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**Voting with public information**

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## **Voting with Public Information**

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# Voting with Public Information

Shuo Liu \*

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## Abstract

We study the effect of public information on collective decision-making in a committee with members of both common and conflicting interests. We show that the set of preferences that allow for the existence of an informative voting equilibrium can be heavily restricted by the presence of a public signal, regardless of the size of the committee and the choice of the voting threshold value. What's worse, the presence of the public information introduces an inefficient equilibrium which robustly exists across different voting rules. To mitigate the harmful effect of the public information, we propose to use a class of more flexible voting rules, whose threshold values depend on both the precision and the realization of the public signal, that may restore the informative voting equilibrium. In particular, in a standard setting with common interest agents, the contingent voting rule that we construct not only always restores the informative voting equilibrium but also achieves full informational efficiency.

**Keywords:** strategic voting, collective decision-making, public information, committee design, optimal voting rule.

**JEL classification:** D72, D82.

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# 1 Introduction

A common argument for voting mechanisms is that they help aggregating the information that agents in a committee privately hold, and thus lead to better decisions compared to the case of a single decision-maker. Indeed, as the celebrated Condorcet Jury Theorem (CJT) suggests, in a standard setting where agents have common interest, under the majority voting rule even strategic agents will find it optimal to simply vote informatively, and the private information dispersed among the committee members is efficiently aggregated (Condorcet, [1785] 1994; Austen-Smith and Banks, 1996). However, Kawamura and Vlaseros (2014) (henceforth KV) make the interesting observation that, as long as there exists a public signal that can be commonly observed by all agents and that is superior to each of their private signals, the majority rule can no longer lead to an equilibrium in which all agents just vote according to their private signals. What's worse, the presence of the public signal opens the possibility for agents to coordinate on the *obedient voting equilibrium*, in which everyone just votes according to whatever the public signal suggests and thus completely disregards the private information. Quite likely, this could be very inefficient since the public signal is rarely perfect. Experimentally, KV find that a large proportion of subjects in the laboratory behave quite consistently with what the obedient voting equilibrium would predict, and the outcome of the collective decision almost always coincides with that in an obedient voting equilibrium.

This observation is highly relevant, because it should be clear that the access to both private and public information for the voters is the rule rather than the exception: in business, members in the board of directors receive (or even ask) advice from the advisory board of the company; in a court, a witness states his/her testimony in front of all members of the jury; the Central Committee of the Communist Party of China, which has only seven members, often invites renowned scholars in the relevant fields to give short presentations when important decisions that affect the well-being of more than 1.3 billion people are needed to be made. If in the end only the public information counts, why should we bother to use the voting mechanism in the first place?

In this paper, we first take this observation one step further. We study the effect of the public signal in a richer setting where agents have both common and conflicting interests: while agents share the common goal of making a collective decision that will be matched to the state, they may have different payoffs from the different types of decision errors that could occur. This allows us to see how the presence of a public signal, even if it is less accurate than any of the private signals, limits the existence of the *informative voting equilibrium*, in which every agent simply votes according to her private signal. More specifically, we show that whenever the public signal is superior to each agent's private signal, the informative voting equilibrium does not exist for *any*  $k$ -voting rule (which includes the majority rule as a special case) and *any* preference profile of the agents. Moreover, even if the public information is less accurate than the private information, the set of preference profiles that allow for informative voting under *some*  $k$ -voting rule is strictly smaller than it would be in the absence of the public signal: for example, if the public signal is just slightly less precise than the private signal, under the standard majority voting rule the informative voting equilibrium only exists if all agents are sufficiently unbiased *ex ante*. Therefore, we substantially generalize KV's result to arbitrary precision of the public signal, a larger class of voting rules and a richer set of preferences of the agents.

We then provide a solution to tackle the harming effect of the public signal. We show that by using a class of more flexible voting rules of which the threshold values depend on the realization of the public signal, existence of the informative voting equilibrium can be restored for a large range of preferences (in particular, when all agents are unbiased). We call such voting rules *contingent  $k$ -voting rules*. Compared to the corresponding standard  $k$ -voting rule that would sustain informative voting as an equilibrium if the public signal were absent, the appropriately chosen contingent  $k$ -voting rule requires less votes for the committee to select the alternative suggested by the public signal. Thus, in the informative voting equilibrium, an agent knows that she is decisive only if the private signals of the others are collectively more against the decision suggested by the public signal, which helps eliminate her incentive to deviate from informative voting. For any preference profile of the agents, we provide a simple algorithm to find the contingent  $k$ -voting

rules that can be used to restore informative voting. We show, for the important limiting cases where agents' preferences are perfectly aligned (but not necessarily unbiased), that such contingent  $k$ -voting rules always exist, provided that the size of the committee is sufficiently large.

Finally, we discuss the welfare properties of the contingent  $k$ -voting rules. We provide a sufficient condition under which the informative voting equilibria under the contingent  $k$ -voting rules are asymptotically efficient. Moreover, we prove that in a standard setting where agents have pure common interest, under the contingent majority rule (which can be obtained by adjusting the threshold value in the standard majority rule to be dependent on the public signal in a symmetric way) the informative voting equilibrium not only always exists, but also maximizes the expected probability of the collective decision being matched to the state, given all the information that is available in the committee. This suggests that with well-designed voting procedures, not only the harming effect of the public signal can be overcome, but also higher efficiency can be achieved compared to the benchmark case where agents have no access to any public information.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the model. In Section 4 we discuss how the presence of a public signal may significantly limit the existence of an informative voting equilibrium. We show in Section 5 how to restore the existence by using a class of voting rules that are contingent on the public signal, and how the use of such voting rules can enhance informational efficiency. Finally, Section 6 concludes. All proofs are contained in the Appendix.

## 2 Related Literature

There is an extensive literature on strategic voting since the seminal paper of Austen-Smith and Banks (1996). Many of the papers in this line of literature study how informational efficiency of various voting mechanisms is affected by the agents' strategic behavior (see, for example, Feddersen and Pesendorfer (1997) on simultaneous voting rules and Dekel and Piccione (2000) on sequential voting rules). Among all of them, the most closely related paper besides KV is actually Austen-Smith and Banks (1996). In one sec-

tion of the paper (pp. 41-43) they extend the standard voting model to the case with both public and private information, and they conclude that in such a setting sincere voting cannot be both informative and rational (p. 42, Theorem 3). While this to some extent already implies the difficulty of information aggregation in a voting game with both private and public information, it does not address the question of existence of an (individually) informative voting equilibrium. Moreover, it is not immediately clear what drives this impossibility result, because in the benchmark case without public signal, each agent has access to only *one* private signal, while in the extension there are *two* instead, which both enlarges the set of equilibria and makes the information that an agent can draw from pivotality less straightforward. Therefore, neither the results in our paper nor in KV follow directly from Austen-Smith and Banks (1996). In addition, our paper considers a much richer set of preferences of the agents, which allows us to investigate how the existence result changes with respect to the changes of the precision of the public signal and the preferences of the agents.<sup>1</sup>

Several papers study the effect of pre-voting deliberation (e.g., Coughlan, 2000; Austen-Smith and Feddersen, 2006; Gerardi and Yariv, 2007). In these models, agents can communicate their private information before the vote actually takes place, thus public information *endogenously* arises. Our model differs from them in at least two main aspects. First, in the models with deliberation, conflicts between an agent's private signal and the public signal usually do not matter because the former has already been incorporated in the latter. In the current paper, however, such conflicts have a direct effect on agents' provision of private information, which can lead to a severe loss of informational efficiency. Secondly, unlike in the obedient voting equilibrium in the current paper, in these models it is actually socially efficient for the agents to always follow the public information, conditional on their private information being credibly revealed in the deliberation stage.

Finally, there is a third strand of literature on committee design and optimal voting rules. For example, Persico (2004) studies the optimal size and threshold value for simultaneous voting rules when agents' private information is endogenous. Subsequently, Ger-

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<sup>1</sup>A few other papers also study the effect of public information in a voting environment, e.g., Gersbach (2000), Taylor and Yildirim (2010), Tanner (2014). However, both the models and the focus of these papers are quite different from ours.

shkov and Szentes (2009) show that when information is costly, the optimal direct mechanism can actually be implemented by a random, sequential reporting/voting scheme, which suggests in general that the use of more flexible voting rules can be welfare-enhancing. This insight is also shared by Gersbach (2004, 2009), who shows that allowing the voting rule to depend on the proposal to be determined may yield efficient outcomes for classic social choice problems such as provision of public projects and division of limited resources among agents. More recently, Gershkov et al. (2014) show that in an environment where agents have single-crossing preferences, a successive voting rule with a descending threshold achieves the highest utilitarian efficiency among all anonymous, unanimous and dominant strategy incentive-compatible mechanisms. Our paper contributes to this line of literature by showing that in a setting where both private and public information exists, efficiency can also be significantly enhanced by using more flexible voting rules that are contingent on the public information.

## 3 The Model

### 3.1 Players, actions and payoffs

Consider a committee of  $n$  members (agents) indexed by  $i \in \mathcal{I} = \{1, \dots, n\}$ . We assume  $n$  is odd and  $n \geq 3$ . Agents need to make a collective decision  $d \in \mathcal{D} = \{0, 1\}$  over a binary set of alternatives. For concreteness, one could think of a setting in which a board of directors is choosing between two business proposals.

Each agent can cast a vote to support one of the alternatives. We denote  $v_i = 1$  if agent  $i$  votes in favor of the decision  $d = 1$ , and  $v_i = 0$  otherwise. We further denote  $\mathcal{V}_i = \{0, 1\}$  as agent  $i$ 's action set and  $\mathbf{v} = (v_1, \dots, v_n) \in \mathcal{V} = \prod_{i=1}^n \mathcal{V}_i$  as the agents' voting profile. A collective decision rule is a function,  $g : \mathcal{V} \rightarrow \mathcal{D}$ , that assigns every voting profile to a collective decision. In this paper, we restrict our attention to a class of collective decision rules called  $k$ -voting rules. Formally, if we set the alternative associated with  $d = 0$  as the default option, under a given  $k$ -voting rule the alternative associated with  $d = 1$  will be



chosen if and only if there are at least  $k \in \{1, \dots, n\}$  votes in favor of it:

$$g(\mathbf{v}) = \begin{cases} 1 & \text{if } \sum_{i=1}^n v_i \geq k, \\ 0 & \text{otherwise.} \end{cases}$$

Each  $k$ -voting rule is uniquely characterized by its threshold value  $k$ . If  $k = (n + 1)/2$ , we are in a setting of simple majority voting, while  $k > (n + 1)/2$  corresponds to some super-majority rule.

With equal probability, the state of the world  $\theta$  is drawn from a binary set  $\Theta = \{0, 1\}$ . In the context of the board of directors and business proposals, one could think of  $\theta$  as the uncertain (relative) quality of the two proposals, where  $\theta = 1$  means the proposal associated with  $d = 1$  is of higher prospective revenue, while the other is better if  $\theta = 0$ . We assume agent  $i$ 's utility function  $u_i : \mathcal{D} \times \Theta \rightarrow \mathbb{R}$  takes the following form (see also Coughlan, 2000; Kojima and Takagi, 2010; Iaryczower and Shum, 2012):

$$u_i(d, \theta) = \begin{cases} 0 & \text{if } d = \theta, \\ -q_i & \text{if } d = 1, \theta = 0, \\ -(1 - q_i) & \text{if } d = 0, \theta = 1, \end{cases}$$

where  $q_i \in [0, 1]$ . In words, we assume the agents in the committee have common interest in matching the collective decision to the state (i.e., choosing the proposal of higher quality), and we normalize the payoff of successfully choosing  $d = \theta$  to zero. However, we allow the agents' payoffs to differ when committing different types of decision errors (i.e.,  $(d, \theta) = (1, 0)$  or  $(d, \theta) = (0, 1)$ ). We also allow these differences to be heterogeneous across agents. Note that in our setting, each agent's utility function is uniquely characterized by the parameter  $q_i$ , which is a measure of how biased agent  $i$  is towards the default option *ex ante*: if  $q_i = 1/2$ , agent  $i$  is unbiased and indifferent between the two alternatives; if  $q_i < 1/2$ , agent  $i$  is inclined to choose  $d = 1$ ; however, if  $q_i > 1/2$ , agent  $i$  would prefer  $d = 0$  if there is no further information to be revealed. In addition, if  $q_i \neq q_j$ , the two agents  $i$  and  $j$  may strictly prefer different alternatives even when they have exactly the

same information. Hence, we interpret  $q_i \neq q_j$  as a conflict of interest between the two agents. We will call the vector  $\mathbf{q} = (q_i)_{i \in \mathcal{I}}$  the preference profile of the agents.

Note that, given the above specification of payoffs, if agent  $i$  assigns a posterior probability  $\pi \in [0, 1]$  to the event  $\theta = 1$ , she would prefer  $d = 1$  over  $d = 0$  if and only if

$$-(1 - \pi)q_i \geq -\pi(1 - q_i) \iff \pi \geq q_i,$$

that is, whenever the evidence of the state being 1 is sufficiently strong.

### 3.2 Information structure and timing

Before casting their votes, each agent receives a binary, private signal  $s_i \in S_i = \{0, 1\}$ . The private signals are i.i.d. across agents and are drawn according to the conditional probability distribution  $\Pr(s_i = 1|\theta = 1) = \Pr(s_i = 0|\theta = 0) = \alpha$ . In addition, all agents commonly observe a public signal  $s_p \in S_p = \{0, 1\}$ . The public signal is independent of the agents' private signals and is drawn according to the conditional probability distribution  $\Pr(s_p = 1|\theta = 1) = \Pr(s_p = 0|\theta = 0) = \beta$ . In the context of the board of directors and business proposals, one can think of the public signal as the opinion expressed by the advisory board to all directors before the vote takes place. We call  $\alpha$  ( $\beta$ ) the precision or the informativeness of the private (public) signal. Without loss of generality, we assume  $\alpha, \beta \in [1/2, 1]$ . Denote the signal profile of the voters by  $\mathbf{s} = (s_1, \dots, s_n)$  and let  $m_{\mathbf{s}} = \sum_{i=1}^n s_i$ . Note that if an agent could observe all the realizations of signals, the posterior probability that she assigns to the event  $\theta = 1$  will be

$$\pi = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{m_{\mathbf{s}}} \left(\frac{\alpha}{1-\alpha}\right)^{n-m_{\mathbf{s}}} \left(\frac{1-\beta}{\beta}\right)^{\mathbb{1}_{s_p=1}} \left(\frac{\beta}{1-\beta}\right)^{\mathbb{1}_{s_p=0}}}. \quad (3.1)$$

Since the above formula for posterior odds will appear frequently in the results and the proofs of this paper, we provide its formal derivation in the Appendix.

The timing of the voting game is as follows. First, Nature draws  $\theta$ . After that, each agent observes her own private signal and, in addition, the public signal. Agents then cast their votes, and the collective decision  $d$  is determined according to the voting profile and

the voting rule. Finally, the state is revealed and agents collect their payoffs.

### 3.3 Strategies and equilibrium

In the voting game, a (pure) strategy of agent  $i$  is a function  $v_i : S_i \times S_p \rightarrow \mathcal{V}_i$  that maps from the Cartesian product of the private and public signal spaces to the action space. We are particularly interested in the following two types of voting strategies (see also KV):

**Definition 1** A strategy is (individually) **informative** if  $v_i(s_i, s_p) = s_i, \forall s_i \in S_i, s_p \in S_p$ .

**Definition 2** A strategy is **obedient** if  $v_i(s_i, s_p) = s_p, \forall s_i \in S_i, s_p \in S_p$ .

The solution concept we use throughout the paper is Bayes Nash Equilibrium. We call an equilibrium in which all agents play the informative strategy an *informative voting equilibrium*. Similarly, an equilibrium in which all agents play the obedient strategy will be called an *obedient voting equilibrium*. For a given preference profile  $\mathbf{q}$ , if there exists a  $k$ -voting rule under which the informative voting equilibrium exists, we say that such a preference profile allows for the existence of the informative voting equilibrium or simply allows for informative voting.

In the absence of public information, if  $q_i \in [1 - \alpha, \alpha] \forall i \in \mathcal{I}$ , it is easy to check that under the majority voting rule (i.e.,  $k = (n + 1)/2$ ), the informative voting equilibrium exists and the CJT holds (Coughlan, 2000). If all agents are highly biased towards one of the alternatives, a threshold value  $k \neq (n + 1)/2$  may need to be adopted in order to help sustaining informative voting as an equilibrium. For example, if all agents are highly biased towards the decision  $d = 0$  such that  $q_i \in [\alpha, \alpha^3/(\alpha^3 + (1 - \alpha)^3)] \forall i \in \mathcal{I}$ , one can show that the informative voting equilibrium still exists in a voting game with the supermajority rule  $k = (n + 3)/2$ , and the CJT continues to hold as  $n$  becomes sufficiently large. However, as we will see in the next section, if a public signal is introduced to the voting game, whether it is more informative than the private signals or not, the set of preferences that allow for the existence of the informative voting equilibrium may shrink drastically.

## 4 Impossibility of Informative Voting

In order to show how the presence of a public signal could affect the equilibrium outcome of the voting game, and in particular how it limits the existence of the informative voting equilibrium and opens the possibility for agents to coordinate on an inefficient equilibrium, we first state the following proposition, which characterizes a necessary and sufficient condition for the existence of the informative voting equilibrium under a given  $k$ -voting rule:

**Proposition 1** *Given a  $k$ -voting rule, the informative voting equilibrium exists if and only if*

$$\forall i \in \mathcal{I}, q_i \in \left[ \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2} \frac{1-\beta}{\beta}}, \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \frac{\beta}{1-\beta}} \right]. \quad (4.1)$$

The proof of Proposition 1 is based on two simple observations: first, under a given  $k$ -voting rule, an arbitrary agent  $i$  is only pivotal when there are exactly  $k-1$  other agents who vote in favor of the decision  $d = 1$ , while the remaining  $n-k$  agents choose to vote in favor of the decision  $d = 0$ ; secondly, if agent  $i$  prefers to vote according to her private signal even when it conflicts with the public signal, she will also prefer to do so when the two signals agree. Assuming all other agents  $j \neq i$  follow the informative voting strategy, for a given  $k$ -voting rule, the left endpoint of the interval in (4.1) is the posterior probability that a Bayesian agent  $i$  will assign to the event  $\theta = 1$  conditional on  $s_i = 0, s_p = 1$  and being pivotal, while the right endpoint of the interval in (4.1) is the posterior probability conditional on  $s_i = 1, s_p = 0$  and being pivotal. Since a rational agent cares only about the case in which she is decisive about the final voting outcome, we can conclude that all  $q_i$  lying between the above two posterior probabilities is a necessary and sufficient condition for the existence of the informative voting equilibrium under the given  $k$ -voting rule.

KV observe that if the public signal is more accurate than each of the private signals ( $\beta > \alpha$ ), informative voting for agents of pure common interest cannot constitute an equilibrium under the majority rule. The next two corollaries, which follow Proposition 1 immediately, generalize this observation to arbitrary precision of the public signal, the

whole class of  $k$ -voting rules, and a much larger set of preferences.

**Corollary 1** *Suppose  $\beta > \alpha$ . For any threshold value  $k$  and any preference profile  $(q_i)_{i \in \mathcal{I}}$ , the informative voting equilibrium does not exist.*

**Corollary 2** *Suppose  $\beta \leq \alpha$ . If there exist  $i, j \in \{1, \dots, n\}$  such that*

$$q_i < \frac{1}{1 + \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta}}, \quad q_j > \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta}},$$

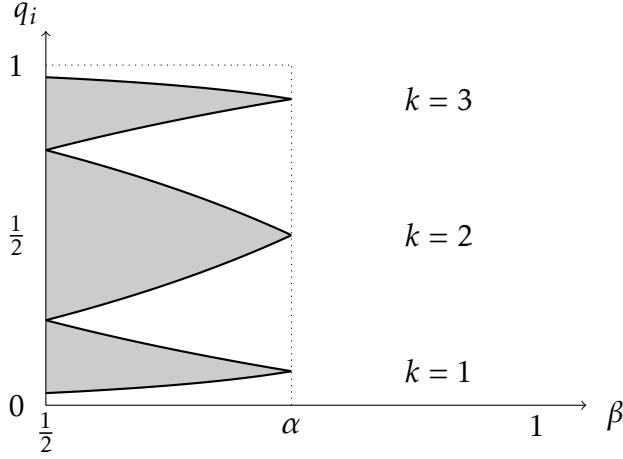
*then, for any  $k$ -voting rule, the informative voting equilibrium does not exist.*

In short, Corollary 1 confirms that whenever the public signal is strictly more precise than each of the private signals, it is hopeless to obtain the informative voting equilibrium under any standard  $k$ -voting rule. Meanwhile, Corollary 2 implies that even if the public signal is less accurate, it is still hard to guarantee the existence of the informative voting equilibrium as long as there are two or more agents who are biased (even just slightly) toward *different* alternatives ex ante (note that when  $\beta$  is close to  $\alpha$ , both  $1/(1 + \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta})$  and  $1/(1 + \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta})$  are close to  $1/2$ ).

The intuition behind both corollaries can be understood via the following simple example of three agents with heterogeneous preferences, such that

$$q_1 \in \left[ 1 - \alpha, \frac{1}{1 + \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta}} \right), \quad q_2 = \frac{1}{2}, \quad q_3 \in \left( \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta}}, \alpha \right].$$

Assume that the collective decision is made according to the majority rule ( $k = 2$ ). In the absence of public information, one can check that informative voting constitutes an equilibrium, even though the first and third agents are biased toward different alternatives ex ante. Suppose now that the public signal is more informative than each of the private signals and consider the unbiased agent 2. If agent 2 assumes that the other two agents will vote informatively, she could infer that the only situation in which she is pivotal is when agent 1 and 3 receive conflicting signals, but this implies that the others' private signals are collectively uninformative about the state. Hence, in this case, agent 2 would



**Figure 1:** The graphs of the correspondences  $Q^{\alpha,k}(\beta)$  given  $n = 3, \alpha = 0.75$ .

make her voting decision by comparing the observed public signal and her own private signal, and simply follows the public one because of its higher precision. Now suppose, conversely, the public signal is less informative than the private signal. While now it is rational for the unbiased agent 2 to vote informatively (assuming the other two agents do so as well), this is not the case for the two biased agents. For example, agent 1 will still strictly prefer to choose  $v_1 = 1$  if  $s_1 = 0$  and  $s_p = 1$ , even when she assumes that the other two agents are voting informatively. This is because the public signal, albeit less informative, is still in favor of her preferred alternative. Moreover, this problem cannot be resolved by using the unilateral ( $k = 1$ ) or unanimity rule ( $k = 3$ ) instead. For example, suppose all three agents are unbiased and the public signal is just slightly more informative than the private signal. While adopting the unanimity rule can successfully encourage agents to vote informatively whenever  $s_p = 0$ , it provides even stronger incentives for the agents to disregard their private information whenever  $s_p = 1$ .

Figure 1 interprets the above results graphically. Suppose for a given  $k$ -voting rule, an agent  $i$  with  $q_i$  will find it optimal to play the informative voting strategy when assuming that all other agents  $j \neq i$  are voting informatively. Let  $Q^{\alpha,k}(\beta) \subseteq [0, 1]$  denote the set of all such  $q_i$ , for given  $k, \alpha$  and  $\beta$ . Clearly, for a given preference profile  $\mathbf{q}$ , the informative voting equilibrium exists under a given  $k$ -voting rule if and only if  $q_i \in Q^{\alpha,k}(\beta), \forall i \in \mathcal{I}$ . Note that, for a given  $\alpha$ ,  $Q^{\alpha,k}(1/2)$  corresponds to the set of preferences that allow for

informative voting under a given  $k$ -voting rule when the public signal is absent. For fixed parameter values  $n = 3$  and  $\alpha = 0.75$ , the top, middle, and bottom part of the gray area in Figure 1 corresponds to the graph of  $Q^{\alpha,3}(\beta)$ ,  $Q^{\alpha,2}(\beta)$  and  $Q^{\alpha,1}(\beta)$ , respectively. As the precision of the public signal increases, the measure of each  $Q^{\alpha,k}(\beta)$  decreases. In particular, when  $\beta > \alpha$ ,  $Q^{\alpha,k}(\beta) = \emptyset, \forall k = 1, 2, 3$ .

Besides shrinking the set of preference profiles that allow for the existence of the informative voting equilibrium, the presence of the public signal opens the possibility for the agents to coordinate on playing the obedient voting equilibrium:

**Proposition 2** *For any given  $k$ -voting rule, the obedient voting equilibrium exists if either of the following two conditions is satisfied:*

1.  $1 < k < n$ .
2.  $k = 1$  or  $k = n$ ,  $\beta \geq \alpha$ , and  $\forall i \in \mathcal{I}, q_i \in \left[ \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta}}, \frac{1}{1 + \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta}} \right]$ .

Clearly, the obedient voting equilibrium could be highly inefficient, especially when the public signal is less accurate or just moderately more accurate than each of the private signals. In other words, unlike endowing its members with better private signals, introducing a public signal may actually lead to a worse performance of the committee. This is similar to one of the most striking findings in the global game literature, namely the heterogeneous effect of public and private information. For example, in a highly influential paper, Morris and Shin (2002) show that in a setting where agents' actions are strategic complements, additional public information can have negative social value. Although agents in the current setting have no intrinsic motive of coordination, our results suggest similarly that the conventional wisdom that additional information is always beneficial for decision-makers should be carefully examined.

One might question why agents would coordinate on such an inefficient equilibrium, especially when it is not unique. In fact, KV prove that if the public signal is more precise than the private signal and all agents are unbiased, under the majority voting rule there exist at least two other equilibria with intuitive properties: one is a symmetric mixed-strategy equilibrium, in which each agent votes according to her private signal with some

positive probability; the other one is an asymmetric pure-strategy equilibrium, in which only a small subset of the agents vote obediently while the remaining agents vote informatively. Unfortunately, KV also experimentally show that a very large proportion of agents tend to vote according to the public signal instead of their private signals much more frequently than either of the above two equilibria would predict. Consequently, the collective decisions coincided with what the public signal suggested most of the time. In other words, the experimental evidence suggests that the presence of a public signal can indeed lead to a significant welfare loss. An important reason for this inefficient outcome may be that, compared to the symmetric mixed-strategy equilibrium and the asymmetric pure-strategy equilibrium, the obedient voting equilibrium requires conceivably less sophisticated coordination from the agents. We are therefore interested in the question whether there exist more complex voting rules that can help restore the existence of the informative voting equilibrium, which arguably requires even less coordination than the obedient voting equilibrium. In the next section, we will show that the answer to the above question is yes if we allow the voting rule to be contingent on the public signal.

## 5 Reversing the Impossibility

### 5.1 Contingent $k$ -voting rules

We construct a new class of voting rules that we call *contingent  $k$ -voting rules*, which can be obtained by adjusting the standard  $k$ -voting rules in an intuitive way. In particular, the threshold values in such voting rules will be no longer fixed but a function of the realization of the public signal:

$$k(s_p) = \begin{cases} k_0 & \text{if } s_p = 0, \\ k_1 & \text{if } s_p = 1, \end{cases} \quad (5.1)$$

where  $k_0, k_1 \in \{1, \dots, n\}$ . Any standard  $k$ -voting rule amounts to a special case of our contingent  $k$ -voting rules.

We first provide a necessary and sufficient condition for the existence of the informa-



tive voting equilibrium under a given contingent  $k$ -voting rule:

**Proposition 3** *Given a contingent  $k$ -voting rule, the informative voting equilibrium exists if and only if*

$$\forall i \in \mathcal{I}, q_i \in \left[ \max\{\pi_0^0, \pi_0^1\}, \min\{\pi_1^0, \pi_1^1\} \right], \quad (5.2)$$

where

$$\begin{aligned} \pi_0^0 &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2} \frac{\beta}{1-\beta}}, & \pi_1^0 &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n} \frac{\beta}{1-\beta}}, \\ \pi_0^1 &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2} \frac{1-\beta}{\beta}}, & \pi_1^1 &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n} \frac{1-\beta}{\beta}}. \end{aligned}$$

The proof of Proposition 3 is very similar to that of Proposition 1. The main difference is that now in the informative voting equilibrium, the information about the other agents' private signals that an agent can infer from her pivotality depends on the realization of the public signal. In particular, depending on the choices of  $k_0$  and  $k_1$ , it is no longer necessarily the case that an agent will have stronger incentives to deviate from the informative voting equilibrium when her private signal differs from the public signal than when the two signals agree. This is why the interval in (5.2) involves taking the maximum and minimum of the posterior probabilities that an agent will assign to the event  $\theta = 1$  in different cases.

To see how the contingent  $k$ -voting rules can help restore the informative voting equilibrium, consider a simple example with  $n = 5$  and  $q_i = 1/2, \forall i = 1, \dots, 5$ . If there is no public signal, the informative voting equilibrium exists under the standard majority rule. Now let us introduce a public signal whose informational content about the state is twice as much as that of a private signal.<sup>2</sup> Because the public signal is more informative than each of the private signals, according to Corollary 1, the informative voting equilibrium no longer exists for any standard  $k$ -voting rule. However, consider the contingent  $k$ -voting rule with the threshold values  $k_0 = 4$  and  $k_1 = 2$ . Assuming all other agents  $j \neq i$  are voting informatively, if  $s_p = 1$ , agent  $i$  is only pivotal when three of the other agents

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<sup>2</sup>We will formally define the measure of the informativeness of the public signal relative to the private signal in the next section.

draw  $s_j = 0$  and the remaining one draws  $s_j = 1$ . Given the above assumption on the informativeness of the public signal, these private signals are collectively uninformative about the state when they are combined with the realization of the public signal. Thus, voting according to her own private signal is a best response for agent  $i$ . Similarly, if  $s_p = 0$ , agent  $i$  is only pivotal under the contingent  $k$ -voting rule when there are three private signals in favor of  $d = 1$  and one in favor of  $d = 0$  among all others' private signals. Again, the collective informational effect of all  $s_j, j \neq i$ , will be exactly counterbalanced by the fact that  $s_p = 0$ , which makes it optimal to vote according to her own signal for agent  $i$ .

Intuitively, the public signal introduces a common random shock to all agents' prior, which has a very similar effect as a common shock on the preference profile in our model. As we have seen in Section 3.3, when agents are commonly biased towards one of the two alternatives, the informative voting equilibrium can still exist if the threshold value  $k$  is appropriately chosen. However, since the shock due to the public signal is random, the direction of the correction of the threshold value must depend on the realization of the public signal. In particular, when the public signal is in favor of one of the alternatives, the threshold value should be adjusted in such a way that this alternative is more likely to be chosen. At first glance, this might be counter-intuitive because the negative welfare impact of the public signal, as discussed in Section 4, is exactly due to the problem that agents are tempted to follow whatever the public signal suggests. However, what we are doing here is to change the information that agents can infer from pivotality: under the contingent  $k$ -voting rule that is chosen in the above example, whenever an agent is pivotal, it must be the case that the private signals of the other agents are collectively more against the alternative favored by the public signal. This restores the incentive for agents to vote according to their own signals.

## 5.2 The choice of $k_0$ and $k_1$

We are now interested in the question what would be suitable choices of  $k_0$  and  $k_1$  if one wants to restore the existence of the informative voting equilibrium by using a contingent  $k$ -voting rule that takes the form of (5.1). Similar to the definition of  $Q^{\alpha,k}(\beta)$ , let  $Q^{\alpha,k_0,k_1}(\beta) \subseteq [0,1]$  denote the set of  $q_i$  such that given the precision of the signals  $\alpha$  and

$\beta$  and a contingent  $k$ -voting rule with the threshold values  $k_0$  and  $k_1$ , an agent  $i$  with  $q_i$  would find it optimal to play the informative voting strategy if she assumes that all other agents  $j \neq i$  are voting informatively. We first ask what choices of  $k_0$  and  $k_1$  would lead to a non-empty  $Q^{\alpha, k_0, k_1}(\beta)$  for given  $\alpha$  and  $\beta$ . To answer this question, we introduce the following measure of (relative) informativeness of the public signal:

$$r = \frac{\ln \beta - \ln(1 - \beta)}{\ln \alpha - \ln(1 - \alpha)}. \quad (5.3)$$

For given  $\alpha$  and  $\beta$  the value of  $r$  is uniquely determined, and we will say that the public signal is  $r$ -times as informative as the private signal. For example, if  $\alpha = 0.6$ , then  $\beta = 0.55, 0.69, 0.77$  correspond to the cases where the public signal is 0.5-, 2- and 3-times as informative as the private signal, respectively. Note that (5.3) can be rearranged as  $\beta/(1 - \beta) = (\alpha/(1 - \alpha))^r$ . Hence, intuitively, the measure  $r$  tells us how many private signals of opposite realization would counter-balance the informational effect of the public signal.<sup>3</sup> We are now ready to state the next corollary, which follows from Proposition 3 and relates non-emptiness of the set  $Q^{\alpha, k_0, k_1}(\beta)$  to the relative informativeness of the public signal.

**Corollary 3**  $Q^{\alpha, k_0, k_1}(\beta) \neq \emptyset$  if and only if  $r - 1 \leq k_0 - k_1 \leq r + 1$ .

Given the result of Corollary 3, in the remainder of the paper we will focus only on the contingent  $k$ -voting rules with  $k_0$  and  $k_1$  such that the inequality  $r - 1 \leq k_0 - k_1 \leq r + 1$  is satisfied. If  $r \leq 1$ , any pair of integers  $k_0$  and  $k_1$  such that  $k_0 = k_1$  or  $k_0 = k_1 + 1$  satisfies the above inequality. If  $1 < r \leq n$ , it is easy to see that there still exists at least one pair of integers  $k_0$  and  $k_1$  that satisfies the inequality and leads to a non-empty  $Q^{\alpha, k_0, k_1}(\beta)$ . On the other hand, for the case  $r > 1$ , we know by Corollary 1 that  $Q^{\alpha, k}(\beta) = \emptyset$  for any  $k$ . Hence, it is clear that introducing the contingent  $k$ -voting rules can *strictly* enlarge the set of preferences that allow for the existence of the informative voting equilibrium, as

<sup>3</sup>Interestingly, (5.3) can be also related to the conditional entropy of  $\theta$ . Note that the conditional entropy of  $\theta$  given the private signal  $s_i$  is  $H(\theta|s_i) = 1 - \alpha \ln \alpha - (1 - \alpha) \ln(1 - \alpha)$ , and similarly the conditional entropy of  $\theta$  given the public signal  $s_p$  is  $H(\theta|s_p) = 1 - \beta \ln \beta - (1 - \beta) \ln(1 - \beta)$ . Hence, it immediately follows that

$$\frac{\partial H(\theta|s_p)/\partial \beta}{\partial H(\theta|s_i)/\partial \alpha} = \frac{\ln \beta - \ln(1 - \beta)}{\ln \alpha - \ln(1 - \alpha)}.$$

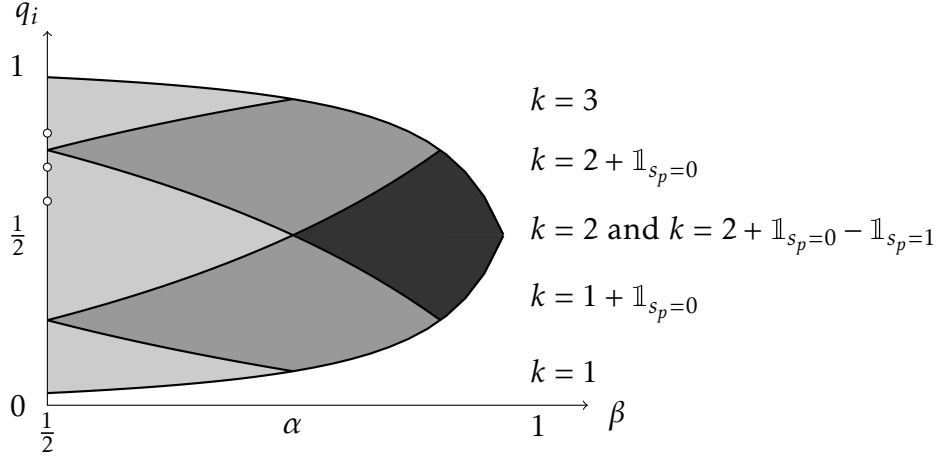
This gives a formal justification for using  $r$  as a measure of relative informativeness of the public signal.

long as the public signal is not extremely precise ( $r \leq n$ ).

When the size of the committee is large, there might be many contingent  $k$ -voting rules whose threshold values satisfy the inequality in Corollary 3. Therefore, for a given preference profile  $\mathbf{q}$  it can be very tedious to go through every pair of these threshold values, and see with which the condition in Proposition 3 will be satisfied. Fortunately, we can rely on the following algorithm. First, define  $\underline{q} = \min_{i \in \mathcal{I}} q_i$  and  $\bar{q} = \max_{i \in \mathcal{I}} q_i$  for the given preference profile. Second, invert the functions  $\pi_0^0$  and  $\pi_1^0$  of  $k_0$  and the functions  $\pi_0^1$  and  $\pi_1^1$  of  $k_1$  that are defined in Proposition 3. Note that this is feasible because all these functions are strictly increasing in either  $k_0$  or  $k_1$ . Third, apply the inverse functions  $(\pi_0^0)^{-1}$  and  $(\pi_1^0)^{-1}$  to  $\bar{q}$  and  $(\pi_1^1)^{-1}$  and  $(\pi_0^1)^{-1}$  to  $\underline{q}$ . Finally, pick an integer that lies in the interval  $[(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})]$  and an integer that lies in the interval  $[(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})]$ , and we obtain a pair of voting threshold values  $k_0$  and  $k_1$  that can sustain informative voting as an equilibrium for the given preference profile.

Note that both  $(\pi_0^0)^{-1}$  and  $(\pi_1^0)^{-1}$  are strictly increasing in  $\bar{q}$ , while both  $(\pi_1^1)^{-1}$  and  $(\pi_0^1)^{-1}$  are strictly increasing in  $\underline{q}$ . Hence, it is very likely that both of the intervals  $[(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})]$  and  $[(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})]$  contain no integer if  $\bar{q}$  is sufficiently larger than  $\underline{q}$ . Intuitively, if the degree of conflict of interest between the agents is too large, it is very difficult to find a voting rule that ensures the incentive for *all* agents to vote informatively, even if we allow the voting threshold value to be flexibly contingent on the public signal. One might therefore expect that it would be easier to find a contingent  $k$ -voting rule that helps restore the existence if the agents' preferences are more aligned. This is *not* true in general because it also depends on the *exact* values of  $\underline{q}$  and  $\bar{q}$ . Nevertheless, for the important limiting cases where agents' preferences are perfectly aligned (e.g., Feddersen and Pesendorfer, 1998; Persico, 2004; Koriyama and Szentes, 2009), we are able to prove that there always exists a contingent  $k$ -voting rule under which the informative voting equilibrium exists, as long as the size of the committee is sufficiently large:

**Proposition 4** *Suppose  $\forall i \in \mathcal{I}, q_i = q$ . There exists  $\bar{n}(q)$ , such that for each  $n \geq \bar{n}(q)$ , there exists a contingent  $k$ -voting rule that can sustain informative voting as an equilibrium.*



**Figure 2:** The graphs of the correspondences  $Q^{\alpha, k_0, k_1}(\beta)$  given  $n = 3, \alpha = 0.75$ .

Figure 2 illustrates the main findings of this section graphically. Consider the same parametric example that we discussed in Section 4, where  $n = 3$  and  $\alpha = 0.75$ . Besides the three standard  $k$ -voting rules, we know from Corollary 3 that there are three other contingent  $k$ -voting rules that may lead to a non-empty  $Q^{\alpha, k_0, k_1}(\beta)$  (depending on the exact value of  $r$ ), with the threshold values  $k(s_p) = 1 + \mathbb{1}_{s_p=0}$ ,  $k(s_p) = 2 + \mathbb{1}_{s_p=0} - \mathbb{1}_{s_p=1}$  and  $k(s_p) = 2 + \mathbb{1}_{s_p=0}$ , respectively. Figure 2 above plots the graphs of the correspondences  $Q^{\alpha, k_0, k_1}(\beta)$  for all these contingent  $k$ -voting rules. As in Figure 1, the top, middle and bottom part of the light gray area corresponds to the graph of  $Q^{\alpha, 3}(\beta)$ ,  $Q^{\alpha, 2}(\beta)$  and  $Q^{\alpha, 1}(\beta)$ , respectively. In addition, the graphs of the correspondences  $Q^{\alpha, 2, 1}(\beta)$  and  $Q^{\alpha, 3, 2}(\beta)$  are represented respectively by the top and the bottom part of the deep gray area. Finally, the dark area corresponds to the graph of  $Q^{\alpha, 3, 1}(\beta)$ . Given a preference profile  $\mathbf{q}$  and the precision of the public signal, the informative voting equilibrium can be sustained by some contingent  $k$ -voting rule if and only if all  $q_i$  lie on a vertical line that is entirely contained in one of the different segments in the figure. The algorithm that we described before is just an analytical way for us to find a segment that contains all  $q_i$ . Comparing Figure 1 and Figure 2, it is clear that allowing the voting threshold value to be contingent on the realization of the public signal can help restore the informative voting equilibrium in many circumstances.

Interestingly, we observe that for some preference profiles, one can actually induce the informative voting equilibrium by introducing a public signal of certain accuracy and using some contingent- $k$  voting rule, even though this would not be possible for any standard  $k$ -voting rule in the absence of the public signal. For example, suppose  $n = 3$  and the agents' preference profile corresponds to the three dots that lie on the vertical axis in Figure 2. If there is no public signal, the informative voting equilibrium does not exist for any fixed  $k$ -voting rule, since not all  $q_i$  are located in the same segment when  $\beta = 1/2$ . However, it is clear from the graph that when  $\beta$  is sufficiently close to  $\alpha$ , all  $q_i$  lie on a line that is entirely contained in the top deep gray area, thus the informative voting equilibrium exists if we adopt the threshold value  $k(s_p) = 2 + \mathbb{1}_{s_p=0}$ .

### 5.3 Welfare analysis

In this section, we investigate the welfare implications of the contingent  $k$ -voting rules. This is not a trivial task, because the threshold value in a contingent  $k$ -voting rule depends on the realization of the public signal, and it is well-known that different threshold values can lead to different probabilities of the final collective decision being matched to the state (Feddersen and Pesendorfer, 1998; Duggan and Martinelli, 2001).

Since the negative welfare impact of the public signal is due to the inefficient equilibrium that it induces, we are first interested in comparing the efficiency of the informative voting equilibria under the contingent  $k$ -voting rules to that of the obedient voting equilibrium. Intuitively, when the size of the committee is sufficiently large, the information privately held by the agents must be collectively much more informative about the state compared to the public signal. Hence, one may expect that the informative voting equilibria under the contingent- $k$  voting rules are more likely to outperform the obedient voting equilibrium as the size of the committee increases. However, by adding new members to the committee, both the amount of private information and the degree of conflict of interest of it may be increased at the same time. The latter effect is problematic because it may destroy the existence of a contingent  $k$ -voting rule that can incentivize the agents to vote informatively, even if this is not a problem at all for the initial committee. To avoid this complication, we consider only the type of expansions of the committee that

do not exaggerate the initial degree of conflict of interest. Formally, let  $\mathbf{q} = (q_1, \dots, q_n)$  be a preference profile with  $\bar{q} = \max_{i \in \mathcal{I}} q_i$  and  $\underline{q} = \min_{i \in \mathcal{I}} q_i$ . We say a sequence of preference profiles  $\{\mathbf{q}^s = (\hat{q}_1, \dots, \hat{q}_n, \dots, \hat{q}_{n+s})\}_{s \in \mathbb{N}}$  preserves the conflict of  $\mathbf{q}$  if  $\forall \mathbf{q}^s$ ,  $\max_{j \in \{1, \dots, n+s\}} \hat{q}_j \leq \bar{q}$  and  $\min_{j \in \{1, \dots, n+s\}} \hat{q}_j \geq \underline{q}$ . For this type of expansions of the committee, the following proposition provides a sufficient condition under which the informative voting equilibria under the contingent  $k$ -voting rules are asymptotically more efficient than the obedient voting equilibrium (as long as the public signal is not perfect):

**Proposition 5** *Suppose  $\alpha > 1/2$ . Assume, for a given preference profile  $\mathbf{q}$  with  $\underline{q}, \bar{q} \in (0, 1)$ , that there exists a contingent  $k$ -voting rule that can sustain informative voting as an equilibrium. Then, for any sequence of preference profiles  $\{\mathbf{q}^s\}_{s \in \mathbb{N}}$  that preserves the conflict of  $\mathbf{q}$ :*

- (1)  *$\forall \mathbf{q}^s$ , there exists a contingent  $k$ -voting rule that can sustain informative voting as an equilibrium.*
- (2) *As  $s \rightarrow \infty$ , the ex ante probability of the collective decision being matched to the state in the informative voting equilibrium under the corresponding contingent  $k$ -voting rule becomes arbitrarily close to 1.*

In the standard setting with pure common interest agents, KV show that if the public signal is  $r$ -times as informative as the private signal, where  $r \leq (n-1)/2$ , then under the standard majority rule there exists an asymmetric equilibrium in which  $r^* = \mathbb{N} \cap (r-1, r]$  agents vote according to the public signal, while the remaining  $n-r^*$  agents vote according to their private signals. This  $r^*$ -asymmetric equilibrium is shown to be more efficient than both the obedient voting equilibrium and the mixed-strategy equilibrium, as well as all other asymmetric pure-strategy equilibria in the same voting game. In the following, we will show that one can always construct a contingent  $k$ -voting rule that not only ensures the existence of the informative voting equilibrium, but also leads to higher efficiency than the  $r^*$ -asymmetric equilibrium in such a setting.

Specifically, consider a contingent  $k$ -voting rule with the following threshold value:

$$k(s_p) = \begin{cases} \frac{n+1}{2} + \left[ \frac{r-1}{2} \right]^+ & \text{if } s_p = 0, \\ \frac{n+1}{2} - \left[ \frac{r-1}{2} \right]^+ & \text{if } s_p = 1, \end{cases}$$

where  $\lceil (r-1)/2 \rceil$  denotes the smallest integer that is larger or equal to  $(r-1)/2$ . For convenience, we will call this rule the contingent majority rule. Note that the contingent majority rule is well-defined whenever  $r \leq n$ . The next corollary, which follows from Proposition 3, justifies our focus on this particular contingent  $k$ -voting rule:

**Corollary 4** *The informative voting equilibrium exists under the contingent majority rule if and only if*

$$\forall i \in \mathcal{I}, q_i \in \left[ \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{|r-2\lceil (r-1)/2 \rceil - 1}}, \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{-|r-2\lceil (r-1)/2 \rceil + 1}} \right]. \quad (5.4)$$

Note that  $1/2$  always belongs to the above interval, since  $|r-2\lceil (r-1)/2 \rceil| \in [0, 1]$  for all  $r \geq 0$ . In other words, for the special case where all agents are unbiased, which is mostly studied in the literature and, in particular, in KV, one can always use the contingent majority rule to ensure the existence of the informative voting equilibrium. In fact, for this case the contingent majority rule is actually the unique contingent  $k$ -voting rule that restores the existence (unless  $r$  is an odd integer that is strictly less than  $n$ ).<sup>4</sup> The next proposition states that the informative voting equilibrium under the contingent majority rule, if exists, achieves the first-best informational efficiency.

**Proposition 6** *Given all the available information, the informative voting equilibrium under the contingent majority rule maximizes the probability of the collective decision being matched to the state.*

To gain some intuition, consider a simple example of  $n = 5$  and  $r = 2$ . Assume all agents are unbiased. Imagine that we introduce two additional phantom agents on top of the existing five real agents. Moreover, these phantom agents are programmed so that they simply vote in line with the public signal. Suppose now we use a standard majority rule to decide which alternative will be chosen. One can easily show that (1) all *real*

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<sup>4</sup>From the proof of Proposition 4, we know that for given  $\mathbf{q}$  and  $r$ , the contingent- $k$  voting rule that can be used to restore the informative voting equilibrium, if exists, is unique if and only if either of the following two conditions is met: (a)  $\underline{q} < \bar{q}$ ; (b) both  $(\pi_1^0)^{-1}(\bar{q})$  and  $(\pi_1^1)^{-1}(\bar{q})$  are *not* integers. If  $\underline{q} = \bar{q} = 1/2$  and  $r$  is an odd integer, both  $(\pi_1^0)^{-1}(1/2) = (n+r)/2$  and  $(\pi_1^1)^{-1}(1/2) = (n-r)/2$  are integers. In this case, there will be four pairs of integers  $k_0, k_1 \in \{1, \dots, n\}$  that satisfy the condition in Proposition 3 if  $r < n$ .



agents voting informatively constitutes an equilibrium in this game, (2) the equilibrium outcome is identical to that of the informative voting equilibrium under the contingent majority rule without the phantom agents, and (3) the equilibrium outcome maximizes the probability of the decision being matched to the state, given all available information. Intuitively, by allowing the threshold value to be dependent on the public signal and by encouraging agents to vote informatively, the contingent majority rule aggregates both the private and the public information efficiently.

On the contrary, in the  $r^*$ -asymmetric equilibrium with the standard majority rule, inefficiency still prevails because there are  $r^*$  agents who always disregard their valuable private information. To see this issue more clearly, consider again the above example. Since  $r^* = 2$ , under the standard majority rule there exists an asymmetric equilibrium in which two agents play the obedient strategy, while the remaining three agents vote informatively. Without loss of generality, assume the first two agents are the obedient voters. Consider the signal profile

$$\mathbf{s} = (1, 1, 0, 1, 1), s_p = 0.$$

In equilibrium, such a realization of signals will lead to a collective decision  $d = 0$ . However, from a benevolent social planner's point of view, given all the available information, the welfare maximizing decision should be  $d = 1$ . Therefore, the  $r^*$ -asymmetric equilibrium is strictly less efficient than the first-best.

We close this section with an important remark about the implementation of the contingent  $k$ -voting rules, which is an intuitive yet non-obvious implication of Proposition 6. In practice, it might be difficult to contract a voting rule that is contingent on the public information, especially when the source of the public information is ambiguous ex ante. However, if agents have pure common interest, we can instead implement an arguably more contractible two-stage voting mechanism, in which, after observing the private and the public signals, the agents first vote about which voting threshold value  $k \in \{1, \dots, n\}$  to use and then vote about which collective decision to take. Independent of the specific procedure of the first-stage voting rule and the off-equilibrium beliefs and

behaviors that we assume for the agents, in the two-stage voting game there must exist a Perfect Bayesian Nash equilibrium that is outcome-equivalent to the informative voting equilibrium under the contingent majority rule, i.e., an equilibrium in which agents first collectively vote to agree on the threshold value that would be chosen by the contingent majority rule, and then they vote informatively in the second stage. The reason being is that, according to Proposition 6, the expected social welfare is maximized when such a voting threshold value is in use. Since each agent's interest is perfectly aligned with the social welfare, any deviation in the first stage will only yield a lower expected payoff to an agent.

## 6 Conclusion

Our paper makes two main contributions. First, we show that the presence of a public signal may significantly limit the existence of an informative voting equilibrium in a setting where agents in a committee have both common and conflicting interests. It is also shown that the public signal opens the possibility for the agents to coordinate on playing an inefficient equilibrium that robustly exists across many different voting rules. Given KV's experimental evidence, we believe these two results to be of high policy relevance. Second, we show that by allowing for more flexible voting rules, of which the threshold values are contingent on the realization of the public signal, the existence of the informative voting equilibrium can be restored and the efficiency of the committee can be enhanced in many circumstances. Generally, the public information in our model can be interpreted as a random shock to the agents' common prior. Although it is known that whether a given voting rule can sustain informative voting as an equilibrium partially depends on the agents' prior beliefs about the state (see, e.g., Coughlan, 2000; Laslier and Weibull, 2010; Ben-Yashar and Nitzan, 2014), to the best of our knowledge, no previous paper in the voting literature actually assumes a random common prior, let alone analyzes its implications for the existence of informative voting equilibrium and the efficiency of different voting rules.

In terms of future research, theory-wise, an interesting extension of our model would

be to drop the assumption that the public information is exogenous and consider it to be strategically chosen by a public agent who has private interest over the potential alternatives. Some recent papers look at the question how a biased public agent can optimally persuade *uninformed* voters by designing the informational content of a public signal (Alonso and Câmara, 2014; Wang, 2015). The results of the current paper suggest that even when we allow the voters to have access to some private information, the collective decision can still be heavily influenced by the public agent who has control over the public information. Experiment-wise, given that in the standard setting with unbiased agents our theory predicts that the contingent majority rule can always ensure the existence of the informative voting equilibrium and achieve first-best informational efficiency, it would be very interesting to test it in the laboratory and see whether it actually outperforms the standard majority rule.

In general, our results suggest that in a voting environment where both private and public information exists, the voting procedures matter and the optimal voting rule should reflect the information that is contained in the public signal. For example, if the advisory board indicates that one of the business proposals is more promising than the other, it might be desirable for the board of directors to set up a voting rule that is more in favor of the acceptance of that proposal. Nevertheless, the design of optimal decision rules in more general social choice environments with public information remains an open and important research question.

# Appendix

## Derivation of Equation (3.1)

Conditional on observing the whole profile of private signals and the public signal, the posterior probability that a Bayesian agent would assign to the event  $\theta = 1$  is given as follows:

$$\begin{aligned}
 \Pr(\theta = 1 | \mathbf{s}, s_p) &= \frac{\Pr(\theta = 1, \mathbf{s}, s_p)}{\Pr(\mathbf{s}, s_p)} \\
 &= \frac{\Pr(\mathbf{s}, s_p | \theta = 1) \Pr(\theta = 1)}{\Pr(\mathbf{s}, s_p | \theta = 1) \Pr(\theta = 1) + \Pr(\mathbf{s}, s_p | \theta = 0) \Pr(\theta = 0)} \\
 &= \frac{\alpha^{m_s} (1 - \alpha)^{n - m_s} \beta^{\mathbb{1}_{s_p=1}} (1 - \beta)^{\mathbb{1}_{s_p=0}}}{\alpha^{m_s} (1 - \alpha)^{n - m_s} \beta^{\mathbb{1}_{s_p=1}} (1 - \beta)^{\mathbb{1}_{s_p=0}} + (1 - \alpha)^{m_s} \alpha^{n - m_s} (1 - \beta)^{\mathbb{1}_{s_p=1}} \beta^{\mathbb{1}_{s_p=0}}} \\
 &= \frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right)^{m_s} \left(\frac{\alpha}{1 - \alpha}\right)^{n - m_s} \left(\frac{1 - \beta}{\beta}\right)^{\mathbb{1}_{s_p=1}} \left(\frac{\beta}{1 - \beta}\right)^{\mathbb{1}_{s_p=0}}},
 \end{aligned}$$

where the first equality follows from Bayes rule and the third equality comes from the independence assumption of the signals.

## Proof of Proposition 1

Suppose all agents  $j \neq i$  play  $v_j(s_j, s_p) = s_j$ . First, note that if  $v_i(1, 0) = 1$  is rational for agent  $i$ , so is  $v_i(1, 1) = 1$ ; similarly, if  $v_i(0, 1) = 0$  is rational for agent  $i$ , so is  $v_i(0, 0) = 0$ . Hence, we only need to consider the optimality of the informative voting strategy in the cases where  $s_i \neq s_p$ .

Secondly, note that agent  $i$  is only decisive when and only when there are  $k - 1$  agents who observe a positive signal ( $s_j = 1$ ) and each of the remaining  $n - k$  agents observes an opposite signal ( $s_j = 0$ ). Hence, given  $s_i = 1, s_p = 0$  and being pivotal, the posterior probability that agent  $i$  assigns to the event  $\theta = 1$  is:

$$\pi_1^0 = \frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right)^k \left(\frac{\alpha}{1 - \alpha}\right)^{n - k} \frac{\beta}{1 - \beta}}.$$

Similarly, given  $s_i = 0, s_p = 1$  and being pivotal, the posterior probability that agent  $i$  assigns to the event  $\theta = 1$  is:

$$\pi_0^1 = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{k-1} \left(\frac{\alpha}{1-\alpha}\right)^{n-k+1} \frac{1-\beta}{\beta}}.$$

To have informative voting as an equilibrium, it is both necessary and sufficient to have  $\forall i \in \mathcal{I}, q_i \in [\pi_0^1, \pi_1^0]$ , i.e.,

$$\forall i \in \mathcal{I}, q_i \in \left[ \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2} \frac{1-\beta}{\beta}}, \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \frac{\beta}{1-\beta}} \right].$$

□

### Proof of Corollary 1

Note that the interval  $[\pi_0^1, \pi_1^0]$  as defined in the proof of Proposition 1 is non-empty if and only if

$$\left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2} \frac{1-\beta}{\beta} \geq \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \frac{\beta}{1-\beta} \iff \alpha \geq \beta.$$

□

### Proof of Corollary 2

Suppose there exists a  $k$ -voting rule under which the informative voting equilibrium exists. According to Proposition 1, the preferences of agents  $i$  and  $j$  must satisfy

$$q_i, q_j \in \left[ \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2} \frac{1-\beta}{\beta}}, \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \frac{\beta}{1-\beta}} \right]. \quad (\text{A.1})$$

Moreover, (A.1) and  $q_i < \frac{1}{1 + \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta}}$  implies

$$\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2} \frac{1-\beta}{\beta}} < \frac{1}{1 + \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta}} \iff k < \frac{n+1}{2}. \quad (\text{A.2})$$

Similarly, (A.1) and  $q_j > \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta}}$  implies

$$\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \frac{\beta}{1-\beta}} > \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta}} \iff k > \frac{n+1}{2}. \quad (\text{A.3})$$

Clearly, (A.2) and (A.3) cannot both be satisfied at the same time. Hence, we can conclude that the informative voting equilibrium does not exist under *any*  $k$ -voting rule.  $\square$

## Proof of Proposition 2

If  $1 < k < n$ , given all other agents  $j \neq i$  are simply obeying the public signal, agent  $i$  will never be pivotal, thus playing  $v(s_i, s_p) = s_p$  for all  $(s_i, s_p)$  is clearly a best response.

If  $k = n$  and  $\beta \geq \alpha$ , assuming all other agents  $j \neq i$  are voting obediently, agent  $i$  will be pivotal whenever  $s_p = 1$ . However, agent  $i$  cannot infer anything about the others' private signals conditional on her pivotality, because they are just obeying the public signal. Thus agent  $i$  simply compares her private signal and the public signal and follows the one of higher precision, as long as she is not too biased. Hence, the obedient voting equilibrium exists if the preference profile satisfies

$$\forall i \in \mathcal{I}, q_i \in \left[ \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta}}, \frac{1}{1 + \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta}} \right],$$

where the left endpoint of the interval is agent  $i$ 's posterior of the event  $\theta = 1$  conditional on  $s_i = 1$  and  $s_p = 0$ , and the right endpoint is her posterior conditional on  $s_i = 0$  and  $s_p = 1$ . The proof for the case  $k = 1, \beta \geq \alpha$  is analogous.  $\square$

## Proof of Proposition 3

Suppose  $s_p = 1$ . Under the stated contingent  $k$ -voting rule, the threshold value for choosing  $d = 1$  is  $k_1$ . Assume all agents  $j \neq i$  are playing the informative voting strategy. Conditional on being pivotal, the posterior probability that agent  $i$  would assign to the event

$\theta = 1$  if  $s_i = 0$  or  $s_i = 1$  are, respectively:

$$\begin{aligned}\pi_0^1 &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{k_1-1} \left(\frac{\alpha}{1-\alpha}\right)^{n-k_1+1} \frac{1-\beta}{\beta}} \\ &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2} \frac{1-\beta}{\beta}}\end{aligned}$$

and

$$\begin{aligned}\pi_1^1 &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{k_1} \left(\frac{\alpha}{1-\alpha}\right)^{n-k_1} \frac{1-\beta}{\beta}} \\ &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n} \frac{1-\beta}{\beta}}.\end{aligned}$$

Now suppose  $s_p = 0$ . Under the stated contingent  $k$ -voting rule, the threshold value for choosing the decision  $d = 1$  is  $k_0$ . Assume all agents  $j \neq i$  are playing the informative voting strategy. Conditional on being pivotal, the posterior probability that agent  $i$  would assign to the event  $\theta = 1$  if  $s_i = 0$  or  $s_i = 1$  are, respectively:

$$\pi_0^0 = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2} \frac{\beta}{1-\beta}} \quad \text{and} \quad \pi_1^0 = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n} \frac{\beta}{1-\beta}}.$$

It is easy to see that the informative voting equilibrium exists if and only if  $\forall i \in \mathcal{I}$ ,  $q_i \geq \max\{\pi_0^0, \pi_0^1\}$  and  $q_i \leq \min\{\pi_1^0, \pi_1^1\}$ .  $\square$

### Proof of Corollary 3

For a contingent  $k$ -voting rule with the threshold value  $k(s_p) = \mathbb{1}_{s_p=1}k_1 + \mathbb{1}_{s_p=0}k_0$ , it is clear that  $Q^{\alpha, k_0, k_1}(\beta)$  is non-empty if and only if the interval in (5.2) is non-empty, i.e.,  $\max\{\pi_0^0, \pi_0^1\} \leq \min\{\pi_1^0, \pi_1^1\}$ .

Note that

$$\begin{aligned}
\max\{\pi_0^0, \pi_0^1\} = \pi_0^1 &\iff \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2} \frac{1-\beta}{\beta}} \geq \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2} \frac{\beta}{1-\beta}} \\
&\iff \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2} \left(\frac{1-\alpha}{\alpha}\right)^r} \geq \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2} \left(\frac{\alpha}{1-\alpha}\right)^r} \\
&\iff \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2+r} \leq \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2-r} \\
&\iff k_0 - k_1 \leq r,
\end{aligned}$$

$$\begin{aligned}
\min\{\pi_1^0, \pi_1^1\} = \pi_1^0 &\iff \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n} \frac{\beta}{1-\beta}} \leq \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n} \frac{1-\beta}{\beta}} \\
&\iff \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n} \left(\frac{\alpha}{1-\alpha}\right)^r} \leq \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n} \left(\frac{1-\alpha}{\alpha}\right)^r} \\
&\iff k_0 - k_1 \leq r.
\end{aligned}$$

Therefore, for the case  $k_0 - k_1 \leq r$ , the interval in (5.2) is non-empty if and only if

$$\begin{aligned}
\pi_0^1 \leq \pi_1^0 &\iff \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2} \left(\frac{1-\alpha}{\alpha}\right)^r} \leq \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n} \left(\frac{\alpha}{1-\alpha}\right)^r} \\
&\iff k_0 - k_1 \geq r - 1.
\end{aligned}$$

Similarly, for the case  $k_0 - k_1 > r$ , the interval in (5.2) is non-empty if and only if

$$\begin{aligned}
\pi_0^0 \leq \pi_1^1 &\iff \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2} \left(\frac{\alpha}{1-\alpha}\right)^r} \leq \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n} \left(\frac{1-\alpha}{\alpha}\right)^r} \\
&\iff k_0 - k_1 \leq r + 1.
\end{aligned}$$

We can now conclude that  $Q^{\alpha, k_0, k_1}(\beta)$  is non-empty if and only if  $r - 1 \leq k_0 - k_1 \leq r + 1$ .  $\square$



## Proof of Proposition 4

From the algorithm that we described in Section 5.2, we know that for a given preference profile  $\mathbf{q}$ , there exists a contingent  $k$ -voting rule that can sustain informative voting as an equilibrium if and only if there exists a pair of integers  $k_0, k_1 \in \{1, \dots, n\}$  that satisfies the inequalities  $(\pi_1^0)^{-1}(\bar{q}) \leq k_0 \leq (\pi_0^0)^{-1}(\underline{q})$  and  $(\pi_1^1)^{-1}(\bar{q}) \leq k_1 \leq (\pi_0^1)^{-1}(\underline{q})$ . These four inverse function are given as follows:

$$\begin{aligned} (\pi_0^0)^{-1}(\underline{q}) &= \frac{1}{2} \left( \frac{\ln\left(\frac{1-\underline{q}}{\underline{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 + r \right), & (\pi_1^0)^{-1}(\bar{q}) &= \frac{1}{2} \left( \frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + r \right), \\ (\pi_0^1)^{-1}(\underline{q}) &= \frac{1}{2} \left( \frac{\ln\left(\frac{1-\underline{q}}{\underline{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 - r \right), & (\pi_1^1)^{-1}(\bar{q}) &= \frac{1}{2} \left( \frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n - r \right). \end{aligned}$$

As a result, we have

$$\begin{aligned} (\pi_0^0)^{-1}(\underline{q}) - (\pi_1^0)^{-1}(\bar{q}) &= \frac{\ln\left(\frac{1-\underline{q}}{\underline{q}}\right) - \ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{2 \ln\left(\frac{1-\alpha}{\alpha}\right)} + 1, \\ (\pi_0^1)^{-1}(\underline{q}) - (\pi_1^1)^{-1}(\bar{q}) &= \frac{\ln\left(\frac{1-\underline{q}}{\underline{q}}\right) - \ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{2 \ln\left(\frac{1-\alpha}{\alpha}\right)} + 1. \end{aligned}$$

If  $\underline{q} < \bar{q}$ , it is easy to check that both  $(\pi_0^0)^{-1}(\underline{q}) - (\pi_1^0)^{-1}(\bar{q})$  and  $(\pi_0^1)^{-1}(\underline{q}) - (\pi_1^1)^{-1}(\bar{q})$  are strictly less than one, which implies that both the interval  $[(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})]$  and the interval  $[(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})]$  can contain *at most* one integer. If  $\underline{q} = \bar{q} = q$ , then  $(\pi_0^0)^{-1}(\underline{q}) - (\pi_1^0)^{-1}(\bar{q}) = (\pi_0^0)^{-1}(\underline{q}) - (\pi_1^0)^{-1}(\underline{q}) = (\pi_0^0)^{-1}(\underline{q}) - (\pi_1^0)^{-1}(\underline{q}) = 1$ . Thus, in this case both the interval  $[(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})]$  and the interval  $[(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})]$  will contain *at least* one integer.<sup>5</sup> Note that these results are independent of the size of the committee. Hence, it remains to be shown that if  $\forall i \in \mathcal{I}, q_i = q$ , then for each of these two intervals, at least one of the integers that are contained in it must belong to the set  $\{1, \dots, n\}$  when  $n$  is sufficiently large, so that the

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<sup>5</sup>If (and only if)  $(\pi_1^0)^{-1}(\bar{q})$  is an integer, the interval  $[(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})]$  will contain exactly two integers. Similarly, there will be two integers in the interval  $[(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})]$  if and only if  $(\pi_1^1)^{-1}(\bar{q})$  is an integer.

corresponding contingent  $k$ -voting rule is well-defined. For this, it suffices to have

$$\begin{aligned} (\pi_1^1)^{-1}(q) \geq 0 &\iff \frac{1}{2} \left( \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n - r \right) \geq 0 \\ &\iff n \geq r - \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} \end{aligned}$$

and

$$\begin{aligned} (\pi_0^0)^{-1}(q) \leq n + 1 &\iff \frac{1}{2} \left( \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 + r \right) \leq n + 1 \\ &\iff n \geq r + \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}, \end{aligned}$$

since  $(\pi_1^1)^{-1}(q) \leq (\pi_1^0)^{-1}(q)$  and  $(\pi_0^0)^{-1}(q) \leq (\pi_0^1)^{-1}(q)$  for all  $q \in [0, 1]$  and  $r \geq 0$ . Let

$$\bar{n}(q) = \left[ \max \left\{ r - \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}, r + \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} \right\} \right]^+,$$

where  $[x]^+$  denotes the smallest integer that is larger or equal to the real number  $x$ . We can now conclude that when agents' preference are perfectly aligned, there exists a threshold value  $\bar{n}(q)$ , such that for each  $n \geq \bar{n}(q)$ , there exists a contingent  $k$ -voting rule under which the informative voting equilibrium exists.  $\square$

## Proof of Proposition 5

Pick any  $\mathbf{q}^s$  from the sequence and let  $\bar{q}^s = \max_{j \in \{1, \dots, n+s\}} \hat{q}_j$  and  $\underline{q}^s = \min_{j \in \{1, \dots, n+s\}} \hat{q}_j$ . From the algorithm that we described in Section 5.2, we know that for such a preference profile, there exists a contingent  $k$ -voting rule that can sustain informative voting as an equilibrium if and only if there exists a pair of integers  $k_0^s, k_1^s \in \{1, \dots, n + s\}$  that satisfies the

inequalities  $(\pi_1^0)^{-1}(\bar{q}^s) \leq k_0^s \leq (\pi_0^0)^{-1}(\underline{q}^s)$  and  $(\pi_1^1)^{-1}(\bar{q}^s) \leq k_1^s \leq (\pi_0^1)^{-1}(\underline{q}^s)$ , where

$$\begin{aligned} (\pi_0^0)^{-1}(\underline{q}^s) &= \frac{1}{2} \left( \frac{\ln\left(\frac{1-q^s}{\underline{q}^s}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + s + 2 + r \right), & (\pi_1^0)^{-1}(\bar{q}^s) &= \frac{1}{2} \left( \frac{\ln\left(\frac{1-\bar{q}^s}{\bar{q}^s}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + s + r \right), \\ (\pi_0^1)^{-1}(\underline{q}^s) &= \frac{1}{2} \left( \frac{\ln\left(\frac{1-q^s}{\underline{q}^s}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + s + 2 - r \right), & (\pi_1^1)^{-1}(\bar{q}^s) &= \frac{1}{2} \left( \frac{\ln\left(\frac{1-\bar{q}^s}{\bar{q}^s}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + s - r \right). \end{aligned}$$

Because  $\ln\left(\frac{1-q}{q}\right)/\ln\left(\frac{1-\alpha}{\alpha}\right)$  is increasing in  $q$  and  $\bar{q} \geq \bar{q}^s$  and  $\underline{q} \leq \underline{q}^s$ , comparing the above four functions to the four functions  $(\pi_1^0)^{-1}(\bar{q})$ ,  $(\pi_0^0)^{-1}(\underline{q})$ ,  $(\pi_1^1)^{-1}(\bar{q})$  and  $(\pi_0^1)^{-1}(\underline{q})$  in the proof of Proposition 4, it is clear that if there is a pair of integers  $k_0, k_1 \in \{1, \dots, n\}$  that are respectively contained in the intervals  $[(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})]$  and  $[(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})]$ , i.e., there exists a contingent  $k$ -voting rule that can sustain informative voting as an equilibrium with the preference profile  $\mathbf{q}$ , there will be also a pair of integers  $k_0^s, k_1^s \in \{1, \dots, n+s\}$  that are respectively contained in the intervals  $[(\pi_1^0)^{-1}(\bar{q}^s), (\pi_0^0)^{-1}(\underline{q}^s)]$  and  $[(\pi_1^1)^{-1}(\bar{q}^s), (\pi_0^1)^{-1}(\underline{q}^s)]$ .

For asymptotic efficiency, note that  $\forall q \in (0, 1)$ , all  $(\pi_1^0)^{-1}(q)/(n+s)$ ,  $(\pi_0^0)^{-1}(q)/(n+s)$ ,  $(\pi_1^1)^{-1}(q)/(n+s)$  and  $(\pi_0^1)^{-1}(q)/(n+s)$  converge to  $1/2$  as  $s \rightarrow \infty$ . Hence, after adding sufficiently many members to the committee, the probability that the collective decision made in the informative voting equilibria under the corresponding contingent  $k$ -voting rules coincide with that in the informative voting equilibrium under the standard majority rule becomes arbitrarily close to 1. Since the informative voting equilibrium under the standard majority rule is asymptotically efficient if the agents' private signals are informative ( $\alpha > 1/2$ ), so are the informative voting equilibria under the contingent  $k$ -voting rules.  $\square$

## Proof of Corollary 4

Plugging  $k_0 = (n+1)/2 + [(r-1)/2]^+$  in the formulas of  $\pi_0^0$  and  $\pi_1^0$ , one can easily verify that for all  $r \geq 0$ ,

$$\max\{\pi_0^0, \pi_1^0\} = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{|r-2[(r-1)/2]^+|-1}}.$$

Similarly, with  $k_1 = (n+1)/2 - [(r-1)/2]^+$ , we have

$$\min\{\pi_1^0, \pi_1^1\} = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{-|r-2[(r-1)/2]^+|+1}}$$

for all  $r \geq 0$ . The result of the corollary thus immediately follows Proposition 3.  $\square$

## Proof of Proposition 6

Consider a social planner who observes the whole vector of private signals  $\mathbf{s} = (s_1, \dots, s_n)$  and the public signal  $s_p$ . Suppose the public signal is  $r$ -times more informative than the private signal, where  $r \geq 0$ . Recall  $m_{\mathbf{s}} = \sum_{i=1}^n s_i$ . To maximize the probability that his decision will be matched to the state, the social planner would choose the following optimal decision rule:

$$d^* = \begin{cases} 1 & \text{if } m_{\mathbf{s}} - (n - m_{\mathbf{s}}) + r \mathbb{1}_{s_p=1} - r \mathbb{1}_{s_p=0} > 0, \\ \{0, 1\} & \text{if } m_{\mathbf{s}} - (n - m_{\mathbf{s}}) + r \mathbb{1}_{s_p=1} - r \mathbb{1}_{s_p=0} = 0, \\ 0 & \text{if } m_{\mathbf{s}} - (n - m_{\mathbf{s}}) + r \mathbb{1}_{s_p=1} - r \mathbb{1}_{s_p=0} < 0. \end{cases}$$

Under the contingent majority rule,  $k(s_p) = \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+$  if  $s_p = 1$  and  $k(s_p) = \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+$  if  $s_p = 0$ . Hence, when  $s_p = 1$ , in the informative voting equilibrium,  $d = 1$  if and only if

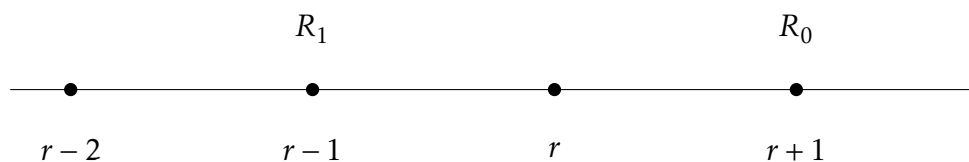
$$m_{\mathbf{s}} \geq \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+ \iff (n - m_{\mathbf{s}}) - m_{\mathbf{s}} \leq 2 \left[\frac{r-1}{2}\right]^+ - 1 =: R_1,$$

while when  $s_p = 0$ ,  $d = 1$  if and only if

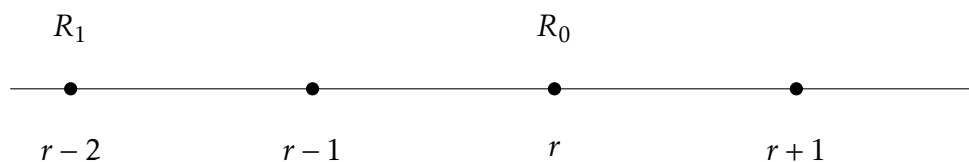
$$m_{\mathbf{s}} \geq \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+ \iff m_{\mathbf{s}} - (n - m_{\mathbf{s}}) \geq 2 \left[\frac{r-1}{2}\right]^+ + 1 =: R_0.$$

There are four possible scenarios for the relative locations of  $r, [r]^+, R_1$  and  $R_0$  on the real line:

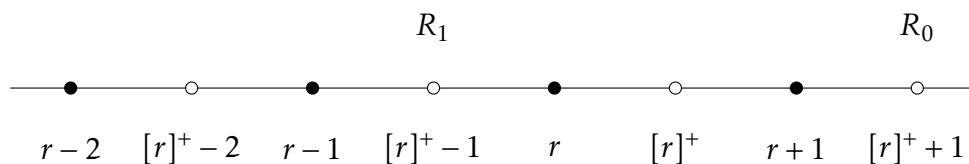
1.  $r$  is an even integer,



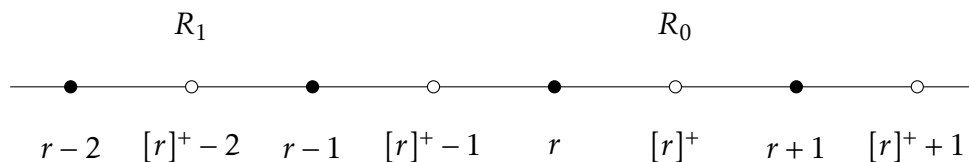
2.  $r$  is an odd integer,



3.  $r$  is not an integer and  $[r]^+$  is even,



4.  $r$  is not an integer and  $[r]^+$  is odd,



Since  $|m_s - (n - m_s)|$  is odd, the above four figures jointly show that the collective decision achieved by the contingent majority rule always coincides with the social planner's choice. □

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