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Growth with Unused Capacity and Endogenous Depreciation

by

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ABSTRACT
This paper contributes to the debate on income growth and distribution from a nonmainstream perspective. It looks, in particular, at the role that the degree of capacity utilization plays in the process of growth of an economy that is not perfectly competitive. The distinctive feature of the model presented in the paper is the hypothesis that the rate of capital depreciation is an increasing function of the degree of capacity utilization. This hypothesis implies analytical results that differ somewhat from those yielded by other Kaleckian models. Our model shows that, in a number of cases, the process of growth can be profit-led rather than wage-led. The model also determines the value to which the degree of capacity utilization converges in the long run.

Keywords: Kaleckian Models of Growth; Capital Accumulation; Capital Depreciation; Income Distribution and Growth

JEL Classifications: E12, E25, O40
1. INTRODUCTION

In recent years, many economists in the non-mainstream camp have been involved in a lively debate on growth and distribution.¹ Most of the models used by the participants in the debate are characterized by equilibria associated with a certain amount of unused capacity. In this context, attention has been paid to the distinction between the degree of capacity utilization determined by the level of aggregate demand and the planned (normal) degree of capacity utilization that, in a non-perfectly competitive environment, depends on factors other than aggregate demand.

Economists who more directly draw their inspiration from Kalecki develop their analysis by using a model of growth that, at least in its simplest version, does not contain any notion of a normal, or planned, degree of capacity utilization. The basic features of this model are that the equilibrium degree of capacity utilization is determined endogenously; the so-called Keynesian “paradox of thrift” holds true (an increase in the economy’s propensity to save has negative demand effects); the process of growth is wage-led, i.e. a distribution of income more favorable to wages is associated with a higher rate of growth.

This position is criticized by others, who argue that, in the long period, the economy must tend to some normal degree of capacity utilization, which is the one that profit-maximizing firms plan to have. Some Kaleckians have responded to these criticisms by accepting the idea that there exists a normal degree of capacity utilization but by arguing, at the same time, that it is not unique. The normal or planned degree of capacity utilization is determined endogenously and tends to adjust to the actual degree of utilization rather than the other way around.

The present paper intends to offer a contribution to this debate by proposing a novel approach to the problem of capacity utilization and the role it plays in the process of growth. The distinctive feature of the model that we present in this paper is the assumption that the rate of capital depreciation varies with the rate at which capital is used. The rate of depreciation is increasing in the degree of capacity utilization.

¹ For a representative collection of recent non-mainstream models of growth and distribution, see for example Setterfield (2010) and Hein et al. (2011).
We introduce a normal degree of capacity utilization, which firms plan to have, but we show that firms may be unable to realize their plans even in the long period. This amounts to regarding the degree of capacity utilization as endogenously determined also in the long period. From this viewpoint, our results are similar to those obtained by other Kaleckian models, although reached by following a different analytical path.

The models developed by the participants in the debate are generally based on the hypothesis of constant marginal costs up to full capacity, to which firms apply a mark-up to set prices. Since the average total cost varies inversely with the degree of capacity utilization and reaches its minimum at full capacity, the rate of profit is increasing in the degree of capacity utilization. These are the only considerations concerning the relation between the degree of capacity utilization and conditions of production and profitability made in the debate.

In our opinion, the relation between capacity utilization and conditions of production and profitability deserves more attention. More in particular, it is important to look at the relation between the degree of utilization of capital and its rate of depreciation. The hypothesis of a constant rate of capital depreciation is, in our view, unsatisfactory and unrealistic.

In everyday life, nobody would hold that, for example, the speed at which a car depreciates over time is independent of the number of hours it is driven each year. Economic modeling should retain this common-sense idea and remove the hypothesis of a constant rate of depreciation. Our model is based on the hypothesis that the rate of depreciation of capital is an increasing function of its utilization in production rather than being constant.

The hypothesis of a varying rate of depreciation not only renders the growth model more realistic; it has also some relevant analytical implications. The introduction of a varying rate of depreciation implies that the Kaleckian results regarding the paradox of thrift and the wage-led nature of growth hold under more restrictive conditions than those usually considered.

A varying rate of depreciation can also give rise to a process through which firms vary their degree of capacity utilization to bring it closer to the value they plan to have. The latter, however, is actually realized only in special cases. Thus, in general, the long-period equilibrium degree of capital utilization is endogenously determined.

The paper is organized as follows. In section 2 the basic Kaleckian model of growth is presented and the implications of introducing a notion of a normal degree of utilization are discussed. Section 3 is devoted to the presentation of our alternative model with a varying rate of depreciation. Section 4 briefly looks at the historical background of the current debate on
capacity utilization and the relation between the intensity of use of capital and conditions of production, costs and profitability. Section 5 concludes.

2. THE STANDARD KALECKIAN MODEL OF GROWTH WITH UNUSED CAPACITY

Most contemporary Kaleckian economists deal with the problem of excess capacity in the context of equilibrium growth models, in which the degree of capacity utilization is an endogenous variable, as it is determined by the level of demand (see, e.g., Dutt, 2011). In the basic version of the Kaleckian model, which is presented below, the notion of a normal degree of capacity utilization does not play any role.

2.1 The Basic Model

Consider a closed economy with no public sector. Aggregate demand at \( t \) is

\[
Y_t^{D} = C_t + I_t
\]

(1)

The economy’s aggregate output is

\[
Y_t = qL_t = \sigma u_t K_t
\]

(2)

where \( q \) is the productivity of labor, \( \sigma \) is the technical output/capital ratio\(^2\) and \( u_t \) is the degree of capacity utilization (\( \bar{Y} \)).

The workers’ propensity to consume is assumed to be 1, while the propensity to consume out of profits (\( s \)) is less than 1. By expressing both saving and investment in terms of ratios to the capital stock \( K \), we have

\[
\frac{S_t}{K_t} = s\pi \sigma u_t
\]

(3)

where \( \pi \) denotes the profit share of income.\(^3\)

Investment, which has an autonomous component \( \alpha \), depends on demand, expressed by the degree of capacity utilization, and the rate of profit \( r_t = \pi \sigma u_t \).

\[
\frac{I_t}{K_t} = \alpha + \beta u_t + \gamma r_t
\]

(4)

---

\(^2\) \( \sigma = \bar{Y}/K \) where \( \bar{Y} \) is the maximum output that can be produced with the capital stock \( K \).

\(^3\) For simplicity, we use a linear saving function.
The rate of capital accumulation and growth at $t$ is
\[ g_t = \dot{K} = \frac{I_t}{K_t} - \delta \] (5)
where $\delta$ is the constant rate of capital depreciation.

At equilibrium, it is
\[ \frac{S_t}{K_t} = \frac{I_t}{K_t} \]
which implies that the equilibrium degree of capacity utilization is
\[ u_* = \frac{\alpha}{s\pi\sigma - \beta - \gamma\pi\sigma} \] (6)
The positivity of $u_*$ is ensured by $s\pi\sigma > \beta + \gamma\pi\sigma$, which is certainly true if the so-called Keynesian stability condition is fulfilled, that is to say if it is assumed that saving is more sensitive than investment to changes in demand.4

It is immediate to see that the equilibrium degree of capacity utilization is decreasing in $s$ and that the Keynesian stability condition also implies that it is decreasing in $\pi$.5

The economy’s equilibrium rate of growth, obtained from (3), (5) and (6), can be written as
\[ g_* = \frac{\alpha}{1 - \frac{\beta + \gamma\pi\sigma}{s\pi\sigma}} - \delta \] (7)
The sign of the first derivatives of $g_*$ in $s$ and $\pi$ are the same as the signs of the first derivatives of $\frac{\beta + \gamma\pi\sigma}{s\pi\sigma}$ in $s$ and $\pi$, and they are all negative.

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4 For the stability condition to hold, it must be
\[ \frac{\partial S}{\partial u} > \frac{\partial L}{\partial u} \]
which amounts to impose that $s\pi\sigma > \beta + \gamma\pi\sigma$.

5 The derivative of $u_*$ with respect to $\pi$, $\frac{\partial u_*}{\partial \pi}$ is negative if $s > \gamma$, which is certainly true if the Keynesian stability condition is fulfilled (see footnote 4).
Therefore, the Keynesian paradox of thrift holds also in the context of a model of growth. An increase in the economy’s overall propensity to save has a negative impact on the economy’s growth rate. The economy’s process of growth is also “wage-led”: a decrease in the profit share \( \pi \) (an increase in the wage share) determines an increase of both \( u_* \) and the equilibrium rate of growth \( g_* \).

### 2.2 Introducing a Normal Degree of Capacity Utilization

The Kaleckian model presented above has been criticized by several economists for not using any notion of normal (planned) degree of capacity utilization. According to this line of criticism, in the context of equilibrium models of growth it must be assumed that, in the long period, profit-maximizing firms are able to realize their planned degree of capacity utilization.

In models of equilibrium growth the existence of excess capacity can be contemplated only if the notion of a normal degree of utilization below full capacity is introduced and regarded as independent of the level of aggregate demand. If the equilibrium degree of capacity utilization is endogenously determined, the results can be paradoxical and counter-intuitive: the solutions of the model may well imply that the degree of capacity utilization is very high and, none the less, firms are content with it at any degree, and keep on accumulating at a constant rate, rather than attempting to adjust it to a “normal” level.\(^6\)

If the Kaleckian model above is modified by introducing an (exogenous) normal degree of capacity utilization \( (u_N) \), the investment function (4) must be modified to take account of the fact that firms react to deviations of the actual degree of capacity utilization from its normal (planned) degree. Equation (4) then transforms into

\[
\frac{I_t}{K_t} = \alpha + \beta(u_t - u_N) + \gamma r_t
\]

\(^6\) Skott expresses this criticism in very clear terms: ‘Since capital stocks adjust slowly and demand expectations are not always met, actual utilization may deviate from desired rates in the short run. It would be unreasonable, however, to assume that demand expectations can be persistently and systematically falsified in steady growth. Consequently, it is hard to conceive of a steady-growth scenario in which firms are content to accumulate at a constant rate despite having significantly more (or less) excess capacity than they desire.’ (Skott, 2008, p. 6). As a way out from this difficulty, Skott proposes an alternative model, which is inspired by Harrod’s approach to growth. At the heart of Harrod’s analysis there is the hypothesis that firms aim at profit maximization and, therefore, they must have a desired (normal) degree of capacity utilization.
Firms increase (decrease) investment when the actual degree of capacity utilization is above (below) its normal level, in the attempt to restore the latter.

The equilibrium degree of capacity utilization and rate of growth can be determined in the same way as above. We now obtain

\[ u_* = \frac{\alpha - \beta u_N}{s \pi \sigma - \beta - \gamma \pi \sigma} \]  \hspace{1cm} (9)

and

\[ g_* = \frac{\alpha - \beta u_N}{1 - \frac{\beta + \gamma \pi \sigma}{s \pi \sigma}} - \delta \]  \hspace{1cm} (10)

The paradox of thrift holds and the process of growth is wage-led. The first derivatives of \( u_* \) and \( g_* \) in \( s \) and \( \pi \) are all negative provided that the normal degree of utilization \( u_N \) is not higher than a certain value.\(^7\) Thus, introducing the notion of normal degree of capacity utilization does not seem to imply substantial alterations of the standard Kaleckian model.

However, solutions (9) and (10) differ from (6) and (7) in an important sense. Whereas (6) and (7) are general solutions of the model, (9) and (10) only denote what, for brevity, can be called a ‘short-period equilibrium’, which we define as a state of the economy where the equality between saving and investment can be associated with a degree of capacity utilization different from its normal value, which firms plan to realize.\(^8\)

Once the distinction between short and long period equilibria is introduced, there arises the problem of the convergence of the short-period equilibrium degree of utilization to its long-period value. If the degree of utilization that firms plan to have, \( u_N \), is taken as exogenously given, it is evident from (9) that it is only by a fluke\(^9\) and there is no force at work that makes \( u_* \) converge to \( u_N \).

Alternatively, although acknowledging that the short-period equilibrium degree of

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\(^7\) It must be \( u_N < \frac{\alpha}{\beta} \).

\(^8\) A more correct distinction between the two equilibria should be between long-period equilibrium and medium-period (or provisional) equilibrium, in which, differently from the short period, the economy’s productive capacity is however growing. On the notion of provisional equilibrium, see Chick and Caserta (1997). The difference between solutions (6)-(7) and solutions (9)-(10) can be expressed also by referring to Harrod (1973, pp.16-20), who distinguishes between a generic growth rate ensuring the equality of saving to investment and the warranted rate of growth, which ensures an equilibrium at which the ratio of capital to output is at the level that firms desire to realize.

\(^9\) Given the parameters \( s, \pi, \alpha, \sigma, \gamma \), \( u_* = u_N \) only if it is \( u_N = \frac{\alpha}{\pi \sigma (s-\gamma)} \).
utilization converges with a long-period value, it can be argued that such a value is determined endogenously and not necessarily equal to $u_N$. Many Kaleckians have followed this line of analysis.

They argue that if the economy must converge to a certain degree of capacity utilization, it need not be that the short-period equilibrium degree converges to a degree of utilization exogenously given. It rather is the other way around: the normal degree of capacity utilization, which must be realized in the long period, converges to the short-period equilibrium degree of capacity utilization.

If, in the short period, the equilibrium degree of capacity utilization diverges from its normal value ($u_s \neq u_N$), this gives rise to a process in which $u_N$ adjusts to $u_s$. Therefore, also the long-period (normal) degree of capacity utilization is endogenously determined.

Several arguments in support of the endogeneity of the normal degree of utilization have been put forward. Lavoie (1995, 1996) and Dutt (1997) introduce the notion of expected rate of growth and argue that, in such a way, it is possible to ensure a stable long-period equilibrium solution. Nikiforos (2012b, 2012a) criticizes these positions and proposes his own explanation of the endogeneity of the normal degree of utilization, which is based on the assumption of returns to scale that increase at a diminishing rate.\textsuperscript{10}

In this paper, we approach the problems of equilibrium and the convergence of the degree of capacity utilization to its long-period value by following a different route, based on the hypothesis that the rate of capital depreciation is variable and endogenously determined. In this analytical context, we show that the short-period equilibrium degree of capacity utilization converges to an endogenously determined long-period equilibrium value, which does not necessarily coincide with $u_N$.

\textsuperscript{10} For a survey of the debate on the endogeneity of the normal degree of utilization, see Hein et al. (2011, 2012) and Nikiforos (2012b).
3. A DIFFERENT APPROACH TO GROWTH AND CAPACITY UTILIZATION

Despite their differences about normal capacity utilization, all participants in the debate on growth and unused capacity share the same hypothesis that the rate of capital depreciation is constant and independent of the intensity of the use of the capital stock. We abandon this hypothesis and assume that the rate of capital depreciation is variable and determined by the intensity with which capital is used, expressed by the degree of capacity utilization. In other words, we make the hypothesis that the rate of capital depreciation is endogenously determined.\(^{11}\)

The assumption of a constant rate of depreciation is questionable. In whatever way the degree of capacity utilization is measured, it is reasonable to assume that the velocity at which the capital stock depreciates depends on the extent and intensity with which it is used during a certain period of time. If capacity utilization is measured by the number of hours that the existing machinery is used, it seems quite obvious that the wear and tear of the capital stock increases with the number of hours during which machines are used. If, alternatively, capacity utilization is measured by the number of machines in use, it is still obvious that the rate of depreciation varies with the number of used machines. Idle machines presumably depreciate more slowly than machines in use.

Therefore, we establish a functional relation between the rate of capital depreciation \(\delta\) and capital utilization \(u\). More precisely, we assume that \(\delta\) is at its “normal” value \(\delta_N\) when also the degree of capacity utilization is at its normal value \(u_N\).\(^{12}\) When \(u\) deviates from \(u_N\), also \(\delta\)

\(^{11}\) The hypothesis of an endogenous rate of depreciation has been introduced into several mainstream models. See, for example, Nickell (1978), Schmalensee (1974), Nelson and Caputo (1997), Licandro and Puch (2000) and Angelopoulou and Kalyvitis (2012).

\(^{12}\) When capacity is not at its normal degree of utilization, production takes place at a ratio of capital to labor
deviates from $\delta_N$: it increases (decreases) when the actual degree of capacity utilization is above (below) its normal level.

A more exhaustive analysis of capital depreciation would require taking account also of the expenditures on capital maintenance, as firms can affect the rate at which their capital stock wears out by varying their expenditure on maintenance. Here, for simplicity, we do not consider maintenance, but this simplification does not have significant analytical implications because our approach is based on the notion of the cost of using capital.\(^{13}\)

An increase in the degree of capacity utilization implies a higher rate of capital depreciation and, hence, an increase in the cost of using capital. Introducing the possibility to alter the rate of depreciation through maintenance would not affect the fact that the cost of using capital increases as a consequence of its being employed more intensely.\(^{14}\)

A varying rate of depreciation can affect the economy’s growth rate in two ways. First, it can affect firms’ investment decisions if they depend on the net rate of profit. Second, if a higher rate of utilization of capital implies that it depreciates more rapidly, the positive effect on capital accumulation of a higher level of demand (a higher degree of capacity utilization) can be at least partly offset by the larger share of total investment to be devoted to the replacement of the worn-out capital.

As for the first aspect, we assume that what is relevant for firms’ investment decisions is the net rate of profit which, at time $t$, is

$$r^N_t = \pi \sigma u_t - \delta_t$$  \hspace{1cm} (11)

\(v = \frac{K}{L}\) different from the corresponding ratio $\frac{u_N}{u}$ when capacity utilization is at its normal value. Therefore, at time $t$ it is $v_t = \frac{K_t}{L_t} = \frac{\eta}{\sigma u_t} = \frac{u_N}{u_t}$.  

\(^{13}\) A similar relation between $\delta$ and $u$ can be obtained also in an analytical context with capital of different vintages, in which changes of the net rate of return determined by changes in $u$ affect the determination of the optimum truncation of capital processes; see, for example, Patriarca (201)2.

\(^{14}\) The problem of maintenance is more relevant in mainstream analyses of depreciation, as they are essentially concerned with the problem of the firms’ optimal choice between new investment and lengthening the duration of their existing capital stock. See, e.g., Schmalensee (1974), Nickell (1978) and Angelopoulou and Kalyvitis (2012).
Accordingly, the investment function (8) is changed into

\[
\frac{I_t}{K_t} = \alpha + \beta(u_t - u_N) + \gamma r_t^N
\]  

(12)

As a result, the equilibrium solutions for the degree of utilization and the growth rate respectively become

\[
u_* = \frac{\alpha - \beta u_N - \gamma \delta}{s \pi \sigma - \beta - \gamma \pi \sigma}
\]  

(13)

and

\[
g_* = \frac{\alpha - \beta u_N - \gamma \delta}{1 - \frac{\beta + \gamma \pi \sigma}{s \pi \sigma}} - \delta
\]  

(14)

It is immediate to see that, in so far as \(\delta\) is assumed to be constant, using the gross or net rate of profit in the investment function does not have any significant implications for the equilibrium solutions of the model.\(^{15}\)

Things are different when \(\delta\) is assumed to be variable: both the utilization rate and the rate of growth are negatively (positively) affected by an increase (decrease) of \(\delta\) through its effect on net investment. Below, we consider three different hypotheses about the form of the functional relation between \(\delta, u\) and \(u_N\).

3.1. A Simple Linear Function of \(\delta\)

We first consider the case of a linear functional relation between \(\delta, u\) and \(u_N\),\(^{16}\) so that it is

\[
\delta_t = \delta_N + \psi(u_t - u_N)
\]  

\((\pi \sigma > \psi > 0)\)  

(15)

From (15) and (11), we have that the net rate of profit is\(^{17}\)

\[\text{The impact of depreciation on investment } (\gamma \delta) \text{ is analogous to the impact of the constant term } \alpha \text{ in (13).}\]

\[\text{Which amounts to assume that the rate of depreciation adjusts instantaneously to the degree of capacity utilization.}\]

\[\text{To ensure that an increase in the degree of utilization does not imply a decrease in the net rate of profit via}\]
\[ r_t^N = u_t(\pi\sigma - \psi) - \delta_N + \psi u_N \]

By substituting (15) in (13), the equilibrium solution for the degree of utilization, we obtain
\[
u_* = \frac{\alpha - u_N(\beta - \gamma \psi) + \gamma \delta_N}{s\pi\sigma - (\beta - \gamma \psi) - \gamma \pi \sigma} \tag{16}\]

Provided that the numerator of (16) is positive,\(^{18}\) the Keynesian stability condition still implies that the equilibrium degree of capacity utilization \(u_*\) is decreasing both in \(s\) and \(\pi\).\(^ {19}\)

What is more interesting is the relationship between \(s\), \(\pi\) and \(g_*\). When the hypothesis (15) is introduced into the model, the equilibrium growth rate becomes\(^ {20}\)
\[
g_* = \frac{\alpha - (\beta - \psi)u_N + \gamma \delta_N}{1 - \frac{(\beta - \gamma \psi) + \gamma \pi \sigma - \psi}{s\pi \sigma - \psi}} - \delta_N + \psi u_N \tag{17}\]

The sign of the first derivatives of \(g_*\) in \(\pi\) and \(s\) is the same as that of the first derivatives of \(\frac{(\beta - \gamma \psi) + \gamma \pi \sigma - \psi}{s\pi \sigma - \psi}\):
\[
\text{sgn} \left( \frac{\partial g_*}{\partial s} \right) = \text{sgn} \left[ \frac{\psi - (\beta - \gamma \psi) - \gamma \pi \sigma}{(s\pi \sigma - \psi)^2} \right] = \text{sgn} \left[ \frac{\psi - (\beta - \gamma \psi) - \gamma \pi \sigma}{(s\pi \sigma - \psi)^2} \right] \tag{18}\]
\[
\text{sgn} \left( \frac{\partial g_*}{\partial \pi} \right) = \text{sgn} \left[ \frac{(s\pi \sigma - \psi)\gamma \sigma - (\beta - \gamma \psi) + \gamma \pi \sigma - \psi}{(s\pi \sigma - \psi)^2} \right] \tag{19}\]

The sign of the effect of a change in \(s\) and \(\pi\) on the equilibrium rate of growth depends on the following conditions:

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\(^{18}\) That is to say, provided it is \(u_N < \frac{\alpha - \delta_N}{\beta - \psi}\).

\(^{19}\) It is easy to see that \(u_*\) is decreasing in \(\pi\) if \(s\pi \sigma > (\beta + \gamma \pi \sigma) - \gamma \psi\), that is to say the investment reaction to changes in the degree of utilization \((\beta + \gamma \pi \sigma)\) minus the effect of the variation of the net rate of profit due to the change in \(\delta(\gamma \psi)\) is lower than the direct reaction of gross saving \((s\pi \sigma)\).

\(^{20}\) We have
\[
g_* = s\pi \sigma u_* - \delta_* = s\pi \sigma u_* - \delta_N - \psi(u_* - u_N) = \frac{\alpha - (\beta - \psi)u_N + \gamma \delta_N}{s\pi \sigma - (\beta - \gamma \psi) - \gamma \pi \sigma} - \delta_N + \psi u_N \]
\[
\frac{\partial g_s}{\partial s} > 0 \iff \beta - \gamma \psi + \gamma \pi \sigma < \psi
\]  
(20)

\[
\frac{\partial g_s}{\partial \pi} > 0 \iff \beta - \gamma \psi + \gamma \pi \sigma < \psi + \frac{\gamma}{s}(s \pi \sigma - \psi)
\]  
(21)

The left hand side of the two inequalities above denotes the sensitivity of investment to an increase in the degree of utilization \(\left(\frac{\partial f_k}{\partial u}\right)\), which embodies also the impact of the increase of the rate of depreciation on the net rate of profit \((\gamma \psi)\).

Condition (20) states that the rate of growth is decreasing in \(s\) if the sensitivity of investment to changes in \(u\) is larger than the sensitivity \((\psi)\) of \(\delta\) to changes in \(u\). Condition (21) is not as easy to explain. The expression in brackets \((s \pi \sigma - \psi)\) in (21) is the effect of a change in \(u\) on net saving\(^{21}\), therefore the term \(\frac{\gamma}{s}(s \pi \sigma - \psi)\) is the effect on net investment produced by the change in the net rate of profit (expressed by \(\gamma\)) and the change in net saving magnified by the multiplier effect \(1/s\).

We can now compare conditions (20) and (21) with the Keynesian stability condition (KSC), which can be written as

\[
\text{K.S.C.} \iff \beta - \gamma \psi + \gamma \pi \sigma < s \pi \sigma
\]  
(22)

The relation between \(s \pi \sigma\) and \(\psi\) is crucial: when \(\psi > s \pi \sigma\), the sensitivity of net saving to a change in utilization \((s \pi \sigma - \psi)\) is negative because the effect on \(\delta\) of the same change is strong. In this (extreme) case, the validity of the K.S.C. implies that both these conditions are fulfilled\(^{22}\). Therefore, the process of growth is profit-led and the paradox of thrift does not hold.

When, instead, the reaction of the depreciation is less strong and \(\psi < s \pi \sigma\) things are different. In this case, the K.S.C. is not sufficient to define the properties of the economy. In fact, according to the two conditions (20) and (21), we can have three different cases.

1. If \(\psi > \frac{\beta}{1-\gamma^{1-s}}\) the economy is profit-led and the paradox of thrift does not hold.

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\(^{21}\) That is to say the change in saving minus the change in capital depreciation.

\(^{22}\) If \(\psi > s \pi \sigma\), and since \(\gamma < s\), we have

\[
\psi + \frac{\gamma}{s}(s \pi \sigma - \psi) = s \pi \sigma - s \pi \sigma + \psi + \frac{\gamma}{s}(s \pi \sigma - \psi) = s \pi \sigma + (\psi - s \pi \sigma)(1 - \frac{\gamma}{s}) > s \pi \sigma
\]

From the K.S.C we obtain

\[
\beta - \gamma \psi + \gamma \pi \sigma < s \pi \sigma < \psi + \frac{\gamma}{s}(s \pi \sigma - \psi) < \psi
\]
2. If \( \frac{\beta - \gamma \pi \sigma}{1 + \gamma} < \psi < \frac{\beta}{1 - \gamma \frac{\pi \sigma}{\psi}} \), the economy is still profit-led but the rate of growth is decreasing in \( s \). The effect of a decrease in \( s \) are now positive, but a decrease in \( \pi \) has still a negative effect because of the direct impact of \( \pi \) on investment through the rate of profit.

3. If \( \psi < \frac{\beta - \gamma \pi \sigma}{1 + \gamma} \), we are back to a case of wage-led growth and the paradox of thrift holds.

In conclusion, the introduction of a varying rate of depreciation implies that the conditions for a wage-led economy are more restrictive than in standard Kaleckian models. The higher is \( \psi \), the sensitivity of the depreciation of capital to the degree of capacity utilization, the more likely is for the process of growth to be profit-led. In other words, the positive effect of demand on the rate of growth is more than compensated by the negative effect that a high degree of capacity utilization has on the depreciation of capital. Whenever the economy is wage-led, the paradox of thrift holds.

As for the problem of the convergence to the long-period equilibrium, the assumption that the rate of depreciation is increasing in \( u \) does not ensure that \( u_* \) in (16) is equal to \( u_N \). Like in the previous case in 2.2, \( u_* = u_N \) only by a fluke. To have a process of convergence of \( u_* \) towards \( u_N \), it is necessary to adopt a different functional relation between \( \delta \) and \( u \).

### 3.2 Variations of \( \delta \) in Time

We now assume that the functional relation between the rate of depreciation, \( u_* \) and \( u_N \) takes the form

\[
\dot{\delta}_t = \phi (u_* - u_N)
\]

\((\phi > 0)\)

where \( u_* \) denotes the equilibrium value of \( u \) at time \( t \).

The rationale of this hypothesis is that a divergence between \( u_* \) and \( u_N \) causes not only a variation of \( \delta \), but it also implies that \( \delta \) keeps on varying over time in so far as it is \( u_* \neq u_N \). In other words, when capital is used with an intensity different from normal, its depreciation keeps on changing until the intensity of use has gone back to normal.\(^{23}\)

\(^{23}\) If, for example, it is \( u_* > u_N \), the above-normal intensity of use of the capital stock determines its more and more rapid wear and tear. The process comes to its end when the normal intensity of use of capital is restored.
As a consequence of (23), the economy moves along ‘short period equilibria’ of the same type as that denoted by equations (13) and (14). The laws of motion of \( u_* \) and \( g_* \) respectively are

\[
\dot{u}_* = -\frac{\gamma}{s \pi \sigma - \beta - \gamma \pi \sigma} \dot{\delta}_t = -\frac{\gamma \phi}{s \pi \sigma - \beta - \gamma \pi \sigma} (u_t - u_N) \tag{24}
\]

and

\[
\dot{g}_* = -\frac{\gamma}{1 - \frac{\beta + \gamma \pi \sigma}{s \pi \sigma}} \dot{\delta}_t - \dot{\delta}_t = -\phi \left( \frac{\gamma}{1 - \frac{\beta + \gamma \pi \sigma}{s \pi \sigma}} + 1 \right) (u_t - u_N) \tag{25}
\]

There is a unique steady-growth solution at \( u_* = u_N \).24 At the normal degree of utilization, the rate of depreciation does not vary any longer; it reaches its long-period equilibrium value \( \delta_* \), which is

\[
\delta_* = \frac{1}{\gamma} \left[ \alpha - u_N \pi \sigma (s - \gamma) \right] \tag{26}
\]

obtained by setting \( u_* = u_N \) in equation (13).

The corresponding long-period equilibrium rate of growth is

\[
g_* = u_N s \pi \sigma - \delta_* \tag{27}
\]

By considering the first derivatives of (26) and (27), it is easy to see that \( \frac{\partial g_*}{\partial s} > 0 \) and \( \frac{\partial g_*}{\partial \pi} > 0 \).

The economy’s long-period equilibrium growth rate is increasing in \( s \) and the process of growth is profit-led. A decrease in \( \pi \) raises the short-period equilibrium degree of utilization and the rate of growth, but the increase in \( u \) determines also an increase in the depreciation rate.

The result of these effects is that the economy moves to its normal degree of capacity utilization associated with a larger rate of capital depreciation. The larger \( \delta \) has a negative impact on the equilibrium growth rate to which the economy converges with a speed proportional to \( \dot{\phi} \). In conclusion, the process of growth is wage-led in the short period (when also the paradox of thrift holds), but profit-led in the long period.

The process depicted by (23) is cumulative; in fact when the degree of capacity utilization has converged to its normal value, the equilibrium value of capital depreciation takes on a value that is different from the value \( \delta_N \), initially associated with \( u_N \). There is no process of convergence of \( \delta_* \) to \( \delta_N \).

This latter result clearly is a drawback of hypothesis (23). It would be reasonable to

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24 Which is globally stable because it is \( \frac{\partial u_*}{\partial (u_t - u_N)} < 0 \).
expect that as the intensity of use of capital goes back to its normal level, its rate of depreciation should behave similarly. To deal with the problem in a more satisfactorily way, below we introduce a modified function of \( \dot{\delta}_t \).

### 3.3 A Generalization

The case considered here can be regarded as a generalization of the previous two. We modify (23) by introducing a factor that 'forces' the actual rate of depreciation not to diverge excessively from its normal value \( \delta_N \):

\[
\dot{\delta}_t = \phi(u_s - u_N) - \nu(\delta_t - \delta_N)
\]

where \( \nu \) is positive and small relatively to \( \dot{\phi} \). A divergence between \( u_s \) and \( u_N \) still triggers a cumulative process that, in the long period, yields a rate of capital depreciation \( \delta_t \) different from \( \delta_N \) but the divergence between them is constrained.

The economy still moves along short period wage-led equilibria like those of equations (13) and (14), even though the laws of motion of \( u_t \) and \( h_t \) are more complex than those in (24) and (25). However, it is now simpler to obtain the solutions for the long-period equilibrium to which the economy converges.

From (28), we have that, at the long-period equilibrium, it is

\[
\delta_t = \delta_N + \phi/\nu(u_s - u_N)
\] (29)

Equation (29) is similar to equation (15). This means that, once we substitute \( \phi/\nu \) for \( \psi \), the

25 From (28), it is evident that the larger is the difference between \( \delta \) and \( \delta_N \) at time \( t \), the stronger is the negative impact on its variation.

26 The laws of motion of \( u_s \) and \( h_s \) are:

\[
\begin{align*}
\dot{u}_{st} &= -\gamma \frac{\phi}{s \pi \sigma - \beta - \gamma \pi \sigma} [u_t - u_N - \frac{\nu}{\phi} (\delta_t - \delta_N)] \\
\dot{g}_{st} &= -\frac{\gamma}{1 - \beta + \gamma \pi \sigma} [\delta_t^\phi] - \dot{\delta}_t = -\phi(\gamma \frac{\delta_t^\phi}{s \pi \sigma - \beta - \gamma \pi \sigma} + 1) [u_t - u_N - \frac{\nu}{\phi} (\delta_t - \delta_N)]
\end{align*}
\]

27 By substituting (13) in (28) we have \( \frac{\partial^2 \delta_t}{\delta_t} < 0 \); thus the steady growth solution is globally stable since \( \frac{\partial^2 \delta_t}{\delta_t} < 0 \).
properties of the long-period equilibrium are similar to those of the first case (3.1). The larger is $\frac{\phi}{\nu}$, the more likely it is to have a profit-led process of growth. The long-period equilibrium degree of capacity utilization is endogenously determined and different from $u_N$.

Thus, when the hypothesis (28) is adopted, we have that in the long period both the degree of capacity utilization and the rate of capital depreciation are endogenously determined. In particular, as for the degree of capacity utilization, we obtain a result that is similar to that obtained in other Kaleckian models: in the long period the degree of capacity utilization generally converges to a different value from $u_N$. Consequently, the value to which $u_*$ tends in the long period should be interpreted as a new normal degree of utilization. However, our result is essentially explained by technological factors rather than by changes in firms’ expectations about the rate of growth.

3.4 Summing-Up
In conclusion, we have shown that if we make the hypothesis that a gap between actual and normal degrees of utilization is not ineffective (i.e. the term ‘normal’ is not meaningless) and we express this through variations of the rate of depreciation, the short and long-period dynamics of a simple Kaleckian model can have different properties in terms of sensitivity to changes in the propensity to save or income distribution.

In the first case considered (when the steady-growth equilibrium and the corresponding endogenous $\delta$ realize immediately) and in the third case (when $\delta$ varies in time increasing in $u$ and decreasing in its gap from $\delta_N$) the properties of the economy depend on the sensitivity of depreciation to the degree of capacity utilization: the higher is such sensitivity, the higher the probability to be in a case of profit-led growth and the higher the probability that the paradox of thrift does not hold.

In the third case, which is a generalization of the other two cases, the wage-led property of the process of growth certainly holds in the short period, but it can be that, in time, the initial positive effect of a decrease in $\pi$ is offset by the variations of $\delta$ associated with the changes in $u$. In the second case, which is a particular extreme case of the third ($\nu = 0$), there always occurs a shift from wage to profit-led growth when moving from the short to the long period.

The matrix below summarizes the signs of the derivatives of $u_*$ and $\phi_*$ in all the cases considered.
Case 1: \( \dot{\delta}_t = \delta_N + \psi(u_* - u_N) \); Case 2: \( \dot{\delta}_t = \phi(u_* - u_N) \); Case 3: \( \dot{\delta}_t = \phi(u_* - u_N) - \nu(\delta_* - \delta_N) \).

The convergence of \( u_* \) to \( u_N \) (but not of \( \delta_* \) to \( \delta_N \)) in the long period is ensured only in the second case (3.2), which is based on the most extreme hypothesis about the behavior of \( \delta \). In this case the long-period process of growth necessarily is profit-led.

### 4. EXCESS CAPACITY, EQUILIBRIUM, COSTS OF PRODUCTION: THE HISTORICAL BACKGROUND

#### 4.1. Long-Period Equilibria with Excess Capacity

The idea that the economy may be characterized by the existence of a certain degree of unused capacity (excess capacity) also when it is in its long-period equilibrium is connected to the notion of imperfect competition, which was introduced in the 1920s and 1930s.28

Kalecki did not participate directly in that debate, but he accepted the idea that excess capacity can persist in the long period. Earlier in the 1930s, Kalecki argued that the existence of excess capacity was essentially due to the presence of economies of scale and indivisibilities.29 Later on, however, he seemed to be content with ascribing the persistence of excess capacity over time to variations of aggregate demand over the business cycle and to variations in income distribution which, in turn, affect demand (see Kalecki, 1965, pp. 61, 129-31 and 156).

It was Steindl who introduced a notion of equilibrium in non-perfectly competitive conditions in which there is a certain degree of unused capacity that does not directly depend on the current level of demand but on other factors.30

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28 On the debate on excess capacity and imperfect competition in the 1930s, see Sardoni (1999).
29 ‘It may be asked how it is possible for surplus capacity to exist in the long-run equilibrium without inducing firms to curtail their plant. The answer is that large-scale economies prevent the firms from reducing their plant below a certain limit, a state of affairs described by those writers who have shown that imperfect competition must cause equipment in the long run to be used below the “optimum point” ’ (Kalecki, 1990, p. 245).
30 Steindl (1976, pp. 9-14) provides also an interesting discussion of the measurement of the degree of capacity utilization. He concludes that the most satisfactory measurement is based on the number of machine hours, i.e. the ratio of the actual number of hours that machines are used to the maximum number of
For Steindl, the firms’ choice to normally have a certain amount of unused capacity is strictly related to their strategies to face the menace of new entrants into the industry. Entrepreneurs in a non-perfectly competitive environment want to have excess capacity to be able to neutralize the threat of potential new entrants into the industry when there is an increase in demand.31

For Steindl, differently from Kalecki, the existence of excess capacity in the long period cannot be explained simply by the level of demand and its variations over the cycle. In other words, excess capacity in the long period cannot be explained in the same way as in the short period. Keynes, in the 1940s, had made this point by criticizing Kalecki and Joan Robinson.

In 1941, Kalecki submitted an article for publication in *The Economic Journal*, where he assumed that firms always work below capacity, and Keynes observed: ‘Is it not rather odd when dealing with “long run problems” to start with the assumption that all firms are always working below capacity?’ (1983, p. 829).

For Joan Robinson, who was defending Kalecki’s view, it was not at all odd, because that was ‘part of the usual bag of tricks of Imperfect Competition Theory’ (cited in Keynes, 1983, p. 830). Keynes replied: ‘If he [Kalecki] is extending the General Theory beyond the short period but not to the long period in the old sense, he really must tell us what the sense is. (…) To tell me that as for under-capacity working that is part of the usual pack of tricks of imperfect competition theory does not carry me any further. For publication in the *Journal* an article must pass beyond the stage of esoteric abracadabra. (Keynes, 1983, pp. 830-1).

To try to convince Keynes that the issue of excess capacity in the long run was not ‘esoteric abracadabra’, Joan Robinson should have argued that long-period excess capacity cannot, and should not, be explained in the same way as it can be explained in the analytical context of the short period.

Our model accepts the idea that there is a degree of capacity utilization that firms plan to have for reasons related to the non-perfectly competitive nature of the markets in which they operate. However, we showed that introducing a normal degree of utilization does not ensure

31 ‘The producer wants to be in on a boom first, and not to leave the sales to new competitors who will press on his market when the good time is over.’ (Steindl, 1976, p. 9). Moreover, producers cannot adjust their capacity step by step with the demand for their products because of indivisibilities of plants and equipment and because of their durability. More recently, others have dealt with normal excess capacity in non-perfectly competitive industries in a similar way to Steindl. See, for example, Sylos Labini (1967, 1969); also Spence (1977) relates the existence of excess capacity to the strategy of incumbent firms to defend themselves from the threat of potential new entrants into the industry.
that firms are actually able to realize it. The process of growth can have such characteristics that the economy converges to a long-period equilibrium capacity utilization different from that which firms plan to have. This outcome, in turn, can induce firms to revise and adjust their notion of normal degree of capacity utilization.\textsuperscript{32}

4.2 Degree of capacity utilization and conditions of production

Kalecki carried out his analysis of capitalist dynamics under the hypothesis that firms’ short-period returns are constant up to full capacity, i.e. that individual prime cost curves have a reversed-L shape (Kalecki, 1938, 1991). Kalecki’s hypothesis of constant short-period returns was very similar to the conclusion at which Kahn had arrived in the 1920s (Kahn, 1989). By dealing only with the short period, Kahn questioned the traditional hypothesis of U-shaped cost curves and argued that in many important industries, an adequate description of the behavior of firms’ prime costs is given by a reverse L-shaped cost curve (Kahn, 1989, pp. 49-59).\textsuperscript{33}

If it is assumed that prime costs are constant up to capacity and then they rise vertically, the obvious implication is that the average total cost varies inversely with the degree of capacity utilization and reaches its minimum at full capacity. The critics of the Marshallian hypothesis of decreasing returns did not contemplate the possibility that different degrees of utilization of the firm’s productive capacity may affect costs in other ways. More precisely, they paid no attention to the possibility that different degrees of capacity utilization may affect the cost of using fixed capital. This issue, instead, was considered and analyzed by Keynes.

Keynes introduced the notion of user cost of output, to which he attached great importance (Keynes, 1936, pp. 66-73). The user cost $U$ of output $A$ is a measure of the total sacrifice involved in its production and it is defined as the difference between the maximum net value that the firm’s capital would have if production were not carried out ($G' - B'$, with $B'$

\textsuperscript{32} Also Steindl (1976, pp. 11-13), on the other hand, was aware of the difficulty to distinguish between desired (planned) and undesired excess capacity and, hence, the difficulty to determine a particular normal degree of utilization. However, for Steindl, this problem is less important than the general idea that investment is a function of the degree of capacity utilization.

\textsuperscript{33} From this it follows that, in general, the average prime cost is independent of the level of output and, therefore, the marginal prime cost is equal to the (constant) average prime cost. If the marginal cost curve is perfectly elastic to the point of full capacity, insofar as the price is above the prime cost, a firm in perfect competition would produce to capacity. But in reality firms also work by leaving part of their capacity unused. A satisfactory explanation of this can be found by abandoning the hypothesis of perfect competition, i.e. the hypothesis that firms face a perfectly elastic demand curve (Kahn, 1989, pp. 59-60). If the demand curve is downward sloping, the firm can be in equilibrium also by producing less than its maximum output. Sylos Labini has accepted and developed Kahn’s and Kalecki’s criticism of the hypothesis of decreasing short-period returns (see, for example, Sylos Labini, 1967, 1969, 1988). Sylos Labini also observes that the notion of unused capacity makes really sense only under the hypothesis of constant prime costs. (Sylos Labini, 1988).
denoting the expenditure on the maintenance of the firm’s capital) and the actual value of the firm’s capital at the end of the period during which $A$ is produced ($G$) plus the amount spent to buy finished outputs from other firms ($A_1$):

$$U = (G' - B') - G + A_1$$

Keynes’s notion of user cost is subject to some important criticisms (Sardoni, 2011, pp. 102-3), but this does not imply that his idea that costs of production can be affected by variations in the use of capital is to be rejected altogether. Our model in section 3 takes up Keynes’s idea that the value of the capital stock at the end of the production period depends on the level of output realized by firms. More precisely, we convert Keynes’s idea into the hypothesis that the capital’s wear and tear is a function of the degree of its utilization, i.e. of the intensity with which it is used.

5. CONCLUSION

The main feature of the model presented in this paper is the hypothesis of a varying rate of capital depreciation, which is its most innovative aspect with respect to other Kaleckian models of growth.

The hypothesis of a varying $\delta$ has some significant implications concerning the nature of the equilibrium process of growth. In the short period, the process of growth is wage-led under more restrictive conditions than the Keynesian stability condition. In the long period, the economy can be characterized by a wage-led process of growth only if the rate of capital depreciation cannot vary freely but it is somewhat ‘anchored’ to its normal value $\delta_N$ (case 3.3 above).

The rationale of these results is simple. When there is a distributional change more favorable to wages or the capitalists’ propensity to save is lower, the economy in the short period experiences a higher level of aggregate demand. This determines an increase in the degree of capacity utilization and, hence, an increase in the rate of depreciation. The increase in the rate of depreciation has, in turn, a negative impact on accumulation and, hence, on the equilibrium rate of growth.

As for the problem of normal capacity utilization, our results are not dissimilar from those obtained by other Kaleckian models. Introducing a normal degree of utilization per se does not imply substantial changes of the main results obtained with the basic Kaleckian model. The short-period solutions of the model essentially retain the same properties as those of the basic
Kaleckian model. However, introducing a normal degree of utilization has another important consequence.

Once a normal degree of utilization is introduced, the typical Kaleckian equilibrium solutions for the degree of capacity utilization have to be regarded as short-period solutions and there arises the problem of the convergence to the economy’s long-period equilibrium. The problem has been acknowledged also by several Kaleckian economists, who put forward several possible solutions, all based on the idea that the long-period solution for capacity utilization is endogenously determined.

We approach this problem in our model with endogenous depreciation. In our analytical context, the long-period equilibrium degree of capacity utilization converges to its, exogenously given, normal value only when the most extreme hypothesis on the rate of depreciation is made (case 3.2 above). In such a case, moreover, the process of growth is profit-led in the long period. In all cases considered, the endogenously determined rate of capital depreciation never converges in the long period to the value associated with the normal degree of capacity utilization.

Thus, as for the problem of equilibrium capacity utilization in the long period, we obtain results that are not too far from those characterizing other Kaleckian models. The long-period equilibrium degree of capacity utilization is endogenously determined. We arrive at these results by following a different path, based on the reasonable hypothesis that the rate at which capital wears depends on the intensity with which it is used in production.
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