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Equality of Opportunity: Theory and Evidence

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Equality of Opportunity: Theory and Evidence

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Institute for the Study
of Labor

Equality of Opportunity: Theory and Evidence

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Discussion Paper No. 8994
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ABSTRACT

Equality of Opportunity: Theory and Evidence*

Building on earlier work by political philosophers, economists have recently sought to define a concept of equity that accommodates the fairness of reward to individual responsibility and effort, while allowing for the existence of some inequalities which are unfair and should be compensated. This paper – commissioned as a chapter for the *Oxford Handbook of Well Being and Public Policy* – provides a critical review of the economic literature on equality and inequality of opportunity. A simple 'canonical model' of equal opportunity is proposed, and used to explore the two fundamental concepts in this (relatively) new theory of social justice: the principles of compensation and reward. Ex-ante and ex-post versions of the compensation principle are presented, and the tensions between them are discussed. Different approaches to the measurement of inequality of opportunity – and empirical applications – are reviewed, and implications for the measurement of poverty and of the rate of economic development are discussed.

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1. Introduction

This paper critically reviews a branch of literature that has recently developed in the intersection between normative economics and distributional analysis, and which concerns itself with the definition and measurement of inequality of opportunity. This literature was inspired by a debate that took place in political philosophy during the 1970s and 1980s. Beginning with John Rawls (1971), a number of authors renewed and reinvigorated the egalitarian project by progressively moving the "demand for equality" from the realm of individual achievements to the space of opportunities. This was achieved by proposing a version of egalitarianism that does not deny but, on the contrary, explicitly recognizes the role of individual responsibility. As Cohen (1989) writes, this literature "[...] has performed for egalitarianism the considerable service of incorporating within it the most powerful idea in the arsenal of the anti-egalitarian right: the idea of choice and responsibility" (Cohen 1989, p.993). For this very reason, it is argued, equality of opportunity is now the prevailing conception of social justice in contemporary western societies.

Economists soon followed where political philosophers had first treaded: Beginning in the 1990s, considerations of personal responsibility and opportunities began to feature in formal economic theories of the evaluation of social states. After influential contributions by John Roemer and Marc Fleurbaey, mainly devoted to the identification of opportunity-egalitarian policies and allocation rules, in the last fifteen years a substantial literature has emerged which seeks to assess the degree of inequality of opportunity in different countries, and to evaluate the opportunity-equalizing effects of social and economic policies.

The literature is now sizable. A number of different measurement and evaluation methodologies have been proposed and an even broader array of empirical applications has been undertaken. Inequality of opportunity has been analyzed in different spheres of human life and for different domains of public policy, ranging from income distribution and income taxation; to health and health care; educational achievement; and anti-poverty policy. In light of this broad scope, this paper makes no attempt at being exhaustive. We aim only to provide a flavour of recent research in this field, and interested readers are also referred to two excellent recent surveys by Roemer and Trannoy (2013), and Ramos and Van de Gaer (2012).

The paper is structured as follows. The next section contains a very brief overview of the philosophical foundations of the theory of equality of opportunity (E.Op.). Section 3 then reviews the two main families of approaches to the economic analysis of E.Op.: the *direct approach*, which seeks to characterize people's opportunity sets directly; and the *indirect approaches*, which define equality of opportunity in terms of a distinction between two categories of determinants of individual achievement: those within the realm of personal responsibility, and those beyond it. The family of indirect approaches is anchored on two fundamental principles of equality of opportunity: the compensation and reward principles, which are also discussed in Section 3.

After these conceptual intricacies are ironed out, Section 4 turns to the main empirical methods that have been proposed for measuring inequality of opportunity within the indirect approach. Section 5 reviews a number of applications of some of the empirical methods described in Section 4. Section 6 briefly discusses implications of the concept of equality of opportunity for the measurement of poverty and for the definition and empirical assessment of economic development. Section 7 concludes.

2. The philosophical background: choosing the space for the assessment of equity

Distributional analysis in economics, and the social sciences more broadly, has traditionally been anchored - explicitly or implicitly - on the welfarist paradigm, within which equity assessments are formulated on the basis of the distribution of some individual achievement – welfare, utility or preference satisfaction – over the population. A particularly influential version of the welfarist tradition is the utilitarian approach, which uses an additive aggregation of individual achievements as the social objective function. Jeremy Bentham, one of the most influential eighteenth-century utilitarians, viewed society's objective as the pursuit of "the greatest good for the greatest number", and our continued use of per-capita Gross Domestic Product as a dominant metric by which social or economic progress is measured reflects its remarkable staying power.

But the dominance of utilitarianism – and welfarism more broadly – as the ethical basis for the assessment of social progress has not gone unquestioned. Far from it: the last half century has seen mounting challenges, both in political philosophy and in normative economics. John Rawls (1971), Ronald Dworkin (1981a,b) and Amartya Sen (1980) have been among the most influential critics of the welfarist paradigm. Each of these authors, in his own particular way, has placed at center stage the choice of the *space* in which equity criteria ought to be formulated. It is important to note that these critiques went beyond merely the *aggregation rule* across individual achievements. They did not question only the absence of inequality-aversion implicit in the sum-based metric of utilitarianism (which could be addressed within a welfarist perspective by introducing inequality aversion in the space of utilities, say). Instead, these critics challenged the very *informational basis* of welfarism, by questioning whether social justice could be defined only with reference to the distribution of individual preference satisfaction. The very space upon which equity and justice were to be assessed became a matter for debate. As Sen (1980) famously put it, the crucial question became: "equality of what?"

Rawls (1971), who first focused on the problem of choosing the correct *equalisandum*, proposed the notion of *primary goods*, such as basic liberties and rights, access to political and other offices, income and wealth. But primary goods - Sen (1980, 1985, 1992) argues - are still means to an end, rather than ends in themselves. In many cases primary goods are still commodities, which do not tell us what an individual can really do and be, given her own personal characteristics. The same amount of income does not translate into an equal capacity for movement, for example, for disabled and able-bodied people. In Sen's view the correct space to

consider is not that of commodities, but that of *functionings*, defined as observable "doings and beings" of persons, such as literacy, nutrition and health status. Moreover - Sen argues - what society should promote and equalize is not some specific vector of functionings – a form of achievement – but the (positive) freedom a person enjoys, intended as the opportunity of attaining the relevant achievement. Hence the proposal of *capabilities* as the basal space for Sen's theory of justice, where capabilities are sets of attainable functionings, from which the individual is free to choose.

In two influential papers, Dworkin (1981a, b) formulates a different criticism of the use of individual welfare as the appropriate basis for equity judgments: a society committed to equalizing the welfare of individuals would have to give more resources to those with expensive tastes, and this is ethically unacceptable. As a consequence, Dworkin proposes resources – both transferable and non-transferable – as the correct basal space for a theory of justice. The existence of non-transferable resources led him to conceive of a complex insurance market, carried out behind a veil of ignorance, as the right device for identifying the equitable distribution of resources.

Richard Arneson (1989) and Gerald Cohen (1989), in turn, argue that the relevant distinction for equity purposes is not between preferences and resources, but between factors that lie within the realm of individual responsibility, and those that lie beyond it. Hence, they propose opportunities, defined as "chance[s] of getting a good if one seeks it" (Arneson, 1989, p.85), as the appropriate basal space for a theory of justice, and equality of opportunity as the corresponding social objective.

Rawls's theory of justice, with its emphasis on basic liberties and primary goods; Sen's capability approach; Dworkin's views on equality of resources; and Arneson's and Cohen's defense of equal opportunities are each unique in many ways, and there are substantive differences among them. Nevertheless, these approaches do have something in common, something that sets them apart from the prevailing welfarist paradigm that was dominant before them. That common core is the idea that an equitable society is not necessarily a society that makes all people equally happy, or equally rich, or equally educated. It is rather a society that secures for all of its members an equal chance to attain the outcomes they care about. Thus opportunity, rather than achievements[1], becomes the appropriate "currency of egalitarian justice" (Cohen, 1989). Once the means or opportunities to reach a valuable outcome have been equally allocated, which particular opportunity the individual chooses to seize, from among those open to her, lies outside the scope of justice.[2]

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[1] In this paper we use the terms "outcome", "achievement" and "advantage" interchangeably, to denote the realization of any variable that is intrinsically valuable to individuals, such as income, educational attainment, health status, well-being, etc.

[2] This brief review of the philosophical debate on opportunity egalitarianism is evidently not comprehensive. For a more detailed reconstruction of this intellectual history, see Roemer (1996) and Fleurbaey (2008).

Consequently, the equity (or fairness) of a given distribution of achievements (say utilities, or incomes) cannot be judged by observing only the degree of inequality present in that distribution. Distributive judgments now require an extended informational basis. Information is required, in particular, on *how* the observed outcomes were derived from the choice sets available to individuals. The same inequality in a distribution of outcomes can sometimes be judged equitable and sometimes not, depending on whether they reflect differences in choice sets (that lie beyond individual responsibility), or different choices from within those sets (for which individuals can be held responsible). For Dworkin, Arneson and Cohen, in particular, justice (or equity) requires that all factors influencing the individual's final achievement for which she cannot be held responsible be equalized - or compensated - by society. Such factors are generally labeled the individual's *circumstances*.

Clearly, any theory of equality of opportunity remains an empty box until one defines which factors lie beyond, as opposed to within, the realm of individual responsibility. In other words, for the theory of equality of opportunity to become operationally or empirically meaningful, one must decide which factors should be classified as *circumstances,* and which should be counted as choices for which individuals are to be held responsible. That classification remains a contested matter conceptually, and is additionally subject to the vagaries of data availability, empirically.

In general, a conservative (or "right-wing") interpretation of the theory tends to limit the scope of circumstances, and attribute much to individual choice, whereas a progressive (or "left-wing") interpretation tends to recognize the importance of individual responsibility in principle, but to downplay the ability of individuals to make real choices: in this view, individual choices tend be considered as completely determined by social circumstances (Barry, 1991).

A first crucial disagreement concerns the status of individual preferences. Do they lie within or beyond the realm of individual responsibility? Should a person be held responsible for being lazy, any more than he is responsible for being, say, male? Dworkin argues that justice requires equality of resources, and that preferences are irrelevant, in the sense that they are within the individual responsibility; his favourite examples involve the presence of expensive tastes. Cohen and Arneson, on the other hand, argue that one should consider the process through which individual preferences are generated and, in particular, the presence of adaptive or endogenous preferences: individual tastes themselves can be partly determined by the external environment. In their view, the key determinant of whether or not a particular source of inequality is to be compensated by society is whether or not the individual has *control* over it.

As we shall see, the contrast between the control view (Cohen) and the preference view (Dworkin) has important implications for the empirical analysis of equality of opportunity: for instance, in Fleurbaey's model the individual is held responsible for her choices if they are based

on preferences with which she identifies, whereas Roemer's influential proposal to model individual effort is an expression of the control view.

A second fundamental issue in the implementation of the theory of equality of opportunity concerns the treatment of randomness or luck. Luck can take different forms. A first distinction (Dworkin, 1981a, b) can be made between *brute luck* and *option luck*. Brute luck is defined as a situation where the individual cannot alter the probability that an event takes place, while option luck arises when individuals deliberately take risk, which is assumed to be calculated, anticipated and avoidable.

Brute luck can be either *initial* or *later brute luck*. Within initial brute luck, one can further distinguish between *social background luck*, due to factors related to family background or social origin, and *genetic luck*, due to factors such as ability, talent and other inherited characteristics. *Later brute luck* refers to random and unexpected events that take place later in life, such as natural disasters, wars, or accidents.

Different authors have suggested different normative evaluations of these various kinds of luck. As far as brute luck is concerned, by definition the individual is not responsible for such events and thus it seems reasonable to argue in favor of full compensation. However, while there is almost universal agreement on considering social background luck as a circumstance deserving of compensation, the same cannot be said of genetic luck: for instance, Nozick (1974)'s argument for self-ownership would imply that individuals should benefit from their inborn traits. The treatment of later brute luck is also debatable, since the occurrence of later brute luck is rarely observable in data sets commonly available to economists and other social scientists. In practice, many authors dealing with dynamic models (see Aaberge et al. 2011) implicitly refer to the distinction between initial and later brute luck by making a distinction between circumstances at birth, constant over time, and idiosyncratic factors, which are variable over time.

The evaluation of option luck is not uncontroversial either. Since, by definition, risks involved in option luck are taken deliberately, most authors argue that the resulting differences in outcomes should not warrant compensation. But Fleurbaey (2008), for example, argues that when small errors of judgment involve disproportionate penalties, these disproportionate outcome differences would deserve compensation even under option luck.

So, different authors have distinguished between forms of luck that require full, partial or no compensation. Beside normative reasons, in empirical analysis the inclusion of different forms of luck within the domain of compensation may depend on the availability of information. Some have proposed to include luck as a third determinant of advantage outcomes, separately from circumstances and effort (see e.g. Lefranc et al., 2009). In the remainder of this paper we take the earlier, alternative view that luck must be classified into either the circumstance or responsibility domain, based on the above distinctions, rather than being treated as a third entity.

3. Economic models of equality of opportunity

A. The direct approach

Some of the first attempts to formalize the notion of equality of opportunity within economic models sought to treat opportunities *directly*, or explicitly. A number of papers that address the question of how social states should be ranked according to equality of opportunity have models in which each individual is endowed with a given (abstract) set of opportunities, treated as non-rival goods and assumed to be observable. Society is then represented as a profile of opportunity sets. The problem of measuring the degree of inequality of opportunity (I. Op.) is then handled by characterizing inequality measures (or rankings) of joint distributions of the elements of individual opportunity sets. Inequality, along with other aspects of that joint distribution, such as the first moment, then form the basis for an overall social evaluation function to be used in the assessment of social states. See, for example, Kranich (1996, 1997); Herrero et al. (1998); Ok (1997); Ok and Kranich (1998); Savaglio and Vannucci (2007); and Weymark (2003).

Of course, the solution to this problem necessarily depends on the particular metric one chooses for evaluating the individual opportunity set. Hence, the literature on ranking profiles of opportunity sets is naturally linked to the literature on ranking individual opportunity sets, which was very active in the 1990s, and was mainly inspired by Sen's discussion of freedom and capabilities. This literature started with the seminal paper by Pattanaik and Xu (1990), who characterized the simple cardinality ordering, according to which a set A is preferred to a set B if A contains more elements then B[3]. Indeed, the set of opportunities which is the object of the direct approach can be interpreted very much in the same way as Sen's set of capabilities, from which individuals choose a particular vector of functionings.

In a seminal paper, Kranich (1996) proposes a simple framework, involving only two agents, in which opportunity sets are evaluated on the basis of their respective cardinalities (foolowing Pattanaik and Xu, 1990). On the basis of this assumption, which allows one to have full cardinal comparability of individual positions, several appealing properties for the inequality ranking rule are introduced, and a unique equality relation - namely the cardinality difference relation - is characterized. According to this ranking, an opportunity distribution O is more equitable than O' if the difference between the cardinalities of the agents' opportunity sets in O is less than the difference in O'. In the case of an arbitrary finite number of agents, the absolute mean difference relation is axiomatically derived, according to which the fairness of an opportunity distribution is inversely related to the average (pairwise) cardinality difference.

[3] For a comprehensive survey of the literature on ranking opportunity sets see Barberà et al (2004). For a discussion of the connections between these two branches of literature, one on ranking sets of opportunities and the other on ranking distributions of opportunity sets, see Peragine (1999).

Subsequent papers (see in particular Ok and Kranich, 1998 and Ok, 1997) tried to extend the standard concepts used in the income inequality literature, such as the Pigou-Dalton transfer principle and the Lorenz partial ordering, to the case of distributions of opportunity sets. In so doing, they generally relied on the cardinality relation for ranking individual opportunity sets. An interesting alternative, due to Weymark (2003), proposes instead to judge the relative desirability of opportunity sets according to the simple rule of set inclusion: a set *A* is preferred to another set *B* if all the elements in *B* are contained in *A*. On this basis, Weymark is able to provide an axiomatic characterization of the generalized Gini equality of opportunity orderings[4].

The approach followed in this branch of literature – to model opportunity sets explicitly – is both natural and intuitive. However, its informational requirements have generally proved too strong to be met in empirical applications. In addition to problems of measurability and comparability of individual positions, which are common to all distributional analyses in a multidimensional setting[5], the basic difficulty facing empirical applications of the theoretical approach summarized above is that opportunities are inherently unobservable. And that is because they are, by definition, a set of hypothetical options, some of which are exercised – and become factual – while others are not exercised, and become counterfactual. It is perhaps not surprising, then, that the *direct* approach to equality of opportunity does not seem to have inspired any empirical applications.

As suggested above, the problem of ranking opportunity profiles addressed in this branch of the literature is closely analogous to the problem of ranking capability sets in Sen's framework. Although there are a number of empirical applications of Sen's approach, they generally focus on the space of observed functionings (i.e. the choices that individuals make from within their sets), rather than on actual profiles of capability sets.

B. The indirect approach

The second family of approaches to a formal treatment of inequality of opportunity in economics is *indirect*. These approaches are more structural and *consequentialist* in nature: the focus is not on the distribution (and redistribution) of opportunities themselves, but rather on the *consequences* of a given distribution of opportunities as manifested in the joint distribution of an observable advantage and a number of individual characteristics. Roemer (1993), Van de Gaer (1993), Fleurbaey (1994) and Bossert (1995) independently introduced different models in which

[4] Once the domain of opportunity profiles has been defined as the evaluative space, different rankings, not necessarily focused on inequality criteria, can be defined. For example, Peragine et al. (2009) address the problem of ranking opportunity profiles in terms of poverty; they characterize poverty rankings which generalize the standard headcount and poverty gap measures, mostly used in unidimensional poverty analyses, to the space of opportunities.

[5] In that respect, the problem of describing inequality of opportunity sets is similar to that of describing multidimensional inequality in a welfarist approach, in which individuals are characterized by a set of welfare attributes. See, in particular, the seminal papers by Kolm (1977) and Atkinson and Bourguignon (1982), who laid the foundations for the study of multidimensional inequality.

the link between outcomes, opportunities, effort and exogenous circumstances is explicitly formalized.

Roemer's (1993) influential contribution was mainly concerned with invigorating the egalitarian project by offering a version of egalitarianism that focused primarily on reducing those inequalities perceived as most unfair in western liberal societies: those associated with race, gender, family background, or ethnicity. Roemer's favorite metaphor to convey the main message of opportunity egalitarianism is that of "leveling the playing field". He proposes a model of optimal taxation in which the social planner's evaluation function incorporates an aversion to inequality due to inherited circumstances.

Fleurbaey (1994), on the other hand, was more concerned with the definition of fair division rules that explicitly recognize the role of individual responsibility. In a series of papers, Marc Fleurbaey, François Maniquet and Walter Bossert have proposed a number of allocation rules that compensate for the effect of individual characteristics deemed ethically irrelevant (circumstances), without interfering with the effects of individual responsibility. This literature, which is similar in spirit (but different in detail) to Roemer's approach, is summarized in Fleurbaey (2008).

Although there are important differences among these models, they share a key feature: in all of them the distribution of opportunities is *indirectly deduced* from models of functional relationships between (potentially observable) individual circumstances, effort and achievements. The idea of these consequentialist approaches is that of modeling opportunities as the set of outcomes that an individual can access *given* her exogenous circumstances, depending on her choice of effort level (or of different responsibility characteristics).

Furthermore, an important contribution of this indirect approach was the formulation of the two fundamental ethical principles upon which opportunity egalitarianism rests. The first is the notion that inequality due to circumstances beyond individual responsibility is ethically unjustified. Known as the *principle of compensation* (Fleurbaey 1995a), this first sub-goal requires society to compensate individuals for differences in outcomes which are due to factors beyond their control. The second principle deals with the apportioning of outcomes to individual effort (or, more broadly, to the exercise of individual responsibility) and, in some formulations, it states that outcome inequalities due to differences in efforts are ethically legitimate. Known as the *reward principle*, this second sub-goal is the *locus* of the introduction of the ethics of responsibility into the egalitarian project. It requires societies to respect rewards to individual effort, and not to compensate outcome differences that arise as a result.

The two principles are clearly independent, and it is easy to think of policy rules in which one is observed, while the other is violated. A completely outcome-egalitarian policy, for example, would satisfy the compensation principle, but violate the reward principle. Conversely, a completely laissez-faire policy would satisfy the reward principle, but violate the compensation

principle. In fact both principles can be formalized in various different ways and, in a number of those versions, it turns out that they are logically inconsistent (see Bossert, 1995 and Fleurbaey, 1995a). Much of the subsequent literature that has followed these seminal papers (see Fleurbaey, 2008 and Fleurbaey and Maniquet, 2012) can be seen as an exploration of social orderings that satisfy weaker versions of axioms inspired by the compensation and reward principles.

In the subsequent literature, one particular distinction between two ways of formulating the compensation principle turns out to matter a great deal: That is the distinction between *ex-ante* and *ex-post compensation*. In essence, the distinction revolves around whether compensation is to be accorded to individuals (conceptually) *prior* to the determination of their effort levels, or *after* their efforts are revealed[6]. When compensation is due prior to the realization of efforts, the compensation principle requires equalization of some valuation of the opportunity sets available to everyone, regardless of their circumstances. This is known as the *ex-ante* version of the compensation principle. When compensation is instead due after efforts are realized, the requirement is that all individuals exerting the same level of effort (or making the same choices) receive the same outcomes, regardless of their circumstances. This is the *ex-post* version of the compensation principle.

Although the two versions might sound similar at first brush, their formal and practical implications are quite different. As it turns out, ex-post compensation and the reward principle inspire axioms which are often mutually logically inconsistent. In contrast, there is no clash between (most known formulations of) the reward principle and ex-ante compensation. In order to illustrate the two fundamental principles of E. OP. theory – compensation and reward – and their relationships to each other under different formulations, the remainder of this section sketches out a "canonical" model of the indirect approach to equality of opportunity. This model is loosely based on the ideas of Bossert (1995), Fleurbaey (1994), Roemer (1993) and Van de Gaer (1993). Though there are important differences among the three references, we focus first on the common structure which they share. Then we use the basic model to explore some of the differences.

C. The "canonical" E. Op. model

Consider a population over which a distribution of an individual achievement, or advantage, variable x is defined. Assume that this advantage variable is universally desired, with no satiation; that is, it is an economic *good* (such as income, consumption, or some measure of health status), more of which is preferred to less by all. Suppose that all determinants of x, including the various different forms of luck, can be classified into *either* a vector of *circumstances* $\boldsymbol{C}$ that lie beyond individual control, *or* as responsibility characteristics, summarized by a variable e, denoted *effort*. For simplicity, let us henceforth refer to the

[6] The usage of the ex ante/ex post distinction here is analogous to that in the risk literature, where it refers to the timing of the resolution of uncertainty. Here there is no uncertainty, and the conceptual timing refers to the choice of effort by the individual.

individual advantage variable x as "income", even though, as we shall see below, the framework has been applied much more broadly, to a variety of other advantage indicators.[7]

Circumstances belong to a finite set Ω. For example, suppose that the only circumstance variables are race, which can only take values in the set {black, white}, and parental education, that only takes values in the set {college education, high school education}. In this case the set Ω would be the following: Ω = ({black, parents with high school education}, {black, parents with college education}, {white, parents with high school education}, {white, parents with college education}). Effort may be treated either as a continuous or a discrete variable belonging to the set Θ. The outcome of interest is generated by a function $\boldsymbol{g}: \boldsymbol{\Omega} \times \boldsymbol{\Theta} \Rightarrow \mathbb{R}$, such that:

$$x = g(C, e) \tag{1}$$

This can be seen as a reduced-form model in which incomes are exclusively determined by circumstances and effort, such that all individuals having the same circumstances and the same effort obtain the same income. Neither opportunities themselves, nor the process by which some particular outcomes are chosen, are explicitly modelled in this framework. The idea is to infer the opportunities available to individuals by observing joint distributions of circumstances, effort and outcomes, under the assumption that larger values of x are preferred by all. Although this is clearly indirect, it has the practical advantage that the elements used in formal modelling are either all potentially observable or – as we will see – inferable from other characteristics of observed distributions. This is why, we argue, the indirect approach to E. Op. has spawned a large number of empirical applications.

Roughly speaking, the source of unfairness in this model is that circumstance variables (which lie beyond individual responsibility) affect outcomes. From a *prescriptive* viewpoint, the aim is to correct such unfairness either by removing the differences in circumstances (such as family wealth), or by compensating the effects of circumstances on outcomes. From a *descriptive* viewpoint, the extent of inequality of opportunity is assessed not by computing a measure of inequality for the distribution of opportunity sets (as in the direct approach), but by imposing one additional bit of structure, namely the assumption that any inequality in the distribution of achievements which can be attributed to differences in circumstances is identified as inequality of opportunity. Of course, the identification of the share of inequality associated with circumstances is not unique and, as we shall see, different models within this framework propose different practical solutions to these identification and compensation problems

Thus, we have a population of individuals, each of whom is fully characterized by the triple (x, C, e). For simplicity, treat effort e, as well as each element of the vector of circumstances, $\boldsymbol{C}$, as discrete variables. Then this population can be partitioned in two ways: into *types* T_i , within which all individuals share the same circumstances, and into *tranches* T_j, within

[7] In fact, although most applications we are aware of use a unidimensional outcome, there is no reason why x could not denote a vector of indicators of well-being, including subjective ones.

which everyone shares the same degree of effort. Denote by x_{ij} the income generated by circumstances $\boldsymbol{C_i}$ and effort e_j. Suppose there are n types, indexed by $i=1,...,n$, and m tranches, indexed by $j=1, ...,m$. In this discrete setting[8], the population can be represented by a matrix $[X_{ij}]$ with n rows, corresponding to types, and m columns, corresponding to tranches:

Table 1

	e_1	e_2	e_3	…	e_m
C_1	x_{11}	x_{12}	x_{13}	…	x_{1m}
C_2	x_{21}	x_{22}	x_{23}	…	x_{2m}
C_3	x_{31}	x_{32}	x_{33}	…	x_{3m}
…	…	…	…	…	…
C_n	x_{n1}	x_{n2}	x_{n3}	…	x_{nm}

To the nxm dimensional matrix $[X_{ij}]$ in Table 1, let there be associated an nxm dimensional matrix $[P_{ij}]$ where each element p_{ij} gives the proportion of total population with circumstances C_i and effort e_j. With this very basic toolkit, we are ready to understand the main (indirect) approaches that have been proposed to rank distributions in terms of equality of opportunity in a given society, or to study policies or allocation rules designed to equalize opportunities. In particular, in this section we will discuss social rankings of distributions, where the aim is to maximize social welfare, and allocation rules. The next section will be devoted to the measurement of inequality (of opportunity).

First we introduce the distinction between the compensation and reward principles, and between the ex-ante and ex-post approaches to compensation. As noted above, the egalitarian component of the E.Op. theory is embodied in the compensation principle, which requires that "inequalities due to circumstances should be eliminated". There are two main versions of this principle in the literature. The ex-ante approach to compensation (associated with Van de Gaer, 1993) re-introduces the concept of an opportunity set, but in a very specific sense: it defines a person k's opportunity set as the set of possible achievement levels x_{ij} that she can attain given her circumstances $\boldsymbol{C_i|k \in T_i}$. It then seeks to evaluate the opportunity set faced by each individual k. According to this approach, inequality of opportunity would be eliminated if all individuals faced opportunity sets with the same value (a form of normalization). Hence the focus becomes the counterfactual distribution where each person's income is replaced by the value of his or her opportunity set, v_i.

But how should opportunity sets be valued? A major contribution of this approach is the proposal to evaluate the opportunity set faced by individuals in a given type i, by reference to some feature of the type's income distribution. That is, the set of income prospects of individuals in the same type is interpreted as the set of opportunities open to each individual in that type. It

[8] In an alternative formulation, that would treat effort as a continuous variable, $F_i(x)$ would denote the advantage distribution in type i and $\boldsymbol{q_i}$ denote its population share. The overall distribution for the population as a whole would be $\boldsymbol{F(x)} = \sum_{i=1}^{n} \boldsymbol{q_i F_i}\,(\boldsymbol{x})$.

represents the set of income levels which can be achieved, by exerting different degrees of effort, starting from the same circumstances C_i. Looking at the matrix $[X_{ij}]$, this approach entails focusing on the rows of the matrix: each row i represents the opportunity set available to individuals with circumstances C_i.

Different methods to evaluate such distributions and to assess the distance between them have been proposed in the literature. Lefranc et al. (2009), Peragine (2002) and Peragine and Serlenga (2008) propose different ordinal approaches based on stochastic dominance conditions. An alternative, cardinal version of the ex-ante approach, extensively used in empirical analyses, evaluates the opportunity set associated with each row of the matrix by a single summary statistic such as its mean, $\boldsymbol{\mu_i} = \sum_{j=1}^{m} \boldsymbol{p_{ij} x_{ij}}$. [9] In that case, $\boldsymbol{v_i} = \boldsymbol{\mu_i}, \boldsymbol{i} = \mathbf{1}, \dots, \boldsymbol{n}$. One plausible interpretation of this particular statistic is to think of the frequency distribution in the rows of $[P_{ij}]$ as proxying for the probability distribution associated with each outcome for an individual in type i, in the hypothetical position of being prior to the decision of how much effort to choose.[10] $\boldsymbol{\mu_i}$ can then be seen the expected value of the individual's income prospects, conditional on belonging to type i.

Adopting the mean-based evaluation of individual opportunity sets, and expressing extreme aversion to inequality between types, Van de Gaer (1993) proposes the following social welfare criterion to rank distributions:

$$\boldsymbol{min_i}(\mu_1, \dots, \mu_n) \qquad (2)$$

In words, given two distributions A and B, van de Gaer's rule would rank A higher than B if the lowest mean income for any type in A was higher than the lowest mean income for any type in B. The optimal allocation subject to feasibility constraints which the social planner should choose is that in which the lowest mean income for any type is maximized. This is known as the "min of means" rule. The min of means rule can be generalized by replacing the maximin criterion by an additive and (between-types) inequality-averse social welfare function (see Peragine, 2004a).

The ex-post approach to compensation (associated with Roemer, 1993), on the other hand, proposes that inequalities should be eliminated among individuals who exert the same degree of effort. Under this approach there is no need to evaluate opportunity sets but, on the other hand, one must observe (or agree on a measure of) effort. The ex-post approach therefore focuses on inequalities in classes of individuals that spend identical effort levels – that is: on inequality within the columns of $[X_{ij}]$, or inequality within tranches.

[9] Instead of the mean, one could alternatively use the equally-distributed-equivalent income (EDEI), see Atkinson (1970), or other welfare indicators.
[10] The choice of effort level is then seen as incorporating a stochastic component.

Roemer's specific proposal expresses extreme inequality aversion with respect to the inequality within tranches. Hence, his proposal for the opportunity egalitarian planner is given by the following rule: choose the policy that maximizes the minimum outcome within every tranche. If such policy does not exist (as a different policy might well be required for each tranche) then the compromise solution proposed by Roemer is to choose the policy that maximizes the average of the *minima* of every tranche[11]. This is known as the "mean of mins" rule, which postulates the following as the social welfare criterion for ranking distributions $[X_{ij}]$:

$$\left(\frac{1}{m}\sum_{j=1}^{m} \min_i (x_{1j}, \ldots, x_{nj})\right) \qquad (3)$$

The mean of mins rule can also be generalized by replacing the maximin criterion by an additive and (within-tranches) inequality-averse social welfare function (see Peragine 2004b).

The ex-ante and ex-post versions of the compensation principle are therefore quite different. The ex-ante approach focuses on the opportunity sets of individuals, represented by the type-specific income distributions: the rows of $[X_{ij}]$. Then, the ex-ante compensation principle mandates a reduction – in the limit, the elimination – of inequality among the values of such sets. The ex-post compensation principle instead mandates a reduction – in the limit, elimination – of inequality within tranches: the columns of $[X_{ij}]$.

It is perhaps unfortunate, but not surprising, therefore, that the ex-ante and ex-post approaches to compensation are mutually incompatible. This incompatibility was hinted at in a series of publications (see Ooghe et al. 2007, Fleurbaey 2008, Checchi and Peragine, 2010) and recently proved in a general setting by Fleurbaey and Peragine (2013). Some versions of this incompatibility result rest on a specific evaluation of the opportunity set of individuals (typically a summary statistics of the type's income distribution), and hence on a specific version of the ex-ante compensation principle. The formulation provided by Fleurbaey and Peragine (2013), however, is more general and rests on the following formulation of the ex ante and ex post compensation principles:

i) *Ex Ante Compensation*: consider one type that has an unambiguously better prospect than another type. That is, at each effort level the outcome of the former type is higher than the outcome of the latter type. Then worsening the situation of an individual in the advantaged type, while improving that of an individual in the disadvantaged type, would improve the situation.

ii) *Ex Post Compensation*: reducing inequalities between individuals having the same level of effort but different levels of outcome would improve the situation.

[11] Notice that if there is an unambiguously worst off type, that is, for each effort level there is a type *i* that is worst off, then Roemer's and Van de Gaer's program coincide.

Fleurbaey and Peragine (2013) show that the two principles above are incompatible.

Let us now briefly turn to the reward principle which, as noted earlier, is the second sub-goal of opportunity egalitarianism. Whereas the compensation principle is concerned with (eliminating unfair) inequalities between types or within tranches, the reward principle is concerned with (preserving fair) inequalities within types. In other words, it is concerned with ensuring that inequalities arising from the differential application of effort are preserved.

This second principle can also be formalized in different ways, the two most prominent ones being *liberal reward* and *utilitarian reward.*[12] Liberal reward seeks to minimize redistribution related to differential effort levels, and therefore advocates making equal transfers to individuals with identical circumstances. Utilitarian reward recommends an evaluation of outcome distributions within types that is neutral with respect to inequality, and therefore simply focuses on the sum of outcomes in order to evaluate a change affecting one type. Clearly utilitarian reward is respected by Van de Gaer's "min of means" rule, which in fact evaluates the type distribution by the mean.

It is worth noting that utilitarian reward is a consequentialist idea, as the items being assessed are consequences, described in terms of efforts, circumstance, and final achievements. By contrast, in liberal reward the focus is on the treatment of "equals" (defined in this context as individuals with the same circumstances): hence on taxes and transfers implemented by the government.

To discuss the liberal reward principle more generally, and to summarize the main results of the literature on fair allocation that explored the consequences of this principle in conjunction with compensation (see Fleurbaey 2008), it is necessary to expand the canonical model by explicitly recognizing the existence of external resources, which the social planner can allocate at will. In this richer model, individual outcomes are assumed to be a function of three different categories of factors (circumstances, external resources (r), and effort):

$$x=g(C,r,e) \qquad (1')$$

The literature initiated by Fleurbaey (1994, 1995a) and Bossert (1995) was devoted to the design of rules which - given the distribution of circumstances and effort - would allocate the external resources in order to satisfy the compensation and the liberal reward principle. See Fleurbaey (2008) for a survey and Fleurbaey and Maniquet (2012) for a collection of papers on the topic. A first result of this literature was that, if the outcome function $g(.)$ is not separable in extended resources (circumstances plus external resources) and effort, then ex-post

[12] Alternative formulations of the reward principle have recently been proposed in the literature. These include the *inequality averse reward* (Ramos and Van de Gaer, 2012), the *arithmetic average reward* (Roemer and Trannoy, 2013), and the *minimal reward* (Fleurbaey and Peragine, 2013).

compensation and liberal reward clash. The intuition for the clash can be grasped by constructing an outcome matrix analogous to the one introduced in Table 1, but in which the outcomes are now the results of circumstances, effort and resources allocated. The principle of compensation requires that inequality within columns in the outcome matrix be eliminated (columns should be constants), while the principle of liberal reward demands that the transfers be constant within the rows. It is clear that these two commands can conflict. Although the result can be proved more generally, its essence can be illustrated by a simple example involving two types and two tranches. Consider distribution X, describing a *status quo*, where the individual outcomes are due to effort, circumstances and a given allocation of external resources corresponding to a *laisser-faire* situation:

$X =$

	e_1	e_2
C_1	*10*	*40*
C_2	*20*	*30*

In this very simple case, ex-post compensation requires a transfer from type 2 to type 1 in the first tranche and from type 1 to type 2 in the second tranche. The liberal reward principle requires equal transfers within each type. Those two things cannot both be achieved[13].

The way in which the literature on fair allocation has dealt with the conflict between ex-post compensation and liberal reward has been to propose weaker versions of either (or both) principles and, by using an axiomatic approach, to characterize compromise solutions that might resolve the inconsistency.

Two prominent compromise rules have emerged, which give different weights to the principles of liberal reward and of compensation. A common feature of these solutions is to define a reference value either for effort or for circumstances. Then, the principle that is sacrificed in the compromise is satisfied at least for the reference effort or circumstance. The first allocation rule, giving priority to liberal reward, is *conditional equality*: consider a situation where all individuals exert a "normal" or reference level of effort, but are characterized by different circumstances. The preferred allocation is the one that equalizes the value of the individual outcomes in this specific case. With this allocation liberal reward is respected everywhere while ex-post compensation is satisfied at least for the reference effort level.

The second allocation rule, which gives priority to compensation, is *egalitarian equivalence.*[14] It can be considered as dual to conditional equality*:* consider a counterfactual

[13] Fleurbaey and Peragine (2013) prove that the clash between ex post compensation and reward extends to the principle of utilitarian reward and to weaker versions of the reward principle than liberal reward.
[14] The egalitarian-equivalence criterion was introduced by Pazner and Schmeider (1978).

where each individual faces the same circumstances but exerts his own effort. Then fix a reference level of circumstances *and* find an allocation of the resource which exhausts the amount of resource available, and which equalizes the value of the objective, for every individual, to what her objective would be at the reference circumstances. With this allocation, circumstances are fully compensated for, while transfers obey the natural liberal principle only for the reference circumstances. Of course, both solutions will depend upon the choice of the reference value of circumstances or effort.

The compensation - reward clash has played an important role in the axiomatic literature on fair allocations. However, in a recent paper Fleurbaey and Peragine (2013) show that such clash is an expression of the deeper tension between the ex-ante and the ex-post perspectives in evaluating opportunities. They prove that while the reward principle clashes with ex-post compensation, there is no such clash between ex-ante compensation and reward. This asymmetry is due to the fact that the reward principles can also be interpreted as ex-ante compensation requirements applied to individuals with identical circumstances.

Although some of the most influential authors in the E. Op. field have been partial to the ex-post approach (notably John Roemer and Marc Fleurbaey), the compatibility between the principles of compensation and (liberal and utilitarian) reward within the ex-ante approach could be seen as an argument in favour of the latter. On the other hand, in a series of writings, John Roemer has questioned the validity of the reward principle as necessary ingredient of the E.Op. theory. He advocates an agnostic view with respect to what the proper reward to effort should be and suggests that "considerations outside the realm of equality of opportunity must be brought to bear to decide upon how much inequality with respect to differential effort is allowable".

Quite apart from theoretical considerations, in practice the choice of approach to inequality of opportunity is often severely constrained, if not dictated, by the availability of data. In this respect, as we shall see below, a number of practical and empirical considerations also seem to militate in favour of the ex-ante approach.

One – if by no means the only – use to which the economic model(s) of equality of opportunity summarized above are often put is to assess the extent of inequality of opportunity in a given society. The next section briefly reviews the literature on the measurement of inequality of opportunity.

4. The measurement of inequality of opportunity

In essence, the measurement of inequality of opportunity can be thought of as a two-step procedure[15]: first, the actual distribution $[X_{ij}]$ is transformed into a counterfactual distribution

[15] For an extensive survey on the different approaches and methodologies that have been proposed for the measurement of inequality of opportunity see Ramos and Van de Gaer (2012). See also Pignataro (2012).

$[\widetilde{\boldsymbol{X}}_{ij}]$ that reflects *only and fully* the unfair inequality in $[X_{ij}]$, while all the fair inequality is removed. In the second step, a measure of inequality[16] is applied to $[\widetilde{\boldsymbol{X}}_{ij}]$.

In principle, the construction of the counterfactual distribution $[\widetilde{\boldsymbol{X}}_{ij}]$ should reflect both the compensation and the reward principles – the two subcomponents of opportunity egalitarianism. As discussed above, there is a deep tension between ex-post compensation and reward. It is therefore impossible to construct a distribution $[\widetilde{\boldsymbol{X}}_{ij}]$ fully consistent with reward and with ex post compensation. It is however possible to construct a distribution $[\widetilde{\boldsymbol{X}}_{ij}]$ that satisfies ex ante compensation and reward.

Below we review four of the most influential measurement approaches proposed in the literature: as we will see, they are inspired by the social welfare criteria and the allocation rules discussed in the previous section. The first two subscribe to an ex-ante view of compensation - "between-types inequality" and "direct unfairness" – while the second pair pursue the ex-post compensation principle – "within-tranches inequality", and "fairness gap".

Versions of the *Between-Types Inequality* approach were variously proposed by Peragine (2002), Bourguignon, Ferreira and Menéndez (2007), Checchi and Peragine (2010) and Ferreira and Gignoux (2011). The approach is inspired by the "min of means" criterion proposed by van de Gaer (1993). The counterfactual distribution it relies on, ($\widetilde{X}_{BT}$), is obtained by replacing each individual income x_{ij} by the average income of the type she belongs to. This smoothing transformation is intended to remove all inequality within types. Formally:

Between types ($\widetilde{X}_{BT}$): *For all $j \in \{1,...,m\}$ and for all $i \in \{1,...,n\}$,* $\widetilde{x}_{ij} = \mu_i$.

Table 2: Between-types inequality *(n=m=3)*

	e1	e2	e3
C1	μ_1	μ_1	μ_1
C2	μ_2	μ_2	μ_2
C3	μ_3	μ_3	μ_3

It is immediate to notice that *between-types inequality* is consistent with the principle of utilitarian reward: the types of $\widetilde{X}_{BT}$ are made up of replications of the same outcome, the mean, and therefore the artificial distribution does not reflect any inequality within type – the kind of

[16] This section will focus on inequality measures. For different approaches based on dominance analyses, which give more robust but incomplete rankings, see Lefranc et al. (2009), Peragine (2002, 2004a,b), Peragine and Serlenga (2008), Rodriguez (2008).

inequality which is fair according to the reward principle, and thus should be cleansed in $[\tilde{\boldsymbol{X}}_{ij}]$. It is also consistent with ex-ante compensation, as the inequality between types (evaluated as the inequality between the means of each type) is preserved. On the other hand the $\tilde{X}_{BT}$ does not necessarily reflect the original inequality within tranches and hence it does not guarantee consistency with ex post compensation. By making use of standard inequality decomposition techniques (Theil, 1979a,b; Bourguignon, 1979), some authors (notably Checchi and Peragine, 2010 and Ferreira and Gignoux, 2011) have used this approach to propose a useful decomposition of overall inequality into two terms: between types inequality, to be interpreted as inequality of opportunity, and within types inequality, interpreted as inequality due to effort.

An alternative counterfactual distribution which is still consistent with ex-ante compensation was proposed by Fleurbaey and Schokkaert (2009). Theirs is a norm-based distribution, called *Direct Unfairness*, which is inspired by the conditional equality solution described in the previous section. The direct unfairness distribution ($\tilde{X}_{DU}$) is obtained by replacing each individual outcome $\boldsymbol{x_{ij} = g(c_i, e_j)}$ with the outcome that would be generated by a reference effort $\tilde{e}$, given the function g and the circumstances c_i. Formally:

Direct unfairness ($\tilde{X}_{DU}$)*: take $\tilde{e}$ as the reference effort. Then $\tilde{x}_{ij} = g(c_i, \tilde{e})$, $\forall i \in \{1, ..., n\}$ and $\forall j \in \{1, ..., m\}$.*

Table 3: Direct unfairness (with $\tilde{e}$=1 and *n=m=3*)

	e1	e2	e3
C1	x_{11}	x_{11}	x_{11}
C2	x_{21}	x_{21}	x_{21}
C3	x_{31}	x_{31}	x_{31}

Basically, direct unfairness creates a counterfactual distribution analogous to that of between-types inequality with the difference that, instead of allocating the type's mean income to all individuals in each type, it allocates a particular (reference) income from someone in that type. Direct unfairness is consistent with ex-ante compensation, as inequality between types (now evaluated as inequality between type reference values) is preserved. It is also broadly consistent with the principle of reward in spirit, as $\tilde{X}_{DU}$ does not contain any inequality within types, although it does not fully satisfies utilitarian reward (which implies giving the average outcome to everyone in a type).

On the other hand direct unfairness does not guarantee ex-post compensation: the columns of $\tilde{X}_{DU}$ reflects unfair distances between individuals only in the reference tranche which, because of the non-separability of *g(c,e)*, will generally differ from distances in the other tranches.

Let us turn now to the two ex-post measurement approaches. The first is inspired by Roemer's (1993) "mean of mins" criterion. Checchi and Peragine (2010) and Aaberge et al (2011) propose the *Within-Tranches* counterfactual distribution ($\tilde{X}_{WTR}$), which is obtained by replacing each individual outcome x_{ij} in a given tranche with the ratio between such outcome and the average income of that tranche: $\boldsymbol{\nu_j = \sum_{i=1}^{n} p_{ij} x_{ij}}$. This normalization procedure is intended to remove all inequalities between tranches and to leave unchanged the inequality within tranches. Formally:

Within tranches ($\tilde{X}_{WTR}$***):*** *For all* $j \in \{1,...,m\}$ *and for all* $i \in \{1,...,n\}$, $\tilde{x}_{i,j} = g(c_i, e_j)/\nu_j$.

Table 4: Within tranches inequality *(n=m=3)*

	e1	e2	e3
C1	$x_{11}/\ \nu_1$	$x_{12}/\ \nu_2$	$x_{13}/\ \nu_3$
C2	$x_{21}/\ \nu_1$	$x_{22}/\ \nu_2$	$x_{23}/\ \nu_3$
C3	$x_{31}/\ \nu_1$	$x_{32}/\ \nu_2$	$x_{33}/\ \nu_3$

It is easy to see that *Within-Tranches* is consistent with ex-post compensation: each tranche is obtained simply by rescaling original incomes by a constant ($1/\nu_j$). Therefore $\tilde{X}_{WTR}$ accounts for all of the original (relative) inequality within tranches. On the other hand, compliance with the reward principle is not guaranteed, since Table 4 does in general contain inequality within types: for at least one *i* and a couple *j,h*, $\tilde{x}_{i,j} = g(c_i, e_j)/\nu^j \neq g(c_i, e_h)/\nu^h = \tilde{x}_{i,h}$. Also in this case Checchi and Peragine (2010) propose a useful decomposition of overall inequality into two terms: within tranches inequality, to be interpreted as inequality of opportunity, and between tranches inequality, interpreted as inequality due to effort.

A fourth approach to constructing the "unfair inequality" counterfactual distribution, $[\boldsymbol{\tilde{X}}_{ij}]$, is again due to Fleurbaey and Schokkaert (2009). Adopting an ex-post perspective, these authors propose another norm-based distribution, called *Fairness Gap*, which is linked to the egalitarian equivalence solution described in the previous section. Fairness Gap ($\tilde{X}_{FG}$) is obtained by replacing each individual income $\boldsymbol{x_{ij} = g(c_i, e_j)}$ by the ratio[17] between that income and the

[17] In fact Fleurbaey and Schokkaert (2009) adopt an absolute, i.e., translation-invariant approach. However in this paper we translate their approach into relative terms in order to better compare it with the rest of the literature on inequality measurement, which generally adopts a scale-invariant approach.

income that would be generated by a reference circumstance $\tilde{c}$, given the function g and the effort e_j . Formally:

Fairness gap ***($\tilde{X}_{FG}$)****: take $\tilde{c}$ as the reference circumstance. Then let $\tilde{x}_{i,j} = g(c_i, e_j) / g(\tilde{c}, e_j)$, $\forall i \in \{1,...,n\}$ and $\forall j \in \{1,...,m\}$.*

Table 5: Fairness gap (with $\tilde{c}$=1 and *n=m=3*)

	e1	e2	e3
C1	1	1	1
C2	x_{21}/x_{11}	x_{22}/x_{12}	x_{23}/x_{13}
C3	x_{31}/x_{11}	x_{32}/x_{12}	x_{33}/x_{13}

As in the case of *Within-Tranches*, the *Fairness Gap* counterfactual distribution $\tilde{X}_{FG}$ is consistent with ex-post compensation: in fact the columns of $\tilde{X}_{FG}$ fully account for (relative) inequality within the tranche as they are made by the actual outcome divided by a reference outcome. On the other hand, $\tilde{X}_{FG}$ once again does not guarantee reward: except in the reference type, where $\tilde{X}_{FG}$ is made up of ones, the other rows do contain some within-type inequality. Hence an inequality measure applied to distribution $\tilde{X}_{FG}$ would capture an element of fair inequality (which should ideally be excluded from the counterfactual matrix), for all other $c_i \neq \tilde{c}$, $\tilde{x}_{i,j} = g(c_i, e_j) / g(\tilde{c}, e_j) \neq g(c_i, e_h) / g(\tilde{c}, e_h) = \tilde{x}_{i,h}$.

These four approaches to the measurement of inequality of opportunity, with the relevant welfare criteria and allocation rules that implicitly or explicitly inspire them, are summarized in Table 6 below.

Table 6: Welfare criteria, allocation rules and inequality measures

Approaches	**Welfare criteria and allocation rules**	**Inequality measures**
Ex ante	*Min of means*	*Between types*
	Conditional equality	*Direct unfairness*
Ex post	*Mean of mins*	*Within tranches*
	Egalitarian equivalence	*Fairness gap*

In sum, the exercise of measuring the extent of inequality of opportunity in a given society consists of constructing a distribution that eliminates (or seeks to eliminate) all the "fair"

inequality, and contains all the unfair inequality in the society. Then any suitable inequality index can be applied to that distribution. Within the realm of indirect approaches to E. Op. theory, which inequalities are deemed fair and which unfair is guided by the two key principles of compensation and reward. When an ex-ante view is taken of the compensation principle, it is possible to construct counterfactual distributions that do eliminate all fair inequality, and contain all unfair inequality. Examples include the between-types and direct unfairness distributions in Tables 2 and 3. When an ex-post view of compensation is adopted instead, it is not possible to construct counterfactual distributions that completely eliminate all fair inequality, and contain all unfair inequality. Two examples of distributions that are consistent with ex-post compensation, but not with the reward principle, are the within-tranches and fairness gap matrices in Tables 4 and 5.

5. Empirical applications[18]

As noted in Section 3, the informational requirements for applying the direct approach to the measurement of inequality of opportunity – involving the observation of full choice sets – are very demanding and, to the best of our knowledge, that approach has never been applied empirically. Indirect approaches, on the other hand, rely on the observation of actual circumstances, outcomes and (possibly) efforts, and have been taken to real data. Indeed, empirical applications of all four approaches reviewed in Section 4 exist.

Almas et al. (2011) and Devooght (2008) use norm-based measures, based on Direct Unfairness and Fairness Gap. Checchi and Peragine (2010) and Aaberge et al. (2011) apply the within-tranches measure (as well as the between-types inequality) to estimate inequality of opportunity in Italy and in Norway respectively. But neither ex-post compensation nor norm-based measures have been computed systematically, or comparably, across many countries.[19] Most measures of inequality of opportunity computed in practice have followed ex-ante approaches and, in particular, the between-types approach.

The version of the ex-ante approach with $v_i = \mu_i$ – that is, the between-types measure $I(\tilde{x}_{BT})$ – has been applied to at least forty countries, by a number of authors. The specific inequality index $I(.)$ does vary across some of the papers but most use the mean logarithmic deviation, following Checchi and Peragine (2010) and Ferreira and Gignoux (2011). In a few cases, as noted below, the Gini index, the Theil (T) index and even the variance have been used.

Despite these differences, as well as a variety of caveats on data comparability across – or even within – studies, there is a set of eight papers that report the between-types measure and which constitutes, to our knowledge, the most closely comparable sources on actual I. Op. measures across countries. These eight papers - Checchi et al. (2010); Ferreira and Gignoux

[18] This section draws heavily on Brunori, Ferreira and Peragine (2013).
[19] Though see Björklund et al. (2012) for an interesting application of ex post inequality measures to long-run incomes in Sweden.

(2011); Ferreira et al. (2011); Pistolesi (2009); Singh (2011); Belhaj-Hassine (2012), Cogneau and Mesple-Somps (2008) and Piraino (2012) - cover forty-one countries, ranging from Guinea and Madagascar (with annual per capita GNIs of PPP$980), to Luxembourg, with a per capita GNI of almost PPP$ 64,000. This section briefly reviews their results.

All of these papers use a measure of economic well-being - household per capita income, household per capita consumption, or individual labor earnings - as the advantage indicator. For this reason, Brunori et al. (2013) – on which this section draws heavily – refer to the between-types measure of I. Op. in these studies as an index of *Inequality of Economic Opportunity* (IEO). In fact, two closely related versions of the index are often reported: the absolute or level estimate of inequality of opportunity (IEO_L) is given simply by the inequality measure computed over $\tilde{X}_{BT}$, i.e. by $I(\tilde{x}_{BT})$. The ratio of IEO_L to overall inequality in the relevant advantage variable (e.g. household per capita income) yields the relative measure, IEO_R:

$$\mathrm{IEO_R} = \frac{\mathrm{I}(\tilde{\mathrm{x}}_{\mathrm{BT}})}{\mathrm{I}(\mathrm{x})} \tag{4}$$

The partition of types varies across studies, ranging from six to 7,680 types (although in four of the eight studies, the range is a more comfortable 72-108 types).[20] Because in some cases the data sets are not large enough to yield precise estimates of $\boldsymbol{\mu_i}$ for all types, some authors compute IEO_L using a parametric approximation. After estimating the reduced-form regression of income on circumstances:

$$\mathrm{x} = \mathrm{C}\beta + \epsilon \tag{5}$$

and obtaining coefficient estimates $\hat{\beta}$, these authors use predicted incomes as a parametric approximation to the smoothed distribution:

$$\boldsymbol{I}(\widehat{\boldsymbol{x}}_{\boldsymbol{BT}}), \text{ where } \widehat{\boldsymbol{x}}_{\boldsymbol{i}} = \boldsymbol{C}_{\boldsymbol{i}}\widehat{\boldsymbol{\beta}} \tag{6}$$

Parametric estimates are also presented either as levels (IEO_L) or ratios (IEO_R), analogously. This approach follows Ferreira and Gignoux (2011), which in turn draws on Bourguignon et al. (2007). Empirically, parametric estimates of inequality of opportunity tend to be a little lower than their non-parametric counterparts but, at least in the case of Latin America, the differences are not large: proportional differences between the two average 6.6% in Ferreira and Gignoux (2011).

It is important to note that these empirical estimates of "between-types" I. Op. – whether estimated parametrically or non-parametrically – are, in each and every case, *lower-bound estimates* of inequality of opportunity. A formal proof of the lower-bound result is contained in Ferreira and Gignoux (2011), but the intuition is straight-forward: The set of circumstances which is observed empirically - and used for partitioning the population into types - is a strict

[20] See Table 7 below.

subset of the set of all circumstance variables that matter in reality. The existence of unobserved circumstances – virtually a certainty in all practical applications – guarantees that these estimates of I.Op. could only be higher if more circumstance variables were observed.

As discussed in Ferreira and Gignoux (2011), the existence of effort variables, observed or unobserved, is entirely immaterial to this result, since (5) is a reduced-form equation, where any effect of circumstances on incomes through their effects on efforts is already captured by the regression coefficients, and hence reflected in $\boldsymbol{I}(\hat{\boldsymbol{x}}_{\boldsymbol{BT}})$. Equation (5) therefore captures the reduced-form influence of circumstances on advantages, both directly and indirectly through efforts. By construction, the only omitted variables that matter for IEO are omitted circumstances, and those variables can only bias estimates of IEO_L and IEO_R downwards.[21]

Recently, an upper-bound estimator of between-types inequality of opportunity has also been proposed, which requires panel data sets where the same individuals – or households – are observed at different points in time. If all circumstances are time-invariant (e.g., because they are determined at birth), then Peichl (2014) shows that the share of inequality associated with individual fixed effects in such data can provide an upper-bound estimate for between-types I.Op.. Intuitively, if all circumstances are time-invariant, their contribution to inequality is fully captured by the fixed effects. If, in addition, some component of individual responsibility (or effort) is also time-invariant, then that estimate is upwardly-biased – hence an upper bound. One limitation of this approach, of course, is that it rules out the existence of time-varying circumstances, such as later brute luck.

Table 7 presents the estimates of IEO-L and IEO-R for each of the forty-one countries studied by the eight aforementioned papers. The table also lists their gross national income (GNI) per capita and overall income inequality. Overall inequality is measured by whatever index was used in the construction of the IEO indices for each country. Except where indicated, this measure was the mean logarithmic deviation, also known as the Theil-I index, a member of the generalized entropy class of inequality measures. Whereas overall inequality, IEO-L and IEO-R come from the eight studies mentioned above, GNI per capita comes from the World Bank's World Development Indicators database.

[PLEASE PLACE TABLE 7 HERE]

Table 7 should be read in close conjunction with Table 8, which provides some basic information on each of the eight studies used to construct the inequality of opportunity estimates in Table 7. Table 8 describes which countries are studied in each paper; the specific data sets

[21] Of course, this does not hold for the estimates of the individual coefficients $\hat{\beta}$. First, these coefficients are reduced-form, rather than structural, estimates. In addition, they are likely to be biased (upwards or downwards) even as reduced-form estimates, due to the omission of unobserved circumstances. The lower-bound result applies only to the overall measures of inequality of opportunity, IEO-L and IEO-R, whether parametrically estimated or not. See Ferreira and Gignoux (2011).

(including survey year); the precise income and circumstance variables used; whether the estimation was parametric or otherwise, and the number of types included in each calculation. The table highlights a number of problems for comparability across these studies. First is the nature of the advantage variable (*x*) itself: whereas Pistolesi (2009), Singh (2011) and Belhaj-Hassine (2012) use labor earnings, Checchi et al. (2010), Ferreira and Gignoux (2011) and Piraino (2012) use incomes, Cogneau and Mesple-Somps (2008) use consumption, and Ferreira et al. (2011) use imputed consumption. In addition, because of the usual differences across survey questionnaires and data collection protocols, the definitions of earnings and incomes cannot be assumed to be exactly the same across each of these papers either.

[PLEASE PLACE TABLE 8 HERE]

These distinctions are not immaterial: in a comparison of six Latin American countries, Ferreira and Gignoux (2011) found substantially higher estimates of IEO-R for consumption expenditure than for income distributions, in the same countries.[22] They attributed this finding to the fact that income measures are thought to contain greater amounts of measurement error, as well as transitory income components, which are less closely correlated with circumstances than permanent income or consumption might be. Bourguignon et al. (2007) also found differences between estimates for individual earnings and for household per capita incomes, which they attributed to the fact that unequal opportunities affect the latter not only through earnings, but also through assortative mating, fertility decisions, and non-labor income sources.

Second, the studies differ in the number of types used for the decomposition and, naturally, in the exact set of circumstances used in each case. At one extreme, the Cogneau and Mesple-Somps study has a mere three types for Uganda, based on father's occupation and education levels, while on the other Pistolesi has 7,680 types, constructed on the basis of information on age (20 levels), parental education (4 levels for the mother and 4 for the father), occupational group of the father (6 categories), individual ethnic group (2 categories), individual region of birth (2 categories). Fortunately, there is a core set of studies - which account for most countries in the sample - with 72 to 108 types each. Nevertheless, results for Africa and the US should certainly be interpreted with caution, in light of the number of types used in each case. Finally, a third comparability caveat, on which we have already dwelled, is the fact that some studies use non-parametric estimates while others use parametric ones.

Bearing these caveats in mind, Table 7 nevertheless illustrates the substantial variation in inequality levels – both in advantages and in opportunities - across countries. The mean log deviation for incomes (or the corresponding advantage indicator) ranges from 0.083 in Denmark to 0.675 in South Africa. Norway, Slovenia and Sweden also have comparatively low levels of overall inequality, while Brazil and Guatemala stand out at the upper end. Inequality of opportunity levels (IEO_L) range from 0.003 in Norway and 0.005 in Slovenia to 0.199 in

[22] Similarly, Singh (2011) finds a higher IEO-L for consumption than for earnings in India.

Guatemala and 0.223 in Brazil. In other words, the (lower-bound) level of inequality in the distribution of values of opportunity sets across types in Brazil is almost three times as high as the total inequality (measured by the same index) in the distribution of *actual* incomes in Denmark. One can also observe substantial differences in IEO-L among countries at closer levels of development, and more methodologically comparable: Madagascar's level of inequality of opportunity is twice that of Ghana; those of the US and the UK are ten times that of Norway and almost four times higher than Denmark's.

The ratio of these two inequality measures, i.e. the (lower-bound) share of the overall inequality due to inequality of opportunity (IEO_R), also varies substantially, from 0.02 in Norway to 0.34 in Guatemala. Slovenia also has a low inequality of opportunity ratio, at 0.05, while Brazil closely follows Guatemala in the upper tail, at around 0.32.

Brunori et al. (2013) also investigate how these measures of inequality of opportunity correlate with some other important variables, such as output per capita, overall income inequality, and measures of intergenerational mobility. They report a non-linear relationship between inequality of opportunity and the level of development, as measured by the logarithm of per capita income levels. In fact, the association appears to have an inverted-U shape, much as the "Kuznets curve" that used to be hypothesized for the relation between income inequality and the "level of development". The regression of IEO_R on a quadratic of log GNI per capita produces a coefficient on the linear term of 0.32 (p-value: 0.05), and that on the quadratic term is -0.017 (p-value: 0.05). While the poorest countries in the sample are all located in Africa, the middle income countries near the turning point of the inverted-U include a number of Latin American countries, as well as Egypt, South Africa and Turkey. The richer part of the sample is dominated by European countries and the United States. Although these tend to be more E. Op. egalitarian, there is still a considerable spread among them.

Another question that naturally arises is whether there is any observable empirical association between inequality of opportunity and income inequality. Since in the between-types approach the former is measured as a component of the latter, there is a mechanical aspect to the relationship in levels, but it is not obvious that there is any mechanical reason to expect a correlation between income inequality levels and the *relative* extent of inequality of opportunity. Brunori et al. (2013) find a significant positive association between overall inequality (in economic advantage) and the share of that inequality associated with inequality of opportunity (IEO_R). The correlation coefficient is 0.523 (p-value: 0.0004).

A number of possible mechanisms might drive this positive correlation. One that appears eminently plausible is the notion that today's outcomes shape tomorrow's opportunities: large income gaps between today's parents are likely to imply bigger gaps in the quality of education, or access to labor market opportunities, among tomorrow's children (Ferreira, 2001). Naturally, the reverse direction of causation probably holds too: if opportunity sets differ a great deal among people, then individual outcomes are also likely to be more unequal. Inequalities in

income and opportunities are both endogenously determined, and likely to be mutually reinforcing. While the positive association reported by Brunori et al. (2013) is no proof of this mutual reinforcement, it is certainly consistent with it.

6. Inequality of opportunity, poverty and economic development

As noted earlier, measuring the extent of inequality of opportunity in a particular society is merely one of many possible uses for the concepts and theory of equality of opportunity. As noted in Section 2, introducing a role for personal responsibility in our assessment of equity changes the very space in which social justice can properly be defined and evaluated. It is to be expected that, to the extent that such a fundamental transformation is absorbed by economists, other economic concepts and ideas are also influenced.

There are now incipient signs that this process is beginning. In this section, we briefly report on some recent attempts to apply notions of equality of opportunity to two specific economic ideas: the definition and measurement of poverty, and the concept and measurement of economic development.

One way in which poverty could be redefined in light of the models discussed in Section 3 would be simply to apply poverty measures to the counterfactual distributions that reflect only unfair inequalities, defined in Section 4. Instead of computing inequality of opportunity as $I([\tilde{\boldsymbol{X}}_{ij}])$, one could compute "opportunity poverty" by means of an index $P([\tilde{\boldsymbol{X}}_{ij}])$, where $P(.)$ satisfies the usual axioms for poverty measures, including monotonicity and focus. Such an approach was implicitly suggested by Ferreira and Gignoux (2011) for the between-types case: these authors defined an "opportunity-deprivation profile" as the set of individuals whose counterfactual incomes $\tilde{x}_{ij} = \mu_i$ were below a certain threshold. If such a threshold is interpreted as an "opportunity-poverty line", that profile is nothing but the set of the "opportunity poor". It would then have been a small step to apply a particular poverty index $P(.)$ to the full counterfactual matrix.[23]

The "opportunity poverty" approach sketched above is most appealing, perhaps, in the context of the ex-ante view favored by van de Gaer (1993), where the type means μ_i which populate $\tilde{X}_{BT}$ are interpreted as the value of a person's opportunity set and, in that sense, are the true ex-ante value of her opportunities, prior to the realization of effort. But one might not subscribe strictly to that particular interpretation of the E. Op. model and still wish to incorporate some aversion to inequality of opportunities in poverty measures that are applied to the conventional income space (i.e. to the matrix $[X_{ij}]$). This is the path followed by Brunori, Ferreira, Lugo and Peragine (2013). Rather than measuring poverty in the space of opportunities, these authors propose to identify the poor in terms of income, and to evaluate the poverty in a

[23] A related approach was advocated by Peragine et al. (2009) in a setting similar to that introduced by Kranich (1996), where opportunity sets are treated as abstract sets and societies are modeled as profiles of opportunity sets.

given society by considering the degree of inequality of opportunity among the poor. Hence they propose two classes of "opportunity-sensitive poverty measures" (applied to the space of incomes) and identify poverty dominance conditions corresponding to each class.

In particular, they define a broad class of opportunity sensitive poverty measures (OSPM), which satisfies the standard axioms of monotonicity, focus and additivity, as well as new axioms of within-type anonymity, inequality of opportunity aversion, and (weak) inequality aversion within types. A narrow OSPM subclass is also defined, for the case when weak inequality aversion within types is replaced by a more stringent axiom of inequality neutrality within types. A specific family of indices is also defined and computed.

In an application to poverty comparisons across eighteen European countries, Brunori, Ferreira, Lugo and Peragine (2013) find that there are non-trivial differences between their family of indices and the standard Foster, Greer and Thorbecke (1984) poverty indices, even though both are applied to exactly the same income distributions, and are built upon a similar structure. Corresponding members of the two families are positively correlated, but a large number of re-rankings is observed, and some are substantial. Germany, for example, ranks as the sixth least-poor country in terms of the standard headcount measure (poorer than the Netherlands), but second least-poor in terms of the opportunity-senstive headcount (less poor than the Netherlands). Incorporating a specific axiom of aversion to inequality of opportunity within a poverty index can therefore lead to different poverty rankings than those produced by standard poverty measures, including those that are averse to inequality only in outcomes.

A second economic idea that has been redefined (by some authors) to incorporate the notion of equality of opportunity as the "currency of egalitarian justice", is the concept of economic development. We briefly review three separate examples. Bourguignon, Ferreira and Walton (2007) propose that "equitable" policy-makers should have as their objective to maximize the expected present discounted value (PDV) of the future stream of advantage (income x_{ij}, in our notation) for the least advantaged type, where 'least advantaged' is also defined in terms of the expected PDV of future advantage. This objective should be pursued subject to two constraints: first, that policies employed should belong to a permissible set, defined not only by standard feasibility constraints, but also in terms of the ethical acceptability of the policies themselves. Second, that along the future path of the economy, no individual is ever below an absolute minimum income level, to be socially agreed upon.

In bringing equity considerations to the definition of the development problem, Bourguignon, Ferreira and Walton make three contributions. First, they extend the van der Gaer "min of means" rule to a dynamic framework under uncertainty. While this is trivial from a formal point of view, it helps social choice theorists adapt to the reality faced by development economists, where economic growth and economic risk are always paramount considerations. A social program that ignores the trade-offs between current redistribution and future growth, for example, is unlikely to be taken seriously by those working on development issues. Second, the

"no absolute deprivation" constraint incorporates an element of aversion to extreme inequality in the space of outcomes: the State should pursue equal (and ampler) opportunities, in a lexicographic and dynamic sense, but without permitting extreme poverty among its citizens, regardless of what their initial opportunity sets were.[24] Finally, the introduction of a permissible policy set as a constraint incorporates a concern with process fairness: there may well be certain individual liberties (e.g. the choice of a spouse) which society decides that policies should not interfere with (even though forcing rich people to marry poor spouses could do a lot to promote equal opportunities for the next generation).[25]

A second example takes a different perspective: in a recent paper, Peragine et al. (2014) argue that a better understanding of the distributional effects of growth can be obtained by shifting the analysis from the space of final achievements to the space of opportunities. If two growth processes have, say, the same impact in terms of poverty and inequality reduction, but in the first case, all members of a certain ethnic minority - or all people whose parents are illiterate - experience the lowest growth rate, whereas poverty reduction in another case is uncorrelated with differences in race or family background, our current arsenal of measures does not readily allow us to distinguish them. These authors address this measurement problem by proposing a framework and a set of simple tools that can be used to investigate the distributional effects of growth from an opportunity-egalitarian viewpoint. In particular, with reference to a given growth episode, they address the following questions: is growth reducing or increasing the degree of I.Op.? Are some socio-economic groups systematically excluded from growth?

To answer these questions, Peragine et al. (2014) depart from the concept of the growth incidence curve (GIC) proposed by Ravallion and Chen (2003), and extend it to the space of opportunities. They introduce the concept of the Opportunity Growth Incidence Curve (OGIC), which is intended to capture the effect of growth from the EOp perspective. They further distinguish between an "individual OGIC" and a "type OGIC": the former plots the rate of growth of the (value of the) opportunity set given to individuals in the same position in the distributions of opportunities. The latter plots the rate of income growth for each sub-group of the population, where the sub-groups are defined in terms of initial exogenous circumstances. These tools capture distinct phenomena: the individual OGIC enables one to assess the pure distributional effect of growth on aggregate inequality of opportunity, whereas the type OGIC allows one to track the evolution of specific groups of the population in the growth process, to detect the existence of possible inequality traps. The authors apply these tools to different growth spells in Brazil and Italy.

[24] Another way to present this constraint is that it moves the elimination of the most extreme forms of deprivation from an objective to a constraint.

[25] Without necessarily subscribing to the specific freedoms and rights that Nozick (1974) defended, this allowance for fairness in process is nevertheless a nod to his contribution. On incorporating equality of opportunity considerations into the definition of development itself, see also Roemer (2014).

A third example of the incorporation of ideas from the literature on equality of opportunity to the problem of economic development can be found in the Human Opportunity Index (HOI) of Barros et al. (2009, 2011). This index is defined over a different set of advantages (which, confusingly, are sometimes referred to as 'basic opportunities'), namely access to certain basic services, such as piped water, electricity or sanitation.

For a particular service l the HOI, denoted by $\boldsymbol{H^l}$, is simply the average access rate in the population, penalized by the degree of dissimilarity in that coverage of that particular service across types. It is clearly analogous to the Sen welfare function, where mean outcomes are adjusted by one minus a measure of inequality. Sometimes an aggregate index is calculated as an average of $\boldsymbol{H^l}$ across a number of different services.[26] Various versions of the HOI have now been computed for at least 39 countries, and the index is popular at the World Bank.

The motivation behind the HOI, as initially proposed by Barros et al. (2009), was to measure the extent to which children in various developing countries have access to "basic opportunities". Although the authors do not motivate it this way, one could view the index as an example of the ex-ante approach applied to a multidimensional advantage space, with each dimension corresponding to access to a particular service – such as water or schooling – and the valuation of the opportunity set of each type being given by the coverage of the service in that type. The particular inequality index applied to that smoothed distribution of probabilities is the dissimilarity index, commonly used by sociologists. While the dissimilarity index might therefore be seen as a measure of inequality of opportunity, the HOI itself is closer in nature to a "development index" (like the Human Development Index – HDI - propounded by the United Nations Development Program), that incorporates an aversion to inequality of opportunity. [27] Indeed, Brunori et al. (2013) show that the HOI and HDI are very highly correlated over the current sample of countries for which they are available.

7. Conclusions

What is, then, the state of the economic literature on individual responsibility and equality of opportunity? In one sense, it is possible to claim that much has been achieved in the twenty years since the pioneering works of Roemer, van de Gaer and Fleurbaey. After all, changing the space in which judgments of fairness are made across the social sciences is no small task, and incorporating respect for individual responsibility into an egalitarian framework

[26] However, see Ravallion (2011) on the potential pitfalls of such arbitrary aggregate indices or, as he calls them, "mashup indices" of development.

[27] A possible caveat with viewing the dissimilarity index within the HOI as a measure of inequality of opportunity is that the index is typically calculated "for children". This justifies the use of certain variables - like geographic location or education of the adults in the household - as circumstances, which are clearly in the realm of choices for adults. The argument is that the index applies to children, and these are circumstances from their perspective. But this then raises the issue of the age of responsibility, and whether or not all inequalities in access to services for children below a certain age should not be considered inequality of opportunity. Under that view, unequal access to water or sanitation among five-year olds within the same type (i.e. sharing identical observed circumstances) should also be counted as inequality of opportunity.

no mean feat. The length of our reference list –as well as the fact that this paper is but one of three surveys of this literature being readied at the time of writing - are themselves indications of some impact, at least at the scholarly level. Ideas first developed by philosophers such as Arneson, Dworkin and Cohen, and by economists such as Sen, have been translated into simple, coherent and powerful models by a number of economists since the 1990s. And these models, in turn, have spawned a subsidiary literature on measurement, with a growing number of empirical applications. Some have now gone beyond the measurement of inequality of opportunity and of implications for public finance, and into issues related to poverty and the assessment of economic growth and development.

Yet at another level, there is still a long way to go. Although it is growing in importance, the literature on inequality of opportunity has not yet been fully incorporated into the economics mainstream. Only a minuscule fraction of the applied literature on impact evaluation, for example, examines the effects of policies on the distribution of opportunity.[28] Although policymakers the world over pay lip service to (or, in some cases, genuinely embrace) equal opportunities, it is not clear that much of that discourse has been informed by the literature reviewed above.[29] We close this chapter by highlighting three challenges that, in our view, face this field of normative economics in its quest for greater relevance.

The first is *robustness*. The sheer number of different approaches to quantifying inequality of opportunity – direct, indirect, ex-ante, ex-post, etc. – is understandable, considering the inherent difficulty in measuring a concept that refers to unobservable sets of possible counterfactuals as well as the novelty of the field. Yet, it is easy to see that it can appear bewildering to potential users. It is perfectly true, for example, that the ex-ante and ex-post approaches to the compensation principle are distinct and valid alternative ways to defining the egalitarian component of E. Op. Nevertheless, if the concept of inequality of opportunity is to appear as more than mere academic arcana to applied social scientists and policymakers, the field would greatly benefit from a few simple unifying principles that provide robust assessments or comparisons – much as Lorenz dominance placed the measurement of income inequality on a much firmer footing in the early 1970s.

The second is *accuracy*. The empirically dominant approach to measuring I.Op., as highlighted in Section 5, is currently the between-types specification of the ex-ante indirect approach. Yet, despite its simplicity, this approach suffers from a serious practical limitation: it generates only lower-bound estimates, and how far these lower-bound estimates are from the truth is very much an open question. And, to make it worse, it is an open question that is largely driven by the quality and nature of the data (both on advantage and circumstance variables) available in each application. This naturally makes comparisons across different countries

[28] Brunori et al. (2012), Jones et al. (2014), Van de Gaer et al. (2014) are some examples of this small vanguard.
[29] One exception was the World Bank's 2006 *World Development Report* on Equity and Development.

problematic and, in our view, is one factor behind some of the critiques that have been levelled against it (see, e.g. Kanbur and Wagstaff, 2014).

The third challenge we'd like to mention is *dimensionality*. At the risk of considerable over-simplification, the intellectual "anti-welfarist" revolution that took place in the 1970s and 1980s contained two fundamental elements: one was the need to incorporate an ethical role for individual responsibility in our assessments of social states. The other was the need to recognize the multi-dimensionality of well-being: that it could not always or productively be captured simply by some measure of preference satisfaction, such as "utility". In addition to Sen's capability approach, this second element has led to a surge in studies of multidimensional well-being, poverty, and inequality. Yet, presumably, the two aspects – individual responsibility (translated into seeking equality in the domain of opportunities) and the multi-dimensionality of well-being – are related. Despite a few tentative starts here and there, we are aware of no systematic attempt to extend the theory of E.Op. to a world with many advantage variables. We look forward to future contributions where those two pieces of the puzzle can be brought together.

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Table 7: Inequality of opportunity and income inequality in 41 countries

Country	GNI per capita PPP	Total inequality	IEO-L	IEO-R
Austria *(1)*	39,410	0.1800	0.0390	0.2167
Belgium *(1)*	37,840	0.1450	0.0250	0.1724
Brazil *(3)*	10,920	0.6920	0.2230	0.3223
Colombia *(3)*	9,000	0.5720	0.1330	0.2325
Cyprous *(1)*	30,160	0.1700	0.0510	0.3000
Czec Rep. *(1)*	23,620	0.1760	0.0190	0.1080
Denmark *(1)*	40,140	0.0830	0.0120	0.1446
Ecuador *(3)*	9,270	0.5800	0.1500	0.2586
Egypt *(5)*	5,910	0.4230	0.0491	0.1160
Estonia *(1)*	19,500	0.2430	0.0260	0.1070
Finland *(1)*	37,180	0.1360	0.0130	0.0956
France *(1)*	34,440	0.1630	0.0210	0.1288
Germany *(1)*	38,170	0.1910	0.0350	0.1832
Ghana *(2)*	1,600	0.4000	0.0450	0.1125
Greece *(1)*	27,360	0.2000	0.0340	0.1700
Guatemala *(3)*	4,610	0.5930	0.1990	0.3356
Guinea *(2)*	980	0.4200	0.0560	0.1333
Hungary *(1)*	19,280	0.2080	0.0210	0.1010
India *(8)*	3,560	0.4218	0.0822	0.1949
Ireland *(1)*	32,740	0.1880	0.0420	0.2234
Italy *(1)*	31,090	0.1960	0.0280	0.1429
Ivory Coast *(2)*	1,650	0.3700	0.0500	0.1351
Latvia *(1)*	16,360	0.2290	0.0280	0.1223
Lithuania *(1)*	17,880	0.2280	0.0350	0.1535
Luxemburg *(1)*	63,850	0.1480	0.0350	0.2365
Madagascar *(2)*	980	0.4400	0.0920	0.2091
Netherlands *(1)*	42,580	0.1920	0.0360	0.1875
Norway *(1)*	57,130	0.1300	0.0030	0.0231
Panama *(3)*	12,980	0.6300	0.1900	0.3016
Peru *(3)*	8,940	0.5570	0.1560	0.2801
Poland *(1)*	19,020	0.2710	0.0250	0.0923
Portugal *(1)*	24,710	0.2470	0.0300	0.1215
Slovakia *(1)*	23,140	0.1320	0.0180	0.1364
Slovenia *(1)*	26,970	0.1040	0.0050	0.0481
South Africa *(6)*	10,280	0.6750	0.1690	0.2504
Spain *(1)*	31,550	0.2160	0.0420	0.1944
Sweden *(1)*	39,600	0.1060	0.0120	0.1132
Turkey *(4)*	14,580	0.3620	0.0948	0.2620
Uganda *(2)*	1,230	0.4300	0.0400	0.0930
UK *(1)*	36,580	0.2040	0.0420	0.2059
US *(7)*	47,020	0.2200	0.0409	0.1860

Notes: The source for inequality and IEO measures for each country is given in parentheses after the country's name, and refers to the studies below. GNI per capita is from the World Bank's World Development Indicators, for the year 2010, using PPP exchange rates for 2005. Total inequality is measured by the mean logarithmic deviation in all cases except those from source (2), which use the Theil-T index. IEO indices are always based on the same inequality measure used for total inequality in that country.

(1) Checchi et al. (2010)
(2) Cogneau and and Mesple-Somps (2008)
(3) Ferreira and Gignoux (2011)
(4) Ferreira et al. (2011)
(5) Belhaj-Hassine (2012)
(6) Piraino (2012)
(7) Pistolesi (2009)
(8) Singh (2011)

Table 8: Comparing eight studies of ex-ante inequality of opportunity across 41 countries.

	References	Countries	Data sources	Outcome	Method	Circumstances	Number of types
1	Checchi et al. (2010)	Austria, Belgium, Czech Republic, Germany, Denmark, Estonia, Greece, Spain, Finland, France, Hungary, Ireland, Italy, Lithuania, Latvia, Netherlands, Norway, Poland, Portugal, Sweden, Slovenia, Slovakia, United Kingdom.	EU-Silc 2005	post-tax individual incomes	parametric	parental education, parental occupation, gender, nationality, geographical location	72
2	Cogneau and Mesple-Somps (2008)	Ivory Coast, Ghana, Guinea, Madagascar, Uganda.	Ivory Coast, EPAMCI, 1985-88 Ghana, 1998, GLSS Guinea, 1994, EICVM Madagascar, 1993, EPAM Uganda, 1992, NIHS	per capita household consumption	non parametric	3 groups based on father's occupation and education, region of birth	6 (3 Uganda)
3	Ferreira and Gignoux (2011)	Brazil, Colombia, Ecuador, Guatemala, Panama, Peru	Brazil, PNAD 1996; Colombia, ECV 2003; Ecuador ECV 2006; Guatemala, ENCOVI 2000; Panama, ENV 2003; Peru, ENAHO 2001	household per capita income	parametric	gender, ethnicity, parental education, father's occupation, region of birth.	108 (54 Peru)
4	Ferreira, Gignoux, Aran (2011)	Turkey	TDHS 2003-2004 and HBS 2003	imputed per capita consumption	parametric	urban/rural, region of birth, parental education, mother tongue, number of sibling	768
5	Belhaj-Hassine (2012)	Egypt	ELMPS 2006	total monthly earning	non parametric	gender, father's education, mother's education, father's occupation, region of birth.	72
6	Piraino (2012)	South Africa	NIDS 2008-2010	Individual gross income	parametric	race, father's education	24
7	Pistolesi (2009)	US	PSID 2001	individual annual earnings	semiparametric	age, parental education, father's occupation, ethnicity, region of birth	7,680
8	Singh (2011)	India	IHDS 2004–2005	household per capita earnings	parametric	father's education, father's occupation, caste, religion, geographical area of residence.	108