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The Role of Lenders' Trust in Determining Borrowing Conditions for Sovereign Debt: An Analysis of One-Period Government Bonds with Default Risk

Yanling Guo

Abstract
In this paper, the author considers the sovereign debt in the form of one-period government bonds with default risk, which can be purchased by and traded among domestic and foreign investors. She shows that the weight assigned to the lenders' interest by the borrowing government at the time of debt repayment, which captures the lenders' trust in the government's propensity to repay the debt and is denoted as α, also determines the default risk: a higher α means a lower default risk ceteris paribus which leads to a lower risk premium, and vice versa. Since this relationship only holds in the "good equilibrium", the author further shows that the "good equilibrium" is the only stable equilibrium under some quite general assumptions while the "bad equilibrium" is an unstable one – a possible reason why in practice rather a negative correlation between α and the default risk as well as the corresponding risk premium is observed.

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Keywords Public debt; sovereign debt; sovereign default; domestic debt; external debt; fiscal policy; government bond; government borrowing

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1 Introduction

Recently, the term sovereign default has received much attention both in the literature and in the public discussion. The sovereign default is defined as the default on the sovereign debt. In this paper, I focus on the analysis of the so-called one-period government bond, which is the basic form of the sovereign debt and hence has received the most attention in the literature. The one-period government bond is defined as a non-collateralized and non-committable public debt, which promises state-non-contingent repayment after one period. Here I only consider the government bond purchased by and traded among the private investors, although the result can, after some adaptation, also be applied to sovereign debt owned by public lenders.

Earlier literature modeling this particular form of sovereign debt considers either domestic or external debt, classified by whether the debt is owned by residents or by foreigners, respectively. Recent literature also models government bonds, which are a mixture of the two, since the globalization on the financial market makes it more likely that both residents and foreigners may purchase and trade the government bonds with each other – a modeling strategy that is also adopted in this paper. This modeling strategy necessarily introduces a new parameter, here called α, which represents the weight assigned to the average lender by the borrowing government. In this paper, I focus on the effect of α, which has not yet been thoroughly analyed in the literature. This choice of focus has both its theoretical and practical reasons.

To date, the literature mostly concentrates on the analysis of the effect of the outstanding amount of debt on the default risk. Most empirical research, e. g. Reinhart and Rogoff (2011), shows that an increasing amount of outstanding debt increases the default risk, though the relationship is not linear. This empirical finding has its theoretical foundation, led by Eaton and Gersovitz (1981) who proved this positive correlation analytically. Another strand of literature, led by Calvo (1988), adopted a different modeling strategy and also obtained this positive correlation, but only in the so called "good equlibrium". Calvo (1988) demonstrated that there is also a so called "bad equilibrium" in which all effects are reversed, i. e. with rising amount of outstanding debt, the default risk can even fall. Though this theoretical concept of the "bad equilibrium" is not backed by empirical findings, it can weaken the political will to reduce debt since governments in trouble can argue that they are simply struck by the "bad equilibrium". In this paper, I show that while the "bad equilibrium" indeed exists, it is an unstable equilibrium and hence occurs with the probability of almost surely zero.

My finding that only the "good equilibrium" is a stable equilibrium while the "bad equilibrium" is unstable is supportive of the appeal to more fiscal consolitation or more austerity as a remedy for a debt crisis. However, in practice, governments in trouble often only recognize that they have accumulated too much debt when it is already too late, i. e. although fiscal consolidation can reduce the default risk, it is often difficult to conduct due to a lack of support from the citizens who have to suffer most when the government cuts expenditures or raises taxes. One alternative solution suggested by the literature is to raise the expected output since a rising output reduces the debt as a quotient of GDP and raises the tax base, hence lowers the interest cost in the "good equilibrium". In appendix B.1 I also show with an example that more debt can even benefit the economy when it can generate a higher yield than the agreed interest cost. However, governments which have difficulties to repay their debt often face a weak economy at the same time, or put

differently, if they were able to generate a high yield from borrowed money, be it in the form of direct investment or output-increasing fiscal policy, they would have no problem to access the financial market which always seeks good investment opportunities. Of course, after all, the government has to raise the productivity of the economy to render the debt level sustainable, hence structural reforms are inevitable if the govenment cannot reduce its debt. But sometimes structural reforms are just as difficult to realize for the government as cutting debt. Hence, here I show an alternative solution by considering the effect of α. After proving that the "good equilibrium" is the only stable equilibrium, I show that raising α can also reduce the default risk and hence mitigate the sovereign debt problem. However, raising α could also be difficult, hence what I show here is just an additional possible solution, and the government has to choose which solution it prefers: to reduce debt, to raise the output or to raise α. Of course, it can also combine these three solutions to find a policy mix which seems optimal.

The detailed discussion about α follows in section 4, here I only want to give a brief presentation. When looking at the model in section 2 you can see that α is the weight assigned to the average lender by the government, which represents how well the investors' interests are regarded by the borrowing government. Indeed, a government with a debt problem is by definition a government which is in need of capital from the lenders, and not seldom the lenders are reluctant to lend because they are concerned whether their claims are properly protected. Since the government itself is an institution with changing personnel, the personal promise by a particular governor often does not suffice to gain trust from the lenders, hence the promise of better lender protection needs to be institutionalized in order to be credible. One possiblity is a reliable legal system protecting the lenders' interests. Indeed, the data in Reinhart and Rogoff (2011) suggests that governments which can borrow under domestic law, which implies that even foreign investors have trust in the laws executed by the borrowing government, face lower default risk given the same debt level. Another possibility is through the voting right of the lenders. Since in a democracy the government is elected, the literature believes that the debt is less risky if it is mainly held by domestic investors who are at the same time voters. However, in the case of government bonds, which are freely tradable so that the domestic investors can purchase the government debt from the foreigners when default risk emerges, it is rather the domestic wealth that matters. In section 4 I also briefly discuss the possiblity of a union membership which may reduce the default risk. However, how to credibly raise α is not the subject of this paper. Here I only prove analytically the positive effect of a higher α. Neither do I assert that raising α is the only way to solve the sovereign default problem. As mentioned, it is only an alternative solution which can be combined with other solutions like more fiscal consolidation or structural reforms.

Actually, it has already been briefly discussed in Calvo (1988)[1] that α can co-determine the default risk. Calvo's finding regarding default in the form of debt repudiation is in line with my paper, namely that in the "good equilibrium" a lower α means more *ex post* default risk and at least the same risk premium. However, since his focus is on the domestic debt, he only considers the case when α is close to 1.[2] Besides, in his "bad equilibrium" the effects on interest cost of all variables including α have the opposite sign as in the "good equilibrium". Gennaioli et al. (2010)

[1] In Calvo (1988), the α is coded as γ.

[2] In appendix B.1 I will use the original Calvo model as a special case to illustrate the effect of α.

have also considered an economy borrowing from both residents and foreigners, but they did not analyze the effect of α.

The following text is organized as follows: section 2 sets up the model in a fairly general manner, incorporating both the Calvo type model and the Eaton and Gersovitz type model regarding sovereign default. Section 3 proves the positive impact of α on the repay propensity of the borrowing government and shows that the good equilibrium is the only stable equilibrium and hence a higher repay propensity almost surely leads to better borrowing conditions for the government. Section 4 briefly discusses the application of my analysis in practice, and section 5 concludes.

2 The Model Setup

2.1 The preambles

The model studies a small open economy in a *de facto* monetary union in the sense that this small open economy has only fiscal authority, while the monetary power is concentrated to a union-level institution. Further I assume that this small open economy is so small that its performance has no influence on the monetary policy of as well as the economic development in the union. In particular the union-wide reference interest rate is taken as given for this small open economy. Here I use this "monetary union" setting as an analytical device to abstract from monetary policy accommodation possibilities like "inflating away" the debt and thereby to focus on the effect of α. Hence the "monetary union" term used here should not be confused with a real world monetary union such as the EMU (European Monetary Union) which I will refer to in this paper as an "explicitly declared monetary union". Indeed, the small open economy under study does not necessarily need be a country at all but can be any administrative region with its own fiscal authority like a state in the USA or a province in Canada[3], and I only sometimes refer to this small open economy as a "country" to make the text shorter. Further, a country which has adopted the currency from another country and in this way given up its own monetary authority can also be viewed as a member in a *de facto* monetary union. In short: the monetary union in this paper is defined as a collection of sovereign bodies sharing the same currency and the same monetary policy authority which is not under influence of this small open economy while each union member maintains its own fiscal policy authority.

The government is assumed to be benevolent and attempts to maximize the welfare of the residents. The residents are modeled as the representative agent, as usual. To achieve its goal, the government can choose in each period t its expenditure g_t and income tax τ_t. The difference between g_t and τ_t is lent to or borrowed from the financial market. The debt has to be repaid at the end of each period, and immediately after that the new debt contract is signed at the beginning of the next period. Following the standard literature, I assume that the government cannot commit and decides on its policy instruments anew for every period. The available policy instruments include beside g_t and τ_t also θ_t, the default rate as a fraction of outstanding amount of debt. The

[3] Literature which regards USA or Canada as a monetary union includes for instance Rockoff (2000) and Landon and Smith (2007)

government aimes to maximize the social welfare in term of current and (discounted) future utility of the representative agent. Its policy choice may be constrained in different ways, e. g. g_t may be held constant to reflect a pre-determined fiscal stance, or τ_t is bounded from above by y_t if y_t constitutes the sole tax base.

Since the government cannot commit, there is a non-negative probability of default on the outstanding debt. If the investors from the financial market anticipate a positive default probability for the debt being negotiated, they will charge a higher interest rate to compensate for the possible loss due to default or refuse to lend if the expected debt repayment ratio is strictly below the market return. Denote the gross reference interest rate or the market return as R_t and the contracted gross interest rate for government debt as z_t, then in an arbitrage-free world there should be $z_t E_t(1-\theta_{t+1}) = R_t$ with $\theta_t \in [0,1]$.

If the government chooses to default then the economy will incur some default cost p. This default cost may or may not be of economical nature. As non-economic cost it may stand for the effort to keep a good name, and as economic cost p may stand for negotiation cost, retaliatory actions like trade embargoes, or reduction in trade credit or bank credit,[4] etc. Here I take all kinds of default cost as possible and model p as the default cost, which will be incurred in the default period and possibly also in the following periods, and a positive p will reduce the social welfare in the corresponding period. Although exclusion from the financial market is often regarded in the literature as default cost, I do not model it as a part of p but as a constraint in the government's policy mix choice, i. e. the amount which can be borrowed from the financial market will be constrained to 0 immediately following a default decision. Beside default cost, there may also be cost arising from taxation, referred to in the literature as deadweight loss and denoted here as $x(\tau_t)$.

The parameter α lies in the interval $[0,1]$ and represents to what extent the lenders' interest is considered by the borrowing government. Following the conventional wording, I sometimes use phrases like "a portion of $1-\alpha$ of the debt is held by foreign investors", although in this context the term "foreign investors" does not necessarily mean investors from a foreign country but rather refers to the lenders whose wealth does not, at least not completely, enter the borrowing government's objective function.

2.2 The objective function

Following is the objective function of the borrowing government:

$$\mathbb{V}(y_t, z_{t-1}b_{t-1}, \vec{p}_t; \alpha) = \sup_{b_t \in B_t, \theta_t \in \Theta_t, g_t \in G_t, \tau_t \in \Upsilon_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}(y_{t+1}, z_t b_t, \vec{p}_{t+1}(\theta_t); \alpha)\} \quad \text{s. t.} \tag{1}$$

$$c_t = y_t - x(\tau_t) - \tau_t + \alpha(1-\theta_t)z_{t-1}b_{t-1} - \alpha b_t \tag{2}$$

$$(1-\theta_t)z_{t-1}b_{t-1} + g_t = \tau_t + b_t - p(\theta_t) - \vec{p}_t \tag{3}$$

$$z_t = \inf[z:\ z_t E_t(1-\theta_{t+1}) = R_t] \tag{4}$$

[4] Gennaioli et al. (2010) argue that sovereign default will lead to deterioration in domestic banks' balance sheet and hence reduce credit supply in the domestic market.

Equation (1) describes the representative agent's value function which the government attempts to maximize using the political instruments new debt b_t, default rate on old debt θ_t, government expenditure g_t and tax revenue τ_t. The political instruments can only be chosen within the eligible sets B_t, Θ_t, G_t and Υ_t, respectively. According to the specific model setting, B_t, Θ_t, G_t and Υ_t can be differently defined. For instance, many models assume government expenditure to be exogenous and hence restrict G_t to be a singleton so that $G_t = \{\bar{g}\}$, while other papers allow g_t to be any non-negative value and hence $G_t = \mathbb{R}_+$. Usually $\Theta_t \equiv [0,1]$, but in models interpreting inflation as an implicit default, Θ_t can also include negative values as in Calvo (1988), while in models in which θ_t needs to be flexible since other political instruments are strongly constrained, Θ_t can also include values above one.[5] Here we have $\Theta_t \equiv [0,1]$ since there is no need to consider inflation or deflation as implicit default for a government without own monetary authority. To make a decision on θ_t possible, I only consider Υ_t which is constrained loosely enough so that an optimal policy mix that satisfies the government budget constraint and the constraints put on other eligible sets of political instruments always satisfies the constraint on Υ_t. Indeed, many papers do not put any constraint on Υ_t, and some set $\Upsilon_t \equiv (-\infty, y_t - x(\tau_t)]$ when interpreting τ_t as income tax.

The eligible sets of the political instruments can also be a function of another political instrument as almost all papers assume that new debt taking is restricted to zero, i. e. $B_t(\theta_t) = \{0\} \; \forall \; \theta_t > 0$, if there is a default in the current period and the default is in the form of a contract violation which can entail financial market exclusion.[6] For models in which the exclusion from the financial market may also take place in the following periods after the initial default, the previous constraint will become $B_i(\theta_s) = \{0\} \; \forall \; \theta_s > 0$ and $i \in [s,t]$ with s and t denoting the first and last period of the default era, and t may be ∞ which means a permanent exclusion from the financial market as in Eaton and Gersovitz (1981). These constraints may also appear in expectational form i. e. one can assume that future exclusion from the financial market happens with some positive probability as in Arellano (2008) in which $\Pr(B_i(\theta_s) = \{0\} \mid B_{i-1}(\theta_s) = \{0\}) = \text{const} > 0 \; \forall \; \theta_s > 0$ and $i \in [s+1, \infty)$.

The periodic utility of the representative agent is derived from the absorption of private consumption c_t and public goods provisioning g_t. Equation (2) describes the financing source of private consumption: the average citizen consumes his after tax income $y_t - \tau_t - x(\tau_t)$, plus government bond repayment which is possibly partially repudiated, $\alpha(1-\theta_t)z_{t-1}b_{t-1}$, minus purchase of new government bond αb_t. Here I do not consider the private external borrowing or lending since it is an activity which cannot be influenced by the government using the political instruments available here, hence the aggregate saving or dissaving appears in the form of government bond purchase and the government can adjust b_t to smooth the economy-wide consumption as long as b_t is not restricted to 0 due to a default decision. Note that here the before-tax income is $y_t - x(\tau_t)$ and not just y_t since the distortionary effect from taxation may

[5] An example is Juessen et al. (2011) which models non-strategic government default in which both τ_t and g_t are predetermined and hence θ_t may sometimes be above one to meet the budget constraint.

[6] Of course this does not necessarily hold for models in which default takes place implicitly in the form of inflation or surprising levy of capital income, here of bond repayment receipt. In this case it is plausible that the financial market is ready to continue lending to the government even in the case of a *de facto* default.

reduce the output. Therefore, y_t should rather be interpreted as endowment or potential output, i. e. output which could be achieved if there was no distortion arising from the income tax.

Equation (3) is the budget constraint of the government which says that the repayment of old debt $(1-\theta_t)z_{t-1}b_{t-1}$ and government expenditure g_t is financed by tax τ_t and new debt taking b_t net of default cost which is the punishment imposed by the investors on current default, $p(\theta_t)$, or on past default, $\vec{p}_t$. Usually $p(\theta_t)$ and $\vec{p}_t$ do not co-exist, i. e. when there is still some cost due to past default then we say that this government is further in a default period and cannot make new debt which it could default on, consequently, $p(\theta_t)=0$. And only after the end of the default period, which implies that there is no *burden of the past*, $\vec{p}_t$, the government can again make new debt and would incur default cost in the next period if it would again repudiate the debt contract. This consideration about non-co-existence of $\vec{p}_t$ and $p(\theta_t)$ is reasonable, but loosening this assumption does not matter much analytically since $\vec{p}_t$ is a kind of *sunk costs* and will not affect the current trade-off between different political choices. Nonetheless, here I stick to the non-co-existence assumption so that the government can only optimize on b_t or θ_t if $\vec{p}_t=0$, i. e. only after the last default is resolved through settlements with the investors, the government can take on new debt and possibly again default on it.

The gross contracted interest rate z_t is non-state-contingent while the *ex post* interest rate $z_t(1-\theta_{t+1})$ is state-contingent since the choice of θ_{t+1} will depend on the circumstances in the next period. After choosing the optimal values for b_t, θ_t, g_t and τ_t, the welfare, expressed as the value function of the representative agent, will depend on the existing debt burden $z_{t-1}b_{t-1}$ and possibly on the burden of the past $\vec{p}_t$, as well as on the current endowment y_t which does not depend on the past debt taking and repayment decisions of the government by assumption. Further the value function also depends on α, though α is rather a parameter and not a state variable.

Equation (4) differs from the usual participation constraint equation in the literature in the point that it assumes that among all contracted interest rates which give the lenders the market return R_t in expectational form, the smallest possible interest rate will always be contracted. So here I assume that the government is initiating a debt contract $\{z_t, b_t\}$ and will always offer the lowest possible z_t, which makes the financial market ready to lend the amount b_t which is desired by the government to maximize the social welfare. The government will always choose the smallest possible interest rate because a lower z_t means less debt burden and less default cost due to lower default probability in the next period, and is thus preferable to the government compared to a larger z_t which sustains the same amount of b_t.[7] In the case that the debt contract is not initiated by the government but by the financial market, the lowest possible z_t will also be proposed by the financial market if the investors are under competition as suggested in Eaton and Gersovitz (1981). According to the authors, the competition among the investors will lead them to suggest the most favorable borrowing condition – here a smallest possible z_t – to the government in order to get the debt contract, given that they can get the market return in expectational form under this suggested borrowing condition. Hence, under different institutional assumptions, the lowest possible interest rate can be rationalized. Therefore, I only consider the lowest possible contracted interest rate, which by definition rules out the multiple equilibria problem which arises when the set $\{z:\ z_t E_t[1-\theta_{t+1}]=R_t\}$ contains more than one element. In subsection 3.3 I will

[7] Arellano (2008) has also pointed out that a government always prefers higher bond price to lower bond price.

show that this smallest possible contracted interest rate conjecture also holds in much more general cases even in absence of the institutional assumptions just made.

To summarize: the value of the value function depends on the state variables endowment y_t, outstanding debt amount $z_{t-1}b_{t-1}$, possible inherited default cost, $\vec{p}_t$, as well as the parameter α. To improve the legibility, in the following I will leave out y_t and α as arguments for $\mathbb{V}$ as long as they are not necessary for the understanding.

2.3 Deriving the FOCs

By inserting equation (3) into (2), the objective function of the government is simplified to:

$$\mathbb{V}(z_{t-1}b_{t-1},\vec{p}_t) = \sup_{b_t\in B_t,\theta_t\in[0,1],g_t\in G_t}\{\mathbb{U}(c_t,g_t)+\beta E_t\mathbb{V}(z_tb_t,\vec{p}_{t+1}(\theta_t))\} \quad \text{s. t.} \tag{5}$$

$$c_t = y_t - x(\tau_t) - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} + (1-\alpha)b_t - g_t - p(\theta_t) - \vec{p}_t \tag{6}$$

$$\tau_t = (1-\theta_t)z_{t-1}b_{t-1} + g_t - b_t + p(\theta_t) + \vec{p}_t \tag{7}$$

$$z_t = \inf[z:\ z_tE_t[1-\theta_{t+1}] = R_t]$$

In equation (5) the government is not trying to adjust τ_t since τ_t is determined by the budget constraint (3), now rewritten in (7). Equivalently, here one could choose another political instrument instead of τ_t which does not serve as an optimizer, one candidate could be θ_t as is done in example B.1. Equations (6) and (7) are not really constraints in the sense that they do not put further constraints on the optimizers but merely describe how c_t and τ_t are determined. Equivalently, one could plug them into equation (5) to eliminate the corresponding variables, and I only write them down here separately for better legibility. Equation (6) states that the private consumption c_t is equal to the output, possibly reduced by the distortion due to tax load, $y_t - x(\tau_t)$, net of debt repayment to the foreign investors, $(1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}$, plus new funding received from them, $(1-\alpha)b_t$, minus government expenditure g_t as well as the penalty cost imposed on current and past default, $p(\theta_t)+\vec{p}_t$. Equation (7) says that the tax is used to cover total debt repayment, $(1-\theta_t)z_{t-1}b_{t-1}$, government expenditure g_t as well as default costs $p(\theta_t)+\vec{p}_t$, net of new debt taking b_t.

Since b_t can be constrained to zero when default occurs in the form of contract violation, so if exclusion from the financial market is one punishing instrument available to the investors, there will be a "jump" in the value function when the government switches between default and non-default, as long as the decision maker is not in the so-called last period in which $b_t = 0$ regardless of the default or non-default decision. Hence, in general, one has to distinguish between the value function given default decision, $\mathbb{V}^d$, and the value function given non-default decision, $\mathbb{V}^n$. Given access to the financial market, the value function is henceforth: $\mathbb{V}^f = \max\left(\mathbb{V}^d,\mathbb{V}^n\right)$ which implies that $\vec{p}_t = \emptyset$.

Given the non-default decision, the government can choose g_t and b_t to optimize:

$$\begin{aligned}\mathbb{V}^n(z_{t-1}b_{t-1}) = \sup_{b_t\in B_t, g_t\in G_t}\{\mathbb{U}(c_t,g_t)+\beta E_t\mathbb{V}^f(z_t b_t)\} \quad \text{s.t.} \\ c_t = y_t - x(\tau_t) - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b_t - g_t \\ \tau_t = z_{t-1}b_{t-1} + g_t - b_t \\ z_t = \inf[z:\ z_t E_t[1-\theta_{t+1}] = R_t]\end{aligned} \tag{8}$$

For any function $f(x_1,x_2,\cdots)$, denote $f_i(x_1,x_2,\cdots)$ as the *i-th* first derivative, i. e. $f_i(x_1,x_2,\cdots) \equiv \frac{\partial f(x_1,x_2,\cdots)}{\partial x_i}$, and use f'_t or $f'(x_t)$ as a shortcut for $\frac{\partial f(x_t)}{\partial x_t}$, then the first-order conditions for (8) with respect to b_t reads as follows:

$$\mathbb{U}_1(c_t,g_t) * (x_1(\tau_t)+1-\alpha) = -\beta E_t \mathbb{V}^{f\prime}(z_t b_t) * (z_t + z'_t b_t) \tag{9}$$

The above equation describes the optimal decision about new debt taking b_t as an inter-temporal trade-off. The left-hand side captures the benefit from one additional unit of b_t: by taking one more unit of debt, the economy can get $(1-\alpha)$ units of additional transfer from abroad, and the total output can be raised by $x_1(\tau_t)$ as more debt financing means less tax to finance the government expenditure and leads to less distortion for output, and the total increase in consumption resulting from borrowing abroad and less distortionary tax will enhance the social welfare as each additional unit of consumption can raise the current utility by $\mathbb{U}_1(c_t,g_t)$. This benefit from debt-taking for today's well-being has its cost for the future as each unit of new debt raises the amount of outstanding debt for tomorrow by $(z_t + z'_t b_t)$,[8] and each additional unit of debt repayment obligation will reduce the expected future social welfare by $-E_t\mathbb{V}^{f\prime}(z_t b_t)$, discounted with β. At the optimal amount of new debt taking, the LHS should be equated to the RHS.

Now consider the FOC w. r. t. g_t:

$$\mathbb{U}_2(c_t,g_t) = \mathbb{U}_1(c_t,g_t) * (x_1(\tau_t)+1) \tag{10}$$

Equation 10 describes the optimal decision of government expenditure g_t as an intra-temporal trade-off. One more unit of public goods provisioning will directly increase the current utility by $\mathbb{U}_2(c_t,g_t)$. But government expenditure also needs to be financed by tax and thus will reduce the after-tax income one-by-one. In addition, the before-tax income will be reduced by $x_1(\tau_t)$ due to more distortionary tax, and each additional consumption decrease resulting from more tax and less total income will reduce the current utility by $\mathbb{U}_1(c_t,g_t)$. In the case that the government expenditure is restricted to a singleton, the above equation simply drops out.

Now consider $\mathbb{V}^d$, the value function in the case of default. Given the default decision, the government can choose g_t and θ_t to maximize:

[8] Note that $z_t(b_t)$ is an increasing function as proved in Eaton and Gersovitz (1981), hence one more unit of debt does not only raise the amount of outstanding debt for tomorrow by z_t, but also by $z'_t b_t$, the increased interest rate cost due to more debt taking.

$$\mathbb{V}^d(z_{t-1}b_{t-1}) = \sup_{\theta_t\in[0,1],g_t\in G_t} \{\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(0)+(1-\kappa)E_t\mathbb{V}^a(\vec{p}_{t+1}(\theta_t))\right]\} \quad \text{s.t.} \tag{11}$$

$$\begin{aligned} c_t &= y_t - x(\tau_t) - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} - g_t - p(\theta_t) \\ \tau_t &= (1-\theta_t)z_{t-1}b_{t-1} + g_t + p(\theta_t) \end{aligned}$$

Equation (11) says that the default decision will possibly put the economy in an autarky state from the next period on, whose value function is denoted as $\mathbb{V}^a$. Being in an autarky state means that in each period, the economy is excluded from the financial market, but with some predetermined non-negative probability κ it may regain access to the financial market with no old debt in the next period, and this probability is the same at which an economy can return to the financial market with no old debt directly after a default event. $\mathbb{V}^a$ with inherited default cost $\vec{p}$ is expressed as follows:

$$\begin{aligned} \mathbb{V}^a(\vec{p}) &= \sup_{g_t\in G_t} \{\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(0)+(1-\kappa)E_t\mathbb{V}^a(\vec{p})\right]\} \quad \text{s.t.} \\ c_t &= y_t - x(\tau_t) - g_t - \vec{p} \\ \tau_t &= g_t + \vec{p} \end{aligned}$$

The idea of a random return to the financial market has been introduced in Arellano (2008) to capture the observed different lengths of default era. Before her, and following Eaton and Gersovitz (1981), the literature considering financial market exclusion as a way of punishment for defaulting governments often sets $\kappa = 0$, i. e. once excluded from the financial market, the economy will stay in autarky forever.

The optimal policy mix (θ_t, g_t) in the case of default decision is given by the first-order conditions of equation (11). The FOC w. r. t. θ_t is:

$$\begin{aligned} &\mathbb{U}_1(c_t,g_t) * (x_1(\tau_t)(z_{t-1}b_{t-1} - p_1(\theta_t)) + (1-\alpha)z_{t-1}b_{t-1} - p_1(\theta_t)) \\ =& -\beta(1-\kappa)E_t\mathbb{V}_1^a(\vec{p}_{t+1}(\theta_t)) * \vec{p}'_{t+1}(\theta_t) \end{aligned} \tag{12}$$

The above equation says that an increment in default rate can reduce the tax by the amount of outstanding debt, $z_{t-1}b_{t-1}$, net of the resulting increase in penalty cost $p_1(\theta_t)$, and each unit of reduction in tax can raise the output by $x_1(\tau_t)$; besides, one more unit of default also increases the domestic wealth by $(1-\alpha)z_{t-1}b_{t-1}$, net of increase in penalty cost $p_1(\theta_t)$, and each unit of the resulting increase in consumption will raise the current utility by $\mathbb{U}_1(c_t,g_t)$. This "benefit" from default will be traded off against its cost, namely the rise in future penalty cost $\vec{p}'_{t+1}(\theta_t)$ which reduces the future welfare in case of autarky by $-E_t\mathbb{V}_1^a(\vec{p}_{t+1}(\theta_t))$ for each additional unit of penalty cost. This loss of welfare in the future will enter with probability $(1-\kappa)$ and is discounted by β. Since θ_t is constrained in the interval $[0,1]$ the above equation may not hold in equality in which case no repudiation or full repudiation will occur. To ensure that no multiple solutions

exist, it is sufficient to let $\mathbb{V}^d$ be non-convex in θ_t which is satisfied in the majority of models in which $x_(\tau_t)$ is convex and $p_(\theta_t)$ as well as $\vec{p}_{t+1}(\theta_t)$ are non-concave.

Analogous to the case with $\mathbb{V}^n$, I also derive the optimality condition for $\mathbb{V}^d$ w. r. t. g_t, which should be considered if G_t is no singleton:

$$\mathbb{U}_2(c_t, g_t) = \mathbb{U}_1(c_t, g_t) * (x_1(\tau_t) + 1)$$

It turns out that the above FOC condition is functionally the same as (10).

From solving the first-order conditions, we have the optimal policy mix (θ_t, g_t, τ_t) and (b_t, g_t, τ_t) conditional on default or non-default decision, respectively. Using $\mathbb{V}^f = \max\left(\mathbb{V}^d, \mathbb{V}^n\right)$ yields the optimal policy mix $(b_t, \theta_t, g_t, \tau_t)$ either in the form of $(0, \theta_t, g_t, \tau_t)$ or in the form of $(b_t, 0, g_t, \tau_t)$, according to whether default or non-default is optimal for this small open economy. In the case that default will not cause financial market exclusion in the current period we can simplify this procedure by directly optimizing $\mathbb{V}^f$ over $(b_t, \theta_t, g_t, \tau_t)$ using the first-order conditions (9), (12) and (10) as well as the budget constraint (7).

After setting up the general model, it is interesting to see how the state variables y_t[9] and $z_{t-1}b_{t-1}$ as well as the model parameter α affect the debt repayment behavior of the government. By taking the inductive approach and analyzing the two main types of models, each of which is a special form of the general model I have set up here, it is easy to verify that a lower $z_{t-1}b_{t-1}$ will reduce the default risk, and a higher $z_{t-1}b_{t-1}$ will increase the default risk – a result just as expected and in line with the existing literature, hence not elucidated here. The effect of α has not yet been extensively studied in the literature, hence the following section will be devoted to it. To take the result in advance: a higher α will lead to *cet. par.* lower or at least not higher default risk, and vice versa. Note that this knowledge is not about the equilibrium outcome and merely states that after the interest rate is contracted, then in the next period, when all state variables have been realized and when the government has to make the default or repayment decision, a government with a higher α will have less incentive to default. But without the "smallest-possible interest rate will be contracted" assumption made above, this knowledge of less *ex post* default incentive may lead to even higher interest rate cost in the so called "bad equilibrium" as defined in Calvo (1988), which may in turn raise the default risk in the equilibrium and with it the risk premium. In section 3.3 I will explore under which conditions we can make sure that an *ex post* less default incentive due to factors like lower $z_{t-1}b_{t-1}$ or higher α will lead to *ex ante* less default risk i. e. lower default probability or lower default rate in the equilibrium.

3 The effects of regarding the lenders' interests

In this section, I show how α, a parameter representing how well the lenders' interests are regarded, affects the repay propensity of the government and henceforth the borrowing conditions it gets from the financial market.

[9] If output is autocorrelated as in e. g. Aguiar and Gopinath (2006).

3.1 Model of the default probability

The first type of models which are mainly used to explain external default was first introduced in Eaton and Gersovitz (1981), and then further developed, among others, by Aguiar and Gopinath (2006) and Arellano (2008). In this kind of models, the default cost, including exclusion from the financial market and possibly output drop during the default era, is assumed to be independent from the fraction of debt being repudiated. Together with the increasing "benefit" from default due to the wealth transfer effect, a typical government in this model world will always choose to default on its whole stock of outstanding debt whenever default is preferable to non-default, as I will show below. Consequently the expected default rate is equal to the probability of default, denoted by the parameter λ: $E_t(\theta_{t+1}) = \Pr(\theta_{t+1} > 0) \equiv \lambda_t$. Therefore, I will refer to this type of model as "model of the default probability" when analyzing the correlation between the default risk, here λ, and the parameter α. Besides, this kind of model usually does not consider dead weight loss from taxation since the wealth transfer effect is enough to explain the existence of default, a modeling strategy which I will maintain to simplify the analysis.

Under the above assumptions, the objective function of the government is a special form of the value function (5):

$$\begin{aligned}\mathbb{V}(y_t,(1-\alpha)z_{t-1}b_{t-1},\vec{p}_t) = \sup_{b_t\in B_t,\theta_t\in[0,1],g_t\in G_t} &\{\mathbb{U}(c_t,g_t)+\beta E_t\mathbb{V}(y_{t+1},(1-\alpha)z_tb_t,\vec{p}_{t+1})\} \quad \text{s.t.}\\ c_t &= y_t-(1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}+(1-\alpha)b_t-g_t-p_t-\vec{p}_t\\ z_t &= \inf[z:\ z_tE_t[1-\theta_{t+1}]=R_t]\end{aligned}$$

In words: the value function is a function of the state variables endowment y_t, debt service to foreigners $(1-\alpha)z_{t-1}b_{t-1}$, and inherited default cost from last default event $\vec{p}_t$ which is null for $\mathbb{V}^f$ and some constant $\vec{p}$ for $\mathbb{V}^a$ which is usually equal to the current default cost p_t.[10] Here (7) drops out because τ_t no longer directly affects c_t due to the assumption of no dead weight loss, hence it is unnecessary to model it explicitly.

Given default decision, the corresponding value function is:

$$\begin{aligned}\mathbb{V}^d(y_t,(1-\alpha)z_{t-1}b_{t-1}) = \sup_{\theta_t\in[0,1],g_t\in G_t} &\{\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(y_{t+1},0)+(1-\kappa)E_t\mathbb{V}^a_{t+1}(y_{t+1},\vec{p})\right]\} \quad \text{s.t.}\\ c_t &= y_t-(1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}-g_t-p_t\end{aligned}$$

And the value function in autarky is:

[10] To see that the value function solely depends on the three state variables, just optimize the value function over $(1-\alpha)b_t$, $1-\theta_t$ and g_t which are one-by-one mappings of the optimizers shown in the formula.

$$\mathbb{V}_t^a(y_t,\vec{p}) = \sup_{g_t\in G_t}\{\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(y_{t+1},0)+(1-\kappa)E_t\mathbb{V}_{t+1}^a(y_{t+1},\vec{p})\right]\} \quad \text{s. t.}$$
$$c_t = y_t - g_t - \vec{p}$$

By plugging in $\mathbb{V}^a$, the value function for default can be written in a more parsimonious way as:

$$\mathbb{V}^d(y_t,(1-\alpha)z_{t-1}b_{t-1}) = \sup_{\theta_t\in[0,1],g_t\in G_t}\{\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(y_{t+1},0)+(1-\kappa)E_t\mathbb{V}_{t+1}^a(y_{t+1},\vec{p})\right]\} \quad \text{s. t.}$$
$$c_t = y_t-(1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}-g_t-p_t$$
$$\mathbb{V}_\tau^a(y_\tau,\vec{p}) = \mathbb{U}((y_\tau-g_\tau^a-\vec{p}),g_\tau^a)+\beta\left[\kappa E_\tau\mathbb{V}^f(y_{\tau+1},0)+(1-\kappa)E_\tau\mathbb{V}_{\tau+1}^a(y_{\tau+1},\vec{p})\right]\forall\tau\geq t$$

In the above expression, g_τ^a stands for the optimal government expenditure in autarky at time τ, and g_τ^a is a function of y_τ and $\vec{p}$ only.[11]

The first-order derivative of $\mathbb{V}^d$ over θ_t is:

$$\mathbb{U}_1(c_t,g_t)*(1-\alpha)z_{t-1}b_{t-1}$$

The marginal utility from consumption $\mathbb{U}_1(c_t,g_t)$ is always positive. Further, $z_{t-1}b_{t-1}>0$ whenever the government contemplates default. Therefore the above term will be strictly positive whenever $\alpha<1$, i. e. if we are not dealing with purely domestic debt. Hence the optimal value of θ_t is always one given a default decision, i. e. whenever the government chooses to default, it will choose to default on the whole stock of debt.

In the case of purely domestic debt i. e. $\alpha=1$ default does not make sense since it does not transfer any wealth to the domestic economy. With the additional assumption that the government will only choose to default more when doing so can render the representative agent better off, the government will always choose $\theta_t=0$, given default decision.[12] But since default can trigger financial market exclusion and possibly also other default cost, for $\alpha=1$ the default probability λ will be zero, i. e. the government will never default, and hence does not have to pay any risk premium on its bond issuance and we have $z_t=R_t$.

Now again consider the more interesting case in which $\alpha<1$ and hence $\theta_t=1$ given default decision. Plug $\theta_t=1$ into the expression of $\mathbb{V}^d$ and we get:

[11] This statement also holds in cases in which the distribution of $y_{\tau+1}$ may depend on y_τ as in Aguiar and Gopinath (2006).

[12] Actually θ_t is not exactly equal to 0 but the point right to it, i. e. near zero, but positive, so that it is regarded as default.

$$\mathbb{V}^d(y_t,(1-\alpha)z_{t-1}b_{t-1}) = \sup_{g_t\in G_t}\{\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(y_{t+1},0)+(1-\kappa)E_t\mathbb{V}^a_{t+1}(y_{t+1},\vec{p})\right]\} \quad \text{s.t.}$$
$$c_t = y_t - g_t - p_t$$
$$\mathbb{V}^a_\tau(y_\tau,\vec{p}) = \mathbb{U}((y_\tau - g^a_\tau - \vec{p}),g^a_\tau) + \beta\left[\kappa E_\tau\mathbb{V}^f(y_{\tau+1},0)+(1-\kappa)E_\tau\mathbb{V}^a_{\tau+1}(y_{\tau+1},\vec{p})\right]\forall\tau\geq t+1$$

Denote the optimal choice of g_t given default as g^d_t. By checking the above expression we can see that g^d_t is independent from $(1-\alpha)z_{t-1}b_{t-1}$ since this term disappears after plugging in $\theta_t = 1$. Then write the above expression in an even more compact way:

$$\mathbb{V}^d_t = \mathbb{U}((y_t - g^d_t - p_t),g^d_t)+\beta\left[\kappa E_t\mathbb{V}^f(0)+(1-\kappa)E_t\mathbb{V}^a_{t+1}(\vec{p})\right] \quad \text{s.t.}$$
$$\mathbb{V}^a_\tau(\vec{p}) = \mathbb{U}((y_\tau - g^a_\tau - \vec{p}),g^a_\tau)+\beta\left[\kappa E_\tau\mathbb{V}^f(0)+(1-\kappa)E_t\mathbb{V}^a_{\tau+1}(\vec{p})\right]\forall\tau\geq t+1$$

As is evident from above, the value from default, $\mathbb{V}^d$, does not depend on α. Note that $\mathbb{V}^d$ does depend on y_t, and I have only dropped y_t as an input argument $\forall\, t$ to make the expression look less messy and more legible, what is innocuous in this context since we are not interested in the exogenous variable y_t.

Now consider the value function given non-default:

$$\mathbb{V}^n((1-\alpha)z_{t-1}b_{t-1}) = \sup_{b_t\in B_t, g_t\in G_t}\{\mathbb{U}(c_t,g_t)+\beta E_t\mathbb{V}^f((1-\alpha)z_tb_t)\} \quad \text{s.t.}$$
$$c_t = y_t - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b_t - g_t$$
$$z_t = \inf[z:\ z_t(1-\lambda_t) = R_t]$$

In the expression above, the contracted interest rate is a function of the default probability $\lambda_t \equiv \Pr(\mathbb{V}^d_{t+1} > \mathbb{V}^n_{t+1})$[13] with $\mathbb{V}^n_{t+1}$ depending on $(1-\alpha)z_tb_t$. An implication is that z_t is a function of $(1-\alpha)z_tb_t$ too or equivalently a function of $(1-\alpha)b_t$. Here I have replaced $E_t[1-\theta_{t+1}]$ by $(1-\lambda_t)$ since the expected default rate is interchangeable with the default probability λ_t as explained before.

By denoting the optimal values of b_t and g_t given non-default as b^n_t and g^n_t, respectively, we obtain $\mathbb{V}^n$ in a more compact way:

$$\mathbb{V}^n((1-\alpha)z_{t-1}b_{t-1}) = \mathbb{U}((y_t - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b^n_t - g^n_t),g^n_t)+\beta E_t\mathbb{V}^f((1-\alpha)z_tb^n_t) \quad \text{s.t.}$$
$$z_t = \inf[z:\ z_t(1-\lambda_t) = R_t]$$

[13] The assumption that only the smallest possible interest rate will be contracted has ensured that z_t can be expressed as a function of other terms.

Analogous to the proof used in Eaton and Gersovitz (1981) I show that $\mathbb{V}^n$ increases with α, which is summarized in **Theorem 1**:

Theorem 1: for any $0 \leq \alpha_1 < \alpha_2 < 1$ and any given outstanding amount of debt $z_{t-1}b_{t-1} \geq 0$, it holds that $\mathbb{V}^n((1-\alpha_1)z_{t-1}b_{t-1}) \leq \mathbb{V}^n((1-\alpha_2)z_{t-1}b_{t-1})$.

The formal proof of **Theorem 1** can be found in Appendix A.1, here only a sketch of the underlying idea: as α increases, i. e. more lenders' wealth position is internalized by the government in its objective function, the debt repayment will reduce the current consumption of the representative agent by a smaller amount, and hence repayment is more worthwhile for the government. Besides, when this attitude towards repayment is anticipated by the financial market, then the government will face a lower interest rate for its current borrowing b_t^n, or differently put, for the same future repayment obligation $z_t b_t^n$, the government can borrow more today, which increases the current consumption additionally. Therefore, a higher α makes the repayment decision more valuable to the government.

Since the value from repayment, $\mathbb{V}^n$, increases with α, while the value from default, $\mathbb{V}^d$, is independent from α, as a consequence, the default probability λ_t which is the probability of $\mathbb{V}^n$ being smaller than $\mathbb{V}^d$ will decrease with α. Together with the previously gained knowledge that $\lambda_t = 0$ for $\alpha = 1$ we can say that for all $\alpha \in [0,1]$ the default probability λ_t will decrease or at least not increase with increasing α, i. e. the higher weight assigned to the lenders' wealth position in the objective function makes the government less prone to default, given the same outstanding amount of debt $z_{t-1}b_{t-1}$ and other state variables.

However, due to the interaction between z_t and λ_t, i. e. z_t will increase with increasing λ_t due to the market participation constraint while λ_t will increase with increasing z_t since $\mathbb{V}^n$ decreases *cet. par.* with increasing z_t, there may be a "good equilibrium" characterized by low z_t and low λ_t and a "bad equilibrium " characterized by high z_t and high λ_t given the same conditions. And in the "bad equilibrium" the contracted interest rate z_t may increase with increasing α. Using the smallest-possible-interest-rate-will-be-contracted assumption I have already ruled out the "bad equilibrium" and thereby ensured that the lower default propensity due to a higher α shown above indeed leads to more favorable borrowing conditions for the government in the form of a lower z_t. In section 3.3 I will tackle the multiple equilibria problem in a more general way and show that only the "good equilibrium" is a stable equilibrium which can exist in a world afflicted with shocks. Before doing so, I will first prove in the next section that a higher α will also lower the default propensity in a Calvo type model in which the default rate is a continuous function of the state variables.

3.2 Model of the default rate

The second type of models mainly used to explain domestic default was first presented in Calvo (1988) as a two-period-model which focuses on the default rate decision as a trade-off between dead weight loss due to taxation and the default cost from debt repudiation. The default cost may be linear or convex and can stand for dispute or renegotiation or some other penalty cost; it can also be the inflation cost, etc. In either case, the default decision will not trigger the exclusion from the financial market. With other words, this kind of model is characterized by the independence of the maximal available amount of new debt from the current default decision, hence there will be

no "jump" in the value function of the representative agent when switching between default and non-default decisions. Accordingly, the default rate can be determined as a continuous function of the underlying variables. Consequently, I will refer to this type of model as "model of the default rate" when analyzing the correlation between α and the default risk θ_t. The recent literature about non-strategic default, which can be found in e. g. Uribe (2006), Schabert (2010) and Juessen et al. (2011), also falls into this category. However, since they have set both g_t and τ_t to be exogenous in order to model a given fiscal stance, the default rate θ_t will rather be derived from the exogenous fiscal and monetary stance and hence be independent from α, since θ_t is not the result of an optimizing process. But if we interpret the fiscal stance itself to be the result of an optimization process, then α may still have an impact on the equilibrium default rate and hence on the government borrowing condition. Here I only consider the Calvo type model in which τ_t is constrained loosely enough so that the government can optimize over θ_t, and I do it first in a two-period setting; but the result can also be extended to an infinite-horizon model as is shown in appendix A.3.

In the two-period model without inherited penalty cost, the debt is taken in period 0 while the repayment decision is made in period 1. So the value function in period 1 is again a special form of (5):

$$\begin{aligned}\mathbb{V}(z_0 b_0) = & \sup_{\theta_1 \in [0,1], g_1 \in G_1} \{\mathbb{U}(c_1, g_1)\} \quad \text{s. t.} \\ & c_1 = y_1 - x(\tau_1) - (1-\alpha)(1-\theta_1) z_0 b_0 - g_1 - p(\theta_1) \\ & \tau_1 = (1-\theta_1) z_0 b_0 + g_1 + p(\theta_1)\end{aligned}$$

Note that here we cannot optimize over b_1 not because a possible default decision has triggered the exclusion from the financial market but because this is the last period and hence no new debt can be contracted regardless whether the government fully repays or defaults on (part of) its debt.

The contracted interest rate in period 0, z_0, must satisfy the following market participation constraint, which will be taken as given by the government in period 1, the period of debt repayment:

$$z_0 = \inf[z : \; z_0 E_0[1-\theta_1] = R_0]$$

The first-order condition to determine the optimal default rate θ_1 reads as follows:

$$\mathbb{U}_1(c_1, g_1) * (x_1(\tau_1)(z_0 b_0 - p_1(\theta_1)) + (1-\alpha) z_0 b_0 - p_1(\theta_1)) = 0 \tag{13}$$

The LHS of the above expression is a decreasing function of α for any positive debt stock $z_0 b_0 > 0$. And it is also a decreasing function of θ_1 around the optimum due to the assumption made about the curvature of the deadweight loss function and the penalty cost function. Consequently the optimal default rate θ^* will decrease or at least not increase with α. This can be formally expressed as:

Theorem 2: In a two-period model, given $\mathbb{U}_1(c,g) > 0$, $x'(\tau) > 0$, $x''(\tau) > 0$ and $p''(\theta) \geq 0$ in the vicinity of the equilibrium, it holds that $\frac{d\theta^*}{d\alpha} \leq 0$.

The formal proof of **Theorem 2** can be found in Appendix A.2, here only a brief depiction of the underlying idea: Each additional unit of default rate θ will raise the consumption by reducing the tax needed to finance the debt service by $z_0 b_0 - p_1(\theta_1)$, which is the outstanding debt obligation net of increase in default cost. Each unit of reduced tax service brings in turn less deadweight loss by the amount of $x_1(\tau_1)$. This marginal "benefit" of default will be smaller the larger θ is, since $x(\tau)$ and $p(\theta)$ are convex and non-concave respectively by assumption, and in optimum $x_1(\tau_1)$ is non-negative. The marginal cost of one additional unit of default is $p_1(\theta_1) - (1-\alpha)z_0 b_0$, the marginal change in default cost net of the wealth transfer from abroad. This effect is larger the larger θ is, since $p(\theta)$ is assumed to be non-concave. So when θ rises then the marginal change in consumption, which is the marginal "benefit" minus the marginal cost, will fall, and with it also the marginal utility because the marginal utility from consumption is positive. When α rises/falls, then the marginal utility will fall/rise because a higher/lower α reduces/enhances the wealth transfer effect. In order to let the marginal utility again rise/fall to zero, the optimal default rate θ^* then needs to fall/rise, so a higher α will lower the optimal default rate θ^* until it hits the lower bound, and a lower α will raise θ^* until it hits the upper bound. In other words, a higher weight assigned to the lenders' wealth position in the objective function will lower the government's propensity to default; inversely, a lower α will increase the default propensity.

Indeed, if the penalty cost is assumed to be a constant fraction of the repudiated debt then $p(\theta_1) = \omega\theta_1 z_0 b_0$ so that $p_1(\theta_1) = \omega z_0 b_0$ is a constant. In this case, $\alpha = 0$ will make the government always choose to fully default on its debt since (13) will become $\mathbb{U}_1(c_1,g_1) * (x_1(\tau_1)+1)(1-\omega)z_0 b_0$ which is positive for any $\theta_1 \in [0,1]$. A more detailed analysis of this case of linear default cost can be found in the example in appendix B.1.

Although I have shown the negative correlation between α and θ^* first for the two-period model, this result also holds for the corresponding infinite-horizon model, the proof of which can be found in appendix A.3:

Theorem 3: In an infinite-horizon model, given $\mathbb{U}_1(c,g) > 0$, $x'(\tau) > 0$, $x''(\tau) > 0$ and $p''(\theta) \geq 0$ in the vicinity of the equilibrium, it holds that $\frac{d\theta^*}{d\alpha} \leq 0$.

As in the last section, the negative correlation between α and the default rate θ_t is based on a predetermined contracted interest rate z_{t-1}. And since θ_t itself and its expectation will have influence on z_{t-1} through the market participation constraint, there may exist multiple equilibria, and in the so-called "bad equilibrium" a higher α may even raise z_{t-1}. Although the institutional assumption made in section 2 can already rule out this bad equilibrium and ensure that a higher α will indeed lower the interest cost for the government, I will show in section 3.3 that the negative correlation between α and z also holds under much more general assumptions.

3.3 The multiple equilibria

The term "multiple equilibria" can have various meanings depending on the context. Here I use this terminology as in Calvo (1988). More precisely: the set of contracted interest rates $\{z:\ z_t E_t[1-\theta_{t+1}] = R_t\}$ contains two elements. The lower equilibrium interest rate is referred to

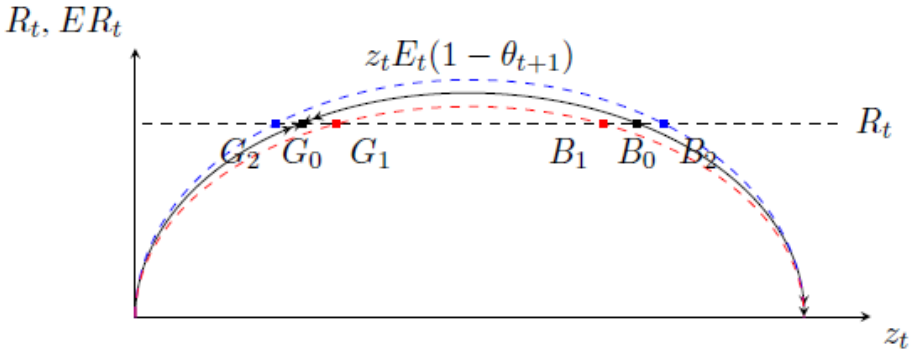

Figure 1: The multiple equilibria

as the "good equilibrium" and the other one is called the "bad equilibrium". As most authors have noticed, the higher "bad equilibrium" has several counter-intuitive features like an interest rate decreasing with increasing outstanding amount of debt, which is rarely observed in reality. Indeed, taking a closer look at the properties of the equilibria, I have found out that the "bad equilibrium" is an unstable equilibrium while the "good equilibrium" is a stable equilibrium as is illustrated in figure 1.

From the previous sections we know that in general the default rate θ_{t+1} will rise and at least not fall with rising contracted interest rate z_t since a higher z_t means a higher debt obligation for tomorrow, given the contracted debt amount b_t. Hence, the expected return $ER_t \equiv z_t E_t(1-\theta_{t+1})$ will first rise with rising z_t but with a lower and lower pace since the expected default rate $E_t\theta_{t+1}$ also rises with the rising interest cost, and after some point, the increase in expected default may outweigh the increase in contracted interest rate and the expected return will fall with further rising z_t, as illustrated in figure 1. Note that the exact form of the expected return as a function of the contracted interest rate z_t may not look exactly the same, and in example B.1 we will see this function as a kinked straight line rather than a smooth curve as in figure 1, but the qualitative effect of the contracted interest rate on the expected return remains the same.

In figure 1, the expected return $z_t E_t(1-\theta_{t+1})$ is represented by the black solid curve which first rises and then falls with rising z_t. At some point, the expected default $E_t\theta_{t+1}$ will reach its maximum of one so that the expected return will become zero and lie on the x-axis. This part is not of interest and hence not plotted here, and we just consider the part above the x-axis. In equilibrium, the expected return should be equated to the market return R_t which is represented by the horizontal black dashed line, and the intersections of R_t with the black curve $z_t E_t(1-\theta_{t+1})$ constitute the two equilibrium points G_0 and B_0. The point G_0 is the so called "good equilibrium" since it maintains the same market return with a lower, and hence more favorable interest cost for the government, compared to the "bad equilibrium" represented by the point B_0.

The "bad equilibrium" has some odd features, e. g. if some shock raises the exogenous market return R_t and thus pushes the horizontal line upwards, then B_0 will move to the left, i. e. a tightening monetary policy would even lower the interest cost z_t in the bad equilibrium, which is counter intuitive and rarely observed. It also predicts that a rise in the amount of outstanding debt

can even reduce the interest cost – a prediction apparently not shared by most researchers who recommend fiscal consolidation to reduce the public debt.[14] Regarding the correlation between α and the borrowing cost: in sections 3.1 and 3.2 I have already shown that a higher α will lower the default propensity of the government and hence push the expected return curve upwards to the blue dashed one while a lower α will raise the default propensity and therefore push the expected return curve downwards to the red dashed one. If the economy is in the good equilibrium then a higher α will lower the contracted interest rate since G_2 is to the left of G_0; and a lower α will raise the contracted interest rate as G_1 is to the right of G_0. But if the economy would be in a bad equilibrium, then a higher α would even raise the contracted interest rate since B_2 is to the right of B_0; and a lower α would lower the contracted interest rate as B_1 is to the left of B_0.

The explanation for the opposite behavior of the two equilibria is the following: in the vicinity of the good equilibrium, the change in expected return is mainly determined by the change in the contracted interest rate, hence, after a positive shock like a higher α or lower $z_{t-1}b_{t-1}$ which pushes the expected return above the market return, the contracted interest rate will fall to bring the expected return down to the market return; and after a negative shock like a higher R_t, the contracted interest rate will rise to raise the expected return to the market return. In the vicinity of the bad equilibrium, the expected return is dominated by the default risk $E_t\theta_{t+1}$ which is positively correlated with z_t. So a positive shock like a higher α will require a higher $E_t\theta_{t+1}$ which means a higher z_t, and a negative shock will lower the interest cost z_t. With the "smallest-possible-interest-rate-is-always-contracted" assumption we can make sure that this economy is always in the good equilibrium and hence a higher α will lower the contracted interest rate and a lower α will raise the contracted interest rate – a conclusion that is in line with our intuition.

But why should the economy ever be in a "bad equilibrium"? By taking a closer look at the expected return curve, we can see that the good equilibrium G_0 is a stable equilibrium while the bad equilibrium B_0 is an unstable one. Say, if some shock would push the contracted interest rate to a value between G_0 and B_0, then there would be an over-demand for the government bond since the expected return is higher than the market return, which would lead to a lower and lower z_t until it converges to G_0.[15] And if the initial interest cost z_t would lie below G_0, then there would be no demand since the expected return is below the market return, and z_t would rise and rise until it converges to G_0 – so the good equilibrium G_0 is also a stable equilibrium, and after any small deviation from it, possibly due to some external shocks, it will soon converge back to G_0 as long as the price adjustment mechanism works in the usual way, namely that an excessive demand will raise the price or lower the yield of the government bond while an excessive supply will work in the opposite way. If the shock would push the interest cost to be higher than B_0, then either the lack of demand would push z_t higher and higher so that no debt can be contracted in the equilibrium, or, in the case that the market participants are aware of the existence of the multiple equilibria problem, they may realize that the reason why no debt can be contracted is that they are bargaining at a "too high" interest rate, and hence they switch to a low z_t and the contracted interest rate again converges to G_0 – in any case, the economy will not return to B_0, so B_0 is an

[14] Cogan et al. (2013) have shown with a DSGE model how to implement such a fiscal consolidation which does not only increase the long-run output but also has a short-run stimulative impact.

[15] The arrows along the curve represent the direction of the movement of z_t.

unstable equilibrium: if any force would push the economy away from this point, then no return to B_0 would follow. Since B_0 is just one point among a continuum of points on the curve, the probability that the economy would ever be in the bad equilibrium is actually almost zero.[16]

Now consider again the correlation between α and the equilibrium borrowing cost. Initially, the economy is very probably at G_0 as I have explained above. A rising α which pushes the expected return curve upwards to the blue curve would cause an over-demand and thus lower the equilibrium borrowing cost to G_2; and a falling α which pushes the expected return curve downwards to the red curve would cause an under-demand and thus raise the equilibrium borrowing cost to G_1 – a negative correlation between α and the borrowing cost as expected. And in the rather unlikely case in which the economy was initially at B_0: a rising α will cause an over-demand since the blue curve is above the black curve, and the over-demand will lower and lower z_t until G_2 has been reached. And a falling α will cause an under-demand which will either raise and raise z_t so that no debt can be contracted or lead the economy to "jump" to some fairly low z_t and from there converge to G_1 – so the negative correlation between α and the borrowing cost will be maintained in the case of a rising α; and a falling α will either lead to no equilibrium, i. e. a complete lending stop which means the worst possible borrowing condition for the government, or a one-time positive correlation between α and borrowing cost as the economy has now switched to the good equilibrium after repeatedly failed negotiations which made the economy aware that it was in the vicinity of a bad equilibrium. But since now the economy is in a good equilibrium, at least from now on the negative correlation between α and borrowing cost will hold. Hence, the negative correlation between α and borrowing cost almost surely holds in a fairly general setting, even without the "smallest-possible-interest-rate-is-always-contracted" assumption. In other words, a better regard of the lenders' wealth position will in general lower the equilibrium borrowing cost of the government, while a lower α will rather raise its borrowing cost.

4 Discussion

4.1 The measurement of α

One reason for researchers to not pay much attention to α hitherto might be a general belief that whenever default becomes likely the foreigners will simply sell their debt claims to the domestic investors, maybe even at par.[17] In such a case there would be no need to consider α explicitly since this parameter is expected to be 1 if default is imminent. Indeed, in the case of tradable assets like government bonds, the parameter α should be measured at the time point of the debt repayment. When issuing the bonds, the investors can only gauge it. Since in the imminent default case the domestic investors are most likely to buy, their wealth relative to the outstanding

[16] Note that the validity of the conclusion that the "bad equilibrium" is unstable hinges on the model assumption that the government cannot commit. If this assumption is violated, there may be a stable "bad equilibrium". One example is Corsetti and Dedola (2014) in which the government can commit with a not too low probability and hence there is a stable "bad equilibrium".

[17] See e. g. Broner et al. (2006).

amount of debt as well as the concentration of their wealth[18] would largely determine the value of α. Following this logic, in the case of small economies with large scale government debt it is questionable whether the domestic investors have sufficient wealth to take over all public debt at par. Thus, given that the government bond is freely tradable, a capital-intensive economy can in general enjoy more favorable borrowing conditions compared to an assimilable economy endowed with less capital. The reason is that when default looms, the debt claim holders of the former economy can more easily find buyers who take over the debt claims in the belief that their wealth position will be better regarded by the borrowing government.

The parameter α is so far interpreted as the fraction of public debt held by domestic residents – a standard interpretation following the literature. The underlying assumption for interpreting α this way is that the government acts as an agent of the principals, its voters or citizens, and hence its task is to maximize the aggregate welfare of the voters, who are usually the residents. Therefore the domestic investors, who are often voters at the same time, dare to buy the bonds from the foreign investors when default looms since their government has more incentive to repay them. However, due to the globalization and integration process of the financial market, it is common that a fiscal authority borrows from other fiscal areas in order to overcome the capital shortage in its own economy. If α would be strictly interpreted as the fraction of debt held by residents in the borrowing economy as is done in the traditional way, then the corresponding default risk would be high for most of the smaller economies in the world, due to the low α value. Though small economies with less capital are indeed often viewed as riskier borrowers, it seems unrealistic to assume that they all have higher default propensity. This is especially the case when we consider that this small open economy may also be a local government with its own fiscal authority. The reason for investors often deeming a borrowing county which is part of a wealthy country more reliable than a borrowing smaller country might be that they intuitively include investors outside the boundary of the borrowing county but within the same country when gauging α. This intuition seems to be reasonable, because α actually represents the weight put to the investors by the borrowing government. There is no reason why a borrowing local government should inevitably discriminate investors from other counties or states since they may well have the same importance for the government as the local investors. There are many possible reasons for non-local and local investors being of equal importance for the local government. For instance, the capital can flow freely within the country, so local and non-local investors actually make the same contribution to back the local finance. Or they can freely chose the location of their business and hence contribute the same to the local labor market. Actually these reasons could also be applied to investors from other countries, possibly to a lesser extent. For example, countries in an explicitly declared union may also be so much integrated that investors from other countries are not discriminated so much by the borrowing government, so that the borrowing government no longer regards the default on the foreign investors' claims as a welcome wealth transfer to the domestic citizens. Of course, what really matters is the actual regard of investors' wealth position by the borrowing government and not the mere membership in a country or in a union. In a model with different voter groups as in Alesina and Tabellini (1990), α would be re-interpreted as the

[18] According to Broner et al. (2006), a low concentration of the wealth is necessary in order to rule out the possibility that the domestic investors collude and thus lower the repurchase price of the bonds.

part of the public debt which is held by the relevant voter group if there is no interconnection or interaction as mentioned above among the different voter groups.

Since what really matters is the regard of the investors' interest by the borrowing government, it is also possible for an economy to have a high α value without the membership in a wealthy country or union. One such alternative is a legal system which protects the investors' interest. As pointed out in the literature, a government with debt problems may be tempted to turn off or manipulate the legal restrictions in order to facilitate default. Whether this is feasible certainly depends on the specific construction of the legal system with regard to factors like how easily it can be manipulated or how investor-friendly it is. In appendix B.1 I show how the domestic welfare can increase when the government internalizes the lenders' interest more than the residents do – possibly through a rigorous lender-protecting law system which facilitates the economy to acquire capital which is necessary for the growth. In that example, I model the growth improving program as a direct investment with higher yield than the interest cost, but it could also be in the form of a reduction of the distortionary tax which raises the output as modeled in Cogan et al. (2013). There may also be other reasons why a small open economy may have a high α value, but this is not subject of this paper. This paper only shows that, first, a higher α value is associated with a lower default risk and lower borrowing cost, second, only the α value at the time point of debt repayment matters, and third, the α value corresponds to the weight assigned to the lenders by the borrowing government in its objective function. These three points should also hold even if the government debt is not tradable or only difficult to trade, with the only difference that in that case the domestic wealth would matter less in determining α.

4.2 The applicability of α for real world issues

The finding that a higher α can reduce default propensity seems to be confirmed empirically as Reinhart and Rogoff (2011) have observed that domestic default occurs less frequently and often happens under much more difficult economic conditions than external default does. Since their data covers several forms of public debt and is not limited to one-period government bonds which are analyzed in this paper, it would be interesting to see in further research whether the analytical analysis of other forms of sovereign debt yields qualitatively the same results as the findings shown here. Reinhart and Rogoff (2011) actually distinguish between domestic and external debt according to under which juridical law the debt is issued. This may suggest that rather institutional features like the legal system matter and not predominantly the country's wealth or a union membership, since a sovereign body can only borrow from abroad under domestic law if its legal system is held as trustworthy by worldwide investors. On the other hand, as the authors noticed, the status of domestic or external debt remains largely the same when one switches the classifying criterion from juridical governance to other criteria like citizenship of the lenders or currency denomination. Anyway, their empirical result is compatible with the hypothesis that a higher α reduces default risk while the higher α may result from a good legal system or from a large proportion of domestic lenders.

The model applied here assumes rational expectation – which is standard in the literature but has been questioned in the recent discussion about modeling strategy. It would be interesting to see whether the model outcome would change if this assumption was violated. For instance, when

arguing that a non-local investor may contribute the same to the domestic economy as the local investors, it is actually assumed that people can correctly gauge the indirect ownership, i. e. when residents work for or own stake of a non-local investing institution like a bank, then they can correctly anticipate that default on that bank has the same wealth effect as default on the residents. If the non-local investor is from the same country, it is more likely that the residents can have a feeling for such indirect links, since they identify themselves with people from their own country, even if they are from other regions. But if the non-local investor is from another country, then it is questionable whether the residents can still see this point. So it is possible that at the time point of debt repayment, people may underestimate α and hence default too readily, and only in the aftermath when the wealth loss becomes more and more evident they will realize that the default was not such a good idea, and a defaulting politician might have to resign, as documented in Borensztein and Panizza (2009). However, in this example, the possible violation of the rational expectation assumption only affects the factor determining α and not α itself. So the statement that a higher α can reduce the default risk still holds, only the economic links between lender and borrower countries may no longer correlate positively with α.

Another assumption underlying this model is that all the public debt is pooled together and hence not distinguishable from each other. The fact that the purchase and possession of government bond is often regarded as anonymous by the financial market justifies this assumption. This anonymity assumption also lets some investors have doubt about whether their interest will be regarded by the government at all since the government has no means to make sure whether its bond is purchased by the voters or not. However, the government can trace the sentiment of the average voter by e. g. looking at the poll results,[19] and the poll results, if representative enough, have already incorporated the congruence between the public debt and the wealth position of the voters.[20] By the way, the anonymity assumption is stronger than the non-distinguishability assumption, hence the result also holds in situations in which the public debt holding is not anonymous but nonetheless non-distinguishable, i. e. the government cannot discriminate the lenders even if it knows about their identity, possibly because such discriminatory behavior in the form of selective default is prohibited by law. Taking a look at the real world, this non-distinguishability assumption seems to be plausible and the norm, since a selective default could destroy the trust of the foreign investors so that they would not invest in this economy at all. Consequently a developed economy often sets up some juridical framework which makes such discrimination difficult. A less obvious form of a selective default is to default on all debt but only rescue the domestic industry, as described in Gennaioli et al. (2010). However, if the economic integration in the union is symmetric, i. e. the domestic citizens are stakeholders not only of domestic firms but also of foreign firms, then such a *de facto* selective default will be ineffective since rescuing the domestic industry and rescuing the foreign industry have the same welfare effect for the domestic citizens.

[19] The likely significance of the poll results for the default or repayment decision of the government is for instance documented by Tomz (2002).

[20] Of course, if the voters are unable to see the indirect ownership as in the above example, then only their direct government bond ownership would enter α.

4.3 A selection of related literature

Admittedly, the analysis here is rather simplified and stylized in order to focus on the effect of α which has not been extensively studied before. Factors like inertia in debt distress as shown in Binder et al. (2015) are not considered. So the conclusions drawn here apply in the first place to the kind of sovereign debt modeled in this paper, namely the one period government bond. Whether the same relationship holds for other types of public debt remains to be shown by further research. I consider the study of the effect of α in alternative model settings worthwhile, since the model outcome here suggests α to be a potentially new determinant of default risk of sovereign debt.[21] Furthermore, the model only considers borrowing from the financial market, and hence the market return consideration is a key factor in determining the debt contract. However, many developing countries, especially during financial distress, heavily rely on borrowing from international organizations like the IMF or the World Bank. Whether the findings in this paper concerning the effect of α still hold for public lending which does not necessarily require the maintenance of the market return but nonetheless requires a minimum return, remains to be shown. For further reading about public lending the interested reader may consult e. g. Binder and Bluhm (2010) about IMF program participation.

In order to focus on the effect of α, I have adopted a partial equilibrium (PE) analysis. Though it is not the purpose of this paper to do a general equilibrium (GE) analysis, I have set up the model in the way to facilitate its implementation in a GE. So instead of specifying a particular functional form, I have used a general functional form with some specifications to its first and second derivatives, so that it can be easily fit into another model with specific functional forms. For instance, the deadweight loss from taxation is taken as exogenous in this PE analysis; when implemented in a GE analysis, the deadweight loss can well be an endogenous outcome like reduced labor supply due to higher income tax as in Schabert (2010), and the model outcome will be qualitatively the same as long as the deadweight loss function in the reduced form is convex, as specified in this paper.[22] Consequently, the result here is rather complementary to the existing work and can be used as a building block to be implemented in addition to other relevant factors in a more full-fledged framework like a DSGE model or an agent based model to quantitatively assess the risk and value of government borrowing. The model can in general also be implemented in a model comparison approach as in Taylor and Wieland (2009).

5 Conclusion

In this paper, I have investigated the relationship between α, a parameter representing how well the average lender's wealth position is regarded by the borrowing government, and the default risk as well as the associated borrowing cost for the government. The main finding is that a higher α

[21] Surely the α value is not the sole factor affecting the default risk of government bond. Other well-known factors include a. o. the output, the debt burden or the term structure as e. g. in Borgy et al. (2011) etc. For a survey of the different aspects studied in the existing sovereign default literature one can read e. g. Stähler (2011).

[22] Whether the model result still holds when one or more of the specifications of the functional forms are violated has not been checked here. Hence the specifications are rather sufficient conditions than necessary conditions.

can lower the default propensity of the government. By showing that the good equilibrium is the only stable equilibrium, I demonstrate that a lower default propensity also leads to a lower default risk and hence more favorable borrowing conditions which in turn enhance the government's ability to repay. This theoretical finding is in line with empirical results from the literature, though further research is still needed in order to extend the model for a broader scope of application. In the case that α can change over time, e. g. when the government debt is freely tradable, only the α value at the time point of debt repayment matters and thus needs to be gauged at the time point of debt issuing.

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[23] The names are sorted alphabetically according to the surname.

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A Appendix

A.1 Proof of Theorem 1

Consider two different values of α with $0 \leq \alpha_1 < \alpha_2 < 1$. Given the outstanding amount of debt $z_{t-1}b_{t-1} \geq 0$ and any endowment y_t, denote the corresponding value function given non-default decision for α_1 and α_2 as $\mathbb{V}^n((1-\alpha_1)z_{t-1}b_{t-1})$ and $\mathbb{V}^n((1-\alpha_2)z_{t-1}b_{t-1})$, respectively. Further denote the associated optimal policy choice for new debt taking and government expenditure as (b^n_{1t}, g^n_{1t}) and (b^n_{2t}, g^n_{2t}), respectively. The contracted interest rate z_t is a function of $(1-\alpha)b_t$: $z_t = z((1-\alpha)b_t)$. Denote the equilibrium contracted interest rates for α_1 and α_2 as z_{1t} and z_{2t} with $z_{1t} \equiv z\left((1-\alpha_1)b^n_{1t}\right)$ and $z_{2t} \equiv \left((1-\alpha_2)b^n_{2t}\right)$. Then we have:

$$
\begin{aligned}
&\mathbb{V}^n((1-\alpha_2)z_{t-1}b_{t-1}) \\
=&\mathbb{U}((y_t - (1-\alpha_2)z_{t-1}b_{t-1} + (1-\alpha_2)b^n_{2t} - g^n_{2t}), g^n_{2t}) + \beta E_t \mathbb{V}^f((1-\alpha_2)z_{2t}b^n_{2t}) \\
\geq&\mathbb{U}((y_t - (1-\alpha_2)z_{t-1}b_{t-1} + (1-\alpha_2)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t} - g^n_{1t}), g^n_{1t}) \\
&\quad + \beta E_t \mathbb{V}^f((1-\alpha_2)z\left((1-\alpha_2)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t}\right)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t}) \\
\geq&U(y_t - (1-\alpha_1)z_{t-1}b_{t-1} + (1-\alpha_1)b^n_{1t} - g^n_{1t}), g^n_{1t}) + \beta E_t \mathbb{V}^f((1-\alpha_1)z_{1t}b^n_{1t}) \\
=&\mathbb{V}^n((1-\alpha_1)z_{t-1}b_{t-1})
\end{aligned}
$$

A.2 Proof of Theorem 2

From equation (13) we have that:

$$x'(\tau_1)(z_0b_0 - p'(\theta_1)) + (1-\alpha)z_0b_0 - p'(\theta_1) = 0$$

First look at the first term $x'(\tau_1)(z_0b_0 - p'(\theta_1))$: From the above equation we see that $x'(\tau_1)(z_0b_0 - p'(\theta_1))$ and $(1-\alpha)z_0b_0 - p'(\theta_1)$ should have different signs. At the optimum we have $x'(\tau_1) > 0$ since otherwise welfare can be increased by e. g. increasing tax financed government expenditure, hence the sign of $x'(\tau_1)(z_0b_0 - p'(\theta_1))$ is determined by the sign of $z_0b_0 - p'(\theta_1)$, therefore $z_0b_0 - p'(\theta_1)$ and $(1-\alpha)z_0b_0 - p'(\theta_1)$ will have different signs. Because $z_0b_0 - p'(\theta_1) > (1-\alpha)z_0b_0 - p'(\theta_1)\ \forall\ z_0b_0 > 0$ so $z_0b_0 - p'(\theta_1) > 0$. And from the budget constraint we have that the first derivative of τ_1 over θ_1 is $-z_0b_0 + p'(\theta_1) < 0$, so around the optimum, increasing θ_1 always lowers τ_1, and hence also lowers $x'(\tau_1)$ since $x''(\tau) > 0$. Further, when θ_1 rises then $-p'(\theta_1)$ will fall or at least not rise since the penalty cost function $p(\theta_1)$ is assumed to be non-concave. So $z_0b_0 - p'(\theta_1)$ also falls with θ_1. Consequently, the whole first term will fall with rising θ_1. Now look at the second term $(1-\alpha)z_0b_0 - p'(\theta_1)$, we see that the second term also moves in the opposite direction as θ does since $p''(\theta_1) \geq 0$ by assumption. Hence the whole LHS decreases with θ. On the other hand, a higher α will lower the LHS at the rate z_0b_0, which needs to be offset by a lower θ_1 to let the equation further hold until θ_1 hits its lower bound, and vice versa. Consequently we have that $\frac{d\theta}{d\alpha} \leq 0$, and the adjustment of θ continues until the upper or lower bound is hit. When denoting the LHS as a function f, then we can express the proof in a more formal way:

$$\frac{\partial f}{\partial \theta} = -x''(\tau_1)(z_0 b_0 - p'(\theta_1))^2 - x'(\tau_1)p''(\theta_1) - p''(\theta_1) < 0$$
$$\frac{\partial f}{\partial \alpha} = -z_0 b_0 < 0 \Rightarrow$$
$$\frac{d\theta}{d\alpha} = -\frac{\partial f/\partial \alpha}{\partial f/\partial \theta} < 0$$

So when the optimal default rate θ^* lies in the interval $[0,1]$ then we will have $\frac{d\theta^*}{d\alpha} < 0$. And when the upper or lower bound has been hit, we must have $\frac{d\theta^*}{d\alpha} = 0$. Take it together, we have $\frac{d\theta^*}{d\alpha} \leq 0$.

A.3 Proof of Theorem 3

Consider the value function for the infinite-horizon model:

$$\begin{aligned}\mathbb{V}(y_t, z_{t-1}b_{t-1}; \alpha) = \sup_{b_t \in B_t, \theta_t \in [0,1], g_t \in G_t} & \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}(y_{t+1}, z_t b_t; \alpha)\} \quad \text{s. t.} \\ c_t &= y_t - x(\tau_t) - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} + (1-\alpha)b_t - g_t - p(\theta_t) \\ \tau_t &= (1-\theta_t)z_{t-1}b_{t-1} + g_t - b_t + p(\theta_t) \\ z_t &= \inf[z:\ z_t E_t[1-\theta_{t+1}] = R_t]\end{aligned}$$

Take the FOC over θ_t, we get the same equation as (13). Then follow the same steps as in proof of **Theorem 2**, we also get the same result, namely $\frac{d\theta^*}{d\alpha} \leq 0$.

B Examples

I illustrate the previous analysis in section 3 by re-considering two well-known models from the earlier literature and extending them with the parameter α. Both models can be interpreted as special cases of the rather general framework set up before and each represents one type of sovereign default model. The first example comes from the seminal work in Calvo (1988) and serves as representative of the model of the default rate as treated in section 3.2.

B.1 The two-period model by Calvo

In Calvo's model about default in form of contract violation, there are two periods: the debt contract is signed in the first period and the repayment or repudiation decision is made in the second period. A default decision will incur default cost as a fraction of the repudiated amount. Since there is no third period, no new loan will be taken in the second period, and the value function of the second period (the period of repayment decision) collapses to the utility function:

$$\mathbb{V}(zb) = \sup_{\theta\in[0,1],g\in\{\bar{g}\},\tau\in\mathbb{R}} \{\mathbb{U}(y-\tau-x(\tau)+\alpha(1-\theta)zb,g)\} \tag{14}$$

$$\equiv \sup_{\theta\in[0,1],\tau\in\mathbb{R}} \{y-\tau-x(\tau)+\alpha(1-\theta)zb\} \quad \text{s. t.} \tag{15}$$

$$(1-\theta)zb+g = \tau - p(\theta) = \tau - \omega\theta zb \tag{16}$$

For notational simplicity, I have dropped out all time subscripts as Calvo did in his paper which does not disturb reading since this is a two-period model and each relevant variable is determined only one time. What we should keep in mind is that contracted gross interest rate z and debt amount b are both determined in period 0 while all other variables in period 1, the period of debt repayment.

Equation (14) is the utility function of the representative agent in the repayment period, which is assumed to be a function of after tax final wealth which will be all consumed and of government expenditure. Because government expenditure is constant, this maximization problem collapses to maximize the after tax final wealth which is the sum of total income y (including labor income and capital income) and wealth receipt $\alpha(1-\theta)zb$, minus tax τ and dead weight loss $x(\tau)$, as stated in equation (15). Since Calvo's model is about domestic debt, he has set $\alpha = 1$, but I want to investigate the difference in repayment decision due to different α values, hence I will further allow α to vary within the interval $[0,1]$

The contracted gross interest rate from last period must give the investors the market return R:

$$zE_0[1-\theta] = (1-\theta)z = R$$

Here the expectation symbol can be dropped out because we are dealing with deterministic case.

And as explained before, I will only consider the minimum interest rate which satisfies the participation constraint of the financial investors, i. e. I only consider the "good equilibrium" in the Calvo model.[24] Hence the above equation becomes to:

$$z = \inf[z : (1-\theta)z = R] \tag{17}$$

The deadweight loss function due to taxation satisfies the following:

$$\begin{aligned} &x(0) = x'(0) = 0 \\ &x"(\tau) > 0 \quad \forall\tau \\ &\lim_{\tau\to\infty} x"(\tau) = \infty = -\lim_{\tau\to-\infty} x"(\tau) \end{aligned} \tag{18}$$

[24] Indeed, as Calvo has noticed, the equilibrium interest rate in the "bad equilibrium" will decrease with increasing outstanding amount of debt which is "paradoxically". The reason why a "bad equilibrium" with the just mentioned characteristic is rarely observed has been detailed in section 3.3.

Rearranging equation (16) yields:

$$\theta = \frac{zb + g - \tau}{(1-\omega)zb} \tag{19}$$

Equation (19) tells that the default rate θ is decreasing in the chosen tax load τ which is intuitive: the more tax is raised, then given government expenditure, the more debt can be repaid, further the less default cost will be incurred which in turn increases the repay ability of the government.

Plugging the government budget constraint (16) into the government maximization calculus (15) to eliminate θ,[25] and focus on the part which can be influenced by the government policy, it can be shown that the maximization problem of the government is equivalent to the following minimization calculus:

$$\inf_{\tau \in \mathbb{R}} \{x(\tau) - \frac{\omega - (1-\alpha)}{1-\omega}\tau\}$$

Hence, the desired tax τ^* is characterized by $x'(\tau^*) = \frac{\omega-(1-\alpha)}{1-\omega}$. In case of domestic debt, i. e. $\alpha = 1$ the result is the same as in Calvo (1988): $x'(\tau^*) = \frac{\omega}{1-\omega}$. As α decreases, relatively more weight in the policy decision will be put on the interest of the domestic tax payers than of the lenders; as a consequence, $x'(\tau^*)$ will decrease, which means less desired tax load τ due to the assumed property of the deadweight loss function as stated in (18), which in turn means that more will be repudiated as θ is a decreasing function of τ (equation (19)), and θ will increase so far until it hits its upper bound. Hence, a government within higher α will repudiate less or equal given all other factors constant, as I have already shown in section 3.2. Note that the θ here is not necessarily on the equilibrium path, but the institutional assumption made in section 2 assures that the equilibrium repudiation ratio never increases with increasing α; and as I have shown in 3.3, the negative correlation between θ and α also holds almost surely under much more general assumption. What we will see the next is that the maximal available loan amount for this small open economy strictly increases with α, too. With other words, a government with a higher α is less credit constrained than a comparative government with lower α.

To illustrate the impact of α on the repayment behavior and the associated borrowing condition for this small open economy, I have drawn Figure 2 which is in the style of the Figure 2 in Calvo (1988) and which is then modified somewhat to highlight the different repayment decision and borrowing condition due to different α values.

The two dashed lines radiating in direction north east represent the boundary of τ^*, the tax level chosen by the government of this small open economy: since θ is constrained in the interval $[0,1]$, so τ^* is constrained in the interval $[\bar{g} + \omega bz, \bar{g} + bz]$. Hence for α below some threshold value, the desired taxation τ^* will be so low that full repudiation will occur for any b. Since the

[25] Here I do not optimize over θ as I did in the main analysis but optimize over τ instead, to facilitate the comparison with the result from Calvo (1988), also to show that the conclusion concerning the effect of α holds irrespective of the way of solving the model.

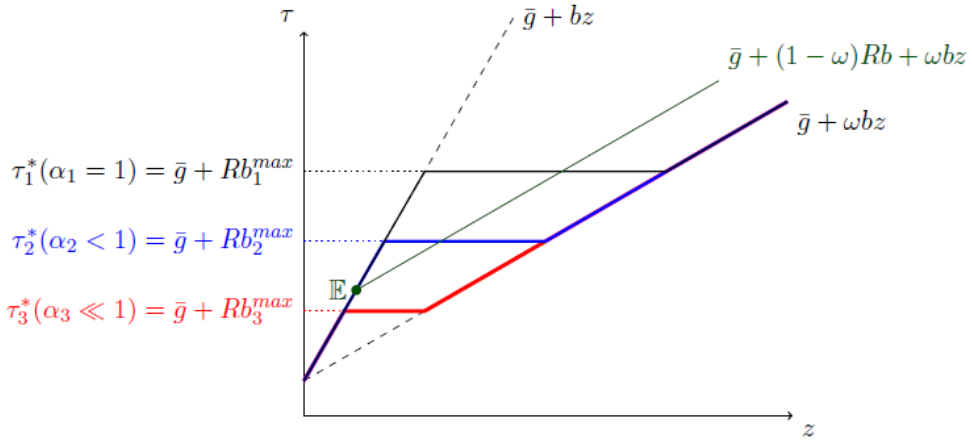

Figure 2: The equilibria under different α

lenders anticipate that, the government of this small open economy will be completely constrained from the financial market and cannot borrow at all as the low α value cannot maintain enough trust for the lenders to invest in this economy. Indeed, suppose $\alpha = 0$, i. e. the government does not consider the welfare of the lenders at all, then more repudiation always dominates less repudiation since that means reduction in wealth transfer to the lenders which exceeds the repudiation cost incurred since the repudiation cost is assumed to be a fraction of the repudiated amount of debt. Further, more default also means reduction of deadweight loss due to less tax load which enables this small open economy to consume more. Hence in the last period the government will always choose full repudiation. And when the financial market participants anticipate this, they will not lend to the government of this small open economy at all.

When $\alpha = 1$, Figure 2 will be the same as in Calvo (1988): the wealth loss of the lenders resulting from a default is fully taken into account in the government's decision making. Since a default is nothing but a wealth transfer from lenders to tax payers which does not increase the welfare of the representative agent, the only reason why the government wants to default is to reduce deadweight loss resulting from too high taxation. Therefore, the government will only default if the gain from reducing deadweight loss exceeds the default cost, while wealth transfer from lenders to tax payers plays no role here. As a consequence, the government will have a higher desired tax level, represented by τ_1 (the black line), and is less inclined to default so that the financial market participants are more willing to grant loan to the government since they anticipate a higher repay ratio.

As α decreases, the horizontal part in the solid line, which presents the desired tax level τ^*, will get lower and lower. All other lines remain in the same position since they are derived from the government budget constraint and the financial market participation constraint which are independent from α. Therefore, as α gets to be small enough, the solid line will lie completely below the line representing the "consistency condition", $\bar{g}+(1-\omega)Rb+\omega bz$, i. e. no contract $\{z,b\}$ can fulfill the market participation constraint for the financial investors, so the government is fully debt constrained. One of such unsustainable desired tax level is depicted as the red line τ_3 and the tax level taking account of the boundary of θ corresponds to the red path .

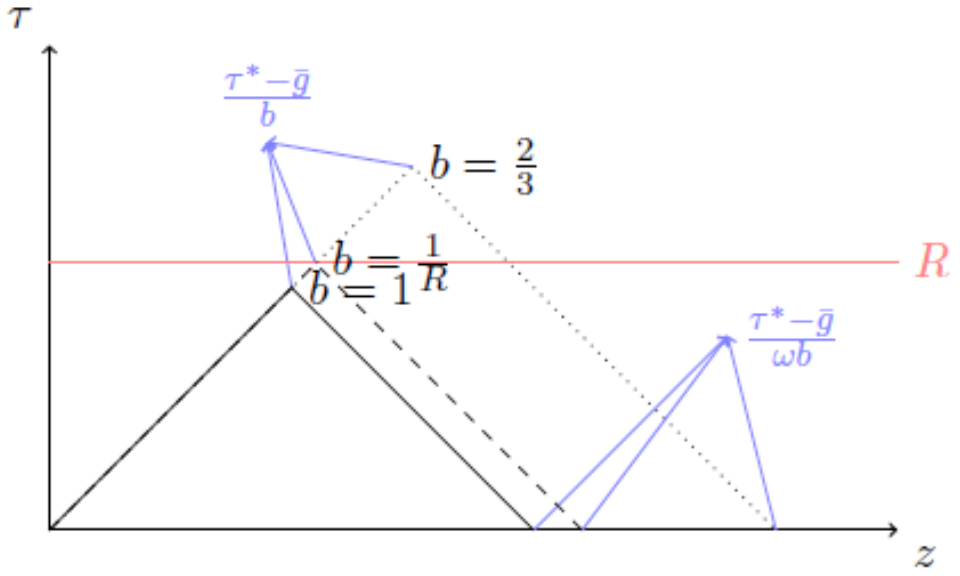

Figure 3: The equilibria under different b

For all values of α which support an equilibrium contract,[26] the contracted interest rate z always equals the market return R since it lies on the full-repayment path $\bar{g}+bz$ and hence no repudiation will be expected under this condition.[27] However, the maximal available debt amount will increase with increasing α. To see this, note $\bar{g}+zb \leq \tau^*$ is the necessary condition for an equilibrium contract to exist, hence the maximal available debt amount b^{max} is equal to $\frac{\tau^*-\bar{g}}{z} = \frac{\tau^*-\bar{g}}{R}$. Since τ^* increases with increasing α, so does b^{max}. In words: a higher α lets the investors have more trust in the government's ability and willingness to repay its debt, hence they are ready to lend more to the government.

Figure 3 shows how the expected return $(1-\theta)z$ varies with increasing contracted interest rate z under different debt amount b:

From equation (19) we have that θ is an increasing function of z. However, since θ is lower-bounded by zero, so for z less than or equal to $\frac{\tau^*-\bar{g}}{b}$, the expected return $(1-\theta)z$ is equal to the contracted interest rate z, which means that the first part of the expected return curve is a 45° straight line. After this peak point has been reached, the expected return curve will become a downwards sloping line as $(1-\theta)z = \frac{-\omega zb-\bar{g}+\tau^*}{(1-\omega)b}$ is a linear decreasing function of z. When z gets to be as high as $\frac{\tau^*-\bar{g}}{\omega b}$, θ will reach its upper bound of one, from here on the expected return curve will overlap with the x-axis which I do not plot here. Although figure 3 does not look the same as figure 1 in section 3.3, but it maintains the feature of having a first increasing and then decreasing

[26] Here I have only depicted two of them: the full internalization of lender's interest, $\alpha_1 = 1$, and a smaller α_2 which also allows the maintenance of expected market return

[27] The other intersection of the consistency condition line with the tax path is the so called "bad equilibrium", and can be ruled out as shown in 3.3.

part above the x-axis, hence the analysis in 3.3 with regard to the multiple equilibria problem also applies here.

In figure 3 I have set $\tau^* = 2$, $\bar{g} = 1$, $R = 1.11$ and $\omega = 0.5$. Then I vary b between $\frac{2}{3}$, $\frac{1}{R}$ and 1. For $b = \frac{2}{3}$, the expected return curve cuts the market return R at two points, and I have shown in 3.3 that only the left intersection for which $\theta = 0$ and $z = R$ can be a stable equilibrium. As b increases, the expected return curve moves downwards or inwards. When $b = \frac{1}{R}$ which equals the maximal debt amount $b^{max} = \frac{\tau^* - \bar{g}}{R}$, the expected return curve has only one tangent point with the red line representing the market return, which is also the only equilibrium in this case. And as we see, before b^{max} has been reached, the stable equilibrium always lies in the point in which $\theta = 0$ and $z = R$. When b further increases e. g. to $b = 1$, the expected return curve will lie below the market return line, and there exists no equilibrium. Since τ^* increases with increasing α, a higher α will push all the curves upwards or outwards, so that a curve associated with $b > \frac{1}{R}$ will eventually get to be tangent to R, thus I show that a higher α will enable a higher b^{max}.

Usually, a more favorable borrowing condition (here in form of less credit constraint, i. e. a higher b^{max}) also means a welfare increase (at least not decrease) for the respective economy. However, in the original Calvo model, this welfare effect cannot be seen directly since only the last period is considered, in which more debt rather means more duty to repay, hence a higher b will rather reduce the average consumption, here a proxy for social welfare. To let government borrowing make sense, we need to go one period back and try to figure out why this government wants to borrow at all.

In period 0, the government chooses to borrow b because the desired debt amount can make the economy better off. If the government is constrained in borrowing, then it will simply borrow as much as it can. The welfare enhancement from borrowing can arise for many reasons: from consumption smoothing to inter-temporal allocation of tax load to some profitable investment opportunity which needs to be financed by debt, etc. For simplicity here I only consider the case of favorable investment opportunity, i. e. with one unit borrowed money which is contracted under the market return R, the government can conduct some investment which will increase its next period's revenue by R' with $R' > R$. Hence the last period's consumption reads as follows:

$$\begin{aligned} c &= y - \tau - x(\tau) + \alpha R b \\ &= y + R'b - g - (1-\alpha)Rb - x(g + Rb - R'b) \end{aligned}$$

Taking the first derivative yields:

$$\frac{\partial c}{\partial b} = R' - (1-\alpha)R + x'(g + Rb - R'b)(R' - R)$$

By assumption we have that $R' - R > 0$ and hence $R' - (1-\alpha)R > 0$. Further $x'(\tau) > 0 \; \forall \; \tau > 0$, hence $\frac{\partial c}{\partial b}$ is strictly positive, i. e. the consumption in the last period increases with increasing debt taken in the previous period so that the government will always take as much debt as possible, hence a looser borrowing constraint, i. e. a higher b^{max} always makes the economy better off since it is now able to consume more.

Another interesting finding here is that even if we assume that the domestic economy (not the government) cannot internalize the foreign lenders' interest as their own and only considers the domestic consumption in the last period $c^d = y + R'b - g - Rb - x(g + Rb - R'b)$,[28] then also this term strictly increases with increasing b and hence more consideration of lenders' interest i. e. a higher α can increase the domestic economy's ability to consume and make them better off, although now the government has incorporated (partially) the foreign lender's interest and no more focuses solely on the domestic citizen's welfare. The reason is that here I assume rationality of the financial market and hence the foreign lenders can always correctly anticipate the government's repay behavior in the next period. If they feel that their interest will not be adequately considered in the next period and hence there is a higher default risk, then they will raise the contracted interest rate and/or, as here, reduce maximal available loan amount to the government. As a result, the domestic economy will always pay for the contracted debt amount, if they can get any, the expected market return, but if they consider more the lender's interest, then they can borrow more here due to perceived lower default risk, so that a higher α will make the domestic citizens better off. To be precise: the final domestic consumption is $c^d = y + (R' - R)b^{max} - g - x(g - (R' - R)b^{max})$ which is increasing in b^{max}. As mentioned above, a higher α will increase (and at least not decrease) b^{max}, hence more consideration of lenders' interest will also make the borrowing economy better off whose welfare increases due to more favorable borrowing condition.

B.2 The model by Eaton and Gersovitz

In Eaton and Gersovitz (1981), purely external debt has been considered. As mentioned, I extend this model by the parameter α so that purely external debt can be viewed as an extreme case in which $\alpha = 0$. But the feature that there is either full repayment or full repudiation remains, so that the expected default rate is equal to the default probability as elaborated in 3.1.

To the model: In each period, a random output y_t has been drawn, from which the old debt b_{t-1} has to be repaid (possibly with repudiation on the fraction of θ_t). In case of full repayment, new debt $b_t \in B_t$ can be taken, with $B_t = 0 \; \forall \, t \geq \tau$ if $\theta_\tau > 0$, i. e. one time (possibly partial) default will exclude the government from financial market participation forever. In Eaton and Gersovitz (1981), the absorption c_t is $y_t - (1 - \theta_t)z_{t-1}b_{t-1} + b_t$ since any repayment of outstanding debt will reduce the economy's absorption one by one. But I want to show how does α affect the repay propensity of the government, hence in the extended model, the economy's absorption reads as follows:

$$c_t = y_t - (1 - \alpha)(1 - \theta_t)z_{t-1}b_{t-1} + (1 - \alpha)b_t \tag{20}$$

The government's objective function is to maximize the current and discounted future utility from absorption minus some penalty cost P_t which is imposed in case of default in addition to the financial market exclusion:

[28] This could be the case if the lenders' interest is internalized by the government through an investor-friendly legal system.

$$\max_{\theta_t \in [0,1]} E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_\tau - P_\tau) \right]$$

One straightforward conclusion from equation (20) is that, in case of purely domestic debt, i. e. $\alpha = 1$, the government of this small open economy will have no incentive to repudiate in any state, since repudiation is just the wealth transfer from one citizen to another citizen and will not affect the economy-wide absorption at all. Anticipating this, the financial market is always willing to lend to the government given the market return, regardless of the contracted debt amount as well as of the expected level and volatility of the future output. But in this case, the optimal debt amount for the government, b_t, is also undetermined since this model assumes that there is no any storage technology, nor is there any dead weight loss due to taxation, and hence all debt is immediately absorbed, so that borrowing money from domestic lenders just lead to the same absorption and consequently it has no impact on the welfare of the domestic citizens.

For $\alpha < 1$, the government will prefer in some states default whenever this decision can enhance the welfare of the representative agent. And whenever the government chooses to default, it will default on the whole outstanding amount of debt, i. e. set $\theta_t = 1$, since both exclusion from the financial market and the possible penalty cost P_t will occur for any $\theta_t > 0$, independent from the amount being repudiated. Denote the probability of default in the next period as λ_t, then in equilibrium:

$$z_t(1-\lambda_t) = R_t \tag{21}$$

In the following I will show that the default probability λ_t is a decreasing function of α, i. e. if the financial investors expect their interest to be more regarded by the government, then they will anticipate a lower default probability.

Denote the value function in case of default as $\mathbb{V}_t^D$, then $\mathbb{V}_t^D$ for a government with outstanding amount of debt $z_{t-1}b_{t-1}$ and the parameter α is given by:

$$\mathbb{V}_t^D = E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(y_\tau - P_\tau) \right] \tag{22}$$

Since the above expression is independent from α, the value of default for a government, $\mathbb{V}_t^D$, is also independent from α.

Denote the value function in case of full repayment as $\mathbb{V}_t^R$, then the value function for period t with access to financial market is given by $\mathbb{V}_t^f = \max(\mathbb{V}_t^D, \mathbb{V}_t^R)$. And $\mathbb{V}_t^R$ for a government with outstanding amount of debt $z_{t-1}b_{t-1}$ and the parameter α can be expressed as:

$$\mathbb{V}_t^R((1-\alpha)z_{t-1}b_{t-1}) = \sup_{b_t} U(y_t - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b_t) + \beta E_t[\mathbb{V}_{t+1}^f((1-\alpha)z_t b_t)]$$

Now consider two governments with different α values, say, $\alpha_1 > \alpha_2$, and all other variables being equal, then it can be shown that the government with a higher α value will also have a higher $\mathbb{V}_t^R$:

$$\begin{aligned}
\mathbb{V}_t^R((1-\alpha_1)z_{t-1}b_{t-1}) &= U(y_t - (1-\alpha_1)z_{t-1}b_{t-1} + (1-\alpha_1)b_{1t}^*) + \beta E_t[\mathbb{V}_{t+1}((1-\alpha_1)z_{1t}^* b_{1t}^*)] \\
&\geq U(y_t - (1-\alpha_1)z_{t-1}b_{t-1} + (1-\alpha_1)\frac{1-\alpha_2}{1-\alpha_1}b_{2t}^*) + \beta E_t[\mathbb{V}_{t+1}((1-\alpha_1)z_{2t}^*\frac{1-\alpha_2}{1-\alpha_1}b_{2t}^*)] \\
&\geq U(y_t - (1-\alpha_2)z_{t-1}b_{t-1} + (1-\alpha_2)b_{2t}^*) + \beta E_t[\mathbb{V}_{t+1}((1-\alpha_2)z_{2t}^* b_{2t}^*)] \\
&= \mathbb{V}_t^R((1-\alpha_2)z_{t-1}b_{t-1})
\end{aligned} \tag{23}$$

In the above expression, b_{it}^* with $i \in \{1,2\}$ is the optimal new debt amount chosen by the respective government and z_{it}^* with $i \in \{1,2\}$ is the contracted interest rate associated with the respective α_i and b_{it}^*. In case of positive outstanding amount of debt, i. e. $z_{t-1}b_{t-1} > 0$, the second $\geq$ will hold with strict inequality.

The inequality (23) tells that a government with a higher α will always value the full repayment more (at least not less) than a government with a lower α does, while the value from full default remains the same according to equation (22). Therefore, in any state, a government with a higher α will always pay back its debt whenever the comparative government with a lower α chooses to repay its debt; and the government with a higher α will only choose to default if the comparative government with a lower α also decides to default. Since $\lambda_t \equiv \Pr(\mathbb{V}_{t+1}^D > \mathbb{V}_{t+1}^R)$, hence λ_t is a decreasing function in α. And from 3.3 we know that a government with lower propensity to default can enjoy a more favorable borrowing condition and therefore the borrowing cost of the government, here the contracted interest rate z_t, will also decrease with increasing α.