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Working Paper

Stock market cycles and supply side dynamics

FinMaP-Working Paper, No. 45

Provided in Cooperation with:
Collaborative EU Project FinMaP - Financial Distortions and Macroeconomic Performance, Kiel University et al.

Suggested Citation: de Grauwe, Paul; Gerba, Eddie (2015) : Stock market cycles and supply side dynamics, FinMaP-Working Paper, No. 45, Kiel University, FinMaP - Financial Distortions and Macroeconomic Performance, Kiel

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Stock Market Cycles and Supply Side Dynamics

by: Paul de Grauwe and Eddie Gerba
The agent-based (behavioural) model is extended to include a financial friction on the supply side. Firms finance capital purchases using external financing, but need to pay for it in advance. In addition, firm financing constraint and net worth are determined by stock market prices, which can (and will) deviate from the fundamental value. The result is that production, supply of credit and the share that firms pay to capital producers heavily depends on the stock market cycles. During phases of optimism, credit is abundant, access to production capital is easy, the cash-in-advance constraint is lax, the risks are undervalued, and production is booming. But upon reversal in market sentiment, the contraction in all these parameters is deeper and asymmetric. This is even more evident in the behavioural model since cognitive limitations of economic agents result in exacerbation of the contraction. Lastly, the behavioural model matches much of the data, including the interest rate, inflation, firm credit, firm financing spread, and bank net worth. It is also successful in matching several supply-side relations (capital-firm credit, inflation-interest rate) as well as their autocorrelations. The results from the empirical validation are favourable to the behavioural model.

**Keywords:** Supply-side, beliefs, financial frictions, model validation

**JEL:** B41, C63, C68, E22, E23, E37

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Stock Market Cycles and Supply Side Dynamics

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June 18, 2015

Abstract

The agent-based (behavioural) model is extended to include a financial friction on the supply side. Firms finance capital purchases using external financing, but need to pay for it in advance. In addition, firm financing constraint and net worth are determined by stock market prices, which can (and will) deviate from the fundamental value. The result is that production, supply of credit and the share that firms pay to capital producers heavily depends on the stock market cycles. During phases of optimism, credit is abundant, access to production capital is easy, the cash-in-advance constraint is lax, the risks are undervalued, and production is booming. But upon reversal in market sentiment, the contraction in all these parameters is deeper and asymmetric. This is even more evident in the behavioural model since cognitive limitations of economic agents result in exacerbation of the contraction. Lastly, the behavioural model matches much of the data, including the interest rate, inflation, firm credit, firm financing spread, and bank net worth. It is also successful in matching several supply-side relations (capital-firm credit, inflation-interest rate) as well as their autocorrelations. The results from the empirical validation are favourable to the behavioural model.

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1 Motivation

Broadbent (2012) finds that the main reason for the most recent contraction in the UK economic cycle has been a contraction in the supply side, rather than the standard demand-side optics emphasized in most financial friction models. He argues that a combination of uneven demand across sectors combined with an impaired financial system (due to its inability to effectively reallocate resources sufficiently quickly to respond to shocks) has lead to a reduction in aggregate output per employee. While Chadha and Warren (2012) find an equally important role for the efficiency or labor wedge as the key driver for the most recent UK contraction, they however reason that it is not necessarily the shocks originating from the supply side of the economy that lie at the centre. Their business cycle accounting exercise shows that asset price shocks might equally appear in the supply-side wedges, suggesting that the supply side works as a propagator for shocks originated elsewhere. At the same time, Manasse (2013) argues that the cause for the most recent recession in Italy (and some other Eurozone countries) is a weak and anemic supply side. A lack of reform in the product, labor and credit markets has resulted in weak productivity and competitiveness performance for more than a decade.

On the contrary, many financial friction models have concentrated on the demand-side effects from financial cycles. Many have investigated the impact of asset prices and/or risks on the demand for credit, investment, demand for mortgages, consumption, labour supply, demand for capital, etc. In (most of) these models, financial prices alter the value of collateral, perception of risks, probability of default, or future propensity to save, which alters the aggregate demand allocations.

However, in the current paper we wish to investigate the impact of financial swings on the supply side of the economy. In particular, we wish to examine how imperfect financial markets coupled with (imperfect) stock market beliefs affect the allocations on the production side of the economy. In addition, we wish to de-couple and analyse in detail the impacts that (imperfect) financial markets have on the supply of capital and credit, demand for labour, and technology from the effects that limited information (or limited cognitive abilities) has on the same. We therefore apply these extensions in a behavioural (or bounded rationality) framework in order to understand and evaluate the importance of each channel in the regularities of the business cycles. We perform model validations using impulse response analyses,

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statistical comparisons, moment matching and business cycle comparisons using
more than 60 years of filtered macroeconomic and financial data.

Impulse response analyses show that (temporary) supply-side shocks do not only
improve the fundamentals in the economy but cause a brief wave of market sentiment
(or animal spirit), which in the case of a positive shock result in a more-than propor-
tional increase in output, capital supply, bank equity, and fall in interest rate.
Moreover, credit supply to entrepreneurs is permanently increased, which means
that firms can access a higher external financing in the future. This means that firm
productivity is permanently improved.

The statistical validation of the model, and moment matching show that the
model is capable of capturing many of the supply-side relations found in the data.
This includes supply-side and financial variables such as the (risk-free) interest rate,
inflation, credit to firms, deposits, firm financing spread and net worth of banks.
It is also successful in matching several supply-side relations (capital-firm credit,
inflation-interest rate) as well as their autocorrelations (output, capital and infla-
tion). Moreover, we find a strong co-movement between asset prices on one hand,
and net worth and the financing spread. During stock market booms, net worth rises
which increases the firm’s collateral value and reduces its probability of default, and
so it reduces the external financing spread (as it is less risky for banks to lend to
firms).

2 Model set-up

To incorporate a supply side with an asset price bubble and financing constraints in
an agent-based framework, we apply the following modifications to De Grauwe and
Macchiarelli (2015). The first modification is an extension of the financial accelerator
mechanism onto input markets. We allow a firm’s purchasing position on the input
markets to directly depend on their financial state. A higher value of net worth
means that the collateral constraint the producing firm faces is lower. As a result it
can borrow more, which will press the marginal costs down, and therefore it will be
able to buy capital inputs at a relatively lower price.

The second modification is a pay-in-advance constraint on the input market. We
impose the condition that (a share of) the cost of capital must be paid in advance
of purchase in order to insure capital good producers that they will sell what they
produce. It is a kind of depository insurance. Firms will finance it with a share of
the (liquid) external financing that they get. Since this in turn depends on the cash
position that they will hold in the next period, the expected (stock) market price
will de facto reflect the price they have to pay in advance for the capital. A higher
expected value of the (stock) market price improves the borrowing conditions of the
firm already today, meaning that it can already today commit to pay more for the
inputs. This will increase the quantity of output produced in the next period. Once
the firm reaches the next period, the (stock) market price will be realised, pushing
the firm’s net worth up and therefore it will be able to repay its debt in full. We
make the share of the cost of capital to be pre-paid time varying over the business
cycle in order to capture the asymmetries in financial (or liquidity) positions over
the cycle.

The third modification we introduce is a rate of utilization of capital. Produc-
ing firms, apart from choosing the amount of capital to purchase and use in the
production, choose also the rate at which capital will be used in the production.
The higher the share, the more effective use is made of capital in the production
function and the more (intermediate) products can be produced for a fixed amount
of capital. However, increasing the capital utilization cost is also costly because it
causes a faster rate of capital depreciation. Hence in this modified version of the
model, the firms do not only choose the quantity of capital to be purchased from
the capital good producers, but also the rate at which they will use this capital in
production.

We proceed in the next subsection by incorporating these mechanisms in the
behavioural model. We will also describe the incomplete information and learning
framework in section 2. The model is solved in section 3 and the derived quantitative
results are analyzed in section 4. Section 5 concludes.

### 2.1 Supply side and financial frictions

In what follows, we will disentangle capital production from capital utilization rate,
and introduce variable capital usage in an otherwise standard financial accelerator
mechanism (augmented with stock market cycles) as in DeGrauwe and Macchiarelli
(2015). Capital good producers produce capital which they rent to entrepreneurs at
cost $R^*_t$. Entrepreneurs use the newly purchased capital and labor to produce final
goods. Whereas capital good producers operate in perfectly competitive good mar-

\footnote{For the remaining model set-up, we refer to aforementioned paper and DeGrauwe (2008, 2012).}
kets, retailers are monopolistically competitive. Therefore they price discriminate, resulting in price frictions on the aggregate supply side (Phillips curve).

2.1.1 Capital Good Producers

Following Gerali et al (2010), perfectly competitive capital good producers (CGP) produce a homogeneous good called 'capital services' using input of the final output from entrepreneurs \((1 - \delta)k_{t-1}\) and retailers \((i_t)\) and the production is subject to investment adjustment costs. They sell new capital to entrepreneurs at price \(Q_t\).

Given that households own the capital producers, the objective of a CGP is to choose a \(K_t\) and \(I_t\) to solve:

\[
\max_{K_t, I_t} E_0 \Sigma_{t=0}^{\infty} \Lambda_{0,t} [Q_t (K_t - (1 - \delta)K_{t-1}) - I_t] \tag{1}
\]

subject to:

\[
K_t = (1 - \delta)K_{t-1} + \left[1 - \frac{\kappa_t}{2} \left(\frac{i_t q_k}{i_{t-1}} - 1\right)^2\right] I_t \tag{2}
\]

where \(1 - \frac{\kappa_t}{2} (\frac{i_t q_k}{i_{t-1}} - 1)^2\) is the adjustment cost function. \(\kappa_t\) denotes the cost for adjusting investment and \(\epsilon_t q_k\) is a shock to the efficiency of investment. Including adjustment costs of investment in the production of capital solves the so-called 'investment puzzle' and produces the hump-shaped investment in response to a monetary policy shock (Smets and Wouters, 2007 and Christiano et al, 2011).

2.1.2 Entrepreneurs

Perfectly competitive entrepreneurs produce intermediate goods using the constant returns to scale technology:

\[
Y_t = A_t [\psi(u_t) K_t]^\alpha L^{1-\alpha} \tag{3}
\]

with \(A_t\) being stochastic total factor productivity, \(u_t\) the capacity utilization rate, and \(K_t\) and \(L_t\) capital and labor inputs. Capital is homogeneous in this model.\(^3\) We assume a fixed survival rate of entrepreneurs in each period \(\gamma\), in order to ensure a constant amount of exit and entry of firms in the model. This assumption also

\(^3\)We could have made capital firm-specific, but the set-up would have to be much more complex without altering qualitatively the results. Using homogeneous capital assumption is standard in these type of models, see for instance Bernanke et al (1999), Gerali et al (2010), Gertler et al (2012).
assures that firms will always depend on external finances for their capital purchases, and so will never become financially self-sufficient.

Just as in the financial accelerator model (Bernanke, Gertler and Gilchrist, 1999) we will continue to work under the framework that all earnings (after paying the input costs) from production are re-invested into the company such that a constant share is paid out to shareholders. This is why entrepreneurs will maximize their value function rather than their production function.

Entrepreneurs also choose the level of capacity utilization, $\psi(u_t)$. As is standard in the capital utilization literature, the model assumes that using capital more intensively raises the rate at which it depreciates. The increasing, convex function $\psi(u_t)k_t$ denotes the (relative) cost in units of investment good of setting the utilization rate to $u_t$. This is chosen before the realization of the production shock (see Auernheimer and Trupkin (2014) for similar assumption). This timing assumption is important because it separates the choice of the stock of productive factor $K_t$, taken before the revelation of the states of nature, from the choice of the flow of factor $u_tK_t$, taken during the production process.

The choice of the rate of capital utilization involves the following trade-off. On the one hand, a higher $u_t$ implies a higher output. On the other hand, there is a cost from a higher depreciation of the capital stock. Therefore this rate can be understood as an index that shows how much of the stock of capital is operated relative to the steady state, per unit of time, given a capital-labor services ratio. Moreover we specify the following functional form for $\psi(u_t)$:

$$
\psi(u_t) = \xi_0 + \xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2
$$

in line with Schmitt-Grohe and Uribe (2006), Gerali et al (2010), and Auernheimer and Truphin (2014). To understand how a firm’s financial position influences its purchasing power in the capital input market, we need to understand the costs it faces. A firm minimizes the following cost function:

$$
S(Y_t) = \min_{k,l} [R_t^sK_t + w_tL_t]
$$

---

4In our exercises, we will in line with Bernanke et al (1999) set this share to 0.

5And so $y_t$ is not a direct argument of the function.

6We could equally assume a fixed rate of capital depreciation and impose a cost in terms of output of using capital more intensively, as in Christiano et al (2005) or Gerali et al (2010).

7In the simulations, $u_t$ will be normalized such that $\psi(u_t) = 0.80$. 
The real marginal cost is therefore \( s(Y_t) = \frac{\partial S(Y_t)}{\partial Y_t} \), which is:

\[
s(Y_t) = \frac{1}{1 - \alpha} \frac{1}{\alpha} (r_t^s)^\alpha (w_t)^{1-\alpha}
\]

The gross return on capital is defined as \( R_s = \frac{E_t[S_t]-S_{t-1}}{S_{t-1}} \). Keeping the wage rate constant, an increase in the expected (stock) market value of capital reduces the (relative) cost of capital service inputs, purchased at today’s capital price.\(^8\)

This is easier to see in the entrepreneur’s budget constraint:\(^9\)

\[
\partial E_t[S_{t+1}]K_{t+1} + w_tL_t + \psi(u_t)K_{t-1} + R_tB_{t-1} + (1 - \vartheta)S_tK_t = \frac{Y_t}{X_t} + B_t + S_t(1 - \delta)K_{t-1} \Rightarrow
\]

\[
\partial E_t[S_{t+1}]K_{t+1} + w_tL_t + \psi(u_t)K_t + R_t[S_tK_t - N_t] + (1 - \vartheta)S_tK_t = \frac{Y_t}{X_t} + [E_t[S_{t+1}]K_{t+1} - N_{t+1}] + E_{t-1}[S_t](1 - \delta)K_{t-1}
\]

with \( \delta \) being the depreciation rate of capital, \( \psi(u_t)K_{t-1} \) the cost of setting a level of the utilization rate, \( \vartheta \) is the share of capital purchases required to be paid in advance by CGP, and \( \frac{P^f_t}{P_t} = \frac{1}{X_t} \) is the relative competitive price of the final good in relation to the capital good (i.e. mark-up).\(^10\) An increase in the expected (stock) market price (right-hand side) has two effects.\(^11\) First, it reduces the relative cost of capital purchases today since firms can borrow more and pay a higher pre-payment share \( \vartheta \) of capital. Second, a higher market price means that the probability of default of an entrepreneur reduces (since the value of the firm is higher) and so CGP will expect entrepreneurs to be solvent in the next period and will therefore require a smaller pre-payment (i.e. \( \vartheta \) on the left-hand side will fall). Let us explain the second mechanism in further detail.

\(^8\)In line with the costs that intermediate firms face in the model of Christiano et al (2005).
\(^9\)We assume that entrepreneurs borrow up to a maximum permitted by the borrowing constraint.
\(^10\)Note that \( \partial E_t[S_{t+1}]K_{t+1} \leq E_t[S_{t+1}]K_{t+1} \).
\(^11\)Notice that, just as in De Grauwe and Macchiarelli (2015), the share price is derived from the stable growth Gordon discounted dividend model: \( S_t = \frac{E_t[\Lambda_{t+1}]}{R_t^s} \) where \( \Lambda_{t+1} \) are expected future dividends net of the discount rate, \( R_t^s \). Agents in this set-up assume that the 1-period ahead forecast of dividends is a fraction \( f \) of the nominal GDP one period ahead, and constant thereafter in \( t+1, t+2 \), etc. Since nominal GDP consists of a real and inflation component, agents make forecast of future output gap and inflation according to the specification in subsection 2.3. This forecast is reevaluated in each period. As a result, in order to get the expected (stock) market price, the expected output gap and inflation needs to be defined.
As a form of depository insurance, CGP will in some periods require entrepreneurs to pay in period 't' a share of the total capital produced and delivered to entrepreneurs in period 't+1'. In particular, when CGP suspect that entrepreneurs will face liquidity problems in the next period, a lower production, or a lower collateral value in the next period, they expect the firm to be less solvent (in relative terms). Because the default probability of entrepreneurs rises, CGP become suspicious of the entrepreneur’s ability to pay for the entire capital produced. Therefore, as an insurance mechanism, CGP will ask the entrepreneur to pay in advance a share of its capital production.\footnote{We could equivalently assume that legal conditions/constraints stipulate that entrepreneurs need to pay in advance for their inputs as in Champ and Freedman (1990, 1994). Our approach is analogue to the one taken in Fuerst (1995) or Christiano and Eichenbaum (1992) for labor input costs.}

Therefore, the share to be paid is strongly contingent on the amount that the entrepreneur can borrow on the credit market.

If the firm’s value is expected to increase in the next period, the financing constraint it faces will loosen, and thus it can borrow more. Since it can borrow more, it has more money to purchase the inputs (i.e. the marginal cost of a unit of capital decreases, \textit{ceteris paribus}) and therefore produce more outputs. This will push the price of capital up in the future. The CGP anticipating this, will require a smaller share of capital production to be pre-paid. On the other end, if the value of the firm is expected to decrease, then the cost of external financing for firms will increase and the firm will be able to borrow less. Because it can borrow less, it has less money to purchase inputs, and this will push the price of capital down in the future. In that case, CGP in anticipation of this will require a higher pre-payment. Hence, we expect the share $\vartheta$ to vary over the business cycle. Formally, the pay-in-advance constraint that entrepreneurs face in the input market is:

$$E_t[S_{t+1}]K_{t+1} \leq \vartheta_t B_t \equiv \vartheta_t [E_t[S_{t+1}]K_{t+1} - N_t] \quad (8)$$

Hence the share of capital purchases that needs to be pre-paid will depend on the entrepreneur’s financial position $B_t$. We can equivalently express it in terms of the additional external funds that the entrepreneur needs for its capital purchases (right-hand side in the above expression) using the fact that an entrepreneur will borrow up to a maximum and use it to purchase capital.\footnote{See Bernanke et al (1999) and Gerba (2014) for a more profound discussion of the entrepreneur’s capital demand behaviour.} We allow $\vartheta$ to vary over time in order to capture the variations in CGP’s pre-cautionary motive over the
business cycle. A value of 1 means that the entrepreneur will need to use all of 1 his external finances (loan) to pay for the capital purchases since CGP expects its 2 financial (cash) position to worsen in the next period. Equivalently, a value of 0 3 means that no pre-payment is required as CGP expects the entrepreneur to be able 4 to pay in full for its purchases in the next period. As a result, the constraint will 5 not be binding.

Both the individual and aggregate capital stock evolves according to:

\[ K_t = (1 - \delta)K_{t-1} + \Psi\left(\frac{I_t}{K_t}\right)K_{t-1} \]  \hspace{1cm}(9)

where \( \Psi\left(\frac{I_t}{K_t}\right)K_{t-1} \) are the capital adjustment costs in the usage of capital. \( \Psi(\cdot) \) is increasing and convex, and \( \Psi(0) = 0 \). 14

2.2 Aggregate dynamics

Since we have introduced a production economy in the baseline behavioural model, 11 we also need to adapt the aggregate equations. First we need to link the capital 12 accumulation with the real interest rate. Linking the investment demand equation 13 from DeGrauwe and Macchiarelli (2015):

\[ i_t = i(\rho) = e_1\tilde{E}_ty_{t+1} + e_2(\rho - \tilde{E}_t\pi_{t+1}); \quad e_2 < 0 \]  \hspace{1cm}(10)

with the aggregate capital accumulation 9, we find that the relation between 15 capital and the real rate is:

\[ k_t = (1 - \delta)k_{t-1} + \Psi\left(\frac{i_t}{i_{t-1}}\right)i(\rho) = (1 - \delta)k_{t-1} + \Psi\left(\frac{i_t}{i_{t-1}}\right)e_1\tilde{E}_ty_{t+1} + 2(r_t - \tilde{E}_t\pi_{t+1}); \quad e_2 < 0 \]  \hspace{1cm}(11)

Incorporating a supply side into the aggregate equations - by means of equations 17 3, 9 and 4 - gives:

\[ y_t = a_1\tilde{E}_ty_{t+1} + (1 - a_1)y_{t-1} + a_2(r_t - \tilde{E}_t\pi_{t+1}) + (a_2 + a_3)x_t + (a_1 - a_2)\psi(u_t)k_t + Adj + \epsilon_t; \quad (a_1 - a_2) > 0 \]  \hspace{1cm}(12)

14The log-linearized version of this expression is: \( k_t = (1 - \delta)k_{t-1} + \delta\bar{i}_t \), as in Bernanke, Gertler and Gilchrist (1999) or Gerba (2014) and the one used in the simulations. \( \delta\bar{i}_t \) is the steady state version of \( \Psi\left(\frac{I_t}{K_t}\right)K_{t-1} \).
The aggregate demand now also depends on the usable capital in the production, \( u_t k_t \) but discounted for the cost of financing \( (x_t) \). Christiano et al. (2005), Smets and Wouters (2007), and Gerali et al. (2010) arrive at the same resource constraint expression in their models. There is an adjustment cost in investment, which we capture by \( Adj_t \). However, it will be calibrated in such a way to equal \( \delta \), as in standard DSGE models.

The reader will notice that aggregate demand also depends on the external finance (or risk) premium \( x_t \). This is a reduced form expression for investment, since investment is governed directly by this premium, and therefore it is the dependent variable (see DeGrauwe and Macchiarelli (2015) for a derivation of this term).

The aggregate supply (AS) equation is obtained from the price discrimination problem of retailers (monopolistically competitive):

\[
\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \nu_t
\] (13)

As explained in DeGrauwe and Macchiarelli (2015), \( b_1 = 1 \) corresponds to the New-Keynesian version of AS with Calvo-pricing (Woodford (2003), Branch and McGaugh (2009)). Setting \( 0 < b_1 < 1 \) we incorporate some price inertia in the vein of Gali and Gertler (1999). Equally, the parameter \( b_2 \) varies between 0 and \( \infty \) and reflects the degree of price rigidities in the context of a Calvo pricing assumption (DeGrauwe, 2012). A value of \( b_2 = 0 \) corresponds to complete price rigidity and \( b_2 = \infty \) to perfect price flexibility (firms have a probability of 1 of changing prices in period \( t \)).

2.3 Expectations formation and learning

Under rational expectations, the expectational term will equal its realized value in the next period, i.e. \( E_t X_{t+1} = X_{t+1} \), denoting generically by \( X_t \) any variable in the model. However, as anticipated above, we depart from this assumption in this framework by considering bounded rationality as in DeGrauwe (2011, 2012). Expectations are replaced by a convex combination of heterogeneous expectation operators \( E_t y_{t+1} = \tilde{E}_t y_{t+1} \) and \( E_t \pi_{t+1} = \tilde{E}_t \pi_{t+1} \). In particular, agents forecast output and inflation using two alternative forecasting rules: *fundamentalist rule* vs. *extrapolative rule*. Under the fundamentalist rule, agents are assumed to use the steady-state value of the output gap - \( y^\ast \), here normalized to zero against a naive forecast based on the gap’s latest available observation (extrapolative rule). Equally
for inflation, fundamentalist agents are assumed to base their expectations on the central bank’s target - $\pi^*$ against the extrapolatists who naïvely base their forecast on a random walk approach.\textsuperscript{15} We can formally express the fundamentalists in inflation and output forecasting as:

$$\tilde{E}^f_t \pi_{t+1} = \pi^*$$  \hspace{1cm} (14)  

$$\tilde{E}^f_t y_{t+1} = y^*$$  \hspace{1cm} (15)

and the extrapolists in both cases as:

$$\tilde{E}^e_t \pi_{t+1} = \theta \pi_{t-1}$$  \hspace{1cm} (16)  

$$\tilde{E}^e_t y_{t+1} = \theta y_{t-1}$$  \hspace{1cm} (17)

This particular form of adaptive expectations has previously been modelled by Pesaran (1987), Brock and Hommes (1997, 1998), and Branch and McGough (2009), amongst others, in the literature. Setting $\theta = 1$ captures the ”naïve” agents (as they have a strong belief in history dependence), while a $\theta < 1$ or $\theta > 1$ represents an ”adaptive” or an ”extrapolative” agent (Brock and Hommes, 1998). For reasons of tractability, we set $\theta = 1$ in this model.

Note that for the sake of consistency with the DSGE model, all variables here are expressed in gaps. Focusing on their cyclical component makes the model symmetric with respect to the steady state (see Harvey and Jaeger, 1993). Therefore, as DeGrauwe and Macchiarelli (2015) show, it is not necessary to include a zero lower bound constraint in the model since a negative interest rate should be understood as a negative interest rate gap. In general terms, the equilibrium forecast/target for each variable will be equal to its’ steady state value.

Next, selection of the forecasting rule depends on the (historical) performance of the various rules given by a publically available goodness-of-fit measure, the mean square forecasting error (MSFE). After the time ‘t’ realization is revealed, the two predictors are evaluated \textit{ex post} using MSFE and new fractions of agent types are determined. These updated fractions are used to determine the next period (aggregate) forecasts of output-and inflation gaps, and so on. Agents’ rationality con-

\textsuperscript{15} The latest available observation is the best forecast of the future.
sists therefore in choosing the best-performing predictor using the updated fitness measure. There is a strong empirical motivation for inserting this type of switching mechanism amongst different forecasting rules (see DeGrauwe and Macchiarelli (2015) for a brief discussion of the empirical literature, Frankel and Froot (1990) for a discussion of fundamentalist behaviour, and Roos and Schmidt (2012), Cogley (2002), Cogley and Sargent (2007) and Cornea, Hommes and Massaro (2013) for evidence of extrapolative behaviour, in particular for inflation forecasts).

The aggregate market forecasts of output gap and inflation is obtained as a weighted average of each rule:

\[ E_{t+1}(\pi) = \alpha_f E_{t+1}(\pi)^f + \alpha_e E_{t+1}(\pi)^e \]  
\[ E_{t+1}(y) = \alpha_f E_{t+1}(y)^f + \alpha_e E_{t+1}(y)^e \]  

where \( \alpha_f \) is the weighted average of fundamentalists, and \( \alpha_e \) that of the extrapolists. These shares are time-varying and based on the dynamic predictor selection. The mechanism allows to switch between the two forecasting rules based on MSFE / utility of the two rules, and increase (decrease) the weight of one rule over the other at each \( t \). Assuming that the utilities of the two alternative rules have a deterministic and a random component (with a log-normal distribution as in Manski and McFadden (1981) or Anderson et al (1992)), the two weights can be defined based on each period utility for each forecast \( U_{x,i}^{e,t}, i = (y, \pi), x = (f, e) \) according to:

\[ \alpha_{\pi,t}^f = \frac{\exp(\gamma U_{\pi,t}^f)}{\exp(\gamma U_{\pi,t}^f) + \exp(\gamma U_{\pi,t}^e)} \]  
\[ \alpha_{y,t}^f = \frac{\exp(\gamma U_{y,t}^f)}{\exp(\gamma U_{y,t}^f) + \exp(\gamma U_{y,t}^e)} \]  
\[ \alpha_{\pi,t}^e = 1 - \alpha_{\pi,t}^f = \frac{\exp(\gamma U_{\pi,t}^e)}{\exp(\gamma U_{\pi,t}^f) + \exp(\gamma U_{\pi,t}^e)} \]  
\[ \alpha_{y,t}^e = 1 - \alpha_{y,t}^f = \frac{\exp(\gamma U_{y,t}^e)}{\exp(\gamma U_{y,t}^f) + \exp(\gamma U_{y,t}^e)} \]  

where the utilities are defined as:
\begin{align*}
U^f_{\pi,t} &= -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}^f_{t-k-2} \pi_{t-k-1}]^2 \tag{24} \\
U^f_{y,t} &= -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}^f_{t-k-2} y_{t-k-1}]^2 \tag{25} \\
U^e_{\pi,t} &= -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}^e_{t-k-2} \pi_{t-k-1}]^2 \tag{26} \\
U^e_{y,t} &= -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}^e_{t-k-2} y_{t-k-1}]^2 \tag{27}
\end{align*}

and $w_k = (\rho^k(1-\rho))$ (with $0 < \rho < 1$) are geometrically declining weights adapted to include the degree of forgetfulness in the model (DeGrauwe, 2012). $\gamma$ is a parameter measuring the extent to which the deterministic component of utility determines actual choice. A value of 0 implies a perfectly stochastic utility. In that case, agents decide to be one type or the other simply by tossing a coin, implying a probability of each type equalizing to 0.5. On the other hand, $\gamma = \infty$ imples a fully deterministic utility, and the probability of using the fundamentalist (extrapolative) rule is either 1 or 0. Another way of interpreting $\gamma$ is in terms of learning from past performance: $\gamma = 0$ imples zero willingness to learn, while it increases with the size of the parameter, i.e. $0 < \gamma < \infty$.

As mentioned above, agents will subject the performance of rules to a fit measure and choose the one that performs best. In that sense, agents are 'boundedly' rational and learn from their mistakes. More importantly, this discrete choice mechanism allows to endogenize the distribution of heterogeneous agents over time with the proportion of each agent using a certain rule (parameter $\alpha$). The approach is consistent with the empirical studies (Cornea et al, 2012) who show that the distribution of heterogeneous agents varies in reaction to economic volatility (Carroll (2003), Mankiw et al (2004)).

### 2.4 Firm equity

To complete the model, we need to characterize the evolution of net worth. In DeGrauwe and Macchiarelli (2015), it is shown that:

$$n^{f,m}_{t} = \frac{1}{\tau}(L^D_{t-1} + \iota_t) \tag{28}$$
\[ n_{f,m}^{f,m} = \bar{n}_t S_t \]  

(29)

where \( \bar{n}_t \) represents the number of (time-varying) shares of the firm and \( S_t \) is the current (stock) market price. Combining the two, we get that the number of shares is:

\[ \bar{n}_t = \frac{1}{\tau} (L_D^{t-1} + \bar{i}_t) \]  

(30)

Inserting the investment demand equation \( i(\rho)_t = e_1 \tilde{E}_t(y_{t+1}) + e_2 (r_t + x_t - \tilde{E}_t(\pi_{t+1})) \) from DeGrauwe and Macchiarelli (2015) into the expression above, we get:

\[ S_t \bar{n}_t = \frac{1}{\tau} (L_D^{t-1} + e_1 \tilde{E}_t(y_{t+1}) + e_2 (r_t + x_t - \tilde{E}_t(\pi_{t+1}))) \]  

(31)

We observe three things. First, the net capital (or loans) the firm has after repaying the cost of borrowing is scaled by the inverse leverage ratio. The more it borrows, the smaller will be its equity in the next period. Second, a higher (expected) production increases its revenues and therefore the capital level (via the capital accumulation function). However, a portion of the production is financed by external funds and thus it will need to pay a cost for those funds, represented by the risky interest rate \( r_t + x_t \). However, the more leveraged the firm is, the higher the downpayment on loans and therefore the more ‘exposed’ the firm will be in recessions. Third, a higher expected inflation implies a reduction in the cost of external financing. For a given level of leverage, this reduces firm’s debt exposure today and permits her, ceteris paribus to take on additional loans. Finally, note that the more leveraged the firm is, the higher is the effect from movements in (stock) market prices on the equity (shares) of the firm. This set-up is analogous to the state equation shown in Gerba (2014).

3 Behavioural model derivations

3.1 Model solution in the behavioural model

We solve the model using recursive methods (see DeGrauwe (2012) for further details). This allows for non-linear effects. The model has six endogenous variables,
output gap, inflation, financing spread, savings, capital and interest rate. The first five are obtained after solving the following system:

\[
\begin{bmatrix}
1 & -b_2 & 0 & 0 & 0 \\
-a_2c_1 & 1 - a_2c_2 & -(a_2 + a_3) & 0 & (a_1 - a_2)\psi(u_t) \\
-\psi\tau^{-1}e_2c_1 & -\psi\tau^{-1}e_2c_1 & (1 - \psi\tau^{-1}e_2) & 0 & 0 \\
d_3c_1 & -(1 - d_1 - d_3c_2) & 0 & 1 & 0 \\
0 & 0 & e_2 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
y_t \\
x_t \\
s_t \\
k_t
\end{bmatrix}
= 
\begin{bmatrix}
b_1 & 0 & 0 & 0 & -e_2 \\
-a_2 & 1 - a_1' & 0 & 0 & \Psi\left(\frac{u}{\pi_{t-1}}\right)e_1 \\
-\psi\tau^{-1}e_2 & -\psi\tau^{-1}e_2 & 0 & 0 & 0 \\
d_3 & -d_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{\pi}_{t+1} \\
\hat{y}_{t+1} \\
\hat{x}_{t+1} \\
\hat{s}_{t+1} \\
\hat{k}_{t+1}
\end{bmatrix}
+ 
\begin{bmatrix}
1 - b_2 & 0 & 0 & 0 & 0 \\
0 & 1 - a_1' & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & -(1 - d_1 - d_2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & (1 - \delta)
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1} \\
y_{t-1} \\
x_{t-1} \\
s_{t-1} \\
k_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\psi\tau^{-1}e_2 & \psi\tau^{-1} \\
a_2c_3 & 0 & 0 \\
e_2 & 0 & 0 \\
\psi\tau^{-1}e_2c_3 & \psi\tau^{-1}c_3 \\
d_2c_3 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
r_{t-1} \\
D_{t-1} \\
n_{t-1}^b \\
\end{bmatrix}
+ 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & a_2 & 1 & (a_1 - a_2) \\
0 & 0 & \psi^{-1}\tau e_2 & 0 \\
0 & -d_3 & -(1 - d_1) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\eta_t \\
u_t \\
\epsilon_t \\
u_{cft}
\end{bmatrix}
\]

Using matrix notation, we can write this as: \( AZ_t = B\hat{E}_tZ_{t+1} + CZ_{t-1} + DX_{t-1} + Ev_t \). We can solve for \( Z_t \) by inverting: \( Z_t = A^{-1}(B\hat{E}_tZ_{t+1} + CZ_{t-1} + DX_{t-1} + Ev_t) \) and assuring \( A \) to be non-singular.

Solution for the interest rate \( r_t \) is obtained by substituting \( y_t \) and \( \pi_t \) into the Taylor rule. Investments, utilization costs, bank equities, loans, labor and deposits are determined by the model solutions for output gap, inflation, financing spread, savings and capital.\(^6\)

\(^6\)However, capital, savings and the external financing spread do not need to be forecasted as these do not affect the dynamics of the model (i.e. there is no structure of higher order beliefs as LIE does not hold in the behavioural model). See section 3.1 in DeGrauwe and Macchiarelli (2015)
Expectation terms with a tilde $\tilde{E}_t$ implies that we do not impose rational expectations. Using the system of equations above, if we substitute the law of motion consistent with heterogeneity of agents (fundamentalists and extrapolators), then we can show that the endogenous variables depend linearly on lagged endogenous variables, their equilibrium forecasts and current exogenous shocks.

Note that for the forecasts of output and inflation gap, the forward looking terms in equations 11, 12 and 13 are substituted by the discrete choice mechanism in 18. For a comparison of solutions in the 'bounded rationality' model and rational expectations framework, see section 3.1 in DeGrauwe and Macchiarelli (2015).

### 3.2 Calibration and simulations

To simplify the discussion, we will only present the calibrations of the parameters that are new to this model. A full parameter list can be found in Appendix.

In line with De Grauwe and Macchiarelli (2015), we calibrate the aggregate demand parameters $(d_1, d_2, e_1)$ to $(0.5, 0.15, 0.1)$ which is consistent with standard macroeconomic simulation results. $\tau$ (or a firms’ average leverage ratio) is again set to 1.43, following Pesaran and Xu (2013), and $\kappa$ (or banks’ equity ratio) is, following Gerali et al (2010), set to 0.09.

The parameters specific to this model are set to standard values in the literature. The share of capital in the production $\alpha$ is set to 0.30 as in Boissay et al (2013). Following Christiano et al (2005), Smets and Wouters (2003, 2007) and Gerali et al (2010), we set the capital depreciation rate $\delta$ to 0.025. The elasticity of the capital utilization adjustment cost function $\psi(i_t)$ is parametrized to 0.5 as in Smets and Wouters (2007).

The sensitivity of capital (or investment) to changes in the real interest rate $e_2$ is, in line with the empirical evidence, set to $e_2 < 0$. To conclude, the parameters of the function determining adjustment costs for capacity utilization $(\xi_0, \xi_1, \xi_2)$ are set to $(0.8, 0.3, 0.25)$ in order to capture the estimation results of Smets and Wouters (2005) who find that the capital utilization adjustment costs are between 0.14 and 0.38 (Euro Area 1983-2002) and 0.21 and 0.42 (US 1983-2002), with a mean of 0.25 (Euro Area) and 0.31 (US). If we normalize $u_t$ to 1 (as in Christiano et al (2005), Miao et al (2013) or Auernheimer and Trupkin (2014)), then the cost for utilizing

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for comparison of solutions under rational expectations and bounded rationality ("heuristics").

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17This is equivalent to setting a $\kappa_t$ equal to the estimated range (10.18 – 12.81) as in Gerali et al (2010).
capital will be 0.20 \(1 - \xi_0\), which is well within the estimated intervals of Smets and Wouters (2005).

All shocks, except to the capital utilization, are parametrized as white noise which means that their autoregressive component is set to 0. Likewise the standard deviations of shocks are set to 0.5 across the entire spectrum.\(^{18}\)

### 4 Quantitative results

Our analysis consists of three parts. The first part is an analysis of (model consistent) impulse responses to a set of independent white noise shocks. The second is an examination of the (model generated) second-, and higher-order moments to contrast the fit of the model to the US data. The final part consists of depicting and analyzing the nature of the model variables over the business cycle. For future work, we wish to compare the quantitative results to a standard DSGE model with an equal mechanism.

#### 4.1 Forcing variables

The three shocks we will examine are:

- **Standard (negative) monetary policy shock \(\epsilon\):**

  \[
  r_t = r_{t-1} + \gamma \pi_t + (1 - \gamma) y_t + \epsilon
  \]  
  \[(32)\]

- (Positive) technology (or productivity) shock, \(\epsilon_{z_t}\)

  \[
  y_t = z_t \epsilon_{z_t} K^\alpha L^{1-a}
  \]  
  \[(33)\]

  where \(\epsilon_{z_t}\) is a white noise shock to the technology factor in the Cobb-Douglas technology function.

- (Positive) shock to utilization cost, \(uc_t\), in the utilization cost function:

  \[
  \psi(u_t) = \xi_0 + \xi_1 (u_t - 1) + \frac{\xi_2}{2} (u_t - 1)^2 + uc_t
  \]  
  \[(34)\]

\(^{18}\)The AR-component of the shock to capital utilization cost is set conservatively to 0.1, just enough to generate some persistence in the capital cost structure.
where \( uc_t \) has the following AR structure:

\[
uc_t = \rho_{uc} uc_{t-1} + \epsilon_{uc} \tag{35}
\]

and \( \epsilon_{uc} \) is a white noise shock. In our simulations, we calibrate the AR component \( \rho_{uc} \) to 0.1 in order to strictly limit the possibility of the shock driving the model dynamics. However, a simple white noise utilization cost shock is excessively short-lived, and does not allow us to study the endogenous dynamics in full. All the white noise shock parameters (\( \epsilon, \epsilon_z \) and \( \epsilon_{uc} \)) are calibrated to 0.5.

4.2 Impulse response analysis

Figures I.1 to I.3 depict the median impulse responses to a monetary policy shock, Figures I.4 to I.6 to a technology shock, and Figures I.7 to I.10 to a shock to utilization costs. Note that the numbers on the x-axis indicate number of quarters. All the shocks are introduced in \( t=100 \) and we observe the responses over a long period of 60 quarters (or 15 years). Note that in these figures we depict the median impulse response amongst a distribution of impulse responses generated with different initializations. The full impulse responses with the 95% confidence intervals are depicted in Figures I.11 to I.13. For the sake of clarity and focus, we will only concentrate on the median impulse response for all the shocks, which is a good representation of the overall distribution.

4.2.1 Monetary policy shock

As is standard, an expansionary monetary policy (0.5% fall) leads to a fall in the external finance premium, which relaxes the credit that firms can access and therefore pushes up investment (0.3%). This pushes up capital accumulation (0.4%). This expansion is perceived by agents as a period of positive outlook, which triggers the optimism (animal spirits up 0.2%). This optimism is translated into an increase in

\(^{19}\)Before we begin with the analysis, bear in mind that the behavioural model does not have one steady state that is time invariant for the same calibration (as is standard for the DSGE method). Therefore, following a white noise shock, the model will not necessarily return to a previous steady state. If not the same steady state, it can either reach a new steady state, or have a prolonged response to the initial shock. In other words, there is a possibility for the temporary shock to have permanent effects in the model (via the animal spirits channel). However, due to the methodological proximity to the DSGE analogue and because it is a standard evaluation (and comparison) tool in the literature, we will proceed analyzing the impulse responses in the behavioural model.
deposits (0.25%) and bank equity (0.3%). The expansion leads to an increase in output (0.20%) and a rise in inflation (0.01%), but with a lag of 1 quarter.\footnote{Initially, output falls by 0.25% as well as inflation by 0.05%, but this is reverted after 1 period. This finding is frequent in the literature and denominated as the price puzzle.}

However, this optimism is very brief as the monetary authority raises the interest rate (0.1%) to combat the rising inflation. By the agents, this is perceived as the end of the expansionary phase, resulting in a reversal of the sentiment to pessimism (animal spirits fall by 0.05%). The consequence is a turn in the response of macroeconomic and financial aggregates, leading to return of these variables to the steady state.

Hence in the behavioural model, we see two waves of responses. The first, standard in the DSGE models, driven directly by a monetary policy expansion. The second, on the other hand, is purely driven by animal spirits. The response of the monetary authority to the initial expansion kills and turns the initial optimism into a pessimism (or negative bubble on the financial market). This results in a reversal in the financial and macroeconomic aggregates, making the initial monetary expansion extremely short-lived. This type of market behaviour are difficult to capture in standard DSGE models (but frequently observed empirically).

\subsubsection*{4.2.2 Technology shock}

Let us now turn to the first of the supply side shocks. An improvement in TFP (or equivalently, increase in productivity) of 0.5% results in an inflation reduction (1%) and a more than proportional output expansion (1.15%). This is both a result from the increased capacity in the final goods market, but also from an increase in investment (0.3%) following the heavy fall in interest rate (1.3%) as a response to the falling inflation. Following this general supply-side expansion, deposits and loans to firms also increase (1 and 1.3% respectively) since the value of firm net worth (i.e. collateral) has increased. As a consequence of the lower marginal cost to investment and higher marginal return on capital, capital accumulation increases significantly in the next period (0.5%). This results in a general market optimism (animal spirits rise by 0.1%).

However, as soon as the inflation starts recovering, interest rate react very rapidly to their increase and start rising (0.35%). Because of this rise in cost of capital, coupled with the fall in external financing for firms, investment and output expansion reverts. However, unlike in the DSGE models, the model has eventually reached a
new steady state, where bank loans, deposits and equity are permanently 1.1%, 0.7% and 0.1% above the previous pre-shock level. Hence a technology improvement in the behavioural model will have long-lasting positive effects on the banking sector and financial efficiency.

4.2.3 Shock to utilization costs

The second of the supply side shocks is a 0.5% decrease in the cost of utilizing capital in production (i.e. a positive supply-side shock). This will therefore increase the marginal benefit or return to capital, which will increase the demand for capital. Hence, capital good producers will produce more, and so investment rises (0.02%). The level of capital will also rise significantly (0.2%) as a result of both capital demand and supply expansion. Therefore, output will expand (0.1%). Because of the higher capital (and thus collateral) and the resulting fall in the financing spread, the quantity of credit to firms will expand (0.7%). Since this is an improvement on the supply side, inflation initially falls (0.03%), and the monetary authority reacts by reducing the interest rate (0.15%). This is reverted as soon as the monetary authority increases the interest rate (0.02%) because of the recovery in the inflation. Following 15 years after the shock, in the new steady state, firm credit and deposits are 0.6% and 0.2% above the pre-shock level. Again a temporary supply-side shock is having permanent effects on financial sector activity.

4.3 Distributions and statistical moments over the business cycle

The second part of the model evaluation consists of analysing and validating the model-generated distribution and statistical moments over the business cycle. These are generated using the entire sample period of 2000 quarters. For our purposes, we will use the data on second and higher moments in Tables I.2 and I.3, the evolution of the model variables over the business cycle in Figures I.14 to I.19, as well as histograms of a selection of these variables in Figures I.20 to I.22. Note for the graphs that we are plotting the business cycles over a sub-sample period of 100 quarters.

\footnote{In DSGE models, this is only possible to achieve with permanent or continuous shocks.}
4.3.1 Macroeconomic aggregates

The short-term cycles of output, inflation and the interest rate are asymmetric. While the amplitude of expansions is in general higher for output, the duration of recessions is longer. This is further confirmed by the histogram for output, which is asymmetric and skewed to the right, with a higher probability mass on the left of the mean of the distribution. Moreover, the autocorrelation of output is very high (0.86), as is the volatility (2.17) and it is leptokurtic (kurtosis=10.91).

The opposite applies to inflation. The amplitude of deflationary periods is in general higher, while the duration of inflationary periods is longer. From histogram, the distribution of inflation is slightly skewed to the left. In line with the data, inflation is three times less volatile than output but has a very similar kurtosis to output. Further, inflation is very persistent over time ($\rho = 0.74$) and countercyclical (-0.42), exactly as in the US data (-0.43).

Turning to the (risk-free) interest rate, it is mostly positive and remains above the trend for a longer period over the cycle. It is also highly correlated with the business cycle (0.39) as well as with inflation (0.57), indicating a firm inflation target on the part of the monetary authority. It is almost as volatile as output (0.95), but highly skewed to the left (-4.29) compared to the general business cycle.

4.3.2 Firm and supply-side variables

Looking at Figure I.15, capital stock is mostly positive over the cycle, with a mean-reversion around 1. This is in line with the data on inventories, which shows it is positive mean-reverting. It is highly persistent ($\rho = 0.95$) and correlated with output (0.45). It is also highly positively correlated with animal spirits (0.34). Distribution-wise, it is less volatile than the business cycle (0.413), but heavily skewed to the right (3.48).

The first thing one notes from utilization costs is that while apparently more volatile, it oscillates within a much smaller interval compared to any of the other variables. Hence, the volatility is 4 times smaller compared to output. In addition, it reverts around a mean of approx. 0.5. This is in line with the data, which points towards a largely non-negative cost in utilizing capital over the cycle. It is however weakly countercyclical (-0.1), and symmetric as well as mesokurtic.

The cash-in-advance constraint (or percent of external credit used for capital input purchases) is strictly non-negative and acyclical (0.02). It is also independent
from the cycles of capital- (-0.01), and financing spread (0.01). In addition, the
distribution of $\vartheta_t$ is highly volatile, skewed to the right and leptokurtic. Effectively,
with 95% probability (or higher) $\vartheta_t$ is significantly above zero.

On the other hand, the financing spread for firms is highly countercyclical (-0.41),
as well as negatively correlated with animal spirits (-0.12). This is consistent with
the model set-up and data, which shows that during expansions both the real risk
(via a higher collateral value) and the perceived risk (via the optimistic sentiment)
of loan default falls, which pushes down the risk premium and so the spread. The
opposite holds for recessions. That is why the spread is both negatively correlated
with the business cycle (collateral value), and with the market sentiment (agents’
risk perception). Statistically, the spread is as volatile as the general business cycle,
but highly skewed to the left, meaning that for most of the time the spreads will
be close to zero (or negative). This is further confirmed by the graph in Figure
I.16. However, with some non-negligible probability, the spread can spike, causing
a severe contraction in liquidity, and the banking market.\(^2\) These results are also
in line with the (model-generated) statistical moments on loan supply, which is
procyclical (0.11), positively correlated with animal spirits (0.12) and capital-net
worth (0.28), but negatively correlated with the financing spread (-0.1)

4.3.3 Market sentiment

An important driver of the business cycle is the market sentiment (or animal spirits).
It is highly procyclical (0.84) throughout the entire sample period (see Figure I.24).
Moreover, we observe a higher persistence during the pessimistic interval compared
to the optimistic. This is in line with our previous observation on the general
business cycle (or output) showing that recessions have a longer duration compared
to expansions. Moreover, market sentiment has fat tails on the left and right of the
mean, but is smoother than the general business cycle.

4.4 Moment matching

The next step in model validation consists of matching the (model generated) mo-
mments to the US data. For that, we have calculated the statistical moments for all

\(^2\) However, the spread is not persistent ($\rho = 0.01$) implying an RBC type of frictionless financial
sector, and non-staggered price setting. That is not a surprise for the current model since the
financial market is modeled in reduced form. However, future work should try to extend the model
by modeling a more complex and empirically consistent financial price setting mechanism.
variables using the longest data sample period available from 1953:I - 2014:IV. Following Stock and Watson (1998), we choose 1953:I as the starting year of our sample since the (post-war) quarters prior to 1953 include noise and inaccuracies in the data recording. The sample includes 247 quarters (or 62 years) which is the closest approximation available for the long-run (cyclical) moments that is generated by the model. During this period, the US economy experienced 10 cycles (using NBER business cycle dates), and the average GDP increase (quarter-on-quarter) during expansions was 1.05% while it was -0.036% during recessions. The data were downloaded from Flow of Funds at the Fed St Louis database. These were de-trended using a standard two-sided HP-filter before their moments were calculated. A full list of variables and other details can be found in Table I.

The behavioural model matches precisely the correlations of many supply-side and financial variables. This includes credit to firms, deposits, the (risk-free) interest rate, inflation, and firm financing spread. It is also very successful in reproducing the autocorrelations of output, capital, and inflation, as well as the correlations between capital and credit to firms, and inflation and the (risk-free) interest rate. However, there is room for improvement in matching stock variables, such as firm and bank net worths, some macroeconomic aggregates (investment mainly) as well as the autocorrelation of firm financing spread. While they are all acyclical and not persistent in the model, they are highly procyclical and highly persistent in the data.

Turning to (relative) second-, third-, and fourth moments, the model is highly successful in reproducing the moments of inflation, the (risk-free) interest rate, credit to firms, deposits, and net worth of banks. It is also successful in making net worth of firms more skewed and more leptokurtic than output. However, the moments of the latter are higher in the model compared to US data. On the other hand, capital and investment are smoother in the model.

Another strength of the model lies in reproducing irregular business cycles. In contrast to standard first-, second-, or even third order approximated DSGE models, the behavioural model generates substantial asymmetries between expansions and recessions as well as produces non-Gaussian probability distribution functions for most variables. That is much more in line with the observed pattern in the US cyclical data. Nonetheless, for some variables (net worth, consumption, savings, etc.)

23The most recent data recorded is for 2014:IV using Fed St Louis database on March 2, 2015.
24This is in order to allow for a smoother comparison with the model generated (cyclical) moments.
(risk free) interest rate, and credit to firms) the model generates excessive skewness and/or kurtosis.

To sum up, the model matches most of the US data. This includes supply-side and financial variables such as the (risk-free) interest rate, inflation, credit to firms, deposits, firm financing spread and net worth of banks. It is also successful in matching several supply-side relations (capital-firm credit, inflation-interest rate) as well as their autocorrelations (output, capital and inflation) There is, however, some scope for improvement in matching demand-side variables (such as consumption, savings, investment) as well as stocks (net worths of firms).

4.5 The nature of business cycles

Next, we wish to understand to what extent the model is capable of generating inertias in the business cycles.

As discussed in Milani (2012) and DeGrauwe and Macchiarelli (2015), business cycle movements in a rational expectations environment arise as a result of exogenous shocks (including the autoregressive structure of shocks), leads and lags in the endogenous transmission of shocks (such as lagged or expected output), habit formation, interest rate smoothing, or nominal rigidities (price and wage stickiness). One could therefore call this ‘exogenously created’ business cycle fluctuations. The behavioural model, on the other hand, generates inertia and business cycle fluctuations even in the absence of endogenous frictions, lags in endogenous transmissions, and autocorrelation shock structures, as shown in DeGrauwe (2012). In the current case, however, we have introduced supply-side and financial market frictions, as well as leads and lags in the output, inflation and capital transmission mechanisms. This is in order to set the behavioural model at par with a standard DSGE model, so to facilitate the comparison between the two frameworks.

The evolution of the different model variables over the business cycle are reported in figures I.14 to I.24. The time period covered is 100 quarters, which is enough to cover multiple cycles. The first thing to note is that with this ‘snapshot’ of the business cycle, we have managed to capture one long cycle (with a high amplitude)
followed by several shorter cycles. Not only is the business cycle peak the highest during those 25 years ($t = 295$), but the amplitude is also the widest (between $t = [280 : 300]$ counting from trough to trough). Moreover, the subsequent bust following the high boom is the sharpest, since it takes the economy more than 40 quarters to return to a level above the long-run trend (or above the zero line). In addition, the subsequent expansions are significantly weaker, somewhat implying that some fundamental (or structural) changes occurred in the economy following the preceding boom and bust.\footnote{Compare that to the boom preceding the Great Recession and the subsequent bust in the US.}

Closely related to above observations, we find that the other variables experience similar cycles (inflation, interest rate, capital and the financing spread). Because the model concentrates on the supply side (with a weak demand-side) and we only employ supply side shocks, inflation falls when output rises (and vice versa).\footnote{See Figure I.18 for the correlation between output and inflation during the entire period.} So during the period of sharpest expansion of the business cycle, inflation experiences its sharpest decline. However, in contrast to output, inflation oscillates relatively evenly around zero (i.e. we don’t observe any temporal shifts in the trend).

As expected, the interest rate responds elastically to the evolution of inflation (see Figure I.19). Nevertheless, it is smoother than inflation since we have included an interest rate lag in the Taylor rule (see DeGrauwe and Macchiarelli, 2014), which smoothes the reaction of the interest rate to current inflation. We also observe a lag in the response of inflation to monetary policy over the cycle, in line with observations from the data.

Capital, on the other hand, is positively skewed and is mostly above the zero line during the entire period. Since it is a stock variable, that is to be expected and in line with the US data (see Table I.4. In addition, capital accumulates the most during the long expansionary period discussed above, and contracts under the proceeding episode. Just as the general business cycle, the subsequent capital accumulations are weaker, and the stock of capital is still below its pre-crisis level 40 quarters (or 10 years) after the bust. Contrast that to the Great Recession episode.

In the same vein, utilization costs are also positively skewed (see Figure I.22), but more volatile than output. This is to be expected since utilization cost function is of second order (see equation 4) and depends directly on the production capacity. Therefore the volatility of production will be squared, which increases the fluctua-

\footnote{However, to confirm this fact one would need to perform a structural breaks analysis on the full data, which includes the trend.}
tions in the cost. Also, as Figure I.18 shows, the more capital is accumulated and
used in production, the higher utilization costs the producer will face (due to the
inherent trade-offs explained in subsection 2.1.2). The correlation between the two
is positive throughout the entire period.

The next thing to note is the high degree of asymmetry over the business cycle.
As histograms in Figures I.20 to I.22 confirm, all variables are skewed. Meanwhile
output and asset prices are positively skewed (skewed to the right), inflation, cap-
ital and (in particular) utilization costs are negatively skewed (skewed to the left).
Taking into account the distribution of animal spirits in Figure I.21 and asset prices
in Figure I.22, this implies that pessimistic phases (or busts) dominate optimistic
ones (or booms).

Just as in the DeGrauwe (2011,12) and DeGrauwe and Macchiarelli (2015) mod-
els, output is highly correlated with animal spirits throughout the entire period.
It’s correlation with animal spirits is 0.83 (see figure I.24 and table I.2). We can
interpret the role of animal spirits in the model as follows. When the animal spirits
index clusters in the middle of the distribution we have tranquil periods. There is no
particular optimism or pessimism, and agents use a fundamentalist rule to forecast
the output gap. At irregular intervals, however, the economy is gripped by either
a wave of optimism or of pessimism. The nature of these waves is that beliefs get
correlated. Optimism breeds optimism; pessimism breeds pessimism. This can lead
to situations where everybody has become either optimist or pessimist. The index
then becomes 1 respectively 0. These periods are characterized by extreme positive
or negative movements in the output gap (booms and busts).

Let us continue by examining one of the novelties of this model, the share of
loans used for capital input pre-payment. It is clear from Figure I.16 that when
the economy expands and the stock market booms, the share of loans required by
CGP for the capital pre-payment is very low, and often zero. This is because of the
stock market booms implying a low probability of default for entrepreneurs (since it’s
collateral value is high, or loan-to-value ratio low). Because of this low probability of
default, entrepreneurs will be able to borrow more, increasing their (expected) cash
positions and so CGP will not require a pre-payment. In contrast during busts, and
in particular during a sharp contraction (as in \( t = [295, 300] \)) CGP become wary of
the entrepreneur’s ability to pay their capital purchases in the next period (because
of an expected lower cash position of entrepreneurs, or reduced production), and
therefore require a high share to be pre-paid. The higher the contraction, the higher
the share required to be pre-paid (see lower graph in Figure I.16). The model is capable of generating these asymmetries over the cycle, in particular in relation to the cash position of entrepreneurs.

To conclude, we see a strong co-movement between asset prices on one hand, and net worth and the financing spread. During stock market booms, net worth rises which increases the firm’s collateral value and reduces its probability of default, and so it reduces the external financing spread (as it is less risky for banks to lend to firms).

5 Discussion and concluding remarks

Including credit frictions on the supply side is a novel way of thinking about financial frictions in the macroeconomics literature. Sharp rises in stock prices do not only allow firms to increase their credit and capital demand, but can equally reduce the input costs of firms, or their input-output ratio. Conversely, a sharp drop in asset prices can restrict the supply of credit to firms, which will increase the production costs for firms, reduce the supply of capital, and (over time) reduce their production capacity (or productivity). Including this mechanism in a behavioural model has significantly improved the fit of the model to the data.

Impulse response analyses show that (temporary) supply-side shocks do not only improve the fundamentals in the economy but cause a brief wave of market sentiment (or animal spirit), which in the case of a positive shock result in a more-than proportional increase in output, capital supply, bank equity, and fall in interest rate. Moreover, credit supply to entrepreneurs is permanently increased, which implies that firms can access a higher external financing in the future. This means that firm productivity is permanently improved.

The statistical validation of the model and moment matching show that the model is capable of capturing many of the supply-side relations found in the data. This includes supply-side and financial variables such as the (risk-free) interest rate, inflation, credit to firms, deposits, firm financing spread and net worth of banks. It is also successful in matching several supply-side relations (capital-firm credit, inflation-interest rate) as well as their autocorrelations (output, capital and inflation). Moreover, we find a strong co-movement between asset prices on one hand, and net worth and the financing spread. During stock market booms, net worth rises which increases the firm’s collateral value and reduces its probability of default, and
so it reduces the external financing spread (as it is less risky for banks to lend to firms).

There is, nevertheless, scope for improvement in matching demand-side variables (such as consumption, savings, investment) as well as stocks (net worths of firms). Net worth of banks and firms are more volatile and asymmetric in the model compared to the data.

There are multiple ways in which the current work can be extended. First, it would be highly interesting to contrast the agent-based framework to a DSGE model at par. In particular, it would be of high relevance to quantify the proportion of the results that are directly and exclusively generated by the learning framework. Hence, a rigorous comparison with a rational expectations model is necessary to extract this share.

Taking into account the (global) capital market disruptions of 2008-09 and more recently the sovereign fund disruptions in the Eurozone, a second important extension would be to study the type of market (agent) behaviour or (size of) shock that is necessary within this theoretical set-up in order to generate the financial market disruption that was observed in the Eurozone in 2012.

A methodological extension would be to make use of the growing literature in forecast evaluation of agent-based models, and test the forecast performance of this model, in particular with respect to relevant competing models.

Lastly, we calibrate our parameters in the model. An interesting exercise would be to estimate the parameters of the model in order to get a more accurate representation of the business cycles.

References


Appendices

I  Tables and Figures
Table I.1: Parameters of the behavioural model and descriptions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>( \pi^* )</td>
<td>The central bank’s inflation target</td>
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</tr>
<tr>
<td>( d_1 )</td>
<td>Marginal propensity of consumption out of income</td>
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</tr>
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<td>( e_1 )</td>
<td>Coefficient on expected output in investment eq.</td>
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</tr>
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<td>( d_2 )</td>
<td>Coefficient on expected output in consumption eq. to match ( a_1 = 0.5 )</td>
<td>( 0.5 \ast (1 - d_1) - e_2 )</td>
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<td>( d_3 )</td>
<td>Coefficient on real rate in consumption eq.</td>
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</tr>
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<td>( e_2 )</td>
<td>Coefficient on real rate in investment eq. to match ( a_2 = -0.5 )</td>
<td>((-0.5) \ast (1 - d_1) - d_3)</td>
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<td>( a_1 )</td>
<td>Coefficient of expected output in output eq.</td>
<td>((e_1 + d_2)/(1 - d_1))</td>
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<tr>
<td>( a_4 )</td>
<td>Coefficient of lagged output in output eq.</td>
<td>(d_3/(1 - d_1))</td>
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<td>Interest rate elasticity of output demand</td>
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<td>Coefficient of output in inflation eq.</td>
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<td>Fraction of nominal GDP forecast in expected future dividends</td>
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<tr>
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<td>Number of shares in banks’ balance sheets</td>
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<td>Initial value for number of firms’ shares</td>
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<td>( \beta )</td>
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<td>( \alpha )</td>
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<td>( \gamma )</td>
<td>Switching parameter in Brock-Hommes (or intensity of choice parameter)</td>
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<td>Speed of declining weights in memory (mean square errors)</td>
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<td>Std. deviation of shock in the utilization cost function</td>
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<tr>
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<td>Correlations</td>
<td>Value - behavioural model</td>
<td>Value - DSGE model</td>
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<td>0.18</td>
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<td>$\rho(y_t, n^f_t)$</td>
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<td>0.22</td>
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</table>

Note: GDP deflator was used as the inflation indicator, 3-month T-bill for the risk-free interest rate, the deposit rate as the savings indicator and the Corporate lending risk spread (Moody’s 30-year BAA-AAA corporate bond rate) as the counterpart for the firm borrowing spread in the models. The variables that are left blank do not have a direct counterpart in the data sample. These are also called ‘deep variables’. The only way is to estimate a structural model (using for instance Bayesian techniques) and to derive a value based on a (theoretical) structure. Alternatively, one could also approximate values using micro data. However, this is outside the scope of this paper.
### Table I.3: Second and higher moments - behavioural model

<table>
<thead>
<tr>
<th>Variable (Rel.)</th>
<th>Standard deviation (Rel.)</th>
<th>Skewness (Rel.)</th>
<th>Kurtosis</th>
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<td>0.21</td>
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<tr>
<td>$x_t$</td>
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<td>20.9</td>
<td>27.9</td>
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<td>1.90</td>
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<td>$r_t$</td>
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<td>$i_t$</td>
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<td>9.54</td>
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<td>0.05</td>
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<td>$AD_t$</td>
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<td>$\vartheta_t$</td>
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<tr>
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<td>$n^l_t$</td>
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<td>53.52</td>
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<tr>
<td>$S_t$</td>
<td>1.23</td>
<td>-3.33</td>
<td>53.75</td>
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Note: The moments are calculated taking output as the denominator. Following a standard approach in the DSGE literature, this is in order to examine the moments with respect to the general business cycle.

### Table I.4: Second and higher moments - US data

<table>
<thead>
<tr>
<th>Variable (Rel.)</th>
<th>Standard deviation (Rel.)</th>
<th>Skewness (Rel.)</th>
<th>Kurtosis</th>
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<td>$k_t$</td>
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<td>-</td>
</tr>
<tr>
<td>$d_t$</td>
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<td>1.36</td>
<td>4.54</td>
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<td>$AD_t$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$AS_t$</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\vartheta_t$</td>
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</tbody>
</table>

Note: The moments are calculated taking real GDP as the denominator. These are calculated using the full sample of US data stretching from 1953:I - 2014:IV. During this period, the US economy experienced 10 cycles (using NBER business cycle dates), and the average GDP increase per quarter during expansions was 1.05% while it was -0.036% during recessions. The data were de-trended using a standard two-sided HP filter before the moments were calculated in order to facilitate comparison with the model generated (cyclical) moments. The variables that are left blank do not have a direct counterpart in the data sample. These are also called 'deep variables'. The only way is to estimate a structural model (using for instance Bayesian techniques) and to derive a value based on a (theoretical) structure. Alternatively, one could also approximate values using micro data. However, this is outside the scope of this paper.
Table I.5: US variables and sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>US data name</th>
<th>Frequency</th>
<th>Source</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>Real GDP</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$c_t$</td>
<td>Real Personal Consumption Expenditure</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$i_t$</td>
<td>Real Investment</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$d_t$</td>
<td>Total Savings and Time Deposits of Households</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$s_t$</td>
<td>Net Private Savings Households</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$l_t^c$</td>
<td>Credit Market Instruments for Firms</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$r_t$</td>
<td>Effective Federal Funds Rate</td>
<td>Monthly</td>
<td>Fed St Louis database</td>
<td>1954:II-2014:IV</td>
</tr>
<tr>
<td>$x_t/ef p_t$</td>
<td>Moody’s (30 year) BAA - AAA Corporate Bond Spread</td>
<td>Monthly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$\pi$</td>
<td>GDP Deflator</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
</tbody>
</table>

Note: All variables were downloaded on March 2, 2015. The latest recorded observation for each variable was 2014:IV (except for capital stock).
Figure I.1: Impulse responses to a expansionary monetary policy shock in t=100
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Figure I.3: Impulse responses 3 to a expansionary monetary policy shock in t=100
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Figure I.9: Impulse responses 3 to a shock in utilization cost in t=100
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Figure I.12: Full impulse responses to an expansionary technology shock with 95% confidence interval
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Figure I.17: Evolution of the key aggregate variables
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Figure I.24: Agent behaviour and animal spirits 2