Berentsen, Aleksander; Huber, Samuel; Marchesiani, Alessandro

Working Paper

The societal benefits of a financial transaction tax

Working Paper, No. 176

Provided in Cooperation with:

Department of Economics, University of Zurich

Suggested Citation: Berentsen, Aleksander; Huber, Samuel; Marchesiani, Alessandro (2014) : The societal benefits of a financial transaction tax, Working Paper, No. 176, University of Zurich, Department of Economics, Zurich

This Version is available at:
http://hdl.handle.net/10419/111228

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
The Societal Benefits of a Financial Transaction Tax

Aleksander Berentsen, Samuel Huber and Alessandro Marchesiani

October 2014
The Societal Benefits of a Financial Transaction Tax∗

Aleksander Berentsen
University of Basel and Federal Reserve Bank of St. Louis

Samuel Huber Alessandro Marchesiani
University of Basel University of Bath

October 1, 2014

Abstract

We investigate the positive and normative implications of a tax on financial market transactions in a dynamic general equilibrium model, where agents face idiosyncratic liquidity shocks and financial trading is essential. Our main finding is that agents’ portfolio choices display a pecuniary externality which results in too much trading. We calibrate the model to U.S. data and find an optimal tax rate of 2.5 percent. Imposing this tax reduces trading in financial markets by 30 percent.

1 Introduction

“The introduction of a substantial Government transfer tax on all transactions might prove the most serviceable reform available, with a view to mitigating the predominance of speculation over enterprise in the United States.” - Keynes (1936, p.160).

A financial transaction tax (FTT) is a proportional tax on financial transactions. One of the early advocates was Tobin (1978) who proposed it in order to add some frictions into the “excessively efficient international money markets” (p. 154). Although Tobin’s proposal was a proportional tax on currency transactions, the term “Tobin tax” is commonly used today for a proportional tax levied on any financial asset transaction.1 In the aftermath of the recent international financial crisis, policy makers have expressed renewed interest in FTTs. For example,

∗The views expressed in this article are those of the authors and not necessarily those of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the FOMC. Any remaining errors are the authors’ responsibility.

1 John Maynard Keynes (1936) considered FTTs as remedies for “excessive” volatility in financial markets. Recent supporters have advocated it to discourage short-term speculative trading which they consider to cause excessive volatility in financial markets (see, e.g., Stiglitz, 1989, Summers and Summers, 1989, and Roll, 1989). Opponents argue that there is no evidence that a FTT reduces volatility and that it can even increase volatility (see, e.g., Schwert and Seguin, 1993, Umlauf, 1993, and Jones and Seguin, 1997).
the Fed is currently considering to introduce an exit fee on claims to bonds and the European Commission is proposing to introduce a FTT in January 2016.

The existing theoretical literature on FTTs focuses mainly on historical episodes or provides basic intuition in favor or against such a tax. Although this literature discusses many dynamic issues such as price volatility and liquidity in financial markets, the analysis is most often static. Furthermore, none of these papers studies the underlying frictions that give rise to the need for financial transactions in the first place, and the reader is left puzzled about what distortion a FTT is intended to correct. Finally, very few studies offer a rigorous analysis on the welfare implications of FTTs.

Building on recent advances in monetary theory, we can now address these shortcomings by building a choice-theoretic dynamic general equilibrium model with frictions that make financial trading essential.\(^2\) The model allows us to address important positive and normative questions regarding the impact of a FTT on the real economy: For example, under which conditions is a FTT desirable and what distortion is corrected by such a tax? More generally, what is the optimal FTT and how does it affect trading volumes in financial markets?

In our model, agents face idiosyncratic random consumption and production opportunities and they hold a portfolio of liquid and illiquid assets to smooth consumption. The liquid asset can be directly traded for consumption goods if a consumption opportunity arises; i.e., it serves as a medium of exchange. In contrast, the illiquid asset cannot be used as a medium of exchange.\(^3\) From the agents’ point of view, the random consumption and production opportunities are liquidity shocks. These shocks generate an ex-post inefficient allocation of the medium of exchange: Some agents will hold liquid assets, but have no current need for them, while other agents will hold insufficient liquidity for their liquidity needs. To mitigate this liquidity mismatch, a financial market opens that allows the exchange of illiquid assets for liquid assets. The financial market is an over-the-counter (OTC) market, where agents are matched in pairs and the terms of trades are bargained.

Our main finding is that the portfolio choice of liquid and illiquid assets displays a pecuniary externality. The reason for the pecuniary externality is that an agent does not take into account that, by holding more liquid assets, he not only acquires additional insurance against his own idiosyncratic liquidity risks, but he also marginally increases the value of the liquid asset, which improves the insurance for other market participants, too.\(^4\) This pecuniary externality can be corrected by a FTT. By making it more costly to readjust a portfolio in response to liquidity shocks, agents attempt to hold more of the liquid asset ex-ante. The resulting increase in the demand for liquid assets drives up the value of these assets, and this effect can be so strong that it is welfare-increasing.

---

\(^2\) These frictions include a lack of record-keeping (public communication of individual trading histories) and a lack of commitment. By essential, we mean that financial trading improves the allocation.

\(^3\) This arrangement is efficient. See Kocherlakota (2003), Andolfatto (2011), and Berentsen and Waller (2011) for a detailed discussion on the societal benefits of illiquid bonds.

\(^4\) Another way to look at this externality is that liquidity shares characteristics of a public good. Under this view, liquidity is a public good, holding liquidity is costly, and market participants attempt to free-ride on the liquidity holdings of other market participants. As a result, there is an underprovision of liquidity without government intervention.
It is important to note that our rationale for imposing a FTT has nothing to do with any of the prior reasons put forward to justify a FTT. In particular, our argument does not rely on mitigating the predominance of speculation over enterprise (Keynes, 1936) or excess volatility (Tobin, 1978). Our pecuniary externality arises in the steady state equilibrium and so we provide a novel justification for imposing a FTT. We also show that the welfare-enhancing effect of a FTT depends on the OTC nature of the financial market.

Finally, we calibrate the model to U.S. data. For the calibration, we assume that the FTT is zero. We then perform the following experiment: We search numerically for the tax rate that maximizes welfare. We find that the optimal tax rate is 2.5 percent and that the average U.S. citizen would be willing to pay 0.006 percent of steady state consumption in order to be taxed the optimal tax rate of 2.5 percent. The optimal tax rate reduces the real volume of financial trading by 30 percent. As a robustness check, we also calibrate the model to Germany and find that the optimal tax rate is 2.3 percent and that the average German citizen would be willing to sacrifice 0.046 percent of steady state consumption in order to be taxed the optimal rate.

2 Literature

Our paper is related to Berentsen et al. (2014a) which shows that restricting access to financial markets can be welfare improving. In this paper, we find that a FTT can improve the allocation. As in Berentsen et al. (2014a), the reason is the presence of a pecuniary externality that arises when agents choose their portfolio of liquid and illiquid assets. There are some important differences, however. First, Berentsen et al. (2014a) impose a competitive financial market, while in our paper we consider an OTC market. Furthermore, we show that the search and matching frictions modelled through an OTC market are crucial for the welfare benefits of a FTT. Second, the mechanism at work is different. In Berentsen et al. (2014a), restricting access to the competitive financial market increases consumption variability across agents. In contrast, in this paper a FTT reduces consumption variability. Third, it might not be feasible to restrict access to financial markets, while imposing a FTT is straightforward. In particular, in Berentsen et al. (2014a) access to the financial market is determined randomly, and it is not clear who flips the coin.

Geromichalos and Herrenbrueck (2014) develop a model where agents can trade assets of different liquidity in an OTC market. They find that trading in a frictional asset market, as opposed to trading in a competitive asset market, can be welfare-improving. The policy implication is that removing frictions, say by moving a frictional asset market (OTC-market) to a centrally organized exchange, as often discussed in the aftermath of the financial crisis, can be welfare-decreasing. Andolfatto et al. (2014) investigate whether the practice of reusing an asset that has already been pledged as collateral for a further cash loan can be welfare-improving (rehypothecation). They find that rehypothecation can improve welfare, but there can be too much of it and so restricting rehypothecation can be welfare-improving. The results

---

5See also Geromichalos et al. (2014) for an extension of Geromichalos and Herrenbrueck (2014) to illiquid bonds of different maturities.
of Geromichalos and Herrenbrueck (2014) and Andolfatto et al. (2014) are related to the result in Berentsen et al. (2014a) that restricting trading in financial markets can be welfare improving. None of these papers, however, study under what conditions a FTT can be welfare-improving.

Our framework extends Lagos and Wright (2005) and is, therefore, part of the rapidly expanding literature labeled the “new monetarist economics.” The first paper in this literature that incorporated idiosyncratic liquidity shocks and trading in financial markets is Berentsen et al. (2007). Many more recent papers also add liquidity shocks and financial trading. However, none of these papers studies FTTs and its implications on welfare, which is the main goal of our paper.

Our paper is also related to the macroeconomic literature on overborrowing. In this literature, agents do not take into account how their borrowing decisions affect collateral prices (a pecuniary externality), and through them the borrowing constraints of other agents. As a consequence, the equilibrium is characterized by overborrowing, which leads to credit booms and busts. Our paper differs from this literature, because it is not a model of crisis. The pecuniary externality is present in the unique steady-state equilibrium. Furthermore, the pecuniary externality emerges from the portfolio choices and not from borrowing decisions.

There are also many attempts to assess the effects of FTTs empirically. The main issue in this literature is whether a FTT increases or reduces volatility in financial markets. The results from this literature are ambiguous. There are a few theoretical papers that have studied the effects of a FTT. To our knowledge, only three papers investigated the implications of a FTT on welfare: Subrahmanyam (1998), Dow and Rahi (2000) and Dávila (2013). Subrahmanyam (1998) develops a two-period rational expectations model with noisy observations using the Kyle (1985) framework. Subrahmanyam shows that a transaction tax on financial transactions reduces an agent’s incentive to acquire information before others do so, and eventually increases welfare. The main argument proposed by Subrahmanyam is that agents spend too much effort on information acquisition. This effort constitutes a deadweight loss, since the advantage for society of having the information incorporated in the prices hours in advance is likely to be very small. Hence, a policy that induces agents to scale down their trades, such as a transaction tax, may end up being socially beneficial. Dow and Rahi (2000) study the welfare effects of a transaction tax in a model with informed and uninformed agents. They show that a tax on transactions made by informed agents can be beneficial both for them and for the uninformed agents as well. They also show that these results apply when the tax is levied on all transactions, instead of on transactions made by uninformed agents only. Like Subrahmanyam (1998), Dow and Rahi (2000) build on

---

6 For a discussion of this literature, see Williamson and Wright (2010a and 2010b).

7 There is a rapidly growing literature that studies liquidity shocks and financial intermediation in the Lagos-Wright (2005) framework. A non-exclusive sample of these papers are Berentsen and Monnet (2008), Andolfatto et al. (2014), Geromichalos and Herrenbrueck (2014), Geromichalos et al. (2014), Li (2011), Li and Li (2013), Chiu et al. (2011 and 2012), Chiu and Monnet (2014), and Williamson (2012). For a mechanism design approach to financial intermediation, see Gu et al. (2013a).

8 See, for example, Caballero and Krishnamurthy (2003), Lorenzoni (2008), Bianchi and Mendoza (2011), Jeanne and Korinek (2012), and Korinek (2012). For a more detailed discussion of this literature, see Berentsen et al. (2014a).

9 See Pomeranets (2012) for a detailed discussion of these studies.
Kyle (1985). However, a key difference between Subrahmanyam (1998) and Dow and Rahi (2000) is that liquidity trades are exogenous in the former, while agents can choose the optimal trade in the latter. Dávila (2013) studies FTTs in a model with belief disagreement. He shows that, when heterogeneous beliefs induce investors to trade too much, it is always optimal to levy a FTT. The tax reduces both the amount of fundamental and non-fundamental trading. Since the fundamental trading is based on optimality decisions, while the non-fundamental trading is not, a small tax will generate a second-order welfare loss, due to the reduction of the fundamental trading, and a first-order welfare gain, due to the reduction of non-fundamental trading.

3 Environment

Consider the environment analyzed in Berentsen et al. (2014a). Time is discrete and, in each period, there are three markets that open sequentially. The first market is a secondary bond market where agents trade money for nominal bonds. The second market is a goods market, where agents produce or consume the market-2 goods. The third market is a centralized market, where all agents consume and produce the market-3 goods, and financial contracts are redeemed. We call this market the primary bond market. All goods are perfectly divisible and nonstorable.

The economy is populated by a \([0, 1]\)-continuum of infinitely lived agents. At the beginning of each period, each agent receives an idiosyncratic i.i.d. preference/technology shock that determines whether he is a producer or a consumer in the goods market. With probability \(n \in (0, 1)\), he can produce but not consume, and with probability \(1 - n\), he can consume but not produce. This shock is introduced in order to obtain a liquidity mismatch and hence a role for asset trading in the secondary bond market.\(^{10}\)

In the goods market, consumers and producers are matched according to a reduced-form matching function \(M_2(n, 1 - n)\). We assume that the matching function has constant returns to scale, and is continuous and increasing with respect to each of its arguments. A consumer meets a producer with probability \(\delta_2 = M_2(n, 1 - n)(1 - n)^{-1}\), and a producer meets a consumer with probability \(\delta_2^p = M_2(n, 1 - n)n^{-1}\). Once they meet, the consumer and producer bargain over the quantity of money and goods to be exchanged.

In the goods market, a consumer enjoys utility \(u(q)\) from \(q\) consumption, where \(u(q)\) has the standard properties; i.e., \(u'(q), -u''(q) > 0, u'(0) = \infty,\) and \(u'(\infty) = 0\). Producers incur a utility cost \(c(q)\) from \(q\) production in the goods market. For ease of exposition, we assume linearity of the cost function; i.e., \(c(q) = q\). In the primary bond market, all agents can produce and consume using a linear production technology. In particular, agents can use \(h\) units of time to produce \(h\) units of market-3 goods. The utility of consuming \(x\) units of goods is \(U(x)\) where \(U'(x), -U''(x) > 0, U'(0) = \infty,\) and \(U'(\infty) = 0\). Agents discount between, but not within, periods. The discount factor between two consecutive periods is \(\beta = 1/(1 + r)\), where \(r > 0\) represents the real interest rate.\(^{11}\)

\(^{10}\) This liquidity shock was introduced in Berentsen et al. (2007). In their model, financial intermediation emerges endogenously to mitigate the liquidity mismatch generated by these shocks.

\(^{11}\) It is routine to show that the first-best quantities satisfy \(U'(x^*) = 1\) and \(u'(q^*) = 1\).
There are two perfectly divisible, storable objects: money and nominal bonds. Both money and bonds are intrinsically useless, and are issued by the central bank in the primary bond market. Bonds are issued at a discount, and one bond pays one unit of money in the next-period primary bond market. The central bank has a record-keeping technology over financial transactions. Bonds are intangible objects, and the central bank operates the primary and secondary bond markets and keeps track of ownership. Trading in the goods market requires a medium of exchange. The frictions that make the use of a medium of exchange in the goods market necessary are limited commitment and a lack of record-keeping. In our model, only money can serve as a medium of exchange. The reason is that bonds are intangible objects, and so they are incapable of being used as a medium of exchange in the goods market, hence they are illiquid. Therefore, money is the only means of payment in the goods market.

At the beginning of each period, and after the realization of the idiosyncratic shocks, agents can trade bonds and money in the secondary bond market. In this market agents meet according to a matching function $M_1(n, 1 - n)$, where $M_1$ has the same properties as $M_2$. The probability of a meeting for a consumer and a producer in the secondary bond market are $\delta_1 = M_1(n, 1 - n)(1 - n)^{-1}$ and $\delta_1^p = M_1(n, 1 - n)n^{-1}$, respectively. Once in a meeting, the consumer and producer bargain over the quantity of money and bonds to be exchanged. We refer to agents who are matched in this market as "active", and to those who are not as "passive".

Let $M_t$ be the per-capita stock of money and $B_t$ the per-capita stock of newly issued bonds at the end of period $t$. Let $\rho_t$ denote the price of bonds in the primary bond market. Then, the law of motion of money in period $t$ is given by

$$M_t - M_{t-1} = \tau_t M_{t-1} + B_{t-1} - \rho_t B_t - T_b.$$  \hspace{1cm} (1)

The change in the stock of money at time $t$, $M_t - M_{t-1}$, is affected by four components: the lump-sum money injections, $\tau_t M_{t-1}$, the money created to redeem previously issued bonds, $B_{t-1}$, the money withdrawn from selling newly issued bonds, $\rho_t B_t$, and the revenues from the financial transaction tax in the secondary bond market, $T_b$. We assume there is a strictly positive initial stock of money and bonds; i.e., $M_0, B_0 > 0$.

4 Agent’s Decisions

For notational simplicity, the time subscript $t$ is omitted from now on. Next-period variables are indexed by $+1$, and previous-period variables are indexed by $-1$. In what follows, we study the agents’ decisions beginning in the last market (the primary bond market) and then move backwards within a period to the goods market and finally to the secondary bond market.

---

12 The essential role of a medium of exchange has been studied, for example, by Kocherlakota (1998) and Wallace (2001). Sanches and Williamson (2010) show that an economy with no memory and monetary exchanges may achieve the same equilibrium allocation as an economy with perfect memory and private credit. Limited commitment is important for this result. In a similar fashion, Gu et al. (2013a and 2013b) study issues related to banking and credit.

13 An alternative arrangement that would render bonds illiquid is if they can be counterfeited at no cost (Li et al. 2012).
4.1 Primary bond market

In the primary bond market, bonds are redeemed and agents choose a portfolio of money and newly issued bonds by producing and consuming market-3 goods. An agent entering the primary bond market with \( m \) units of money and \( b \) units of bonds has the indirect utility function \( V_3(m, b) \). His decision problem is

\[
V_3(m, b) = \arg \max_{x,h,m+1,b+1} [U(x) - h + \beta V_1(m+1, b+1)],
\]

subject to

\[
x + \phi m_{+1} + \rho b_{+1} = h + \phi m + \phi b + \phi T,
\]

where \( T = \tau_i M_{t-1} \) is a lump-sum money transfer \((T > 0)\) or a lump-sum tax \((T < 0)\). The first-order conditions with respect to \( m_{+1} \), \( b_{+1} \) and \( x \) are

\[
\beta \frac{\partial V_1}{\partial m_{+1}} = \rho^{-1} \beta \frac{\partial V_1}{\partial b_{+1}} = \phi, \tag{4}
\]

and \( U'(x) = 1 \), respectively. The term \( \beta \frac{\partial V_1}{\partial m_{+1}} \) \((\beta \frac{\partial V_1}{\partial b_{+1}})\) is the marginal benefit of taking one additional unit of money (bonds) into the next period, while \( \phi \) \((\rho \phi)\) is the marginal cost. Due to the quasi-linearity of preferences, the choices of \( b_{+1} \) and \( m_{+1} \) are independent of \( b \) and \( m \). It is straightforward to show that all agents exit the primary bond market with the same portfolio of bonds and money. The envelope conditions in the primary bond market are

\[
\frac{\partial V_3}{\partial m} = \frac{\partial V_3}{\partial b} = \phi. \tag{5}
\]

According to (5), the marginal value of money and bonds at the beginning of the primary bond market is equal to the price of money in terms of market-3 goods. Note that (5) implies that the value function \( V_3 \) is linear in \( m \) and \( b \).

4.2 Goods market

In a meeting in the goods market, the consumer and producer bargain over the terms of trade \((q, d)\), where \( q \) is the quantity of goods and \( d \) is the amount of money exchanged in the match. We assume that the terms of trade are determined according to the proportional bargaining solution (Kalai, 1977).\(^\text{14}\) The net payoff of a producer in a meeting in the goods market is \(-c(q) + V_3(m+d, b) - V_3(m, b)\), while the net payoff of a consumer is given by \( u(q) + V_3(m-d, b) - V_3(m, b) \). Using the linearity of \( V_3 \) with respect to \( m \) and \( b \), the Kalai bargaining problem can be formulated as follows:

\[
\max_{q,d} u(q) - \phi d
\]

\(^\text{14}\)See Rocheteau and Wright (2005) for a formalization of Kalai bargaining, and alternative trading protocols in these models.
subject to

$$(1 - \theta) [u(q) - \phi d] = \theta [-c(q) + \phi d],$$

and $d \leq m,$

where $\theta$ is the consumer’s bargaining weight, and $m$ is the consumer’s money holding. The first constraint is the Kalai constraint, which requires that the producer receives the fraction $1 - \theta$ of the total surplus. The second constraint is the borrowing constraint. If the latter constraint is nonbinding, the Kalai bargaining solution is $d < m$ and $q = q^*$, where $q^*$ satisfies $u'(q^*) = 1$. If the constraint is binding, the solution is $d = m$ and consumption $q$ satisfies

$$\phi m = z(q) \equiv (1 - \theta) u(q) + \theta c(q).$$

The value function of a consumer entering the goods market with $m$ units of money and $b$ units of bonds is

$$V^*_C(m, b) = \delta_2 [u(q) + V_3(m - d, b)] + (1 - \delta_2) V_3(m, b).$$

With probability $\delta_2$, the consumer has a match, and the terms of trade are $(q, d)$. Under these terms, he receives consumption utility $u(q)$, and an expected continuation value $V_3(m - d, b)$. With probability $1 - \delta_2$, the consumer has no match and receives continuation value $V_3(m, b)$. To derive the marginal indirect utilities of money and bonds, take the total derivatives of (9) with respect to $m$ and $b$ and use (5) to replace $\frac{\partial V_3}{\partial m}$ and $\frac{\partial V_3}{\partial b}$ to obtain

$$\frac{\partial V^*_C}{\partial m} = \delta_2 \left[ u'(q) \frac{\partial q}{\partial m} + \phi \left( 1 - \frac{\partial d}{\partial m} \right) \right] + (1 - \delta_2) \phi \quad \text{and} \quad \frac{\partial V^*_C}{\partial b} = \phi.$$ 

(10)

If the constraint in (7) is nonbinding, then $\frac{\partial q}{\partial m} = 0$ and $\frac{\partial d}{\partial m} = 0$. In this case, (10) reduces to $\frac{\partial V^*_C}{\partial m} = \frac{\partial V^*_C}{\partial b} = \phi$. If the constraint is binding, then $\frac{\partial q}{\partial m} = \frac{\phi}{z'(q)}$ and $\frac{\partial d}{\partial m} = 1$. In this case, (10) can be rewritten as

$$\frac{\partial V^*_C}{\partial m} = \delta_2 \phi \frac{u'(q)}{z'(q)} + (1 - \delta_2) \phi \quad \text{and} \quad \frac{\partial V^*_C}{\partial b} = \phi.$$ 

(11)

The first equation simply states that the consumer’s marginal utility of money has two components: With probability $\delta_2$ he has a match and, by spending the marginal unit of money, he receives utility $\phi u'(q)z'(q)^{-1}$ from consumption; with probability $1 - \delta_2$ he has no match, in which case, by spending the marginal unit of money in the primary bond market, he receives utility $\phi$. The second equality states that a consumer’s marginal utility of bonds at the beginning of the goods market is equal to the price of money in the primary bond market, since bonds are illiquid in the goods market.

In the goods market, agents hold different portfolios of money and bonds. Consumers who had a match in the secondary bond market hold more money and fewer bonds than consumers who had no match. Along the same lines, producers hold different portfolios, but since their portfolio is irrelevant for the terms of trade, there are only two types of matches: active matches
and passive matches. A match is called active if the consumer has had a match in the secondary bond market, and a match is called passive if the consumer had no match. The quantities traded in active matches are denoted by the symbol “ˆ”; i.e., ˆq and ˆd.

Accordingly, the value function of a producer entering the goods market with m units of money and b units of bonds is

\[
V_p^2(m, b) = \delta_2^2 \delta_1 \left[ -c(\hat{q}) + V_3(m + \hat{d}, b) \right] + \delta_2^2 (1 - \delta_1) \left[ -c(q) + V_3(m + d, b) \right] + (1 - \delta_2^2) V_3(m, b).
\] (12)

The producer in the goods market faces three possible events: (i) with probability \(\delta_2^2 \delta_1\), he is matched with a consumer who traded in the secondary bond market, in which case the producer produces \(\hat{q}\) units of the market-2 good for \(\hat{d}\) units of money; with probability \(\delta_2^2 (1 - \delta_1)\), he is matched with a consumer who did not trade in the secondary bond market, in which case the producer produces \(q\) units of the market-2 good for \(d\) units of cash; finally, with probability \(1 - \delta_2^2\) he is matched with nobody in the goods market, in which case he does not produce.

To obtain the producer’s envelope conditions in the goods market, take the total derivative of (12) with respect to \(m\) and \(b\), respectively, and use (5) to replace \(\partial V_3^p / \partial m\) and \(\partial V_3^p / \partial b\), to obtain

\[
\frac{\partial V_2^p}{\partial m} = \frac{\partial V_2^p}{\partial b} = \phi.
\] (13)

A producer’s marginal utility of money and bonds at the beginning of the goods market are equal to the price of money in the primary bond market, because a producer has no use for these assets in the goods market.

4.3 Secondary bond market

In the secondary bond market, consumers and producers are matched pairwise, and the gains from trade are split according to the proportional bargaining solution developed by Kalai (1977). To derive the terms of trade, one can consider the case where the consumer chooses the terms of trade in order to maximize his payoff subject to the constraint that the producer receives a given fraction of the total surplus (see Aruoba et al., 2007). In particular, he chooses the quantities \((d_m, d_b)\), where \(d_m\) is the quantity of money received by the consumer for \(d_b\) units of bonds.

Transactions in the secondary bond market are subject to a proportional tax \(\tau\). If the producer accepts the offer, \(d_b\) units of bonds and \(d_m\) units of money change hands, and the consumer pays \(\tau d_m\) units of money to the government. We assume that the government operates the secondary bond market and as such can perfectly enforce tax payment. Participation in the secondary bond market is voluntary so that agents always have the option to avoid the tax by not trading. This contrasts with the scenario for a lump-sum tax where the assumption of perfect enforcement means that agents always have to pay it.\(^{15}\)

\(^{15}\)Andolfatto (2013) studies the case where lump-sum taxation must satisfy participation constraints, which limits the government’s ability to run the Friedman rule.
The consumer’s problem in a meeting in the secondary bond market is

$$\max_{d_m,d_b} \Delta^c$$

subject to

$$(1 - \eta) \Delta^c \leq \eta \Delta^p,$$

$$\phi b - \phi d_b \geq 0,$$

$$\phi m - \phi d_m \geq 0,$$

where $\eta \in [0,1]$ is the consumer’s bargaining weight in a meeting, and $\Delta^c$ and $\Delta^p$ are the consumer’s and producer’s net surplus. The first constraint in the consumer’s problem is the Kalai constraint. The second constraint means that a consumer cannot deliver more bonds than he has, and the third constraint means that a producer cannot deliver more money than he has.

In the Appendix, we derive expressions for $\Delta^c$ and $\Delta^p$. Using these expressions, the consumer’s problem in the secondary bond market can be rewritten as follows:\(^{16}\)

$$\max_{d_m,d_b} \delta_2 [u(\hat{q}) - u(q)] + \phi d_m (1 - \delta_2) (1 - \tau) - \phi d_b$$

subject to

$$\phi d_b \geq (1 - \eta) \delta_2 [u(\hat{q}) - u(q)] + [\eta + (1 - \eta) (1 - \delta_2) (1 - \tau)] \phi d_m,$$

$$\phi b \geq \phi d_b,$$

$$\phi m \geq \phi d_m.$$  

The first constraint is again the Kalai constraint. In any equilibrium, it has to hold with equality, and it is therefore convenient to solve it for $d_b$ to obtain

$$\phi d_b = (1 - \eta) \delta_2 [u(\hat{q}) - u(q)] + [\eta + (1 - \eta) (1 - \delta_2) (1 - \tau)] \phi d_m.$$  

(14)

Use (14) to eliminate $\phi d_b$ from the objective function and the second inequality, and rewrite the consumer’s problem as follows:

$$\max_{d_m} \eta \{\delta_2 [u(\hat{q}) - u(q)] - [1 - (1 - \delta_2) (1 - \tau)] \phi d_m\}$$

subject to

$$\phi b - (1 - \eta) \delta_2 [u(\hat{q}) - u(q)] - [\eta + (1 - \eta) (1 - \delta_2) (1 - \tau)] \phi d_m \geq 0,$$

$$\phi m - \phi d_m \geq 0.$$  

(16)  

(17)

\(^{16}\)The solution to this problem always satisfies the producer’s participation constraint, $\Delta^p \geq 0$. In contrast, the consumer’s participation constraint, $\Delta^c \geq 0$, may not be satisfied. This is, in particular, the case if the tax is high and/or inflation low, which reduces the benefits from having the secondary bond market. In this case, there is no trading and the market shuts down. In what follows, we assume that the tax (or the inflation) is such that there is trading, and later on we verify under which conditions $\Delta^c \geq 0$.  

10
Note that the expression in the curly bracket in the objective function is the total surplus of the match \(\Delta^p + \Delta^c\). Thus, the Kalai proportional solution maximizes the total surplus and is hence efficient.

Denote \(\lambda^c\) and \(\lambda^p\) the Lagrange multipliers for constraints (16) and (17), respectively. As we will demonstrate, the nature of the equilibrium will depend on whether these constraints are binding or not. The first-order condition in the secondary bond market is

\[
\eta \delta_2 u' (\hat{q}) \frac{\partial \hat{q}}{\partial d_m} - \eta [1 - (1 - \delta_2) (1 - \tau)] \phi - \phi \lambda^p - \lambda^c \left[ (1 - \eta) \delta_2 u' (\hat{q}) \frac{\partial \hat{q}}{\partial d_m} + [\eta + (1 - \eta) (1 - \delta_2) (1 - \tau)] \phi \right] = 0.
\]

Finally, the value function in the primary bond market satisfies

\[
V_1 (m, b) = n V^p_1 (m, b) + (1 - n) V^c_1 (m, b),
\]

where

\[
V^c_1 (m, b) = \delta_1 V^c_2 (m + d_m (1 - \tau), b - db) + (1 - \delta_1) V^c_2 (m, b)
\]

\[
V^p_1 (m, b) = \delta^p_1 V^p_2 (m - d_m, b + db) + (1 - \delta^p_1) V^p_2 (m, b).
\]

### 5 Monetary Equilibrium

We focus on symmetric, stationary monetary equilibria, where all agents follow identical strategies and where real variables are constant over time. Let \(\zeta \equiv B/B_{-1}\) denote the gross growth rate of bonds, and let \(\gamma \equiv M/M_{-1}\) denote the gross growth rate of the money supply. In a stationary monetary equilibrium, the real stock of money must be constant; i.e., \(\phi M = \phi_{+1} M_{+1}\), implying that \(\gamma = \phi/\phi_{+1}\). Furthermore, the real amount of bonds must be constant; i.e., \(\phi B = \phi_{-1} B_{-1}\), implying that \(\zeta = \phi/\phi_{+1}\). To simplify notation, let\(^{17}\)

\[
\Phi (q) \equiv \delta_2 u'(q) z'(q) + 1 - \delta_2. \tag{22}
\]

In what follows, we focus on two cases. In the first case, labeled type-I equilibrium, the constraints (16) and (17) do not bind (i.e., \(\lambda^c = \lambda^p = 0\)). In the second case, labeled type-II equilibrium, the producer’s cash constraint binds and the consumer’s bond constraint does not bind (i.e., \(\lambda^p > 0\) and \(\lambda^c = 0\)). Further below, we calibrate the model to U.S. data and find that these are the relevant cases.\(^{18}\)

All equilibria involve the derivation of the marginal values of money and bonds from equation (19). Furthermore, the Kalai equation (14) and the first-order condition in the secondary bond market (18) play a key role. This last equation can be written as follows:

\[
\eta [(1 - \tau) \Phi (\hat{q}) - 1] = \lambda^p + \lambda^c [(1 - \eta) (1 - \tau) \Phi (\hat{q}) + \eta], \tag{23}
\]

\(^{17}\)With Kalai bargaining \(\Phi (q)\) is decreasing in \(q\).

\(^{18}\)The other possibilities (i.e., \(\lambda^c > 0\) and \(\lambda^p = 0\), and \(\lambda^c, \lambda^p > 0\)) are analyzed in a supplementary Appendix that is available on request.
where we have used the budget constraint in the goods market; i.e., $\phi [m + d_m (1 - \tau)] = z (\hat{q})$, to replace $\partial \hat{q}/\partial d_m = \phi (1 - \tau) / z' (\hat{q})$.

### 5.1 Type-I equilibrium

In a type-I equilibrium, an active consumer’s bond constraint does not bind, and an active producer’s cash constraint does not bind. A type-I equilibrium can be characterized by the three equations stated in Proposition 1. All proofs are in the Appendix.

**Proposition 1** A type-I equilibrium is a time invariant path $\{\hat{q}, q, \rho\}$ satisfying

\[
\begin{align*}
1 & = \Phi (\hat{q}) (1 - \tau) \\
\frac{\gamma}{\beta} & = (1 - n) \{\delta_1 \{\Phi (q) + \eta [\Phi (\hat{q}) - \Phi (q)]\} + (1 - \delta_1) \Phi (q)\} + n \\
\frac{\rho \gamma}{\beta} & = 1.
\end{align*}
\]

Equation (24) is derived from the first-order condition (23). The meaning of this equation is that the cost of acquiring one additional unit of money in a meeting in the secondary bond market has to be equal to its benefit. Equation (25) is derived from the marginal value of money in the secondary bond market. The right-hand side of (25) is the marginal benefit of money at the beginning of the period. With probability $(1 - n) \delta_1$, the agent is a consumer who has a match in the secondary bond market. In this case, the marginal unit of money generates $\Phi (q) + \eta [\Phi (\hat{q}) - \Phi (q)]$ utility; i.e., the consumer receives the outside option $\Phi (q)$ plus the fraction $\eta$ of the surplus $[\Phi (\hat{q}) - \Phi (q)]$ that the asset trading generates. With probability $(1 - n) (1 - \delta_1)$, the consumer has no match and so the marginal utility is $\Phi (q)$. With probability $n$ the agent is a producer and the marginal utility is 1. The left-hand side of (25) represents the marginal cost of acquiring one additional unit of money in the primary bond market. Equation (26) is derived from the marginal value of bonds in the secondary bond market. Along the same lines as for (25), equation (26) reflects the fact that the benefit of taking one additional unit of bonds into the secondary bond market must be equal to the marginal cost of acquiring it in the primary bond market.

### 5.2 Type-II equilibrium

The following Proposition 2 characterizes the type-II equilibrium.

**Proposition 2** A type-II equilibrium is a time invariant path $\{\hat{q}, q, \rho\}$ satisfying

\[
\begin{align*}
z (\hat{q}) & = (2 - \tau) z (q) \\
\frac{\gamma}{\beta} & = (1 - n) \{\delta_1 \{\Phi (q) + \eta [\Phi (\hat{q}) - \Phi (q)]\} + (1 - \delta_1) \Phi (q)\} + n \{\delta_1 [(1 - \eta) (1 - \tau) \Phi (\hat{q}) + \eta] + (1 - \delta_1^p)\} \\
\frac{\gamma \rho}{\beta} & = 1.
\end{align*}
\]
To derive equation (27), we compare the budget constraint of an active consumer with the budget constraint of a passive consumer. Furthermore, we use the fact that a producer transfers all his money to the active consumer. The interpretations of (28) and (29) are similar to their counterparts in Proposition 1. It is interesting to compare (28) to its counterpart (25). They are equal except for the marginal value of money for the producer. In (25), the producer’s marginal value of money is 1, while in (28) it is \( \delta_p^* [(1 - \eta) (1 - \tau) \Phi(\hat{q}) + \eta] + (1 - \delta_p^*) > 1 \). The reason is that, in the type-II equilibrium, the producer’s money constraint is binding (\( \lambda_p^* > 0 \)), and so he can earn a strictly positive interest rate on his money holdings.

5.3 Regions of Existence

Proposition 3 characterizes two non-overlapping regions in which the two types of equilibria exist. Let \( \gamma_1 \) denote the value of \( \gamma \) such that \( \hat{q} = q \) holds in the type-I equilibrium. Furthermore, let \( \gamma_2 \) denote the value of \( \gamma \) such that equations (25) and (28) hold simultaneously. In the proof of Proposition 3, we show that such values exist and that they are unique. Furthermore, we show under which conditions \( \beta \leq \gamma_1 \leq \gamma_2 < \infty \).

**Proposition 3** If \( \gamma_1 \leq \gamma < \gamma_2 \), equilibrium prices and quantities are characterized by Proposition 1; and if \( \gamma_2 \leq \gamma \), they are characterized by Proposition 2.

In the type-I equilibrium (\( \gamma_1 \leq \gamma < \gamma_2 \)), consumers and producers are unconstrained in the secondary bond market (i.e., \( \lambda_c^* = \lambda_p^* = 0 \)). In the type-II equilibrium (\( \gamma_2 \leq \gamma \)), active consumers are unconstrained, but the constraint on money holdings of active producers binds (i.e., \( \lambda_c^* = 0 \), \( \lambda_p^* > 0 \)). Thus, in both types of equilibria active consumers do not sell all their bond holdings and thus the price of bonds in the primary bond market, \( \rho \), must equal the fundamental value of bonds, \( \beta/\gamma \).

![Figure 1: Consumed quantities for \( \tau = 0 \) and \( \tau > 0 \)](image)
Figure 1 shows the consumed quantities as a function of $\gamma$.\textsuperscript{19} The figure is drawn for $\theta = 1$. For $\tau = 0$ and $\gamma_1 = \beta \leq \gamma < \gamma_2$, the economy is in the type-I equilibrium, where active consumers get the first-best quantity; i.e., $\hat{q} = q^*$, while passive consumers get $q \leq q^*$. Note that $q = \hat{q} = q^*$ at $\gamma = \beta$ (see the graph on the left-hand side of Figure 1).\textsuperscript{20} For $\tau > 0$ and $\beta < \gamma_1 \leq \gamma < \gamma_2$, both consumption quantities are less than $q^*$ (see the graph on the right-hand side of Figure 1). For $\beta \leq \gamma < \gamma_1$, active agents are better off by not trading in the secondary bond market; i.e., $\Delta^c < 0$. In this case, the quantities are equal and correspond to the consumption quantities obtained in the standard Lagos and Wright (2005) framework (see the region labeled LW in the graph on the right-hand side of Figure 1).\textsuperscript{21} For increasing values of $\tau$, both critical inflation rates, $\gamma_1$ and $\gamma_2$, move to the right. Finally, for $\gamma_2 < \gamma$ both quantities are smaller than $q^*$ and decreasing in $\gamma$.

6 Optimal Tax

The main result of our paper is that imposing a transaction tax in the secondary bond market can be welfare increasing. In this section, we show under which conditions this is the case, and we provide intuition for the result. Let $W$ be the expected life-time utility of the representative agent at the end of the period. Then, welfare $W$ can be written as follows:

$$ (1 - \beta) W = (1 - n) \delta_2 \{ \delta_1 [u(\hat{q}) - \hat{q}] + (1 - \delta_1) [u(q) - q] \} + U(x^*) - x^*, $$

where the term in the curly brackets is an agent’s expected period utility in the goods market, and $U(x^*) - x^*$ is the agent’s period utility in the primary bond market. Differentiating (30) with respect to $\tau$ yields

$$ \frac{1 - \beta}{(1 - n) \delta_2} \frac{dW}{d\tau} = \delta_1 \left[ u'(\hat{q}) - 1 \right] \frac{d\hat{q}}{d\tau} + (1 - \delta_1) \left[ u'(q) - 1 \right] \frac{dq}{d\tau}. $$

The welfare effect depends on the derivatives $\frac{d\hat{q}}{d\tau}$ and $\frac{dq}{d\tau}$. In the type-I equilibrium, from (24), we have $\frac{dq}{d\tau} = \frac{1}{\Phi'(q)} < 0$, and, from (25), we have $\frac{d\hat{q}}{d\tau} = -\frac{\delta_1 \eta}{\eta \delta_1 [1 - \eta] + 1 - \delta_1 |\Phi'(q)|} > 0$.\textsuperscript{22} Thus, whether a transaction tax is welfare-improving depends on which of the two effects dominates.

\textsuperscript{19}Throughout the paper when we consider a change in the transaction tax, we assume that the additional tax income is redistributed lump-sum to the agents in the primary bond market. This means from (1), that a change in the transaction tax has no effect on the inflation rate.

\textsuperscript{20}For $\tau = 0$ and $\gamma = \beta$, we have $q = \hat{q} = q^*$. That is, the Friedman rule ($\gamma = \beta$) implements the first-best allocation. In this case, there is obviously no welfare-enhancing role for a transaction tax. There is not even a role for a secondary bond market, since holding cash is costless and agents do not need to economize their cash holdings. In this paper, we abstract from this well-known result and ask under which conditions, away from the Friedman rule, can be optimal to impose a transaction tax.

\textsuperscript{21}In this region, the consumption quantity satisfies $\hat{q} = (1 - n) \Phi(q) + n$ and $\hat{q} = q$. The bond price is at its fundamental value $\frac{p}{p^*} = 1$.

\textsuperscript{22}Recall that $\Phi(q)$ is decreasing in $q$. 
Proposition 4 In the type-I equilibrium, if

\[
\Theta(q, \hat{q}) = \left[ \frac{(\eta - \eta \delta_1) \Phi'(\hat{q})}{(1 - \eta \delta_1) \Phi'(q)} \right] \left[ \frac{u'(q) - 1}{u'(\hat{q}) - 1} \right] > 1,
\]

then welfare is increasing in \(\tau\).

Proposition 4 formulates a condition under which it is welfare improving to increase the transaction tax in the type-I equilibrium. In general, the first term is smaller than 1 and the second term is larger than 1. The second term approaches infinity as \(\hat{q} \to q^*\), which means that for some preferences and technology parameters the second term dominates the first term.\(^{23}\) For example, the second term is increasing in the bargaining power of the consumer in the goods market \(\theta\). Thus, as we increase \(\theta\), the second term eventually dominates the first term and so \(\Theta(q, \hat{q}) > 1\).

The search frictions play a crucial role for this result. From (32), \(\Theta(q, \hat{q})\) is decreasing in \(\delta_1\) and approaches 0 as \(\delta_1 \to 1\). In the absence of search frictions \((\delta_1 = 1)\), all consumers trade in the secondary bond market, and so all consumers get the same consumption \(\hat{q}\) in the goods market. In this case, adding a tax is strictly welfare-decreasing, since it lowers consumption for all consumers.

This last observation also clarifies why a transaction tax can be welfare-increasing. In the type-I equilibrium, we have \(q < \hat{q}\). Increasing \(\tau\) increases \(q\) and decreases \(\hat{q}\). Thus, the tax has a redistributional effect. The question is why does it increase \(q\)? The reason is straightforward. The role of the secondary bond market is to allocate "idle" money from producers to consumers. In doing so, this market provides insurance to agents against the liquidity shock of becoming a consumer. The drawback of this insurance is that it reduces the incentive to self-insure against the liquidity shocks. This lowers the demand for money in the primary bond market, which depresses its value. This effect can be so strong that it can be optimal to impose a transaction tax in the secondary bond market.

In summary, in this section we have shown that it can be welfare-improving to impose a transaction tax. Such a tax makes trading in the secondary bond market less attractive, which results in an increase in the demand for money and its price.\(^{24}\) The optimal tax rate depends on preferences and technology. In the following section, we calibrate the model to U.S. data to obtain estimates for the optimal transaction tax rate.

7 Quantitative Analysis

We choose a model period as one year. The functions \(u(q)\), \(c(q)\), and \(U(x)\) have the forms \(u(q) = Aq^{1-\alpha}/(1-\alpha)\), \(c(q) = q\), and \(U(x) = \log(x)\). For the matching technologies in the goods

\(^{23}\)In particular, consider an initial tax of \(\tau = 0\), \(\delta_1 < 1\) and \(\gamma_1 < \gamma < \gamma_2\). In this case, for \(\theta = 1\), we have \(\hat{q} = q^*\), and so this condition is satisfied, since \(\lim_{\hat{q} \to q^*} \Theta(q, q^*) = \infty\).

\(^{24}\)In Berentsen et al. (2014a) it is shown that an alternative welfare-enhancing measure is to restrict, but not to eliminate, access to the secondary bond market.
market and in the secondary bond market, we use $\mathcal{M}(B, S) = BS/(B+S)$, where $B = 1-n$ is the measure of consumers and $S = n$ the measure of producers. The parameters to be identified are as follows: (i) preference parameters: $(\beta, A, \alpha)$; (ii) technology parameters: $(n, \tau)$; (iii) bargaining power: $(\theta, \eta)$; (iv) policy parameters: $\gamma$.

To identify these parameters, we use U.S. data from the first quarter of 1995 to the fourth quarter of 2013. All data sources are provided in the Appendix. The calibration targets are shown in Table 1.

<table>
<thead>
<tr>
<th>Target Description</th>
<th>Target Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average real interest rate $r$</td>
<td>0.021</td>
</tr>
<tr>
<td>Average inflation rate</td>
<td>0.024</td>
</tr>
<tr>
<td>Average velocity of money</td>
<td>6.30</td>
</tr>
<tr>
<td>Average elasticity of money demand (SE in parentheses)</td>
<td>-0.21 (0.014)</td>
</tr>
<tr>
<td>Average manufacturing markup</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The four parameters $\gamma$, $\beta$, $n$ and $\eta$ can be set equal to their targets. The gross growth rate of the money supply $\gamma$ matches the average change in the consumer price index. The discount factor $\beta$ is set such that the model’s real interest rate matches the real interest rate in the data, which is measured as the difference between the long-term government bond yield and the change in the consumer price index. In order to maximize the number of matches, we set $n = 0.5$. The baseline calibration, we set the consumer’s bargaining share in the secondary bond market to $\eta = 0.5$. Different values for $\eta$ are then assumed for robustness checks.

The parameters $\alpha$, $A$, and $\theta$ are obtained by matching the velocity of money, the elasticity of money demand, and the manufacturing markup simultaneously. We do this by minimizing the sum of squared differences between the target values and the respective model-generated moments. Following this calibration strategy, we are able to hit the three targets exactly. The average velocity of money, measured as the ratio of GDP to M1, is equal to $v = 6.30$ in the U.S. The elasticity of money demand with respect to the long-term government bond yield, estimated by ordinary least squares and a log-log specification, is equal to $\xi = -0.21$. The manufacturing markup is equal to $\mu = 0.15$, which represents the value estimated by Martins et al. (1996).

---

25Related studies work with the yield on AAA bonds, which consists of bonds with remaining maturities of as close as possible to 30 years. As we analyse Germany in the robustness section, we decided to use the long-term government bond yield, which is available for the U.S. and for Germany, and consists of government bonds with a remaining maturity of 10 years.

26For the U.S., we work with M1 adjusted for retail sweeps, instead of M1. Cynamon et al. (2006) show that the presence of commercial demand deposit sweep programs leads to an underreporting of transactions balances in M1. Furthermore, in Berentsen et al. (2014b), it is shown that M1 adjusted for retail sweeps represents the available transaction media in the economy more accurately than M1. As the stock of money in the model is equal to the stock of transaction media, we map the model’s money demand to the empirical money demand adjusted for retail sweeps.

27Related studies use a retail sector markup of 30 percent for the U.S. (see Faig and Jerez, 2005). Here, we use the manufacturing markup, because Martins et al. (1996) perform an extensive cross-country analysis that allows
The model’s velocity of money is
\[ v = \frac{Y}{\phi M_{-1}} = 1 + (1 - n) \delta_2 [\delta_1 z(\bar{q}) + (1 - \delta_1) z(q)] / z(q), \]
which depends on the interest rate in the primary bond market; i.e., \( i = \frac{1}{\rho} - 1 \), and on \( \alpha \) via the functions \( z(q) \) and \( z(\bar{q}) \).\(^{21}\) As for the empirical elasticity, the model’s elasticity of money demand is estimated by ordinary least squares and a log-log specification. The model’s manufacturing markup is given by the real amount of money exchanged in a bilateral match divided by the production cost,
\[ \mu = \delta_1 z(\bar{q})/c(\bar{q}) + (1 - \delta_1) z(q)/c(q). \]

With probability \( \delta_1 \), a consumer is active and buys \( \bar{q} \) units of market-2 goods for \( \hat{m} \) units of money. The real value of money he spends for goods purchases is \( \phi \hat{m} = z(\bar{q}) \). The producer incurs a real cost \( c(\bar{q}) \) to produce \( \bar{q} \) units of market-2 goods. In this case, the markup is \( z(\bar{q})/c(\bar{q}) \). With probability \( 1 - \delta_1 \), a consumer is passive and consequently consumes \( q \) units of market-2 goods for \( m \) units of money, in which case the markup is \( z(q)/c(q) \).

The targets discussed above, and summarized in Table 1, are sufficient to calibrate all but one parameter: the taxation rate \( \tau \). To address the question of whether it might be optimal to set \( \tau > 0 \), we calibrate the model for \( \tau = 0 \) and then calculate the optimal taxation rate \( \tau^* \). The optimal taxation is calculated as follows. For each set of calibrated parameter values, we numerically search for the value of \( \tau \) that maximizes ex-ante welfare, defined in (30). Furthermore, we calculate \( 1 - \Delta \), which is the percentage of total consumption that agents would be willing to give up in order to be in a steady state with \( \tau = \tau^* \), instead of \( \tau = 0 \). Finally, we are also interested in the share of market-2 consumption that is financed by the sale of bonds, defined as
\[ \kappa(\tau) = \frac{\delta_1 (\bar{q} - q)}{q + \delta_1 (\bar{q} - q)}. \]

In particular, we calculate the change in \( \kappa \) associated to an increase in \( \tau \); i.e., \( \kappa(\tau^*)/\kappa(0) - 1 \).

### 7.1 Baseline Results - U.S.

Table 2 presents the results for the baseline calibration and three robustness checks. The robustness checks are defined as follows: In the calibration labeled “markup”, we target a retail sector markup of 30 percent, estimated by Faig and Jerez (2005), instead of the manufacturing markup of 15 percent; in the calibration labeled “elasticity”, we target an elasticity of money demand of \(-0.30\), instead of \(-0.21\); and in the calibration labeled “velocity”, we target a velocity of money demand of 4.0, instead of 6.30.

\(^{21}\)The model’s velocity of money is derived as follows. The real output in the goods market is \( Y_{GM} = (1 - n) \delta_2 [\delta_1 \phi \hat{m} + (1 - \delta_1) \phi m] \), where \( \phi \hat{m} = z(\bar{q}) \) and \( \phi m = z(q) \), and the real output in the primary bond market is \( Y_{PBM} = 1 \) for \( U(x) = \log(x) \). Accordingly, total real output of the economy adds up to \( Y = Y_{GM} + Y_{PBM} \), and the model-implied velocity of money is \( v = Y/\phi M_{-1} \).
Table 2: Calibration results for the U.S.\(^{a}\)

<table>
<thead>
<tr>
<th>Description</th>
<th>Baseline</th>
<th>Markup</th>
<th>Elasticity</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) utility weight</td>
<td>0.306</td>
<td>0.298</td>
<td>0.436</td>
<td>0.439</td>
</tr>
<tr>
<td>( \theta ) bargaining in GM</td>
<td>0.951</td>
<td>0.891</td>
<td>0.901</td>
<td>0.950</td>
</tr>
<tr>
<td>( \alpha ) relative risk aversion</td>
<td>0.724</td>
<td>0.696</td>
<td>0.549</td>
<td>0.716</td>
</tr>
<tr>
<td>( \tau^* ) optimal tax(^{b})</td>
<td>0.026</td>
<td>0.025</td>
<td>0.024</td>
<td>0.025</td>
</tr>
<tr>
<td>( 1 - \Delta ) consumption delta</td>
<td>0.004%</td>
<td>0.005%</td>
<td>0.006%</td>
<td>0.007%</td>
</tr>
<tr>
<td>( s_{GM} ) goods market size</td>
<td>0.046</td>
<td>0.049</td>
<td>0.049</td>
<td>0.073</td>
</tr>
<tr>
<td>( \kappa(\tau^*_1) - 1 ) decrease in volume</td>
<td>-0.298</td>
<td>-0.296</td>
<td>-0.273</td>
<td>-0.297</td>
</tr>
<tr>
<td>( \gamma_{1*} ) critical value</td>
<td>0.993</td>
<td>0.993</td>
<td>0.992</td>
<td>0.993</td>
</tr>
<tr>
<td>( \gamma_{2*} ) critical value</td>
<td>1.131</td>
<td>1.131</td>
<td>1.080</td>
<td>1.129</td>
</tr>
</tbody>
</table>

\(^{a}\)Table 2 displays the calibrated values for the key parameters \((A, \theta, \alpha)\), the optimal value for the tax, \(\tau^*\), and the size of the goods market, \(s_{GM}\). It also shows the percentage of total consumption that agents would be willing to give up in order to be in a steady state with \(\tau = \tau^*\), instead of \(\tau = 0, 1 - \Delta\). The table also displays the change in the share of market-2 consumption that is financed by the sale of bonds, and the two critical inflation rates. \(^{b}\)The optimal tax \(\tau^*\) is calculated numerically by searching for the welfare-maximizing value of \(\tau\), holding all other parameters at their calibrated values.

Table 2 shows that, for all calibrations, the optimal taxation rate is about 2.5 percent. The gain in total consumption from being in a steady state with \(\tau = \tau^*\), as opposed to \(\tau = 0\), is about 0.004 percent in the baseline calibration. For a higher elasticity of money demand or a lower value of the velocity of money, we find that the gain in total consumption increases to 0.006 percent. In Table 2, we also provide the estimates of the model-implied goods market share, \(s_{GM}\), which, for all calibrations is about 5 percent. This is in line with related studies.\(^{28}\) Table 2 shows that levying a transaction tax decreases the share of market-2 consumption that is financed by the sale of bonds. In particular, for all calibrations, \(\kappa\) decreases by around 30 percent.

The table also shows that the type-I equilibrium exists for inflation rates in the range of \(\gamma_{1*} < \gamma < \gamma_{2*}^*\) for \(\tau^*\) calculated at the calibrated value of \(\gamma\). For \(\beta < \gamma < \gamma_{1*}\), financial intermediation shuts down. From the calibration results presented in Table 2, one can see that a higher elasticity results in a decrease of \(\gamma_{2*}^*\), while changing the other parameter targets has a minor effect.

We now check for the sensitivity of our results with respect to the inflation rate \(\gamma\). For this purpose, we calculate the effect of increasing \(\gamma\) on \(\tau^*\) (Figure 2), and the effect of increasing \(\gamma\) on \(1 - \Delta\) (Figure 3).

\(^{28}\)See for instance Aruoba et al. (2011) or Lagos and Wright (2005).
Figure 2 shows that \( \tau^* \) is increasing in \( \gamma \). Note, that the critical inflation rates depend on \( \tau^* \) and, for each point, we check that we are in the type-I equilibrium. When \( \gamma > \gamma_2^* \), the optimal taxation rate is zero, which is evident from the calibration labeled "elasticity". We find that, for the calibration labeled "elasticity", the type-I equilibrium exists for inflation rates up to 8.4 percent, while, for the other calibrations, it exists for inflation rates of more than 10 percent.

Figure 3 shows that the benefit from being in a steady state with \( \tau = \tau^* \), as opposed to \( \tau = 0 \), is the highest for annual inflation rates of around 10 percent, where we obtain \( 1 - \Delta \approx 0.009 \) percent for the baseline calibration. A lower velocity of money has the biggest impact on \( 1 - \Delta \) and results in \( 1 - \Delta \approx 0.014 \) percent for \( \gamma = 1.1 \).
7.1.1 The consumer’s bargaining share in the secondary bond market

In order to see how a change of the consumer’s bargaining share in the secondary bond market $\eta$ affects our results, we recalibrate each model presented in Table 2 for different values of $\eta \in (0, 1)$, and draw $\tau^*$ and $1-\Delta$ in Figure 4 and Figure 5, respectively.\(^\text{29}\)

![Figure 4: $\tau^*$ for increasing values of $\eta$](image)

We find that the optimal taxation rate is increasing in $\eta$. In particular, $\tau^*$ increases up to $\tau^* \approx 8$ percent for $\eta \to 1$ for all calibrations.

\(^\text{29}\)In this paragraph, we estimate the model’s interest rate elasticity of money demand using a point approximation at the calibrated value of $\gamma$. In Berentsen et al. (2014b) it is shown that when estimating the model’s elasticity of money demand by ordinary least squares and a log-log specification, the standard errors are small. Thus, a point approximation at the calibrated value of $\gamma$ represents a reasonable approximation for the model’s elasticity of money demand.
The same effect is visible for $1 - \Delta$, where we obtain $1 - \Delta \approx 0.035$ percent for $\eta \to 1$ in the baseline calibration, and up to $1 - \Delta \approx 0.055$ percent in the velocity calibration.

### 7.2 Robustness Check - Germany

As a robustness exercise, we now calibrate our model to Germany. The velocity of money in Germany ($v = 0.70$) is substantially lower than that in the U.S. ($v = 6.3$). The elasticity of money demand is similar in both countries. Furthermore, in Germany, the average inflation rate is nearly 1 percent lower, the real interest rate is 0.5 percent higher, and the manufacturing markup is 6 percent higher than in the U.S. Table 3 summarizes the calibration targets.

**Table 3: Calibration targets for Germany**

<table>
<thead>
<tr>
<th>Target Description</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average real interest rate $r$</td>
<td>0.026</td>
</tr>
<tr>
<td>Average inflation rate</td>
<td>0.015</td>
</tr>
<tr>
<td>Average velocity of money</td>
<td>0.70</td>
</tr>
<tr>
<td>Average elasticity of money demand (SE in parentheses)</td>
<td>-0.21 (0.079)</td>
</tr>
<tr>
<td>Average manufacturing markup</td>
<td>0.21</td>
</tr>
</tbody>
</table>

As we did for the U.S., we calibrate the model to the targets presented in Table 3 and perform three robustness checks, where we use exactly the same targets as for the U.S.\(^{30}\) Note that an

---

\(^{30}\) That is, in the calibration labeled “markup”, we target a goods market markup of 30 percent; in the calibration labeled “elasticity”, we target an elasticity of money demand of -0.30; and in the calibration labeled “velocity”, we target a velocity of money demand of 4.0.
average markup target of 30 percent, used in the calibration labeled "markup", is close to the number observed for Germany.\textsuperscript{31} Table 4 shows the calibration results for Germany.

\textbf{Table 4: Results for Germany\textsuperscript{a}}

\begin{tabu}{lcccc}
\hline
\textbf{Description} & \textbf{Baseline} & \textbf{Markup} & \textbf{Elasticity} & \textbf{Velocity} \\
\hline
A & utility weight & 1.731 & 1.681 & 1.691 & 0.479 \\
θ & bargaining in GM & 0.869 & 0.819 & 0.799 & 0.896 \\
α & relative risk aversion & 0.565 & 0.570 & 0.440 & 0.628 \\
τ^\text{optimal} & optimal τ & 0.023 & 0.023 & 0.021 & 0.023 \\
1 − Δ & consumption delta & 0.046% & 0.050% & 0.066% & 0.007% \\
s_{GM} & goods market size & 0.397 & 0.397 & 0.435 & 0.073 \\
\frac{\kappa(\tau^*)}{\kappa(0)}−1 & decrease in volume & -0.263 & -0.286 & -0.253 & -0.296 \\
γ_{1\tau}^\text{critical} & critical value & 0.986 & 0.986 & 0.985 & 0.987 \\
γ_{2\tau}^\text{critical} & critical value & 1.078 & 1.078 & 1.045 & 1.097 \\
\hline
\end{tabu}

\textsuperscript{a}Table 4 is the Table 2’s counterpart for Germany. For a description of the reported variables, we refer the reader to Table 2.

Table 4 shows that, for all but one calibration, the goods market share is around 40 percent in Germany. As a consequence, there is a significantly higher benefit from being in a steady state with \( \tau = \tau^* \) as opposed to \( \tau = 0 \). For example, in the baseline calibration, we obtain \( 1 − Δ \approx 0.046 \) percent, which is more than ten times higher than for the U.S. The main reason behind this result is the lower velocity of money, which becomes clear when we look at the calibration labeled "velocity". When we increase the target of the velocity from \( v = 0.70 \) to \( v = 4.0 \), the benefit of raising the tax from zero to \( \tau = \tau^* \) drops to \( 1 − Δ \approx 0.007 \) percent. Similar to the U.S., the share of market-2 consumption that is financed by the sale of bonds decreases by around 28 percent when \( \tau = \tau^* \) as compared to \( \tau = 0 \).

In Figure 6, we show the effect of increasing \( γ \) on \( \tau^* \) and in Figure 7, the effect of increasing \( γ \) on \( 1 − Δ \). As is evident from Figure 6, in Germany the critical inflation rate \( γ_{2\tau}^* \) is generally lower than in the U.S. In particular, for the calibration labeled "elasticity", the type-I equilibrium exists for inflation rates below 4.6 percent, while for the other calibrations it exists for inflation rates below 8 or 10 percent.

\textsuperscript{31}Christopoulou and Vermeulen (2008) estimate an average markup of 33 percent for Germany.
Figure 6: Germany - $\tau^*$ for increasing values of $\gamma$

Figure 7 shows that the velocity of money is the most important variable that determines the benefit from being in a steady state with $\tau = \tau^*$, as opposed to $\tau = 0$. The lower the velocity is, the higher is the share of goods market consumption relative to total consumption, and thus the larger is the impact of monetary policy.

7.2.1 The consumer’s bargaining share in the secondary bond market

In order to see how a change of the consumer’s bargaining share in the secondary bond market affects the results, we redo each calibration presented in Table 4 for different values of $\eta \in (0, 1)$,
and draw $\tau^*$ and $1 - \Delta$ in Figure 8 and Figure 9, respectively.\footnote{In this paragraph, we estimate the model’s interest rate elasticity of money demand using a point approximation at the calibrated value of $\gamma$.}

The dynamics of the optimal taxation rate for the U.S. and Germany are similar (compare Figures 4 and 8). The behavior of $1 - \Delta$ as a function of $\eta$, however, is different in the U.S. and Germany (compare Figures 5 and 9). At the lower end of Figure 9, we find for the calibration labeled "velocity" that $1 - \Delta \approx 0.05$ percent for $\eta \to 1$. At the upper end, we find for the calibration labeled "elasticity"
that $1 - \Delta \approx 0.48$ percent for $\eta \to 1$. In between, we find for the calibrations labeled "baseline" and "markup" that $1 - \Delta \approx 0.32$ percent for $\eta \to 1$. Hence, for $\eta \to 1$, the representative German is willing to give up up to 0.5 percent of steady state consumption in order to be taxed at the optimal rate as opposed to $\tau = 0$.

8 Conclusion

Since the initial proposition of a financial transaction tax (FTT) by John Maynard Keynes in 1936, economists remained astonishingly silent in developing a theory in support for such a policy measure. In this paper, we present a theoretical framework that embeds the fundamental frictions that give rise to trading in financial markets. Furthermore, we highlight the distortions caused by such markets and show how a FTT proves competent in correcting them.

The initial arguments of Keynes (1936) and Tobin (1978) in support for a FTT were mainly aimed at taming "excessive" volatility in financial markets. In this paper, we reveal fundamentally different justifications in favor of such a policy, which can be summarized as follows. The introduction of a FTT discourages agents from trading in financial markets, inducing them to increase their money holdings. This marginally increases the price of money and thereby improves the insurance for all market participants. Through this mechanism, the consumption of agents who do not readjust their portfolio increases, while the consumption of agents who decide to do so drops, due to the duty levied by the FTT. This results in an overall decline in consumption variability, which is clearly welfare-improving. Beside this, we show that the introduction of a FTT results in a decrease in the share of consumption that is financed through the sale of illiquid assets in financial markets.

Our arguments strictly rely on the OTC nature of financial markets. Without search and matching frictions, our results cease to hold. Hence, the recent advances in the “new monetarist economics” were key to developing our framework.
9 Appendix

Derivation of \( \Delta^c \) and \( \Delta^p \). They are defined as follows:

\[
\Delta^c \equiv V_2^c \left[ m + d_m (1 - \tau), b - d_b \right] - V_2^c (m, b), \\
\Delta^p \equiv V_2^p \left( m - d_m, b + d_b \right) - V_2^p (m, b).
\]

Let us start with \( \Delta^c \) and write it as follows

\[
\Delta^c = \delta_2 \left\{ u(\hat{q}) + V_3 \left[ m + d_m (1 - \tau) - \hat{d}, b - d_b \right] - [u(q) + V_3 (m - d, b)] \right\} \\
+ (1 - \delta_2) \left\{ V_3 \left[ m + d_m (1 - \tau), b - d_b \right] - V_3 (m, b) \right\}.
\]

The consumer’s surplus, \( \Delta^c \), is given by his payoff from trading bonds for money in a match in the secondary bond market minus his payoff from not trading. If a trade occurs in the goods market (first line), the consumer’s payoff is \( u(\hat{q}) + V_3 \left[ m + d_m (1 - \tau) - \hat{d}, b - d_b \right] \) if he trades in the secondary bond market, while it is \( u(q) + V_3 (m - d, b) \) if he does not. If a trade does not occur in the goods market (second line), his payoff is given by \( V_3 \left[ m + d_m (1 - \tau), b - d_b \right] \) if he trades in the secondary bond market, and \( V_3 (m, b) \) if he does not.

If a trade takes place, the consumer receives \( d_m (1 - \tau) \) units of money for \( d_b \) units of bonds, and he spends \( \hat{d} = m + d_m (1 - \tau) \) units of money and consumes \( \hat{q} \) units of goods if he is matched with a producer in the goods market, which occurs with probability \( \delta_2 \). If no trade takes place in the secondary bond market, the consumer spends \( d = m \) units of money and consumes \( q \) units of goods if he is matched with a producer in the goods market. If the consumer is not matched with a producer in the goods market, he will just wait for the primary bond market to open. Simplifying and rearranging terms, the consumer’s net surplus \( \Delta^c \) can be rewritten as

\[
\Delta^c = \delta_2 \left\{ u(\hat{q}) - u(q) \right\} + \phi d_m (1 - \delta_2) (1 - \tau) - \phi d_b,
\]

where we have used \( \hat{d} = d + d_m (1 - \tau) \), the consumer’s value function in the goods market, \( V_2^c \), and the linearity of \( V_3 \).

Let us now derive \( \Delta^p \) and write

\[
\Delta^p = \delta_2^p \delta_1 \left\{ \left[ -c(\hat{q}) + V_3 \left( m - d_m + \hat{d}, b + d_b \right) \right] - \left[ -c(q) + V_3 \left( m + \hat{d}, b \right) \right] \right\} \\
+ \delta_2^p (1 - \delta_1) \left\{ \left[ -c(q) + V_3 \left( m - d_m + d, b + d_b \right) \right] - \left[ -c(q) + V_3 \left( m + d, b \right) \right] \right\} \\
+ (1 - \delta_2^p) \left\{ V_3 \left( m - d_m, b + d_b \right) - V_3 (m, b) \right\}.
\]

The producer’s surplus, \( \Delta^p \), is given by his payoff from trading bonds for money in a match in the secondary bond market minus his payoff from not trading. A producer faces three possible events in the goods market: (i) he is matched with a consumer who traded in the secondary bond market (first line); (ii) he is matched with a consumer who did not trade in the secondary bond market (second line); (iii) he is matched with nobody (third line). In case (i), the producer’s payoff is \( -c(\hat{q}) + V_3 \left( m - d_m + \hat{d}, b + d_b \right) \) if he trades in the secondary bond market, while it
is \( -c(q) + V_3\left(m + \hat{d}, b\right) \) if he does not. In case (ii), the producer’s payoff is given by \( -c(q) + V_3(m - d_m + d, b + d_b) \) if he trades in the secondary bond market, while it is \( -c(q) + V_3(m + d, b) \) if he does not. In case (iii), the producer’s payoff is given by \( V_3(m - d_m, b + d) \) if he trades in the secondary bond market, and \( V_3(m, b) \) if he does not.

The producer’s production in the goods market is not affected by his trading activity in the secondary bond market. This is because the terms of trade in the goods market depend on the consumer’s money holdings, and do not depend on the producer’s money holdings. Rearranging and simplifying terms, we obtain

\[
\Delta^P = \phi d_b - \phi d_m.
\]

Note that this constraint implies that the interest rate in the secondary bond market is non-negative, since the interest rate is \( \frac{d_b}{d_m} - 1 \).

**Proof of Proposition 1.** Derivation of (24). In a type-I equilibrium, both the producer’s cash constraint and the consumer’s bonds constraint are nonbinding in the secondary bond market (i.e., \( \lambda^p = \lambda^c = 0 \)). In this case, the first-order condition in the secondary bond market (23) reduces to \( 0 = (1 - \tau) \Phi(\hat{q}) - 1 \).

**Derivation of (25).** In a type-I equilibrium, the marginal value of money of an agent entering the secondary bond market, before the realization of the idiosyncratic shock, is

\[
\frac{\partial V_1(m, b)}{\partial m} = (1 - n) \frac{\partial V_1^c(m, b)}{\partial m} + n \frac{\partial V_1^p(m, b)}{\partial m}.
\]

In the supplementary Appendix, we show that

\[
\frac{\partial V_1^c(m, b)}{\partial m} = \phi \left\{ \delta_1 [(1 - \eta) \Phi(q) + \eta \Phi(\hat{q})] + (1 - \delta_1) \Phi(\hat{q}) \right\}
\]

\[
\frac{\partial V_1^p(m, b)}{\partial m} = \phi
\]

Using (4) updated one period, we obtain (25).

**Derivation of (26).** In a type-I equilibrium, the marginal value of bonds of an agent entering the secondary bond market, before the realization of the idiosyncratic shock, is

\[
\frac{\partial V_1(m, b)}{\partial b} = (1 - n) \frac{\partial V_1^c(m, b)}{\partial b} + n \frac{\partial V_1^p(m, b)}{\partial b}.
\]

In the supplementary Appendix, we show that \( (1 - n) \frac{\partial V_1^c(m, b)}{\partial b} + n \frac{\partial V_1^p(m, b)}{\partial b} = \phi \). Using (4) updated one period, we get (26).

**Proof of Proposition 2.** Derivation of (27). In a type-II equilibrium, a producer is cash-constrained in the secondary bond market, thus \( d_m = m = M \). Also note that \( \phi m + \phi (1 - \tau) d_m = z(\hat{q}) \) and \( \phi m = z(q) \). Eliminate \( \phi m \) in the former equation using the latter, then use \( d_m = M \), and rearrange terms to get (27).

**Derivation of (28).** The marginal value of money of an agent entering the secondary bond market, before the realization of the idiosyncratic shock, can be written as follows

\[
\frac{\partial V_1(m, b)}{\partial m} = (1 - n) \frac{\partial V_1^c(m, b)}{\partial m} + n \frac{\partial V_1^p(m, b)}{\partial m}.
\]
In the supplementary Appendix, we show that
\[
\frac{\partial V^c_t (m, b)}{\partial m} = \phi \{ \delta_1 [(1 - \eta) \Phi (q) + \eta \Phi (\hat{q})] + (1 - \delta_1) \Phi (q) \},
\]
\[
\frac{\partial V^p_t (m, b)}{\partial m} = \phi \{ \delta_1 [(1 - \eta) (1 - \tau) \Phi (\hat{q}) + \eta] + 1 - \delta_1 \}.
\]

Using (4) updated one period, we obtain (28).

**Derivation of (29).** In a type-II equilibrium, the marginal value of bonds of an agent entering the secondary bond market, before the realization of the idiosyncratic shock, is
\[
\frac{\partial V_1 (m, b)}{\partial b} = (1 - n) \frac{\partial V^c_1 (m, b)}{\partial b} + n \frac{\partial V^p_1 (m, b)}{\partial b}.
\]

In the supplementary Appendix, we show that \((1 - n) \frac{\partial V^c_1 (m, b)}{\partial b} + n \frac{\partial V^p_1 (m, b)}{\partial b} = \phi\). Using (4) updated one period, we obtain (29). ■

**Proof of Proposition 3.** The critical value \(\gamma_1\) is the value of \(\gamma\) such that \(\hat{q} = q\) in Proposition 1. If \(\hat{q} = q\), from (24) and (25) we have
\[
\gamma_1 = \beta \frac{1 - n \tau}{1 - \tau}.
\]

Since \(\Phi (q)\) is strictly decreasing in \(q\), \(\gamma_1\) is unique. Note that for \(\tau = 0\), \(\gamma_1 = \beta\) and for \(\tau > 0\), we have \(\gamma_1 > \beta\).

The critical value \(\gamma_2\) is the value of \(\gamma\) such that the quantities \(q\) and \(\hat{q}\) in Proposition 2 are equal to the respective quantities \(q\) and \(\hat{q}\) in Proposition 1. To that end, from (24), we have \(\Phi (\hat{q}) (1 - \tau) = 1\), which, since \(\Phi (\hat{q})\) is strictly increasing in \(q\), can be solved for \(\hat{q} (\tau) = \Phi^{-1} [(1 - \tau)^{-1}]\). Next, from (27) we have \(z (\hat{q}) = (2 - \tau) z (q)\), which can be solved for \(q (\hat{q}, \tau) = z^{-1} [z (\hat{q}) (2 - \tau)^{-1}]\).

Use the previous expression to replace \(\hat{q}\) to obtain \(q (\tau) = z^{-1} [z (\hat{q} (\tau)) (2 - \tau)^{-1}]\). Finally, use these expressions in (25) or (28) to obtain
\[
\gamma_2 = \beta \left\{ (1 - n) \left\{ \delta_1 \left( (1 - \eta) \Phi (q (\tau)) + \frac{\eta}{1 - \tau} \right) + (1 - \delta_1) \Phi (q (\tau)) \right\} + n \right\}.
\]

Note that \(\gamma_1 \leq \gamma_2\) if \(1 \leq (1 - \tau) [1 - \eta \delta_1] \Phi (q (\tau)) + \eta \delta_1\), which requires that the tax is not too high. ■

**Proof of Proposition 4.** Use \(\frac{d\hat{q}}{d\tau} = \frac{1}{\Phi (\hat{q})}\) and \(\frac{dq}{d\tau} = -\frac{\delta_1 \eta}{\delta_1 (1 - \eta) + 1 - \delta_1} \Phi (\hat{q})\) to replace \(\frac{d\hat{q}}{d\tau}\) and \(\frac{dq}{d\tau}\) in (31) and rearrange the resulting expression to get
\[
\frac{1 - \beta}{(1 - n) \delta_2} \frac{dW}{d\tau} = \delta_1 \left[ u' (\hat{q}) - 1 \right] \frac{1}{\Phi (\hat{q})} \left\{ 1 - \frac{(1 - \delta_1) [u'(q) - 1]}{\delta_1 [u'(\hat{q}) - 1]} \right\} \frac{\delta_1 \eta \Phi' (\hat{q})}{\delta_1 (1 - \eta) + 1 - \delta_1 \Phi' (\hat{q})}.
\]

Since \(\delta_1 [u'(\hat{q}) - 1] \frac{1}{\Phi (\hat{q})} < 0\), \(\frac{dW}{d\tau} > 0\) requires that the term in curly brackets is negative. This is the case if (32) holds. ■
Data Source. The data we use for the calibration is downloadable from the St. Louis FRED® database, except for the consumer price index of Germany, which we downloaded from Bloomberg. Table A.1. gives a brief overview of the data sources for the U.S. and Germany.

Table A.1: Data source

<table>
<thead>
<tr>
<th>Country</th>
<th>Description</th>
<th>Identifier</th>
<th>Period</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>Long-term gov. bond yield</td>
<td>IRLTLT01USM156N</td>
<td>95:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>U.S.</td>
<td>M1 adj. for retail sweeps</td>
<td>M1ADJ</td>
<td>95:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>U.S.</td>
<td>Nominal GDP</td>
<td>GDP</td>
<td>95:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>U.S.</td>
<td>Consumer price index</td>
<td>CPIAUCSL</td>
<td>95:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>Germany</td>
<td>Long-term gov. bond yield</td>
<td>IRLTLT01DEM156N</td>
<td>95:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>Germany</td>
<td>M1</td>
<td>MYAGM1DEM189S</td>
<td>95:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>Germany</td>
<td>Nominal GDP</td>
<td>DEUGDPNQDSMEI</td>
<td>95:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>Germany</td>
<td>Consumer price index</td>
<td>GRCP20YY Index</td>
<td>95:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
</tbody>
</table>

For the consumer price index of Germany, we applied the same aggregation method to calculate the quarterly values as the St. Louis FRED® database, which is defined as the average of the monthly data.
References


