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Merger Remedies in Oligopoly Under a Consumer Welfare Standard*

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June 2015

Abstract

We analyze the welfare effects of structural remedies on merger activity in a Cournot oligopoly if the antitrust agency applies a consumer surplus standard. We derive conditions such that otherwise price-increasing mergers become externality-free by the use of remedial divestitures. In this case, the consumer surplus standard ensures that mergers are only implemented if they increase social welfare. If the merging parties can extract the entire surplus from the asset sale, then the socially optimal buyer will be selected under a consumer standard.

JEL-Classification: L13, L41, K21

Keywords: Remedies, Merger control, Consumer standard, Synergies.

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1 Introduction

Remedies are increasingly applied by antitrust agencies (in short: AA) in the US and EU to clear merger proposals which are otherwise subject to serious anticompetitive concerns (see FTC, 1999, EU, 2006, and OECD, 2011, for recent remedy reviews). The US Horizontal Merger Guidelines and the EU Merger Regulation allow for remedial offers to address competitive concerns (see DoJ, 2010, and EU, 2004, respectively). Accordingly, remedies are offered by the merging parties to effectively restore competition and to remove any competition concern the AA may have.

We analyze the impact of remedies on (horizontal) merger activity in oligopolistic industries if the AA follows a consumer surplus standard; that is, the AA blocks mergers which lower consumer surplus.\(^1\) We focus our analysis on such industries which are characterized by barriers to entry and where the amount of productive assets can be regarded as fixed. In these industries, divestitures of critical assets by the merging firms to a competitor firm can be used to increase market competition by reallocation (“structural remedies”).\(^2\) Mergers are assumed to produce scale economies (resulting from combining the capital of the merging firms) and synergies (which directly reduce marginal production costs). Thus, a merger can be desirable from a consumer perspective when the merger synergies are sufficiently large.\(^3\) If synergies fall short

\(^1\)This is in line with recent Industrial Organization literature (e.g., Nocke and Whinston, 2010) which takes the consumer surplus standard for granted. For instance, Whinston (2007) states that the AA’s “enforcement practice in most countries (including the US and the EU) is closest to a consumer surplus standard.” Davies and Lyons (2007) emphasize that AAs have no mandate to use merger review for industrial policy purposes. Hence, remedies should only be applied if there is a threat to competition.

\(^2\)For example, in the retailing sector divestitures concern suitable property and branches which are largely fixed in the industry. A similar role is taken by gasoline stations in the petroleum industry, by landing slots in the airline industry, and by spectrum in the mobile phone industry. In all these examples, the critical assets are largely fixed for some time period, but may change in the longer run because of innovations or entry. Antitrust authorities consider the “foreseeable” future in their decisions (which is typically confined to the next 1-2 years), so that the capital stock in the mentioned industries is usually regarded as fixed. As a consequence, the respective assets also qualify as divestitures which can counter anticompetitive merger effects already right after the execution of the merger.

\(^3\)Our analysis is placed in a Cournot setting in which synergies are necessary to make consumers not worse off after the merger (see Farrell and Shapiro, 1990a; Spector, 2003; Vergé, 2010).
of a certain threshold value, approval by the AA can be achieved with the use of remedies; i.e.,
physical asset sales to rival firms.

The possibility of clearing a merger conditional on remedies is shown to enlarge the set of
profitable and acceptable mergers. More importantly, if a divestiture is necessary to keep prices
from rising, then under reasonable conditions the merging parties will propose a divestiture
which is price-restoring; i.e., the pre-merger price equals the post-merger price. Therefore, any
merger which involves such a structural divestiture is *externality-free* because it leaves consumer
surplus and outsiders’ profits unchanged.⁴ It follows that the consumer surplus standard en-
sures that mergers are only implemented if they increase social welfare. If the merging parties
can extract the entire surplus from the asset sale, then the socially optimal buyer will be selected
given a consumer standard. The reason is that the merged entity becomes the residual claimant
in the asset sales process, so that the choice of the buyer firm maximizes both the merged firm’s
profit and social welfare. These insights reveal a new efficiency rationale of the seemingly inef-
ficient consumer surplus standard (or “price test”) which ignores changes in profits, and hence,
total welfare.

Our model takes care of the following two remedy principles which are stated both in EU
and US regulations (see, EU, 2008, and DoJ, 2011, respectively): *First*, the remedy is designed
and proposed to the AA by the merging firms, while the AA can either reject or accept the offer.
*Second*, the remedy must be proportional to the competitive concern (see EU, 2004, Article 30).
The first property says that the merging firms are supposed to design a *fix-it-first remedy* which
they have to propose to the AA before it decides about the merger proposal.⁵ Accordingly, we
assume that the merging parties have to determine the remedial divestiture and the buyer firm
to the AA in advance. Thereafter, the buyer firm either accepts are rejects the proposal. The sec-
ond property is derived endogenously; in equilibrium, an approvable remedy is always such
that its size is proportional to the anticompetitive effects of the full merger proposal. Hence,

⁴ Outsider firms remain unaffected by the merger as their optimal quantities do not change when the price stays
put.

⁵ The rules are different in the second stage of the merger processes in the US and the EU (see, for instance, Wood,
2003, for a comparison of the US and EU merger control systems and the role of remedies therein, and Farrell, 2003,
who describes the remedy settlement as a bargaining process between the merging parties and the AA).
lower synergies and/or larger capital stocks of the merging firms must induce a larger divestiture to make the merger approvable. Moreover, when the anticompetitive concern increases (either because of lower synergies or because of larger capital stocks of the merging firms), then the scope for mergers approved conditional on remedies is reduced.

We extend our model by comparing the merger outcomes under different selling mechanisms which determine the extent of rent-extraction. If the merging firms’ must sell the divestiture at a fixed price (i.e., rent-extraction is limited), then in equilibrium it is sold to the weakest competitor (that is, typically, the smallest outsider firm). If a price-restoring remedy is sold through an auction it will be acquired by the incumbent competitor with the highest willingness to pay. In general, only perfect selling power (or efficient bargaining between the parties) ensures that the divestiture is acquired by the socially efficient buyer. Fix-it-first remedies create “take-it or leave-it” power for the merging firms in the asset sales process because a rejection by the buyer puts the entire undertaking at risk. Thus, we provide a novel rationale for the efficiency of fix-it-first remedies which are favored both by the EU and the US merger guidelines.

We also examine remedy-dependent synergies such that the acquirer of the assets realizes synergies on its own or if the realized synergy of the merged firm decreases when assets have to be divested. In the former case, small divestitures which are price-decreasing become possible if the realized synergy of the acquiring firm is relatively large. In the latter case, the scope for approvable divestitures is reduced, but we show that our main results remain qualitatively valid.

Our paper contributes to the analysis of mergers in Cournot oligopoly when productive capital in an industry is fixed (Perry and Porter, 1985; Farrell and Shapiro, 1990a,b; McAfee and Williams, 1992). This approach was applied to structural remedies in Medvedev (2007), Vergé (2010), and Vasconcelos (2010). All the latter three works refer to specific Cournot oligopoly models and they invoke specific assumptions concerning functional forms. Vergé (2010) disregards merger synergies. It is shown that only under very restrictive assumptions a re-allocation of productive assets through structural remedies may increase consumer surplus. Medvedev (2007) shows for a three-firm oligopoly that remedies in association with merger synergies extend the scope for approvable mergers. Vasconcelos (2010) analyzes remedies for the case of
a four-firm oligopoly when merger synergies are possible. Each firm owns one unit of capital and a firm’s capital is indivisible. It is assumed that the AA restructures the industry optimally in order to maximize consumer surplus, which is crucial when there are at least three firms involved in a merger. In these instances an “over-fixing” problem associated with remedial divestitures may emerge (see also Farrell, 2003). Over-fixing unfolds adverse effects because a firm may abstain from proposing a (socially desirable) merger with two other firms as the acquirer expects, and correctly so, that the AA will use its power to sell one of the acquired firms to the remaining competitor. Consequently, the acquirer may strategically propose a one-firm takeover which can be worse from a consumer point of view than allowing a takeover of the two other firms.

Cabral (2003) analyzes mergers in a differentiated industry with free entry. When assets are sold to an entrant firm as a remedy then a “buy them off” effect follows, which means that an entrant firm is dissuaded from opening a new store (or introducing a new product variant). This effect may work against the interest of consumers, who are better off the more variants are offered in the market. Chen (2009) analyzes mergers in a three-firm oligopoly model of dynamic capital accumulation. A merger may then have long-run effects that are worse than its short-run effects. We disregard the issue of endogenous entry and endogenous capacities as the capital is assumed to be fixed in the industry.

Our analysis also adds to the literature which identifies circumstances such that a consumer surplus standard is preferable in competition policy (Besanko and Spulber, 1993, Neven and Röller, 2005, and Armstrong and Vickers, 2010). In contrast to existing theories, our point is that a consumer standard in merger control leads to socially efficient remedial divestitures.

The impact of remedies on the effectiveness of merger control has also been investigated empirically (see Duso et al., 2011, and Duso et al., 2013, for the EU and Clougherty and Seldeslachts, 2013, for the US). These works use event studies which identify the anticompetitive effect of a merger by abnormal stock market returns of competing firms. Overall, the results appear to indicate that an upfront-buyer remedy tends to restore the pre-merger competitive situation.

Ormosi (2012) analyzes major EU merger cases and shows that remedial offers and efficiency claims are often strategic to avoid costly delay in litigation processes.
We proceed as follows. Section 2 presents the basic model. In Section 3 we conduct the merger analysis for two different merger control regimes depending on whether or not remedies are feasible. Section 4 presents extensions of our model before Section 5 concludes.

2 The Model

We analyze the effects of remedies in a Cournot oligopoly with homogeneous products by extending the analysis of Farrell and Shapiro (1990a). There are \( n \geq 3 \) firms indexed by \( i \in I = \{1, \ldots, n\} \). All firms produce a homogeneous good with inverse market demand given by a twice differentiable function \( p(X) \), where \( p \) is price, \( X \) is industry output, and \( p'(X) < 0 \). Firm \( i \)'s production costs depend on its output level, \( x_i \), and the capital stock, \( k_i \), it uses for production. Total productive capital of the industry, \( K \), is fixed and distributed among the firms in the industry; i.e., \( k_i > 0 \) for all \( i \in I \) and \( \sum_{i \in I} k_i = K \). Firm \( i \)'s total production cost function is given by \( c^i := c^i(x_i, k_i) \).

We invoke the standard assumption that additional capital lowers the cost and the marginal cost curve; i.e., \( c^i_k < 0 \) and \( c^i_{xx} < 0 \). Firms set their output levels simultaneously (Cournot competition).

Each firm \( i \) maximizes its profit \( \pi_i = p(X)x_i - c^i(x_i, k_i) \) given its rivals' outputs, which yields the first-order conditions

\[
p(X) + x_ip'(X) - c^i_x(x_i, k_i) = 0, \text{ for all } i \in I.
\]

In a Cournot equilibrium, (1) holds for all firms \( i \in I \). From (1) it follows that firm \( i \) produces a larger quantity than firm \( j \) if and only if its marginal production costs are lower; i.e., \( c^i_x < c^j_x \) holds. We assume that each firm’s reaction function slopes downward with a slope between \(-1\) and \(0\), for which it is sufficient to assume that

\[
p'(X) + x_ip''(X) < 0 \quad \text{holds for all } i \in I.
\]

We abbreviate a function’s partial derivative by indexing the respective variable; for instance \( c^i_x \equiv \partial c^i(x_i, k_i)/\partial x_i \).

Inequality (2) holds if the industry demand curve satisfies \( P''(X)X + P'(X) < 0 \). It is standard assumption in Cournot analysis and guarantees the existence of a unique Cournot equilibrium when marginal costs are non-decreasing.
The AA applies a consumer standard when evaluating a merger proposal. Therefore, a merger is approved if and only if the post-merger price level does not exceed the pre-merger equilibrium price $p^*$. We distinguish two different merger control regimes, depending on whether or not remedies are feasible.\footnote{Throughout the analysis we assume that the AA can only impose a remedy on the merging firms that the parties themselves propose. This mirrors legal practice in the EU and in the US (see EU, 2006/2008, and DoJ, 2011).}

- **No-remedy regime** (in short: NR): If the merger guidelines do not allow for a remedial divestiture, then the AA can either approve or block the merger proposal altogether.

- **Remedy regime** (in short: R): The merger guidelines allow for an approval conditional on a divestiture to a competitor if it counters any price-increasing effect of the proposed merger.

We examine a bilateral merger with firm $i$ being the acquirer and firm $j$ the target firm. Firms $i$ and $j$ merge if the merged entity’s profit does not fall short of the sum of the pre-merger profits. A merger allows to recombine the capital of the merging firms to explore economies of scale.\footnote{After the merger it is optimal to bring all the new entity’s capital together rather than leaving it divided among the plants of the pre-merger configuration which is optimal because of $c_{x,k} < 0$ (see also Farrell and Shapiro, 1990b, p. 113).} If firms $i$ and $j$ merge, they generate a synergy, which is measured by the parameter $s := s(i, j) \in [0, 1]$. The synergy rotates the cost function downward such that marginal costs for a given level of output are lowered. More precisely, the merged firm $M = M(i, j)$ (which combines the assets of firms $i$ and $j$) produces with the cost function $c^M(x, k, s)$, where $k$ denotes the merged firm’s capital, possibly reduced by divested assets. Let $c^M(x, k, s)$ be continuous in $s$ with $c^M(x, k, 1) = c^i(x, k)$ and $c^M(x, k, 0) = 0$. Perfect synergies ($s = 0$) imply that the firm’s costs are reduced to zero, while the absence of any synergies ($s = 1$) implies that the merged firm produces with the pre-merger cost function $c^i(x, k)$. We assume that the synergy reduces marginal production costs,

$$\frac{\partial c^M_x(x, k, s)}{\partial s} > 0 \text{ holds for all } x, k > 0.$$  

Let $0 \leq \sigma \leq k_j$ denote the share of firm $j$’s capital which stays under control of the merged firm...
M after a possible divestiture. Accordingly, \( k_j - \sigma \) is the share of firm \( j \)'s capital which goes as a divestiture to another firm, say firm \( l \). Let \( I_M \) denote all firms which are active after the merger; i.e., \( I_M := I \setminus \{ i, j \} \cup M \). Furthermore, denote the total pre-merger equilibrium quantity by \( X^* \) and the post-merger equilibrium quantity by \( X^s(k_j - \sigma) \), the latter depending on the divestiture level \( k_j - \sigma \) and synergy level \( s \).

We impose two independence conditions on the interplay between the synergy level and the remedy. First, we assume that the synergy level \( s(i, j) \) is unaffected by the size of the divestiture. Second, the buyer of the assets does not realize any synergies. Consequently, the merged entity faces overall costs of \( c^M(x_M, k_i + \sigma, s) \), while firm \( l \) operates with the cost function \( c^l(x_l, k_l + k_j - \sigma) \).

In addition, we invoke two more assumptions concerning firms' cost functions. First, all firms (except the merged firm) have access to the same technology. Second, all firms (including the merged firm) have constant marginal costs. Specifically, we suppose that each firm \( i \in I \) produces with the cost function

\[
c^i(x_i, k_i) = \frac{x_i}{k_i}
\]

prior to the merger. If firms \( i \) and \( j \) merge to form the merged entity \( M \), they realize the synergy level \( s \) and (possibly) divest \( k_j - \sigma \) to firm \( l \). The merged firm’s cost function is then given by

\[
c^M(x, k_i + \sigma, s) = \frac{sx}{k_i + \sigma},
\]

\( ^{11} \)We suppose that the acquirer divests parts of the target firm’s assets if the AA requires a remedy to approve the merger. We could also assume that the authority requires that parts of the acquirer’s assets are divested, which would not change the results of our analysis.

\( ^{12} \)Below, we discuss how our results change if we relax each of these requirements.

\( ^{13} \)In our model, the effects of a structural remedy are not burdened with uncertainties which may play a role in practical merger control (see Davies and Lyons, 2007).

\( ^{14} \)This assumption implies that asymmetries among firms (apart from the merged firm) only depend on the amount of productive assets they own. It allows to perform a comparative static analysis with regard to the equilibrium divestiture level.

\( ^{15} \)Even though this assumption restricts the generality of our results, the constant-marginal cost case is important in itself. In fact, many results obtained in the merger literature (for instance, Salant et al., 1983, and Nocke and Whinston, 2010) are based on constant marginal costs.
while the acquiring firm \( l \) produces with the pre-merger cost function
\[
c^l(x, k_l + k_j - \sigma) = \frac{x}{k_l + k_j - \sigma}.
\]
(5)

Note that the synergy level \( s \) enters the merged firm’s cost function (4) in multiplicative form, so that \( c^M(x, k, 1) = c^l(x, k) \) and \( c^M(x, k, 0) = 0 \) follow. We analyze the following merger game under the NR and the \( R \) regime: In the first stage, firms \( i \) and \( j \) decide whether or not to propose a merger to the AA. If they decide to merge, they can also specify a divestiture under regime \( R \) which they sell to a competing firm \( l \in I \setminus \{i, j\} \). In the second stage, the AA either approves or blocks the merger proposal according to a consumer standard. In the third stage, firms compete à la Cournot.

\section{Merger Analysis and Main Results}

First, we examine how a change in capital \( d k := (dk_1, ..., dk_n) \) affects equilibrium quantities \( dx := (dx_1, ..., dx_n) \). Following Farrell and Shapiro (1990a), the total derivative of firm \( i \)'s first-order condition with respect to \( X \) and \( k_i \) can be written as\(^1\)
\[
dx_i = -\lambda_i dX + \delta_i dk_i,
\]
(6)
where
\[
\delta_i := \delta_i(k) := \frac{c^i_{xk}}{p'(X)} > 0
\]
(7)
and
\[
\lambda_i := \frac{p'(X) + x_i p''(X)}{p'(X)} > 0.
\]
(8)
The variable \( \delta_i \) gives the direct effect of capital \( k_i \) on firm \( i \)'s output \( x_i \) and \( \lambda_i \) denotes firm \( i \)'s equilibrium responsiveness to changes in price. There is a direct relationship between \( \lambda_i \) and the slope of firm \( i \)'s reaction function \( R_i \) which is given by \( \lambda_i = -R_i/(1 + R_i) \) (see Farrell and Shapiro, 1990a). Summing up Condition (6) for all firms, yields the following lemma (see Farrell and Shapiro, 1990a, Prop. 2).\(^2\)

\(^1\)Note that we assume \( c_{xx} = 0 \) which simplifies the expressions below when compared with the corresponding expressions in Farrell and Shapiro (1990a).

\(^2\)We present all omitted proofs in the Appendix.
Lemma 1 (Effects of selling units of capital). A sale of a small amount of capital from firm \( j \) to firm \( l \) increases industry output and reduces the market price if and only if \( \delta_l > \delta_j \).

Lemma 1 gives a necessary and sufficient condition under which small asset sales to a rival firm increase consumer surplus. Our main analysis builds on this local result and investigates under which circumstances the divestitures of a merged firm can restore the pre-merger industry output.

Second, we impose a condition under which a merger satisfies the consumer standard at least if the realized synergy is maximal \((s = 0)\). A merger between firms \( i \) and \( j \), which creates synergies \( s \), leads to an output level of the merged firm that is weakly larger than the sum of the firms’ outputs before the merger if and only if\(^{18}\)

\[
c_i^j + c_j^i - c_x^M \geq p(X^*). \tag{9}
\]

Using the cost functions (3) and (4), Condition (9) becomes

\[
\frac{1}{k_i} + \frac{1}{k_j} - \frac{s}{k_i + k_j} \geq p(X^*). \tag{10}
\]

This condition is more likely to be fulfilled if the merging firms’ capital stocks, \( k_i \) and \( k_j \), are not too large. Or, conversely, the smaller the merging firms are, the more likely the consumer surplus standard is met. Similarly, Condition (10) is more likely to be fulfilled the lower the value of the synergy parameter \( s \) becomes. In the following, we assume that Condition (10) is fulfilled when the merger synergies are maximal.

Assumption 1. A merger between firms \( i \) and \( j \) increases the industry output if the merger synergies are maximal; i.e., Condition (10) holds at \( s = 0 \).

3.1 Merger Outcomes with and without Remedies

No-remedy regime (NR). Under the no-remedy regime, the AA can only clear or reject a merger proposal in its entirety. Hence, if a merger is approved, then \( \sigma = k_j \) holds. We obtain the following lemma according to which mergers are approved if and only if the generated synergy does not fall short of a certain threshold value.

\(^{18}\)See Farrell and Shapiro (1990b), Prop. 1, p. 112.
Lemma 2 (Full mergers). Suppose a no-remedy regime (NR). Then, there exists a unique synergy level \( s := s(i, j) \in [0, 1] \), such that a merger of firms \( i \) and \( j \) does not reduce industry output \( X^* \) if and only if \( s \leq s \). Hence, the AA approves a merger proposal between firms \( i \) and \( j \) if and only if \( s \leq s \). Such a merger is strictly profitable for the merging firms.

The critical synergy level \( s \) equals the synergy level for which Condition (10) holds with equality; i.e., it is the synergy level for which consumer surplus does not change after the merger. Lemma 2 then makes use of the fact that for \( s \leq s \) the merged firm produces more than both firms \( i \) and \( j \) together before the merger (with equality holding at \( s = s \)). This implies that the market price does not increase and consumers are not worse off after the merger. Moreover, profitability of the merger follows from noticing that the merged firm’s production costs are reduced over all output levels because of the increased productive capital and the realized synergy. According to Lemma 2, only mergers with relatively large synergies can pass the decision screen of the AA. If the synergy level falls short of the critical value (i.e., \( s > s \) holds), then the post-merger price increases, in which case the AA blocks the merger proposal altogether.

The critical synergy level \( s = s(i, j) \) depends on the capital stocks \( k_i \) and \( k_j \). The left-hand side of Condition (10) monotonically decreases in the parameters \( s \), \( k_i \) and \( k_j \), while the right-hand side does not depend on these parameters. Hence, larger values of \( k_i \) and/or \( k_j \) imply a lower value of the critical synergy level \( s \). Precisely, \( s(i', j) < s(i'', j) \) if \( k_{i'} > k_{i''} \) and \( s(i, j') < s(i, j'') \) if \( k_{j'} > k_{j''} \). Intuitively, this result depends on the fact that the price-raising effect of a merger increases when the merging firms are larger. In addition, a merger of relatively small firms creates a more competitive firm which tends to intensify competition with existing larger firms. Therefore, the critical synergy level \( s \) decreases in the size of the merging firms. It is also noteworthy that the critical synergy level, \( s \), neither depends on the capital stocks of the outsider firms nor on their distribution. The reason is that the merger does not affect the price level at \( s = s \) such that the outsider firms’ equilibrium quantities remain unaffected by the merger.

Remedy regime (R). With a remedy rule at hand, the AA can make a merger proposal conditional on structural remedies. According to the consumer surplus standard, the AA will accept all remedial offers which offset the price-increasing effect of reduced competition. It follows from Lemma 2 that remedies become relevant if the synergy parameter \( s \) is larger than \( s \). In
those instances, the merged firm may offer to divest a share of the target firm’s capital, \( k_j - \sigma \), which suffices to lower the market price or to keep it at the pre-merger level.

**Lemma 3 (Approvability).** Suppose a remedy regime \( R \). If a merger between firms \( i \) and \( j \) yields relatively large synergies with \( s \leq \bar{s} := \bar{s}(i, j) \), it is approved without a remedy. For lower synergy levels, \( s > \bar{s} \), there exists a unique threshold value \( \bar{s}^R \geq \bar{s} \), such that any merger proposal with \( s \in (\bar{s}, \bar{s}^R] \) is approvable with a certain divestiture level. For merger proposals with \( s > \bar{s}^R \), no divestiture level exists which would induce the AA to approve the merger.

Typically, \( \bar{s}^R > \bar{s} \) holds, such that the feasibility of remedies strictly increases the scope for mergers. Then there is an interval of synergies \( (\bar{s}, \bar{s}^R] \) for which divestitures exist that resolve the AA’s anticompetitive concerns such that a merger between firms \( i \) and \( j \) becomes approvable. If, however, the created synergy is too low (i.e., \( s > \bar{s}^R \)), then divestitures cannot outweigh the merger’s anticompetitive effects.

In fact, the proof of Lemma 3 reveals that remedies strictly increase the scope for mergers (i.e., \( \bar{s}^R > \bar{s} \)) if and only if

\[
k_i + k_j > \sqrt{s} \min_{l \in I \setminus \{i, j\}} \{k_l\}
\]  

(11) holds. Condition (11) ensures that there is a firm \( l \) for which the direct effect of capital on output is larger than for the merged firm \( M \), that is, for which \( \delta_l(k_l) > \delta_M(k_i + k_j) \) holds. In that case, selling assets of \( M \) to firm \( l \) increases industry output and therefore increases the scope for mergers. In particular, (11) holds if the merged entity \( M \) is not the smallest firm in the market. If, however, there exists a firm which is smaller than \( M \), then competition can be strengthened through selling capital to this firm.

Note that approvable remedies are most likely to exist for small buyer firms. To see this, assume that firms \( i \) and \( j \) merge and divest \( k_j - \sigma \) to some firm \( l \). Denote the sum of firms’ post-merger marginal costs as \( MC := \sum_{m \in I_M} c^m_x \). Summing up firms’ first-order conditions yields

\[
P'(X)X + (n - 1)P(X) = MC.
\]  

(12) The left-hand side of (12) is decreasing in \( X \) due to Condition (2). Thus, the equilibrium industry output is the larger the smaller the sum of firms’ marginal costs becomes (given that the number
of active firms stays constant). Consequently, if the divestiture \( k_j - \sigma \) is approvable for buyer firm \( l \), then this divestiture is also approvable for buyer firm \( l' \) as long as firm \( l' \) is smaller than firm \( l \); i.e., as long as \( k_{l'} \leq k_l \) holds. In particular, there is a certain threshold value \( \tilde{k} \in \{k_l|l \in I \setminus \{i,j\}\} \), such that there exists an approvable divestiture to be sold to firm \( l \) if and only if \( k_l \leq \tilde{k} \).

So far we have discussed the approvability of mergers involving divestitures. Next, we examine the profitability of a merger which involves divestitures to a competitor. For the remainder of this section, we assume that the merging firms have full bargaining power in the asset sales process.

**Assumption 2.** The merged firm can make take-it or leave-it offers to each firm in the market. It can also tailor the divestiture level to each buyer.

Suppose that a merger involves divestitures to ensure its approval by the AA. The following lemma states that if the synergy is such that an approvable merger (possibly involving a divestiture) exists, then there exists also an approvable and profitable merger for the same synergy level.\(^{19}\)

**Lemma 4 (Profitability).** Suppose a remedy regime \( R \). A profitable and approvable merger generating synergy \( s \) (possibly with a remedy) exists if and only if \( s \leq \bar{s}_R \).

The critical value \( \bar{s}_R \) is derived from the approvability condition. As for any merger with synergies \( s \in (\bar{s}, \bar{s}_R] \) there exists a divestiture which ensures approvability, there is also always a divestiture level which is price-restoring for the same synergy level. Given such a price-restoring divestiture level, the merger is always profitable because of its cost-reducing effect and because of Assumption 2 according to which the merged firm can extract all gains from trade from the buyer firm.

How the exact value of \( \bar{s}_R \) depends on the capital that the merger combines (i.e., \( k_M := k_i + k_j \)), hinges on the following trade-off. On the one hand, the larger the merging firms’ capital stocks are, the lower is the industry output after a full merger for a given synergy level \( s \).

\(^{19}\)This result depends on Assumption 2. If the merged firm does not have perfect selling power, then the profitability condition may restrict the range of synergy parameters for which approvable and profitable divestiture levels exist.
To see this, we compare the sum of all firms’ marginal costs after a merger of firms \( i' \) and \( j \) and after a merger of firms \( i'' \) and \( j \), where \( k_{i'} > k_{i''} \); i.e., firm \( i' \) being larger than firm \( i'' \). Denote the former sum by \( MC(i', j) \) and the latter sum by \( MC(i'', j) \), respectively. Comparing both sums gives

\[
MC(i', j) - MC(i'', j) = \frac{s}{k_{i'} + k_j} + \frac{1}{k_{i''}} - \left( \frac{s}{k_{i''} + k_j} + \frac{1}{k_{i'}} \right) > 0.
\]

As the industry output is the larger the lower the sum of marginal costs \( MC \), a full merger between firms \( i' \) and \( j \) induces a lower post-merger output level than the merger between firms \( i'' \) and \( j \). Thus, ceteris paribus, the larger the merging firms’ capital, the larger is the market power effect which tends to reduce the value of \( \bar{\delta}^R \).

On the other hand, the larger the merging firms are, the larger is the quantity which can be restored through a divestiture to a rival firm. This can be seen from Lemma 1. If the direct effect of capital on output is smaller for the merged firm than for the buyer firm (i.e., \( \delta_M < \delta_i \)), then divesting capital increases the total output until \( \delta_M = \delta_i \). The divestiture to firm \( l \) which maximizes industry output is then given by

\[
k_j - \bar{\sigma} = \frac{1}{\sqrt{s + 1}} \left( k_i + k_j - \sqrt{s}k_l \right). \tag{13}
\]

The size of \( k_j - \bar{\sigma} \) increases in \( k_M = k_i + k_j \), but decreases in the capital stock \( k_l \) of the buyer firm. The larger the divestiture (as long as it does not surpass \( k_j - \bar{\sigma} \) in size), the larger is the restored output level. Therefore, we can conclude that both the merger’s anticompetitive effect and the quantity which can be restored through a divestiture increase when the combined capital of the merging firms increases. Which of these effects dominates depends on the shape of the demand function and on the capital stocks of the firms.

We summarize our results concerning the merger outcome under regime \( R \) as follows.

**Proposition 1 (Implementation).** Suppose a remedy regime \( R \). Then all mergers with relatively large synergies, \( s \leq \bar{s} \), are profitable and are approved without a remedy. For lower synergy levels \( s \in (\bar{s}, \bar{s}^R] \), a merger with a certain divestiture exists which is approved by the AA and which is also profitable. For \( s > \bar{s}^R \), no merger is implemented since there exists no remedy which could fix the AA’s competitive concerns.

Proposition 1 states that there is a monotone relationship between a merger’s synergy level
and the AA’s final decision. Precisely, there exists a unique threshold value of the synergy level, $\bar{s}^R$, up to which mergers (possibly including a certain divestiture) are implementable. Given that Condition (11) holds, this threshold value strictly exceeds $\bar{s}$. If the merger synergies are too low (i.e., $s > \bar{s}^R$), then allowing for remedies does not change the AA’s decision when compared with the NR regime.\(^{20}\) If there are several potential buyer firms for which an approvable remedy exists, then the identity of the buyer firm is not determined yet. In particular, it depends on the asset sales mechanism which can vary between “full rent extraction” (Assumption 2) and “no rent extraction” (selling at a price of zero) as we will examine in detail below.

### 3.2 Social Welfare

According to Proposition 1, the introduction of remedies may change the market structure and, therefore, also social welfare (the sum of consumer surplus and producer surplus), which we denote by $SW$. We compare social welfare if the merger control regime allows for remedies and if it does not. The following analysis is restricted to synergy levels $s \in (\bar{s}, \bar{s}^R]$ for which remedies strictly increase the scope for approvable and profitable mergers.

**Binding consumer surplus standard.** If there is a divestiture such that the consumer surplus constraint is satisfied, then (due to continuity of the cost function) there also exists at least one divestiture level such that the consumer surplus standard binds (i.e., pre- and post-merger prices are the same). We call such a divestiture level a *price-restoring divestiture*. It is externality-free as the consumer surplus and profits of those firms not involved in the merger process are unaffected and remain at their pre-merger levels. In particular, the smallest divestiture level $\min_{\sigma \in [0,k_j]} \{k_j - \sigma\}$ which satisfies the consumer surplus standard condition is such a price-restoring divestiture. While it appears to be intuitive that the merged firm should prefer not to divest more than required by the AA, this is not always the case. The next lemma states conditions under which the merged firm proposes to divest the smallest approvable divestiture which we denote by $k_j - \hat{\sigma}$.

\(^{20}\)Note that if the merged firm is forced to sell the assets at a fixed price, then Lemma 4 and Proposition 1 still hold, however, with a potentially different profitability threshold value $\bar{s}^R$. The proofs stay analogous, so that our insights do not depend qualitatively on assuming perfect selling power (Assumption 2).
Lemma 5 (Price-restoring divestitures). Suppose a remedy regime $R$ and assume $s \in (\bar{s}, \bar{s}^R)$. Then each of the following conditions is sufficient to ensure a price-restoring divestiture.

i) Independent of the merged firm’s selling power, the merged firm proposes the minimal price-restoring divestiture $k_j - \hat{\sigma}$ if $\delta_l(k_l + k_j - \hat{\sigma}) \leq \delta_M(k_i + \hat{\sigma})$.

ii) If the divestiture is sold at a fixed price, then the merged firm proposes the minimal, price-restoring divestiture $k_j - \hat{\sigma}$.

iii) If the merged firm has the entire selling power, then it will propose the minimal, price-restoring divestiture $k_j - \hat{\sigma}$ if

$$2 \left( \frac{x_l}{(k_l + k_j - \sigma)^2} - \frac{s x_M}{(k_i + \sigma)^2} \right) < \left( \frac{1}{(k_l + k_j - \sigma)^2} - \frac{s}{(k_i + \sigma)^2} \right) \frac{(1 + \lambda_l) x_l + (1 + \lambda_M) x_M}{1 + \sum_{m \in I_M} \lambda_m}$$

holds for all $\sigma > \bar{\sigma}$, where $k_j - \bar{\sigma}$ is the remedy which induces the lowest possible post-merger price over the interval $\sigma \in [0, k_j]$.

Parts i)-iii) of Lemma 5 can be explained as follows. Part i): From Lemma 1 we directly observe that the remedy must be price-restoring of size $k_j - \hat{\sigma}$ if $\delta_l(k_l + k_j - \hat{\sigma}) \leq \delta_M(k_i + \hat{\sigma})$ holds as $\delta_l(k_l + k_j - \hat{\sigma})$ and $\delta_M(k_i + \hat{\sigma})$ denote the direct effects of capital on $M$’s and $l$’s output levels after $l$ has acquired the assets $k_j - \hat{\sigma}$. Selling more assets implies a market price which is above $p^*$, so that the consumer surplus condition is violated. Therefore, in equilibrium, the consumer surplus standard must be binding.

Part ii): If the merged firm sells the assets at a fixed price, it has no incentive to divest more than the AA requires. Hence, it proposes to divest the minimal price-restoring divestiture $k_j - \hat{\sigma}$, so that the consumer surplus condition binds.

Part iii): If the merged firm has the entire selling power, then (14) is the condition under which it cannot profitably sell more to firm $l$ than the minimal required asset package $k_j - \hat{\sigma}$. Each of the following two requirements $a)$ and $b)$ is sufficient for (14) to hold.

$\ 
\quad a)$ If $\delta_l(k_l + k_j - \hat{\sigma}) > \delta_M(k_i + \hat{\sigma})$ (i.e., the large bracket on the right-hand side of (14) is positive) and if $c_k^l > c_k^M$ (i.e., the large bracket on the left-hand side of (14) is negative) hold for

\footnote{In the proof of Lemma 3 (see Appendix), we note that the function $\delta_l(k_l + k_j - \sigma) - \delta_M(k_i + \sigma)$ has at most one zero on $\sigma \in [0, 1]$. Therefore, it suffices to require that $\delta_l(k_l + k_j - \sigma) \leq \delta_M(k_i + \sigma)$.}
all $\sigma > \bar{\sigma}$, then the implemented remedy will be price-restoring of size $k_i + \hat{\sigma}$.\footnote{Note that $\delta_i (k_i + k_j - \hat{\sigma}) = -1/[p'(X)(k_i + k_j - \sigma)^2]$ and $\delta_M (k_i + \hat{\sigma}) = -s/[p'(X)(k_i + \sigma)^2]$. Furthermore, $c_k^l = -\sigma = -x_i/(k_i + k_j - \sigma)^2$ and $c_k^M = c_k^M = -sx_M/(k_i + \sigma)^2$.} To see this, note the following. In general, if a firm has a larger $\delta$ than another firm, it also faces higher marginal costs and a lower equilibrium output level. Thus, $\delta_i (k_i + k_j - \hat{\sigma}) > \delta_M (k_i + \hat{\sigma})$ implies $x_M > x_i$. If $x_M > x_i$ holds, then $c_k^l > c_k^M$ is also likely to hold. While $\delta_i (k_i + k_j - \hat{\sigma}) > \delta_M (k_i + \hat{\sigma})$ and $c_k^l > c_k^M$ ensure that (14) holds, we show in the Appendix that (14) is sufficient for the divestiture to be price-restoring. The intuition behind this condition is that the merged firm does not have an incentive to sell more capital than necessary if the capital lowers its own production costs by more than the rival firm’s production costs; i.e., if the merged firm can use the capital more efficiently.

b) If $\delta_i (k_i + k_j - \hat{\sigma}) > \delta_M (k_i + \hat{\sigma})$, the proposed merger will involve the price-restoring divestiture $k_j - \hat{\sigma}$ if $2(1 + \sum_{m \in I_M} \lambda_m) < (1 + \lambda_i)x_i + (1 + \lambda_M)x_M$. For instance, a linear demand function, $p(X) = a - bX$, implies $\lambda_i = 1$, so that the preceding condition is equivalent to $n < x_i + x_M$. This holds if the reservation price $a$ is sufficiently large.

If none of the conditions listed in Lemma 5 holds, then the merging parties may divest more than required by the AA. In that case, prices may strictly decrease and consumers may be strictly better off when remedies are feasible.\footnote{Price-decreasing divestitures may also exist if the buyer of the divested assets experiences synergies on its own (see Section 4 below).} In the following, we restrict our analysis to externality-free mergers. Therefore, for the remainder of our analysis we invoke Assumption 3.

**Assumption 3.** Suppose $s \in (\bar{s}, \bar{s}^R]$. All proposed divestitures are price-restoring.

Given Assumption 3, we can easily derive the proportionality-principle claimed in the remedy guidelines; namely, the remedy’s size should be proportional to the anticompetitive concern. If the merged firm’s synergy level increases, the merger’s anticompetitive effects are smaller such that it has to divest less assets in order to satisfy the consumer surplus standard.

**Lemma 6 (Proportionality principle).** Suppose that $s \in (\bar{s}, \bar{s}^R]$ and Assumption 3 hold. Then the size of the price-restoring divestiture sold to a firm $l$ increases in $s$.\footnote{Price-decreasing divestitures may also exist if the buyer of the divested assets experiences synergies on its own (see Section 4 below).}
If a merger is externality-free, then the first-order conditions of the outsider firms remain unaffected by the merger. As a consequence, the social welfare effect of remedies depends only on a comparison of the total production costs for the firms involved in the merger (firms $i$ and $j$) and firm $l$ which buys the divested assets. We define $W(i, j, s)$ as the set of potential buyers for which a price-restoring remedy exists, where firms $i$ and $j$ are the merging firms which realize synergy $s$. Let $SW(l)$ denote social welfare when firm $l \in W(i, j, s)$ acquires the price-restoring divestiture that was offered to it by the merged entity. Firm $l' \in W(i, j, s)$ is the socially optimal buyer if and only if $SW(l') \geq SW(l'')$ for all $l'' \in W(i, j, s)$. We can state the following proposition.

**Proposition 2 (First efficiency result).** Suppose that $s \in (\bar{s}, \bar{s}^R]$ and Assumption 3 hold. Given a consumer surplus standard, firms merge if and only if the merger raises social surplus. Assume that the merging parties can choose to divest assets to any incumbent competitor. If the merging parties can extract the entire gains from the asset sales (e.g., through a take-it or leave-it offer) then they select the socially optimal buyer.

Proposition 2 shows that a merger control regime which allows for remedies under a consumer surplus standard is always preferable from a social welfare perspective when compared with regime $NR$. The reason for this result is quite general: given that consumer surplus is held fixed, under Cournot competition the market price must be held fixed and therefore the profits of any outsider firm not buying the divestiture assets. Then, the merger only affects the profits of the merged firm and the firm which buys the divested assets. The merging firms’ incentive to select the most efficient buyer is fully aligned with the social welfare-maximizing choice. The merging firms are residual claimants and act socially optimally as they maximize the gains from trade under the remedy constraint. Formally, suppose there is more than one possible buyer $l$ for price-restoring remedies. The merged firm then chooses the buyer for which the sum of the profit changes of the merged firm, $\Delta \pi_M$, and the buyer firm, $\Delta \pi_l$, are maximal. If the consumer surplus standard binds, the change in consumer surplus, $\Delta CS$, is zero which implies that the change of the outsider firms’ profits, $\sum_{k \in I_M \setminus \{M, l\}} \Delta \pi_k$, is also zero. Hence, in this case, maximizing $\Delta \pi_M + \Delta \pi_l$ through the choice of a buyer firm is equivalent to maximizing the change in social welfare $\Delta SW := \Delta \pi_M + \Delta \pi_l + \sum_{k \in I_M \setminus \{M, l\}} \Delta \pi_k + \Delta CS$ because there are no exter-
nalities; i.e., \( \sum_{k \in I \setminus \{M,l\}} \Delta \pi_k + \Delta CS = 0 \) holds. It follows that the consumer surplus standard ensures that the merged entity chooses the social welfare maximizing buyer and social welfare increases strictly. We can generalize this reasoning to any oligopoly market with homogenous products.

**Corollary 1.** Suppose an arbitrary homogenous goods oligopoly market and assume that the merging parties propose a price-restoring remedy to the AA (which uses a consumer surplus standard). Then the following efficiency result holds: If the merging parties can extract the entire gains from the asset sales, then they will pick the socially optimal buyer.

This efficiency result crucially depends on the fact that the AA applies a consumer standard. If, instead, the AA applies a social welfare standard, a similar efficiency result cannot be obtained. According to a social welfare standard, all mergers are approved which do not lower social welfare. Suppose a full merger between two firms strictly lowers social welfare such that a remedy becomes necessary. If we presume that the merged firm always prefers a minimal remedy (in spirit of Lemma 5), then the merged firm always chooses social welfare-restoring remedies such that social welfare cannot increase beyond the pre-merger level. It also follows that the merged firm selects the buyer which maximizes \( \Delta \pi_M + \Delta \pi_l \) which is equivalent to maximizing the negative externality of the merger; namely, \(- \sum_{k \in I \setminus \{M,l\}} \Delta \pi_k - \Delta CS\).

## 4 Extensions and Discussion

We analyze two extensions of our basic setup. *First*, we investigate the equilibrium outcomes under different selling mechanisms to show that our efficiency result concerning the consumer surplus standard (Proposition 2) depends crucially on the merged firm’s ability to extract all rents from selling the assets. *Second*, we examine remedy-dependent synergies according to which the size of the divestiture lowers the merged firm’s synergy or creates a synergy for the buyer firm.
4.1 Different Selling Mechanisms

Different selling mechanisms for a divestiture might induce different post-merger market structures and outcomes. When there are several possible buyer candidates then, depending on the selling mechanism, a different buyer may be chosen. Suppose that $s \in [\bar{s}, \bar{s}]$. We examine remedial asset sales for three different selling mechanisms to show how distortions from the socially optimal choice (according to Proposition 2) can occur. *First*, the divestiture may be sold at a fixed price. *Second*, the divestiture may be auctioned off. *Third*, the merged firm has perfect seller power, so that it can make a take-it or leave-it proposal to a preselected buyer. In each case, we assume that the divested remedy is price-restoring. As before, $W(i, j, s)$ denotes the set of potential buyers for which a price-restoring remedy exists, where firms $i$ and $j$ are the merging firms which realize synergy $s$. Note that any other buyer not in $W(i, j, s)$ will be disregarded by the AA as the consumer surplus standard would be violated for any divestiture level in those instances. Furthermore, note that the size of the price-restoring remedy depends on the buyer itself, that is, each remedy is buyer-specific.

We, therefore, assume the following two-stage procedure. In the first stage, the merged firm determines for each potential buyer firm in $W(i, j, s)$ a price-restoring divestiture. In the second stage, the merged entity sells exactly one of these price-restoring divestitures to the targeted buyer. If the divestiture is to be sold at a fixed price, the merged entity selects one buyer firm and offers the remedy at a pre-determined price. If it is sold through an auction, then each buyer firm bids for the price-restoring divestiture that was offered to it by the merged entity. The firm with the highest bid wins the auction. If the merged firm has perfect selling power, it selects one firm in $W(i, j, s)$ and makes a take-it or leave-it offer for the divestiture that was assigned to that firm.

**Selling at a fixed price.** Assume that the divestiture is sold at a fixed price. In order to ensure that no potential buyer is excluded, we assume that the selling price is zero. As a consequence, the merged firm selects the buyer which leads to the highest post-merger profit level. As a firm produces a larger quantity the lower its marginal costs are, the merged firm’s output will also be the larger the less capital it divests. Therefore, the merged firm selects a buyer firm to minimize the size of the asset sale.
Bidding for the divestiture. An auction does not allow for buyer selection as the divestiture goes to the buyer with the highest bid. For simplicity, we take it for granted that the merged firm can extract the entire willingness to pay for a divestiture from the winning bidder; for example, by setting a reserve price. If the divestiture is sold through an auction in which all buyers bid their maximum willingness to pay, then the divestiture goes to the buyer for which the profit-differential through the acquisition of the remedy is largest. A firm $l$’s maximum willingness to pay equals the difference between its post-acquisition and its pre-merger profit as the sale of a price-restoring remedy to an incumbent competitor is externality-free so that firm $l$’s profit is not affected if it does not acquire the assets. The winner of the auction is likely to be a firm for which the price-restoring divestiture is rather large. A large divestiture weakens the merged firm’s market position and lowers its equilibrium output, but enables the acquirer to steal a rather large proportion of the merged firm’s market share. Consequently, a larger price-restoring divestiture shifts equilibrium output to the acquirer, at the cost of the merged firm. Therefore, the winner of the auction may not be the firm which is either preferred by the seller or from a social welfare point of view, as a relatively large output-share is reallocated to the buyer firm.

Perfect selling power. If the merged firm can commit to making a take-it or leave-it offer to a preselected firm, it extracts all gains from trade as we have shown in the previous section.

Proposition 3 (Second efficiency result). Suppose firms $i$ and $j$ propose a merger with synergy level $s \in (\bar{s}, \bar{s}^R]$, so that the AA requires a divestiture in order to approve a merger proposal. Suppose the divestiture is price-restoring. The outcome of the sales process crucially depends on the selling mechanism.

i) If the divestiture is sold at a fixed price which does not exclude any potential buyer (thus is assumed to be zero), then the merged firm sells the remedy to a firm for which the size of the divestiture is minimal. For a linear demand function, $p(X) = a - bX$, this is the smallest firm within $W(i, j, s)$.

ii) If the divestiture is sold through an auction in which all buyers bid their maximum willingness to pay, then the merged firm sells the remedy to a firm with the largest post- and pre-merger profit differential. For a linear demand function, $p(X) = a - bX$, this is the largest firm within $W(i, j, s)$.

iii) If the merged firm can make a take-it or leave-it offer to a preselected buyer then the divestiture is sold to the socially optimal buyer within $W(i, j, s)$.
Proposition 3 shows that the merged firm’s ability to extract rents from the asset sale determines the divestiture level and the buyer’s identity. If, for some reason, potential buyers can avoid getting absorbed in a bidding race, so that rent extraction is severely limited, then the merging parties minimize the amount of assets to be sold (part i) of Proposition 3). If rent extraction is enhanced, for instance, when the asset sale is structured through an auction-type selling process, then the divestiture should be expected to go to a firm which can run the additional assets most profitably (part ii) of Proposition 3). Even though such a buyer may not be preferred by the merged firm as it may “steal” its market share, the merged firm cannot avoid such an outcome if the remedy is sold through an auction. Finally, part iii) of Proposition 3 shows that the merged firm’s divestiture decision is perfectly aligned with the social welfare maximizing rule whenever it can commit to make a take-it or leave-it offer to a preselected buyer. The merged firm is then able to extract the entire surplus created by the divestiture process. As the sale of a price-restoring remedy is externality-free, it follows that the merged firm will make the socially optimal choice.

In Dertwinkel-Kalt and Wey (2012) we show that it depends on the specific setup and the synergy level s whether the socially optimal buyer is more likely to be an efficient (i.e., large) firm or an inefficient (i.e., small) competitor. A relatively inefficient firm can be regarded as an “entrant” firm which has not yet acquired sufficient capital to get a substantial market share. In contrast, efficient firms can be regarded as incumbent competitors which are established in the market and have built up a considerable capital stock. Therefore, our analysis mirrors a feature of the remedy guidelines, according to which remedies might be sold to an entrant firm or an incumbent competitor. Per se, it cannot be determined which buyer type is optimal from a social-welfare perspective.

The message of Proposition 3 is that the merging parties should have a maximum of power in the asset sales process, because this would lead to the selection of the socially preferred buyer. It is noteworthy that remedy guidelines mirror our findings. For instance, the merger remedy guidelines of the DOJ distinguish between “fix-it-first remedies” and “post consummation sales” (DOJ, 2011, pp. 22-25). Successful fix-it-first remedies eliminate the competitive concerns and allow the AA to clear the merger without the need to file the case in court. In con-
trast, post-consummation sales induce the AA to file the case in court to obtain a consent decree, which allows the remedial provisions to be enforced and monitored because of the court’s contempt power. The guidelines clearly favor an adequate fix-it-first remedy, while the post-consummation sale is much more restrictive (and costly) for the merging parties.\footnote{Quite bluntly, the remedy guidelines state: “For the parties, resolving a merger’s competitive issue with an upfront buyer can shorten the divestiture process, provide more certainty about the transaction than if they (...) must seek a buyer for a package of assets post-consummation, and avoid the possibility of a sale dictated by the Division in which the parties might have to give up a larger package of assets” (DOJ, 2011, p. 22).} With regard to the fix-it-first remedy, the guidelines “provide the parties with the maximum flexibility in fashioning the appropriate divestiture” (DOJ, 2011, p. 22). Accordingly, the merging parties can adjust the divestiture freely, so that the assets can be “tailored to a specific proposed purchaser” (DOJ, 2011, p. 22). In contrast, if a consent decree is needed for a post-consummation sale, then the guidelines build up a credible threat of force. First, a package of assets to be divested must be identified in advance, and second, “crown jewels” must be offered “to increase the likelihood that an appropriate purchaser will emerge” (DOJ, 2011, p. 24).

Those rules increase the commitment value of the merging parties when proposing an asset sale to a potential purchaser to obtain a fix-it-first remedy. First, the guidelines give a maximum of flexibility in adjusting the asset sale to the competitiveness of the purchaser. Second, entering into a consent decree is costly, full of uncertainty, and further burdened with the crown-jewel provision. Those additional costs may make the entire merger unattractive, adding to the commitment value necessary to extract rents in the fix-it-first sales process.

4.2 Remedy-Dependent Synergies

So far we have assumed that only the merging firms realize a fixed synergy level. We discuss two extensions: first, the firm which buys the divested assets may realize synergies itself, and second, the merged firm’s synergy level may depend negatively on the amount of the assets to be divested.

In the first case, the acquirer of the assets may generate a synergy \( t \), which effect is analogous to that of \( s \) (i.e., \( t \in [0, 1] \) enters the acquirer’s cost function as a multiplicative factor).
vector \((k_l, t_l)\) describes the efficiency of an acquiring firm \(l\). Given that the price-restoring condition is fulfilled, the merging firms’ incentives to search for the most efficient buyer are fully aligned with the social welfare-maximizing choice. As before, the reason for this result is that the merged firm maximizes the gains from trading the remedy as long as it has perfect selling power.

**Corollary 2.** Suppose an arbitrary oligopoly market and assume that the merging parties propose a price-restoring remedy to the AA (which uses a consumer surplus standard). Assume that buyers of different efficiencies \((k_l, t_l)\) exist. Then the following efficiency result holds: If the merged firm has perfect selling power then it selects the most efficient buyer.

Note also that a very small divestiture (“\(\varepsilon\)-divestiture”) may become possible which is not price-restoring, but price-decreasing. Even a small divestiture may have a significant impact on competition if the divested assets create a considerable synergy \(t\). As such a divestiture raises the competitor’s efficiency significantly while only marginally lowering the merger’s efficiency, consumer surplus may strictly increase through the merger.

In the second case, the divestiture of assets may lower the synergy level of the merging firms. In the following we show that the scope for mergers must shrink if divestitures reduce the merged firm’s synergies. Assume that Condition (11) holds (i.e., \(\bar{s} < \bar{s}^R\)). Suppose that the synergy level of the merged entity depends on the divestiture; i.e., \(s'(\sigma) = \partial s(\sigma)/\partial \sigma < 0\). Define \(s := s(k_j)\) as the synergy level realized if no capital is divested. The following lemma shows that Proposition 1 remains qualitatively valid when considering a negative impact of divestitures on merger synergies.

**Lemma 7.** Suppose a remedy regime \(R\). Firms \(i\) and \(j\) are the merger candidates and \(\bar{s} < \bar{s}^R\) holds. Let \(s := s(k_j)\) denote the synergy level of a full merger, while divestitures reduce merger synergies; i.e., \(s'(\sigma) < 0\). Then all mergers with relatively large synergies, \(s \leq \bar{s}\), are profitable and are approved without a remedy. There is a threshold value \(\bar{s}^{R,D} \in [\bar{s}, \bar{s}^R]\) such that for merger proposals with \(s \in (\bar{s}, \bar{s}^{R,D}]\), a merger with a certain divestiture is proposed and approved by the AA. If \(s > \bar{s}^{R,D}\), no merger will be implemented. As \(\bar{s}^{R,D} < \bar{s}^R\) holds, the scope for mergers is reduced if divestitures affect synergies negatively (i.e., \(s'(\sigma) < 0\) holds) when compared with the case when they do not affect merger synergies (i.e., \(s'(\sigma) = 0\) holds).
From Condition (13) it can be seen that the scope for mergers shrinks when divestitures reduce merger synergies. Given $s'(\sigma) < 0$, the divestiture $k_j - \bar{\sigma}$ which maximizes industry output is smaller when compared with the case where remedies do not affect merger synergies, so that $\bar{s}^{R,D} < \bar{s}^R$ holds. In the extreme case, when even a very small divestiture erases all merger synergies, the entire range for approvable synergies may vanish (i.e., $\bar{s}^{R,D} = \bar{s}$ holds).

If, however, there is a remedy which suffices to fix the AA’s concerns, then this is even more likely to be price-restoring than in our basic model. This is the case because the merged entity has less incentives to sell more assets than necessary if this affects its realized synergies negatively. Formally, we obtain the following lemma.

**Lemma 8.** Suppose a remedy regime $R$ and assume $s \in (\bar{s}, \bar{s}^{R,D}]$. If the merged firm sells a price-restoring divestiture when synergies do not depend on divestitures, i.e., $s'(\sigma) = 0$, then it also sells a price-restoring divestiture if synergies depend on divestitures, i.e., $s'(\sigma) < 0$.

If the negative effect of remedies on the realized synergy level $s$ is relatively small, then the consumer surplus standard fully unfolds the advantageous effects which we have shown so far. The social welfare standard becomes relatively more attractive when the negative effect of an asset sale is large. Assume a merger with relatively small synergies which is approved fully under a social welfare standard but not under a consumer standard. If the negative effect of assets sales, $\partial s/\partial \sigma$, is strong, then an approvable remedy may fail to exist, so that the merger is blocked under the consumer surplus standard. Hence, such a merger cannot occur under a consumer surplus standard. Under a social welfare standard, however, it goes through and induces possibly strictly larger levels of social welfare.

## 5 Conclusion

We analyzed the effects of remedies on merger activity in a Cournot oligopoly model with homogeneous products under a consumer welfare standard. In general, remedies increase the scope for profitable mergers that do not harm consumers. In particular, if the consumer surplus standard binds, the merger does not change the equilibrium market price and is therefore externality-free. Accordingly, the profits of firms not involved in the merger process do not
change. We derive fairly general conditions under which the consumer surplus standard binds and obtain that remedial offers must be larger when the merger’s synergy level is smaller, which mirrors the proportionality principle in the US- and the EU-remedy guidelines.

Furthermore, we derived several efficiency properties concerning current merger control regimes. The ability of the merging firms to extract the gains from trading the divested assets is critical when the purchaser is endogenously determined. If the merging parties’ ability to extract these gains is maximal, that is, if they can make a take-it or leave-it offer to a preselected buyer, then the socially optimal buyer is selected. The merging firms have strong incentives to search for the socially optimal buyer as this tends to increase the feasible set of mergers and, at the same time, maximizes the gains from trading the divestiture. The consumer surplus standard together with the formulation of merger remedy guidelines yields efficient outcomes with respect to two features. First, a remedy regime in combination with a consumer surplus standard ensures that only those mergers are implemented which are strictly social welfare-enhancing. This is achieved in a way such that no market participant is made worse off through the merger. Second, as endorsed by the guidelines, firms should have a maximum of power in the asset sales process which concerns the selection of the buyer firm, the design of the divestiture asset, and its selling price. It then follows that the socially efficient remedial divestiture is implemented.

However, our model also has some limitations. For instance, we take it for granted that claimed synergies are verifiable, that is, the AA can fully anticipate the size of synergies created through a merger which may be not the case in reality. In addition, we regard industry capital as fixed and abstracted from a long-run perspective where the industry’s capital stock may be endogenous because of innovations and entry.

Appendix

In this Appendix we provide the omitted proofs.

Proof of Lemma 1. This proof is analogous to the proof of Proposition 2 in Farrell and Shapiro (1990a). In order to assess how a change in capital \(dk = (dk_1, ..., dk_n)\) affects equilibrium quan-
Let \( x = (dx_1, \ldots, dx_n) \), we take the total derivative of (1) with respect to \( k_j \) and \( x_j \) which gives
\[
[p'(X) + x_j p''(X)]dx_j + p'(X)dx_j - c_{xk}^i dk_j = 0.
\]
Using (6) and (8) and defining \( \Lambda := \sum_i \lambda_i \) we obtain
\[
dX = \sum_i \delta_i dk_i / (1 + \Lambda).
\] (15)

Let capital \( dk \) be sold from firm \( j \) to firm \( l \), so that the preceding formula simplifies to
\[
\frac{dX}{dk} = \frac{\delta_l - \delta_j}{1 + \Lambda}.
\] (16)
This proves the lemma.

**Proof of Lemma 2.** Let \( x^*_i \) and \( X^* \) denote the pre-merger equilibrium levels of firm \( i \)'s output and of the industry output, respectively. Let \( x^*_M \) and \( X^* \) denote the merged firm’s equilibrium output and equilibrium industry output, respectively, after firms \( i \) and \( j \) have merged and have realized synergy level \( s \). By Assumption 1, industry output increases strictly at \( s = 0 \). Note that the industry output \( X^* \) is strictly monotonically decreasing in the sum of firms’ marginal costs. As the merged firm’s cost function is monotone and continuous in \( s \), it follows that industry output is also monotonically and continuously decreasing in \( s \). If there is no synergy parameter \( s \in [0, 1] \) for which the post-merger industry output falls short of the pre-merger output, i.e., for which \( X^* < X^* \) holds, then we define \( \bar{s} := 1 \). Otherwise, there exists a unique threshold value \( \bar{s} \in [0, 1] \) such that industry output increases, \( X^* > X^* \), if and only if \( s < \bar{s} \), while it decreases, \( X^* < X^* \), if and only if \( s > \bar{s} \), with equality holding at \( s = \bar{s} \). Note that all approvable mergers (for which \( s \leq \bar{s} \) holds) are profitable as the joint output of the merging firms (weakly) increases while marginal and infra-marginal production costs decrease.

**Proof of Lemma 3.** Let firms \( i \) and \( j \) be the merger candidates. Note that merging and selling the divestiture simultaneously is formally equivalent to a two-stage procedure where firms \( i \) and \( j \) merge before they divest \( k_j - \sigma \) to a rival firm \( l \).

If \( s \leq \bar{s} \), then consumers are not be harmed by the merger, and the AA applying a consumer-surplus standard approves a full merger. For lower synergy levels \( s > \bar{s} \), however, a full merger cannot be approved by the AA since the industry’s post-merger equilibrium quantity falls below
the pre-merger industry output level. In order to assess to which extent remedies can enlarge the scope for approvable mergers, we first show that there exists a unique threshold value of the synergy parameter $s$ for each potential buyer $l$ up to which a divestiture increases industry output locally.\textsuperscript{25} Second, we derive a condition under which remedies strictly increase the scope for mergers for a certain buyer firm $l$. Third, we show that the existence of an approvable remedy for a certain buyer firm $l$ is monotone in the realized synergy level. Next, we generalize our findings toward all potential buyers $l$: fourth, we obtain a unique synergy threshold value up to which remedies enlarge the scope of mergers, and fifth, we state a weak condition under which remedies strictly enlarge the scope for mergers.

Step 1 (Local effects of a divestiture on industry output). Let $x^s(\sigma)$ and $X^s(\sigma)$ denote the equilibrium quantities for a given divestiture level $k_j - \sigma \geq 0$ and a given buyer firm $l \in I \setminus \{i, j\}$. Given the specification of firms’ cost functions (3)-(5), we obtain

$$c_M^M(x, k, s) = \frac{s}{k^2},$$

$$c_{2M}(x, k, s) = -\frac{s}{k^2}.$$

After divesting $k_j - \sigma$ to firm $l$, the direct effects of capital on output for firms $M$ and $l$ are given by

$$\delta_M(k_i + \sigma) = \frac{c_M^M}{p'} = \frac{s/(k_i + \sigma)^2}{-p'(X^s(\sigma))} \quad \text{and}$$

$$\delta_l(k_l + k_j - \sigma) = \frac{c_{2M}}{p'} = \frac{1/(k_l + k_j - \sigma)^2}{-p'(X^s(\sigma))},$$

respectively.

Therefore, the difference of the direct effects of capital on output between the merged firm $M$ and the acquirer $l$ of the assets $k_j - \sigma$, which is given by

$$\delta_M(k_i + \sigma) - \delta_l(k_l + k_j - \sigma) = \frac{1}{-p'(X^s(\sigma))} \left[ \frac{s}{(k_i + \sigma)^2} - \frac{1}{(k_l + k_j - \sigma)^2} \right]$$

is continuous and has at most one zero on the interval $\sigma \in [0, k_j]$. If there is no such zero and $\delta_M(k_i + \sigma) > \delta_l(k_l + k_j - \sigma)$ holds for all $\sigma \in [0, k_j]$, then set $\bar{\sigma} = k_j$. If there is no such zero and

\textsuperscript{25}This is the main property which our proof exploits. Therefore, only the exact threshold values which we derive, but not the logic of our proof rely on the constant marginal cost specification which we have imposed.
\(\delta_M(k_i + \sigma) < \delta_l(k_l + k_j - \sigma)\) holds for all \(\sigma \in [0, k_j]\), then set \(\bar{\sigma} = 0\). Otherwise, there is a unique threshold value
\[
\bar{\sigma} = \bar{\sigma}(s) = \frac{1}{\sqrt{s + 1}} (-k_i + \sqrt{s}k_j + \sqrt{s}k_l),
\]  
(17)
for which the direct effect of capital on output is the same for the merged firm \(M\) and the acquirer of the assets \(l\); i.e., for which \(\delta_M(k_i + \sigma) = \delta_l(k_l + k_j - \sigma)\) holds. The threshold value \(\bar{\sigma}\) gives the unique maximum divestiture up to which a divestiture can increase the industry output. We find that
\[
\delta_M(k_i + \sigma) < \delta_l(k_l + k_j - \sigma)
\]
holds if and only if
\[
\sigma > \bar{\sigma}.
\]
(18)
This means that the direct effect of capital on output is larger for the merged firm \(M\) than for buyer \(l\) if and only if the divestiture’s size exceeds the threshold value \(k_j - \bar{\sigma}\). Note that
\[
\frac{\partial \bar{\sigma}}{\partial s} = \frac{1}{2\sqrt{s}} \frac{(k_i + k_j + k_l)}{\left(\sqrt{s} + 1\right)^2} > 0,
\]
such that the threshold value \(\bar{\sigma}\) is strictly increasing in \(s\). Therefore, the range of divestitures \(\{k_j - \sigma | \sigma > \bar{\sigma}\}\) for which \(\delta_M(k_i + \sigma) < \delta_l(k_l + k_j - \sigma)\) holds, strictly increases with a higher synergy level (i.e., a lower parameter value \(s\)).

**Step 2 (Remedies increase the scope for acceptable mergers).** In order to prove that remedies increase the scope for mergers, we have to investigate those potential buyers \(l\) for which \(\delta_M(k_i + \sigma) < \delta_l(k_l + k_j - \sigma)\) holds for a small divestiture \(k_j - \sigma\) and for some \(s > \bar{s}\). For a certain buyer \(l\), Condition (18) holds if and only if \(\sigma \in (\bar{\sigma}, k_j]\), while this interval may be empty. Fix \(l\) and define \(\varepsilon(s) = \max\{k_j - \bar{\sigma}(s), 0\}\). Since \(\bar{\sigma}\) is monotonically increasing in \(s\), the function \(\varepsilon(s)\) is monotonically decreasing in \(s\).

For the moment, we assume that \(\varepsilon := \varepsilon(\bar{s}) > 0\) (which holds if \(k_j - \bar{\sigma} > 0\)). According to (17), this is equivalent to assuming that
\[
k_i + k_j > \sqrt{\bar{s}}k_l
\]
(20)
holds. Given \(\varepsilon > 0\), Condition (18) holds for \(s = \bar{s}\) and for all \(\sigma \in (k_j - \varepsilon, k_j]\). Since \(\delta_l(k_l + k_j - \sigma) - \delta_M(k_i + \sigma)\) is continuous in \(s\) and since \(d\bar{\sigma}/ds > 0\), for each \(\varepsilon' < \varepsilon\) we can define \(s' = s'(\varepsilon') > \bar{s}\).
to be the largest \( \hat{s} \) such that Condition (18) holds for all \( s \in [\hat{s}, \hat{s}] \) and for all \( \sigma \in (k_j - \varepsilon', k_j) \). For each \( \varepsilon' \), define \( \varepsilon'' = \varepsilon''(\varepsilon') = \min_{s \in [\hat{s}, \hat{s}]} [X^*(k_j - \varepsilon') - X^*(k_j)] \). First, \( X^*(k_j - \varepsilon') - X^*(k_j) > 0 \) holds for all \( s \in (\hat{s}, \hat{s}) \). Second, \( \lim_{s \to \hat{s}} X^*(k_j - \varepsilon') - X^*(k_j) = X^*(k_j - \varepsilon') - X^*(k_j) > 0 \) holds and third, \( \lim_{s \to \hat{s}} X^*(k_j - \varepsilon') - X^*(k_j) > 0 \) holds. Therefore, \( \varepsilon'' \) is well defined and \( \varepsilon'' > 0 \).

As for each fixed \( \sigma \), the equilibrium quantity \( X^*(\sigma) \) is continuous in \( s \), there exists a largest \( s \in (\hat{s}, \sigma(\varepsilon')) \) which satisfies \( X^* - X^*(k_j) \leq \varepsilon'' \); we denote this synergy by \( \varepsilon'' = \varepsilon''(\varepsilon') \). Then, \( X^* - X^*(k_j) \leq \varepsilon'' \) holds for all \( s \in [\hat{s}, \sigma] \).

As a consequence, \( X^*(k_j - \varepsilon') - X^* = [X^*(k_j - \varepsilon') - X^*(k_j)] - [X^* - X^*(k_j)] \geq \varepsilon'' - \varepsilon'' = 0 \) for all \( s \in [\hat{s}, \sigma'] \). Thus, for synergy \( s \in (\hat{s}, \sigma'] \) there exists a divestiture which can offset the merger’s negative effect on aggregate output. Consequently, as long as our initial assumption \( \varepsilon(\hat{s}) > 0 \) holds, for all such \( s \in (\hat{s}, \sigma] \) there exists a remedy which fixes the AA’s concerns. We will call such a remedy an approvable remedy.

**Step 3 (Monotonicity and uniqueness).** Clearly, \( \partial X^* / \partial s < 0 \) holds as a larger \( s \) implies a lower synergy and therefore a higher sum of firm’s marginal production costs. If there is an approvable remedy sold to firm \( l \) for a merger which realizes synergy \( s \), then there is an approvable remedy also for higher synergies, i.e., lower \( s \). As a consequence, there is a threshold synergy value \( s_l^R \) such that there exists an acceptable remedy if and only if the merger synergy satisfies \( s \leq s_l^R \).

Precisely, this threshold value can be defined as \( s_l^R := \sup_{\varepsilon' < \varepsilon(\hat{s})} \varepsilon''(\varepsilon') \in [\hat{s}, 1] \) if \( \varepsilon(\hat{s}) > 0 \) and \( s_l^R := \hat{s} \) if \( \varepsilon(\hat{s}) = 0 \).

**Step 4 (Extending toward all potential buyers).** We can repeat the analysis with all potential buyer firms \( l \neq i, j \). Allowing for remedies to be divested to any competitor further increases the scope where remedies can induce a merger’s approval. We define \( \bar{s}^R := \max_{l \in \Gamma \setminus \{i, j\}} s_l^R \in [\bar{s}, 1] \). Therefore, the synergy range where mergers do not harm consumers is strictly increased through remedies if there is a firm \( l \) such that \( s_l^R > \bar{s} \).

**Step 5 (Condition such that remedies increase the scope for mergers strictly).** Extending Condition (20) toward all potential buyers yields that divestitures strictly increase the scope for mergers if and only if

\[
k_i + k_j > \sqrt{\bar{s}} \min_{l \in \Gamma \setminus \{i, j\}} \{k_l\}.
\]

This proves the lemma. \( \square \)
Proof of Lemma 4. For synergy levels \( s \in [\bar{s}, \bar{s}^R] \) there is a merger of firms \( i \) and \( j \) (potentially involving a divestiture sold to some firm \( l \)) which does not increase the final good’s price. Then (due to continuity of the cost function) there also exists at least one divestiture level \( k_j - \sigma \) such that the consumer surplus standard binds (i.e., pre- and post-merger prices are the same). Given this divestiture level, the joint post-merger equilibrium output of firms \( M \) and \( l \), i.e., \( x_M^* + x_l^* \), equals the joint pre-merger output of firms \( i, j \) and \( l \), i.e., \( x_i^* + x_j^* + x_l^* \) (where the superscript “\( s \)” indicates equilibrium outcomes after a merger generating synergy \( s \) and superscript “\( s^* \)” denotes pre-merger equilibrium outcomes), while this output is produced at strictly lower costs after the merger.

Then, the sum of the merged firm’s and the buyer firm’s profits \( \Pi_M^* + \Pi_l^* \) exceeds the sum of firms’ pre-merger profits \( \Pi_i^* + \Pi_j^* + \Pi_l^* \). As the merged firm \( M \) has perfect selling power, it can extract up to \( \Pi_M^* + \Pi_l^* - \Pi_l^* \), which is larger then \( \Pi_i^* + \Pi_j^* \). Thus, for every synergy level \( s \in [\bar{s}, \bar{s}^R] \) there exists an approvable merger (possibly involving a divestiture) which is also profitable.

□

Proof of Lemma 5. Part i) is immediate while part ii) follows directly from Lemma 1. In order to prove part iii), we derive condition (14). Using Equation 13 of Farrell and Shapiro (1990a), the derivative of the sum of firm \( M \)’s and firm \( l \)’s profits with respect to \( \sigma \) can be written as\( ^{26} \)

\[
\frac{d(\Pi_M + \Pi_l)}{d\sigma} = -p'(X) \left( \delta_M x_M - \delta_l x_l + \frac{\delta_l - \delta_M}{1 + \Lambda} ((1 + \lambda_l)x_l + (1 + \lambda_M)x_M) \right) - c_k^M + c_k^l. \tag{21}
\]

Substituting \( \delta_M, \delta_l, c_k^M \) and \( c_k^l \) and re-arranging yields

\[
\frac{d(\Pi_M + \Pi_l)}{d\sigma} = 2 \left( \frac{sx_M}{(k_i + \sigma)^2} - \frac{x_l}{(k_l + k_j - \sigma)^2} \right) + \frac{1}{(k_l + k_j - \sigma)^2} - \frac{s}{(k_i + \sigma)^2} \left( \frac{(1 + \lambda_l)x_l + (1 + \lambda_M)x_M}{1 + \Lambda} \right).
\]

This proves the lemma. □

Proof of Lemma 6. For \( s \in [\bar{s}, \bar{s}^R] \) let \( k_j - \hat{\sigma} \) be the price-restoring equilibrium divestiture to be sold to firm \( l \). Suppose the synergy parameter \( s \) falls marginally. Holding \( \hat{\sigma} \) fixed, the final good price decreases. Due to Assumption 3, the merging firms will adjust the remedy in order

\( ^{26} \)The following equation can be derived by using the total derivatives of firm \( l \)’s and firm \( M \)’s first-order conditions with respect to \( x, X \) and \( \sigma \).
to keep the remedy price-restoring. As \( \delta_l(k_l + k_j - \sigma) \geq \delta_M(k_i + \sigma) \) for synergy \( s \) and all \( \sigma > \hat{\sigma} \), this inequation holds also for a synergy parameter slightly below \( s \) and all \( \sigma > \hat{\sigma} \). Therefore, the respective price-restoring remedy is smaller than \( k_j - \hat{\sigma} \).

\[ \square \]

**Proof of Proposition 3.** We prove each part of the proposition separately.

**Part i)** As the merged firm does not earn any revenues from selling the assets, it maximizes its own market profit. The market profit is maximal if the size of the divestiture is minimal. This must be so as additional capital lowers marginal production costs and as the own equilibrium quantity strictly increases with lower marginal costs. We consider the linear demand function \( p = a - bX \) with parameters \( a, b > 0 \). In order to assess the impact of a divestiture to firm \( l \) on the industry output, we analyze condition (16) and obtain

\[
\frac{dX}{d\sigma} = \frac{\delta_M(\sigma) - \delta_l(\sigma)}{(1 + \Lambda)} = \frac{\frac{s}{(k_i + \sigma)^2} - \frac{1}{(k_l + k_j - \sigma)^2}}{nb},
\]

which is strictly monotonically decreasing in \( k_l \) for all admissible \( \sigma \). Therefore, the size of the price-restoring divestiture, i.e., the divestiture which suffices to restore the pre-merger industry quantity, is smallest if the merged firm divests to the firm \( l \in W(i, j, s) \) which holds the smallest capital stock.

**Part ii)** Let \( \tilde{l} \) be the firm with the largest capital stock within \( W(i, j, s) \); i.e., \( k_{\tilde{l}} \geq k_l \) for all \( l \in W(i, j, s) \). First, note that firm \( \tilde{l} \) produces the largest pre-merger equilibrium quantity, i.e., \( \tilde{l} \in \arg \max_{l \in W(i, j, s)} \{ x_l^* \} \). Second, Equality (22) implies that the price-restoring divestiture \( k_j - \sigma \) is weakly larger for firm \( \tilde{l} \) than for all other firms \( l \in W(i, j, s) \). Third, as the pre-merger industry quantity is required to be restored through the divestiture process and as the quantity produced by the merged firm \( M \) is strictly monotonically decreasing in the size of the divestiture \( k_j - \sigma \), firms of size \( k_{\tilde{l}} \) record the largest increase in equilibrium output through the acquisition of the price-restoring divestiture. This means that \( \tilde{l} \in \arg \max_{l \in W(i, j, s)} \{ \overline{x}_l - x_l^* \} \), where \( \overline{x}_l \) denotes firm \( l \)'s equilibrium output after the acquisition of a price-restoring divestiture. Fourth, a firm \( l \)'s willingness to pay for a price-restoring divestiture, \( WTP(l) \), equals the difference between its profit after the asset’s acquisition, \( \Pi_l \), and its pre-merger profit, \( \Pi_l^* \). For the demand function \( p = a - bX \) with parameters \( a, b > 0 \), a firm \( l \) producing quantity \( x_l \) earns profit \( \Pi_l = bx_l^2 \) (which can be easily seen from inspecting a firm’s first-order condition). It then follows that
\( \tilde{l} \in \arg \max_{l \in W(i,j,s)} WTP(l) \), which proves the claim.

Part iii) This follows from Proposition 2. \(\square\)

**Proof of Lemma 7.** We provide a sketch of the proof. First, we show that the minimal synergy level which fulfills the consumer surplus standard must by lower when divestitures reduce synergies; i.e., \( \tilde{s}^{R,D} < \tilde{s}^R \) holds. Second, we argue that the monotonicity of the AA’s decision rule remains valid (as shown in Proposition 1).

The industry output after a merger of firms \( i \) and \( j \) and divestiture \( k_j - \sigma \) to firm \( l \) is proportional to

\[
MC(s(\sigma)) := \frac{s(\sigma)}{k_i + \sigma} + \frac{1}{k_i + k_j - \sigma} + \sum_{m \in I_M \setminus \{M,l\}} \frac{1}{k_m}.
\]

Provided \( \bar{s} < \tilde{s}^R \), in our main model a merger which generates synergy \( \tilde{s}^R \) is approvable with a certain divestiture. We denote \( MC(\tilde{s}^R) \) as the respective sum of firms’ marginal costs. If \( s'(\sigma) < 0 \) and the full synergy \( s = s(k_j) \) equals \( \tilde{s}^R \), then \( MC(s(\sigma)) > MC(\tilde{s}^R) \) for \( \sigma < k_j \). We have seen that if \( s'(\sigma) = 0 \) holds, then for synergy level \( \tilde{s}^R \) there is no approvable divestiture such that the consumer surplus strictly decreases. Consequently, if \( s'(\sigma) < 0 \), then there is no approvable divestiture for synergy level \( s(k_j) = \tilde{s}^R \). Thus, mergers with a synergy level of \( s(k_j) = \tilde{s}^R \) (and strictly lower synergies) are not approvable. Therefore, the scope for mergers under a consumer surplus standard shrinks if remedies frustrate the merger’s synergy. While the notation will be more complicated, the monotonicity results can be obtained by an analogous analysis as in the proof of Lemma 3. Finally, the profitability condition is satisfied due to the same reasoning as in Lemma 4. \(\square\)

**Proof of Lemma 8.** If \( s'(\sigma) < 0 \), then

\[
\delta_M(k_i + \sigma) = \frac{s(\sigma)(k_i + \sigma)^2 - s'(\sigma)(k_i + \sigma)}{-p'(X^s(\sigma))},
\]

such that Equation (21) can be re-written as

\[
\frac{d(\Pi_M + \Pi_I)}{d\sigma} = 2 \left( \frac{s(\sigma)x_M}{(k_i + \sigma)^2} - \frac{x_I}{(k_i + k_j - \sigma)^2} - \frac{s'(\sigma)x_M}{(k_i + \sigma)} \right) + \left( \frac{1}{(k_i + k_j - \sigma)^2} - \frac{s(\sigma)}{(k_i + \sigma)^2} - \frac{s'(\sigma)}{k + \sigma} \right) (1 + \lambda_l)x_I + (1 + \lambda_M)x_M \sum_{m \in IM} \lambda_m.
\]

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As \( s'(\sigma)/(k + \sigma) < 0 \), we obtain that \( d(\Pi_M + \Pi_l)/d\sigma > 0 \) holds for \( s'(\sigma) < 0 \) (given it holds for \( s'(\sigma) = 0 \)). Thus, mergers are more likely to be price-restoring if divestitures reduce the merger synergies. □

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