

Majumder, Amlan

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Working Paper Series

An alternative measure of economic inequality in the light of optics

Amlan Majumder

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An alternative measure of economic inequality in the light of optics*

Amlan Majumder[†]

Dinhata College, West Bengal, India

Abstract

An ideal state of development, when viewed with fantasy, is nothing but a state or condition where light touches everybody without refraction. The diagonal line of the Lorenz Curve Framework represents such an ideal condition. In the presence of inequality, however, it deviates or refracts from the ideal condition. In this paper, I try to measure economic inequality from the index of refraction. First, I compute such an index for each stratum to evaluate condition in each and then add all to propose an overall measure of economic inequality, which appears to be a standardised measure of the length of the Lorenz Curve relative to that of the diagonal line. The exercise is done utilising data on distribution of income or consumption from the WDI 2014. Results are lively and remarkable. While an index value of less than 1.00 represents an ‘anomalous refraction’ in Optics, such a condition of inequality is true and too common for many of us (60-80%) in reality. In contrast to that, in some countries, the index of refraction of the richest group exceeds that of Diamond (2.42), where an index value of 1.00 depicts an ideal condition that is enviable. In regard to technicalities, it goes at par Gini Index and beyond. Additionally, it makes analysis of economic inequality more sensible. Presumably, the proposed index could be a good substitute of the Gini Index as it is found perfectly correlated with the latter by quadratic equation with an Adjusted R Square value of 1.00.

Keywords: Gini Index, Inequality, Lorenz Curve, Optics, Refractive Index, Snell’s Law.

JEL Classification: D310, D630.

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[†]**Contact details:** Dinhata College, Dinhata, Cooch Behar, W. B. 736135, India, E-mail: amlan@amlan.co.in.

1. Introduction

An ideal state of development, when viewed with fantasy, is nothing but a state or condition where light touches everybody without refraction. Although we insist upon an ideal state of development, in the real-world situation we live in a stratified society with varied living conditions, where light, as a source of goodness, seems to refract while passing from one stratum into another. This paper conceptualises presence of refraction as inequality and tries to capture it from the index of refraction, which is commonly used in Optics¹ to measure the bending of a ray of light in similar conditions. The concept of Gini index under the Lorenz Curve Framework is akin to that of refraction, as it measures the extent to which the distribution of income or consumption expenditure among individuals or groups within an economy deviates from a perfectly equal distribution. If we view the world or a part of it from the perspective of the Lorenz Curve Framework superimposing the ideas of Optics on it, we realise that in case of an ideal condition, light or the Lorenz Curve passes diagonally touching everybody in it without refraction. In the presence of inequality, however, it deviates from the hypothetical line of absolute equality and is seen to refract while passing from one stratum into another. This paper uses simple mathematical tools and follows the well-known Snell's Law² to measure refractive index for each stratum as a measure of inequality in each with respect to the ideal condition, and consequently treats a simple summation of those for all the strata as an overall measure of inequality within the Lorenz Curve Framework. It is further shown that the new proposed index is a standardised measure that can be expressed as a ratio of length of the observed Lorenz Curve to that in the ideal condition. The exercise is done for the World Bank member countries utilising data on distribution of income or consumption from the World Development Indicators (WDI) 2014. In this context it is to be mentioned that although the indices of refraction and the overall measure of inequality are computed with grouped data, the exercise can be extended vividly

to the cases when the curve is smooth and continuous with large number of groups or individuals.

The paper is organised as follows. Section 2 briefly discusses background of the study with focus on simple and alternative derivations. Section 3 demonstrates the methodology of measuring index of refraction and the methods of computing the same for each stratum as well as that for the whole Lorenz Curve Framework. Section 4 computes refractive index for each stratum and presents results for some selected countries. Section 5 computes the overall measure of inequality, which is termed as Optical Inequality Index (OI Index) and presents results as above. Section 6 explores relationship between OI Index and Gini Index and depicts that they are perfectly correlated by quadratic equation. Section 7 discusses properties of the OI Index particularly with focus on Pigou-Dalton condition and policy imperative with indication of convergence of unequal distributions. Section 8 presents conclusion.

2. Background of the study

Economists in field of measurement of inequality have always been in the quest of deriving simple and alternative ways to measure inequality. While any previous attempt to assimilate the idea of refraction of light with that of inequality based on Lorenz Curve is not known, efforts on alternative derivation in different dimensions of Gini Index are too common. In point of fact, the inequality measure under the mean difference approach proposed by Corrado Gini in 1912, which became popular afterward as Gini Index, is also an alternative formulation to the geometric measure proposed by Max O. Lorenz in 1905. Since then alternative formulations under such frameworks or else grew exponentially, and it is fairly impossible today to cover all. Popular survey papers^{3,4} reveal that out of the four broad groups of studies (such as, geometric approach, mean difference approach, covariance approach, and matrix form approach), authors largely concentrate on the first three and establish linkages between the existing and newly developed formulations under them

considering the area based concept of the measure in the first. Such survey papers do not indicate presence of any study on the approach under discussion.

However, in efforts of presenting intuitively simpler derivations, a couple of masterpieces need special mention. For example, with the intention of proposing an alternative and simple measure of Gini Index, B. Milanovic⁵ works out a geometric formula by looking at the vertical height between the 45⁰ line and the Lorenz curve. In the quest of simplicity he goes further to derive a measure under the covariance approach and claims that since all the components in it are easy to calculate, the Gini Index can be obtained using a simple hand calculator⁶.

With similar objectives, in one of my previous efforts, I focus on cotangent and cosecant of the left-hand side complementary angle of each right-angled triangle under the Lorenz Curve (in case of grouped data) to measure inequality with the application of trigonometry⁷. However, lack of robustness in those measures, which seems prominent today, is likely to be substantiated in the present approach that too based on angles of incidence and refraction.

3. Methodology

In optics, Snell's Law of refraction⁸ exhibits the relationship between different angles of light as it passes from one transparent medium into another as follows:

$$I_a * \sin(\theta_a) = I_w * \sin(\theta_w), \quad (1)$$

where I_a is the refractive index of the medium a the light is leaving, θ_a is the angle of incidence, I_w is the refractive index of the medium w the light is entering, and θ_w is the angle of refraction. An illustration of refraction (from air to water) is shown in figure 1.

We may apply formula 1 to the Lorenz Curve Framework as demonstrated in figure 2 (with standard concept and notations), where we have five different strata with y_i as the proportion of income or consumption of one particular stratum such that $\sum y_i = 1$ (for $i = 1, 2, \dots, 5$ or $1, 2, \dots, n$ in general). In that, an ideal condition is the one where light passes

diagonally without refraction. As inequality exists, light refracts five times (as we have considered five different strata) while passing from one stratum into another. From figure 2 we may check that there are five different triangles associated with five different strata. Hypotenuses of all the triangles constitute the Lorenz Curve. If we assume that light passes from the upward direction (from right to left), the perpendicular of a triangle is 0.20 (i.e., $1/n$) and the base is y_i . The hypotenuse of each triangle is:

$$\sqrt{(0.20)^2 + (y_i)^2}, \text{ and} \quad (2)$$

$$\sin(\theta_w) = \frac{0.2}{\sqrt{(0.2)^2 + (y_i)^2}}. \quad (3)$$

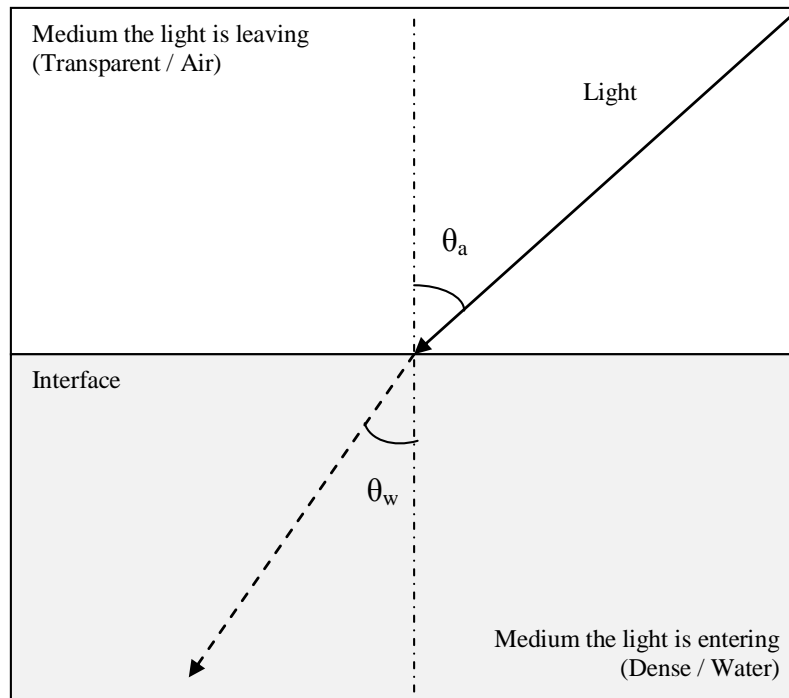


Figure 1. An illustration of refraction (from air to water)

The refractive index of the stratum where light enters may be computed with respect to that of the immediate preceding one or relative to that of the ideal condition, where $\theta = 45^\circ$ with respect to the vertical normal. As the latter seems simple, we compute the index of refraction following the latter. The index of refraction of a particular stratum is (from equation 1):

$$I_w = I_a * \frac{\sin(\theta_a)}{\sin(\theta_w)} . \quad (4)$$

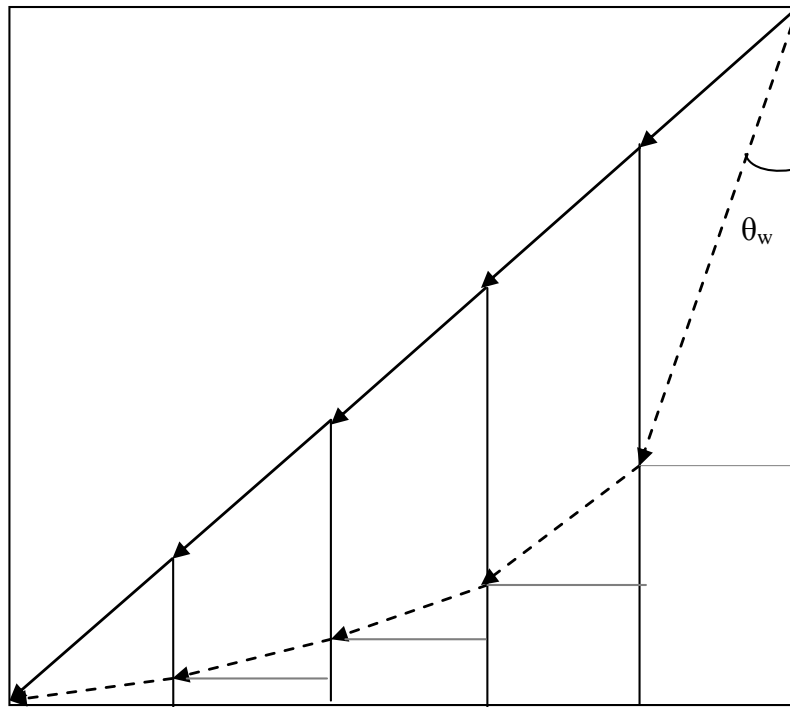


Figure 2. Lorenz Curve Framework with five groups

As in case of a fully transparent medium and / or in ideal condition the refractive index is 1.00 (by assumption) and the angle of incidence (θ_a) is 45° ,

$$I_w = 1.00 * \frac{\sin(45^\circ)}{\frac{0.20}{\sqrt{(0.20)^2 + (y_i)^2}}} . \quad (5)$$

$$= \frac{\sin(45^\circ) \sqrt{(0.20)^2 + (y_i)^2}}{0.20} . \quad (6)$$

As y_i is known and $\sin(45^\circ) = 0.70710678118$, index of refraction of each stratum can be obtained easily from expression 6. In general, as $0.20 = 1/n$ and if we denote the hypotenuse in the numerator (as in expression 2) as h ,

$$I_w = n \sin(45^\circ) h . \quad (7)$$

When $y_i = 0$, $I_w = 0.70710678118 \approx 0.71$ (as obtained from formula 6). When $y_i = 0.2$ (the ideal condition), $I_w = 1.00$. When $y_i = 1$ (the extreme case), $I_w = 3.60555127546 \approx 3.61$.

If we add all the refractive indices (for all the strata) under the Lorenz Curve Framework, we get the overall measure of economic inequality for the particular income distribution. As all the hypotenuses constitute the deviated or observed Lorenz Curve (say, OLC), after summation (for $i = 1, 2, \dots, 5$ or $1, 2, \dots, n$ in general) of all the refractive indices we get:

$$I = n \sin(45^\circ) \text{OLC}. \quad (8a)$$

Equivalently, as for the whole triangle under the line of absolute equality, $\sin(45^\circ)$ is nothing but the base of the triangle (perpendicular in true sense with respect to $\theta = 45^\circ$, whose length is 1.00) divided by the hypotenuse (the diagonal line, whose length is $\sqrt{2}$),

$$I = \frac{n}{\sqrt{2}} \text{OLC}. \quad (8b)$$

Equivalently, as $\sqrt{2} = \text{Lorenz Curve in the ideal condition (say, ILC)}$,

$$I = n \frac{\text{OLC}}{\text{ILC}}. \quad (8c)$$

The length of ILC is: $\sqrt{1^2 + 1^2} = \sqrt{2} = 1.41421356237$. When $n=5$ and in the extreme case, when all resources are given to one group or individual, (in figure 2) the OLC takes an upward turn from point (0, 0.8). So, the maximum length of OLC is: $0.8 + \sqrt{(0.20)^2 + (1)^2} = 1.81980390271$. As, in the ideal case $\text{ILC} = \text{OLC}$, for $n=5$,

$$I_{\min} = 5.00; \quad (9)$$

and in the extreme case,

$$I_{\max} = 5 * \frac{1.81980390271}{1.41421356237} = 6.43397840021 \approx 6.43. \quad (10)$$

If we want results in a normalised 0-100 scale, we define the overall measure of economic inequality (which may be termed as Optical Inequality Index or OI Index) as:

$$I_o = 100 * \frac{I - I_{\min}}{I_{\max} - I_{\min}}. \quad (11)$$

Using formulae 8a or 8b or 8c and 11 we will be able to compute OI Index and from formula 6 or 7 refractive index of each stratum.

Alternatively, the index of refraction (I_w , as in expression 6 or 7) can be computed directly from the slope of the Lorenz Curve (for example, in continuous case). The slope of the tangent line at a particular point on the Lorenz Curve is nothing but $\tan(\theta_c)$, where θ_c is the left-hand side complementary angle of each right-angled triangle below the tiniest straight line portion of the Lorenz Curve.

$$\tan(\theta_c) = \frac{dy}{dp}, \quad (12)$$

where y = proportion of income or consumption and p = proportion of population. In expression 12, $\tan(\theta_c)$ measures change in share of income or consumption due to unit change in proportion of population. For example, if the slope or $\tan(\theta_c) = 0.50$, $dp = 1.00$ and $dy = 0.50$. In that case from expression 3:

$$\sin(\theta_w) = \frac{1.00}{\sqrt{(1.00)^2 + (0.50)^2}}, \text{ and from expression 6} \quad (13)$$

$$I_w = \frac{\sin(45^\circ) \sqrt{(1.00)^2 + (0.50)^2}}{1.00} \approx 0.79. \quad (14)$$

The refractive index in expression 14 is equal to that of the lowest 20 % group in Sweden in 2014. As per WDI 2014, the share of this group (in Sweden) is 0.10. If this share is divided by the proportion of population, 0.20, we get $dy = 0.50$ with respect to $dp = 1.00$.

Further, in continuous case, there is a point on Lorenz Curve where the slope of the tangent line is equal to that of the diagonal one. This is the point of inflection, as it divides the population into two groups with a refractive index of less than 1.00 in the left and more than 1.00 in the right. This concept may be used to derive a line of inequality in accordance with that of poverty.

As the overall measure or OI Index is based on length of the Lorenz Curve in different conditions, in continuous case, it can be obtained from expression 8c.

4. Computation of refractive index for each stratum

We utilise data on distribution of income or consumption from the WDI 2014 for 148 countries (as per completeness of information) and compute refractive index for each stratum using formula 6. Results of some countries (selected arbitrarily) are displayed in table 1.

We learnt that in the ideal condition refractive index is equal to 1.00 (as discussed in relation to formula 7). So, an index value of 1.00 is desirable for each of the strata. Deviation from 1.00 is undesirable. Any value less than 1.00 is strictly undesirable. Standard literature in Optics maintains that an index value of less than 1.00 does not represent a physically possible system⁹. Further, literature in Optics defines an index value of less than 1.00 as ‘anomalous refraction’¹⁰.

Table 1. Refractive Index of each stratum and OI Index for some selected countries in 2014

Country	Refractive index in each stratum					OI Index
	Lowest	Second	Third	Fourth	Highest	
	20%	20%	20%	20%	20%	
China	0.73	0.79	0.88	1.08	1.81	20
India	0.77	0.82	0.90	1.02	1.66	12.7
Italy	0.74	0.82	0.93	1.08	1.64	14.9
Namibia	0.72	0.74	0.78	0.91	2.44	39.9
Slovenia	0.79	0.88	0.95	1.07	1.41	7.2
South Africa	0.71	0.72	0.76	0.91	2.57	46.9
Sweden	0.79	0.86	0.95	1.07	1.45	7.9

Source: Self-elaboration

However, the condition, which does not represent a physically possible system or which is considered ‘anomalous’ in physical science, is true and too common for many of us in reality. For example, in table 1, we see that 60-80 % common mass in each country are

subject to such an ‘anomalous refraction’ and presumably a miserable condition of economic inequality too.

Refractive index with a value of more than 1.00 indicates higher concentration of wealth or income with the upper limit being 3.61 (the extreme case, as discussed in relation to formula 7). However, the refractive index of the highest 20 % group in Namibia in 2014 is 2.44, which is close to that of Diamond (2.42)¹¹. Similarly, the richest group in South Africa, in the same year, commands far more luxury as its refractive index (2.57) is seen to exceed that of Diamond. It is to be noted that in both the countries, 80 % of total population live in an ‘anomalous’ and miserable conditions of inequality.

Among other countries (in 2014), the refractive index of the richest 20 % group in China (1.81) is close to that of Sapphire (1.78)¹². The said index values of India (1.66) and Italy (1.64) are higher than that of Amber (1.55)¹³, and those of Slovenia (1.41) and Sweden (1.45) are close to that of Opal (1.45)¹⁴. Interpretation of results of economic inequality with the refractive indices of precious materials is simply ornamental and has no special scientific meaning. However, from this classification, one may imagine the extent of inequality between the people in miserable condition and luxury commanded by people in the highest income groups in the respective countries.

5. Computation of Optical Inequality Index for the whole framework

Optical Inequality Index (OI Index) is computed using formula 8 and 11. It is nothing but the summation of all the refractive indices of the five different income groups or strata expressed in a 0-100 point normalised scale. Index values are displayed in the final column of table 1. Interpretation of the OI Index is similar to that of Gini Index. However, Gini Index is subject to the ‘first-order’ downward bias associated with small sample or small size or grouping of micro data into smaller number of equally sized parts as discussed precisely in theoretical^{15,16,17} and empirical literature¹⁸. By definition OI Index is free from such (first-

order) downward bias in case of grouped data, and works similarly as some other well-known formulations do^{19,20}. It is further confirmed with numerical exercises (with uniform distribution) that in case of grouping too ‘first-order’ downward bias is absent in OI Index. However, we will understand OI Index better if we relate it with Gini Index and study its properties further.

6. Relationship between Optical Inequality Index and Gini Index

Gini Index and OI Index are perfectly correlated by quadratic equation as shown in table 2 and in figures 3 and 4. As, OI Index is obtained from the grouped data on distribution of income or consumption, the relationship is explored after computing Gini Index from the same data following the standard measure under the mean difference approach^{21, 22}.

First, I estimate a model with the seven countries included in table 1 and then I repeat the exercise with the data of 148 countries as listed in table 5 in the appendix. It is found, in both the models, that 100 % variability in the OI Index is explained by the Gini Index with identical Adjusted R Square value of 1.00.

Table 2. The Summary and goodness of fit statistics of Quadratic models

	Statistic	Value	Standard error	F or t [*]	Sig.
	Adjusted R Square	1.000	0.184	22123.973	0.000
Model I	Constant	0.268	0.892	0.301	0.779
(n=7)	Gini Index	-0.009	0.047	-0.183	0.864
	Gini Index Square	0.013	0.001	23.577	0.000
	Adjusted R Square	1.000	0.150	235621.836	0.000
Model II	Constant	1.499	0.222	6.738	0.000
(n=148)	Gini Index	-0.070	0.012	-6.009	0.000
	Gini Index Square	0.014	0.000	96.970	0.000

* F for adjusted R square, t for the constant and the coefficients
Source: Self-elaboration

The precise relationships as estimated in the models and as depicted by figures 3 and 4 respectively are shown below.

$$\text{OI Index (Model I)} = 0.268 - 0.009(\text{Gini Index}) + 0.013(\text{Gini Index Square}), \quad (15)$$

$$\text{OI Index (Model II)} = 1.499 - 0.070(\text{Gini Index}) + 0.014(\text{Gini Index Square}). \quad (16)$$

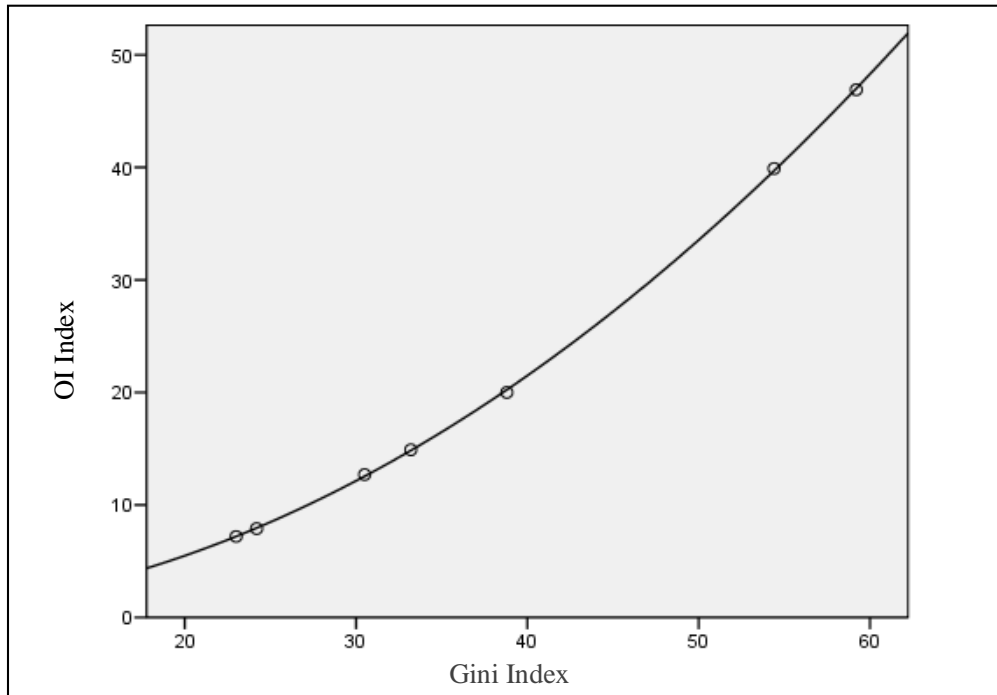


Figure 3. Gini Index vs. OI Index (Model I, n =7)

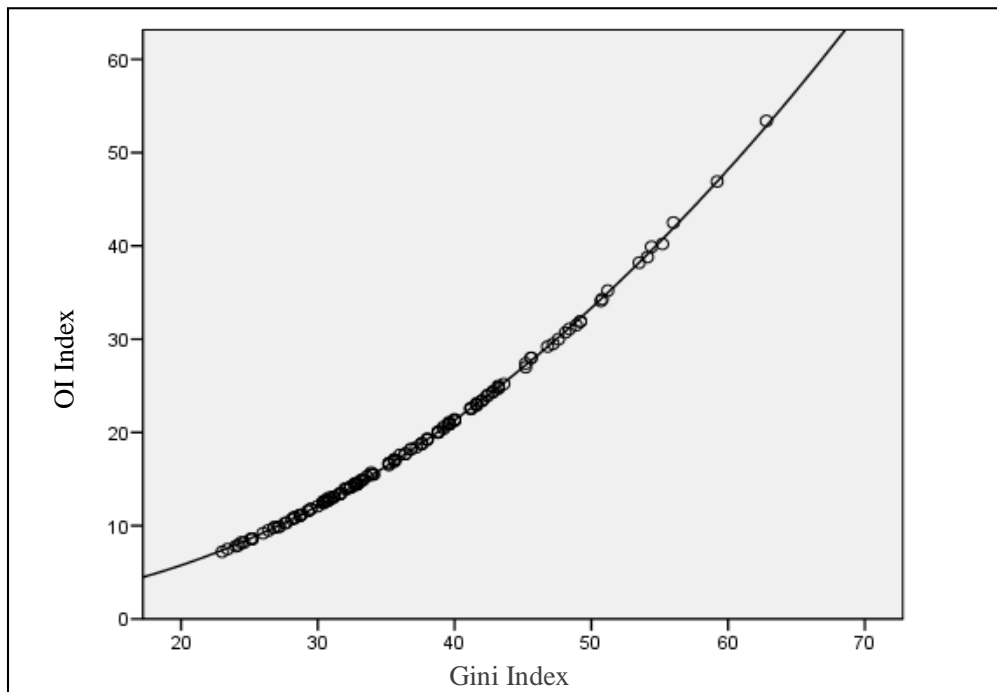


Figure 4. Gini Index vs. OI Index (Model II, n = 148)

7. Properties of Optical Inequality Index

As the OI Index is derived under the Lorenz Curve Framework, it is well-understood that it satisfies the basic desirable properties (as applicable) such as, population-size independence,

and the Pigou-Dalton condition²³. However, the OI Index is far more sensitive to the changes in share of income or consumption both at the lower and the upper ends as demonstrated below. Table 3 shows percentage change in the indices due to one unit change in share of income or consumption from one group to the other. For each country (we need to move along row), we see that sensitivity is higher in OI Index than in Gini Index.

Additionally, as the OI Index is additive, it is confirmed that each component under different strata maintains the spirit of the Pigou-Dalton condition. For example, for the stratum where the index value is more than 1.00, in response to any positive transfer to it, index value increases indicating further escalation of inequality. On the other hand, for the stratum where the index value is less than 1.00, in response to any outward transfer from it, index value decreases aggravating the ‘anomalous’ condition further.

Table 3. Sensitivity of Gini Index and OI Index for one unit change in share

Country	Q ₁ to Q ₅		Q ₂ to Q ₃		Q ₄ to Q ₃		Q ₅ to Q ₁	
	GI	OI	GI	OI	GI	OI	GI	OI
China	4.1	8.7	1.0	2.3	-1.0	-1.7	-4.1	-8.1
India	5.2	10.0	1.3	2.7	-1.3	-1.5	-5.2	-9.2
Italy	4.8	10.6	1.2	2.7	-1.2	-1.5	-4.8	-9.8
Namibia	2.9	5.1	0.7	1.0	-0.7	-1.1	-2.9	-4.8
Slovenia	6.9	15.1	1.7	3.1	-1.7	-2.3	-6.9	-13.7
South Africa	2.7	4.7	0.7	1.1	-0.7	-1.2	-2.7	-4.4
Sweden	6.6	13.9	1.6	3.7	-1.6	-2.1	-6.6	-12.6

Q₁: Lowest 20 %, Q₂: Second 20 %, Q₃: Third 20 %, Q₄: Fourth 20 %, Q₅: Highest 20 %;

GI: Gini Index, OI: Optical Inequality Index

Source: Self-elaboration

I now do an exercise to know whether refractive index gives us any clue towards convergence of the unequal distributions (considering the case of Sweden only). As, $n = 5$, a 20 % share of each group indicates an ideal condition and for which refractive index is 1.00. In table 1, we have a set of refractive indices for Sweden. Index value of the first stratum is 0.79. If a refractive index of 1.00 indicates a share of 0.20, 0.79 indicates a share of 0.16

(0.79×0.20). So, if we multiply the refractive indices of Sweden by 0.2, we get an indicative distribution as shown in the second row of table 4. As our objective is to reduce inequality, at the very first stage we may try to obtain a distribution corresponding to the first iteration. After achieving such a distribution in reality, we may go for the second iteration and so on. Finally, after sixth iteration, we will reach the ideal condition. If we consider a highly unequal distribution, a few more iterations are necessary. This exercise has importance from policy imperative, as it shows that if we work in response to our senses with good intention, we will converge towards an ideal condition.

Table 4. Convergence of unequal distributions: an example in case of Sweden

Iteration	Distribution of income or consumption				
	Lowest 20%	Second 20%	Third 20%	Fourth 20%	Highest 20%
Initial	0.10	0.14	0.18	0.23	0.36
First	0.16	0.17	0.19	0.21	0.29
Second	0.18	0.18	0.19	0.20	0.25
Third	0.19	0.19	0.20	0.20	0.23
Fourth	0.19	0.19	0.20	0.20	0.21
Fifth	0.20	0.20	0.20	0.20	0.21
Sixth	0.20	0.20	0.20	0.20	0.20

Source: First row – WDI 2014, rest – Self-elaboration

8. Conclusion

The inherent objective of the paper has been to propose an alternative measure of economic inequality under the Lorenz Curve Framework, which could be far more lively and responsive to our senses as compared to the Gini Index. We have observed the overall workability of the proposed index with its sensibility in the previous sections and our experience has not been unpleasing. Being overly simple but contented, the proposed measure of economic inequality based on the index of refraction of light could be a good substitute of the said Gini Index and similar ones.

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- ²⁰ The World Bank. (No date), Module 5: Inequality Measures, available at http://info.worldbank.org/etools/docs/library/93518/Hung_0603/Hu_0603/Module5MeasuringInequality.pdf, p. 22 (accessed on 05 November 2014).
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- ²² . The issue of 'first-order' downward bias for using grouped data is set aside, as variability is independent of change in scale and origin.
- ²³ *Ibid.* 21, p. 341.

Appendix

Table 5. List of 148 countries included in regression analyses with computed Gini Index and OI Index *

Country	Gini Index	OI Index	Country	Gini Index	OI Index
<i>Slovenia</i>	<i>23.0</i>	<i>7.2</i>	Mali	30.7	12.7
Slovak Republic	24.6	8.2	Nepal	30.7	12.7
Ukraine	23.4	7.5	Spain	33.9	15.7
<i>Sweden</i>	<i>24.2</i>	<i>7.9</i>	Guinea	30.9	12.8
Belarus	25.2	8.6	Belgium	30.4	12.6
Czech Republic	24.4	8.2	Ethiopia	31.2	13.1
Iceland	24.0	7.8	<i>Italy</i>	<i>33.2</i>	<i>14.9</i>
Norway	25.2	8.6	Jordan	31.2	13.1
Romania	25.2	8.6	Sao Tome and Principe	32.4	14.1
Denmark	25.1	8.6	Sudan	32.4	14.1
Finland	26.0	9.2	Croatia	31.1	13.1
Hungary	26.8	9.8	Algeria	32.9	14.5
Netherlands	26.8	9.8	<i>India</i>	<i>30.5</i>	<i>12.7</i>
Albania	27.2	9.9	Tunisia	32.9	14.5
Kazakhstan	27.1	9.9	West Bank and Gaza	31.7	13.5
Serbia	28.3	10.8	Burundi	30.8	13.0
Iraq	27.7	10.3	Guinea-Bissau	32.8	14.4
Armenia	27.6	10.3	Vietnam	32.8	14.4
Pakistan	26.4	9.5	Latvia	33.9	15.5
Tajikistan	28.8	11.2	Uzbekistan	32.7	14.5
Austria	28.7	11.1	Indonesia	32.9	14.5
Germany	28.7	11.1	Maldives	34.1	15.5
Moldova	28.7	11.1	Mauritius	33.3	14.9
Montenegro	28.7	11.1	Sierra Leone	32.1	13.9
France	29.3	11.6	Mongolia	34.0	15.5
Ireland	29.3	11.6	Syrian Arab Republic	32.8	14.5
Timor-Leste	28.1	10.7	United Kingdom	35.2	16.5
Estonia	30.4	12.5	Yemen, Rep.	32.4	14.2
Japan	30.4	12.5	Lao PDR	33.1	14.8
Lithuania	30.4	12.5	Iran, Islamic Rep.	35.6	16.9
Switzerland	30.0	12.1	Liberia	35.6	16.9
Egypt, Arab Rep.	28.3	10.9	Sri Lanka	33.6	15.3
Niger	28.3	10.9	<i>Bhutan</i>	<i>35.2</i>	<i>16.7</i>
Australia	31.6	13.4	<i>Congo, Rep.</i>	<i>37.2</i>	<i>18.4</i>
Bosnia and Herzegovina	31.6	13.4	<i>Djibouti</i>	<i>36.4</i>	<i>17.7</i>
Bulgaria	32.0	14.0	<i>Tanzania</i>	<i>35.2</i>	<i>16.7</i>
Cameroon	38.0	19.3	<i>Turkey</i>	<i>36.4</i>	<i>17.7</i>
Poland	30.4	12.4	<i>United States</i>	<i>38.0</i>	<i>19.2</i>
Azerbaijan	30.7	12.7	<i>Burkina Faso</i>	<i>35.6</i>	<i>17.1</i>
Bangladesh	29.5	11.8	<i>Thailand</i>	<i>35.6</i>	<i>17.1</i>
Cabo Verde	40.0	21.3	<i>China</i>	<i>38.8</i>	<i>20.0</i>
Greece	32.7	14.5	<i>Israel</i>	<i>38.8</i>	<i>20.0</i>
Kyrgyz Republic	30.7	12.7	<i>Mauritania</i>	<i>37.6</i>	<i>18.8</i>

Russian Federation	36.8	18.2	Togo	42.0	23.4
Senegal	37.6	18.8	Mozambique	41.6	23.1
Turkmenistan	37.6	18.8	Ecuador	43.2	24.7
Uruguay	38.8	20.0	Jamaica	42.4	23.9
Georgia	38.8	20.0	Gambia, The	42.8	24.3
Madagascar	36.0	17.6	Paraguay	43.6	25.2
Morocco	36.8	18.2	Malawi	42.4	24.0
Cambodia	29.5	11.8	Kenya	42.8	24.4
El Salvador	38.0	19.3	Costa Rica	45.2	27.0
St. Lucia	39.2	20.4	Mexico	43.2	25.0
Angola	39.6	20.9	Panama	47.2	29.5
Argentina	39.6	20.9	Papua New Guinea	45.2	27.4
Chad	39.6	20.9	Chile	45.6	28.0
Cote d'Ivoire	39.6	20.9	Guatemala	46.8	29.2
Ghana	39.6	20.9	Rwanda	45.6	28.0
Gabon	38.8	20.1	Swaziland	47.6	30.0
Nigeria	40.0	21.3	Brazil	48.1	30.7
Guyana	41.2	22.6	Suriname	48.9	31.5
Venezuela, RB	41.2	22.6	Belize	48.4	31.1
Bolivia	43.2	24.8	Colombia	49.2	31.9
Canada	31.6	13.4	Lesotho	49.2	31.9
Fiji	39.2	20.6	Honduras	50.7	34.1
Peru	42.0	23.4	Central African Republic	50.8	34.3
Philippines	40.0	21.3	Zambia	51.2	35.2
Malaysia	41.6	22.9	Haiti	54.1	38.8
Uganda	39.6	21.1	Micronesia, Fed. Sts.	55.2	40.2
Benin	40.0	21.4	Botswana	53.5	38.2
Macedonia, FYR	41.2	22.5	<i>Namibia</i>	<i>54.4</i>	<i>39.9</i>
Congo, Dem. Rep.	41.6	23.0	Comoros	56.0	42.5
Nicaragua	41.6	23.0	<i>South Africa</i>	<i>59.2</i>	<i>46.9</i>
Dominican Republic	42.0	23.4	Marshall Islands	62.8	53.4

* Countries are ranked according to the refractive index of the highest 20 % group in each in ascending order, with a low value implying a less alarming situation.

Four colours represent four different self-defined groups (based on the refractive index of the richest 20 %): [Opal: ≤ 1.50], [Amber: 1.51-1.75], [Sapphire: 1.76-1.99], and [Diamond: ≥ 2].

Bold italicised countries (as in table 1) are included in Model I; all the 148 countries are included in Model II.

Gini Index is computed from grouped data and hence it shows lower values than those based on micro data.

Source: Self-elaboration