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Time-scale analysis of sovereign bonds market co-movement in the EU

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Keywords: financial crisis, co-movement, wavelet, sovereign debt crisis, sovereign bonds

JEL: C32, C49, C58, H63

ABSTRACT

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Abstract

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1. Introduction

Since its beginning the Eurozone was created to strengthen the financial and macroeconomic integration between its members. A part of the integration was harmonization of sovereign debt markets which resulted in a high degree of co-movement between yields of sovereign bonds (Laopodis, 2008; Gilmore et al., 2010; Missio, 2013). After the fall of Lehman Brothers the financial and banking crisis spread through the whole world and theories of risk management were shaken. Another shock came later. Greece fiscally undisciplined Eurozone member state admitted that budget deficit will be 12.5% of GDP. Later it was revealed that the government accounting was falsified and the true volume of public debt was systematically lowered in order to achieve the Euro. Progressively more countries became dangerously close to bankruptcy; Spain, Portugal, Italy, Ireland or more recently Cyprus.

Financial crisis shattered trust of lenders toward corporations as well as banks and Greek revelation acted in a similar way toward states of European Union (EU). Suddenly borrowing via government bonds became more expensive, because a textbook assumption that sovereign bonds of developed states are riskless seemed to be obsolete. Important questions arise: how these events affected co-movement of yields on government bond’s market inside the EU? Was some form of contagion and further disintegration present during the crisis?

There are many ways how to measure co-movement - from simple ones (e.g. sample correlation) to more advanced techniques, which are able to capture dynamics of co-movement in time (e.g. DCC-GARCH or dynamic copula to name a few). All of them have one thing in common - they focus only on time-domain aspect of data and usually ignore frequency-domain, hence the results could not be complete.

As financial markets are highly connected, the financial crisis, which erupted in one country, spreads through various channels to other countries. This phenomenon is called contagion. The main problem is how to distinguish between contagion and a normal co-movement. The question how to detect contagion has been presented in economics since seminal works written by Calvo and Reinhart (1996) and Eichengreen et al. (1996). Gallegati (2012) provides a list of various techniques, which are used for its identification (multivariate GARCH, copula or probit to name a few). Moreover, he points out that standard time-domain techniques have problems with distinguishing between a co-movement and a contagion. On the other hand
the wavelet method has got a good ability to detect contagion because of its properties, which allow us to decompose time series into different scales.

According to Kaminsky et al. (2003) contagion accompanied Mexican crisis in 1994, Russian crisis in 1998 and Asian crisis in 1999. Aloui et al. (2011) study the contagion during global financial crisis in 2008 on stock markets. The topic itself is not fully developed and its theory is not fully established. Even the definition of contagion itself is not unified. For example Pericoli and Sbracia (2003) give five different definitions of a contagion. In our analysis we will use the definition of Forbes and Rigobon (2002): “Contagion occurs if there is significant increase in cross-market linkages after a shock to an individual country (or group of countries)”. This type of contagion is sometimes called shift-contagion because contagion arises from the shift of inter–market linkages. Further, literature distinguishes between fundamental–based and pure contagion. The first type refers to a spread of contagion through real linkages, e.g. according to Kumar and Persaud (2002), through trade links or common external shocks, whereas the pure contagion means that the crisis in one country is spread without the change of the fundamentals of financial markets.

The main goal of our work is to enrich the discussion by employing alternative methodology – a wavelet analysis. This method allows us to decompose time series into different frequency bands (wavelet scales), which can be explored separately. Using wavelets we are able to discriminate between short run and long run co-movement. Moreover, we will employ wavelets for detection of contagion in the EU. We will apply wavelet techniques on 10Y sovereign bond yields of 11 member states of the EU - Greece, Spain, Portugal, Italy, Germany, France, Netherlands, Great Britain, Belgium, Denmark and Sweden. Those time series cover period from 1.1.2001 to 31.12.2013 with daily sampling.

The paper is structured as follows. After literature review focused on co-movement and contagion, in Section 3 we describe dataset and motivate the wavelet analysis. Next, in section 4 we present the results from wavelet coherence analysis. Section 5 is dedicated to contagion analysis within the EU. Finally, Section 6 concludes.

2. Literature Review

With the outbreak of financial and sovereign debt crisis, it is important to study whether there is a significant change in degree of co-movement and in the integration of the market. For example, Dias (2012) studies 10Y
daily data from 2007 to 2010 of 19 countries and compares the results with the previous study by Gilmore et al. (2010), who use monthly data from 1993 to 2008. Dias (2012) shows that sovereign bonds market is shattered into smaller groups (Eurozone Core vs Eurozone Periphery) and the co-movement decreased. It is an important change because the older pre-crisis studies show that before the crisis the market was more integrated. Further, Dias (2012) observe that there is a member state which is the most connected with others - Netherlands. Moreover, Netherlands remains strongly connected with France and Germany even in the crisis period. Conversely, Greece, Portugal, Ireland and Spain became isolated.

Antonakakis (2012) analyzes co-movement of sovereign yields spreads (2007 - 2012) in the EU using multivariate DCC-GARCH. He shows that dynamic conditional correlation of spreads often follows an inverted U-shaped curve - sharp rise and subsequent decrease. Dajčman (2013a) uses a rolling window exceedance correlation approach to demonstrate that correlation of sovereign yields is not symmetric. Moreover, he observes that in the crisis period co-movement decreased. This finding is in accordance with findings obtained by Inoue et al. (2013). They show that conditional correlation between sovereign yields of the states most severely hit by the crisis and Germany significantly decreased. Moreover, co-movement among PIIGS states decreased through the crisis, but co-movement between Spain, Portugal and Belgium remained strong even in the crisis period. Similar results related to the three states are obtained by Claeys and Vašíček (2014) using the factor augmented VAR. Furthermore, they observe higher heterogeneity among non-Eurozone members in comparison with its member states. Christiansen (2014) measures integration of the EU members by explanatory power of a portfolio. She finds that in the crisis integration decreased in the Eurozone. For the new members states, such as Greece, Spain and Portugal the effect is stronger than for the old states.

Next, we focus on sovereign bonds markets contagion in the EU, mainly on works using definition of Forbes and Rigobon (2002). Bhanot et al. (2012) analyze 5Y daily data four years before and after June 2007. They show that unconditional correlation between spreads increased, but analysis of impulse response functions revealed that impact of a shock in one country leads to a significantly smaller impact on yields in another country, which goes against the contagion hypothesis. Dajčman (2013b) argues that standard contagion detection methods based on correlation have an important flaw: they do not distinguish between co-movement of normal and extreme
values. He employs a measure of co-exceedance (large positive changes) based on extreme value theory. The results show high co-exceedance in following cases: Ireland-Portugal, Italy-Spain, France-Italy, and Germany-France. On the other hand, the lowest co-exceedance was observed in cases of France-Portugal and Germany-Portugal. Using 10Y daily data from 1999 to 2012 Gómez-Puig and Sosvilla-Rivero (2014) find new Granger causality patterns in the crisis period plus intensification of the older ones, which the authors interpret as a proof of contagion. They use endogenous structural breaking points for each pair of tested time series. The interesting result is that 2/3 of the breaking points were identified after the day of budget deficit revelation. Missio (2013), using 10Y yields and DCC GARCH, observes sharp increase of short-time conditional correlation after July 2010 between Greece-Spain, Greece-Portugal and Greece-Ireland. Conversely, significant drop between Greece-Germany, Greece-Italy and Greece-Netherlands correlation occurred in the same time. A decrease of correlation took place after the fall of Lehman Brothers. To make the literature review complete several studies using large multi-factor models are listed. Arghyrou and Kontonikas (2012) use panel data method and detected contagion in crisis period, especially in Portugal, Spain and Italy. Study of Mink and De Haan (2013) indicate that news about Greece and its bailout affects sovereign spreads of the Eurozone states. Claeys and Vašíček (2014) demonstrate that bilateral spillovers in the EU are larger in the crisis era, but it does not automatically imply contagion.

3. Data

In our analysis we use daily data of bid yields of 10Y sovereign bonds on a secondary market. A bid yield is a minimal yield demanded by an investor, which makes her to buy a bond and thus it mirrors investor’s attitude toward issuer. All data are from Reuters Wealth Manager database. The dataset cover period from January 1, 2001 to December 31, 2013. We chose sovereign yields of 11 states, preliminarily divided into the three groups:

The core of the Eurozone: Germany, France, Netherlands and Belgium. These countries were the founders of European Communities, their economies are considered to be highly developed.
The periphery of the Eurozone: Greece, Spain, Portugal and Italy. The group is often called by acronym PIIGS or GIIPS\(^1\). They are the states, which problems with government debts.

The states outside the Eurozone: Great Britain, Denmark and Sweden. These three countries are highly developed economies, which stand out of the Eurozone. We chose them to test how the relationship between them differs from others.

In general wavelet transformation does not require stationarity, the process should be only a locally stationary. However, to obtain sensible confidence intervals of wavelet correlation we need data without non-stationary features (Gençay et al., 2001). It is important, as we want to test data for the presence of a contagion based on overlapping intervals. Hence we will use log-differences of the bond yields (Figure 1). Descriptive statistic indicates that no time series of yields is normally distributed. It is not surprising, because high excessive kurtosis is observable in all cases, especially in case of Greece, Spain and Portugal (Table 1). In our work we analyze bond yields and not spreads which is not that common. Spreads are used more likely in larger panel data studies with multiple dependent variables. A major part of the literature defines yield spread as a difference between a yield of a country and a yield of German sovereign bonds and we would like to analyze Germany too, because it is probably the most important economy in the EU. Furthermore, we would like to model “real” yields, which are demanded by investors. A yield spread represents risk premium of a bond, thus practically it is a different variable. Large panel data studies focusing on the contagion use spreads. Literature dealing with bivariate co-movement of yields (without additional explanatory variables) is not numerous. Large panel data studies focused on contagion use spreads. While comparing works of Inoue et al. (2013) and Antonakakis (2012) we see that same method (DCC-GARCH) brings different results.

4. Bond yields co-movement

To see how the financial crisis changed the perception of financial risk in the EU we study behavior of sovereign bond yields co-movement. Further,

\(^1\)The last state is Ireland. Original acronym was PIGS and Ireland was not its part. It obtained this status - in our opinion - undeservely because of bubble, not structural weaknesses. Hence we prefered to analyse only old “members” of PIGS.
we also want to examine influence of various investment horizons of these co-moments, for this reason, we test whether these co-movements are scale (frequency) dependent. To do this we employ wavelet coherence to discover time–scale relationship between the states.\textsuperscript{2} This approach allows us to study exact co-movement and correlation structures dynamics in time as well as scales.

Co-movement of sovereign bond yields is depicted at the wavelet coherency graphs, where the horizontal axis shows time while the vertical axis shows the scales (frequency or period). Low scales in the upper part represent high frequency comovement which is related to short investment horizons of periods of several days, while high scales, located in the lower part of the graph, represent low frequency comovement which is related to long investment horizons of periods from several months to a year. Co-movement intensity is represented by a color scale where dark red means

\textsuperscript{2}For the estimation of wavelet coherence we use Matlab toolbox created by A. Grinsted, J.C. Moore and S. Jevrejeva, http://noc.ac.uk/using-science/crosswavelet-wavelet-coherence
Table 1: Descriptive statistic of sovereign bond yields

<table>
<thead>
<tr>
<th>State</th>
<th>Mean</th>
<th>S.D.</th>
<th>Min</th>
<th>Max</th>
<th>Skew.</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR</td>
<td>0.00013399</td>
<td>0.022344</td>
<td>-0.7155</td>
<td>0.1501</td>
<td>-12.214</td>
<td>371.70</td>
</tr>
<tr>
<td>PT</td>
<td>4.2361e-05</td>
<td>0.015953</td>
<td>-0.3019</td>
<td>0.1590</td>
<td>-1.2580</td>
<td>48.933</td>
</tr>
<tr>
<td>SP</td>
<td>-6.4191e-05</td>
<td>0.014848</td>
<td>-0.2541</td>
<td>0.2578</td>
<td>-0.2881</td>
<td>59.548</td>
</tr>
<tr>
<td>IT</td>
<td>-7.7926e-05</td>
<td>0.012406</td>
<td>-0.1411</td>
<td>0.1081</td>
<td>0.0009</td>
<td>10.031</td>
</tr>
<tr>
<td>FR</td>
<td>-0.00021270</td>
<td>0.013125</td>
<td>-0.0816</td>
<td>0.0750</td>
<td>0.0210</td>
<td>3.8502</td>
</tr>
<tr>
<td>GE</td>
<td>-0.00026995</td>
<td>0.017232</td>
<td>-0.1559</td>
<td>0.0958</td>
<td>-0.0058</td>
<td>5.7125</td>
</tr>
<tr>
<td>NL</td>
<td>-0.00023724</td>
<td>0.014302</td>
<td>-0.1147</td>
<td>0.0748</td>
<td>0.1405</td>
<td>4.1635</td>
</tr>
<tr>
<td>GB</td>
<td>-0.00013988</td>
<td>0.015500</td>
<td>-0.0005</td>
<td>0.1152</td>
<td>0.1463</td>
<td>5.8020</td>
</tr>
<tr>
<td>BE</td>
<td>-0.00021027</td>
<td>0.012604</td>
<td>-0.0817</td>
<td>0.0810</td>
<td>0.2169</td>
<td>4.5969</td>
</tr>
<tr>
<td>DM</td>
<td>-0.00028048</td>
<td>0.018666</td>
<td>-0.2122</td>
<td>0.2913</td>
<td>1.1421</td>
<td>32.246</td>
</tr>
<tr>
<td>SW</td>
<td>-0.00019247</td>
<td>0.015776</td>
<td>-0.1338</td>
<td>0.1407</td>
<td>0.2037</td>
<td>7.8546</td>
</tr>
</tbody>
</table>

The highest intensity and dark blue stands for no relationship. Further, there are hazy areas at the beginning and end of the graph. In these areas the so-called edge effect is present, meaning that the wavelet filters analyze nonexistent data. As a consequence, these small parts of the wavelet coherence estimates are not fully reliable. Torrence and Compo (1998) call these areas the cone of influence. For more details about wavelet coherence see Appendix B. As we use daily data, the numbers on coherence plots on the scale/frequency vertical axis represent periods in days. We do not report scales representing periods higher than 256 days as the results above this level is difficult to interpret since the cone of influence would be excessively large.

During the thirteen years of our dataset we are interested mainly in two specific days crucial for the European debt crisis. First one is September 15, 2008 – the day when the Lehman Brothers went bankrupt and which is considered to be the start of the hot phase of the global financial crisis. The second date, October 20, 2009, is the day when the minister of finance, Papakonstantinou, announced that the budget deficit will be much higher than it was previously expected. Both dates are denoted on the coherence plots with yellow vertical lines.

According to Lane (2012) it was the trigger of the sovereign debt crisis too.
4.1. Intra-group analysis

Wavelet coherence plots of intra-group time-scale relationships reveals that the core states were highly integrated since 2001, see Figure 2 a-f. We observe significant coherence almost on all examined scales (red colored regions), thus strong co-movement on all scales (investment horizons) lasted through the whole non-crisis period. In the crisis era after October 20, 2009 the co-movement temporarily disappeared on lower and medium scales, but remained (at least partially) on higher scales. The exception is the Germany-Netherlands pair where the co-moment change in the crisis period is negligible.

Contrary to the core states we do not see such strong co-movement since 2001 on all scales on the periphery (Figure 2 g-l). When the final stage of the Eurozone was established, co-movement increased on the majority of scales, but we see a larger heterogeneity across scales than in the case of core countries, especially on the low scales, there is lower integration on short investment horizons. After the fall of Lehman Brothers coherence decreased (except Spain–Italy). Possible reason why coherence vanished is that the economies of the Peripheral states were more severely hit by the crisis. Moreover, according to some studies e.g. Hagen et al. (2011), Bernoth & Erdogan (2012) or Beirne and Fratzscher (2013) sensitivity on national macroeconomic determinants increased in the crisis, therefore investors more carefully distinguished between the Eurozone states. De Grauwe & Ji (2013) offer an alternative explanation, that the Peripheral states are trapped in the self–fulfilling prophecies as such they are the victims of negative market sentiments. An effect of the global uncertainty can be presented as well.

Coherence plots of the states outside the Eurozone (Figure 2 m-o) show different story. We see that the non-Eurozone states were less integrated than the core and the periphery before the crisis, but in the crisis period their mutual co-movement seems to be stronger than on the periphery. This finding is in accordance with Clayes & Vasicek (2014). The significance of coherence on the lower scales varies, especially between Sweden and Great Britain. Even on the medium scales (8-32 days) the co-movement does not seem to be strong all the time\(^4\). It is hard to judge if the crisis affected the co-movement. After the bankruptcy of Lehman Brothers a decrease

\(^4\)The possible reason, why co-movement on high frequencies is relatively low and volatile, can be the presence of exchange rate risk.
of coherence can be seen, especially on the medium and high scales, but
the effect is not as strong and persistent as in case of the periphery. The
most interesting is the Sweden–Denmark pair. There is a clearly observable
decrease of coherence after the fall of Lehman Brothers as well as an increase
immediately after October 20, 2009.

4.2. Inter-group analysis

In this section is dedicated to the inter-group co-movement analysis. We
focus mainly on the periphery states. Coherence plots in Figure E.7 show
relationships between sovereign yields of Greece and Belgium, Denmark,
France, Great Britain, Germany, Netherlands and Sweden (panels a-g). In
all these coherence plots similar pattern appears. From 2000 to 2005 almost
no significant co-movement can be measured on short investments horizons,
however in the 8-256 day period co-movement is high. After 2005 economic
boom took place in Greece, thus the co-movement on the low scales in-
creased and co-movement on the others remained high. This indicates that
Greece became more integrated with the other states with the only two ex-
ceptions – Great Britain and Sweden (not surprisingly, as these two states
are outside the Eurozone). Further, after the fall of Lehman Brothers co-
movement sharply decreased on all scales. To conclude, we can see that
the crisis shattered integration of Greek sovereign yields with other states.
Surprisingly, the revelation of Greek’s deficit did not played important role
for the Greek government bond disintegration, however as we show next,
for other states this date is important.

Another interesting co-movement dynamics of the bond yields exhibit
pairs of Spain–France and Spain–Belgium (Figure E.7 h,i). After September
15, 2008 co-movement vanished around the period of 8-32 and 128, followed
by other drop after October 20, 2009 on all scales. This second drop of co-
movement is visible especially on Spain–France coherence plot. Dynamics
of Spain–Belgium pair behaved differently (Figure E.7 n). After the fall of
Lehman Brothers a decrease is observable on the medium scales, but the
overall drop of co-movement is much lower, especially on the high scales.
A large amount of co-movement disappeared after October 20, 2009, with
further gradual decrease in 2012, when massive downgrades of EU member
states (Spain and Belgium included) took place.

Now let us focus on yields of Portugal (Figure E.6 a-g). In 2001 its
yields were more integrated than the yields of Greece and Spain. Except low
scales in the beginning of the examined period, significant co-movement is
observable on all periods up to 256 days. Furthermore, after September 15,
Figure 2: Intra-group coherence: the Core, the Periphery and the states outside Eurozone

(a) France-Belgium  (b) France-Netherlands  (c) Germany-Belgium
(d) Germany-France  (e) Germany-Netherlands  (f) Netherlands-Belgium
(g) Greece-Italy  (h) Greece-Portugal  (i) Greece-Spain
(j) Italy-Portugal  (k) Spain-Italy  (l) Spain-Portugal
(m) Great Britain-Denmark  (n) Great Britain-Sweden  (o) Sweden-Denmark
Figure 3: Inter–group coherence (selected results)

(a) Greece-Belgium  
(b) Greece-Denmark  
(c) Greece-Great Britain  
(d) Greece-Germany  
(e) Spain-France  
(f) Spain-Belgium  
(g) Portugal-Great Britain  
(h) Portugal-Germany  
(i) Portugal-Netherlands  
(j) Italy-Great Britain  
(k) Italy-Germany  
(l) Italy-Belgium
2008 co-movement began to decrease on all scales and almost disappeared before October 20, 2009. Similarly to Spain only small regions of significant co-movement occurred in the end of the analyzed period.

Co-movement dynamics between Italian yields and yields of the other states depicts the lower part of Figure E.6 h-n. Italy did not suffer a banking crisis like Spain or Portugal or did not falsify its accounting as Greece, however we observe similar pattern of co-movement as on the Periphery. We clearly see that immediately after the speech of Papakonstantinou co-movement disappears. One of the reasons for this sovereign bond market disintegration may be investors’ feeling that Italy no longer belongs to the core but the periphery after the Greek’s deficit problems. Note that Italy and Spain (Figure 2 k) exhibit strong co-movement in the crisis at high scales (periods 32-256).

Although there is heterogeneity in the three groups we clearly observe that the Core is less disintegrated than the Periphery after the fall of Lehman Brothers. A specific pattern is observable in a group of the states outside the Eurozone. Before the crisis co-movement on the Periphery used to be (at least on the scales representing the periods of 2-8 days) weaker. Situation changed during the crisis, when almost all significant coherence between the Peripheral states vanished.

5. Contagion and disintegration on the sovereign bond market

In the second part of the empirical analysis we test the hypothesis whether the contagion was present during the crisis. We study changes of wavelet correlations between yields of Greece and the other states at different scales that correspond to investment horizons. We use the same two dates as in the wavelet coherence analysis - the fall of Lehman Brothers and the announcement of the new budget deficit. For a brief introduction of discrete wavelet transform and wavelet correlation see Appendix C and Appendix D.

Now, let us briefly describe the testing procedure proposed by Gallegati (2012). At first we select the exact point in time, when we expect the contagion began. We define \( \rho_{XY}^{(I)}(\lambda_j) \) as a correlation at a specific scale \( j \) before the point and \( \rho_{XY}^{(II)}(\lambda_j) \) as correlation at the same after the point. We test null hypothesis that the both values are the same:

\[
H_0 : \rho_{XY}^{(I)}(\lambda_j) = \rho_{XY}^{(II)}(\lambda_j) \quad (1)
\]

\[
H_A : \rho_{XY}^{(I)}(\lambda_j) \neq \rho_{XY}^{(II)}(\lambda_j). \quad (2)
\]
The scales represent again investment horizons. For example, scale $\lambda_1$ represent short investment horizon with the period of 2-4 days. Further advantage of this wavelet based method is its robustness to non-Gaussian features of data (Gallegati, 2012). It is clear that hypothesis will be rejected, and thus change of correlation at scales detected, if confidence intervals of pre-crisis and after-crisis correlation do not overlap. Gallegati’s approach closely follows methods proposed by Bodart and Candelon (2009) and Orlov (2009) where they decompose analyzed time series into different frequencies, because it is the key for distinguishing between contagion and interdependence. Further, we change the Gallegati’s definitions slightly, so we call the change of correlation a contagion only in case the correlation increases in the second examined period, i.e.,

$$H_C : \rho^{(I)}_{XY}(\lambda_j) < \rho^{(II)}_{XY}(\lambda_j).$$

In case where the correlation in the second examined period drop we call the correlation change disintegration, thus

$$H_D : \rho^{(I)}_{XY}(\lambda_j) > \rho^{(II)}_{XY}(\lambda_j).$$

This approach is closer to Forbes and Rigobon (2002), whose definition of contagion is “significant increase in cross-market linkages”\textsuperscript{5}.

Changes of co-movement on higher frequencies (lower scales in wavelet framework) represents temporary changes - a contagion, lower frequencies (higher scales) describe only change of interdependence. For this reason we use the following scales: $\lambda_1$ (2-4 days), $\lambda_2$ (4-8 days) and $\lambda_3$ (8-16 days). We estimate the wavelet correlation on a window with the length equal approximately to one trading year (250 observations) before as well as after 15.9.2008 and 20.10.2009, respectively.

The justification of this approach is following. A detection of the contagion during the bankruptcy of Lehman Brothers is made by comparison between the latent-crisis period (the window before the fall of Lehman Brothers) and the period of the hot phase of the financial crisis. When detecting the contagion near to 20.10.2009 we compare the window partially covering the after-Lehman Brothers subsample from the previous test with another window representing the sovereign debt crisis in the EU. Hence the approach allows us to decompose a potential jump or a drop of correlation between two events\textsuperscript{6}.

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\textsuperscript{5}In a sense, this definition of contagion update is rather technical, as both the alter-
Figure 4: Contagion between Greece and other states after fall of Lehman Brothers on 10Y sovereign bonds market. Blue (red) lines denote wavelet correlation estimates before (after) the contagion date. Dashed lines denote corresponding 95% confidence intervals.
5.1. Lehman Brothers

Wavelet correlations of sovereign bond yields of Greece and other states are depicted on Figure 4. We observe that in the period before the fall of the Lehman Brothers the correlation between Netherlands and Greece used to be high and relatively homogeneous, similarly to Spain, Portugal and Denmark. In the post-Lehman window the situation changed and the correlation significantly decreased in all of these pairs. The confidence intervals are not overlapping, thus we detected change in the correlation structure. It is in not a contagion in a sense of (Gallegati, 2012), but the debt market disintegration. On the other hand, we can say that the contagion of fear disintegrated the sovereign bond market significantly for these four pairs. These results are interesting, because these countries belong to the different groups the Core, the Periphery as well as the states outside the Eurozone.

In case of Italy, Germany, Belgium and France we observe significant increase of correlation at the lowest scale (2-4 days) after the collapse of the Lehman Brothers, but correlation at remaining two scales is similar. Thus, for these four states we detect contagion only at shortest investment horizon.

5.2. Deficit announcement

After announcement that the Greek’s budget deficit will be much higher than it was expected, significant change of correlation was detected only in the highly developed states, see Figure 5. In case of Germany, Netherlands, France, Denmark and Sweden we notice a significant decrease of correlation at all three scales. In most of these cases the correlation after the deficit revelation dropped to negative values, which leads to a conclusion that this event caused total disintegration of sovereign bond market between these countries and Greece. Conversely, an increase of the co-movement on the lowest scale between yields of Greece and Portugal (contagion at the lowest scale) may imply that Portugal stopped being trustworthy and was perceived by investors similarly in short investment horizons as Greece. To complete the analysis, for Spain, Italy, Belgium and Great Britain we do not observe any change in the correlation structure. In general, our results indicate that this second event further undermined the sovereign bond market in the EU.

The analysis is made using R package waveslim created by B. Whitcher. The LA8 wavelet filter will be used and further the wavelet filter D4 will be employed for the robustness check.

natives $H_C$ and $H_D$ are caused by some form of contagion or panic at bond markets.
Figure 5: Contagion between Greece and other states on 20.10.2009 (disclosure of debt by Papakonstantinou) on 10Y sovereign bonds market. Blue (red) lines denote wavelet correlation estimates before (after) the contagion date. Dashed lines denote corresponding 95% confidence intervals.
6. Conclusion

The paper examines co-movement between sovereign bond yields of the EU members. In the centre of our attention are changes of the co-movement during the financial crisis and the sovereign debt crisis. We employed wavelet transformation, which is able to decompose time series into different scales and then co-movement can be estimated for various investment horizons independently. We demonstrated that co-movement and wavelet correlations are frequency dependent, thus time-scale techniques are needed to uncover all dynamic relationships at the sovereign bond market.

Results show that in the crisis period brings rich time-scale dynamics to European debt markets. The co-movement was significant on the majority of scales in the non-crisis period, but after the crisis erupted, strong bond yields co-movement almost completely disappeared, especially on the periphery and between the groups. Later, the bond yields of the Core group restored high degree of co-movement in the end of 2013, but the integration of the yields on the Periphery was shattered and not reestablished, thus it seems that in general the integration of sovereign bonds in the Eurozone suffered hard blow.

We further demonstrate that application of correlation change tests around the two important dates during the crisis brings important results. After the fall of Lehman Brothers disintegration of the debt market occurred in all groups as we observe significant decrease of correlation between Greece and other countries—Spain, Portugal, Denmark and Netherlands. Subsequent decrease of correlation was after the Greeks’ deficit announcement leading even to negative correlation in five pairs, which resulted in a total disintegration of Greek debt market with the Eurozone countries.

References


Appendix A. The Continuous Wavelet Transformation

The section introduces an important tool of our work - continuous wavelet transformation. At first we have to describe wavelet function $\psi(t)$, which is called mother wavelet. A function is a wavelet if it has got several properties. The first and the most important one of them is called the admissibility condition:

$$0 < C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(f)|^2}{f} df < \infty,$$  \hspace{1cm} (A.1)

where $\hat{\psi}(f)$ is a Fourier transformation of the function. If $f \to 0$, then $C_\psi \to 0$ (Addison, 2002). The second necessary condition is a consequence of the admissibility condition. It says that the integral of a wavelet function has to be equal to zero, thus $\int_{-\infty}^{\infty} \psi(t) dt = 0$. Further, we assume the wavelet to be a square integrable function having a unit energy, i.e. $\int_{-\infty}^{\infty} \psi^2(t) dt = 1 < \infty$. 

References


Let us define the continuous wavelet transform (CWT) as:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi^*\left(\frac{t - \tau}{s}\right) \, dt,$$

where $\psi^*\left(\frac{t - \tau}{s}\right)$ denotes modified wavelet function with complex conjugate and $\tau$ and $s$ are location and scaling parameter. As the CWT is an inner product of time series and modified mother wavelet function, the CWT can be interpreted as degree of similarity between the series and mother wavelet in a time-frequency space.

**Appendix B. Wavelet Coherence**

Wavelet coherence allows us to express co-movement between the time series on a time-scale space. We will use standard notation like e.g. Torrence and Compo (1998) or Grinsted et al. (2004). Let us assume that both time series $x(t)$ and $y(t)$ are square integrable and locally stationary. At first we have to define cross-wavelet transformation (XWT) as a product of two wavelet transformations

$$W_{xy} = W_x(\tau, s)W_y^*(\tau, s), \quad (B.1)$$

where $W_x(\tau, s)$ and $W_y(\tau, s)$ are continuous wavelet transformations of both time series and $^*$ denotes complex conjugate. The cross wavelet power spectrum is defined as:

$$CWS_{xy} = |W_{x,y}(\tau, s)|. \quad (B.2)$$

It is a wavelet equivalent of covariance, i.e., covariance at a scale $\tau$. Following Torrence and Compo (1998) we define the squared wavelet coherence as:

$$R^2(\tau, s) = \frac{|(S(s^{-1}W_{xy}(\tau, s)))|^2}{S(s^{-1}|W_x(\tau, s)|^2)S(s^{-1}|W_y(\tau, s)|^2)}, \quad (B.3)$$

where $S$ denotes smoothing operator. In our estimates of wavelet coherence we use the Morlet wavelet (Goupillaud et al., 1984). For a more detailed treatment see Torrence and Compo (1998) and Grinsted et al. (2004).

The squared coherence is defined in the interval $0 \leq R^2 \leq 1$. High level of coherence implies that there is a strong co-movement between the time series, whereas low values mean that the series are linearly independent on a particular scale.
An important information is whether our results are statistically significant. Although there were some formal significance tests developed (see Ge (2008)), according to Aguiar-Conraria et al. (2012) their null hypotheses are too restrictive and hence they are not suitable for wavelet analysis in economics. For our purposes, we use Monte Carlo simulations.\(^7\)\(^1\)\(^0\) The significant regions of the wavelet coherence estimates are inside bold black lines.

Appendix C. Maximal overlap discrete wavelet transform

For computation of wavelet correlation we use a special form of discrete wavelet transform called the maximal overlap discrete wavelet transformation (MODWT). In this section we briefly show an application of the pyramid algorithm in order to obtain the MODWT wavelet and scaling coefficients that are used for wavelet correlation. In practice the pyramid algorithm proposed by Mallat (1989) is based on filtering time series with MODWT wavelet filters; the output after filtering is then filtered again in a subsequent stage to obtain wavelet coefficients on other wavelet scales. For detailed information about the MODWT see Walden and Percival (2000).

Appendix D. Wavelet Correlation

Let \(X_t\) and \(Y_t\) be two stationary stochastic time series. Following Walden and Percival (2000) we define the MODWT wavelet variance at scale \(\lambda_j\) as

\[
\sigma^2_X(\lambda_j) = \frac{1}{\bar{N}_j} \sum_{t=L_{j-1}}^{N-1} \tilde{w}_{X,j,t},
\]

where \(\tilde{w}_{X,j,t}\) denotes the MODWT wavelet coefficient at scale \(\lambda_j\) and \(L_j\) denotes length of wavelet filter of scale \(\lambda_j\). The number of wavelet coefficients unaffected by the boundary conditions is \(\bar{N}_j = N - L_j + 1\).\(^8\) Further we define scale-by-scale covariance of the two series:

\[
\gamma_{XY}(\lambda_j) = \frac{1}{\bar{N}_j} \sum_{t=L_{j-1}}^{N-1} \tilde{w}_{X,j,t} \tilde{w}_{Y,j,t}.
\]

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\(^7\)Following the testing procedure of Grinsted et al. (2004) we use 5% significance levels.

\(^8\)For the discussion about boundary see Gençay et al. (2001), Chapter 4.6.3.
Then we can compute scale-by-scale correlation between the two series:

\[ \rho_{XY}(\lambda_j) = \frac{\gamma_{XY}(\lambda_j)}{\sqrt{\sigma^2_X(\lambda_j)\sigma^2_Y(\lambda_j)}}. \]  

(D.3)

According to Whitcher et al. (2000) the estimator is unbiased, consistent and asymptotically normal. The confidence intervals of the MODWT wavelet correlation are computed via the following formula (Gençay et al., 2001):

\[ \left[ \tanh \left( \tanh^{-1} (\rho_{XY}(\lambda_j)) \pm \frac{\Phi^{-1}(1 - p)}{\sqrt{N_j - 3}} \right) \right], \]

(D.4)

where \( \Phi^{-1}(1 - p) \) is a quantile of the standard normal distribution.

Appendix E. Additional figures
Figure E.6: Inter-group coherence: Portugal and Italy

(a) Portugal-Belgium  (b) Portugal-Denmark  (c) Portugal-France
(d) Portugal-Great Britain  (e) Portugal-Germany  (f) Portugal-Netherlands
(g) Portugal-Sweden  (h) Italy-Denmark  (i) Italy-France
(j) Italy-Great Britain  (k) Italy-Germany  (l) Italy-Sweden
(m) Italy-Netherlands  (n) Italy-Belgium
Figure E.7: Inter-group coherence: Greece and Spain

(a) Greece-Belgium  (b) Greece-Denmark  (c) Greece-France
(d) Greece-Great Britain  (e) Greece-Germany  (f) Greece-Netherlands
(g) Greece-Sweden  (h) Spain-Denmark  (i) Spain-France
(j) Spain-Great Britain  (k) Spain-Germany  (l) Spain-Netherlands
(m) Spain-Sweden  (n) Spain-Belgium