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Working Paper

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Working Paper, No. 14-8

Provided in Cooperation with:

University of California Davis, Department of Economics

Suggested Citation: Shipper, Burkhard C.; Woo, Hee Yeul (2014) : Political awareness, microtargeting of voters, and negative electoral campaigning, Working Paper, No. 14-8, University of California, Department of Economics, Davis, CA

This Version is available at:

<https://hdl.handle.net/10419/109678>

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DEPARTMENT OF ECONOMICS

Working Paper Series

Political Awareness, Microtargeting of Voters, and Negative Electoral Campaigning

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March 11, 2014

Paper # 14-8

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POLITICAL AWARENESS, MICROTARGETING OF VOTERS, AND NEGATIVE ELECTORAL CAMPAIGNING*

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March 11, 2014

Abstract

In modern elections, ideologically motivated candidates with a wealth of information about individual voters and sophisticated campaign strategies are faced by voters who lack awareness of some political issues and are uncertain about the exact political positions of candidates. We study to what extent electoral campaigns can raise awareness of issues and unravel information about candidates' political positions. We allow for microtargeting in which candidates target messages to subsets of voters. A candidate's message consists of a subset of issues and some information on her political position in the multi-dimensional policy subspace spanned by this subset of issues. The information provided can be vague, it can be even silent on some issues, but candidates are not allowed to bluntly lie about their ideology. Every voter votes for the candidate she expects to be closest to her but takes into account only the subspace spanned by the issues that come up during the campaign. We show that any prudent rationalizable election outcome is the same as if voters have full awareness of issues and complete information of policy points, both in parliamentary and presidential elections. We show by examples that these results may break down when there is lack of electoral competition, when candidates are unable to use microtargeting, or when voters have limited abilities of political reasoning. Allowing for negative campaigning restores the positive results if voters' political reasoning abilities are limited. It can even be achieved with just public campaign message in the presidential elections while parliamentary elections still require microtargeting of voters.

Keywords: Electoral competition, multidimensional policy space, microtargeting, dog-whistle politics, negative campaigning, ideological candidates, presidential elections, parliamentary elections, persuasion games, verifiable information, unawareness, framing, prudent rationalizability, forward-induction.

JEL-Classifications: C72, D71, P16.

*We thank Pierpaolo Battigalli, Oliver Board, Giacomo Bonanno, Jon Eguia, Ignacio Esponda, Boyan Jovanovic, Jean-François Laslier, Alessandro Lizzeri, Tymofiy Mylovanov, Joaquim Silvestre, Walter Stone, Thomas Tenerelli and seminar participants at NYU Stern and participants at WEIA 2012 for helpful comments. Burkhard is grateful for financial support from the NSF SES-0647811. An earlier version was circulated under the title "Political Awareness and Microtargeting of Voters in Electoral Competition".

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1 Introduction

We study to what extent electoral competition can effectively promote awareness of political issues and reveal information about political positions of candidates. To stack the deck against a positive result, our model features a stark asymmetry between voters and candidates. Voters may not think about all political issues and face incomplete information about political positions of candidates, while candidates are aware of all political issues, know perfectly the preferences of voters, and can use modern sophisticated campaign strategies including microtargeting to persuade voters. Nevertheless, we will show that election outcomes under voters' unawareness of many political issues and incomplete information about candidates' political positions are equivalent to election outcomes under full awareness of political issues and complete information about candidate's political positions. We also show that this positive result depends crucially on the strength of electoral competition, the ability of candidates to microtarget voters, and voters' political reasoning abilities.

Traditionally, candidates in electoral competition are portrayed as being completely opportunistic in their choice of political positions (i.e., Downs, 1957). While winning the election is an important motivation, we believe that candidates are constrained by their ideology. Politicians emphasize that their agendas reflect personal convictions (Hillygus and Shields, 2008, p. 40). For instance, during the 2000 presidential campaign, George W. Bush insisted that “we take stands without having to run polls and focus groups to tell us where we stand.” (Carney and Dickerson, 2000). Hillygus and Shields (2008) point out that political systems are structured in a way that individuals running for office are more to care about policy outcomes than the average citizen. We believe that parties and lobbies have mechanisms like primary elections that ensure the selection of ideologically motivated candidates (for empirical support, see for instance Brady, Han, and Pope, 2007). Thus, we will assume in our model that each candidate has a fixed political position.

The political positions of candidates may not be obvious to all voters and may pertain to many political issues that are shaping the complex political environment of an election. For instance, the University of Wisconsin Advertising Project identified more than 70 issues in the 2004 presidential campaign including government spending, minimum wages, immigration, abortion, homosexuality, gun control, narcotics, education, terrorism etc. We will assume that candidates have a fixed political position in a multidimensional policy space in which each dimension corresponds to an issue.

Candidates may raise only some political issues in a campaign but may be completely silent on others. On some of political issues raised, candidates may be intentionally vague about their political position while on others they may be completely transparent. Moreover, they

may intentionally communicate some information on some of those issues to some voters only but not to all voters. For instance, in the 2000 presidential campaign, Bush sent a letter to the U.S. Conference of Catholic Bishops, who weight influence over a traditional Democratic constituency, in which he pledged that taxes should not be used to fund research that involves the destruction of human embryos. Yet, stem cell research was not mentioned in nominated speeches of either candidate, not covered on their television advertising, not raised in presidential debates nor displayed on campaign web sites (see Hillygus and Shields, 2008, p. 2). This is an example of political campaigning that targets a subset of voters. Hillygus and Shields (2008, p. 5-6) write “(T)he contemporary information environment has made it easier ... to target issue messages to narrow segments of the population. With a wealth of information about individual voters, candidates are increasingly able to microtarget personalized appeals on the specific issues for which each voter disagrees with the other candidate. This fragmentation of the candidates’ campaign communications leads to *dog-whistle politics* – targeting a message so that it can be heard only by those it is intended to reach, like the high-pitched dog whistle that can be heard by dogs but is not audible to the human ear. By narrowly communicating issue messages, candidates reduce the risk of alienating other voters, thereby broadening the range of issues on the campaign agenda. For instance, our analysis finds that the candidates in the 2004 presidential election staked positions on more than seventy-five different policy issues in their direct-mail communications. Thus, new information and communication technologies have changed not only how candidates communicate with voters, but also who they communicate with and what they are willing to say.” In our model, we will allow for sophisticated targeting of voters with specific campaign messages and issues.¹

In order to successfully tailor campaign messages to voters, candidates need to know political preferences of voters. In the past, it was impossible for candidates to know individual political preferences of voters. Rather, they had to be content with aggregate information about voters’ preferences from opinion polls or similar. Yet, modern information technology allows to

¹This seems to be contrast to most of the theoretical literature on electoral competition. For instance, Laslier (2006), who presents an interesting study of ambiguity in electoral competition, writes that “(p)olitical communication is mass communication. If a politician was able to design a different talk for each elector, maybe each of these talks would be very clear. Actually, politicians can easily give way to the temptation of making different promises to different people.” He further writes “(a)n ambiguous electoral platform may be understood differently by individuals, and politicians would like to target their messages at different electors. For practical reasons, it is impossible to perfectly realize this targeting. From the normative point of view, it is interesting to consider that a party cannot at all target its communication at different voters. This simply corresponds to an hypothesis of equal information of the electors as to the party’s platform.” In light of the empirical evidence about modern election campaign strategies such as microtargeting, we take the view that not all political communication is mass communication in the public sphere but that candidates can communicate also privately with voters. We study what may happen if the traditional assumption of political communication being exclusively public is given up.

collect a wealth of individual data on voters, apply sophisticated data mining tools², and use this information strategically in campaigns. Recent campaigns merged voter registration files with consumer data that include names, addresses, address histories, driving records, criminal records, and consumer purchases like magazine subscriptions, mortgage information, credit-card purchases, gun ownership etc. (see Hillygus and Shields, 2008, p. 159, 161).³ This information was then used to “microtarget messages through direct mail, email, telephone calls, and personal visits.” As Sara Taylor, a strategist for Bush’s 2004 presidential campaign summed up “We could identify exactly who should be mailed, on what issues, and who should be ignored completely.” (quoted from Hillgus and Shields, 2008, p. 161). We reflect this “transparent” voter in our model by assuming that candidates have perfect information about voters’ most preferred policy points.

So far, we painted a picture of elections in a complex political environment with many issues and with candidates who hold fixed political positions, know perfectly voters’ preferences and use sophisticated campaign strategies including microtargeting messages to subsets of voters. On the voters’ side we assume that voters may have limited political awareness in that they do not take all political issues into account when forming their preferences over candidates. Moreover, they face incomplete information about candidates’ political positions on issues they are aware of. In electoral competition, candidates may raise some political issues to some subset of voters and other issues to other voters, and provide more or less precise information about their political positions on those issues to selected subsets of voters. Despite the extreme asymmetric awareness and information between candidates and voters, we will show that in elections with two candidates, election outcomes are the same as under full awareness and complete information. This holds for both presidential elections in which candidates care about

²In 2011, the Obama campaign posted a job ad for a “Predictive Modeling and Data Mining Scientists/Analysts” on KDnuggets, a website who bills itself as “Data Mining Community’s Top Resource for Data Mining and Analytics Software, Jobs, Consulting, Courses, and more”. See <http://www.kdnuggets.com/jobs/11/07-13-obama2012-predictive-modeling-data-mining-scientists-analysts.html>.

³There are several commercial companies in the US like Aristotle, Camelot, and Catalist who collect individual voter records, merge them with other public and commercial data, and provide them for a fee to campaigns. For instance, Catalist claims to maintain a “database of over 265 million persons (more than 180 million registered voters and 85 million unregistered adults)”. The data include “Registered Voters and Non-Registered persons (with contact information)” but also “Commercial and Census Data ...” (see <http://catalist.us>). The company Aristotle claims that “(i)n addition to the wealth of demographics Aristotle already provides for high level micro-targeting, you can now identify your voters based on their interests and hobbies. Aristotle maintains a list of over 5.4 million voters who hold hunting and fishing licenses, as well as individuals who subscribe to a wide array of magazine subscriptions including family, religious, financial, health, culinary and Do-It Yourself publications.” Premium data are priced at \$0.06 per record for over 50,000 records (see <http://www.aristotle.com> or <http://www.voterlistsonline.com>).

only winning the election, as well as parliamentary elections in which candidates care about only their share of voters.

To provide some intuition, we like to sketch here some features of a simplified model. Consider two candidates and just one voter. Focusing on one voter allows us to explain more transparently some features of our model. The policy space is a multidimensional Euclidean space. Each issue corresponds to a dimension and the political positions of the candidates and the most preferred policy point of the voter are points in this space. The voter evaluates a candidate by how far the candidate's policy point is away from his most preferred policy point using Euclidean distance but only in a subspace spanned by the issues that he is aware of. Assume first that the voter is unaware of all but one issue and that candidates can campaign by raising issues to him. Further, we assume for the moment that once an issue has been raised by some candidate, the political positions of both candidates on this issue becomes completely transparent to the voter. Finally, we assume that both candidates know the voter's most preferred policy point and each other's political positions. We claim that in this extremely simplified model candidates will raise enough issues so as to produce an election result that would emerge also under full awareness of all issues. To see this note that candidates face a zero-sum game. If raising an issue is not beneficial to one candidate, it will be to the other. Thus, either all issues will be raised or raising further issues won't change the voting outcome anymore. Now assume that policy points on issues that have been raised do not become automatically transparent. Each candidate can provide some information on her political position. We assume that this information can be vague, that a candidate can be silent on issues, but that she can not bluntly lie in the sense of not including her political position (in the subspace of issues revealed) in the information she provides. This is reminiscent of models of verifiable information à la Grossman (1981), Milgrom (1981), and Milgrom and Roberts (1986)⁴ except that our model involves unawareness of some dimensions of the information and more than one informed party.⁵ We claim that despite unawareness and incomplete information, all "relevant" information is revealed to

⁴See also Okuno-Fujiwara, Postlewaite, and Suzumura (1990), Seidmann and Winter (1997), and Battigalli (2006). Battigalli (2006) employs a solution concept similar to ours. The paper is also related to persuasion games with commitment à la Kamenica and Gentzkow (2011) and Gentzkow and Kamenica (2013). The main difference is that in our model candidates learn their political position first and only then decide what to disclose while in persuasion games with commitment candidates would be required to commit to their disclosure strategy before learning their political position.

⁵The nihilistic popular opinion about politicians may dispute the assumption that politicians do not lie. But we believe that even politicians refrain from lying bluntly but rather resort to being vague about their positions or keeping quiet on some issues (and thus appear to provide "misleading information"), both features that we seek to analyze here. We also like to point out that standard models of electoral competition à la Downs (1957) do assume that politicians do not lie. See Callander and Wilkie (2007) for a rare study of lies in electoral competition.

the voter. To see this, note that a voter should realize that if a candidate does not provide more precise information on an issue that he is aware of, then it is because this information is not favorable to the candidate. This is a version of forward-induction reasoning embodied in the solution concept that we employ.

Different from a model without unawareness, this unraveling result depends crucially on the strength of electoral competition. For instance, in former socialist countries like the German Democratic Republic, “elections” consisted mainly of voting ‘Yes’ or ‘Abstain’ on a single list of candidates. There was no choice between the candidates possible.⁶ Alternatively, our model with one candidate only may be interpreted as model of state censorship, in which a ruler decides to what extent the press is allowed to make citizens aware of issues and provide information on those issues and voters decide whether to go along with the ruler or aim for a regime change. This is similar to Shadmehr and Bernhardt (2013) who study a setting in which a ruler can commit to censorship of information. We show in Section 5.1 that if there is only the choice between a candidate and a passive “status quo” (i.e., a passive candidate who is not campaigning at all), then our result may break down. This is different from standard models of verifiable information without unawareness à la Grossman (1981) and Milgrom (1981) in which even with a single “sender” there is full unraveling of information.

With more than one voter, we potentially face well-known limits to aggregation of voters’ preferences. In particular, since voters can have probabilistic beliefs about candidates’ positions, a candidate may find it useful to keep a voter uncertain about his political position.⁷ In particular, the “preference” of society may not correspond to a von Neumann-Morgenstern utility function even though each voter’s preference is captured by a von Neumann-Morgenstern utility. In Section 5.2 we present a simple example with three voters, in which one and the same candidate is elected if each of the possible political positions of the candidates were commonly known but the other candidate is elected under uncertainty over the candidate’s political positions. In this sense, there is a role for “ambiguity” in electoral competition.⁸ We note that this problem arises only if candidates are forced to provide the same (public) information to all voters. In this case, a voter not receiving more precise information cannot deduce that the

⁶Except for the last election in 1989, upon arrival at the ballot station voters were handed a list with the “Kandidaten der Nationalen Front” that they were supposed to put into the ballot box under the supervision of “helpers”. In the last election in 1989 before the “Wende”, polling booths were provided for the first time. Voters had the choice of using the polling booth to cross out the entire list of candidates, but there was still no alternative list of candidates.

⁷Related problems have been noted in Zeckhauser (1969) and Shepsle (1970, 1972).

⁸This topic lead to an extensive literature with different approaches. See Downs (1957), Shepsle (1972), Alesina and Cukierman (1990), Aragonès and Neeman (2000), Aragonès and Postlewaite (2007), Glazer (1990), Jensen (2009), Laslier (2006), McKelvey (1980), Meirowitz (2005), and Page (1976).

candidate’s political position is not favorable to him because it may just be unfavorable to some other voters and that’s the reason why this more precise information is not communicated by the candidate. Last issue can be circumvented in our model by allowing targeted campaigning as motivated above, in which candidates can provide differently precise information to different voters. This highlights a somewhat unexpected role for targeted campaigning that has previously been viewed as having problematic impacts on democracy. For instance, Hillygus and Shields (2008, pp. 13) write “The fragmentation of campaign dialogue also has potential implications beyond the electoral contest itself. Elections have always been a blunt instrument for expressing the policy preferences of the public but the multiplicity of campaign messages makes it even more difficult to evaluate whether elected representatives are following the will of the people. Microtargeting enables candidates to focus attention on the issues that will help them win, irrespective of whether they are of concern to the broader electorate. ... How does a winning candidate interpret the policy directive of the electorate if different individuals intended their vote to send different policy messages? Can politicians claim a policy mandate if citizens are voting on the basis of different policy promises?” Our observations suggest to be cautious about an entirely negative assessment of microtargeting voters. We highlight a positive role by demonstrating that microtargeting voters with different messages enables effective information revelation as a voter can now deduce from the fact that a candidate has not provided precise information on her political position that her true position is unfavorable to the voter. It also suggests that “ambiguity” in electoral competition may be due in part to imperfect microtargeting of voters.⁹

It may be argued that casual empirical evidence suggests that electoral competition does not reveal sufficiently “relevant” information. In this case, our theory offers a useful map for discovering culprits for the lack of information unraveling. Some of our modeling assumptions must be violated. This is how we view the main contribution of the positive result. In Section 5, we discuss with examples how the result breaks down under various conditions. We previously mentioned that lack of electoral competition and the inability to microtarget messages may limit the unraveling of information in electoral campaigns. There may be another empirically relevant reason: Although we assume that voters may be unaware of some political issues and face incomplete information about candidates’ political positions, we still attribute to them quite sophisticated capabilities of political reasoning that are embodied in our solution concept. We show in Section 5.3 how our positive result may break down if they lack sophisticated political reasoning.

⁹We recently became aware of another paper that features private messages. Eliaz and Forges (2012) focus on a setting with two receivers and one sender who can commit to privately disclosure information before learning his type.

To take stock, we show that the candidates' ability to microtarget voters and voters' sophisticated reasoning capabilities are sufficient for producing election outcomes equivalent to ones under full awareness and complete information. We also show that the lack of sophisticated reasoning capabilities and microtargeting may prevent such outcomes. Are there other features of electoral campaigns that can remedy both impediments? In Section 6 we show that negative campaigning can come to the rescue. According to Geer (2006, p. 23) negative campaigning refers to "any criticism leveled by one candidate against another during the campaign." In order to facilitate for such criticism in our model, we extend our model in Section 6 by allowing candidates not only to provide information to voters on their own policy point but also on the policy point of their opponent. We will show that in this modified model that election outcomes under unawareness and incomplete information are the same as if voters have full awareness and complete information even if voters are not gifted with sophisticated political reasoning capabilities. An interesting difference between the presidential and the parliamentary model emerges under negative campaigning. While for the presidential model the positive result emerges even when we shut-off microtargeting and require candidates to make all campaign messages public, we show that microtargeting in addition to negative campaigning is necessary for unraveling of awareness and information in the parliamentary model when voters have limited political reasoning capabilities.¹⁰ In any case, we conclude that negative campaigning improves the informational efficiency of electoral campaigns. This is in contrast to the public opinion on negative campaigning, who views it as being detrimental to the political process, a view that is echoed widely in the political science literature (for an overview, see Geer, 2006, pp. 2-3, 15-18). An exception is Geer (2006) who argues that negative campaigning improves the "information environment" of elections. Based on a content analysis of presidential campaigns from 1964 to 2000 he shows that negative TV campaign advertisements bring up more issues per ad to voters than "positive" campaign ads. He also argues that for "a negative appeal to be effective, the sponsor of that appeal must marshal more evidence, on average, than for positive appeals."

Allowing for dynamically changing multidimensional policy spaces and voters who are aware of different subspaces only and who may become aware of larger subspaces during the political campaign poses a modeling challenge in terms of tractability.¹¹ To model such limited awareness

¹⁰There is empirical evidence that negative campaigning is less frequent in parliamentary systems (Walter, 2013) although this is viewed as a result of multiparty competition in parliamentary systems.

¹¹Our model can be viewed as an answer to an early critique in political science of the uni-dimensional Downs model. Stokes (1963) postulated that "the space in which political parties compete can be of highly variable structure. Just as the parties may be perceived and evaluated on several dimensions, so the dimensions that are salient to the electorate may change widely over time." He also criticized the assumption of a commonly perceived policy space by stating "(b)ut with the space formed out of perceptions, there is no logically necessary reason why the space of voters and of parties should be identical, and there is good empirical reason to suppose that it often is not."

in a dynamic strategic context, we will make use of generalized extensive-form games with unawareness introduced in Heifetz, Meier, and Schipper (2013).¹² Such a game consists of a collection of game trees partially ordered by a subtree relation. Initially a player may be unaware of some dimension of the problem and perceive the strategic situation as a subtree. During the course of play, he may become aware of more and more dimensions and perceive increasingly richer subtrees as the description of the strategic situation. Because players cannot anticipate on which dimensions exactly they will become aware of in future, there may not be a natural equilibrium convention that could have been learned in the past. Therefore we will make use of prudent rationalizability, a version of extensive-form rationalizability à la Pearce (1984) and Battigalli (1997) that has been introduced for generalized extensive-form games in Heifetz, Meier, and Schipper (2011). It is a strong solution concept that entails forward-induction. In perfect information game it yields outcomes that are equivalent to outcomes reached with iterated admissibility (see Meier, and Schipper, 2012), a solution concept that has a long tradition in political economy models and in voting games (see Farquharson, 1969, Brams, 1975, Moulin, 1979, and Gretlein, 1982). It is an iterative solution concept that also allows us to study behavioral implications for every finite level of rationalization and thus implications of limited political reasoning (see Section 5.3).

The paper is organized as follows: The next section we introduce the baseline model. This is followed by two simple examples in Section 3. In Section 4 we state and prove the main results under microtargeting. Limitations and counterexamples are discussed in Section 5. In Section 6 we explore negative campaigning. We conclude in Section 7, where we also further discuss the related literature. Proofs are collected in an appendix.

2 Model

Let $I = \{1, \dots, m\}$ be a finite set of *political issues*. Examples of issues are “Iraq War”, “Abortion”, “Health Care” etc. Different policies with regard to an issue are associated with different points in the real interval $[0, 1]$, one interval for each issue. Thus, the full-dimensional policy space considered in this model is $[0, 1]^{|I|}$. Since we aim to study electoral competition when some voters may not be aware of all issues, we also need to consider subspaces of the full-dimensional policy space. For any nonempty subset of issues I' and I'' with $\emptyset \neq I' \subseteq I'' \subseteq I$, denote the projection by $r_{I'}^{I''} : [0, 1]^{|I''|} \rightarrow [0, 1]^{|I'|}$. We let Y denote a finite set of policy points in $[0, 1]^{|I|}$. For every nonempty subset I' of issues, $I' \subseteq I$, let $Y_{|I'}$ be the projections of policy points in Y onto the subspace $[0, 1]^{|I'|}$ spanned by I' .

¹²See Halpern and Rego (2013), Feinberg (2012), Grant and Quiggin (2013), and Ozbay (2007) for related work.

There are two candidates, a and b . Each candidate $k \in \{a, b\}$ has a fixed ideological policy point $y^k \in Y$. If candidate k 's political position in Y is y^k , then for any nonempty $I' \subseteq I$ his political position in $Y_{|_{I'}}$ is $r_{I'}^I(y^k)$. We sometimes write $y_{|_{I'}}^k$ for $r_{I'}^I(y^k)$.

There is a finite set of voters denoted by $N = \{1, \dots, n\}$. Each voter $j \in N$ has a unique most preferred policy point x^j in the full-dimensional policy space, $[0, 1]^{|I|}$, with projections denoted by $x_{|_{I'}}^j$ for any nonempty $I' \subseteq I$.

A voter j 's utility from candidate k depends on voter j 's awareness of political issues and is given by the Euclidean distance between his most preferred point and the candidate's policy but only in the subspace of issues that he is aware of. I.e., the utility of voter j from voting for candidate k when voter j is aware of issues in I' with $\{1\} \subseteq I' \subseteq I$ and candidate k 's political position on these issues is $y^k \in Y_{|_{I'}}$ is¹³

$$u^j(I', y^k, x^j) = - \|x^j - y^k\|_{I'} := - \sqrt{\sum_{i \in I'} (x_i^j - y_i^k)^2}. \quad (1)$$

We assume that at the beginning of the campaign voters are aware of only one default issue, which is issue 1. That is, even when neither candidate raises any issue, voters are aware of issue 1 and the set of all possible policy points of candidates, $Y_{|\{1\}}$, regarding issue 1. It allows for well-defined preferences of voters even if no issues are raised in the campaign.¹⁴

We start with describing one game tree denoted by T^I . At the first stage of T^I , nature c (i.e., "chance") moves and selects for each candidate a most preferred policy point in the finite set $Y \subseteq [0, 1]^{|I|}$.¹⁵ We assume that candidates have complete information about each other's political positions (and the voters' most preferred policy points). That is, each of their

¹³Note that the Euclidean distance will typically increase with an increasing number of dimensions. That is, revealing further dimensions to voters may decrease their utility. We could use a "dimension-normalized" Euclidean distance, $\sqrt{\frac{1}{|I'|} \sum_{i \in I'} (x_i^j - y_i^k)^2}$ instead (and our results would follow too). Yet, despite the fact that raising additional dimensions decreases the utility of voters, we will show an unraveling result. The use of the Euclidean distance may be defended with the argument that most political issues are perceived as "problems" and thinking about them may causes disutility per se. Such a "hedonic" argument is misguided when utilities are viewed purely as decision weights. Nothing in our model depends on the interpretation of utilities. This is because voters make decisions to vote for candidate a or b always given *one* awareness level (i.e., subset of political issues) although they can contemplate how they would make decisions given any counterfactual lower awareness level. What matters to them under von Neumann-Morgenstern utilities is their ranking of lotteries *given their awareness level*. We conjecture that our results remain true if we consider more generally single peaked preferences of voters.

¹⁴Our results do not depend on the fact that ex ante all voters are aware of the same default issue. We could allow that ex ante different voters are aware of different subset of issues and our results would remain true.

¹⁵We don't require a (common) prior probability distribution over moves of nature. Our results do not depend on prior beliefs about candidates' most preferred policy points.

information sets is a singleton. After the move of nature, candidates simultaneously campaign for votes. In this campaign, each candidate reveals to each voter some subset of issues and some information (i.e., a nonempty subset of policy points) about her own political position on these issues. The information provided to a voter is observed by this voter only and not by other voters (i.e., microtargeting). Note that as a special case this model encompasses public campaigns in the sense that each candidate is allowed to provide to all voters the same information.

After the campaign, each voter votes for a candidate and the game ends. Each voter takes into account her awareness of issues and the (inferred) information on the candidates' political positions. Since not all issues may have been raised during the campaign, a voter may not be aware of all issues. Consequently, she is unable to think about these issues and does not realize that they could have been raised in a different campaign. This means that the voter's information set emanating from a node in the tree T^I may be a subset of corresponding nodes in a poorer description of the game in which nature chooses policy points of candidates only in space spanned by a subset of issues and where candidates can raise only subsets of this subset of issues during the campaign and provide information on those raised issues to voters. That is, our model involves a collection of trees, $(T^{I'})_{\{1\} \subseteq I' \subseteq I}$, one for each subset of issues that includes the default issue. For each $T^{I'}$, $\{1\} \subseteq I' \subseteq I$, nature selects a profile of candidates' (projected) policy points in $Y_{|I'} \times Y_{|I'} \subseteq [0, 1]^{|I'|} \times [0, 1]^{|I'|}$. In tree $T^{I'}$, candidates are unaware of issues in $I \setminus I'$ but have complete information about their policy points (i.e., singleton information sets). Candidates campaign simultaneously for votes by revealing to each voter some (possibly proper) subset of issues in I' and some information on their own political positions on these issues. After the campaign, voters vote on candidates taking only the subset of issues raised during this campaign and information provided in this campaign into account when forming their expectations. That is, a voter's information set emanating from a node in tree $T^{I'}$ may be a subset of corresponding nodes in an even poorer description of game $T^{I''}$ with $\{1\} \subseteq I'' \subset I'$. Note that an information set of a voter may contain several nodes (within one tree) because of the uncertainty over the policy points of candidates on the issues they are aware of.

To complete the description of the game, we need to specify preferences. We assume that voters vote sincerely. That is, each voter votes for the candidate that is "closest" to her given her awareness and information. She assigns the utility given in equation (1) to any terminal history in which she voted for candidate k , nature selected y^k , and I' is the union of sets of issues that were raised during the campaign by either candidate a or b .¹⁶

¹⁶We assume that voters have complete information about their own most preferred policy on issues once they are aware of them. We don't think that this assumption is crucial. Instead we could allow voters to remain to some extent uncertain about their most preferred policy on issues that they are aware but assume that candidates provide verifiable information on the *distance* between the candidate's and voter's policy points.

For candidates we consider two types of preferences. For any terminal node z , let $\sigma(z)(a)$ be the share of voters voting for candidate a . Candidates *care only about winning the election* if the utility function of candidate a is defined by

$$u^a(z) = \begin{cases} 1 & \text{if } \sigma(z)(a) \geq \frac{1}{2} \\ -1 & \text{otherwise} \end{cases} \quad (2)$$

and candidate b 's utility function is given by

$$u^b(z) = -u^a(z). \quad (3)$$

For simplicity we don't allow for ties in payoffs but confer to candidate a a slight advantage in case both candidates obtain the same number of votes.¹⁷ For reasons motivated in the introduction, we call the game in which candidates care only about winning the election the *presidential election model*. It has been also called the “majority tournament” in the literature (see for instance, Laslier, 2005).

Candidates *care only about the share of voters* if the utility function of candidate a is defined by

$$u^a(z) = \sigma(z)(a) \quad (4)$$

and candidate b 's utility function is given by

$$u^b(z) = 1 - u^a(z). \quad (5)$$

Note that under both specifications of utility functions the game is a strictly competitive for candidates. Again, for reasons discussed in the introduction, we call the game in which candidates care about the share of voters the *parliamentary election model*. It has been also called the “plurality game” in the literature (see for instance, Laslier, 2005).

The collection of game trees partially ordered by set inclusion on the set of issues and the information sets outlined above shall satisfy the properties of generalized extensive-form games introduced in Heifetz, Meier, and Schipper (2013).

For any nonempty subset I' , $\{1\} \subseteq I' \subseteq I$, the I' -partial game is the collection of the tree $(T^{I'})_{\{1\} \subseteq I'' \subseteq I'}$ such that all information sets emanating at nodes in the trees of this collection are contained within trees of this collection.

¹⁷Alternatively, we could have required as often done in the literature the number of voters to be odd. While latter assumption would simplify the proofs slightly, we opted for the first assumption for two reasons. First, in reality there is often an incumbent who may have a slight advantage over the other candidate. Second, we are just interested in comparing election results under incomplete information and unawareness with election results under full information and awareness. The tie breaking assumption is not important as long as it is the same under both scenarios.

For $k \in N \cup \{a, b\}$ we denote by H_k the set of all information sets of player k (across all trees) and by $I(h_k)$ be the set of issues such that information set h_k belongs to the tree $T^{I(h_k)}$.

2.1 Strategies

Let $I^{k,j} \subseteq I$ be the set of issues raised by candidate $k \in \{a, b\}$ to voter $j \in N$. To ease notation we assume that each candidate raises at least the default issue 1 to each voter, i.e., $\{1\} \subseteq I^{k,j}$, $k \in \{a, b\}$, $j \in N$.

When deciding to vote for one candidate or another, each voter j takes into account only issues that are raised to him during the campaign by either candidate (apart from the default issue). For instance, if candidate a campaigned to voter j on issues in the set $I^{a,j}$ and candidate b campaigned to voter j on issues in $I^{b,j}$, then voter j takes into account issues in $I^{a,j} \cup I^{b,j} \subseteq I$. That is, the policy space perceived by voter j is restricted to the domain $[0, 1]^{|I^{a,j} \cup I^{b,j}|}$.

A strategy of voter j is a function that assigns to each information set of voter j a candidate she votes for. That is,

$$s_j : H_j \longrightarrow \{a, b\}. \quad (6)$$

Note that a voter's strategy assigns to each of the voter's information sets in *each tree* the candidate for which he votes. Since a voter may be unaware of many policy issues and consequently may not perceive all trees, he can not "choose" such a strategy ex ante before the game starts. Rather, at each of his information sets the voter chooses an action. Strategies of voters will be used here just as objects of candidate's beliefs. As we will see below, candidates form beliefs about the behavior of voters.

For each candidate $k \in \{a, b\}$, let $y^k(h_k)$ be the policy point selected by nature in $Y_{I(h_k)}$ after which information set h_k of candidate k occurs. That is, $y^k(h_k)$ is candidate k 's policy point selected by nature on the path to h_k . A strategy for candidate $k \in \{a, b\}$, specifies for each information set $h_k \in H_k$ of candidate k which issues and which information on those issues she provides to each voter. We assume that each candidate can not bluntly lie about her policy point but she can be vague. That is, if $I^{k,j} \subseteq I(h_k)$ is the nonempty set of issues provided by candidate k to voter j at information set h_k , then her (projected) policy point $y^k(h_k)|_{I^{k,j}}$ must be in the set of policy points provided by candidate k to voter j at the information set h_k .¹⁸ This assumption generalizes the model of verifiable information à la Grossman (1981) and Milgrom (1981) to a multi-dimensional setting with possible unawareness of some dimensions

¹⁸We could have imposed this restriction on the generalized extensive-form game itself rather than on strategies of candidates. However, in order to avoid additional notation we opted to impose it as a restriction on candidates' strategies.

by the receiver. Note that candidate k in her message to voter j at the information set h_k can be silent on some issues in $I(h_k)$. Finally, we do not require that the same set of issues and information is provided to each voter, i.e., we allow for microtargeting of voters. Formally, a strategy for candidate $k \in \{a, b\}$ is

$$s_k : H_k \longrightarrow \prod_{j \in N} \left[\bigcup_{\{1\} \subseteq I^j \subseteq I} 2^{Y_{|I^j}} \right] \quad (7)$$

such that for every voter $j \in N$, there exists I^j with $\{1\} \subseteq I^j \subseteq I(h_k)$ such that $y^k(h_k)_{|I^j} \in (s_k(h_k))_j \in 2^{Y_{|I^j}}$, where $(s_k(h_k))_j$ is the j th component in the profile $s_k(h_k)$. With this notation, $(s_k(h_k))_j$ is the information provided by candidate k to voter j , i.e., it is a subset of policy points in some policy space that includes candidate k 's “true” policy point in this space. Note that candidate k 's “true” policy point $y^k(h_k)_{|I^j}$ at the information set h_k (subject to possibly being silent on some issues) is required to be in the set of possible policy points $(s_k(h_k))_j$ provided to voter j . Note that although we assume each candidate to be aware of all issues, we require his strategy to assign actions to his information sets even in lower trees where his “unaware incarnations live.” This is because candidates’ strategies are objects of beliefs of voters.

For $k \in N \cup \{a, b\}$ we denote by S_k player k 's set of strategies. Moreover, for any strategy $s_k \in S_k$ and any subset I' of issues with $\{1\} \subseteq I' \subseteq I$, we denote by $s_k^{I'}$ the I' -partial strategy in the I' -partial game induced by s_k . This is the strategy s_k restricted to k 's information sets in the I' -partial game. $S_k^{I'}$ denotes the set of I' -partial strategies of player k .

2.2 Belief Systems

Each voter forms beliefs about candidates’ policy points and (partial) strategies. For every information set of the voter, his belief is restricted to issues that the voter is aware of. Voter j 's belief system is a tuple

$$(\beta_j(h_j))_{h_j \in H_j} \in \prod_{h_j \in H_j} \Delta \left(Y_{|I(h_j)} \times Y_{|I(h_j)} \times S_a^{I(h_j)} \times S_b^{I(h_j)} \right) \quad (8)$$

such that for all h_j , $\beta_j(h_j)$ assigns probability 1 to the subset of candidates’ policy points selected by nature in $Y_{|I(h_j)} \times Y_{|I(h_j)}$ and candidates’ strategy profiles in $S_a^{I(h_j)} \times S_b^{I(h_j)}$ that reach h_j in the $I(h_j)$ -partial game. That is, at every of his information sets h_j , voter j is certain to have reached his information set h_j .

For two information sets h and h' in a given tree $T^{I'}$, we say h precedes h' (or h' succeeds h) if for node $n' \in h'$, there is a path n, \dots, n' in $T^{I'}$ such that $n \in h$.¹⁹

¹⁹We could require a belief system to satisfy Bayesian updating whenever possible. Yet, Bayesian updating

At every information set of candidate $k \in \{a, b\}$, we assume that she knows her policy point and the policy point of the opponent candidate $-k$ selected by nature. She forms beliefs about the other candidate's strategy and the strategies of voters. For $k \in \{a, b\}$, candidate k 's belief system is a tuple

$$(\beta_k(h_k))_{h_k \in H_k} \in \prod_{h_k \in H_k} \Delta \left(S_{-k}^{I(h_k)} \times \prod_{j \in N} S_j^{I(h_k)} \right). \quad (9)$$

For $k \in N \cup \{a, b\}$, we denote by B_k the collection of player k 's belief systems.

2.3 Prudent Rationalizability

In our model, voters may not think about all political issues before the election and consequently may be surprised about the issues arising in the campaign. Thus, it would be inappropriate to assume that voters could have always learned an equilibrium convention that is guiding their behavior. Instead, we will make use of a solution concept that embodies “*political reasoning*” in the sense that voters asked themselves *why* candidates provided them with this or that information and *why* they raised this or that political issue. Our iterative solution concept called prudent rationalizability has been introduced in Heifetz, Meier, and Schipper (2011) for generalized extensive-form games with unawareness. It is a version of extensive-form rationalizability (see Pearce, 1984, and Battigalli, 1997) featuring *cautious* behavior and an extensive-form analogue to iterated admissibility.

For any player $k \in N \cup \{a, b\}$, with a belief system β_k , a strategy s_k of player k is *rational* at information set $h_k \in H_k$, if there exists no other action $s'_k(h_k)$ at h_k such that by only replacing the action $s_k(h_k)$ with action $s'_k(h_k)$ (which results in some new strategy) yields k a strictly higher expected utility.

Prudent rationalizability adapted to our context takes the following form:

Definition 1 (Prudent Rationalizability) For $k \in N \cup \{a, b\}$, let

$$S_k^0 = S_k.$$

For $\ell \geq 1$, define inductively for $k \in \{a, b\}$,

$$B_k^\ell = \left\{ \beta_k \in B_k : \text{For every information set } h_k, \text{ the support of } \beta_k(h_k) \text{ is } S_{-k}^{I(h_k), \ell-1} \times \prod_{j \in N} S_j^{I(h_k), \ell-1} \right\},$$

whenever possible will be implied by our solution concept. See Meier and Schipper (2012) and (for standard games and standard extensive-form rationalizability) Shimoji and Watson (1998).

for $j \in N$

$$B_j^\ell = \left\{ \beta_j \in B_j : \begin{array}{l} \text{For every information set } h_j, \text{ if there exists some profile} \\ \text{of policy points } (y^a, y^b) \in Y_{|I(h_j)} \times Y_{|I(h_j)} \text{ and some profile} \\ \text{of candidates' strategies } (s_a, s_b) \in S_a^{\ell-1} \times S_b^{\ell-1} \text{ such that} \\ (y^a, y^b, s_a, s_b) \text{ reaches } h_j \text{ in the tree } T^{I(h_j)}, \text{ then the support} \\ \text{of } \beta_j(h_j) \text{ is the set of policy profiles and strategy profiles} \\ (y^a, y^b, s_a, s_b) \in Y_{|I(h_j)} \times Y_{|I(h_j)} \times S_a^{I(h_j), \ell-1} \times S_b^{I(h_j), \ell-1} \\ \text{such that } (y^a, y^b, s_a, s_b) \text{ reaches } h_j. \end{array} \right\},$$

and for any player $k \in N \cup \{a, b\}$,

$$S_k^\ell = \left\{ s_k \in S_k^{\ell-1} : \begin{array}{l} \text{There exists } \beta_k \in B_k^\ell \text{ such that for every information set } h_k \\ \text{player } k \text{ is rational at } h_k. \end{array} \right\}.$$

The set of prudent rationalizable strategies of player $k \in N \cup \{a, b\}$ is

$$S_k^\infty = \bigcap_{\ell=1}^{\infty} S_k^\ell.$$

At each round of elimination, a strategy is kept if there exists a full support belief on the remaining strategies of other players and possible moves of nature for which the strategy is rational at every information set of the player. The prudence or cautiousness of players enters through the full support beliefs about the remaining strategies and possible moves of nature. It means that at each level, a player does not completely exclude any of the opponents' remaining strategies. This feature will be essential for our result. See Heifetz, Meier, and Schipper (2011) and Meier and Schipper (2012) for further discussions of the solution concept.

Since the game is finite, existence of a nonempty set of prudent rationalizable strategy profiles follows directly from a result in Heifetz, Meier, and Schipper (2011). Moreover, since the space of policy points Y is finite, at most finite number of eliminations of strategies suffice.

3 Two Examples

Before we state and prove our main results, we like to build some intuition and illustrate the definitions with the help of two simple examples.

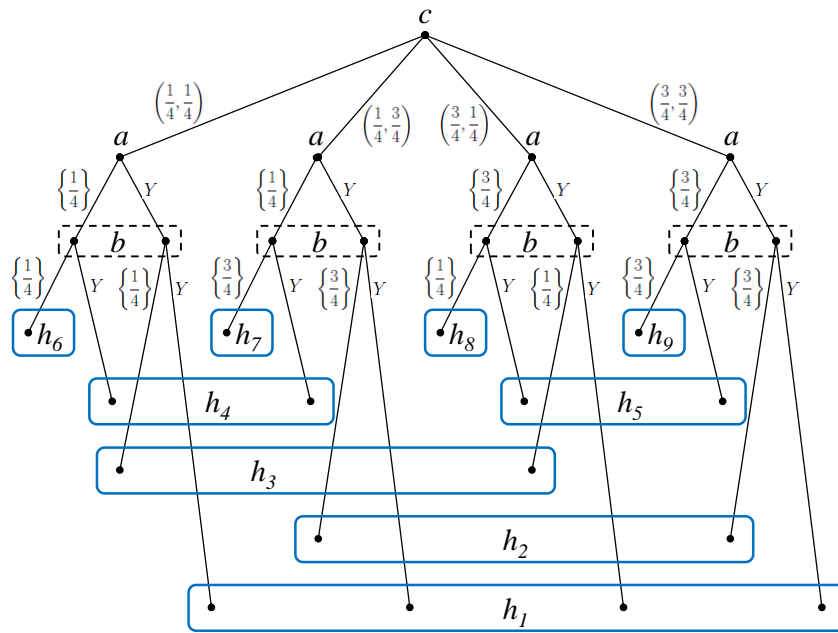
3.1 Uni-Dimensional Case

Consider a model with just one issue and single voter who is aware of this (default) issue. The voter's most preferred point is $\frac{5}{12}$. Further, he is uncertain about the policy points of

the two candidates, which are in the set $Y = \{\frac{1}{4}, \frac{3}{4}\}$. At the first glance, the case of a single voter may look artificially contrived and uninteresting for the study of electoral competition. After all, elections are about the aggregation of preference of a sizable population of voters. Yet, the model has an interesting reinterpretation. Often a political leader such as a president may has to select between two candidates for an important appointment (such as a secretary of state). The two candidates may compete for the appointment by providing more or less precise information about their political preferences on issues that should be considered for this position.

The game form is depicted in Figure 1. Nature moves first and selects the political positions

Figure 1: Game Form of the Uni-dimensional Example



of candidates. Then candidate a provides information about her political position. He is followed by candidate b who also knows the political positions but not the information revealed by candidate a .²⁰ After the moves of candidates, we reach the information sets of the voter indicated by the blue solid ovals. To save space, the game form is truncated after the information sets of the voter.

A strategy of candidate k is a map $s_k : Y \times Y \rightarrow 2^Y$ that assigns to each profile of policy

²⁰In the exposition of the model, we stated that candidates move simultaneously while in Figure 1 we let candidate a move before candidate b but let the latter not know the move of the first. This is done solely for an easier graphical exposition in Figure 1. Inspired by Dubey and Kaneko (1984), generalized extensive-form games with unawareness of Heifetz, Meier, and Schipper (2013) used in this paper do allow for simultaneous moves of players.

points some information candidate k can reveal about her policy point such that $s_k(y^k, y^{-k}) \in \{\{y^k\}, Y\}$ for any $(y^a, y^b) \in Y \times Y$. We let S_k denote the set of all strategies of candidate k .

Note that the voter has nine information sets (see Figure 1). Let H denote his set of information sets. A strategy of the voter assigns to each information set the candidate for whom he votes, i.e., $s_v : H \rightarrow \{a, b\}$. Denote by S_v the set of all voter's strategies.

We assume that each candidate tries to win the election and the voter likes to vote for the candidate whose policy point is closest to his.

We now apply prudent rationalizability to this example by eliminating strategies iteratively. At the first level of induction, any candidate k has full support beliefs about all strategies of the other candidate and the voter. For any strategy in S_k , we can find a full support belief of candidate k such that this strategy is rationalizable. Thus $S_k^1 = S_k$.

For the voter, prudent rationalizability has some bite already at the first level. At information sets h_6 to h_9 , the voter learns precisely the policy points of both candidates. Every first level prudent rationalizable strategy must prescribe to vote for the candidate whose policy point is closest to his at any of those information sets. At the information set h_4 , the voter is certain that $y^a = \frac{1}{4}$ and that y^b is either $\frac{1}{4}$ or $\frac{3}{4}$. Any of his beliefs must assign some strict positive probability to $y^b = \frac{3}{4}$ since beliefs are full support. Thus, every first level rationalizable strategy must prescribe to vote for candidate a at the information set h_4 . A similar argument applies to h_2 , and an analogous argument is used to show that the voter votes for candidate b at information sets h_3 and h_5 . However, at the information set h_1 the voter did not receive any non-trivial information about both candidates' policy points. Thus, for every candidate, there is a full support belief of the voter with which it is rational to vote for this candidate.

At the second level, candidates form full support beliefs on the voter's first level rationalizable strategies and the opponent's strategies. In particular, candidate a , after the move of nature, $(\frac{1}{4}, \frac{1}{4})$, knows now that when candidate b reveals her policy point, $y^b = \frac{1}{4}$, the voter may or may not vote for her if she also truthfully reveals her policy point, $y^a = \frac{1}{4}$, while the voter votes for candidate b if she does not. Candidate a also knows that when candidate b reveals trivial information, Y , the voter votes for her if she truthfully reveals her policy point, $y^a = \frac{1}{4}$, while the voter may or may not vote for her if she does not. Thus, any second level rationalizable strategy of candidate a must prescribe revealing her policy point $y^a = \frac{1}{4}$ after history $(\frac{1}{4}, \frac{1}{4})$. An analogous argument applies to history $(\frac{3}{4}, \frac{3}{4})$. The difference is that now any prudent rationalizable strategy must prescribe no information, Y , at this history. At history $(\frac{1}{4}, \frac{3}{4})$, candidate a knows now that if she truthfully reveals her policy point the voter votes for her no matter how candidate b acts. Otherwise, if she reveals trivial information, Y , then she may win or lose the election depending on whether candidate b reveals and how the voter votes (she has full support beliefs over any of their strategies). Thus, any second level rationalizable

strategy of candidate a must prescribe revealing her policy point $y^a = \frac{1}{4}$ after history $(\frac{1}{4}, \frac{3}{4})$. At history $(\frac{3}{4}, \frac{1}{4})$, candidate a knows that if she reveals her policy point, $y^a = \frac{3}{4}$, then no matter how candidate b acts, the voter's first level rationalizable strategies prescribes voting for b . Yet, if candidate a chooses to reveal trivial information, Y , then the voter may vote for her if candidate b does reveal Y either. Latter action is first level rationalizable for candidate b . Thus, any second level rationalizable strategy of candidate a must prescribe Y after history $(\frac{3}{4}, \frac{1}{4})$. By analogous arguments, we can show that any second level rationalizable strategy of candidate b can prescribe to reveal $y^b = \frac{1}{4}$ whenever his true policy point is $\frac{1}{4}$, while to reveal Y when his true policy point is $\frac{3}{4}$.

Table 1: Prudent Rationalizable Strategies in the Uni-Dimensional Case

	Voter							
Information set	$S_v^1 = S_v^2$							
$h_1 = \{(Y, Y)\}$	a	a	a	a	b	b	b	b
$h_2 = \{(Y, \frac{3}{4})\}$	a	a	a	a	a	a	a	a
$h_3 = \{(Y, \frac{1}{4})\}$	b	b	b	b	b	b	b	b
$h_4 = \{(\frac{1}{4}, Y)\}$	a	a	a	a	a	a	a	a
$h_5 = \{(\frac{3}{4}, Y)\}$	b	b	b	b	b	b	b	b
$h_6 = \{(\frac{1}{4}, \frac{1}{4})\}$	a	a	b	b	a	a	b	b
$h_7 = \{(\frac{1}{4}, \frac{3}{4})\}$	a	a	a	a	a	a	a	a
$h_8 = \{(\frac{3}{4}, \frac{1}{4})\}$	b	b	b	b	b	b	b	b
$h_9 = \{(\frac{3}{4}, \frac{3}{4})\}$	a	b	a	b	a	b	a	b

	Candidate a		Candidate b	
Information set	S_a^1	S_a^2	S_b^1	S_b^2
$(\frac{1}{4}, \frac{1}{4})$	$\{\frac{1}{4}\}$	Y	$\{\frac{1}{4}\}$	Y
$(\frac{1}{4}, \frac{3}{4})$	$\{\frac{1}{4}\}$	Y	$\{\frac{3}{4}\}$	Y
$(\frac{3}{4}, \frac{1}{4})$	$\{\frac{3}{4}\}$	Y	$\{\frac{1}{4}\}$	Y
$(\frac{3}{4}, \frac{3}{4})$	$\{\frac{3}{4}\}$	Y	$\{\frac{3}{4}\}$	Y

The iterated elimination process concludes after two levels of elimination. Table 1 summarizes the prudent rationalizable strategies for every player and every level. The first column shows the information sets. Each remaining column lists a strategy. In order to quickly recognize any differences between strategies, we colored differing components. Although not all information is revealed at every second level rationalizable strategy profile, the voter's prudent rationalizable strategy is also optimal when he were to know all the information.

3.2 Two-Dimensional Case

In the uni-dimensional case, the problem of unawareness of an issue does not come up since there is just one single default issue and the voter is aware of it. Would sufficient information be revealed if the voter may be unaware of an issue? To illustrate the answer to this question, we need a model with at least two issues. Again, issue 1 is the default issue. The most preferred policy point of the single voter in the two-dimensional space is $(\frac{5}{12}, \frac{2}{3})$. The policy points of the candidates are elements of the set $Y = \{(\frac{1}{4}, \frac{1}{4}), (\frac{3}{4}, \frac{3}{4})\}$. Note that the voter would vote for the candidate whose policy point is $(\frac{3}{4}, \frac{3}{4})$ if he is aware of both issues, while he would vote for the candidate whose policy point is $\frac{1}{4}|_{\{1\}}$ if he is aware of only the default issue.

The game form is depicted in Figure 2. Again, in order to save space, we truncate the trees after reaching the information sets of the voter. The lower tree is identical to Figure 1 since in this tree, the voter is unaware of the second dimension. The voter's information set at a node in the upper tree $T^{\{1,2\}}$, let's say after candidate a and b both choose $\frac{1}{4}|_{\{1\}}$, is now in the lower tree $T^{\{1\}}$.

In the upper tree $T^{\{1,2\}}$, strategies of candidate k prescribe to any profile of candidates' policy points $(y^a, y^b) \in Y \times Y$ actions in $\{\{y^k|_{\{1\}}\}, Y|_{\{1\}}, \{y^k\}, Y\}$. As the lower tree, $T^{\{1\}}$, depicts the uni-dimensional case, the set of $T^{\{1\}}$ -partial strategies of candidate k corresponds to her set of strategies in the uni-dimensional case. A strategy of the voter assigns to every information set (in both trees) the candidate for whom he votes.

The payoffs are analogous as before.

We now apply prudent rationalizability to the two-dimensional example. At each step of the iterative procedure, we have to consider information sets in both trees. At the first level, candidate k has full support beliefs about any strategies of the other candidate and the voter. Similar to the uni-dimensional case, any strategy of candidate k is first level rationalizable. Again, similar to the uni-dimensional case, prudent rationalizability has already some bite for the voter at the first level. Consider for instance the voter's information set $(Y|_{\{1\}}, \{(\frac{1}{4}, \frac{1}{4})\})$. Since candidate b reveals her policy point on both issues, the voter is aware of them, even though candidate a reveals (trivial) information on issue 1 only. Because the voter has a full support belief, he must assign some strict positive probability on candidate a having the policy point $(\frac{3}{4}, \frac{3}{4})$, while he is certain that candidate b has policy point $(\frac{1}{4}, \frac{1}{4})$. Thus, any first level rationalizable strategy of the voter must prescribe voting for candidate a at the information set $(Y|_{\{1\}}, \{(\frac{1}{4}, \frac{1}{4})\})$.

Table 2 shows the rationalizable strategies of the voter at each level of the iterative process. A strategy (i.e., column) assigns to each information set of the voter (i.e., row), the candidate for whom the voter votes. Note that at every level of the iterative process, the $T^{\{1\}}$ -partial

Figure 2: Two-dimensional example with uncertainty

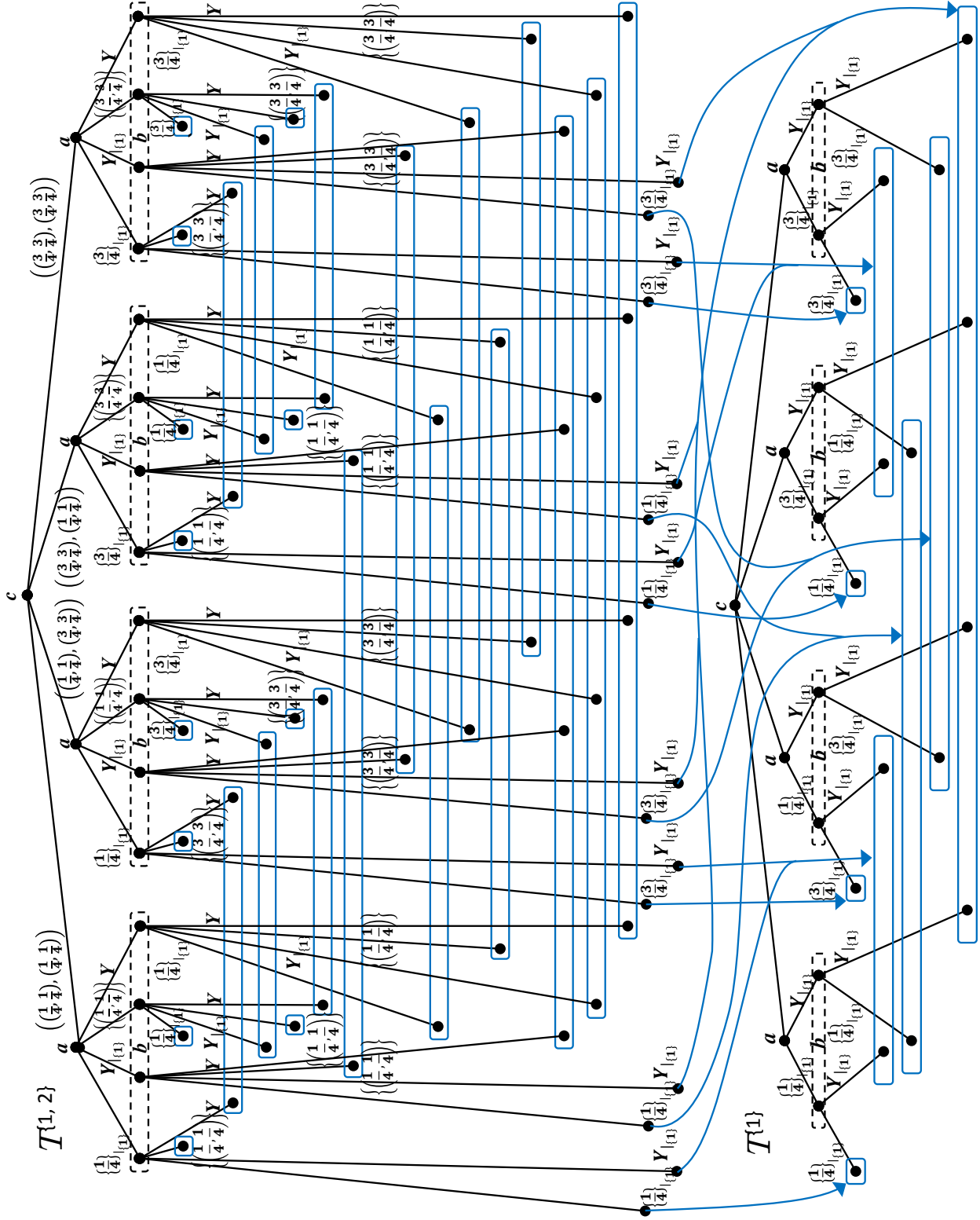


Table 2: Prudent Rationalizable Strategies of the Voter in the Two-Dimensional Case

Partial Game	Information Sets		$S_v^1 = S_v^2$	
	a 's Action	b 's Action		
$T^{\{1,2\}}$	$Y_{ \{1\}}$	Y	a	b
$T^{\{1,2\}}$	$Y_{ \{1\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	a	a
$T^{\{1,2\}}$	$Y_{ \{1\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	b	b
$T^{\{1,2\}}$	Y	$Y_{ \{1\}}$	a	b
$T^{\{1,2\}}$	Y	Y	a	b
$T^{\{1,2\}}$	Y	$\{\frac{1}{4}\}_{ \{1\}}$	a	a
$T^{\{1,2\}}$	Y	$\{(\frac{1}{4}, \frac{1}{4})\}$	a	a
$T^{\{1,2\}}$	Y	$\{\frac{3}{4}\}_{ \{1\}}$	b	b
$T^{\{1,2\}}$	Y	$\{(\frac{3}{4}, \frac{3}{4})\}$	b	b
$T^{\{1,2\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	Y	b	b
$T^{\{1,2\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	a	b
$T^{\{1,2\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	b	b
$T^{\{1,2\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	$Y_{ \{1\}}$	b	b
$T^{\{1,2\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	Y	b	b
$T^{\{1,2\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	$\{\frac{1}{4}\}_{ \{1\}}$	a	b
$T^{\{1,2\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	a	b
$T^{\{1,2\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	$\{\frac{3}{4}\}_{ \{1\}}$	b	b
$T^{\{1,2\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	b	b
$T^{\{1,2\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	Y	a	a
$T^{\{1,2\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	a	a
$T^{\{1,2\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	a	b
$T^{\{1,2\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	$Y_{ \{1\}}$	a	a
$T^{\{1,2\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	Y	a	a
$T^{\{1,2\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	$\{\frac{1}{4}\}_{ \{1\}}$	a	a
$T^{\{1,2\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	$\{(\frac{1}{4}, \frac{1}{4})\}$	a	a
$T^{\{1,2\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	$\{\frac{3}{4}\}_{ \{1\}}$	a	b
$T^{\{1,2\}}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	$\{(\frac{3}{4}, \frac{3}{4})\}$	a	b
$T^{\{1\}}$	$Y_{ \{1\}}$	$Y_{ \{1\}}$	a	b
$T^{\{1\}}$	$Y_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	b	b
$T^{\{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	$Y_{ \{1\}}$	a	a
$T^{\{1\}}$	$Y_{ \{1\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	a	a
$T^{\{1\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	$Y_{ \{1\}}$	b	b
$T^{\{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	a	b
$T^{\{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	a	a
$T^{\{1\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	b	b
$T^{\{1\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	$\{\frac{3}{4}\}_{ \{1\}}$	a	b

prudent rationalizable strategies (see the lower part of the table) correspond exactly to the prudent rationalizable strategies in the uni-dimensional case.

Consider now candidate k . At the second level of the iterative process, she forms full support beliefs about the first level rationalizable strategies of the voter and the other candidate. In particular, at the information set $((\frac{3}{4}, \frac{3}{4}), (\frac{1}{4}, \frac{1}{4}))$ candidate a is now certain that the voter would vote for her if she reveals her policy point in the upmost tree. Otherwise, if she chooses to reveal some trivial information, e.g. $Y_{|\{1\}}$ or Y , then the voter may or may not vote for her depending on whether candidate b reveals some non-trivial information. Moreover, if she reveals her “true” policy point in the lower tree, then the voter may or may not voter for her depending on whether candidate b raises issue 2. Thus, any second level rationalizable strategy of candidate a must reveal non-trivial information in the upmost tree about her policy point at the information set $((\frac{3}{4}, \frac{3}{4}), (\frac{1}{4}, \frac{1}{4}))$.

Table 3 presents the rationalizable strategies of candidates at each level of the iterative process. Since the set of strategies that are remaining after each level of elimination is relatively large (i.e., for each candidate, 4096 strategies at the first level and 27 strategies at the second level), we just mention for each information set of the candidate the actions consistent with those strategies. Note again that at every level of the iterative process, the $T^{\{1\}}$ -partial prudent rationalizable strategies (see the lower part of the table) correspond exactly to the prudent rationalizable strategies in the uni-dimensional case.

Table 3: Prudent Rationalizable Strategies of the Candidates in the Two-Dimensional Case

Information Set	Candidate a		Candidate b	
	Actions consistent with S_a^1	Actions consistent with S_a^2	Actions consistent with S_b^1	Actions consistent with S_b^2
$((\frac{1}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{4}))$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{1}{4}, \frac{1}{4})\}, Y$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}, Y$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{1}{4}, \frac{1}{4})\}, Y$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}, Y$
$((\frac{1}{4}, \frac{1}{4}), (\frac{3}{4}, \frac{3}{4}))$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{1}{4}, \frac{1}{4})\}, Y$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}, Y$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{3}{4}, \frac{3}{4})\}, Y$	$\{(\frac{3}{4}, \frac{3}{4})\}$
$((\frac{3}{4}, \frac{3}{4}), (\frac{1}{4}, \frac{1}{4}))$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{3}{4}, \frac{3}{4})\}, Y$	$\{(\frac{3}{4}, \frac{3}{4})\}$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{1}{4}, \frac{1}{4})\}, Y$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}, Y$
$((\frac{3}{4}, \frac{3}{4}), (\frac{3}{4}, \frac{3}{4}))$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{3}{4}, \frac{3}{4})\}, Y$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{3}{4}, \frac{3}{4})\}$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{3}{4}, \frac{3}{4})\}, Y$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}, \{(\frac{3}{4}, \frac{3}{4})\}$
$(\frac{1}{4}_{ \{1\}}, \frac{1}{4}_{ \{1\}})$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$
$(\frac{1}{4}_{ \{1\}}, \frac{3}{4}_{ \{1\}})$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}$	$Y_{ \{1\}}$
$(\frac{3}{4}_{ \{1\}}, \frac{1}{4}_{ \{1\}})$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}$	$Y_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}, Y_{ \{1\}}$	$\{\frac{1}{4}\}_{ \{1\}}$
$(\frac{3}{4}_{ \{1\}}, \frac{3}{4}_{ \{1\}})$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}$	$Y_{ \{1\}}$	$\{\frac{3}{4}\}_{ \{1\}}, Y_{ \{1\}}$	$Y_{ \{1\}}$

As in the uni-dimensional case, the process stops after the second level. Although not all information is revealed in every second level rationalizable strategy profile, the voter’s prudent rationalizable strategy is also optimal when he were fully aware and knew all the information. For instance, in some prudent rationalizable outcomes after the move of nature, $(\frac{3}{4}, \frac{3}{4})$, the voter votes for candidate a , like at information set $(Y_{|\{1\}}, \{\frac{3}{4}\}_{|\{1\}})$; in others he votes for b like at information set $(Y_{|\{1\}}, \{(\frac{3}{4}, \frac{3}{4})\})$; and at the other outcomes the voter may or may not vote for

candidate a like at information set $(\{(\frac{3}{4}, \frac{3}{4})\}, \{(\frac{3}{4}, \frac{3}{4})\})$. However, the voter's actions are optimal even with respect to full awareness of all political issues and complete information about the candidates' political positions. Compared to the uni-dimensional example, the two-dimensional example demonstrates that unawareness of an issues may not impede unraveling. This is due to the competition among candidates. Since candidates compete essentially in a zero-sum game, one or the other candidate has an incentive to raise an issue unless raising further issues cannot sway the election outcome. In Section 5.1 we show that competition is necessary for unraveling in the presence of unawareness, while we know from Grossman (1981), Milgrom (1981), and Milgrom and Roberts (1986) that it is not necessary in the case of full awareness.

4 Results When Allowing for Microtargeting

Despite voters' unawareness of issues and uncertainty about the candidates' policy points, we claim that electoral competition is sufficient for the emergence of election outcomes that are equivalent to outcomes with full awareness of issues and full information of policy points. Note that we do not claim that all issues or all information are revealed during the campaign. All we claim is that the revelation of further issues and information won't change the election outcome.

Proposition 2 (Parliamentary Model) *At every prudent rationalizable outcome of the parliamentary model with unawareness of political issues and incomplete information about candidates' policy points, if a voter votes for a candidate, then he prefers to vote for the same candidate when having full awareness of all political issues and complete information about the candidates' policy points. Conversely, if a voter strictly prefers to vote for a candidate under full awareness of political issues and complete information about candidates' policy points, then in any prudent rationalizable outcome of the parliamentary model with unawareness of political issues and incomplete information about candidates' policy points, he votes for the same candidate.*

While the proof is naturally somewhat tedious due to the multiplicity of multidimensional policy spaces and the change of dimensions during the play, the basic idea is to show unraveling of sufficient issues and information such that election outcomes are the same as under full awareness and complete information.

Roughly we show that with any first-level prudent rationalizable strategy, the voter who receives from exactly one candidate information that he has her most preferred policy point (in the policy space that she is aware of at that information set) must vote for that candidate. Any second-level prudent rationalizable strategy of candidates must be such that if the candidate has the best policy point for the voter in some subspace and all higher-dimensional spaces,

then he must reveal it to the voter. For any $\ell \geq 1$, at level $(2\ell + 1)$ prudent rationalizable strategies voters vote for the candidate who reveals unambiguously the ℓ -closest or any closer policy point to the voter, while at level $(2\ell + 2)$ prudent rationalizable strategies candidates reveal if possible to voters the ℓ -closest or closer policy point in an appropriate policy space or any higher-dimensional policy space.

The presidential model is slightly more challenging than the parliamentary model since competing for a majority in the presidential model may involve less intense competition than competing for any small improvement in the share of voters in the parliamentary model. In the presidential model, all what a candidate cares about is a majority of voters while in the parliamentary model a candidate cares about every (even small) share of voters. In Section 5 we show that electoral competition is necessary for election outcomes under unawareness to be equivalent to election outcomes under full awareness. Yet, the positive result below shows that the electoral competition in presidential elections is sufficiently “intense”.

Proposition 3 (Presidential Model) *At every prudent rationalizable outcome of the presidential model with unawareness of political issues and incomplete information about candidates’ policy points, if a candidate obtains the majority of votes then he also obtains the majority of votes under full awareness of all political issues and complete information about the candidates’ policy points. Conversely, if a majority of voters strictly prefer to vote for a particular candidate under full awareness of political issues and complete information about candidates’ policy points, then in any prudent rationalizable outcome of the presidential model with unawareness of political issues and incomplete information about candidates’ policy points, this candidate obtains a majority of votes.*

The proofs of both results are contained in the appendix. In fact, we conveniently state the proofs in reverse order. The proof for the presidential model applies with minor modifications also to the parliamentary model. The proofs differ mainly in the set of “relevant” voters that candidates care about. In the parliamentary model, candidates care about every voter while in the presidential model candidates do not necessarily care about subsets of voters larger than a majority.

5 Limits to Unraveling

In the previous section, we presented strong positive results on the informational efficiency of electoral competition with microtargeting of voters due to unraveling of awareness and information. One may question whether elections in the real world achieve unraveling. In this section, we like to shed some light on which assumptions if violated could prevent the revelation

of “relevant” awareness and information. Identifying such assumptions we view as the main contribution of the model.

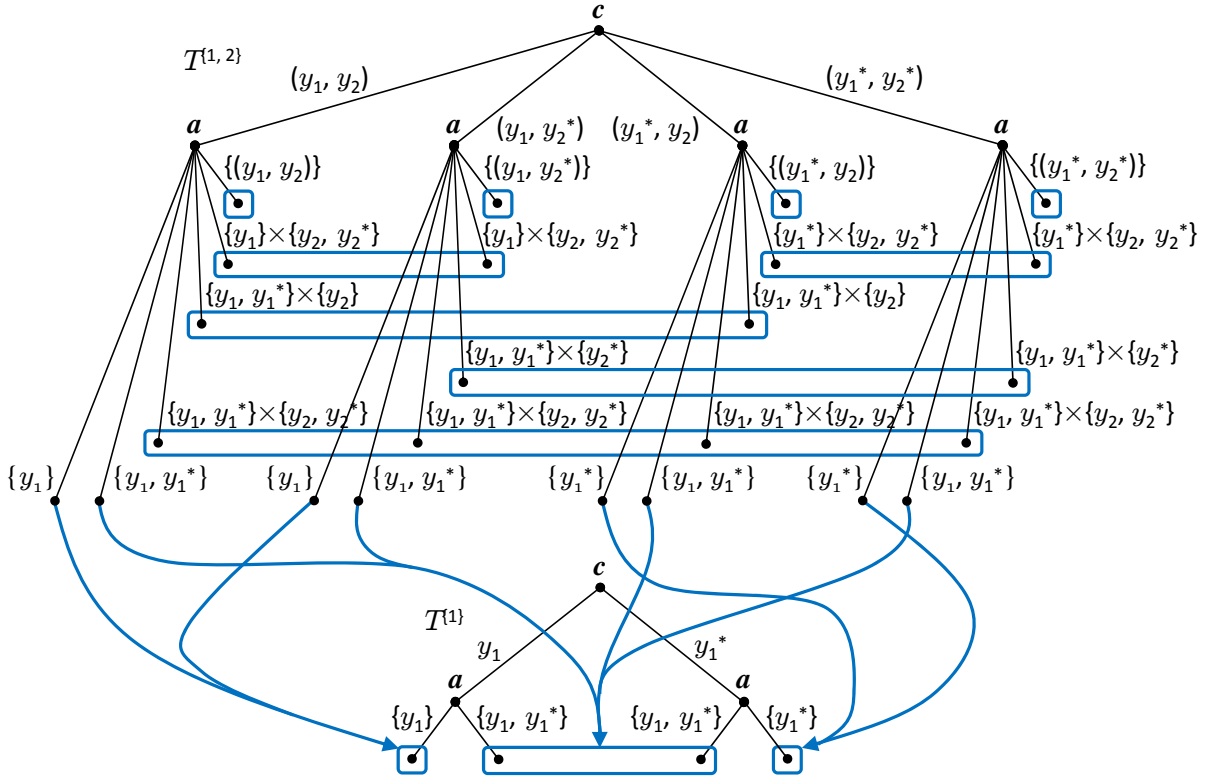
5.1 Lack of Electoral Competition

First, we show that electoral *competition* is necessary for the positive results to hold and that competition is the main driving force that allows for unraveling in the presence of unawareness while it is not necessary under full awareness. Thus, the presence of unawareness makes a difference. Milgrom and Roberts (1986) show that if there is one informed and one uninformed agent and the uninformed agent is “skeptical”, then there is a sequential equilibrium with full unraveling of information (see also Grossman, 1981, and Milgrom, 1981). That is, competition may not be necessary for all the relevant information to be revealed in standard games with uncertainty only but no unawareness. Battigalli (2006) showed that sequential equilibrium can be replaced by a version of extensive-form rationalizability with a restriction on first-order conditional beliefs that requires “weak scepticism” in the sense that the lowest type consistent with a message has positive probability. Heifetz, Meier, and Schipper (2011) show that the unraveling result can be obtained by replacing sequential equilibrium and “scepticism” of the uninformed agent by prudent rationalizability, the solution concept also used in the current paper.²¹ That is, when the uninformed agent is aware of all issues but may be uncertain about the type of the sender, then full unraveling of the information obtains. Yet, Heifetz, Meier, and Schipper (2011) also show that unraveling may break down in the presence of unawareness of the uninformed agent and a single sender. Here we will discuss a version of this example put in the context of elections in order to show that electoral competition is a necessary condition for our positive results to obtain under unawareness.

Consider an example where there is only *one* candidate (i.e., candidate *a* only), hence *no electoral competition*. There are two issues. The policy point of the candidate is $y = (y_1, y_2)$, where y_1 and y_2 are the coordinates on issues 1 and 2, respectively. If the candidate is not elected, a fixed “status quo” policy, $y^* = (y_1^*, y_2^*)$ is implemented instead, with $y_i \neq y_i^*$, $i \in \{1, 2\}$. This “status quo” can be understood as the political position of the opposition who is not allowed to campaign. For simplicity, let there be a single voter only who is initially aware of default issue 1 only, i.e., the subspace $\{y_1, y_1^*\}$. The candidate can either reveal $\{y_1\}$ or $\{y_1, y_1^*\}$

²¹There are advantages and disadvantages for using one or the other rationalizability procedure. Battigalli’s (2006) solution has the nice property that it can be viewed as a reduction procedure on beliefs (that implies a reduction of strategies) while prudent rationalizability is necessarily a reduction procedure on strategies. Yet, prudent rationalizability is not “tailored” to the particular context with an extra restriction on first-order beliefs motivated by the application. It applies essentially to any finite game and for standard extensive-form games it is equivalent to iterated admissibility in the associated normal-form game (see Meier and Schipper, 2012).

Figure 3: Failure of Unraveling in an Example with a Single Candidate



on issue 1, or $\{y\}$, $\{y_1\} \times \{y_2, y_2^*\}$, $\{y_1, y_1^*\} \times \{y_2\}$, or $\{y_1, y_1^*\} \times \{y_2, y_2^*\}$ on issues 1 and 2. Once the candidate reveals anything on issue 2, the voter becomes aware of the entire policy space $\{y_1, y_1^*\} \times \{y_2, y_2^*\}$. Moreover, we assume that once he is aware of the entire policy space, he knows the status quo y^* in the full-dimensional policy space.²² The game form is depicted in Figure 3. Again, to simplify the graphical exposition, we truncated the game form at the voter's information sets. The voter is assumed to strictly prefer y_1 to y_1^* in the uni-dimensional policy space but y^* to y in the full-dimensional policy space. The candidate strictly prefers being appointed to not being appointed. If the candidate is not silent on issue 2, then the voter must assign strict positive probability to y and may not vote for the candidate, while the voter would appoint the candidate for sure if she reveals $\{y_1\}$ and keeps silent on issue 2. Consequently, the candidate keeps silent on issue 2. Unraveling breaks down because the “status quo” is not actively campaigning with y^* . This example illustrates the electoral competition is crucial for unraveling to obtain under unawareness. But as we known from Grossman (1981), Milgrom (1981), and Milgrom and Roberts (1986) it is not crucial under full awareness and complete information. It highlights that unawareness of issues can be overcome with competition.

²²Alternatively, we could assume that the opponent is not allowed to touch new issues on the campaign but can only provide information about his position on issues raised by the first candidate.

5.2 Inability of Targeting Voters

For our positive results, we assume that candidates are able to target different voters with different messages in terms of issues raised and the precision of information provided about their policy points. This feature is motivated by modern sophisticated campaign strategies that involve “microtargeting” of voters and “dog-whistle politics”. The appropriateness of such strategies has been questioned in the political science literature (Hillygus and Shields, 2008). In this section we show that the assumption of such strategies is crucial for our positive results. This assumption is for example violated in a context in which candidates can campaign only on national TV, national radio, or nation-wide newspapers such that any information relayed to voters could reach any voter. Alternatively it may be violated in a situation in which candidates are prevented (either by law or by prohibitive costs) to gather and use information about individual voters in order to form sufficiently precise beliefs about their preferences.

In the following example, we will assume now that each candidate cannot send different messages to different voters but must send the same message to all voters. We can interpret this as public campaign messages sent to all voters. One can easily imagine a well-intentioned regulatory initiative that aims at maximal “transparency” of the election process and whose aim is to make all campaign information public to voters. We will show that such an initiative may be counterproductive in that the outcome may be the opposite to what it intends.

For simplicity, we ignore the issue of unawareness and show that microtargeting is already crucial under uncertainty only. There are three possible policy points of candidates, y_1 , y_2 , and y_3 . Moreover, there are three voters. For simplicity we consider just three information sets of voters in Table 4. The preferences of voters are such that their first-level prudent rationalizable strategies at those information sets are given in Table 4.²³ Voter 3 strictly prefers candidate a

Table 4: First-Level Prudent Rationalizable Strategies of Voters

Information Set		First-level Prud. Rat. Strategies				Winning Candidate	
Message by a	Message by b	Voter 1		Voter 2			Voter 3
$\{y_1\}$	$\{y_2\}$	a	a	b	b	a	a
$\{y_1\}$	$\{y_3\}$	b	b	a	a	a	a
$\{y_1\}$	$\{y_2, y_3\}$	a	b	a	b	a	a or b

with policy point y_1 no matter whether candidate b 's policy point is y_2 or y_3 . Voter 1 prefers candidate a over candidate b if the latter's policy point is y_2 while he prefers candidate b over candidate a if candidate b 's policy point is y_3 . Voter 2 has preferences dual to voter 1.

²³That is, voter 1 strictly prefers y_3 to y_1 to y_2 while voter 2 strictly prefers y_2 to y_1 to y_3 . Voter 3 strictly prefers y_1 to any other policy points.

Consequently, when candidate b reveals $\{y_2, y_3\}$, voters 1 and 2 have full-support beliefs that would make voting for candidate a rational as well as full-support beliefs that would make voting for candidate b rational.

Any second-level prudent rationalizable strategy of candidate b must ascribe to reveal $\{y_2, y_3\}$ to all voters. Finally, with any third-level prudent rational belief, voters 1 and 2 cannot deduce anymore candidate b 's policy point. For instance, voter 1 is uncertain whether candidate b did not reveal $\{y_2\}$ because her policy point is in fact $\{y_3\}$ or because candidate b 's policy point is indeed $\{y_2\}$ but she doesn't want to publicly reveal it because she would lose the election (i.e., the vote of voter 2).

This example turns out to be a special case of a more general preference aggregation paradox of the following kind: Analogous to our example, consider a set of three outcomes $\{x, y, z\}$ and lotteries over those outcomes. There are three voters, each having a preference relation on lotteries over outcomes as follows:

Voter	Preferences		
1	$(1, 0, 0) \succ_1 (0, 1, 0)$	$(1, 0, 0) \prec_1 (0, 0, 1)$	$(1, 0, 0) \prec_1 (0, \frac{1}{2}, \frac{1}{2})$
2	$(1, 0, 0) \prec_2 (0, 1, 0)$	$(1, 0, 0) \succ_2 (0, 0, 1)$	$(1, 0, 0) \prec_2 (0, \frac{1}{2}, \frac{1}{2})$
3	$(1, 0, 0) \succ_3 (0, 1, 0)$	$(1, 0, 0) \succ_3 (0, 0, 1)$	$(1, 0, 0) \succ_3 (0, \frac{1}{2}, \frac{1}{2})$
Majority	$(1, 0, 0) \succ_M (0, 1, 0)$	$(1, 0, 0) \succ_M (0, 0, 1)$	$(1, 0, 0) \prec_M (0, \frac{1}{2}, \frac{1}{2})$

Each voter's preference is consistent with von Neumann-Morgenstern utility. The last line of the table shows the "social choice" using simple majority over pairwise comparisons. Clearly, this social choice is inconsistent with von Neumann-Morgenstern utility.

5.3 Lack of Political Reasoning Capabilities of Voters

In our model we build a stark contrast between candidates and voters. Candidates are aware of all policy issues, know the preferences of voters, and are able to use sophisticated campaign strategies including microtargeting of voters. In contrast, voters are unaware of all policy issues except the default issue and don't know the preferences of candidates. The purpose for the stark contrast was to study whether electoral competition can overcome this stark asymmetry in awareness and information. Yet, we still assumed that voters are rational and use sophisticated political reasoning. In particular, our solution concept, prudent rationalizability, entails forward-induction reasoning by voters. One may question whether all voters are able to sophisticatedly reason about political campaigns. What can we say about a context where voters' rationality is limited in the sense that they still try to do what is best for them but they are unable to use higher-order reasoning in the form of asking *why* this or that information and issue has been revealed by the candidate? That is, in this section we will assume that voters

try to do what is best to them but are oblivious to the strategic intentions of candidates in that they do not necessarily believe that candidates are rational.

In the examples of Section 3, we obtained our strong unraveling result after just two levels of prudent rationalization. That is, essentially both voters as well as candidates are rational and believe both to be rational. We did not need to require that voters believe that candidates believe that voters are rational. It would be misleading however to conclude that just two levels of prudent rationalizability are required to obtain strong unraveling results in general. Both examples of Section 3 are special in that there are just two possible policy points that candidates could have. Consider now a policy space with three possible policy points of candidates, $Y = \{y_1, y_2, y_3\}$, no unawareness, and just a single voter who strictly prefers y_1 to y_2 , and y_2 to y_3 . Focus on the move of nature $(y^a, y^b) = (y_2, y_3)$. In Table 5, the second column denotes all possible actions of candidate a after this move of nature and the second row lists all possible actions of candidate b after this move of nature. Thus, each cell corresponds to an information set of the voter. In each cell, we indicate the actions that first-level prudent rationalizable strategies of the voter can ascribe to this information set. Every second-level prudent

Table 5: First-Level Prudent Rationalizable Actions for the Voter

		Information provided by candidate b			
		$\{y_3\}$	$\{y_1, y_3\}$	$\{y_2, y_3\}$	Y
Information provided by candidate a	$\{y_2\}$	a	a, b	a	a, b
	$\{y_1, y_2\}$	a	a, b	a	a, b
	$\{y_2, y_3\}$	a	a, b	a, b	a, b
	Y	a	a, b	a, b	a, b

rationalizable strategy of a candidate prescribes to fully reveal its policy point if the policy point is y_1 and it never prescribes to fully reveal its policy point if it is y_3 . Any second-level prudent rationalizable strategy of candidate a may prescribe to reveal $\{y_2\}$, $\{y_1, y_2\}$, $\{y_2, y_3\}$, or Y while any second-level prudent rationalizable strategy of candidate b may prescribe to reveal $\{y_1, y_3\}$, $\{y_2, y_3\}$, or Y . Assume that the voter has limited political reasoning capabilities in the sense that he does not form beliefs about the candidates' belief in the voter's rationality. That is, the voter won't necessary believe in the second-level prudent rationalizable strategies of candidates. In this case, the process of eliminating strategies stops after the second level. Note that there are many second-level prudent rationalizable outcomes where the voter votes for candidate b even though he would prefer candidate a over candidate b under complete information. This is because not enough information is revealed after two levels of elimination of imprudent strategies.

It is possible albeit tedious to prove a more general result. For every finite level ℓ of elimi-

nating imprudent strategies, there is a generic policy space (in the sense that the policy points of candidates could be perturbed slightly without affecting the result) with a sufficient large but finite number of possible policy points for candidates and a ℓ -level prudent rationalizable election outcome that differs from the outcome under full awareness and complete information. Thus, the “richer” the policy space, the higher are the demands on the political reasoning capabilities of voters in order for the unraveling results to obtain. We conclude that the political reasoning abilities of voters are very crucial for our results.

6 Negative Campaigning - Unraveling Regained

In the previous section, we observed that limited political reasoning capabilities of voters or the inability of candidates to microtarget voters may prevent unraveling of information in electoral campaigns. In this section, we explore to what extent positive results can be regained when allowing for negative campaigns of candidates. A candidate can now reveal information not only about his own policy point but also about the policy point of its opponent.

We consider first the case in which information revealed is now public to all voters instead targeted to particular voters. Formally, a strategy for candidate $k \in \{a, b\}$ is now redefined

$$s_k : H_k \longrightarrow \bigcup_{\{1\} \subseteq I' \subseteq I} 2^{Y_{|I'}} \times 2^{Y_{|I'}} \quad (10)$$

such that $(s_k(h_k))_k, (s_k(h_k))_{-k} \in 2^{Y_{|I'}} \times 2^{Y_{|I'}}$ for some I' with $\{1\} \subseteq I' \subseteq I(h_k)$ satisfying $(y^k(h_k)_{|I'}, y^{-k}(h_k)_{|I'}) \in (s_k(h_k))_k, (s_k(h_k))_{-k}$, where $(s_k(h_k))_k$ denotes k 's “message” about her own policy points, and $(s_k(h_k))_{-k}$ denotes k 's “message” about her opponent's policy points.

As in the previous model, we assume that while she can be vague, each candidate can not bluntly lie about its own policy points nor about the opponent's. In the definition above, this is reflected in the restriction $(y^k(h_k)_{|I'}, y^{-k}(h_k)_{|I'}) \in (s_k(h_k))_k, (s_k(h_k))_{-k}$. That is, if I' is the nonempty set of issues²⁴ raised by candidate k at information set h_k , then the profiles of (projected) policy points $(y^k(h_k)_{|I'}, y^{-k}(h_k)_{|I'})$ must be in the set of policy points revealed by candidate k to voters at the information set h_k .

At the first glance, the assumption of verifiable information may appear to be stronger than in the previous model because a candidate might be tempted to lie about the opponent's policy point rather than about his own. Yet, as Geer (2006, p. 6) points out “(f)or a negative appeal to be effective, the sponsor of that appeal must marshal more evidence, on average, than for

²⁴In the definition of candidates' strategies above, we require that candidate k reveals the same set of issues for herself and for the opponent. This is without loss of generality since a voter is assumed to reason about any issues that were raised during the campaign.

positive appeals. The public, like our legal system, operates on the assumption of ‘innocent until proven guilty.’ A candidate cannot ... simply assert that their opposition favor a tax increase. They must provide some evidence for this claim ...” This is echoed by some political consultants and campaign managers (see Geer, 2006, pp. 53). Geer (2006, pp. 54) supports he claim with empirical evidence from a content analysis of some television ads of presidential campaigns from the 1964 to 2000. Thus, assuming verifiability of information about opponents may be weaker than assuming verifiability of the candidate’s own information. In any case, we like to point that in this modified model information about the ”entire” policy profiles becomes verifiable, while only “partial” information about policy profiles is assumed to be verifiable in the previous model.

Belief systems in Section 2.2 and prudent rationalizability in Definition 1 are redefined with modified strategies of candidates. We call this the model with negative campaigning because candidates can now reveal unfavorable information about the opponent.

We are now able to show for the presidential model that even when voters lack political reasoning capabilities beyond two rounds of prudent rationalizability (as was assumed in the example of Section 5.3) or candidates lack the ability to microtarget voters, prudent rationalizable outcomes are equivalent to outcomes under full awareness and complete information.

Proposition 4 (Presidential Model with Negative Campaigning) *At every second-level prudent rationalizable outcome of the presidential model with negative campaigning under unawareness of political issues and incomplete information about candidates’ policy points, if a candidate obtains the majority of votes then he could also obtain the majority of votes under full awareness of all political issues and complete information about the candidates’ policy points.*

The proof in the appendix goes roughly as follows: With any first-level prudent rationalizable strategies of a voter, she votes for the candidate whose policy point she is certain to weakly prefer and may even strictly prefer to the other candidate’s policy point. Any second-level prudent rationalizable strategy of candidates must be such that if a candidate has a strictly preferred policy point for a voter in some subspace and all higher-dimensional spaces, then he reveals his own policy point and the other’s.

To gain intuition for why less political reasoning capabilities are required in the model with negative campaigning, recall that candidates face a zero-sum game. If providing certain information is not beneficial to one candidate, it will be to the other. In the baseline model, a candidate with the strictly less preferred policy point for a voter would hide precise information on its policy points since revealing it would benefit the opponent. Hence, sophisticated political reasoning capabilities are required for the voter to reason why this candidate does not provide more precise information. However, in the extended model, even though the candidate may

provide vague information to the voter, the opponent candidate can now reveal the information on the candidate so that the voter has all relevant information for a vote equivalent to one under full awareness and complete information.

To understand why microtargeting is not required anymore, recall from Section 5.2 that without microtargeting a voter may be uncertain whether a candidate did not reveal fully her policy point because her policy point is in fact unfavorable to the voter or because the candidate's policy point is favorable to the voter but unfavorable to another voter. This uncertainty kicks in at the third and higher level of reasoning. With negative campaigning, just two levels of reasoning suffice to reveal sufficient information such that election outcomes do not differ from the case of full awareness and complete information. Thus, negative campaigning is not just a substitute for limited political reasoning capabilities of voters but also for sophistication of candidates' campaign strategies.

What about the parliamentary model? Surprising, a result analogous to Proposition 4 *does not hold* for the parliamentary model. The following counterexample demonstrates that in the parliamentary model, negative campaigning is insufficient for unraveling sufficient information such that election outcomes are equivalent to outcomes under full awareness and complete information of voters if campaign messages are public and no microtargeting of voters is allowed.

Example. Consider the case of a just one-dimensional policy space $Y = \{y^I, y^{II}\}$. There are two voters, $N = \{1, 2\}$, with most preferred policy points $x^1 = y^I$ and $x^2 = y^{II}$. Note that unless the policy points of candidates coincide, the share of voters voting for either one candidate is exactly $\frac{1}{2}$ when voters have complete information about candidates' policy points.

It will suffice to focus on two moves of nature, the case of identical policy points among candidates $(y^a, y^b) = (y^I, y^I)$ as well as the case of different policy points $(y^a, y^b) = (y^I, y^{II})$. The arguments for the other moves of nature are analogous by symmetry. Table 6 shows the first-level prudent rationalizable strategies of voters. The upper table pertains to the case $(y^a, y^b) = (y^I, y^I)$ while the lower is for case $(y^a, y^b) = (y^I, y^{II})$. Each cell refers to a common information set of voters that is reached by a particular combination of actions of candidates a (rows) and b (columns).

Unfortunately, we will show that *any* strategy of each candidate is second-level prudent rationalizable. Consider Case 1: $(y^a, y^b) = (y^I, y^I)$ (upper table). Noteworthy, the strategy of the voter may prescribe to vote for different candidates at different information sets among those 9 information sets at which voters are certain of $(y^a, y^b) = (y^I, y^I)$ because voting for any candidate is rational. Candidate k 's action $\{(y^I, y^I)\}$ is rational at the information set reached by the move of nature, (y^I, y^I) if k believes with sufficiently high probability that the same action is taken by candidate $-k$ and that voters vote for candidate k only at their

Table 6: First-level prudent rationalizable strategies of voters

Case 1 : $(y^a, y^b) = (y^I, y^I)$		Actions of candidate b			
	(y^I, y^I)	$\{(y^I, y^I)\}$	$\{y^I\} \times Y$	$Y \times \{y^I\}$	$Y \times Y$
Actions of candidate a	$\{(y^I, y^I)\}$	any, any	any, any	any, any	any, any
	$\{y^I\} \times Y$	any, any	(a, b)	any, any	(a, b)
	$Y \times \{y^I\}$	any, any	any, any	(b, a)	(b, a)
	$Y \times Y$	any, any	(a, b)	(b, a)	any, any

Case 2 : $(y^a, y^b) = (y^I, y^{II})$		Actions of candidate b			
	(y^I, y^{II})	$\{(y^I, y^{II})\}$	$\{y^I\} \times Y$	$Y \times \{y^{II}\}$	$Y \times Y$
Actions of candidate a	$\{(y^I, y^{II})\}$	(a, b)	(a, b)	(a, b)	(a, b)
	$\{y^I\} \times Y$	(a, b)	(a, b)	(a, b)	(a, b)
	$Y \times \{y^{II}\}$	(a, b)	(a, b)	(a, b)	(a, b)
	$Y \times Y$	(a, b)	(a, b)	(a, b)	any, any

information set reached by the move of nature (y^I, y^I) and the profile of candidates' actions $(\{(y^I, y^I)\}, \{(y^I, y^I)\})$ and otherwise vote for the opponent. By analogous arguments, any other action of candidate k is second-level prudent rationalizable after the move of nature (y^I, y^I) .

Consider now Case 2: $(y^a, y^b) = (y^I, y^{II})$ (bottom table). All actions of candidate k are payoff-equivalent no matter what his opponent does except action $Y \times Y$. Consider for instance actions $\{(y^I, y^{II})\}$ and $Y \times Y$. At any information set of voters reached with candidate k 's action $\{(y^I, y^{II})\}$, exactly one voter votes for each candidate. Action $\{(y^I, y^{II})\}$ is second-level prudent rationalizable for candidate k with a belief system that assigns sufficiently high probability to action $Y \times Y$ of candidate $-k$ and exactly one voter voting for each candidate at their information set reached by those candidates' actions while both vote for $-k$ at the information set reached by the move of nature (y^I, y^{II}) and the candidates' action profile $(Y \times Y, Y \times Y)$. Also action $Y \times Y$ is second-level prudent rationalizable for candidate k with a belief system that assigns sufficiently high probability to action $Y \times Y$ of candidate $-k$ and that both voters vote for k at the information set reached by the move of nature (y^I, y^{II}) and the candidates' action profile $(Y \times Y, Y \times Y)$.

Thus, we have shown that for any strategy of candidate k , there is a full-support belief of candidate k with which the strategy is second-level prudently rational at every of his informa-

tion sets. Moreover, since no strategies of candidates have been eliminated at the first-level of the prudent rationalizability procedure, no strategies of voters are eliminated at the second-level of prudent rationalizability. Hence, prudent rationalizability does not eliminate any candidate's strategies at a higher level and information fails to unravel. \square

Note that the arguments of the previous counterexample do not apply to the presidential model. The crucial difference is that in the presidential model candidates care only about winning the election. Candidate a can now secure his election by choosing the fully revealing action $\{(y^I, y^{II})\}$ in Case 2 above, while due to prudent beliefs there is some uncertainty that candidate a is elected by choosing $Y \times Y$. (Recall that we excluded ties in our model, see p. 12.) This is different from the parliamentary model where candidate a just obtains $\frac{1}{2}$ of all votes when choosing the fully revealing action.²⁵

Microtargeting would help to overcome the unraveling problem of the example in the parliamentary. To see this, note that the only reason why candidate k may use action $Y \times Y$ in Case 2 in the example above is that he can have hopes that candidate $-k$ uses $Y \times Y$ as well and that both voters vote for k . But since he entertains prudent beliefs, there remains some uncertainty whether voters will actually vote like that. Instead, with microtargeting candidate a can ensure that voter 1 votes for him by sending her privately message $\{(y^I, y^{II})\}$ while still remain hopeful on voter 2 by sending her the message $Y \times Y$. Since candidate b acts analogously at the second level of the prudent rationalizability procedure, just two levels of prudent rationalizability are enough to produce election outcomes equivalent to outcomes under full awareness and complete information. It turns out that this holds generally in the parliamentary model.

We now allow again for microtargeting of voters just we did in the baseline model. Formally, a strategy for candidate $k \in \{a, b\}$ is now redefined

$$s_k : H_k \longrightarrow \prod_{j \in N} \left[\bigcup_{\{1\} \subseteq I^j \subseteq I} 2^{Y_{I^j}} \times 2^{Y_{I^j}} \right] \quad (11)$$

such that for every voter $j \in N$, there exists I^j with $\{1\} \subseteq I^j \subseteq I(h_k)$ with $(s_k(h_k))_{j,k}, (s_k(h_k))_{j,-k} \in 2^{Y_{I^j}} \times 2^{Y_{I^j}}$ satisfying $(y^k(h_k)_{|_{I^j}}, y^{-k}(h_k)_{|_{I^j}}) \in (s_k(h_k))_{j,k}, (s_k(h_k))_{j,-k}$, where $(s_k(h_k))_{j,k}$ denotes k 's "message" to voter j on k 's policy points, and $(s_k(h_k))_{j,-k}$ denotes k 's "message" to voter j on $-k$'s policy points.

²⁵Note that a similar counterexample cannot be constructed for the presidential model with an alternative tie-breaking rule. That is, the difference between the parliamentary and the presidential model with respect to negative campaigning is not due to the assumption of the tie-breaking rule in the latter model. Consider for instance an alternative tie-breaking rule according to which each candidate wins with probability 1/2 in case of a tie. This tie-breaking rule mimics the shares of voters in the parliamentary model in the counterexample. In this case, Proposition 4 holds trivially because any candidate can win under full awareness and complete information.

Belief systems in Section 2.2 and prudent rationalizability in Definition 1 are redefined with modified strategies of candidates. We call this the model with negative campaigning and microtargeting.

Proposition 5 (Parliamentary Model with Negative Campaigning and Microtargeting)

At every second-level prudent rationalizable outcome of the parliamentary model with negative campaigning and microtargeting under unawareness of political issues and incomplete information about candidates' policy points, every voter prefers to vote for the same candidate as when he has full awareness of all political issues and complete information about the candidates' policy points.

The proof is contained in the appendix. We conclude that with negative campaigns just “two levels” of political reasoning are required for sufficient unraveling of awareness and information. That is, with negative campaigning it is enough that each voter naively votes for the candidate that she believes is closest to her based on the information that emerges in the campaign and candidates take this into account when choosing their campaign messages. Yet, there is a crucial difference between the presidential and parliamentary model with respect to the sophistication of candidates' campaign strategies. The presidential model achieves unraveling with less sophisticated campaign strategies of candidates that do not require microtargeting of voters while in the parliamentary model unraveling requires microtargeting of voters.

7 Conclusion

We analyzed a model of electoral competition with an extremely stark asymmetry of awareness and information between candidates and voters. Candidates are aware of all policy issues, know the preferences of voters, and are able to use sophisticated campaign strategies including microtargeting of voters. In contrast, voters are unaware of all policy issues except the default issue and don't know the preferences of candidates. The purpose for this extreme model of electoral competition is to study whether electoral competition can overcome this stark asymmetry in awareness and information. We show that despite the stark asymmetries in awareness and information, electoral competition both in the presidential and parliamentary model is strong enough to unravel all “relevant” awareness and information to voters in the sense that the election outcome corresponds to an election outcome under full awareness and complete information of voters. We show by an example that lack of competition impedes unraveling under unawareness of issues.

Somewhat surprisingly, we find that microtargeting voters, a strategy whose appropriateness in elections has been questioned previously in the literature, facilitates unraveling of informa-

tion. In Section 5.2 we illustrated that the candidates' ability to microtarget voters is crucial for our results. Essentially different (although not disjoint) information and awareness is provided to different voters. Wouldn't voters like to share their information with other voters and render microtargeting ineffective? Note that if voters know each others' preferences, they should like to share their own information with "like-minded" voters similar to viral campaigns in online social networks like Facebook or MySpace. They would have no interest to voluntarily provide their information to voters that are much different from them. That is, communication in homophilic social networks formed by like-minded voters is effectively enhancing microtargeting of voters. Lutz (2009), a public relations strategist, claims that Obama's formula of victory over McCain in the presidential election had to do with his 13 million member email-list and 3 million SMS and mobile subscribers, tools that McCain did not effectively use. We leave the detailed analysis of imperfect microtargeting to future research. Similar to social networks, we believe that also lobbies or special interest groups may enhance unraveling. Our analysis considers competition among at most to candidates. One natural extension would be to allow for special interest groups that can raise political issues and provide information about candidates' policies. A special interest group would simply support the candidate closest to her political preferences. There wouldn't be much of a difference between campaign messages of the special interest group and the supported candidate. Another possible extension to our model is to include costs of campaigning. Such costs may trivially prevent candidates to raise awareness of a sufficient number of issues and impede unraveling.

From the view point of the literature on unawareness, we have shown that unraveling of information in electoral competition is robust to voters' unawareness of political issues. It does not imply that unawareness "does not matter" since we show that unraveling under unawareness hinges crucially on the assumption of competition. Moreover, while negative campaigning can overcome some of the impediments to unraveling like the inability to microtarget voters or voters' limited political reasoning capabilities, it cannot overcome the problems for unraveling created by unawareness under lack of competition because negative campaigning seems hard to define in a non-competitive environment. We view it a strength of our framework that it enables us to formulate and analyze questions about the robustness of outcomes to unawareness. Our results are related to Heifetz, Meier, and Schipper (2011) who show that unraveling of information about product quality may break down under unawareness in a model with a monopolist seller and a buyer. Filiz-Ozbay (2012) shows in a different framework that a monopolist insurer may propose incomplete insurance contracts to an insuree who faces unawareness of some relevant contingencies but that competition among insurers leads to completeness. Li, Peitz, and Zhao (2013) study disclosure of product information to consumers under vertical competition in a duopoly when consumers may be unaware of one dimension of the product but otherwise have complete information. Our model and solution concept differs from theirs substantially.

In particular, we allow for unawareness of many dimensions and incomplete information even after some dimensions have been disclosed. This requires us to model carefully the state of mind of (unaware) voters after candidates have made their disclosure decisions. On the other hand, Li, Peitz, and Zhao (2013) allow firms to also set prices and study the effect of timing price and disclosure decisions.

The paper closest to our study is Demange and Van der Straeten (2013). They also model electoral campaigning with a persuasion game à la Milgrom (1981). Candidates can send a signal on each issue about their fixed ideological policy point on that issue. Signals are unbiased, normally distributed with the variance controlled by the candidate. There is just a single voter who is aware of all issues, has a prior belief that is independent across candidates and across issues, and normally distributed. This voter observes both the signals and the variances chosen by candidates. Candidates do not interact strategically as each just considers the effect of her own strategy on the voter. They show unraveling in the sequential equilibrium of the game. Moreover, they show that when voters are naive and take messages at face-value, unraveling breaks down. Their model is very similar to ours. Both use verifiable information. Yet, while in their model candidates chose unbiased signals distributed normally with chosen variance, while in our model candidates chose arbitrary but finite sets of policy points that contain their true policy point. We make no distributional assumptions. Their conclusions are very similar to ours as well, which shows that the results in either paper cannot be an artefact of differing modeling assumptions. Their result on naive voters is akin to our observation that unraveling breaks down when voters have limited political reasoning capabilities. Our analysis suggests that their result could be extended to strategic interaction among candidates. Moreover, our analysis suggests that their results could be extended to unraveling even in the presence of unawareness if strategic interaction among candidates is added. We also show that allowing for more than one voter introduces additional difficulties because unraveling may break down in the absence of microtargeting. The fundamental difference between public campaigns and microtargeting is not exposed in Demange and Van der Straeten (2013). Finally, we study negative campaigning, a feature that cannot be studied without strategic interaction among candidates.

We are not the first who discuss salience of issues in electoral competition. There is a large literature in political science on issue ownership theory starting with Budge and Farlie (1983) and Petrocik (1996). According to issue ownership theory, “candidates emphasize issues on which they are advantaged”. Yet, as Green and Hobolt (2008) emphasize with data from British elections, as parties converge ideologically, their relative competence on an issue becomes more important than ideological considerations. At a first glance, a candidate’s relative competence on an issue as reason for the candidate to campaign on that issue seems different from our model. Yet, if political positions in our model are reinterpreted as degrees competence on the

issues, our model becomes a formal model of issue ownership theory. Presumably in such a setting voters prefer uniformly more competence to less, and hence preferences of voters are homogeneous. Our model predicts then that enough relevant issues and information on the candidates's competence is revealed in electoral competition such that election outcomes are identical to the ones under full awareness of all issues and complete information about the candidates' competence. Note that such a result would not hinge anymore on the candidates' abilities to microtarget voters as all voters have homogeneous preferences over competence. Yet, a lack of electoral competition may still prevent unraveling under unawareness of issues because the candidate could be silent on issues he does not "own".

In economics, Berliant and Konishi (2005) study whether candidates in a multi-dimensional Downs model with linear utility of voters like to announce policies on all issues. They assume that voters know the state of the world and candidates just have a prior distribution over voters' types. They show that if a Nash equilibrium exists, candidates like to announce policies on all issues. Moreover, they show by example that non-salience may emerge when candidates face Knightian uncertainty and maximize minimal expected utility. Our models differ substantially from each other. Besides differing assumptions about ideological versus opportunistic candidates and differing informational assumptions, it is impossible in their model to be salient on an issue without announcing a policy on this issue. There is also no role for microtargeting of voters. Moreover, our focus on a rationalizability procedure rather than Nash equilibrium allows us to shed more light on the importance of political reasoning capabilities of voters. Colomer and Llavador (2012) study electoral competition with salience of issues. Ex ante salience of an issue is proportional to the disagreement of the electorate about the status quo policy of the incumbent on this issue. The ex post salience of an issue is highest when both candidates campaign on it by choosing a policy point. Candidates are allowed to campaign only on one issue each. They show that in subgame perfect equilibrium candidates may not campaign on ex ante most salient issues if there is no policy on this issue that would attract broad agreement among the electorate. Again, our model differs substantially from theirs. Most importantly, in our model candidates can choose to campaign on as many issues as they like and can target different issues to different voters.

Our work is related to the large literature on information aggregation of elections. McKelvey and Ordeshook (1985) present an uni-dimensional Downs model with two candidates. Candidates do not know the preferences of voters and uninformed voters do not know the political positions adopted by candidates but there is also a share of informed voters who know the political positions. All participants can learn from "polls" and "interest group endorsements". They show that in a version of self-confirming equilibrium, in which strategies of participants are optimal with respect to the public information available and the public information is consistent with the strategies of participants, election outcomes correspond to full

information election outcomes. McKelvey and Ordeshook (1987) present sufficient conditions on the number and distribution of informed and uninformed voters for an analogous result in a multi-dimensional setting, in which poll-data must be broken down by subgroups of voters. Feddersen and Pesendorfer (1997) show that large elections with strategic voters and two fixed policy alternatives can aggregate information about a uni-dimensional state variable. Voters are differently informed by some “information services” instead by strategically campaigning candidates. They also show that with higher-dimensional uncertainty, elections may not effectively aggregate information and suggest that future research should “focus on the events that precede elections – nominating procedures, campaigns, polls, etc. – as such events determine the information environment.” Although our model is very different from aforementioned models, it can be viewed as focusing exactly on “events that precede elections”. Gratton (2014) studies electoral competition between two perfectly informed candidates that are faced by voters who have a common value over the policies but he requires them to have some information about what is best for them. Candidates can be of two types, either strategic or truthful. He studies a sequential equilibrium that also entails some forward-induction in that voters can revise beliefs when candidates propose different policies and that leads an election outcome identical to the full information outcome. Heidhues and Lagerlöf (2003) study a model in which voters possess no information. But candidates receive imperfectly correlated private signals about the state of nature. In equilibrium candidates bias their information transmission through the choice of platforms towards the voters’ prior, letting information revelation fail. Laslier and Van der Straeten (2004) show that this conclusion is not robust as soon as the voters have a tiny bit of relevant information. In this case, all equilibria are dismissed by standard refinements except the one in which information revelation occurs.

We are not aware of models of electoral campaigning with microtargeting of voters. Most closely related is Glaezer, Ponzetto, and Shapiro (2005) who study electoral competition à la Downs (1957) but with endogenous voter turnout in which some party members can observe secretly the platform of the candidate on “their side” of the political spectrum while less members of the other parties can observe it. Thus deviating from the median voter does not necessarily alienate voters on the other side while mobilizing support on its own side, producing divergent platforms. Different from our model, there is no role for parties to microtarget “swing voters” that may be traditionally associated with the other party.

We close with a comment on empirical testing of our theory. The difficulty is that preferences, beliefs, and levels of reasoning are not directly observable in the field. Yet, it should be possible to carefully design a laboratory experiment that controls for preferences. This is left for further research.

A Proofs

A.1 Proof of Proposition 2

The proof is analogous to the proof of Proposition 3 with the following modifications:

- Erase (N3).
 - Replace (N4) by
- (N4') if $|N_k^\ell(h_k)| < |N|$ then there is no $N' \subseteq N$ satisfying N1 and N2 in place of $N_k^{(\ell)}(h_k)$ and for which $|N'| > |N_k^{(\ell)}(h_k)|$.
- Redefine $\bar{N} := N_k^{(\ell)}(h_k)$ and erase equations (12) and (13).
 - Essentially replace “majority of voters” by “every voter” or “some voter”, whatever appropriate, throughout.
 - Essentially replace “ k wins the election” by “ k obtains more votes” and “to $-k$ wins the election” by “that $-k$ obtains votes that k could obtain”.

□

A.2 Proof of Proposition 3

Consider first the case in which there is a majority of voters who are indifferent between candidates when all voters are fully aware and have complete information about the candidates' profile of policy points $(y^a, y^b) \in Y \times Y$ in the full-dimensional policy space. For those voters, it is rational to vote for any candidate. Hence, for those voters it is trivially true that if any of them prefers to vote for a particular candidate in a prudent rationalizable outcome of the presidential model with unawareness of political issues and incomplete information about candidates' policy points, then he prefers to vote for the same candidate when having full awareness of all political issues and complete information about the candidates' policy points. Thus, from now on we consider only the case in which there is no majority of voters who are indifferent between the candidates when all voters have full awareness of political issues and complete information about candidates' policy points.²⁶

We also assume $|N| \geq 2$ since otherwise the presidential model is a special case of the parliamentary model.

²⁶Note that it does not imply that there is no majority of voters who are indifferent between candidates under unawareness of some issues but complete information about the policy points in the subspace of issues that they are aware of.

The proof proceeds by induction. Note that the base-case involves the first two levels of prudent rationalizability, while the induction step assumes properties of level $2\ell - 1$ and 2ℓ prudent rationalizable strategies and proves properties of level $2\ell + 1$ and $2\ell + 2$ prudent rationalizable strategies, for $\ell > 1$. This is due to the nature of our two-stage game.

First level: For any candidate $k \in \{a, b\}$ we have $S_k^1 = S_k$. To see this, note that for candidate k , every $s_k \in S_k$ is first-level rationalizable with a belief system β_k such that for every information set h_k the full-support belief $\beta_k(h_k)$ puts sufficiently high probability to strategies of voter j that ascribe voting for k at every information set reached by $s_k(h_k)$ and voting for $-k$ at all other information sets of j , for all $j \in N$.

Before we turn to first-level prudent rationalizable strategies of voters, it will be helpful to introduce the following notation. We say that y^k reaches the information set h_j of voter j if there is a move of nature (y^k, y^{-k}) there is a path (i.e., a sequence of nodes) from (y^k, y^{-k}) to some node in h_j . Let $Y^k(h_j) := \{y^k \in Y_{|I(h_j)} : y^k \text{ reaches } h_j\}$. That is, $Y^k(h_j)$ is the set of candidate's policy points in $Y_{|I(h_j)}$ that voter j considers possible at his information set h_j .

For any voter j and any of his information sets $h_j \in H_j$, define inductively,

$$Y_j^{(1), I(h_j)} := \left\{ y \in Y_{|I(h_j)} : \|x^j - y\|_{I(h_j)} \leq \|x^j - y'\|_{I(h_j)} \text{ for all } y' \in Y_{|I(h_j)} \right\},$$

and for $\ell > 1$,

$$Y_j^{(\ell), I(h_j)} := \left\{ y \in Y_{|I(h_j)} \setminus \left(\bigcup_{\ell' \leq \ell-1} Y_j^{(\ell'), I(h_j)} \right) : \begin{array}{l} \|x^j - y\|_{I(h_j)} \leq \|x^j - y'\|_{I(h_j)} \\ \text{for all } y' \in Y_{|I(h_j)} \setminus \left(\bigcup_{\ell' \leq \ell-1} Y_j^{(\ell'), I(h_j)} \right) \end{array} \right\}.$$

That is, $Y_j^{(1), I(h_j)}$ is the set of voter j 's most preferred policy points of candidates in the policy space that he is aware of at the information set h_j . Similarly, $Y_j^{(\ell), I(h_j)}$ is the set of voter j 's ℓ -most preferred policy points of candidates in the policy space that he is aware of at his information set h_j . Since Y is finite, there is a well-defined set of voter j 's least preferred candidates' policy points that he is aware of at his information set h_j , i.e., a finite largest ℓ .

With these definitions in place, we turn to the first-level prudent rationalizable strategies of voters. For any voter j consider an information set h_j with $Y^k(h_j) \subseteq Y_j^{(1), I(h_j)}$ and $Y^{-k}(h_j) \not\subseteq Y_j^{(1), I(h_j)}$. Note that for any of voter j 's belief system β_j , the support of $\beta_j(h_j)$ is the set of policy points and strategy profiles of candidates that reach the information set h_j . With any such a belief system, any first-level prudent rationalizable strategy of voter j must ascribe voting for candidate k at h_j . This is because with any such a belief system, voter j is certain that candidate k has his most preferred policy point in $Y_{|I(h_j)}$ while he must assign some strict positive probability to policy points of candidate $-k$ that are strictly less preferred.²⁷

²⁷This condition is just a necessary condition for first-level prudent rationalizable strategies of voters.

Second level: Before we turn to second-level rationalizable strategies, the following definition will be helpful. Given an information set h_k of candidate k reached by the move of nature $(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}))$, let for any $\ell \geq 1$, $N_k^{(\ell)}(h_k) \subseteq N$ be a (possibly empty) subset of voters such that

- (N1) every voter $j \in N_k^{(\ell)}(h_k)$ strictly prefers $r_{I(h_k)}^I(y^k)$ over $r_{I(h_k)}^I(y^{-k})$,
- (N2) for every voter $j \in N_k^{(\ell)}(h_k)$, $r_{I(h_k)}^I(y^k) \in Y_j^{(\ell'), I(h_k)}$ for some ℓ' with $\ell \geq \ell' \geq 1$,
- (N3) the cardinality of $N_k^{(\ell)}(h_k)$ is such that²⁸

$$|N_k^{(\ell)}(h_k)| \leq \begin{cases} \lceil \frac{1}{2}|N| \rceil & \text{if } k = a \text{ or } (k = b \text{ and } |N| \text{ is odd}) \\ \frac{1}{2}|N| + 1 & \text{else (i.e., if } k = b \text{ and } |N| \text{ is even),} \end{cases}$$

- (N4) if $|N_k^{(\ell)}(h_k)| \leq \frac{1}{2}|N|$ then there is no $N' \subseteq N$ that satisfies properties N1 to N3 in place of $N_k^{(\ell)}(h_k)$ and for which $|N'| > |N_k^{(\ell)}(h_k)|$.

We claim that if s_k is a second-level prudent rationalizable strategy of candidate $k \in \{a, b\}$ then for any information set h_k of candidate k reached by the move of nature $(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}))$ we have that for any voter $j \in N$, $(s_k(h_k))_j \subseteq Y_{I^j}$ for some $I^j \subseteq I(h_k)$. Moreover, there is (a possibly empty) subset of voters $N_k^{(1)}(h_k) \subseteq N$ such that for all $j \in N_k^{(1)}(h_k)$ and any I' with $I^j \subseteq I' \subseteq I(h_k)$,

- (i) $(r_{I^j}^{I'})^{-1}((s_k(h_k))_j) \subseteq Y_j^{(1), I'}$, and
- (ii) voter j strictly prefers $r_{I'}^I(y^k)$ over $r_{I'}^I(y^{-k})$.

We show that these conditions are necessary for second-level prudent rationalizable strategies of candidates. Consider any information set h_k of candidate k such that k knows $(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}))$. Let s_k be a second-level prudent rationalizable strategy of candidate $k \in \{a, b\}$. By the definition of strategy, we must have that for any $j \in N$, $(s_k(h_k))_j \subseteq Y_{I^j}$ for some $I^j \subseteq I(h_k)$. Suppose to the contrary that for every $N_k^{(1)}(h_k)$ there is a nonempty subset of voters $N' \subseteq N_k^{(1)}(h_k)$ such that for any $j \in N'$ properties (i) or (ii) are violated. (If $N_k^{(1)}(h_k)$ is empty, there is nothing to prove.) That is, for any $j \in N'$, there exists \tilde{I}^j with $I^j \subseteq \tilde{I}^j \subseteq I(h_k)$ such that $(r_{\tilde{I}^j}^{I^j})^{-1}((s_k(h_k))_j) \not\subseteq Y_j^{(1), \tilde{I}^j}$ or j does not strictly prefer $r_{\tilde{I}^j}^I(y^k)$ over $r_{\tilde{I}^j}^I(y^{-k})$.

With any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to strategies of candidate $-k$ that reveal information to any voter $j \in N'$ such that an information set h_j of voter $j \in N'$ in $T^{\tilde{I}^j}$ is reached with $Y^{-k}(h_j) \cap Y_j^{(1), \tilde{I}^j} \neq \emptyset$.

²⁸Recall from Section 2 that a wins if it obtains weakly more than half of the votes, whereas b wins with strictly more than half of the votes. $\lceil x \rceil$ denotes the smallest integer not less than x .

Let $\bar{N} \subset N$ be such that $N_k^{(1)}(h_k) \subseteq \bar{N}$ and

$$|\bar{N}| = \begin{cases} \lceil \frac{1}{2}|N| \rceil & \text{if } k = a \text{ or (if } k = b \text{ and } |N| \text{ is odd)} \\ \frac{1}{2}|N| + 1 & \text{else (i.e., if } k = b \text{ and } |N| \text{ is even).} \end{cases} \quad (12)$$

We can partition the set of voters N into $\{N_k^{(1)}(h_k) \setminus N', N', \bar{N} \setminus N_k^{(1)}(h_k), N \setminus \bar{N}\}$. With any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to first-level prudent rationalizable strategies of voter j such that

$$\text{if } j \in \begin{cases} N_k^{(1)}(h_k) \setminus N' & \text{then } j \text{ votes for } k \\ N' & \text{then } j \text{ votes for } -k \\ \bar{N} \setminus N_k^{(1)}(h_k) & \text{then } j \text{ votes for } k \\ N \setminus \bar{N} & \text{then } j \text{ votes for } -k \end{cases},$$

which implies that candidate k must assign strictly positive probability to $-k$ winning the election. Yet, candidate k can strictly improve his expected payoff at h_k given $\beta_k(h_k)$ by replacing $(s_k(h_k))_j$ for $j \in N'$ with $\{r_{I(h_k)}^I(y^k)\}$, because at any information set h_j of voter $j \in N'$ reached by this modified strategy of candidate k , any first-level prudent rationalizable strategy of voters $j \in N'$ must ascribe voting for k implying that k wins the election, a contradiction.

For all voters $j \in N$, $S_j^2 = S_j^1$ since $S_k^1 = S_k$ for $k \in \{a, b\}$.

Induction step: The following definitions are helpful. For any $\ell > 1$, we say that *strategy s_j of voter j satisfies condition ℓ* if for every h_j such that for some $k \in \{a, b\}$,

1. for some ℓ^k with $\ell \geq \ell^k \geq 1$, $Y^k(h_j) \cap Y_j^{(\ell^k), I(h_j)} \neq \emptyset$ and $Y^k(h_j) \cap Y_j^{(\ell'), I(h_j)} = \emptyset$ for all $\ell' > \ell^k$, and
2. if for some ℓ^{-k} with $\ell \geq \ell^{-k} \geq 1$, $Y^{-k}(h_j) \cap Y_j^{(\ell^{-k}), I(h_j)} \neq \emptyset$ and $Y^{-k}(h_j) \cap Y_j^{(\ell''), I(h_j)} = \emptyset$ for all $\ell'' > \ell^{-k}$, then $\ell^{-k} > \ell^k$,

then voter j votes for k at h_j . Intuitively, this condition states that if candidate k reveals information to voter j that is weakly better than her ℓ -most preferred policy points and $-k$ reveals information to voter j that is not unambiguously better than k 's information, then voter j votes for candidate k .

For any $\ell > 1$, we say that *strategy s_k of candidate k satisfies condition ℓ* if for every $y^k \in Y$ and every information set h_k of candidate k reached by the move of nature $(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}))$ we have that for all voters $j \in N$, $(s_k(h_k))_j \subseteq Y_{I_j}^{I'}$ for some $I^j \subseteq I(h_k)$. Moreover, there is (a possibly empty) subset of voters $N_k^{(\ell)}(h_k) \subseteq N$ such that for all $j \in N_k^{(\ell)}(h_k)$ and any I' with $I^j \subseteq I' \subseteq I(h_k)$,

- (I) there is $\ell^{I'}$ with $\ell \geq \ell^{I'} \geq 1$ such that $r_{I'}^I(y^k) \in Y_j^{(\ell^{I'}), I'}$ (and hence $(r_{I_j}^{I'})^{-1}((s_k(h_k))_j) \cap Y_j^{(\ell^{I'}), I'} \neq \emptyset$) and $(r_{I_j}^{I'})^{-1}((s_k(h_k))_j) \cap Y_j^{(\ell'), I'} = \emptyset$ for all $\ell' > \ell^{I'}$, and

(II) voter j strictly prefers $r_{I'}^I(y^k)$ over $r_{I'}^I(y^{-k})$.

Assume now that we have proved that for every voter $j \in N$ the $(2\ell - 1)$ -level prudent rationalizable strategies of voter j satisfy condition ℓ and that for every candidate $k \in \{a, b\}$ the 2ℓ -level prudent rationalizable strategies of candidate k satisfy condition ℓ . We claim that for any voter $j \in N$, the $(2\ell + 1)$ -level prudent rationalizable strategies satisfy condition $\ell + 1$ and for every candidate $k \in \{a, b\}$, the $(2\ell + 2)$ -level prudent rationalizable strategies of candidate k satisfy condition $\ell + 1$.

Consider a voter $j \in N$ with information set h_j . Suppose that for some ℓ^k with $\ell + 1 \geq \ell^k \geq 1$, $Y^k(h_j) \cap Y_j^{(\ell^k), I(h_j)} \neq \emptyset$ and $Y^k(h_j) \cap Y_j^{(\ell'), I(h_j)} = \emptyset$, for all ℓ' with $\ell' > \ell^k$. Unless we also have that for some ℓ^{-k} with $\ell^k \geq \ell^{-k} \geq 1$, $Y^{-k}(h_j) \cap Y_j^{(\ell^{-k}), I(h_j)} \neq \emptyset$ and $Y^{-k}(h_j) \cap Y_j^{(\ell''), I(h_j)} = \emptyset$, for all ℓ'' with $\ell'' > \ell^{-k}$, then with any $2\ell + 1$ prudent rationalizable strategy, voter j must vote for k at h_j . To see this note that for any belief system of voter j , $\beta_j \in B_j^{2\ell+1}$, the support of the belief $\beta_j(h_j)$ at h_j is the set of 2ℓ -prudent rationalizable strategies of candidates who by assumption satisfy condition ℓ . Thus, the voter is certain at h_j of $y_{|I(h_j)}^k \in Y_j^{(\ell^k), I(h_j)}$. Moreover, since candidates' strategies satisfy condition ℓ , voter j with belief $\beta_j(h_j)$ cannot assign strict positive probability to policy points $y_{|I(h_j)}^{-k}$ of candidate $-k$ that are strictly preferred to k 's policy point since otherwise $-k$ would have revealed it. It follows that voter j 's $(2\ell + 1)$ -level prudent rationalizable strategies satisfy condition $\ell + 1$.

Consider any information set h_k of candidate k such that k knows $(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}))$. Let s_k be a $(2\ell + 2)$ -level prudent rationalizable strategy of candidate k . By the definition of strategy, we must have that for any $j \in N$, $(s_k(h_k))_j \subseteq Y_{|I^j}$ for some $I^j \subseteq I(h_k)$. Suppose to the contrary that for every $N_k^{(\ell+1)}(h_k)$ there is a nonempty subset of voters, $N' \subseteq N_k^{(\ell+1)}(h_k)$, such that for any $j \in N'$ there exists \tilde{I}^j with $I^j \subseteq \tilde{I}^j \subseteq I(h_k)$ for which properties (I) or (II) are violated. (If $N_k^{(\ell+1)}(h_k)$ is empty, there is nothing to prove.) That is, we have for all $\tilde{\ell}$ with $\ell + 1 \geq \tilde{\ell} \geq 1$, $r_{\tilde{I}^j}^I(y^k) \notin Y_j^{(\tilde{\ell}), \tilde{I}^j}$ or $(r_{\tilde{I}^j}^I)^{-1}((s_k(h_k))_j) \cap Y_j^{(\ell'), \tilde{I}^j} \neq \emptyset$ for some $\ell' > \tilde{\ell}$, or j does not strictly prefer $r_{\tilde{I}^j}^I(y^k)$ over $r_{\tilde{I}^j}^I(y^{-k})$.

Note that since $j \in N_k^{(\ell+1)}(h_k)$ we have by N1 that j strictly prefers $r_{I(h_k)}^I(y^k)$ over $r_{I(h_k)}^I(y^{-k})$. Moreover, by N2 we must have $r_{I(h_k)}^I(y^k) \in Y_j^{(\ell''), I(h_k)}$ for some ℓ'' with $\ell + 1 \geq \ell'' \geq 1$.

With any belief system $\beta_k \in B_k^{2\ell+2}$, candidate k must assign strict positive probability to $(2\ell + 1)$ -level prudent rationalizable strategies of candidate $-k$ that reveal information to voter $j \in N'$ such that an information set h_j of voter j in $T^{\tilde{I}^j}$ is reached with $Y^{-k}(h_j) \cap Y_j^{(\tilde{\ell}), \tilde{I}^j} \neq \emptyset$.

Redefine $\bar{N} \subset N$ such that $N_k^{(\ell+1)}(h_k) \subseteq \bar{N}$ and

$$|\bar{N}| = \begin{cases} \lceil \frac{1}{2}|N| \rceil & \text{if } k = a \text{ or (if } k = b \text{ and } |N| \text{ is odd)} \\ \frac{1}{2}|N| + 1 & \text{else (i.e., if } k = b \text{ and } |N| \text{ is even).} \end{cases} \quad (13)$$

Partition the set of voters N into $\{N_k^{(\ell+1)}(h_k) \setminus N', N', \bar{N} \setminus N_k^{(\ell+1)}(h_k), N \setminus \bar{N}\}$. With any belief system $\beta_k \in B_k^{2\ell+2}$, candidate k at h_k must assign strict positive probability to first-level prudent rationalizable strategies of voter j such that

$$\text{if } j \in \begin{cases} N_k^{(\ell+1)}(h_k) \setminus N' & \text{then } j \text{ votes for } k \\ N' & \text{then } j \text{ votes for } -k \\ \bar{N} \setminus N_k^{(\ell+1)}(h_k) & \text{then } j \text{ votes for } k \\ N \setminus \bar{N} & \text{then } j \text{ votes for } -k \end{cases},$$

which implies that candidate k must assign strictly positive probability to $-k$ winning the election. Yet, candidate k can strictly improve his expected payoff at h_k given $\beta_k(h_k)$ by replacing $(s_k(h_k))_j$ for $j \in N'$ with $\{r_{I(h_k)}^I(y^k)\}$, because at any information set h_j of voter $j \in N'$ reached by this modified strategy of candidate k , any $(2\ell + 1)$ -level prudent rationalizable strategy of voters $j \in N'$ must ascribe voting for k implying that k wins the election, a contradiction.

Hence, since Y is finite, there is a finite $2\bar{\ell}$ such that for any $\ell > 2\bar{\ell}$ no strategy is eliminated anymore. Moreover, conditions $\bar{\ell}$ imply that neither candidate can change the election outcome by revealing his policy point in the full-dimensional space at any of his information sets in T^I . Conversely, since we assumed that there is a majority of voters who strictly prefer to vote for one particular candidate when all voters have full awareness of political issues and complete information about the policy points, this candidate must also win in any prudent rationalizable outcome under unawareness and incomplete information. \square

A.3 Proof of Proposition 4

By the same arguments as in the proof of Proposition 3, we just consider the case in which there is no majority of voters who are indifferent between the candidates when all voters have full awareness of political issues and complete information about candidates' policy points.

First level: For the same reason as in the proof of the Proposition 3, we have $S_k^1 = S_k$ for any candidate $k \in \{a, b\}$.

For any voter j , consider an information set h_j such that for any $y^k \in Y^k(h_j)$ and $y^{-k} \in Y^{-k}(h_j)$, $\|x^j - y^k\|_{I(h_j)} \leq \|x^j - y^{-k}\|_{I(h_j)}$ and for some $y^k \in Y^k(h_j)$ and $y^{-k} \in Y^{-k}(h_j)$, $\|x^j - y^k\|_{I(h_j)} < \|x^j - y^{-k}\|_{I(h_j)}$. Note that for any of voter j 's belief system β_j , the support of $\beta_j(h_j)$ is the set of profiles of policy points and strategy profiles of candidates that reach the information set h_j . With any such a belief system, any first-level prudent rationalizable strategy of voter j must ascribe voting for candidate k at h_j . This is because with any such a belief system, voter j at h_j is certain that candidate k is (weakly) preferred to candidate $-k$ for any profile of candidates policy points in $Y_{I(h_j)}$, while he must assign some strictly positive probability to profiles of policy points (y^k, y^{-k}) for which he strictly prefers y^k to y^{-k} .

Second level: For all voters $j \in N$, $S_j^2 = S_j^1$ since $S_k^1 = S_k$ for $k \in \{a, b\}$. Given an information set h_k of candidate k reached by the move of nature $(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}))$, let

$$\bar{N}_k(h_k) = \left\{ j \in N : \text{voter } j \text{ strictly prefers } r_{I(h_k)}^I(y^k) \text{ to } r_{I(h_k)}^I(y^{-k}) \right\}. \quad (14)$$

Denote by m_k the majority of number of voters required for candidate k to win, which is defined by

$$m_k := \begin{cases} \lceil \frac{1}{2}|N| \rceil & \text{if } k = a \text{ or (if } k = b \text{ and } |N| \text{ is odd)} \\ \frac{1}{2}|N| + 1 & \text{else (i.e., if } k = b \text{ and } |N| \text{ is even).} \end{cases}$$

Since we just need to consider the case in which there is no majority of voters who are indifferent between the candidates when all voters have full awareness of political issues and complete information about candidates' policy points, for every move of nature in the upmost tree there is a candidate k and an information set h_k of candidate k reached by this move of nature such that $|\bar{N}_k(h_k)| \geq m_k$. Fix the move of nature that reaches h_k with $|\bar{N}_k(h_k)| \geq m_k$.

We claim that if s_k is a second-level prudent rationalizable strategy of candidate k , then there is a maximal subset of voters $N_k(h_k) \subseteq \bar{N}_k(h_k)$ such that $|N_k(h_k)| \geq m_k$ and for all $j \in N_k(h_k)$ and any I' with $I^k \subseteq I' \subseteq I(h_k)$ (where I^k is the subset of issues revealed to voters by strategy s_k at information set h_k)

- (i) $\|x^j - y'\|_{I'} \leq \|x^j - y''\|_{I'}$ for any $y' \in (r_{I'}^{I^k})^{-1}((s_k(h_k))_k)$ and $y'' \in (r_{I'}^{I^k})^{-1}((s_k(h_k))_{-k})$,
- (ii) voter j strictly prefers $r_{I'}^I(y^k)$ over $r_{I'}^I(y^{-k})$.

Suppose to the contrary that there is no such a $N_k(h_k)$. Then there is subset of voters $N' \subseteq N$ with $|N'| \geq m_{-k}$ and I' such that for any $j \in N'$, $\|x^j - y'\|_{I'} > \|x^j - y''\|_{I'}$ for some $y' \in (r_{I'}^{I^k})^{-1}((s_k(h_k))_k)$ and $y'' \in (r_{I'}^{I^k})^{-1}((s_k(h_k))_{-k})$, or j does not strictly prefer $r_{I'}^I(y^k)$ over $r_{I'}^I(y^{-k})$.

With any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to strategies of candidate $-k$ that reveal information to voters such that an information set h_j of voter $j \in N'$ in I' is reached at which for some $y' \in Y^k(h_j)$ and $y'' \in Y^{-k}(h_j)$, $\|x^j - y'\|_{I'} > \|x^j - y''\|_{I'}$ or $\|x^j - r_{I'}^I(y^k)\|_{I'} \geq \|x^j - r_{I'}^I(y^{-k})\|_{I'}$. Thus, with any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to first-level prudent rationalizable strategies of voter j and strategies of candidate $-k$ such that all voters in N' vote for candidate $-k$, in which case candidate k loses the election. Yet, candidate k can strictly improve her expected payoff at h_k given β_k by replacing $(s_k(h_k))$ with $\left\{ \left(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}) \right) \right\}$, because at any information set h_j of voter $j \in \bar{N}_k(h_k)$ reached by this modified strategy of candidate k , any first-level prudent rationalizable strategy of the voter must ascribe voting for k . Since $|\bar{N}_k(h_k)| \geq m_k$, it implies that k wins the election, a contradiction. \square

A.4 Proof of Proposition 5

First level: The arguments about first-level rationalizable strategies for both candidates and voters are analogous to the proof of Proposition 4.

Second level: For all voters $j \in N$, $S_j^2 = S_j^1$ since $S_k^1 = S_k$ for $k \in \{a, b\}$. We claim that if s_k is a second-level prudent rationalizable strategy of candidate k , then for all $j \in \bar{N}_k(h_k)$ (where $\bar{N}_k(h_k)$ is defined as in (14) in the proof of Proposition 4) and I' with $I^k \subseteq I' \subseteq I(h_k)$ (where I^j is the subset of issues revealed to voter j by $s_k(h_k)$)

- (i) $\|x^j - y'\|_{I'} \leq \|x^j - y''\|_{I'}$ for any $y' \in (r_{I^k}^{I'})^{-1}((s_k(h_k))_{j,k})$ and $y'' \in (r_{I^k}^{I'})^{-1}((s_k(h_k))_{j,-k})$,
- (ii) voter j strictly prefers $r_{I'}^I(y^k)$ over $r_{I'}^I(y^{-k})$.

Suppose to the contrary that there is a voter $j \in \bar{N}_k(h_k)$ and I' with $I^k \subseteq I' \subseteq I(h_k)$ such that $\|x^j - y'\|_{I'} > \|x^j - y''\|_{I'}$ for some $y' \in (r_{I^k}^{I'})^{-1}((s_k(h_k))_{j,k})$ and $y'' \in (r_{I^k}^{I'})^{-1}((s_k(h_k))_{j,-k})$, or j does not strictly prefer $r_{I'}^I(y^k)$ over $r_{I'}^I(y^{-k})$.

With any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to strategies of candidate $-k$ that reveal information to voters in $\bar{N}_k(h_k)$ such that an information set h_j of voter j in $T^{I'}$ is reached at which for some $y' \in Y^k(h_j)$ and $y'' \in Y^{-k}(h_j)$, $\|x^j - y'\|_{I'} > \|x^j - y''\|_{I'}$ or $\|x^j - r_{I'}^I(y^k)\|_{I'} \geq \|x^j - r_{I'}^I(y^{-k})\|_{I'}$. Thus, with any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to first-level prudent rationalizable strategies of voter j and strategies of candidate $-k$ such that voter j votes for candidate $-k$. Yet, candidate k can capture the vote of voter j for sure and thus strictly improve her expected payoff at h_k given β_k by replacing $(s_k(h_k)_{j,k}, s_k(h_k)_{j,-k})$ with $\left\{ \left(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}) \right) \right\}$, because at any information set h_j of voter j reached by this modified strategy of candidate k , any first-level prudent rationalizable strategy of the voter must ascribe voting for k (this follows from the fact that $j \in \bar{N}_k(h_k)$), a contradiction. \square

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