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Working Paper

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Working Paper, No. 139

Provided in Cooperation with:

Department of Economics, University of Zurich

Suggested Citation: Berentsen, Aleksander; Müller, Benjamin (2014) : A tale of fire-sales and liquidity hoarding, Working Paper, No. 139, University of Zurich, Department of Economics, Zurich

This Version is available at:

<https://hdl.handle.net/10419/111196>

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**University of
Zurich** ^{UZH}

University of Zurich
Department of Economics

Working Paper Series
ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 139

A Tale of Fire-Sales and Liquidity Hoarding

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January 2014

A Tale of Fire-Sales and Liquidity Hoarding^{*}

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January 2014

Abstract

We extend the analysis of the interbank market model of Gale and Yorulmazer (2013) by studying a larger set of trading mechanisms. A trading mechanism, which allows for randomized trading, restores efficiency. In contrast to Gale and Yorulmazer, we find that fire-sale asset prices are efficient and that no liquidity hoarding occurs in equilibrium. While Gale and Yorulmazer find that the market provides insufficient liquidity, we find that it provides too much liquidity.

JEL Classification: G12, G21, G33, D83.

Keywords: Fire-sales, lotteries, liquidity hoarding, interbank markets, indivisibility.

1 Introduction

Liquidity hoarding and fire-sales are seemingly closely related phenomena. Consider an economy populated by many banks, which face random liquidity needs. Suppose further that banks anticipate that tomorrow many banks will face liquidity shortages and in order to avoid default are forced to sell assets at fire-sale prices. If assets are cheaper tomorrow than today, liquid banks prefer to hoard liquidity today. This is inefficient, since banks that are illiquid today have no access to liquidity and default. This is essentially the tale of fire-sales and liquidity hoarding developed in the paper by Gale and Yorulmazer (2013, henceforth GY).

^{*}The views expressed in this paper are those of the authors and do not necessarily represent those of the Swiss National Bank.

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The crucial question with this tale is why today’s defaulting banks are not selling their assets at fire-sale prices in order to prevent default. In GY, the reason is that the price of assets, q , cannot always adjust to market conditions. This is due to two assumptions: First, GY assume that in period 1 the portfolio of a liquid bank consist of one unit of the indivisible cash and one unit of the divisible asset, and the portfolio of an illiquid bank consist of one unit of the divisible asset only. Second, they consider a trading mechanism, where an indivisible unit of cash is exchanged for $x \leq 1$ units of the assets. Hence, the asset price cannot fall below one in period 1, since it is constraint to satisfy $q = 1/x \geq 1$. In contrast, the expected asset price in period 2 can fall below one, in which case it is optimal for liquid banks to hoard cash in period 1.¹

In this paper, we argue that the apparent relationship between liquidity hoarding and fire-sales in GY is driven by the use of an inefficient trading mechanism. In particular, we extend the interbank market model of GY by introducing randomized trading (lotteries) and we derive a mechanism that restores efficiency, where efficiency requires that assets and cash are not wasted. In contrast to GY, we find that fire-sales are efficient and that no hoarding occurs in equilibrium.

The intuition for our result is straightforward. With lotteries, the asset price is $q = \lambda/x$, where λ is the probability that the indivisible unit of cash is exchanged for $x \leq 1$ units of assets. Apparently, with lotteries there is no lower bound on the asset price, since for $\lambda < x$, we have $q < 1$. We show that under the efficient mechanism, if banks anticipate a fire-sale tomorrow, there is a fire-sale today, which eliminates the incentives to hoard liquidity. Thus, we show that fire-sales are efficient and restore efficiency in the GY framework.

The indivisibility of cash in GY introduces a nonconvexity into their environment. It is well known from theory that in economic environments with indivisibilities or other nonconvexities, agents can often do better using randomized rather than deterministic trading mechanisms, and so it is interesting to ask if there is a role for lotteries in the GY model, too. Analyses of nonconvexities and lotteries include Prescott and Townsend (1984a, 1984b), Rogerson (1988), Diamond (1990), Shell and Wright (1993), and Chatterjee and Corbae (1995). These authors typically justify the use of lotteries because of welfare considerations as expressed by Rogerson (1988, p. 11): “One of the reasons for adding lotteries to the consumption set was the potential gain in welfare. In essence, making labor indivisible creates a barrier to trade and the introduction of lotteries is one way to overcome part of this barrier.”

Models of monetary exchange that allow for randomized trading include Berentsen, Molico and Wright (2002) and Berentsen and Rocheteau (2002). In a model where agents meet pairwise at random, Berentsen and Rocheteau demonstrate that indivisible money generates a *no-trade* inefficiency, where no trade takes place in bilateral

¹A price below one in period 2 is interpreted by GY as a fire-sale price.

meetings even though it would be socially efficient to trade. They show that agents prefer not to trade if they expect to receive more goods for the indivisible unit of cash tomorrow. This *no-trade* inefficiency is closely related to the *hoarding* inefficiency in GY, since liquid banks prefer not to trade if they expect to receive more assets for the indivisible unit of cash tomorrow. Berentsen and Rocheteau also show that randomized trading eliminates the *no-trade* inefficiency. Along the same line, we show that lotteries eliminate the *hoarding* inefficiency in GY, too.²

Our paper does not refute the phenomenon of liquidity hoarding, since empirical evidence suggests that liquidity hoarding was indeed a problem during the latest financial crisis.³ However, we do not think that the pricing frictions, emphasized by GY, were at the origin of this phenomenon. Rather, we speculate that private information problems generated severe counterparty risks (see, for instance, Afonso, Kovner and Schoar; 2011).

The remainder of this paper is structured as follows. Section 2 summarizes the GY environment. Section 3 studies the banks' decisions. Section 4 derives an efficient incentive-feasible mechanism for the GY environment. Section 5 describes the portfolio choice of banks in the initial period. Section 6 relates our results to GY. Finally, Section 7 concludes.

2 Environment

Our environment is identical to GY. Nevertheless, for ease of reference we summarize it below.

The economy is populated by a continuum of identical and risk neutral utility-maximizing banks indexed by $i \in [0, 1]$. There are four periods ($t = 0, 1, 2, 3$) and two assets: An indivisible asset (henceforth called *cash*) which has a return of one unit of consumption at date 3 and an divisible asset (henceforth called *asset*) which has a return of $R > 1$ units of consumption at date 3. Both assets are storable.

In period 0, banks have an initial portfolio consisting of one unit of the asset and one unit of cash $\{1, 1\}$. A bank's utility function is defined as follows:

$$U(c_0, c_3) = \rho c_0 + c_3. \quad (1)$$

The interpretation of the utility function is as follows: banks can either consume the indivisible unit of cash in period 0 or in period 3. The asset can only be turned into

²It is important to note that it is not sufficient to make cash divisible to restore efficiency in the GY framework. The reason is that avoiding default requires precisely one unit of cash. Thus, it is the use of lotteries that restores efficiency.

³See, for instance, Heider, Hoerova and Holthausen (2009), Acharya and Merrouche (2013) or Ashcraft, McAndrews and Skeie (2011).

consumption in period 3.⁴ Banks prefer to consume cash in period 0 because of the opportunity cost $\rho > 1$ of holding cash after period 0. Banks which consume their cash in period 0 are called *illiquid* banks and those which keep it, *liquid* banks.

Every bank is required to pay one unit of cash either in period $t = 1, 2, 3$. Formally, in period 1 and 2 banks experience a liquidity shock which is modelled as a random cost for a bank to maintain its portfolio. Banks which experience a liquidity shock have to deliver one unit of cash. Otherwise, the bank's assets are liquidated and the liquidation cost is exactly equal to the remaining value of the portfolio. In order to avoid liquidation a bank can either use its initial cash endowment or sell assets to acquire one unit of cash in a competitive interbank market which opens in period $t = 1, 2$. If a bank is required to pay in period 3, it can make the repayment out of the asset return R .

Let θ_t denote the probability that a liquidity shock arrives at date t . The random variable θ_t has a density function $f(\theta_t)$ and a cumulative distribution function $F(\theta_t)$, where $t = 1, 2$. The liquidity shocks θ_1 and θ_2 are assumed to be independent. A bank can only receive one liquidity shock. With probability θ_1 a bank receives a liquidity shock in period 1. With probability $(1 - \theta_1)\theta_2$ the liquidity shock arrives in period 2. With probability $(1 - \theta_1)(1 - \theta_2)$ a bank receives no shock either period 1 or 2 and repays in period 3.

2.1 Planner allocation

As GY, we assume that the planner's objective is to maximize the total expected surplus. From (1), utility can be generated by consuming cash in period 0 and by carrying forward assets to period 3, where the return can be turned into consumption.

In contrast to GY, we characterize the first-best allocation which refers to the *unconstrained*-efficient allocation in GY. In particular, we allow the planner to redistribute assets between banks in period $t = 1, 2, 3$. As suggested by GY, if the planner is able to redistribute assets between banks, the planner can assign all assets to those banks with no liquidity shock in a period.⁵ In this case, all assets can be carried forward to period 3 and, since no cash is needed to do so, all cash holdings can be consumed in the initial period. Hence, there is no waste of assets and cash. Thus, welfare of the *unconstrained*-efficient allocation is

$$W^P = R + \rho. \tag{2}$$

⁴GY call cash the liquid asset because cash can be turned into consumption utility in period 0 already, while the asset is called illiquid because it yields utility in period three only.

⁵On page 298, GY call this solution a trivial outcome and therefore "[...] restrict the planner's actions to accumulating cash at date 0, distributing cash at dates 1 and 2, and redistributing the consumption good at date 3. It is because of this constraint that we refer to the solution of the planner's problem as a constrained-efficient allocation."

2.2 Trading mechanism

In GY, the trading mechanism in the interbank market is restricted to the exchange of one unit of indivisible cash for x units of divisible assets. Here, we allow for lotteries which is a more general trading mechanism.

A lottery $\lambda \in [0, 1]$ is the probability that an indivisible unit of cash has to be delivered in exchange for x units of assets, whereas a lottery $\tau \in [0, 1]$ is the probability that an indivisible unit of cash is received in exchange for x units of the asset. Without loss in generality, we do not consider lotteries on the asset, since the asset is divisible.

Let x_t denote the quantity of assets which has to be delivered in order to get one unit of cash with probability τ_t in periods $t = 1, 2$. Furthermore, let \tilde{x}_t denote the quantity of assets received in exchange for delivering one unit of cash with probability λ_t in periods $t = 1, 2$.

Definition 1 *A mechanism for period t is denoted by the list $(x_t, \tau_t, \tilde{x}_t, \lambda_t)$, where $t = 1, 2$. A mechanism for the entire game is denoted by the list $(x_t, \tau_t, \tilde{x}_t, \lambda_t)_{t=1,2}$.*

A feasible mechanism is a mechanism that satisfies physical constraints. Feasibility in period t requires that the measure of banks which receives one unit of cash is equal to the measure of banks which delivers one unit of cash. Then, feasibility requires that

$$m_t^s \lambda_t = m_t^d \tau_t. \quad (3)$$

The left-hand side of equation (3) represents the cash delivered in period t : m_t^s is the measure of banks that supplies cash and they have to deliver it with probability λ_t . The right-hand side of equation (3) represents the cash received in period t : m_t^d is the measure of banks that demands cash and they receive it with probability τ_t .

Feasibility in period 1 also requires that the quantity of assets delivered is equal to the quantity of assets received. Then, feasibility requires that

$$m_t^s \tilde{x}_1 = m_t^d x_1. \quad (4)$$

The left-hand side of equation (4) represents the assets received, since m_t^s is the measure of banks that receives \tilde{x}_t units of assets. The right-hand side of equation (4) represents the assets delivered, since m_t^d is the measure of banks that delivers x_t units of assets.

Finally, feasibility also requires that

$$x_t \leq e_t \text{ and } \tau_t, \lambda_t \leq 1, \quad (5)$$

where e_t denotes the asset holdings of a bank which supplies assets in period $t = 1, 2$. Since banks have an initial endowment of one unit of the asset, $e_1 = 1$ in the first

period. However, e_2 can exceed one, since liquid banks can acquire assets in exchange for cash in period 1. Note that there is no such constraint on \tilde{x}_t . A bank can receive one unit of the asset in exchange for one unit of cash which it has to deliver with probability λ_t only. Hence, a bank can acquire more than one unit of the asset by offering one unit of cash.⁶

Definition 2 *A feasible mechanism is a mechanism $(x_t, \tau_t, \tilde{x}_t, \lambda_t)_{t=1,2}$ that satisfies (3), (4) and (5).*

An implication from (3) and (4) is

$$p_t \equiv \frac{\tilde{x}_t}{\lambda_t} = \frac{x_t}{\tau_t}. \quad (6)$$

The quantities \tilde{x}_t/λ_t and x_t/τ_t can be interpreted as the price of cash or as the value of cash. A bank that delivers cash in exchange for assets receives \tilde{x}_t assets and has to provide one unit of cash with probability λ_t . With risk neutral agents, this is similar to a trade, where a bank receives \tilde{x}_t/λ_t assets for one unit of cash. Similarly, a bank in need of cash delivers x_t assets and receives one unit of cash with probability τ_t . Again, with risk neutral agents this is similar to a trade, where a bank delivers x_t/τ_t assets for one unit of cash. Thus, feasibility implies that the price of cash is the same for cash suppliers and cash demanders.

Note that our trading mechanism encompasses the mechanism applied in GY. In their paper, $\tau_t = \lambda_t = 1$ which reduces the set of feasible mechanisms and leads to liquidity hoarding.

3 Decisions

We now characterize the banks' decisions in period $t = 1, 2, 3$.

Period 0 In period 0, banks choose whether to consume their cash holdings or not. Let $0 \leq \alpha \leq 1$ denote the measure of illiquid banks. The α illiquid banks end period 0 with portfolio $\{1, 0\}$ and the $(1 - \alpha)$ liquid banks with portfolio $\{1, 1\}$.

⁶The reader might ask the question how such a mechanism could be implemented in reality. Imagine that ATM machines can be programmed to randomly deliver assets and cash. A cash demander inserts x units of the asset into the ATM machine and the ATM machine ejects one unit of cash with probability τ . The cash supplier inserts one unit of cash and the ATM machine ejects \tilde{x} units of assets and one unit of cash with probability $1 - \lambda$.

We think 'gambling for resurrection' has this flavour (see, for instance, Dewatripont and Tirole, 1994). A distressed bank has an incentive to gamble if there is a chance to survive.

Period 1 Consider a mechanism $(x_1, \tau_1, \tilde{x}_1, \lambda_1)$ for period 1. At the beginning of period 1, a fraction θ_1 of banks receives a liquidity shock. Figure 1 displays the game tree.

First, illiquid banks with a shock in period 1 have measure $\alpha\theta_1$ and pre-trade portfolio $\{1, 0\}$. To avoid default, they need to acquire cash in period 1. If they trade, they have portfolio $\{1 - x_1, 0\}$ with probability τ_1 and portfolio $\{0, 0\}$ with probability $(1 - \tau_1)$. Otherwise, they default and the portfolio is $\{0, 0\}$.

Second, illiquid banks without a shock in period 1 have measure $\alpha(1 - \theta_1)$. Since they hold no cash, they remain inactive and end period 1 with portfolio $\{1, 0\}$.

Third, liquid banks with a shock in period 1 have measure $(1 - \alpha)\theta_1$ and pre-trade portfolio $\{1, 1\}$. Since they hold cash, they can avoid default by using their own cash holdings and end period 1 with portfolio $\{1, 0\}$.

Finally, liquid banks with no shock in period 1 have measure $(1 - \alpha)(1 - \theta_1)$ and pre-trade portfolio $\{1, 1\}$. They can either buy assets in exchange for cash in period 1 or not trade. GY call the former banks *buyers* and the latter banks *hoarders* and we will keep their language.

Buyers are the potential suppliers of cash in period 1. With probability λ_1 they have to deliver one unit of cash and they end period 1 with portfolio $\{1 + \tilde{x}_1, 0\}$. We call these banks λ_1 -*buyers*. With probability $(1 - \lambda_1)$ they don't need to deliver cash and they end period 1 with portfolio $\{1 + \tilde{x}_1, 1\}$. We call these banks $(1 - \lambda_1)$ -*buyers*. Hoarders end period 1 with portfolio $\{1, 1\}$.

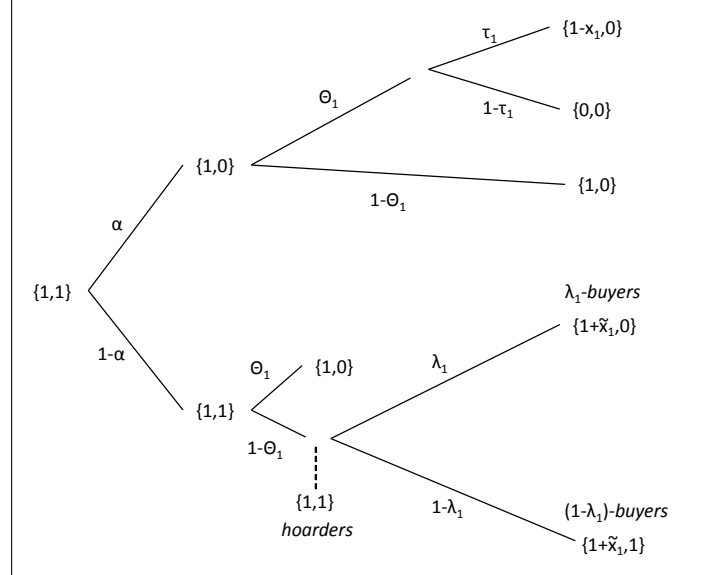


Figure 1: Event tree for $t = 1$

We will focus on equilibria, where all banks are willing to trade. Hence, ‘not trading’ is an out-of-equilibrium action. The ‘not trading’ decision which is labelled as ‘hoarding’ in GY is indicated by the dashed line in figure 1. All other possible ‘not trading’ actions are ignored for the moment (‘not trading’ is a possible action at any node of the tree, where a trading decision has to be made).

Period 2 Consider a mechanism $(x_2, \tau_2, \tilde{x}_2, \lambda_2)$ for period 2. At the beginning of period 2, a fraction θ_2 of banks receive a liquidity shock. Banks with a shock in the previous period remain inactive in period 2, since they hold no cash. Figure 2 displays the game tree for period 2.

First, illiquid banks with no shock in period 1, but with a shock in period 2 have measure $\alpha(1 - \theta_1)\theta_2$ and pre-trade portfolio $\{1, 0\}$. To avoid default, they need to acquire cash in period 2. If they trade, they have portfolio $\{1 - x_2, 0\}$ with probability τ_2 and portfolio $\{0, 0\}$ with probability $(1 - \tau_2)$. Otherwise, they default and the portfolio is $\{0, 0\}$.

Second, illiquid banks with no shock in period 1 and 2 have measure $\alpha(1 - \theta_1)(1 - \theta_2)$. Since they hold no cash, they remain inactive and end period 2 with portfolio $\{1, 0\}$.

Third, λ_1 -buyers with a shock in period 2 have measure $(1 - \alpha)(1 - \theta_1)\lambda_1\theta_2$ and pre-trade portfolio $\{1 + \tilde{x}_1, 0\}$. They need to acquire one unit of cash in period 2 to avoid default. If they trade, they have portfolio $\{1 + \tilde{x}_1 - x_2, 0\}$ with probability τ_2 and portfolio $\{0, 0\}$ with probability $(1 - \tau_2)$. If they don’t trade, they default and the portfolio is $\{0, 0\}$.

Fourth, λ_1 -buyers with no shock in period 2 have measure $(1 - \alpha)(1 - \theta_1)\lambda_1(1 - \theta_2)$. Since they hold no cash, they remain inactive and end period 2 with portfolio $\{1 + \tilde{x}_1, 0\}$.

Fifth, $(1 - \lambda_1)$ -buyers with a shock in period 2 have measure $(1 - \alpha)(1 - \theta_1)(1 - \lambda_1)\theta_2$ and pre-trade portfolio $\{1 + \tilde{x}_1, 1\}$. Since they hold cash, they can avoid default by using their own cash holdings and end period 2 with portfolio $\{1 + \tilde{x}_1, 0\}$.

Sixth, $(1 - \lambda_1)$ -buyers with no shock in period 2 have measure $(1 - \alpha)(1 - \theta_1)(1 - \lambda_1)(1 - \theta_2)$ and pre-trade portfolio $\{1 + \tilde{x}_1, 1\}$. If they trade, they have portfolio $\{1 + \tilde{x}_1 + \tilde{x}_2, 0\}$ with probability λ_2 and portfolio $\{1 + \tilde{x}_1 + \tilde{x}_2, 1\}$ with probability $(1 - \lambda_2)$.

Finally, consider the hoarders (recall, this is out-of-equilibrium) with pre-trade portfolio $\{1, 1\}$. If they receive a shock in period 2, they use their cash holdings to meet their required payment and end the period with portfolio $\{1, 0\}$. If they do not receive a liquidity shock they can supply cash. If they trade, they have portfolio $\{1 + \tilde{x}_2, 0\}$ with probability λ_2 and portfolio $\{1 + \tilde{x}_2, 1\}$ with probability $(1 - \lambda_2)$.

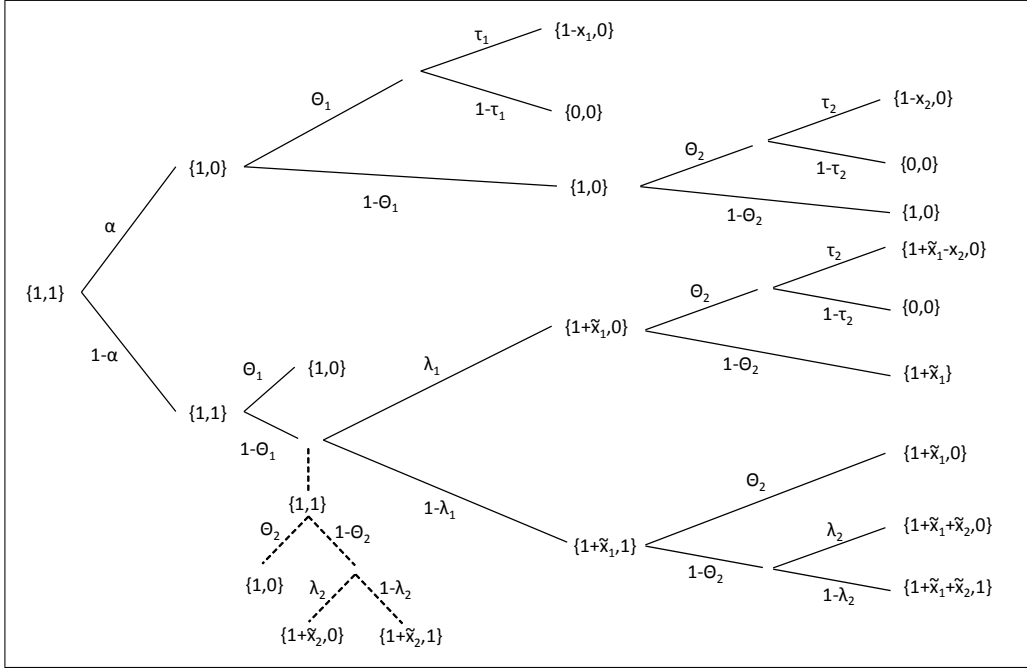


Figure 2: Event tree for $t = 1, 2$

Period 3 In period 3, the return of the various portfolios carried forward from period 2 realizes. Each bank that has not yet received a liquidity shock meets its required payment either from the realization of the asset return or the cash holdings. Then, the economy ends.

4 An efficient incentive-feasible mechanism

An efficient incentive-feasible mechanism is a feasible mechanism that satisfies participation constraints. The participation constraints require that banks are willing to go along with the proposed mechanism; i.e., we allow them to opt out of a mechanism. Efficiency requires that there is no waste of assets or cash.

4.1 An efficient incentive-feasible mechanisms for period 1

Participation constraints in period 1 The supply of cash comes from liquid banks without a shock in period 1. They can either buy assets or hoard. They are willing to buy and hence to supply cash if

$$X_1^S \geq Y_2^S, \quad (7)$$

where X_1^S is a buyer's expected payoff and Y_1^S is a hoarder's expected payoff.

The demand for cash comes from the illiquid banks with a shock in period 1. They are willing to sell assets in exchange for cash if

$$X_1^D \geq Y_1^D, \quad (8)$$

where X_1^D is the expected payoff for trading and Y_1^D the expected payoff of default.

In what follows and without loss in generality, we restrict our attention to incentive-feasible mechanisms. Hence, all banks accept the mechanism. Later on we will verify that the participation constraints (7) and (8) are satisfied.

Feasibility in period 1 Feasibility in period 1 requires that the measure of banks which receives one unit of cash is equivalent to the measure of banks which delivers one unit of cash. If (7) and (8) hold, then from (3), feasibility in period 1 requires that

$$(1 - \alpha)(1 - \theta_1)\lambda_1 = \alpha\theta_1\tau_1. \quad (9)$$

The left-hand side of equation (9) represents the cash supply in period 1: $(1 - \alpha)(1 - \theta_1)$ is the measure of buyers that provide cash with probability λ_1 . The right-hand side of equation (9) represents the cash demand in period 1: $\alpha\theta_1$ is the measure of illiquid banks with a shock in period 1 that receive cash with probability τ_1 .

From (4), feasibility also requires that the quantity of assets delivered is equal to the quantity of assets received. Then, feasibility in period 1 requires also that

$$(1 - \alpha)(1 - \theta_1)\tilde{x}_1 = \alpha\theta_1x_1. \quad (10)$$

The left-hand side of equation (10) represents the assets received: $(1 - \alpha)(1 - \theta_1)$ is the measure of buyers and each of them receives \tilde{x}_1 units of assets. The right-hand side of equation (10) represents the assets delivered: $\alpha\theta_1$ is the measure of illiquid banks with a shock in period 1 and each of them delivers x_1 units of assets.

Finally, from (5), feasibility also requires that

$$x_1, \tau_1, \lambda_1 \leq 1 \quad (11)$$

As discussed above, there is no such constraint on \tilde{x}_1 , since a buyer can receive more than one unit of assets in exchange for one unit of cash.

An implication from (9) and (10) is

$$p_1 \equiv \frac{\tilde{x}_1}{\lambda_1} = \frac{x_1}{\tau_1}.$$

Hence, feasibility implies that the price in period 1 is the same for cash demanders and cash suppliers.

Incentive-feasible mechanism for period 1 An incentive-feasible mechanism for period 1 is a mechanism $(x_1, \tau_1, \tilde{x}_1, \lambda_1)$ that satisfies the participation constraints (7) and (8) and the feasibility conditions (9) through (11).

It is important to note that there are many mechanisms for period 1 that will satisfy these conditions. In what follows, we will choose an incentive-feasible mechanism that will get us as close to an efficient allocation as possible.

An efficient incentive-feasible mechanism for period 1 Efficiency in period 1 requires that no assets and no cash holdings are wasted.

Assets are wasted if a bank that holds assets defaults. Recall that in GY, by assumption the asset holdings of a defaulting bank are liquidated and the liquidation cost is equal to the remaining portfolio value. There are two possibilities to avoid a waste of assets in period 1. First, if each illiquid bank with a liquidity shock receives one unit of cash, there is no default and no assets are wasted.⁷ Second, if all assets are transferred from illiquid banks to liquid banks, then default is not costly, since defaulting banks hold portfolio $\{0, 0\}$. Cash is wasted if it is used to save a portfolio with zero value.

We will restore efficiency with a mechanism that transfers all assets from illiquid banks to liquid banks and which ensures that all cash rest with the liquid banks. In particular, consider the mechanism

$$(x_1, \tau_1, \tilde{x}_1, \lambda_1) = \left(1, 0, \frac{\alpha\theta_1}{(1-\alpha)(1-\theta_1)}, 0\right). \quad (12)$$

This mechanism is efficient, since all assets are transferred from illiquid to liquid banks; i.e., $x_1 = 1$, which implies from (10) that $\tilde{x}_1 = \frac{\alpha\theta_1}{(1-\alpha)(1-\theta_1)}$. Furthermore, no cash is wasted. That is, all cash should rest with the liquid banks, since they might have a need for it in period 2. This is attained by setting $\lambda_1 = 0$, which implies from (9) that $\tau_1 = 0$. Furthermore, it is easy to verify that the mechanism is feasible, since it satisfies conditions (9) through (11).

4.2 An efficient incentive-feasible mechanisms for period 2

Now, we derive an incentive-feasible mechanism for period 2 given that the mechanism in period 1 satisfies (12).

Participation constraints in period 2 The supply of cash comes from the $(1 - \lambda_1)$ -buyers without a shock in period 2. They are willing to supply cash in exchange

⁷Since in GY, one unit of cash has to be delivered in order to get assets, the only way to obtain efficiency in market 1 is to transfer $\alpha\theta_1$ units of cash to the $\alpha\theta_1$ illiquid banks. If there is not enough cash, efficiency cannot be obtained.

for assets if

$$X_2^S \geq Y_2^S, \quad (13)$$

where X_2^S is the expected payoff of trading and Y_2^S the expected payoff of remaining inactive.

The demand for cash comes from illiquid banks with a shock in period 2. They are willing to supply assets in exchange for cash if

$$X_2^D \geq Y_2^D, \quad (14)$$

where X_2^D is the expected payoff for trading and Y_2^D the expected payoff of default.

In what follows and without loss in generality, we restrict our attention to incentive-feasible mechanisms. Hence, all banks accept the mechanism. Later on we will verify that the participation constraints (13) and (14) are satisfied.

Feasibility in period 2 Feasibility in period 2 requires that the measure of banks which receives one unit of cash is equivalent to the measure of banks which delivers one unit of cash. If (13) and (14) hold, then, feasibility requires that

$$(1 - \alpha)(1 - \theta_1)(1 - \lambda_1)(1 - \theta_2)\lambda_2 = \alpha(1 - \theta_1)\theta_2\tau_2.$$

Given the mechanism for period 1, $\lambda_1 = 0$. Then, the above equation simplifies to

$$(1 - \alpha)(1 - \theta_1)(1 - \theta_2)\lambda_2 = \alpha(1 - \theta_1)\theta_2\tau_2. \quad (15)$$

The left-hand side of equation (15) represents the cash supply in period 2: $(1 - \alpha)(1 - \theta_1)(1 - \theta_2)$ is the measure of liquid banks that provide cash with probability λ_2 . The right-hand side of equation (15) represents the cash demand in period 2: $\alpha(1 - \theta_1)\theta_2$ is the measure of illiquid banks with a shock in period 2 that receive cash with probability τ_2 .

From (4), feasibility also requires that the quantity of assets delivered is equal to the quantity of assets received. Then, feasibility in period 2 also requires that

$$(1 - \alpha)(1 - \theta_1)(1 - \lambda_1)(1 - \theta_2)\tilde{x}_2 = \alpha(1 - \theta_1)\theta_2x_2.$$

Again, taking into account that $\lambda_1 = 0$, the above equation yields

$$(1 - \alpha)(1 - \theta_1)(1 - \theta_2)\tilde{x}_2 = \alpha(1 - \theta_1)\theta_2x_2. \quad (16)$$

The left-hand side of equation (16) represents the assets received: $(1 - \alpha)(1 - \theta_1)(1 - \theta_2)$ is the measure buyers and each of them receives \tilde{x}_2 units of assets. The right-hand side of equation (16) represents the assets delivered: $\alpha(1 - \theta_1)\theta_2$ is the measure of illiquid banks with a shock in period 2 and each of them delivers x_2 units of assets.

Finally, feasibility also requires that

$$x_2, \tau_2, \lambda_2 \leq 1. \quad (17)$$

Again, as discussed above, there is no such constraint on \tilde{x}_2 , since a buyer can receive more than one unit of assets in exchange for one unit of cash.

An implication from (15) and (16) is

$$p_2 \equiv \frac{\tilde{x}_2}{\lambda_2} = \frac{x_2}{\tau_2}.$$

As for period 1, feasibility implies that the price in period 2 is the same for cash demanders and cash suppliers.

Incentive-feasible mechanisms for period 2 An incentive-feasible mechanism for period 2 is a mechanism $(x_2, \tau_2, \tilde{x}_2, \lambda_2)$ that satisfies the participation constraints (13) and (14) and the feasibility conditions (15) through (17).

As for period 1, there are many mechanisms that will satisfy these conditions. In what follows, we will choose an incentive-feasible mechanism that will get us as close to an efficient allocation as possible.

An efficient incentive-feasible mechanism for period 2 As for period 1, efficiency in period 2 requires that no assets and no cash holdings are wasted.

Again, assets are wasted because of the liquidation costs of default. Hence, to attain efficiency set $x_2 = 1$ which implies that $\tilde{x}_2 = \frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)}$. Furthermore, as in period 1 no cash should be wasted. That is, all the cash should rest with the liquid banks, since they have a need for it in period 3. This can be attained by setting $\lambda_2 = 0$ which implies from (15) that $\tau_2 = 0$. Hence, consider the mechanism

$$(x_2, \tau_2, \tilde{x}_2, \lambda_2) = (1, 0, \frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)}, 0). \quad (18)$$

It is easy to verify that by construction it satisfies (15) through (17).

4.3 Efficiency in period 1 and 2

The efficient incentive-feasible mechanism (12) and (18) satisfies (9) through (11) and (15) through (17). Moreover, it satisfies participation constraints (7), (8), (13) and (14). The mechanism is efficient, since no assets and no cash holdings are wasted.

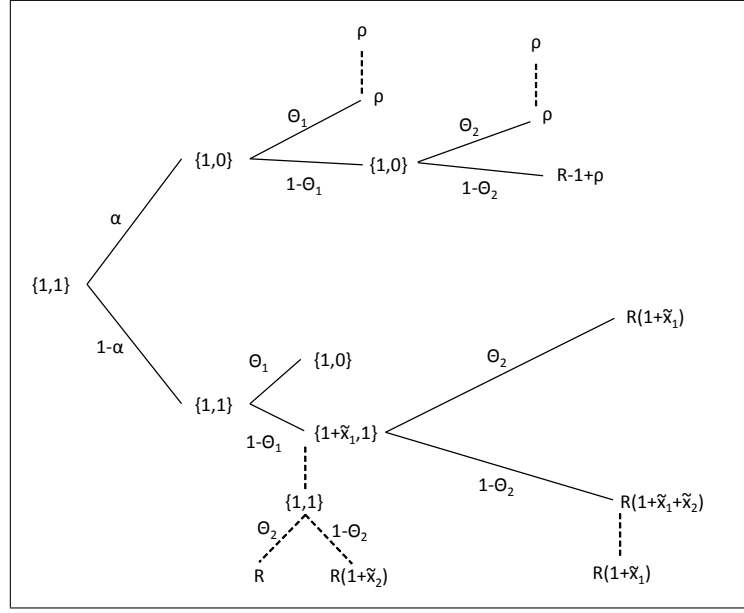


Figure 3: Final payoffs

Figure 3 displays the reduced game tree and the final payoffs given the mechanisms (12) and (18) for period 1 and 2. The reduced game tree in Figure 3 also includes all ‘not trading’ decisions indicated by the dashed lines. Note that the fact that no cash has to be delivered ($\lambda_t = 0$) simplifies the analysis of the game tree considerably.

Proposition 3 *Given α , the mechanism*

$$(x_t, \tau_t, \tilde{x}_t, \lambda_t)_{t=1,2} = \left(1, 0, \frac{\alpha \theta_t}{(1-\alpha)(1-\theta_t)}, 0\right)_{t=1,2} \quad (19)$$

is an efficient incentive-feasible mechanism for period 1 and 2.

Proof. From the above derivations we know that the mechanisms for period 1 and 2 are feasible and efficient. In what follows, we will show that the proposed mechanism also satisfies the participation constraints (7), (8), (13) and (14).

Consider first, (7). With our mechanisms for the two periods

$$\begin{aligned} X_1^S &= \int_0^1 \{\theta_2 R(1 + \tilde{x}_1) + (1 - \theta_2) R(1 + \tilde{x}_1 + \tilde{x}_2)\} f(\theta_2) d\theta_2 \\ Y_1^S &= \int_0^1 \{\theta_2 R + (1 - \theta_2) R(1 + \tilde{x}_2)\} f(\theta_2) d\theta_2, \end{aligned}$$

where X_1^S represents the expected utility of a buyer and Y_1^S the expected utility of a hoarder. It is easy to see that (7) is always satisfied. Note that this is simply a statement that no hoarding occurs with our mechanism.

Second, (8). With our mechanism (19) for the two periods

$$\begin{aligned} X_1^D &= \tau_1 R(1 - x_1) = 0 \\ Y_1^D &= 0, \end{aligned}$$

where X_1^D represents the expected utility an illiquid bank with a shock which trades and Y_1^D represents the expected utility of the same bank which does not trade and consequently defaults. Given (19), the former group of banks has expected payoff $\tau_1 R(1 - x_1) = 0$, since $\tau_1 = 0$ and $x_1 = 1$. Hence, also (8) is satisfied.⁸

Third, (13). With our mechanisms for the two periods

$$\begin{aligned} X_2^S &= R(1 + \tilde{x}_1 + \tilde{x}_2) \\ Y_2^S &= R(1 + \tilde{x}_1). \end{aligned}$$

Since (7) is satisfied, there are no hoarders. Hence, X_2^S represents the expected payoff of a buyer without a liquidity shock in either period which decides to trade. Y_2^S is the expected payoff of a buyer without a liquidity shock in either period which decides not to trade. It is straightforward to see that (13) is satisfied.

Finally, consider (14). With our mechanisms for the two periods

$$\begin{aligned} X_2^D &= \tau_2 R(1 - x_2) = 0 \\ Y_2^D &= 0. \end{aligned}$$

Again, since (8) is satisfied, there are only illiquid banks with a shock in period 2 which demand cash. They can either accept the mechanism (19) or not and consequently default. Banks which accept have expected payoff $\tau_2 R(1 - x_2) = 0$, since $\tau_2 = 0$ and $x_2 = 1$ given (19). Again, following the same argumentation from above, the participation constraint (14) is satisfied. This concludes the proof. ■

5 Portfolio Choice

Proposition 3 describes an efficient mechanism given α . In what follows, we derive the portfolio choice of banks in period 0.

⁸If an illiquid bank does not trade, it defaults and has a continuation payoff is zero. So, it is indifferent between accepting the mechanism (19) and default. It is standard in game theory that the illiquid bank is willing to accept this trade since we can always offer some additional marginally small utility.

In period 0, a fraction α of banks becomes illiquid and a fraction $(1 - \alpha)$ of banks remains liquid. For $0 < \alpha < 1$, banks must be indifferent between the two alternatives. In what follows, we show under which conditions this must hold. In order to so, we compare the expected payoff of an illiquid bank and the expected payoff of a liquid bank under the efficient mechanism (19).

The expected utility of a liquid bank is

$$\begin{aligned} \Psi_{LI} \equiv & \int_0^1 \int_0^1 \{ \theta_1 R + (1 - \theta_1) [\theta_2 R(1 + \tilde{x}_1) \\ & + (1 - \theta_2) R(1 + \tilde{x}_1 + \tilde{x}_2)] \} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2. \end{aligned}$$

With probability θ_1 , a liquid bank receives a liquidity shock in period 1. The bank can avoid default by using its own cash holdings and carry forward the remaining portfolio to period 3, where the return R is realized. With probability $(1 - \theta_1)\theta_2$, a liquid bank receives a liquidity shock in period 2. In period 1, \tilde{x}_1 units of assets were acquired and given (19), no cash had to be delivered in exchange. Hence, the bank can avoid default by using its own cash holdings and carry forward the remaining portfolio to period 3, where the return $R(1 + \tilde{x}_1)$ is realized. Finally, with probability $(1 - \theta_1)(1 - \theta_2)$, a liquid bank receives no liquidity shock in period 1 and 2. In periods 1 and 2, \tilde{x}_1 and \tilde{x}_2 units of assets were acquired and given (19), no cash had to be delivered in exchange. In period 3, the bank can make the required repayment by using its own cash holdings and hence the return $R(1 + \tilde{x}_1 + \tilde{x}_2)$ is realized.

The expected utility of an illiquid bank is

$$\Psi_{IL} \equiv \rho + \int_0^1 \int_0^1 \{ (1 - \theta_1)(1 - \theta_2)(R - 1) \} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2. \quad (20)$$

An illiquid bank receives ρ from consuming its one unit of cash in period 0. In period 1 and 2, the surplus is zero so that only if no shock is experienced in both periods the bank receives payoff $(R - 1)$.

The expected payoffs are equal if $\Psi_{IL} = \Psi_{LI}$.

Proposition 4 *There exists a critical value $1 < \rho_0 < R$ such that if $\rho \leq \rho_0$, $\alpha = 0$ and if $\rho > \rho_0$, $\alpha \in (0, 1)$.*

Proof. Using the expressions for \tilde{x}_1 and \tilde{x}_2 from (19) we can rewrite the expected utility of a liquid bank as follows

$$\Psi_{LI} = \int_0^1 \int_0^1 \left\{ R + \frac{\alpha\theta_1}{(1 - \alpha)} R + (1 - \theta_1) \frac{\alpha\theta_2}{(1 - \alpha)} R \right\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2. \quad (21)$$

The expected payoffs are equal if

$$\begin{aligned} & \rho + \int_0^1 \int_0^1 \{(1 - \theta_1)(1 - \theta_2)(R - 1)\} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2 \\ &= \int_0^1 \int_0^1 \left\{ R + \frac{\alpha\theta_1}{(1 - \alpha)}R + (1 - \theta_1)\frac{\alpha\theta_2}{(1 - \alpha)}R \right\} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2. \end{aligned} \quad (22)$$

Note that the left-hand side of (22) is independent of α and the right-hand side is increasing in α , approaching ∞ as $\alpha \rightarrow 1$. Thus, for an interior solution, we need that the right-hand side of (22) at $\alpha = 0$ is smaller than the left-hand side of (22); i.e.,

$$\rho + \int_0^1 \int_0^1 \{(1 - \theta_1)(1 - \theta_2)(R - 1)\} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2 > R.$$

Accordingly, the critical value ρ_0 is defined by

$$\rho_0 = R - \int_0^1 \int_0^1 \{(1 - \theta_1)(1 - \theta_2)(R - 1)\} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2$$

Hence, if $\rho > \rho_0$, $0 < \alpha < 1$ must hold in equilibrium. Otherwise, if $\rho \leq \rho_0$, $\alpha = 0$ must hold in equilibrium, since banks prefer to remain liquid in period 0. ■

Note that an interior value for α is more likely if ρ increases or R decreases. Thus, as in GY, if ρ is not too high relative to R , $\alpha \in (0, 1)$.

6 Discussion

In this section, our results from above are compared with the findings of GY.

6.1 Fire-sales, liquidity hoarding and efficiency

In GY, fire-sales are the source of liquidity hoarding. Liquid banks acquire assets in period 1 by selling cash. If these banks receive a liquidity shock in period 2, they are able to offer more assets in exchange for one unit of cash than was feasible in period 1. Hence, they will drive down asset prices so that assets are traded at fire-sale prices. As a result, if the expected asset price is lower in period 2 than in period 1, liquid banks react by hoarding cash in period 1.⁹

⁹On page 293, GY explain the connection between fire-sales and hoarding as follows: “Asset-price volatility results from the use of the asset market as a source of liquidity. When liquid bankers first supply cash in exchange for assets, they create an imbalance in the system. They are increasing their holdings of illiquid assets and reducing their holding of liquid assets. If these large, illiquid bankers are subsequently hit by a liquidity shock, they have even more assets to dump on the market, producing a greater fire sale and reducing asset prices further. A laissez-faire equilibrium is inefficient because the incentive to hoard are simply too high.”

Our results are very different. With our mechanism, no hoarding occurs since all liquid banks trade in period 1. Furthermore, our mechanism is efficient, since no assets and no cash is wasted. The origin of these contradictory results is the absence of lotteries in the pricing of assets in GY. To see this, define the price of assets as follows (which is of course simply the inverse price of cash p_t defined in (6)):

$$q_t \equiv \frac{\lambda_t}{\tilde{x}_t} = \frac{\tau_t}{x_t}. \quad (23)$$

In period 1, the asset price in GY is restricted to $q_1 \geq 1$. To see this, set $\lambda_t = \tau_t = 1$ in (23) and note that from (9) through (11) it follows that $\tilde{x}_t = x_t \leq 1$. In contrast, the expected asset price in period 2 can be smaller than one; i.e., $E(q_2) < q_1$. In this case, liquid banks hoard liquidity in anticipation for a lower asset price in period 2. With our mechanism, the asset price in period 1 can fall below one, since λ_1 and τ_1 can be smaller than one. If assets are sold at a fire-sale price in period 1, there is no reason to wait for a fire-sale price in period 2.

Interestingly, our results imply that fire-sales are efficient. They are needed to allocate all assets from the illiquid banks to the liquid banks. It is clear that our mechanism allows for a fire-sale asset price in period 1 ($q_1 \geq 0$), while a fire-sale price in GY is not feasible ($q_1 \geq 1$). In contrast, fire-sale prices are possible in GY as well in period 2 ($q_2 \geq 0$). Table 1 summarizes this relation of GY and our solution (BM).

	$t = 1$	$t = 2$	Efficient
GY	Liquidity hoarding ($q_1 \geq 1$)	Fire-sale prices ($q_2 \geq 0$)	No
BM	Fire-sale prices ($q_1 \geq 0$)	Fire-sale prices ($q_2 \geq 0$)	Yes

Table 1: Comparison of GY and BM

6.2 Welfare and market liquidity

We define welfare $W(\alpha)$ to be the expected utility of a bank at the beginning of period 0. Since the fraction of illiquid banks is α and the fraction of liquid banks is $(1 - \alpha)$, we have

$$W(\alpha) = \alpha \Psi_{IL} + (1 - \alpha) \Psi_{LI}, \quad (24)$$

where Ψ_{IL} and Ψ_{LI} are defined in (20) and (21), respectively.¹⁰

In what follows, we will discuss welfare and market liquidity. Given the mechanism (19), two important questions arise: First, is our mechanism welfare maximizing and second, what is the welfare maximizing level of liquidity in this economy?

¹⁰It can be verified that for $\alpha \in (0, 1)$, $W(\alpha) = \Psi_{IL} = \Psi_{LI}$.

Simplifying (24) yields

$$W(\alpha) = R + \alpha\rho - \alpha \int_0^1 \int_0^1 (1 - \theta_1)(1 - \theta_2)f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2. \quad (25)$$

$W(\alpha)$ is increasing in α . Hence, it is optimal to set α as close as possible to one, which is equivalent to say that the entire stock of cash should be consumed in the initial period.¹¹ For $\alpha = 1$, we get

$$W(1) = R + \rho - \int_0^1 \int_0^1 (1 - \theta_1)(1 - \theta_2)f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2.$$

Note that this expression is identical to the planner's solution W^P defined in (2), but for the term

$$\int_0^1 \int_0^1 (1 - \theta_1)(1 - \theta_2)f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2.$$

This term reflects that some illiquid banks experience no shock in period 1 and 2, but have to make a payment of one unit of cash in period 3. They can do so from the realized return of their asset holdings.

The planner avoids this payment by redistributing the asset holdings from illiquid banks with no shock in period 1 and 2 to those banks that already experienced a liquidity shock and let the former banks default. Our mechanism cannot do this because there is no interbank market in period 3.

Again, this reveals two interesting aspects. First, in GY, the market solution implies that the aggregate level of cash in the economy is too low compared to their constrained-efficient planner solution, whereas our aggregate level of liquidity is too high compared to the planner solution. Second, in addition to the usual feasibility constraints, the planner in GY operates under the constraint that he cannot transfer assets between banks.¹² Note that lotteries allow transferring assets exactly in the way that GY restrict the planner's allocation.

6.3 Welfare without liquidity shocks in period 3

Here, we maintain all assumption, but we assume that in period 3, there are no costs of maintaining the portfolio. Hence, with this assumption banks that had no liquidity shock in period 1 or 2 receive the return of their asset holdings and don't need to repay one unit of cash in period 3. In this case, the expected utility of an illiquid bank is

$$\rho + \int_0^1 \int_0^1 \{(1 - \theta_1)(1 - \theta_2)R\} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2,$$

¹¹When we set $\alpha = 1$, we effectively mean $\alpha \rightarrow 1$, since for the mechanism to work, the stock of cash has to be strictly positive, but arbitrarily small.

¹²See Footnote 5.

and the expected utility of a liquid bank is

$$\int_0^1 \int_0^1 \left\{ R + \frac{\alpha\theta_1}{(1-\alpha)}R + (1-\theta_1)\frac{\alpha\theta_2}{(1-\alpha)}R + (1-\theta_1)(1-\theta_2) \right\} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2.$$

Hence, the welfare function is represented by the following expression

$$\begin{aligned} W(\alpha) = & \alpha \left(\rho + \int_0^1 \int_0^1 \{ (1-\theta_1)(1-\theta_2)R \} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2 \right) \\ & + (1-\alpha) \left(\int_0^1 \int_0^1 \left\{ R + \frac{\alpha\theta_1}{(1-\alpha)}R \right. \right. \\ & \left. \left. + (1-\theta_1)\frac{\alpha\theta_2}{(1-\alpha)}R + (1-\theta_1)(1-\theta_2) \right\} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2 \right), \end{aligned}$$

which can be simplified to yield

$$W(\alpha) = \alpha\rho + R + \int_0^1 \int_0^1 \{ (1-\alpha)(1-\theta_1)(1-\theta_2) \} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2.$$

It is easy to see that if $\alpha = 1$,

$$W(1) = \rho + R,$$

which equals the welfare level that the planner can achieve (see (2)).

6.4 Adding back creditors into the welfare criterion

In an earlier version of the paper (GY, 2011), GY characterized the planner solution under the assumption that the initial cash holdings had to be borrowed from a creditor. In that case, the liquidity shock was modelled as the random demand of the creditor for repayment of a callable bond. Creditors, as in Diamond and Dybvig (1983), are uncertain about their time preferences, but they want to consume at precisely one of the dates $t = 1, 2, 3$. With probability θ_1 , the creditor wants to consume in period 1, with probability $(1-\theta_1)\theta_2$ in period 2 and with probability $(1-\theta_1)(1-\theta_2)$ in period 3. The creditor's expected utility function is given by

$$V(c_1, c_2, c_3) = \theta_1 c_1 + (1-\theta_1)\theta_2 c_2 + (1-\theta_1)(1-\theta_2)c_3.$$

If the creditors enter the planner's objective function, welfare under the first-best allocation satisfies

$$W_{Creditors}^P = R + \rho - 1. \quad (26)$$

Here, the term $\rho - 1$ reflects the fact that consuming one unit of cash in the initial period, but not paying it back to the creditor, yields the net surplus of $\rho - 1$ to society (without creditors, the net surplus is just ρ ; see (2)).

Note that adding back creditors does neither affect the pricing nor the demand and supply in period 1 and 2, respectively. Hence, our analysis continues to hold. However, the welfare calculation needs some adjustments. In particular, with our market solution $\Delta \equiv \alpha \int_0^1 \int_0^1 [\theta_1 + (1 - \theta_1)\theta_2] f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2$ banks default and they don't repay one unit of cash to the creditor, and so, welfare is reduced by $-\Delta$. Accordingly, the welfare function (25) needs to be adjusted as follows:

$$W(\alpha) = R + \alpha\rho - \alpha \int_0^1 \int_0^1 \{(1 - \theta_1)(1 - \theta_2) - [\theta_1 + (1 - \theta_1)\theta_2]\} f_1(\theta_1)f_2(\theta_2)d\theta_1d\theta_2.$$

This expression can be simplified to

$$W(\alpha) = \alpha(\rho - 1) + R,$$

which, for $\alpha = 1$, satisfies

$$W(1) = \rho + R - 1,$$

which equals the welfare level with creditors that the planner can achieve (see (26)).

7 Conclusion

We generalize the interbank market model of GY which features indivisible cash and divisible assets by introducing randomized trading (lotteries). We derive an efficient mechanism under which no hoarding occurs. Rather, the economy is characterized by fire-sale asset prices. Counterintuitive, fire-sales ensure efficiency in the GY framework. We also find that with our efficient mechanism, too much liquidity is provided by the market compared to the planner solution, while the market liquidity in GY is insufficient compared to the social optimum.

During the recent financial crisis, markets for liquidity were subject to severe stress which heavily impaired the ability of banks to transform illiquid portfolios into liquid portfolios. As a result, a number of banks became illiquid or even insolvent. Our results suggest that the mechanism proposed by GY might not be at the origin of these phenomena. Rather, we speculate that private information problems generated severe counterparty risks that made liquidity hoarding an optimal choice. However, our believe needs to be subject to further research in order to improve our understanding of the markets for liquidity.

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