discontinuous precisely at the points where the agency head switches between the contract designs. In Figure 2 (left), there is a jump in welfare at points \( D^\ast_1 \) and \( D^\ast_2 \), since the agency head decides to stop rewarding the complex task when the budget drops below the critical value \( D^\ast_2 \), yet reopens the task once \( D \) drops below \( D^\ast_1 \). Importantly, the jumps in social welfare caused by the change in the head’s strategy can be of a very substantial size - around half of total welfare in our example. The size of the jump at points \( D^\ast_1 \) and \( D^\ast_2 \) in Figure 2 (left) is given by \( \Delta W_{A,B} = D^\ast \times \frac{\theta^{2\alpha_A d_A - \theta^{2\alpha_B d_B}}}{\gamma} \), holding every time there is a shift between tasks \( A \) and \( B \) for budget values below \( R^u_{B} \).\(^{32}\)
If we assume that the part of the budget that is spent on non-case specific activities does generate welfare, our general findings carry through, with some adjustments. First, if society values the impression management activities connected to opening big cases, the overall welfare increases. The head’s incentive to open high-profile cases, $\phi d_i$ in the head’s utility function, may be mirrored by $\phi^S d_i$ in the social welfare function - $\phi^S$ then captures the instantaneous social welfare gain of opening a case. The agency’s behavior remains unaffected by any change in the social welfare function and the generated welfare increases by a constant $\phi^S d_i$, dependent on the cases being performed under the given budget. In addition, for the parts of the budget where complex cases are being performed, this constant has a higher value. The social welfare functions in Figure 2 shift up by $\phi^S d_i$, more for the intervals where task $A$ is being performed (since $\phi^S d_A > \phi^S d_B$). Immediately then the size of the discontinuous jumps decreases. As long as society values the impression management utility from opening tasks less than the agency’s head ($0 < \phi^S < \phi$), the existence and sign of the jumps remains unchanged.\footnote{In the unlikely opposite scenario $\phi^S > \phi$, the sign of the jump depends also on the head’s (and society’s) valuation of the discretionary budget spent on non-case-specific activities.}

Second, the discontinuity of welfare in $D$ remains also when impression management financed by the discretionary budget spent on non-case-specific activities is valued by society at $V^S$. Social welfare would then have the form $E[W] = p_i d_i - V^S p_i R_i + V^S D - D + \phi^S d_i$, similar to the head’s utility function. This has an effect on the shape of the social welfare function which seizes being linear and becomes concave on its continuous parts. As a borderline case, if welfare generation effects of impression management activities mirror the head’s valuation (either because the agency head is fully benevolent, or as a mere coincidence), i.e. if $\phi = \phi^S$ and $V = V^S$, the social welfare function becomes continuous since it is exactly the same as the head’s utility function up to a constant $-D$. Except from this extreme case, there is always a wedge between the head’s utility and the social welfare, amounting to jumps in the social welfare function. Again, the sign of the jumps remains unchanged for the most relevant (and defendable case) $\phi^S < \phi$ and $V^S < V$, where the agency’s head gains utility from the impression management in addition to the social welfare generated by it.\footnote{For $\phi^S > \phi$ and $V^S > V$, the jumps go in the opposite direction. The remaining cases are ambiguous and depend on other parameter values.} Society’s valuation of the residual budget has, however, an effect on the size of the jumps in the welfare function - they become smaller since $V^S \neq 0$ effectively makes the welfare function more like the head’s utility. The differences between $(V, \phi)$ and $(V^S, \phi^S)$ determine the size of the jump.

Similarly, the discontinuity of welfare in $D$ remains also when any budget residue would be returned to society - somehow: as discussed in the introduction, bureaucracies tend to exhaust their resources and spend budget surplus to avoid future budget cuts. Social welfare would then be $E[W] = p_i d_i - p_i R_i + \phi^S d_i$ since $V^S = 1$, while the objective of the head is $E[U^H] = p_i d_i - V p_i R_i + VB + \phi d_i$, thus still leaving a wedge between the head’s and society’s disutility of paying the official. This difference remains for the arguably more likely head’s objective function $E[U^H] = p_i d_i + \phi d_i$, which reflects that the budget residue is not at the head’s discretion and therefore $V = 0$.  

20
Finally, consider the implications of different views on what government agencies produce for society. The directions of the welfare jumps of the type $D^*_2$ and $D^*_1$ described by Proposition 2 rest on the assumption that the head and society value the benefits generated by the tasks in the agency’s portfolio in the same way, that is, the values of $d_A$ and $d_B$ are common. The social good that comes from the control tasks of the government agencies modeled, however, is not always obvious to the public. If society would instead perceive the benefits of the tasks differently, say as $d^p_i$ different from the actual welfare benefit $d_i$, the jumps in the social welfare can in principle have any sign and size. In particular does it seem reasonable to consider the case in which the agency’s behavior is driven by society’s perceived tasks’ benefits $d^p_i$, since the head will be evaluated by budget-setter with that perception. While a mission-motivated agency’s head might then still use the true $d_i$ as a measure for determining the agency’s performance, a head who is more interested in the public perception - which would ultimately affect future budgets - of his agency’s activity would instead aim at $d^p_i$.

To see some of the possible welfare consequences of an asymmetric understanding of society’s benefits, consider a head who is interested in an appearance of performance ($p_i d^p_i$), and a public that considers more difficult tasks to be of higher significance than they actually are - for example because of their greater exposure in the media - i.e. $d^p_A > d_A$. Such a public misconception has a negative impact on welfare. From a social welfare point of view, when the head already stimulates too much take-up of the more difficult task $A$, a public over-assessment $d^p_A > d_A$ further increases the rewarding of task $A$ at the expense of task $B$, with possible negative consequences for social welfare.

Similarly, $d^p_B > d_B$ (or equivalently $d^p_A < d_A$) can cause the jumps in perceived and actual social welfare to have opposite signs. Consider the case $d^p_A = d_A$, while $d^p_B > d_B$. If the public overestimates the impact of the simpler tasks and the agency head adjusts his choice accordingly, the jumps in the perceived social welfare smoothen, while the jumps in the actual welfare remain governed by the same formula as before - only do they happen at different budget levels if the agency’s head considers the perceived benefits of tasks’ completion in his objective. For instance if $\phi > 0$, and if the perceived welfare gains from the simpler tasks become much higher than what they actually are, the jumps in the perceived welfare function would have an opposite sign. Clearly, the existence of asymmetries between actual welfare effects of cases and their public perception can have deeply detrimental effects with agency heads concerned with perceived effects.

5 Agency with Several Officials and Multiple Tasks

The main insights derived from the basic model above extend straightforwardly to an agency employing a number of officials and having several types of tasks that it can perform in addition to its regular

\[35\text{Assuming } d^p_B = d_B.\]

\[36\text{See the proof to Proposition 2 for a derivation of the values for } d^p_i \text{ for which the jump in the perceived welfare function disappears. } d^p_i \text{ being above or below these values then directly determines the direction of the jump.}\]
duties, under the following three assumptions. First, we assume that contracts can be individualized and the head can fully discriminate among his officials. This is in line with the examples of secondary and tertiary personal rewards given above, as well as with the assumption that the agency head has perfect information about his staff’s qualities. Second, if two officials perform one and the same type of task, their probabilities of task completion are independent, that is, there is no interaction or economies of scale or scope - which can be thought of as each official individually working on a different task of the same type. Third, interpreting the discretionary budget as an administrative constraint on the head’s rewarding options, we assume that it is an upper limit on the reward for each official - that is, in case the agency employs $m$ officials, the head uses up to $1/m$ of the total discretionary resources to motivate each official. At the end of this section, we briefly discuss why alternative specifications, while introducing considerable complexity of analysis, do not change our results qualitatively.

Throughout this section, we will assume $\phi \geq 0$ for clarity of exposition. Examples with $\phi < 0$ can be constructed analogically. The mechanics of the head’s switching between the available tasks remains unchanged, including the formulas for critical values of budget.\textsuperscript{37} The head’s most preferred task for each official without a binding budget is given by Lemma 2. The main difference when more tasks are available to the agency is that there can be more critical values of the budget at which the head switches the agency’s focus, as these can happen among multiple pairs of tasks for different budget values. One of these tasks will still be more difficult and yield higher benefits upon completion.\textsuperscript{38} The head always has one preferred task for the official to perform for a given budget value, and the changes in priorities are always between two tasks, just as described in Proposition 2. Moreover, the intuition of Proposition 1 directly implies that for the lowest budget values the head will always incentivize the most complex tasks, as they have the highest fixed reward for opening. Only if $\phi = 0$ will the agency perform continuously simpler tasks as the discretionary budget depletes towards $D = 0$. For an agency with $k$ officials and $n$ tasks to choose from, there are between 0 and $k \times n$ jumps in performance, as the head’s level of discretion decreases.\textsuperscript{39}

Figure 3 (left) depicts a simple parametrization, in which the agency entails two officials and there are three types of tasks that each of them can perform.\textsuperscript{40} The two officials $O_1$ and $O_2$ differ in their talent level, $\theta_{O_1} < \theta_{O_2}$, while the tasks are ordered by difficulty as $(d_A, \psi_A) > (d_B, \psi_B) > (d_C, \psi_C)$ for official $O_1$ and $(d_X, \psi_X) > (d_Y, \psi_Y) > (d_Z, \psi_Z)$ for official $O_2$. For each budget interval, staff might

\textsuperscript{37}See proof to Proposition 1 in the Appendix.

\textsuperscript{38}Lemma 3 holds for each pair from any number of available tasks.

\textsuperscript{39}For each official, there can be up to $n - 1$ jumps from the more difficult task to simpler tasks, as well as in addition, for the lowest budget values, a jump back to the most complex task with the highest fixed reward for opening a case.

\textsuperscript{40}In the example in Figure 3, the agency’s head considers the perceived tasks’ benefits in his objective and the parameter values are: \begin{align*} \theta_{O_1} &= 0.5; \quad (\psi_A, \psi_B, \psi_C) = (2,1,0.2); \quad (d_A^p, d_B^p, d_C^p) = (40,20,10); \quad (d_A, d_B, d_C) = (80,40,20); \\ \theta_{O_2} &= 0.6; \quad (\psi_X, \psi_Y, \psi_Z) = (2,1.07,0.2); \quad (d_X^p, d_Y^p, d_Z^p) = (30,18,10); \quad (d_X, d_Y, d_Z) = (60,25,20); \quad \gamma = 5; V = 1; \quad \phi = 0.05. \end{align*} The head uses half of the budget to incentivize each official, that is $D_1 = D_2$. Note that in this example, there is a difference between the tasks’ benefits as society perceives them, and the actual welfare that these tasks generate upon successful completion. The perceived benefits enter the head’s utility function, while the actual benefits determine the social welfare as discussed in Section 4. Similar figures can be constructed in which the two sets of benefits coincide, much like in Figure 1.
get "assigned" to a different task via the contract design. The contracts offered to each of the two officials and the resulting tasks picked up are still governed by Lemma 2 and Proposition 1.\footnote{Lemma 2 determines the task that will be picked up by the official when the budget constraint is not binding. Take the official $O_1$ and the most complex task he can perform, $A$. The critical budget values derived in the proof to Proposition 1 then determine whether a simpler task will be performed for some budget constraints - we know that task $A$ will be performed for the lowest budget values as well. However, there are now up to six possible critical budget values instead of just two: a jump from task $A$ to task $B$, from $A$ to $C$, from $B$ to $C$, and each of them back. Their ordering determines which task will be performed for a particular budget value. There can be an interval in which task $B$ is preferred over task $A$, an interval in which task $C$ is preferred over task $B$, and an interval in which task $C$ is preferred over task $B$. For instance, the official $O_1$ could perform task $A$ for the highest and lowest budget values and task $C$ for some intermediate values without ever performing task $B$. This would happen if the interval in which $B$ is preferred over $A$ is a subset of the interval in which $C$ is preferred over $B$.} In our example, both officials are offered contracts that induce them to perform the most complex task from their portfolio when the discretionary budget constraint is non-binding, and are gradually pushed to tasks with a lower level of complexity when the discretionary reward that a the head can offer goes down.\footnote{This is a result of the parametrization that we have chosen for our numerical example. It is plausible to construct examples where officials are induced to perform only one type of task or just a few of them. For instance substantially increasing the talent level of official $O_1$ would make him skilled enough so that the agency’s head would make him perform task $A$ no matter the budget constraint.} For the lowest ranges of $D$, it becomes worthwhile for the head to make every official pick the most complex task, even those that are not cut out for them, because of the head’s incentive to open cases for exposure. For low enough budgets, part of the impression management incentive captured by $\phi$, rather than productivity of the official, becomes the most important determinant of what is being rewarded by the agency’s head.

Figure 3 (right) displays the welfare function to the above illustration of agency’s performance, assuming again that impression management is socially unproductive on the whole. On every continuous part of the social welfare function, each official picks one task as is indicated in the figure. The jumps in welfare follow a pattern similar to that in the two-task case. There is an increase in welfare when the budget falls below a critical value for other than the lowest values, caused by a decrease in the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Probabilities of task completion (left) and possible welfare effects of discretionary spending (right) in an agency with two officials, $O_1$ (solid lines) and $O_2$ (dashed lines), and three tasks.}
\end{figure}
expected resources spent on impression management. However, for lower discretionary budget values, the incentive to open big cases for presentation purposes causes a prioritization towards complex tasks to the detriment of social welfare.

For budgets above $R^A$, prioritizing the complex task $A$ generates negative welfare, since incentivizing it serves the head's personal preferences for impression management, but this type of task is too complex for society's good.\(^{43}\) The much higher benefits that would materialize upon its completion cannot offset the low probability of success for this hard task. Similarly, the social benefits from task $X$ are decreasing in resources above $R^X$, where the reward for official $O_2$ reaches its cap, below zero for large enough budgets. Social welfare can also be negative for low or intermediate budget values. In such cases, society would be better off dismantling the agency altogether, rather than getting the discretionary budget constraint wrong.

The absolute size of the welfare jump discontinuities is governed by the same formula as in the case of one official choosing between two tasks only, yet each jump is smaller relative to overall welfare, because generically each is caused by a single official switching between tasks, while the others remain on theirs. The fact that in the case of a still relatively simple agency the number of local maxima of the welfare function generated by the discretionary spending is already large underscores the importance of considering task prioritization when devising the budgeting policy by any agency's superiors. For agencies with more officials and tasks, the jumps become less pronounced, but the overall welfare function is in general not monotonic.

As the number of officials rises, the welfare effect of individual jumps becomes less pronounced, but there is likely to be more of them. More tasks affects the agency's behavior depending on their characteristics relative to the existing portfolio. Adding a simplest task or a task of intermediate difficulty may have no effect at all. Adding a new task that is more complex and beneficial than any existing one always has an effect on both the tasks being picked up and welfare, since it will be performed by all officials for the lowest budget values, and may be performed by the most talented officials for other budget values. If tasks of intermediate difficulty and yield are added, the size of the welfare jumps might decrease, for example because some jumps among tasks with very differing characteristics can be replaced by two smaller-sized jumps among similar tasks.

A full continuum of available tasks with differing characteristics effectively flattens out the welfare function, in the sense that at every point of the budget constraint there is an infinitesimal small jump and each task is performed only for a specific budget value.\(^{44}\) Note that still in that case task switching determines welfare, in the sense that neglecting the head's changes of mind will underestimate the effects from discretionary budget changes - discretely so for any finite number of tasks as soon as the

\(^{43}\)The difficulty of task $A$ is set to generate zero social welfare up to $R^A$ when performed by an official with talent level $\theta_{O_1}$, so that all welfare on intervals where task $A$ is performed comes from the activity of the official $O_2$.

\(^{44}\)Do note that it cannot simply be the case that doubles $(d_i, \psi_i)$ cover the whole $R^+\cup$, since then the task with $\psi = 0$ and $d = \infty$ would dominate all others and always be chosen by the head. A continuum of tasks could instead look for instance like $\{ (d_i, \psi_i); \psi_i = \frac{1}{2} d_i \& d_i \in (1,100) \}$.\(^{24}\)
budget is moved over a jump discontinuity threshold. Since each discontinuity is effectively the result of one (or more) officials being incentivized to perform a different task, the jumps will be affected by introducing society’s nonzero valuation of the impression management in the same manner as in Section 4. The sign of the jumps remains unchanged, as long as the agency’s head gains some utility from the impression management in addition to its welfare generating effects. The size of the jumps then again depends on the wedge between society’s and the head’s valuation of these activities.

Now consider variations of the three simplifying assumptions made at the beginning of this section. Should, for legal reasons for example, the agency have to offer all, or classes of its officials the same incentive contracts, so that the head cannot exercise full discretion in awarding his staff, he could still write one or several universal contracts that include cut-off values in the reward structure. Effectively, officials with talent in certain intervals would choose to perform certain tasks. Because of the incentive constraints, the agency’s head might then have to leave information rents to some officials in order to induce them to perform certain tasks. Yet the main results carry through. The same is true for allowing officials to jointly work on a case and so affect its probability of successful completion. This would highly complicate the analysis, yet still return abrupt shifts in performance caused by changing the budget to the head’s discretion. Finally, the budget constraint could be modeled alternatively as the maximum total money spent if every official who is offered a reward is successful in completing its task. If such reallocation of resources among officials becomes part of the head’s decision space, the model dynamics would change substantially, since a change in the budget can then amount to changes in any number of contracts between the head and the officials, and the agency head would get to decide for which officials or tasks the budget is effectively (non)binding. Again, while considerably more complex to assess where and by what extent, is would result in jumps in the agency’s priorities and welfare all the same.

6 Optimal Discretionary Authority

Society employs the agency head for his expertise. Yet while the head’s private information about his staff’s best talent-task matches gives him the ability to maximize his agency’s contribution to social welfare, his personal tastes for self-presentation through non-case-specific activities with less obvious social benefits means he has to be kept in check. Apart from appointing a head whose incentives are closest to the public interest, society has the budget part over which it gives the head discretion, $D$, to do so. The political decision on a government agency’s budget is two-fold: it concerns the agency’s total budget, as well as its division between the non-discretionary part and the discretionary part $D$. To the extent that this division is determined outside the agency, it defines the discretionary space of the agency’s head. While an authority’s budget-setter is unlikely to have the information required to determine $D$ socially optimally, there are several qualitative insights to go by.

The socially optimal split of the total budget depends crucially on the welfare that is generated by
generic tasks, relative to what can be obtained in addition through discretionary spending. Let \( F \) be the part of the generic budget in which welfare is smooth, including expenses for work facilities and support staff, as well as fixed wages for regular agency’s activities that are readily assigned by law and require no special expertise beyond the common agency standard of professionalism or prioritizing, such as common random inspections to monitor compliance. For any agency that is socially productive, it seems reasonable to assume that the welfare function in generic resources spent on regular agency activities is concave, without discontinuities and with a maximum above which the marginal benefit of funding the agency’s generic tasks is lower than its marginal costs. Suppose that \( F \) has diminishing returns to society, that is, let net welfare as a function of \( F \) be \( W_F(F) \), strictly concave with a maximum at point \( F = F^* \).

The optimal division between \( D \) and \( F \), given a fixed total budget, then depends on the shape of the welfare function generated by the discretionary spending, relative to the shape of the welfare function generated by the performance of the generic task. A budget-setter with perfect information about the shape of both of the (expected) welfare functions \( W_D(D) \) and \( W_F(F) \) would in principle want to divide any budget total so that the marginal welfares generated by the two budget chapters are equalized. There are two caveats to this. First, such a split might not exist due to the discontinuities in the welfare function of the discretionary spending. In that case, the division should be made so that the discretionary funds are kept on the "right side" of the jumps as discussed in Section 4, and the rest of the funds is assigned to the performance of the generic tasks. Second, equating the marginal welfare gains is not sufficient for attaining the optimal division, since \( W_D \) is non-monotonic and typically has several local maxima. A welfare-maximizing division of a given total budget thus has to be determined on a case-by-case basis, requiring immense information about the agency’s inner workings, its various tasks and the characteristics of those people performing them - exactly the type of information only a head would have and a government would hire him for.

Figure 4 provides a graphical illustration of the problem faced by the budget-setter for the baseline case in which the non-case specific spending is unproductive.\(^{45}\) Note that for \( F = 0 \) (front box plain), the welfare function is the same as in Figure 2 (left), hence the size of the jumps and types of tasks being performed on different intervals of \( D \). For \( D = 0 \) (right box plain) social welfare is solely a function of the generic spending with the above mentioned properties. Suppose first that the budget-setter has perfect information. The optimal total budget and the optimal budget split are simply found using the global maxima of both \( W_D \) and \( W_F \). The socially optimal total budget is \( F^* + D^* \) and the socially optimal budget division is \( F = F^* \) and \( D = D^* \).

Whenever the optimal total budget is not available, the optimal split can be determined by moving alongside the \((D, F)\) plain (bottom box plain). If, for instance, \( W_D'(0) > W_F'(0) \), as is the case in our illustration, all resources should to channeled towards the discretionary spending for the lowest

\(^{45}\)In the example in Figure 4, the discretionary budget \( D \) follows the welfare function in Figure 2 (left), which has a maximum \( D^* \) at a point where the agency performs task \( B \) at the maximum level, i.e \( D^* = D_B^* \). The welfare function to the generic budget is given as \( W_F(F) = 6\ln(3F + 1) - 3F^2/100 \) with a maximum at \( F^* \approx 9.83 \).
values of $F + D$. As the total resource constraint increases, both tasks might get financed provided that at some point the two marginal welfare gains are equal. Whenever a discontinuous jump in total welfare occurs alongside the discretionary budget $D$, a perfectly informed budget-setter will keep the discretionary resources on the "right side" of the jump and finance the generic tasks with the remainder. For instance, if the discontinuous increase in the welfare function alongside $D$ is sufficiently high, the perfectly informed budget-setter will at some point abruptly start allocating large part of the total resources towards the discretionary budget and decrease the financing of the generic task $F$. In Section 5 we have shown that there can be large number of points of discontinuity in $W_D(D)$. There can be many different values of the total budget at which the budget-setter would decide to reallocate resources between $D$ and $F$ in bulk. Determining the optimal budget split thus requires an immense amount of information about the workings of the agency and the motivations of the agency’s head, in particular for agencies with multiple discretionary tasks to perform and several experts to chose among them.

![Figure 4: Welfare generated by the total budget split into generic and discretionary spending.](image)

Now suppose the budget-setter has imperfect information, so that the first-best budget division is out of reach. Still, some information about the officials’ costs or an estimate of the head’s preference for impression management, together with the knowledge of $W_F$, can go a long way in setting a reasonably good level of discretion. A strong preference for impression management implies that the maximum
incentive contract rewards will be relatively low, speaking for narrowing the discretionary space. At the same time there may be a negative impact of tightening the head’s discretionary budget too much, since it can edge him towards having high-profile cases being pursued unsuccessfully for the exposure they generate. Overall, the jump discontinuities in \( W_D \) remain the prime determinants of the optimal division of the budget, but the budget-setter is unlikely to have full knowledge of \( W_D \).

If the agency’s superiors have limited information about the shapes of \( W_F \) and \( W_D \), the question who should determine the budget total and the budget’s division gains relevance, and it comes down to the characteristics of the agency’s head. A fully benevolent head would himself promote the optimal budget division and assign appropriate funds to generic tasks as their performance also enters his utility function. A partially benevolent agency’s head with preference for discretionary spending might still allocate part of the resources to the generic tasks’ performance, depending on his relative utility gain from the agency’s welfare generation and advancement of his private goals via less productive discretionary spending. Whether or not the agency’s head should then be allowed to determine the budget division depends both on the quality of the budget-setter’s information about the head’s motivations and about the shapes of \( W_F \) and \( W_D \). Both how much discretionary space a given agency head should have, and whether he should be able to partially determine the extent of it himself, all depends on the level of alignment between the head’s motivations and society’s interests. Moreover, in determining the discretionary space of the agency head, the budget-setter can limit the extraction of resources, but thereby also reduces the benefits from the head’s superior information on how to incentivize the officials. If impression management activities would also generate welfare, the heuristics of finding the optimal total budget and optimal budget division do not change, even though the shape of \( W_D \) is altered. The more social benefits come from impression management, the more discretionary space and influence over the budget split the agency’s head should be given.

7 Implications for Institutional Design

Our findings underline the importance of socially optimal institutional design and budgeting for government agencies. Policy makers should consider not only the effects of budget changes on the scale of the agency’s activities, but also on the type. In that, the size of the discretionary budget is a control tool with important welfare implications. Discussion about which tasks and tools to make available to a government agency should not be separate from determining the resources it will have at its disposal. They are complementary, both directly and indirectly. Moreover, the optimal sets of tasks and tools for an agency are subject to such realities as availability of skilled staff, means of secondary and tertiary performance rewards, and the personality of the agency’s head.

Any agency superior, at federal or state level, should be aware of its crucial role in tasking its agencies. To a government that has to save a certain amount across different agencies, our model suggests that these cuts be allocated where there is a bigger chance for a higher welfare jump upwards
- that is, where agencies have taken on high-profile cases too ambitious for their limited means. In practice, however, it will be difficult to tell how close to a welfare jump any given agency is, and so what would be optimal cuts and reassignments. In addition, in many agency practices the truly discretionary budget is stochastic, as high-priority cases - be it a terrorism threat, a tax scandal, or a merger notification - present themselves unannounced and then must be dealt with immediately.

One possible instrument to better control priorities is to compartmentalize the discretionary budget, earmarking parts for designated classes of cases. The organization may be setup to this effect, with departments that are given dedicated tasks and matching resources. To do so and improve welfare, however, requires a considerable amount of information that budget-setters typically would not have. While requirements on an agency to return left-over budget appear appealing to impose, they may not be effective either. Even if government were able to tell what amount of the budget was not spent on which cases, the return requirement would lead to rewards going up, as the head would no longer care about a residue and rather spend the entire budget. This might lead to a different type of wasteful spending, even though the incentives of the agency’s head are now more aligned with society’s interests. Moreover, the head’s instantaneous incentive to open up high-profile cases for impression management purposes remains.

Another possible institutional design element that could help counter the head’s urge to extract budget is to feed back part of the revenues from fines imposed by the agency, directly into its budget. This introduces a different type of potentially perverse incentives for agencies. It would encourage picking low hanging fruits with little social harm for their fine revenues, if not to foster a steady crop of violations to harvest later on. Yet, a fine return would in principle counter the head’s inclination to spend resources on cases that are a likely loss for their short-run impression management features. The more indirect feedback from agency success into budget increases over time may therefore be a better instrument for curbing excessive impression management. Yet it presupposes ability in government to properly evaluate what constitutes agency success.

Institutional mergers are an invasive form of agency reform. In the U.S., the debate on merging the DoJ’s Antitrust Division with the FTC into one competition authority has been long, yet rather academic. In Europe, meanwhile, several Member State authorities, including those in the United Kingdom, The Netherlands and Spain, have recently gone through extensive institutional re-organizations, that also included mergers with other agencies such as sector regulators and consumer authorities. An emerging literature studies the effectiveness of such institutional changes for market oversight by looking at the costs of the merger itself versus merger specific efficiencies in eliminating dual enforcement and expected gains from complementarities, the importance of a unified mission, and effects from regulatory competition lost.  

Our formalized approach points at the importance of the interaction between the combined talent pools and resources of the previously separate agencies, together with the new head’s objectives, in

\[\text{See Crane (2011) and Blumenthal (2013).}\]
determining the emphases that the merged agency will lay in the execution of its enlarged set of tasks. While there may be synergies in enforcement, an institutional merger can result in the more expertise and resource demanding duties being largely abandoned, if the two original agencies differed sufficiently in their tasks and talent pools, so that the smaller new budget is channeled to incentivize mostly simple basic cases, at the expense of the complex major tasks. Such a shift in the agency’s performance may be accompanied by a sudden unanticipated increase in social welfare. On the other hand, the head can possibly use the joined discretionary budgets across the merged agencies to increase a reward for a particular set of tasks, while decreasing other rewards, to ambiguous welfare effects. Amongst other things, our model can advise on the types of tasks that would be best combined under one roof.

Our findings reveal how both institutional design and budget can be used to pursue political goals or promote private interests. By either steeply reducing the budget or, instead, over-financing an agency ran by an ambitious head, its focus of attention can be shifted from low-risk welfare increasing tasks to high-profile cases that will ultimately fail. A more reserved head, instead, can be pushed into anergy by just slightly cutting its agency’s budget. Similar effects follow from extending the spectrum of tasks the agency is made responsible for, without also offering a matching budget. Parliaments better control their governments not to abuse these mechanisms, when they value the independence of their government agencies. While instructions and the administrative procedures of an agency are instruments to do this, so is replacing the agency head.

Competition authorities illustrate also how institutional design and budget assignments can fundamentally affect the political independence of government agencies. Gal (2004) observes that there are important differences in the effectiveness of competition law enforcement among developing countries, even if their legal background is similar, in large part due to the fact that

"[...] decision makers may not properly fund and structure the competition agency in order to reduce its ability to enforce law in practice." Gal (2004), p.7.

Choke of resources turning performance may be bluntly significant in developing countries, the European Commission recently recognized it "high time" that the independence of national competition authorities in the Member States is guaranteed in specific regulation, after having observed, amongst other things:

"[...] the misuse of NCA budgets by governments to gain leverage or as retaliation measure when decisions do not please them, for instance, by reducing or limiting budgets."47

According to the European Commission:

"It is necessary to ensure that NCAs can execute their tasks in an impartial and independent manner. For this purpose, minimum guarantees are needed to ensure the independence of NCAs and their management or board members and to have NCAs endowed with sufficient human and financial resources. Important aspects in this respect are the grant of a separate budget with budgetary autonomy for NCAs [...]."  

Our model shows that the transmission from budget to priorities is more subtle also in well-resourced agencies as well. By edging the discretionary part of the agency’s budget over certain threshold values, a budget-setter can qualitatively affect the agency’s task pick-up to its liking, ranging from the pursuit of complex sure failures to low hanging fruits. The budget so acts as an indirect instrument of political control over agencies.

8 Concluding Remarks

This paper offers a formal model to study task prioritization under a binding budget constraint in government agencies with multiple tasks to be picked up by staff with varying talent that is managed by a head who balances several interests. We find that the size of the agency’s discretionary budget influences not only the scale, but also the type of tasks it will engage in. Social welfare is non-monotonic and discontinuous in the agency’s budget. Small changes in the budget over certain thresholds may cause extensive restructuring from major to minor tasks, or vice versa. For lower binding budgets, the head who values exposure continues to sub-optimally incentivize work on complex tasks, when the agency should have shifted down to simpler tasks. A head who rather prefers his agency to remain low profile will even stop case handling altogether at low budgets. Such a head may need to be nudged towards pursuing welfare improving high-profile cases with more resources. In any event, looking locally at marginal welfare can give a budget-setter the wrong idea about socially optimal budget changes. A budget cut can improve welfare more than extra budget would, even if resources are below the welfare-maximizing level. By determining the size of the discretionary space of the agency head, the budget-setting body can indirectly control the type of tasks being pursued.

A number of extensions of our analysis present themselves. A public agency might be able to influence the skill level of its employees through recruitment, training and on the job skill growth. Staff quality is also endogenous in the sense that an agency that continuously does menial work will lose high quality staff and cannot hire better, whereas in a challenging institutional environment, the quality of work may spiral up, as a booming agency attracts talent. The quality of the agency’s

\footnote{Communication from the Commission to the European Parliament and the Council, Ten Years of Antitrust Enforcement under Regulation 1/2003: Achievements and Future Perspectives, European Commission, Brussels, 9 July 2014, at recital 29.}
talent pool directly affects its responsiveness to incentives and hence budget changes. A fuller model would include endogenous dynamics, as well as choices on human resources management within the agency as part of the head’s discretionary space. Increasing talent need not necessarily be beneficial, however: while within-case productivity may go up, in addition to the cost of training, the head may stimulate the more complex tasks more, leading to ambiguous welfare effects. Also, the head need not necessarily have the best intentions in this respect either: depending on his preferences and the agency’s circumstances, the head may prefer a staff that is below the socially optimal standard - which he can cheaply induce to take on complex cases his staff will not be able to complete successfully.

One possible institutional safeguard against a head’s preferences dictating his agency’s priorities is to install an executive committee or board to lead the agency instead of a single head, which is often the case. While this would introduce extra complexities of joint decision making, there is a priori no reason to think that some members of such a committee would have strong motivations to counter a typical head’s incentives. As long as sufficiently many committee members would value impression management activities for the agency, our basic results remain. In principle, the same is true for more complex multi-layered organizations, with division heads and a central agency head, although some of the personal gains from impression management are to be split among the committee members. While interesting questions about countervailing arguments in delegating arise, the type of findings we obtain seems rather robust. Likewise would a fuller description of the agent as a case-handling team of officials enrich our analysis.

In addition, officials may draw motivations outside of their contract terms. Their goals can imaginably contain elements similar to those of the agency’s head, including exposition from being a lead officer in charge of a high-profile case. Demonstrating high ability by taking on a difficult task may further ones career within the institution and beyond, for example through a revolving door into private sector jobs. A zealous official bent on serving welfare would lean towards opening landmark cases, despite it possibly being less preferred and rewarded by the agency’s head. Since the contract terms become of limited impact, the agency’s head will have less control, the fewer resources he has at his disposal. As long as the contract terms matter somewhat, our results remain. However, for higher budget values, a zealous staff can partially offset a head’s concern with impression management, and so improve the agency’s welfare yield. Intrinsically motivated to pursue high profile cases, the official’s high effort level may skew the head’s task preference towards the official’s favorite endeavor, as the head values high level of official’s activity, especially if it comes cheap. Such an agency will perform society’s bidding for larger ranges of budget constraints and with higher probability of success. Just like skill level, intrinsic staff motivation blooms in the right institutional structure, to the benefit of social welfare.

The non-monotonicity of the welfare function in the budget is related to the limited set of cases of certain discrete sizes that the agency can choose from. A more continuous set of tasks for each official to perform can smoothen the welfare function. Yet a policymaker will still likely over- or underestimate
the welfare impact of a budget change on the upper welfare envelope parts where shifts towards less complex tasks occur with a budget cut. Moreover, while the nature of the tasks performed by the type of government agencies considered in this paper is that they are discrete and of a certain, and typically large, minimum size, so that each task commits a chunk of resources to complete once opened, more cooperative types of enforcement, such as settlements, may also reduce the sizes of the jumps. Following public prosecutors plea bargaining in criminal cases, competition authorities increasingly seek to settle cartel cases, or obtain commitments. By effectively reducing the resources and time that need to be committed to a case, settlements make it possible to pursue more cases with the same overall budget and same number of staff. To accommodate such a scenario, our model could be extended to officials that can pick up more that one task and split their effort among them. This may reduce the sizes of the welfare jumps at agency focus shift points, in a similar way as adding more officials into the baseline setup does, but will not eliminate them. The discontinuities will become less pronounced, as each task's performance constitutes a lower share of the total expected welfare. Yet, some level case discretion will always remain, as settlements still require extensive case preparation.

Our results are obtained for a head who has perfect information about his officials’ talents. A natural extension is to assume that the talent level is private information of the official and the agency head knows only the ex ante talent distribution. Under asymmetric information, our qualitative findings remain. It implies that the individualized contract offered to an official is driven by the head's expected utility of the official's choice of tasks multiplied by their respective probabilities of being performed. In a setting with one difficult and one basic task, the agency head uses the contract design to set a "cut-off value" of the draw from the official's talent distribution above which the official chooses to perform the difficult task, and below which he performs the basic task. The official thus ex ante performs each task with some probability. If the budget is binding, this comes at a cost: in order to satisfy the incentive constraint of an official with a high talent draw, the reward for the basic task has to be set below the head's desired level. The difference increases as the discretionary budget becomes tighter, because the official with high talent draw is rewarded less and less optimally from the head's point of view. Once this cost becomes higher than the head's utility of having the officials with higher talent draws perform the more difficult task, the agency's head stops rewarding the complex task altogether and increases the reward for the simple task, because he is no longer bound by the ex ante incentive constraint of the official. This will generate a jump in the probability of performing each task and a discontinuity in the social welfare function. Moreover, if the agency’s head gains nonzero utility from opening a big case, there will be a shift towards performing complex tasks for the lowest budget values, just as in the model without information asymmetry. Introducing the information asymmetry thus changes the critical budget values, but the main message of the model holds: the head's shifting between task types creates discontinuities in the social welfare function. Moreover, agency performance remains suboptimal when the head gains utility from socially less productive impression management.

Another ready extension is asymmetric information about the characteristics of the available tasks;
their complexity and the benefits they bring to society and the agency’s head. We assume an "expert" agency head who has perfect information about the agency’s possible undertakings. If the head would not be an expert, a wedge is driven between formal and real authority in the agency, similar to that in Aghion & Tirole (1997). While the head retains the formal authority - that is, the right to overrule the official’s selection of the task to perform - the officials would have a real authority over the task pickup, whenever some level of authority delegation is optimal for the agency’s head, for example when the costs of obtaining the relevant information is too high. In our basic setup, the official’s private benefits of task completion were set at zero, for simplicity of analysis. If instead there is a difference between the private benefits of the tasks’ completion of the head and those of the official, and the officials have private information about the tasks’ characteristics, the effects of the agency’s prioritization with a shift in the discretionary budget assignment are ambiguous. The head is bound by the incentive constraint of the official, unless he can discover the relevant information himself - at a cost. Should these costs be too high for some types of tasks, the head may stop rewarding them as feasible missions completely. This would effectively decrease the number of tasks in the official’s choice set, and amount to more abrupt shifts in the agency’s performance and the resulting welfare. Alternatively, the agency’s head may need to leave the better informed official a rent in order to satisfy his incentive constraint, possibly resulting in a less efficient allocation of resources.

Finally, our model lays foundations for the political economy of budget assignment, focusing on the relationship between the head and his (direct and indirect) superiors, politicians, who have their own incentives. We noted that politicians may abuse the agency budget to steer its task take-up. One possible reason for a politician to may want to do so is to please his constituency. A lobby from industry with the responsible Ministry against the competition authority’s perceived aggression on discovering and sanctioning cartels may result in either budget cuts or enlargements - depending on where the agency is on the case type spectrum. Another mechanism is to modify the head’s incentives by changing the criteria by which he will be assessed. On the other hand, the head has tools to influence public opinion, which also interests politicians. Through impression management, the head can produce public support, which may translates into pressure on politicians to enlarge the agency’s budget. To start understanding these and other mechanisms of political influence requires an additional principal-agent model, on top of the one analysed in this paper, in which government is the principal and the agency head the agent.

References


Appendix

Proof of Lemma 1

The official knows his own talent level $\theta$, the difficulty of all possible tasks $\{\psi_1, ..., \psi_n\}$, and the contract offered to him, with rewards for completing each of these tasks $\{R_1, ..., R_n\}$. If the official chooses to perform task $i$, he has expected utility

$$U^O = p_i R_i - \frac{1}{2} \gamma a^2,$$

where $p_i = a \times \theta^{\psi_i}$ and $R_i$ is the reward offered for completing task $i$. The official determines the effort he will put in by maximizing expected utility

$$E[U^O] = p_i R_i - \frac{1}{2} \gamma a^2 = a \theta^{\psi_i} R_i - \frac{1}{2} \gamma a^2,$$

which leads to first-order condition

$$\frac{\partial U^O}{\partial a} = \theta^{\psi_i} R_i - \gamma a = 0$$

and so to

$$a^* = \frac{\theta^{\psi_i} R_i}{\gamma},$$

which immediately from $p_i = a \times \theta^{\psi_i}$ gives the probability of tasks’ completion under the official’s optimal choice of effort as

$$p_i = \frac{\theta^{2 \psi_i} R_i}{\gamma}.$$

Hence, an official that performs task $i$ and exerts his optimal effort level has expected utility

$$E[U^O] = a^* \theta^{\psi_i} R_i - \frac{1}{2} \gamma a^2 = \frac{\theta^{2 \psi_i} R_i^2}{\gamma} - \frac{1}{2} \gamma \frac{\theta^{2 \psi_i} R_i^2}{\gamma^2} = \frac{\theta^{2 \psi_i} R_i^2}{2 \gamma^2} = \frac{(\theta^{\psi_i} R_i)^2}{2 \gamma}.$$

Maximizing this expression over the set of tasks $i \in \{1, ..., n\}$, taking into account the contract offered to the official $\{R_1, ..., R_n\}$, the official chooses to perform the task for which utility

$$\theta^{\psi_i} R_i$$

attains the highest value. ■
**Proof of Lemma 2**

The head’s utility generated by rewarding a given task \(i\) is

\[
U^H = p_id_i + \phi d_i + V(D - p_iR_i) = \frac{\theta^{2\psi_i}R_id_i}{\gamma} + \phi d_i + V\left(D - \frac{\theta^{2\psi_i}R_i^2}{\gamma}\right).
\]

The optimal reward \(R_i\) for task \(i\) follows from

\[
\frac{\partial U^H}{\partial R_i} = \frac{\theta^{2\psi_i}d_i}{\gamma} - 2V \frac{\theta^{2\psi_i}R_i}{\gamma} = 0,
\]

as

\[R_i^u = \frac{d_i}{2V}.
\]

Substituting this optimal reward level back into the head’s utility function we get

\[
U^H = \frac{\theta^{2\psi_i}d_i^2}{2V\gamma} + \phi d_i + V\left(D - \frac{\theta^{2\psi_i}d_i^2}{4V^2\gamma}\right) = \frac{\theta^{2\psi_i}d_i^2}{4V\gamma} + \phi d_i + VD.
\]

Since the fixed component is immaterial for his choices, the task that maximizes the head’s utility is the task that maximizes \(\frac{\theta^{2\psi_i}d_i^2}{4V^2\gamma} + \phi d_i\). The head’s utility-maximizing probability of completing this task is then implied by the reward \(R_i^u = \frac{d_i}{2V}\) and Lemma 1. Finally, the head may also decide not to incentivize any task yielding the utility of using all resources for impression management activities, \(VD\). He will do so if \(VD > \frac{\theta^{2\psi_i}d_i^2}{4V^2\gamma} + \phi d_i + VD\) for all tasks \(i \in I\). □

**Proof of Lemma 3**

First we establish that the agency head’s utility from performance of task \(B\) is non-increasing in the task’s difficulty.

For \(D \geq R_B^u\), the head’s utility from choosing and optimally rewarding task \(B\), that is, by offering the reward \(R_B^u\), is

\[
E[U_B^H] = \frac{\theta^{2\psi_B}d_B^2}{4V\gamma} + \phi d_B + VD,
\]

therefore

\[
\frac{\partial E[U_B^H]}{\partial \psi_B} = \frac{d_B^2}{4V\gamma} \times \frac{\partial \theta^{2\psi_B}}{\partial \psi_B} = \frac{2d_B^2\theta^{2\psi_B}}{4V\gamma} \times \ln(\theta),
\]

which is clearly non-positive given \(\theta \in [0, 1]\).

For \(D < R_B^u\), the head’s utility from choosing and optimally rewarding task \(B\), that is, by offering
the reward $D$, is
\[ E[U_B^H] = \frac{\theta^2 \psi B D d_B}{\gamma} + \phi d_B + V \left( D - \frac{\theta^2 \psi B D^2}{\gamma} \right), \]
therefore
\[ \frac{\partial E[U_B^H]}{\partial \psi B} = \frac{\partial \theta^2 \psi B}{\partial \psi B} \times \frac{D}{\gamma} (d_B - V D) = \ln(\theta) \times \frac{2 D \theta^2 \psi B}{\gamma} (d_B - V D), \]
where the last bracket is positive since $D < R_B^n = \frac{d_B}{2}$, thus making the whole derivative non-positive for $\theta \in [0, 1]$.

It is now sufficient to prove the lemma for $\psi_A = \psi_B$, since that is the lower bound for task’s $B$ difficulty given by the assumptions of the lemma. Proving the lemma for the lowest possible difficulty of task $B$, that is, when task $B$ is “most attractive” for the head, then proves it for all higher difficulties. Moreover, we only need to prove the lemma for $\phi < 0$, since for non-negative values of $\phi$ task $A$ is preferred over task $B$ for all budget values trivially. Assuming $\psi_A = \psi_B$ and $\phi < 0$, we will prove the lemma for three ranges of $D$ with different forms of head’s utility: non-binding ($D > R_A^n$), partially binding ($R_B^n \leq D \leq R_A^n$), and fully binding ($D < R_B^n$).

**Non-binding** ($D > R_A^n$). Task $B$ will be preferred over both task $A$ and no task iff $E[U_B^H] > E[U_A^H]$ and $E[U_B^H] \geq E[U_0^H]$, i.e.
\[ \frac{\theta^2 \psi B d_B^2}{4V\gamma} + \phi d_B + V D > \frac{\theta^2 \psi A d_A^2}{4V\gamma} + \phi d_A + V D, \]
\[ \frac{\theta^2 \psi B d_B^2}{4V\gamma} + \phi d_B + V D \geq V D. \]

The minimal value that $\phi$ can have in order for the condition $E[U_B^H] \geq E[U_0^H]$ to be satisfied is $\phi = -\frac{\theta^2 \psi B d_B}{4V\gamma}$. With this minimal value for $\phi$ and $\psi_B = \psi_A$, the condition $E[U_B^H] > E[U_A^H]$ becomes
\[ 0 > \theta^2 \psi A d_A - \theta^2 \psi B d_B, \]
which is clearly not possible since $d_B < d_A$. Either task $A$ or no task is preferred over task $B$ in this budget range.\(^{49}\)

**Partially binding** ($R_B^n \leq D \leq R_A^n$). Task $B$ will be preferred over both task $A$ and no task iff $E[U_B^H] > E[U_A^H]$ and $E[U_B^H] \geq E[U_0^H]$, i.e.
\[ \frac{\theta^2 \psi B d_B^2}{4V\gamma} + \phi d_B + V D > \frac{\theta^2 \psi A d_A^2}{4V\gamma} + \phi d_A + V D, \]
\[ \frac{\theta^2 \psi B d_B^2}{4V\gamma} + \phi d_B + V D \geq V D. \]

\(^{49}\)Considering the minimal value of $\phi$ is sufficient, since higher values of $\phi$ make the first inequality even less likely to be satisfied, as $\phi < 0$.  

40
\[ \frac{\theta^{2\psi_B} d_B^2}{4V\gamma} + \phi d_B + VD \geq \frac{\theta^{2\psi_A} D d_A}{\gamma} + \phi d_A + V \left( D - \frac{\theta^{2\psi_A} D^2}{\gamma} \right), \]

\[ \frac{\theta^{2\psi_B} d_B^2}{4V\gamma} + \phi d_B + VD \geq VD. \]

The minimal value that \( \phi \) can have in order for the condition \( E[U^H_B] \geq E[U^H_0] \) to be satisfied is \( \phi = -\frac{\theta^{2\psi_B} d_B}{4V\gamma} \). With this minimal value for \( \phi \) and \( \psi_B = \psi_A \), the condition \( E[U^H_B] > E[U^H_A] \) becomes

\[ 0 > \frac{\theta^{2\psi_B}}{\gamma} (Dd_A - \frac{d_B d_A}{4V} - VD^2), \]

which is never satisfied for \( D \) in the relevant range \( R_u^B \leq D \leq R_u^A \), i.e. \( \frac{d_B}{2V} \leq D \leq \frac{d_A}{2V} \).

**Fully binding** \( (D < R_u^B) \). Task \( B \) will be preferred over both task \( A \) and no task iff \( E[U^H_B] > E[U^H_A] \) and \( E[U^H_B] \geq E[U^H_A] \), i.e.

\[ \frac{\theta^{2\psi_B} D d_B}{\gamma} + \phi d_B + V \left( D - \frac{\theta^{2\psi_B} D^2}{\gamma} \right) > \frac{\theta^{2\psi_A} D d_A}{\gamma} + \phi d_A + V \left( D - \frac{\theta^{2\psi_A} D^2}{\gamma} \right), \]

\[ \frac{\theta^{2\psi_B} D d_B}{\gamma} + \phi d_B + V \left( D - \frac{\theta^{2\psi_B} D^2}{\gamma} \right) \geq VD. \]

For \( \psi_B = \psi_A \), the condition \( E[U^H_B] > E[U^H_A] \) is satisfied when \( D < \frac{\theta^{2\psi_B}}{\gamma} \), but that directly violates the condition \( E[U^H_B] \geq E[U^H_A] \) which concludes the proof that either task \( A \) or no task will always be preferred over task \( B \) when \( d_A > d_B \) and \( \psi_B \geq \psi_A \).

**Proof of Proposition 1**

The agency will always perform the head’s most favorite activity for a given budget constraint - either task \( A \), task \( B \) or no task at all. To find the budget points for which the head’s order of preferences changes, we do a pairwise comparison of the activities’ utilities. The agency will then change its focus whenever the head’s utility from his two most preferred activities at that budget value is equal.

For the two tasks \( A \) and \( B \) with \( (d_B, \psi_B) < (d_A, \psi_A) \), Lemma 2 implies that \( R_A^u > R_B^u \). Consider two cases separately: first the case when both the head’s most preferred rewards are unaffordable, and second the case in which only \( R_B^u \) is affordable under the budget constraint.
Case 1: Fully binding budget constraint.\footnote{In case of two tasks \( A \) and \( B \) such that \( d_A > d_B \) we call the budget “fully binding” when none of the head’s preferred rewards is affordable, i.e. \( D < R_B'' \), and “partially binding” when \( R_B'' \leq D \leq R_A'' \).} For the budget values \( D < R_B'' \), none of the u-rewards is affordable, and the agency head will just reward his most preferred task by offering the maximal reward \( D \) for completion, since head’s utility from rewarding performing task \( i \) is increasing on interval \((0, R_i'')\). The head’s expected utility generated by the task \( i \in \{A, B\} \) then amounts to

\[
E[U_i] = \frac{\theta^{2\psi_i} D}{\gamma} + \phi d_i + V\left(D - \frac{\theta^{2\psi_i} D^2}{\gamma}\right).
\]

Task \( B \) will be preferred by the agency’s head for a given budget \( D \) if

\[
\frac{\theta^{2\psi_B} D d_B}{\gamma} + \phi d_B + V\left(D - \frac{\theta^{2\psi_B} D^2}{\gamma}\right) > \frac{\theta^{2\psi_A} D d_A}{\gamma} + \phi d_A + V\left(D - \frac{\theta^{2\psi_A} D^2}{\gamma}\right),
\]

amounting to

\[
0 < (\theta^{2\psi_A} - \theta^{2\psi_B})VD^2 + (\theta^{2\psi_B} d_B - \theta^{2\psi_A} d_A)D + \gamma \phi (d_B - d_A).
\]

The roots of the quadratic equation are

\[
D_1^* = \frac{\theta^{2\psi_B} d_B - \theta^{2\psi_A} d_A - \sqrt{(\theta^{2\psi_B} d_B - \theta^{2\psi_A} d_A)^2 - 4(V\theta^{2\psi_A} - V\theta^{2\psi_B})\gamma \phi (d_B - d_A)}}{2V(\theta^{2\psi_B} - \theta^{2\psi_A})},
\]

\[
D_2^* = \frac{\theta^{2\psi_B} d_B - \theta^{2\psi_A} d_A + \sqrt{(\theta^{2\psi_B} d_B - \theta^{2\psi_A} d_A)^2 - 4(V\theta^{2\psi_A} - V\theta^{2\psi_B})\gamma \phi (d_B - d_A)}}{2V(\theta^{2\psi_B} - \theta^{2\psi_A})},
\]

where task \( A \) is preferred by the agency’s head in the interval \([D_1^*, D_2^*]\) and task \( B \) otherwise. There is a switch among the performed tasks for (one of) these budget values if they fall in the \((0, R_B'')\) interval. A necessary condition for either of these critical budget values to exist is their existence in real numbers, i.e. \((\theta^{2\psi_B} d_B - \theta^{2\psi_A} d_A)^2 - 4(V\theta^{2\psi_A} - V\theta^{2\psi_B})\gamma \phi (d_B - d_A) \geq 0\). This condition is violated whenever \( \phi >> 0 \) and task \( A \) is then the head’s most preferred task for all budget values amounting to no switches in the task performed.

Assuming \( D_1^* \in \mathbb{R}, D_1^* \in (0, R_B'') \) iff \( \theta^{2\psi_A} d_A < \theta^{2\psi_B} d_B \) together with \( \phi \geq 0 \). If \( \phi < 0 \), we have that \( D_1^* < 0 \) and the agency will never switch to performing task \( A \) for the lowest budget values as it will perform no task. If \( \phi \geq 0 \) and \( \theta^{2\psi_A} d_A \geq \theta^{2\psi_B} d_B \), the task \( A \) is again the head’s most preferred task for all budget values.

Assuming \( D_2^* \in \mathbb{R}, D_2^* < R_B'' \) iff \( (\theta^{2\psi_A} + \theta^{2\psi_B})d_A^2 - \theta^{2\psi_A} 2d_A d_B + 4V \gamma \phi (d_B - d_A) < 0 \). Moreover, \( D_2^* > 0 \) if \( \theta^{2\psi_B} d_B > \theta^{2\psi_A} d_A \) or if \( \theta^{2\psi_B} d_B < \theta^{2\psi_A} d_A \) and \( \phi < 0 \).

So far we were concerned with the head’s preferences over tasks \( A \) and \( B \). As long as \( \phi \geq 0 \), the
above conditions are the necessary and sufficient conditions for $D_1^*$ and/or $D_2^*$ to exist in the fully binding budget interval as the critical budget values for the agency as a whole.

When $\phi < 0$, $D_1^*$ will not exist, but both $D_2^*$ and $D_0^*$ may. We define $D_{0,i}$ as a critical budget point below which the agency performs no task at all, that is, for all budget values $D \in (0, D_{0,i})$, and at (and for some values above) which the agency performs task $i$. It has the form

$$D_{0,i}^* = \frac{\theta^2 \psi_i d_i - \sqrt{(\theta^2 \psi_i d_i)^2 + 4V \theta^2 \psi_i \gamma \phi d_i}}{2V \theta^2},$$

where the head’s utility of rewarding task $i$, i.e. $E[U^H_i]$ equals to rewarding no task at all, i.e. $VD$. $D_{0,i}^*$ will then represent a shift in the agency’s behavior for fully binding budget range as long as it exists and $D_{0,B}^* \leq D_{0,A}^*$ (or equivalently $D_{0,A}^* \leq D_{0,B}^*$). More generally, as long as $\phi < 0$ there will exists one critical budget point $D_{0,i}^*$ if the agency performs some task $\{A, B\} \in I$ for some budget value in the fully binding budget range.

**Case 2: Partially binding budget constraint.** For the budget values $R_{B}^u \leq D \leq R_{A}^u$, the optimal affordable reward for task $A$ is $D$, since the head’s utility generated by rewarding task $A$ is increasing in the reward offered until $R_{A}^u$. $R_{A}^u$ is the point above which the marginal benefit of further increasing reward for task $A$ is lower than marginal benefit of keeping the money, $V$. For task $B$, $R_{B}^u$ is still affordable. For the budget values $R_{B}^u \leq D \leq R_{A}^u$, the head’s expected utilities generated by rewarding the two tasks, given the optimal choices of both the agency’s head and the official, are thus

$$E[U^H_A] = \frac{\theta^2 \psi_A d_A}{\gamma} + \phi d_A + V \left( D - \frac{\theta^2 \psi_A D^2}{\gamma} \right),$$

$$E[U^H_B] = \frac{\theta^2 \psi_B d_B^2}{4V \gamma} + \phi d_B + V D.$$

Task $B$ will be preferred by the agency’s head if

$$\frac{\theta^2 \psi_B d_B^2}{4V \gamma} + \phi d_B + V D > \frac{\theta^2 \psi_A d_A}{\gamma} + \phi d_A + V \left( D - \frac{\theta^2 \psi_A D^2}{\gamma} \right),$$

amounting to

$$0 < (4V^2 \theta^2 \psi_A) D^2 + D(-4V \theta^2 \psi_A d_A) + (-4V \gamma \phi d_A + \theta^2 \psi_B d_B^2 + 4V \gamma \phi d_B).$$

The roots of the quadratic equation are
\[ D_{1,2} = \frac{\theta^\psi d_A \pm \sqrt{\theta^{2\psi} d_A^2 + 4V\gamma \phi d_A - \theta^{2\psi} d_B^2 - 4V\gamma \phi d_B}}{2V\theta^\psi}. \]

The positive root never falls within the interval \([R_B^u, R_A^u]\) since it is never smaller than \(R_A^u = \frac{d_A}{2\psi}\) and does not represent a point of change in head’s or agency’s priorities.

The negative root, denoted \(D_2^*\) to represent a same type of switch from task \(A\) to task \(B\) as \(D\) decreases since task \(A\) is preferred over task \(B\) for budget values \(D \in [D_2^*, \tilde{D}_1]\), does exist if it is in real numbers - ensured by the condition \(\frac{\theta^{2\psi} d_A^2}{4V\gamma} + \phi d_A > \frac{\theta^{2\psi} d_B^2}{4V\gamma} + \phi d_B\) stating that task \(A\) is preferred by the agency’s head over task \(B\) when the budget is not binding - if \(D_2^* > R_B^u\), ensured by the condition \((\theta^{2\psi} + \theta^{2\psi} B)d_B^2 - \theta^{2\psi} 2d_A d_B + 4V\gamma \phi (d_B - d_A) > 0\), and if \(0 \notin \max\{Q_A, Q_B, 0\}\), ensuring that performing task \(i\) is preferred by the agency’s head over performing no task for \(D = R_A^u\).

Finally, if \(Q_A \geq 0 > Q_B\) and \(D_{0, A}^* > R_B^u\), there will exist exactly one critical budget value in the partially binding budget range, \(D_{0, A}^*\), representing a shift in the agency’s priorities from performing task \(A\) to performing no task at all with a decrease in the available budget \(D\). \(Q_A \geq 0 > Q_B\) ensures that task \(A\) is performed when \(D = R_A^u\). In order for no task to be preferred over task \(B\) at point \(D_{0, A}^*\) in the partially budget range, no task has to be preferred over task \(B\) also at point \(R_B^u\), hence the need for \(0 > Q_B\).

**Proof of Proposition 2**

For the two tasks \(A\) and \(B\) with \((d_B, \psi_B) < (d_A, \psi_A)\), when the official performs task \(i\), social welfare is

\[ E[W_i] = p_i d_i - D = a\theta^\psi d_i - D = \frac{\theta^\psi R_i}{\gamma} \theta^\psi d_i - D, \]

which without a binding discretionary budget, when \(R_i = R_i^u = \frac{d_i}{2\psi}\), is

\[ E[W_i^u] = \frac{\theta^\psi R_i}{\gamma} \theta^\psi d_i - D = \frac{\theta^\psi d_i}{2V\gamma} \theta^\psi d_i - D = \frac{\theta^{2\psi} d_i^2}{2V\gamma} - D. \]

Moreover, social welfare when no task is being performed is always \(W_0 = -D\). Society therefore prefers that the official performs the task that has maximal \(\theta^\psi d_i\), hence the agency head's interests are aligning with those of society only when \(\phi = 0\). However, society would prefer to always pay the full \(D\) instead of \(R_i^u\), given that \(D\) is already determined and any residue is lost.

**Proposition 2 (iii)** follows from \(E[W_i] = \frac{\theta^\psi R_i}{\gamma} \theta^\psi d_i - D\) and \(W_0 = -D\). The size of the welfare jump is then \(\Delta W_{i, 0} = E[W_i] - W_0 = \frac{\theta^\psi R_i}{\gamma} \theta^\psi d_i\).
Proposition 2 (ii) concerns jumps of the type $D_1^*$, which can only happen in the fully binding budget range where

$$E[W] = p_i d_i - D = a \theta^{\psi_i} d_i - D = \frac{\theta^{\psi_i} R_i}{\gamma} \theta^{\psi_i} d_i - D = \frac{\theta^{2\psi_i} d_i}{\gamma} - D,$$

and therefore the size of the jump is

$$\Delta W_{B,A} = D_1^* \left( \frac{\theta^{2\psi_B} d_B}{\gamma} - 1 \right) - D_1^* \left( \frac{\theta^{2\psi_A} d_A}{\gamma} - 1 \right) = D_1^* \frac{\theta^{2\psi_B} d_B - \theta^{2\psi_A} d_A}{\gamma}.$$

From Proposition 1 $\theta^{2\psi_A} d_A < \theta^{2\psi_B} d_B$ is a necessary condition for $D_1^*$ to exist and the jump is thus always from a higher social welfare to a lower level of social welfare with a budget decrease below $D_1^*$.

Proposition 2 (i) requires two comparisons of welfare jumps separately for the fully and the partially binding budget range.

Case 1: Fully binding budget constraint. For the fully binding budget range, the size of the welfare jump is analogically to Proposition 2 (ii)

$$\Delta W_{A,B} = D_2^* \theta^{2\psi_A} d_A - \theta^{2\psi_B} d_B.$$

Whenever $\phi \geq 0$ existence of $D_2^*$ in Proposition 1 requires $\theta^{2\psi_B} d_B > \theta^{2\psi_A} d_A$ and the social welfare increases with a budget cut below $D_2^*$ and a shift to performance of task $B$. Whenever $\phi < 0$, $D_2^*$ may exist for both $\theta^{2\psi_B} d_B > \theta^{2\psi_A} d_A$ and $\theta^{2\psi_B} d_B < \theta^{2\psi_A} d_A$.

Note that the sign of the welfare effect of the jump is independent of $D_2^*$, but the size of the jump is not. Also note the difference between the society’s preference (for the task with maximum $\theta^{2\psi_i} d_i$) and the task that the agency head chooses- the task with maximum $\theta^{2\psi_i} D(d_i - DV)$ as long as $\phi = 0$. If furthermore $V \rightarrow 0$, society’s and the head’s preferences are aligned as far as the task selection goes, and the task with the highest $\theta^{2\psi_i} d_i$ is always performed on the interval $D \in (0, R_B)$.

Furthermore, the expressions $\Delta W_{A,B}^p$ and $\Delta W_{B,A}^p$ in the perceived welfare function will be zero if the public subjectively evaluates task $B$ as

$$d_B^p = d_A \theta^{2(\psi_A - \psi_B)},$$

instead of $d_B^p = d_B$. Higher or lower (subjective) value assigned to this task’s completion by society then determines the sign of jump in perceived welfare.
**Case 2: Partially binding budget constraint.** With a partially binding budget, i.e. \( R_A \geq D \geq R_B \), the head offers \( D \) as a reward for completing task \( A \), and \( R_B \) for task \( B \). Social welfare generated by the performance of the two tasks is then given by

\[
E[W_A] = p_A d_A - D = a\psi_A d_A - D = \frac{\theta \psi_A R_A}{\gamma} \theta \psi_A d_A - D = \frac{\theta^2 \psi_A D}{\gamma} d_A - D,
\]

\[
E[W_B] = p_B d_B - D = a\psi_B d_B - D = \frac{\theta \psi_B R_B}{\gamma} \theta \psi_B d_B - D = \frac{\theta^2 \psi_B D_B^2}{2V\gamma} - D.
\]

The jump at a point \( D^*_2 \) between tasks \( A \) and \( B \) amounts to welfare difference

\[
\Delta W_{B,A} = \frac{\theta^2 \psi_B d_B^2}{2V\gamma} - D^*_2 - \frac{\theta^2 \psi_A D^*_2}{\gamma} d_A + D^*_2 = \frac{\theta^2 \psi_B d_B^2}{2V\gamma} - \frac{\theta^2 \psi_A d_A D^*_2}{\gamma}.
\]

The partially binding budget range of such \( D^*_2 \) together with the conditions for its existence from Proposition 1 ensure again that \( \Delta W_{B,A} > 0 \) whenever \( \phi \geq 0 \) and the sign depends on other parameter values whenever \( \phi < 0 \).

Finally, \( \Delta W^p_{B,A} \) will be zero if society subjectively evaluates the benefits from completing task \( B \) as

\[
d^p_B = \sqrt{2V\theta^2 \psi_A - 2\psi_B d_A D^*_2}.
\]

Higher or lower (subjective) value assigned to this task’s completion by society then determines the sign of jump in the perceived welfare. ■