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A Theory of the Boundaries of Banks with Implications For Financial Integration and Regulation
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A Theory of the Boundaries of Banks with Implications for Financial Integration and Regulation*

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Abstract

We offer a theory of the "boundary of the firm" that is tailored to banking, as it builds on a single inefficiency arising from risk-shifting and as it takes into account both interbank lending as an alternative to integration and the role of possibly insured deposit funding. Amongst others, it explains both why deeper economic integration should cause also greater financial integration through both bank mergers and interbank lending, albeit this typically remains inefficiently incomplete, and why economic disintegration (or "desychronization"), as currently witnessed in the European Union, should cause less interbank exposure. It also suggests that recent policy measures such as the preferential treatment of retail deposits, the extension of deposit insurance, or penalties on "connectedness" could all lead to substantial welfare losses.

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1 Introduction

We offer a theory of the "boundary of the firm" that is tailored to banks as it recognizes the relevance of both (insured) deposit financing and that of interbank lending as a possible substitute for integration. Our theory relies on a single inefficiency that has been at the core of banking theory: risk-shifting incentives in the interest of banks' shareholders. Still, our model is capable of delivering (i) a number of, mostly new, empirical predictions, (ii) a theory of the limits to financial integration both through interbank lending and the reallocation of funds within a merged bank, (iii) as well as normative implications closely related to the current financial crises and the respective proposed or already implemented policy measures, such as the preferential treatment of retail deposits and penalties for "interconnectedness".

In our baseline model local banks have specific skills in collecting funds and making loans, so that a reallocation of funds across geographically segmented markets relies on interbank lending when banks remain non-integrated or the operation of an "internal capital market" inside an integrated bank. The extent to which financial integration is achieved through these two channels and when bank integration replaces interbank lending, will both depend on, first, the extent through which funding relies on insured deposits and, second, on how well the two markets are already integrated economically, as expressed by the correlation in their lending markets. The key mechanism at work is the following: Greater reallocation of funds across markets, both through integration and through interbank lending, generates co-insurance benefits for depositors, so that an insufficient exploitation of this potential is an expression of risk-shifting to the benefits of shareholders. We next summarize the results of our analysis in terms of its different implications.

Between non-integrated banks, we find that interbank lending is larger, though never inefficiently high, when markets are already more closely aligned, as represented by the correlation between local lending markets. This holds as then the co-insurance benefit that a reallocation of funds across markets has for depositors is smaller. Importantly, the positive relationship between interconnectedness and correlation is thus not an inefficient consequence of banks' incentive to bailed out together (Acharya and Yorulmazer (2006, 2007); cf. also Wagner (2010) for a similar logic).
a contagious effect on the creditor bank. From this we derive the observation that there should be a tendency towards either relatively low or relatively high levels of interbank exposure between individual banks.

Changes in the correlation of lending markets can derive from an increase or a decrease in economic integration. Our model would thus predict that greater economic integration, such as within in the European Union before the crisis, should itself trigger also more interbank lending (as well as mergers in the banking industry, as we see shortly), while disintegration (or the "de-synchronization" of economic activity) should reduce interbank exposure at the expense of allocative efficiency. The latter observation clearly throws a somewhat different light on the current financial disintegration in the European Union, notably between banks at its core and its periphery.

More generally, our theory thus contributes to a better understanding of the patterns and limits of global financial integration. Such greater financial integration yields potentially large welfare benefits given cross-regional differences in net savings, in productivity, and in exposures to output shocks both on a global scale but also within relatively homogenous areas such as the Euro zone and the U.S.\textsuperscript{4} Various researchers have, however, noted that the extent to which such financial integration has been achieved is still limited. Surprisingly, this observation seems to apply not only to global financial integration, which is still restrained by regulation, but also to the financial integration in the Euro area, where de jure obstacles to financial integration have been largely removed.\textsuperscript{5} To understand this puzzle, it is important to understand the incentives of banks as they play a key role both in collecting funds from households, notably through deposits, and in investing, notably in smaller and medium-sized companies where local proximity is (still) of major importance. We find that incentives for financial integration are typically inefficiently low. This holds notably also for mergers between banks.\textsuperscript{6}

\textsuperscript{4}For evidence and measurements see, for instance, ECB (2013a, p. 96-107), Kalemli-Ozcan et al. (2003), or Bonfiglioli (2008).

\textsuperscript{5}For a discussion of the evidence for the limited effect of financial globalization see Stulz (2005). Lane (2009) and more recently van Beers et al. (2014) discuss this for the Euro area.

\textsuperscript{6}The role of banks for financial integration, both through cross-border asset holdings and interbank lending as well as through cross-border mergers, has indeed been largely documented in the empirical literature. Globally, Milesi-Ferretti and Tille (2011) argue that the cross-border activity of banks plays a dominant role for financial integration (cf. also Figure 1 in Fecht et al. (2012) for the role of interbank lending). Even within the Euro area the pre-crisis growth in cross-border asset holdings and financial integration was predominantly driven by the internationalization of European banks (cf. van Beers et al. 2014) and interbank lending (Sapir and Wolff 2013). There is also a large literature showing that the deregulation of cross-regional banking improved diversification and capital allocation even though other financial markets were already de facto integrated before. See, for instance, Black and Strahan (2002), Acharya et al. (2006), and Acharya et al. (2010).
Our model predicts a high degree of fragmentation among banks that rely much on insured deposits, such as savings and loan banks or cooperative banks with a strong local retail presence. The "boundary of the bank" also depends on the economic integration of the respective markets: Economic integration that increases the correlation between lending markets makes a bank merger more likely. Notably, as in our model there are no exogenously assumed advantages or disadvantages to the allocation of funds either through interbank lending or within an integrated firm, a bank’s boundary is determined solely by the following force: The choice between integration or non-integration generates commitment vis-a-vis the providers of uninsured funding in terms of the subsequent reallocation of funds and the thereby achieved coinsurance benefit. Interestingly, though our theory builds on a single inefficiency, that is risk-shifting, the trade-off between integration and non-integration is resolved differently depending on the correlation between the respective lending markets: For low correlation an integrated bank would achieve a less efficient allocation of funds than non-integrated banks relying on interbank lending alone, while for high correlation the allocation is more efficient in the integrated bank.

Rather than excessive interconnectedness or excessive integration to form "too-big-too-fail" banks, our parsimonious model of banking predicts the opposite: Too little exposure through interbank lending and too little financial integration through mergers and acquisitions among banks. This is why, in our model, further disincentives, arising for instance from a "tax" or from other penalties on size or interconnectedness, may have negative first-order effects on allocative efficiency and thus welfare. This should throw a new light on several policy initiatives that strive to discourage interbank lending and aim at either directly limiting bank size or imposing additional levies on larger banks.

Yet another policy implication relates to the extension of deposit insurance in the wake of the financial crisis. In our model, this would reduce the commitment role of a bank merger vis-a-vis providers of uninsured funding, so that the extension of deposit insurance can reduce financial integration and welfare. On the other side, as implicit and explicit insurance of bank debt holders are a subtle disincentive to bank mergers in our model, this suggests that the new EU Bank Recovery and Resolution Directive, which increases the

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7 Clearly, "too-big-to-fail" as well as "too-connected-to-fail" could generate additional moral hazard problems, from which we abstract. An important insight of our analysis is, however, that there may also be strong disincentives working the opposite way and those effects need to be considered when determining the optimal degree of regulation.

8 According to the BIS (2011) banks considered as global systemically important financial institutions (G-SIFIs) will be required to hold up to 3.5% additional equity against their risk based assets. Whether a bank is considered a G-SIFI depends among other things on its wholesale funding ratio. On limiting the size of banks see also the respective provisions in the Dodd Frank Act, Section 622.
bail-in of bank debt holders, could increase Euro area banks' incentives to merge - possibly counteracting the objective of preventing banks from becoming "too-big-too-fail". In yet another twist, current regulatory initiatives that encourage banks' reliance on insured retail deposits, such as their preferential treatment in liquidity coverage ratios and stress tests, would again have the opposite effect of reducing incentives for greater financial integration.

Our paper is embedded in a large banking theory literature, as surveyed for instance in Freixas and Rochet (2009). We share with this literature the following key features of our model: i) The importance of deposit financing, both insured and uninsured, for banks; ii) Banks' role as local and "skilled" collectors of funds and providers of loans; and iii) risk-shifting as the important inefficiency and friction. Much fewer papers have considered more than one bank and allowed for interbank lending. While our model focuses on the improvement of allocative efficiency through interbank lending, papers such as Bhattacharya and Gale (1987), Allen and Gale (2000), and Freixas et al. (2000) stress the role of the (short-term) interbank market in liquidity risk sharing. None of these papers poses the question of the "boundary of the firm", which is of course addressed in a large separate body of literature.

While a number of empirical papers on multinational banks draw largely on this theoretical literature, regarding both the operation of an internal capital market and the benefits of integration, this literature does however not consider the specificities of the banking sector. As noted above, this concerns the reliance on often insured deposits as well as the use of interbank lending. We also focus exclusively on risk shifting as the sole inefficiency, following much of the banking literature, and thereby do not assume other frictions that could provide an (exogenous) disadvantage for integration, such as limits to managerial control, greater conflicts of interest and scope for "rent seeking" in larger organizations, or internal agency conflicts that could be exacerbated under integration.

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9 See, for instance, Campello (2006) and Cetorelli and Goldberg (2012).
10 As discussed below, our specification of deposit financing, which we share with the banking literature, allows us to abstract from the endogenization of leverage (cf. Lewellen (1971), Leland (2007), [Ottaviani...2013] or more generally the financial claims issued by integrated and non-integrated firms (Inderst and Müller ).
11 The case where integration leads to a more extreme allocation of funds across markets (more risk-taking) is related to the analysis in Dewatripont and Mitchel (2005), in which a financial conglomerate would maximize risk by choosing perfectly positively correlated projects. In Freixas et al. (2007) conglomerates with an integrated balance sheet have excessive risk taking incentives due to the deposit insurance while conglomerates with a holding structure practice regulatory arbitrage. In such a setting with subsidiary vs. branch structures Lóránth and Morrison (2007) solve for the optimal capital requirements.
12 These have been addressed, for instance, in [Rajan/Zingales; Scharfstein/Stein ; Inderst/Klein:]. Notably, Stein () considers the interaction of an internal capital market and internal agency problems in a banking context.
that could provide an (exogenous) advantage, such as asymmetric information or limited contractibility across the boundaries of firms.\footnote{[Example]. With respect to the role of non-contractibility, of course, the seminal approach in Hart [...] , which focuses on incentives and hold-up, should be noted.}

The rest of this paper is organized as follows. In Section 2 we introduce our model of segmented funding and lending markets. Section 3 sketches the plan of the further analysis. The analysis with non-integrated banks is contained in Section 4. Section 5 considers the allocation of funds within an integrated bank and compares this with interbank lending. This comparison is then used in Section 6 to endogenize the decision whether to integrate or not. Section 7 collects the key positive and normative implications of our model and Section ?? concludes. The Appendix contains all proofs as well as additional material notably on the analysis with a competitive retail deposit market.

## 2 The Model

Based on the motivation provided in the Introduction, we consider a stylized model of segmented lending and funding markets. We also build into our model a role for banks in both collecting savings from households and making informed investment decisions through loans. The various assumptions that we thereby make follow closely the large extant literature on banking,\footnote{See, for instance, Freixas and Rochet (2009).} which is why the following presentation of our model focuses on those ingredients that are more novel and decisive for our subsequent results.

**Markets and Technologies.** There are two locally segmented markets, \( n = A, B \). Each market is populated by a mass one of households. In market \( A \), each household has funds of size \( M_A \). As there is a mass one of households, this also represents the measure of the total funding potential when funds are raised solely in market \( A \). In market \( B \), each household has funds of size \( M_B \). We assume without loss of generality that \( M_A \geq M_B \geq 0 \). The interesting case will be that where the local funding potential differs across markets. To derive for this a convenient measure, we denote total available funding by \( M_A + M_B = 2M \) and write \( M_A = M + z \) and \( M_B = M - z \) with \( z > 0 \). When analyzing the role of banks to allocate funding across markets, we will conduct a comparative statics analysis in \( z \).

To streamline the model, we abstract from modelling consumption and saving decisions of households and thus take as given that households set aside the respective funds \( 2M \) for later consumption. Next to a storage technology, which simply preserves the value of
funds, we introduce a risky investment technology in each of the two markets. For this we suppose that in each market there is one penniless entrepreneur who has access to a real investment opportunity, as specified next, and that there is at the same time a large number of fraudulent entrepreneurs who will abscond with any funds that they receive. By specifying that only one locally active bank has the necessary (soft) information to screen out fraudulent entrepreneurs, we grant each local bank monopoly power in the lending market and also preclude any forms of non-intermediated financing. In the case of an integrated bank, \( AB \), we suppose that, by acquiring the respective technology, the integrated bank inherits this knowledge across both markets.

The project of the (non-fraudulent) entrepreneur, on whom we can focus, is risky as it only succeeds with probability \( p \). In case of success, when having received funds of size \( F \), the project pays back \( L(F) \), while it pays back zero when it fails. The (production) function \( L(\cdot) \) satisfies \( L'(\cdot) > 0 \) and \( L''(\cdot) < 0 \). As we stipulate that the monopolistic local bank can make a take-it-or-leave-it offer to the local entrepreneur, the function \( L(F) \) also represents the local lending (or loan-making) potential. By assuming that it is symmetric across markets, we can focus our analysis on banks’ role to bridge funding differences across markets. A crucial parameter in our analysis, however, will be the extent to which the performance of loans in the two markets is correlated. We denote the respective correlation coefficient by \( \rho \) and allow for values \( 0 \leq \rho \leq 1 \). As is immediate, the likelihood with which loans in both markets perform is then given by \( p^2 + \rho p(1-p) \), which becomes \( p^2 \) when projects are fully independent (\( \rho = 0 \)) and \( p \) when projects are perfectly (positively) correlated (\( \rho = 1 \)).

Next to \( z \), which captures the difference in the local funding base, \( \rho \) will be our main comparative variable in what follows.

We further want to focus our analysis on the case where local funding is never in excess, so that we assume throughout that

\[
pL'(M + z) > 1. \tag{1}
\]

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\[15\] In practice, this should hold notably for smaller and medium-sized companies where local proximity is (still) of major importance. See, for instance, Petersen and Rajan (1994) and more recently Degryse and Ongena (2005).

\[16\] Hence, we abstract from any agency related inefficiencies that larger (merged) banks could have in generating and processing the necessary local information (cf. Stein 1997).

\[17\] Note at this point that our specification of a single loan opportunity in each market can also be interpreted as a perfect positive correlation for loans in a local market. What is essential for our following arguments is that, in this case, loans in the bank’s own portfolio are more correlated than loans across banks’ loan portfolios. Incorporating additional flexibility to allow for more general correlations for a given local loan portfolio has, however, proved to make the analysis much less transparent and at points intractable.
Further, to create scope for default and contagion when interbank loans are not repaid, we suppose that
\[ L(M) < 2M. \quad (2) \]
In words, when only half of all available funding, \( M \), is invested in one market, then in case of success the resulting payoff is insufficient to pay back all available funding, \( 2M \).

**Strategies and Timing.** A key part of the analysis in this paper is an endogenization of banks’ integration decision and the derivation of its key determinants. We thus start the game at \( t = 0 \) with banks’ decision whether to integrate or not. This as well as all further decisions are made in the interest of banks’ shareholders. The subsequent game then unfolds depending on whether integration took place or not. We first take the case where banks remained separate.

In \( t = 1 \) funding can be collected from households. Given our preceding discussion, households will either invest in the storage technology or invest in risky projects through one of the two banks. In our baseline analysis, we further stipulate that households in market \( n \) can only invest through bank \( n \), albeit we can extend results to the case where banks compete for funding across markets.\(^{18}\) Our key assumption is that households’ claims on banks’ assets will be senior to those of shareholders. We comment shortly on this assumption. We will refer to these claims as deposits, so that in our baseline model at \( t = 1 \) bank \( n \) offers a deposit rate \( r_n \) in its local market and attracts deposits of total size \( R_n \leq M_n \). Non-integrated banks can arrange interbank lending in \( t = 2 \), which prescribes a transfer of funds \( W_n \) from bank \( n' \) to bank \( n \) in exchange for a promised repayment \( w_n \). To make our baseline analysis as transparent as possible, we stipulate that the (lending) bank with a higher funding base can make a take-it-or-leave-it offer.\(^{19}\) In \( t = 3 \) banks extend loans in their local market. Payoffs are realized in \( t = 4 \). When banks have chosen to integrate in \( t = 0 \), the subsequent game simplifies as there is no need to arrange interbank lending in \( t = 3 \). All parties are risk neutral and we abstract from discounting.

Before proceeding to the analysis, it remains to comment on our specification of banks’ contracts with households. The key assumption in what follows is that these claims are senior, both to those of shareholders and to any interbank loan. The assumption of such

\(^{18}\) In the respective analysis in Appendix B we still endow the local bank with an advantage: Households who invest in a non-local bank will incur switching costs.

\(^{19}\) However, we show in Appendix C that the key results for interbank lending are unchanged when we stipulate instead a game of Nash bargaining with a more symmetric distribution of bargaining power. Note also that a fully competitive (fragmented) market would seem at odds with the arrangement of interbank lending, while an analysis of a network of interbank lending is beyond the scope of this paper.
deposit financing is again shared with a large literature in banking. Though it is there often assumed exogenously as well, it is well known that seniority can be given a microfoundation (cf. Diamond and Rajan 2005). However, we abstain from enriching our model in such ways, thereby focusing on what is novel in our analysis compared to the extant literature.

3 Plan of Analysis and Overview of Results.

The plan of our further analysis is as follows. Section 4 contains our main analysis of interbank lending to bridge funding differences between segmented banking markets. Our main result will be that of a persistent gap despite the operation of a frictionless interbank lending market (Section 4.1). Funding differences will only be smoothed out completely when a further (marginal) increase of interbank lending no longer offers coinsurance benefits to depositors, which will be the case either when an interbank loan is already large enough so as to have a contagious effect or when loan portfolios are perfectly correlated. These insights further give rise to various comparative statics results, notably for the size of interbank lending, depending on the characteristics of banks’ segmented funding and lending markets (Section 4.2).

Section 5 considers the operation of an integrated bank. Importantly, integration does not per se reduce frictions in the allocation of funds, as the interbank lending market is notably not plagued by adverse selection or moral hazard in our model. Still, we show how it can substantially alter the allocation of funds across local markets. The fundamental difference is that with integration depositors in both markets, A and B, have a claim that needs to be repaid out of the proceeds of the bank’s lending in both markets, rather than only in one market as in the case of separate banks. We show how, depending notably on the correlation between the local lending markets, this can lead to more or less efficient allocation of funds through the risk-shifting channel that is at the heart of this paper: The

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20 Clearly, by taking an agency perspective the role (e.g. with respect to information acquisition) of different lenders would have to be clarified as well. Other aspects of deposit financing, such as a "first-come-first-serve" feature that provides liquidity but can give rise to bank runs are however not at the core of our analysis.

21 Recall also from our discussion in the Introduction that the seniority of retail deposits as well as the immediacy with which they can be withdrawn are not necessarily undone by households’ inertia compared to institutional investors, notably in our model the creditor bank in case of an interbank loan. One reason for this is the discussed longer maturity of these claims.

22 Funding differences are also persistent when deposit markets are no longer fully segmented, but - given the low granularity of retail deposits - subject to frictions in the form of switching costs. To streamline the exposition of results, however, we noted already that an extension to competitive retail deposit markets is contained in Appendix B.
merger opens up new risk-shifting opportunities on the one hand but reduces risk-shifting incentives that arise from the coinsurance benefits of depositors on the other hand.

The insights from Section 4.1 (non-integrated banks) and Section 5, while being of independent interest, are then tied together to answer in Section 6 the question of the "boundaries of the firm", that is whether banks become integrated or stay non-integrated in equilibrium. When deposits are insured, a merger will never arise, such that banking will remain highly (regionally) fragmented. We show that this result also extends when a sufficiently large fraction of funding, though not necessarily all of it, is insured. Neither bank mergers nor the operation of an interbank market will then ensure the efficient allocation of funds to bridge funding differences. When deposits are (at least to a large part) uninsured, however, our predictions are more nuanced and we obtain clear-cut results on when integration arises, which will then always increase efficiency. With respect to uninsured depositors, the choice of the boundary of the firm, despite the operation of an interbank market, serves as a commitment against risk-shifting. We collect implications from all our results in Section 7.

4 Non-Integrated Banks

4.1 Shortfall of Interbank Lending

In this Section we consider the determination of interbank lending in $t = 2$, taking as given the retail deposit funds $R_n$ that each bank $n$ has already attracted through promising an interest rate $r_n$. Note also that for the present analysis it does not matter whether deposits are insured or not. The main result in this Section will be a characterization of optimal interbank lending and its key determinants.

As is intuitive (and formally derived in the proof of Proposition 1), in equilibrium there will be at most one interbank loan, i.e., in our model there is no scope for both a loan of bank $A$ to bank $B$ and vice versa. As the purpose of interbank lending is to better align banks’ funding with their loan-banking opportunities, it is equally intuitive that an interbank loan will be made, if at all, by the bank with higher initial funding $R_n$ to that with lower funding. We presently suppose that this is bank $A$, so that $R_A \geq R_B$. Denote thus by $W_B = W \geq 0$ the interbank loan that bank $A$ makes to bank $B$ and by $w_B = w \geq 0$ the respective agreed repayment. Recall that in this Section we stipulate that the terms of interbank lending are determined through a take-it-or-leave-it offer made by bank $A$ to bank $B$ (cf. however Appendix C).
Banks are managed in shareholders’ interest. Take first bank $A$. For given (remaining) funding, $F_A = R_A - W$, provided that this is then used to make a loan of the same size, the expected profits of shareholders are

$$\pi_A = \left[ p^2 + p\rho(1-p) \right] [L(F_A) + w - R_A(1 + r_A)]$$

$$+ p(1-p)(1-\rho) \left[ \max \{0, L(F_A) - R_A(1 + r_A)\} + \max \{0, w - R_A(1 + r_A)\} \right].$$

The first line in (3) accounts for the state where all loans are successful. That is, with the respective probability, $p^2 + p(1-p)\rho$, the loan portfolios of both bank $A$ and bank $B$ perform. This also enables bank $B$ to repay $w$ to bank $A$.\footnote{For instance, when loan portfolios are independent, so that $\rho = 0$, the respective probability becomes simply $p^2$.} Note that we implicitly assume that the total repayment to bank $A$, arising from both its own (corporate) loan and the loan made to bank $B$, is sufficient to cover the repayment that bank $A$ promised to its depositors, $R_A(1 + r_A)$. This will always be the case in equilibrium. The second line in (3) accounts jointly for two states that are equally likely: that where only the loans of bank $A$ perform (captured by the first part) and that where only the loans of bank $B$ perform (captured by the second part). When both loan portfolios do not perform, then clearly shareholders of bank $A$ realize zero profits.

Expression (3) for the payoff of bank $A$’s shareholders thus contains various cases, depending on whether the repayment of the bank’s own loans, the repayment of its loan to bank $B$, or only both together are sufficient to cover claims to its own depositors, $R_A(1 + r_A)$. When $L(F_A) > R_A(1 + r_A)$, then there is a positive payout to the shareholders of bank $A$ even when bank $B$ cannot repay its interbank loan. This case is more likely if bank $A$’s funds are mostly invested locally, so that the size of the interbank loan $W$ and consequently also the respective promised repayment $w$ are small relative to $L(F_A)$. The other subcases are those where a failure of repayment from bank $B$ causes default of bank $A$, i.e., interbank lending can then have a contagious effect. While then the proceeds from its own loans, $L(F_A)$, allow bank $A$ to make some repayment to depositors, when its loan to bank $B$ is not paid back this is no longer sufficient to allow for a payout to shareholders as well. Proposition 1 below characterizes the outcome for all possible cases. Which case arises in equilibrium is determined further below, as it depends on the initial allocation of funds $R_n$, as well as on the interest rates promised to depositors, $r_n$.

We next state the profits of shareholders of bank $B$,

$$\pi_B = p [L(F_B) - w - R_B(1 + r_B)],$$
where \( F_B = R_B + W \). Shareholders of bank \( B \) only receive a positive payout when the bank’s own loans perform. That profits are positive in this case will naturally arise in equilibrium, so that we can safely restrict consideration to this case. Given the presently assumed take-it-or-leave-it offer by bank \( A \), we have next that

\[
w = L(F_B) - L(R_B).
\]  

(5)

Hence, in case there is a loan of size \( W \) from bank \( A \) to bank \( B \), the respective repayment \( w \), as specified in (5), ensures that bank \( B \)'s profits are just equal to the "standalone payoff" \( L(R_B) - R_B(1 + r_B) \).

**Proposition 1** Consider stage \( t = 2 \), where banks can arrange for an interbank loan \( W \) from bank \( A \), which has more retail funding as \( R_A \geq R_B \), to bank \( B \). There are two cases to consider.

**Case 1:** The loan size \( W \) and the repayment \( w \) are chosen sufficiently small so that a failure of repayment does not itself cause the insolvency of the creditor bank \( A \), as \( L(R_A - W) \geq R_A(1 + r_A) \). Then, there exists a threshold \( \rho_0 \), such that \( W = W_1^* \) uniquely solves

\[
pL'(R_A - W_1^*) = [p^2 + \rho p(1 - p)]L'(R_B + W_1^*)
\]  

for \( \rho > \rho_0 \) and the corner solution \( W_1^* = 0 \) applies for \( \rho \leq \rho_0 \).

**Case 2:** \( W \) and \( w \) are, instead, sufficiently large so that from \( L(R_A - W) < R_A(1 + r_A) \) a failure of repayment causes insolvency also of the creditor bank \( A \). Then, \( W = W_2^* \) uniquely solves

\[
L'(R_A - W_2^*) = L'(R_B + W_2^*)
\]  

(7)

For a discussion, note first that an efficient reallocation of funds through an interbank loan would require that \( W = W^{**} \) with \( W^{**} \) solving \( L'(R_A - W^{**}) = L'(R_B + W^{**}) \) - or, expressed differently, \( W^{**} = (R_A - R_B)/2 \), so that the same amount of funding is allocated to either market. This is the case in condition (7), where thus \( W = W_2^* = W^{**} \), but not in condition (6), where \( W = W_1^* < W^{**} \) and not if \( W = 0 \). In Case 1, unless banks’ loan portfolios are perfectly correlated, so that \( \rho = 1 \), the respective size of the interbank loan \( W_1^* \) still remains inefficiently low. As a consequence, more of the total available funding, \( R_A + R_B \), is allocated to loans in market \( A \) than to loans in market \( B \).

As a particular case, suppose for an illustration that loan performance across the two banks is independent (\( \rho = 0 \)). Then, the negotiated interbank loan, if positive at all, is such that at this level the non-risk-adjusted return from loans of the creditor bank \( A \) is equal to
the risk-adjusted return from loans of the debtor bank $B$: \( L'(R_A - W_1^*) = pL'(R_B + W_1^*) \).

In Case 2 of Proposition 1, instead, the resulting allocation is efficient.

The results of Proposition 1 arise from the incentives of leveraged shareholders to engage in risk shifting. Precisely, as long as the correlation between the corporate loan portfolios of bank $A$ and $B$ is not perfect, as $\rho < 1$, interbank lending diversifies the overall loan exposure of bank $A$. That is, when bank $A$’s own (corporate) loans fail, depositors can still be (partly) paid back when the loans in market $B$ perform, as this will result in a repayment of the interbank loan. Thus, the resulting diversification that reduces bank $A$’s own loan portfolio, but generates an exposure to loans in market $B$ tends to make the claims of its depositors safer. In Case 1 of Proposition 1, this positive externality of diversification for bank $A$’s depositors generates a wedge between the allocation of funds that would be efficient (through choosing $W = W^{**}$) and the allocation of funds that results as an outcome of optimal interbank lending in shareholders’ interest ($W = W_1^* < W^{**}$).

This wedge is intuitively smaller when banks’ loan portfolios become more (positively) correlated, in which case depositors of bank $A$ have less to gain from such coinsurance of their deposits through interbank lending.\(^{24}\) Consequently, the optimally arranged interbank loan $W_1^*$ increases in Case 1 as banks’ loan portfolios become more correlated. The characterization of Case 1 would thus predict a positive correlation between the size of interbank lending and the correlation of the local loan portfolios of the involved banks. The resulting increase in $W$ enhances efficiency.

**Corollary 1** Suppose that Case 1 from Proposition 1 applies. Then, as the correlation between banks’ loan portfolios increases (higher $\rho$), the size of the interbank loan $W = W_1^*$ increases as well.

Corollary 1 conducts a comparative analysis only for Case 1. Once we have derived the equilibrium for the full game, we will show that our model predicts a robust positive relationship between interbank lending and the correlation of banks’ loan portfolios. For now, however, we postpone a further discussion of this implication.

The allocation of funding becomes efficient in Case 2 of Proposition 1. The reason is as follows. In this case the exposure of bank $A$ to the risk of bank $B$ is sufficiently large such that failure of repayment of the interbank loan would make bank $A$ insolvent as well, regardless of the performance of its own loan portfolio. Then, $W = W_2^*$ solves (7).

\(^{24}\)Of course, under full deposit insurance these benefits would be reaped rather by the deposit insurance institution than by insured depositors themselves, other than in the case without deposit insurance. These considerations will prove important later when the equilibrium interest rate is determined.
Intuitively, once the interbank loan is sufficiently large, so that a failure of repayment has such a "contagious effect", a marginal adjustment of the loan has no longer the discussed positive externality on depositors of bank $A$.

4.2 Equilibrium Analysis: The Case with Insured Deposits

We now turn to stage $t = 1$ of our model. Recall that presently we have $R_n = M_n$ and that banks are non-integrated, so that the only way to reallocate funds between the two markets is through interbank lending, as analyzed in the preceding section.

**Proposition 2** Suppose that deposits are insured and banks non-integrated. We have the following comparative results for the (generically uniquely determined) interbank loan in equilibrium, $W^*$: There exists a threshold $\hat{\rho}$, such that $0 \leq W^* < z$, according to Case 1 in Proposition 1, when $\rho < \hat{\rho}$ and $W^* = z$, according to Case 2 in Proposition 1, when $\rho \geq \hat{\rho}$. Overall, when $z > 0$, $W^*$ is increasing in $\rho$.

With deposit insurance, the equilibrium analysis is heavily simplified by the fact that equilibrium deposit rates do not depend on depositors’ expectations about the size of the interbank exposure of bank $A$. Then, the characterization and comparative analysis in Proposition 2 follows immediately from combining the results in Proposition 1 and Corollary 1. For our subsequent discussion of empirical implications, we next state an additional comparative result:

**Proposition 3** Suppose that deposits are insured and banks non-integrated. When $\rho < 1$, there exists a threshold $0 < \tilde{z} \leq M$, such that $0 \leq W^* < z$, according to Case 1 in Proposition 1, when $z < \tilde{z}$ and $W^* = z$, according to Case 2 in Proposition 1, when $z > \tilde{z}$. Overall, when $\rho < 1$, $W^*$ is thus increasing in $z$.

As the difference in the size of the two deposit markets, $z$, increases, there are two reasons for why the interbank loan should increase in size as well, holding now the correlation $\rho$ fixed. A larger interbank loan is then needed to reduce the gap between available local funding in the two markets. The second reason is that as $z$ increases, we are more likely to be in Case 2 (of Proposition 1), given that then the outstanding claims of depositors in market $A$ are larger.
Illustration. Take a linear-quadratic loan-value function, $L(F) = bF - aF^2$, for which we can now explicitly derive both the resulting interbank loan and the allocation of funds as well as when the different cases arise. For this, we normalize the size of funds so that, when there is symmetry, each bank has a potential deposit base of mass one: $M = 1$. For the following Figure ??, we choose $\rho = 0.875$ and allow for different values for the initial funding difference: $z = 0.5$, and $z = 0.9$. The case with contagious interbank lending only arises when the asymmetry of retail deposits is sufficiently large. Note that then, as the correlation increases, $W^*$ jumps upwards (at $\rho = \hat{\rho}$). Further below we will make use of this feature to derive additional implications on observed exposures through interbank lending.

4.3 Equilibrium Analysis: The Case with Uninsured Deposits

The case without deposit insurance is complicated by the fact that now the equilibrium deposit rate depends on the bank’s riskiness, which in case of bank $A$ depends on its exposure not only to its own loan market but also to that of bank $B$ in case $W > 0$. In fact, when the depositors of bank $A$ can expect to be co-insured through the repayment from an interbank loan, we have $r_A < 1/p - 1$, while when $W = 0$ bank $A$ must pay $r_A = 1/p - 1$ to ensure that depositors just break even in expectation.

The derivation of an equilibrium is, however, simplified by the observation that while the outstanding repayment obligation affects which case of Proposition 1 applies, the optimal choice of $W$ in any given case is not affected. We then denote by $q(r_A)$ the probability with which, for given $r_A$, the bank optimally chooses $W = W^*_2$, i.e., to be in Case 2. Intuitively, there exists a threshold $\hat{r}_A$ so that $q(r_A) = 0$ when $r_A < \hat{r}_A$, $q(r_A) = 1$ when $r_A > \hat{r}_A$, and $q(r_A) \in [0,1]$ when $r_A = \hat{r}_A$. Depending on the anticipated value of $q$, denote by $r_A(q)$ the interest rate at which bank $A$’s non-insured depositors just break even in expectation (cf. the subsequent proof for an explicit derivation). Intuitively, given the co-insurance effect, $r_A(q)$ is strictly decreasing. An equilibrium (in possibly mixed strategies) is now described by a fixed point for $(q,r_A)$, at which both $q = q(r_A)$ and $r_A = r_A(q)$. The left-hand panel in Figure ?? depicts the stylized case of a pure-strategy equilibrium where the equilibrium value $W = W^*$ is characterized by Case 2, so that $W^* = z$. The right-hand panel of Figure ?? depicts the case of a mixed-strategy equilibrium.

Proposition 4 extends the comparative result of Proposition 2 to the case without deposit insurance. now both the interbank loan $W^* < z$ that solves (6) and the probability
that $W^* = z$ is chosen increase in the correlation between the two markets.

**Proposition 4** Suppose that deposits are uninsured and banks non-integrated. Then, there is a unique equilibrium, where the size of the equilibrium interbank loan $W^*$ depends on the correlation of banks’ local loan portfolios as follows. There are two thresholds $\hat{\rho}_l < \hat{\rho}_h$, such that $0 \leq W^* < z$ according to Case 1 in Proposition 1 when $\rho < \hat{\rho}_l$ and $W^* = z$ according to Case 2 in Proposition 1 when $\rho \geq \hat{\rho}_h$. When $\hat{\rho}_l < \rho < \hat{\rho}_h$, the bank mixes between the following outcomes: It chooses $W^* = z$, according to Case 2 in Proposition 1, with probability $q^* \in (0, 1)$ and with probability $1 - q^*$ it chooses $0 \leq W^* < z$, according to Case 1 in Proposition 1. The probability $q^*$ increases in $\rho$ (strictly so for $\rho \in (\hat{\rho}_l, \hat{\rho}_h)$) such that also the expected interbank loan increases in $\rho$.

Finally, we state the analogous comparative result to Proposition 3:

**Proposition 5** Suppose that deposits are uninsured and banks non-integrated. When $\rho < 1$, there are now two thresholds $0 < \hat{z}_l < \hat{z}_h \leq M$, such that $0 \leq W^* < z$, according to Case 1 in Proposition 1, when $z < \hat{z}_l$ and $W^* = z$, according to Case 2 in Proposition 1, when $z > \hat{z}_l$. When $\hat{z}_l < z < \hat{z}_h$, the bank mixes between the following two outcomes: It chooses $W^* = z$, according to Case 2 in Proposition 1, with probability $q^* \in (0, 1)$ and with probability $1 - q^*$ it chooses $0 \leq W^* < z$, according to Case 1 in Proposition 1. The probability $q^*$ increases in $z$ (strictly so for $z \in (\hat{z}_l, \hat{z}_h)$) such that also the expected interbank loan increases in $z$.

## 5 Allocation of Funds within an Integrated Bank

**Objective of the Integrated Bank.** We now suppose that a single bank operates across both markets, $A$ and $B$. We will ask how the resulting allocation of funds differs from that achieved when markets are served by separate banks. While the present analysis will be of interest on its own, as we notably derive conditions for when an integrated bank may either achieve more or less efficiency in its lending, it will also form the background for our subsequent analysis of endogenous integration.

When a single bank, $AB$, operates, the question of whether retail deposit markets are fully segmented or not becomes superfluous. Also, as the repayment of all deposits is served by all of the bank’s assets, as long as all depositors obtain the same level of deposit insurance (or not), in $t = 1$ the integrated bank will now offer the same interest rate $r_{AB}$ to depositors in both markets. As there is no interbank lending, the game then proceeds
to \( t = 3 \), where the bank allocates its aggregate funds over the two segmented lending markets, choosing \( F_A \) and \( F_B \). Payoffs are again realized in \( t = 4 \).

The integrated bank’s shareholders’ profits are given by

\[
\pi_{AB} = \left[ p^2 + \rho p (1 - p) \right] \left[ L (F_A) + L (F_B) - R_{AB} (1 + r_{AB}) \right] + p (1 - p) (1 - \rho) \max \{0, L (F_A) - R_{AB} (1 + r_{AB})\}. \tag{8}
\]

Note first that without loss of generality we restrict attention to cases where weakly more funds are allocated to market \( A \): \( F_A \geq F_B \). The first line in (8) accounts for the outcome where loans in both markets are successful.\(^{25}\) With respect to the second line in (8), note first that the case where the repayment from loan market \( B \) alone would already be sufficient for the integrated bank to remain solvent is excluded by condition (2) and by \( F_B \leq F_A \leq M \). Hence, when only one loan portfolio performs, then from \( F_A \geq F_B \) shareholders of the integrated bank can only expect to receive a payout when loans in market \( A \) perform. The case distinction in the second line of (8) is then whether this is indeed sufficient (or not) to make depositors whole, i.e., whether \( L (F_A) > R_{AB} (1 + r_{AB}) \) indeed holds (or not).

**Integrated Bank with Insured Deposits.** As in the case of interbank lending, the optimal allocation of funds across the two markets is driven by two considerations: On the one hand, the maximization of total profits and thus efficiency, which obtains when \( M \) is allocated to either market, and, on the other hand, the reduction of a co-insurance effect (or, likewise, the maximization of risk-shifting) to the benefits of shareholders, though it reduces the value of depositors’ claims.

Note now that for the following proposition we relabel the threshold for the case distinction with separate banks from Proposition 2 by \( \hat{\rho}^S \). Recall that \( \hat{\rho}^S \) denotes the threshold for the correlation between loan portfolios so that for \( \rho \geq \hat{\rho}^S \) interbank lending is large enough to make the allocation of funds efficient. With integration the corresponding threshold for when efficiency is obtained will be denoted by \( \hat{\rho}^I \) in Proposition 6.

**Proposition 6** Suppose that deposits are insured and banks integrated. There exists a threshold \( \hat{\rho}^I \), such that the (generically unique) equilibrium allocation of funds \( F_B^* \) and \( F_A^* \) with \( F_B^* + F_A^* = 2M \) is uniquely characterized as follows: When \( \rho < \hat{\rho}^I \) it is inefficient with \( F_B^* < F_A^* \) as

\[
p L' (F_A^*) = \left[ p^2 + \rho p (1 - p) \right] L' (F_B^*) \tag{9}
\]

\(^{25}\) Again, as in the case of separate banks, we abbreviate the analysis by stipulating that in this case the bank can indeed fully repay its depositors. This will clearly be the case in equilibrium.
or $F_B^* = 0$ holds and when $\rho \geq \bar{\rho}_I$ it is efficient with $F_A^* = F_B^* = M$ when $\rho \geq \bar{\rho}_I$. The allocation of funds thus becomes more symmetric and efficient as $\rho$ increases ($F_A^* - F_B^* \geq 0$ decreases as $F_B^*$ increases and $F_A^*$ decreases).

Though the characterization in (9) is analogous to that in (6) without integration, the efficiency properties of the two cases can be markedly different. We first report the respective comparison before then providing an intuition also for the characterization in Proposition 6.

**Proposition 7** Suppose that deposits are insured. When $\rho > \bar{\rho}_I$, then the equilibrium allocation of funds across markets is more symmetric and thus more efficient in the integrated bank, while for $\rho < \bar{\rho}_I$ the allocation is less symmetric and thus less efficient in the integrated bank compared to when banks are non-integrated and a reallocation of funds is thus achieved through interbank lending.

The comparison in Proposition 7 derives clear-cut conditions for when an allocation of funds inside an integrated bank is more efficient than that achieved through interbank lending. To our knowledge, such a comparison has not yet been undertaken. Though our analysis is admittedly highly stylized, the respective simplifications allow to clearly isolate incentives for risk shifting by leveraged shareholders as the driving force between the difference in allocations. Incentives and the scope for risk shifting, as manifested by a more asymmetric allocation of funds between the two markets, can both be lower and higher in an integrated bank, depending on the correlation between the loan-making opportunities in the two markets, $\rho$. We return in Section 7 to possible normative and positive implications of Proposition 7.

To understand the key differences between the allocation of funds through the interbank market and that in an integrated bank, the difference in the treatment of depositors is key. When banks are non-integrated, it is only through the interbank loan from $A$ to $B$ that a deposit may be repaid both from loans in market $A$ and loans in market $B$. When no interbank loan is made, deposits in bank $A$ and deposits in bank $B$ will only be repaid when the loan in the respective local market performs. Instead, all deposits in the integrated bank represent senior claims, compared to those of shareholders, to the proceeds from loans in both market $A$ and market $B$. The key difference lies thus in the "status quo" regarding the treatment of deposits, which for separate banks means that each bank’s deposits are secured only by the assets of this bank, while for an integrated bank depositors in either market have senior access, compared to shareholders, to repayments of loans made in both
markets. When banks are non-integrated and the co-insurance externality is large, as \( \rho \) is low, the case with \( W = 0 \) provides thus the limit of risk-shifting through a lack of reallocation of funds across markets, as then no such co-insurance externality exists. But this is different in an integrated bank, where the case with \( W = 0 \) for a non-integrated bank corresponds to \( F_A = M_A \) and \( F_B = M_B \) and thus still involves co-insurance benefits for depositors. This is the reason why the allocation in the integrated bank is less efficient when \( \rho < \tilde{\rho}_I \). On the other hand, the larger repayment obligations of the integrated bank make it more likely that there is "contagion", i.e. that the failure of loans in one market completely eradicate the claims of shareholders. Recall however that from this threshold onwards, a further increase in reallocation does not generate an additional co-insurance externality. This the reason for why the allocation in the integrated bank is more efficient when \( \rho > \tilde{\rho}_I \). The trade-off in terms of efficiency in 7 thus derives from a single distortion due to the risk-shifting incentives of shareholders.

**Integrated Bank with Uninsured Deposits.** We show next how the basic insights of the comparison with insured deposits extend to the case with uninsured deposits. The key difference between the cases with and without deposit insurance will be uncovered only subsequently when we ask whether and when integration will arise in equilibrium. For a characterization recall that without integration there was a mixed strategy equilibrium for intermediate values of the correlation coefficient \( \rho \). We now denote the respective boundaries with a superscript \( S \) and the analogous boundaries under integration with a superscript \( I \). The following result comprises both a characterization and a comparison with the case of non-integration.

**Proposition 8** Suppose that deposits are uninsured and banks integrated. Then, there exist two thresholds \( \tilde{\rho}_I^l < \tilde{\rho}_I^h \), such that the equilibrium allocation of funds is inefficient when \( \rho \leq \tilde{\rho}_I^l \), as \( F^*_B = 0 \) or as (9) holds, while it is efficient with \( F^*_A = F^*_B = M \) when \( \rho \geq \tilde{\rho}_I^h \). When \( \tilde{\rho}_I^l < \rho < \tilde{\rho}_I^h \), the bank mixes between the following outcomes: It chooses \( F^*_A = F^*_B = M \) with probability \( q' \in (0, 1) \) and with probability \( 1 - q' \) it chooses \( F^*_A \) and \( F^*_B \) according to (9), where \( q' \) strictly increases in \( \rho \). Furthermore, there exists a unique threshold \( \hat{\rho} \) such that when \( \rho \geq \hat{\rho} \), the expected amount of funds allocated to market B is larger in the integrated bank, while for \( \rho \leq \hat{\rho} \) the expected amount of funds

\(^{26}\text{This clearly does not hold strictly for values of } \rho \text{ where efficiency is obtained also for non-integrated banks.}\)
allocated to market B is smaller in the integrated bank compared to when banks are non-integrated and a reallocation of funds is thus achieved through interbank lending.

6 Endogenous Integration

As discussed previously, integration can - at least when correlation of loan portfolios is sufficiently high - lead to a more efficient reallocation of funds from depositors in market A, which has a larger deposit base, to loans made in market B. On the other hand, we showed as well how integration can lead to greater risk shifting when \( \rho \) is low. Integration has, in addition, the immediate effect of providing co-insurance for all deposits, as depositors then have jointly a claim on all assets of A and B, albeit the scope of such co-insurance depends on the ensuing equilibrium allocation of funds across markets. In general, as there are no other frictions in our model, interbank lending and the internal allocation of funds in an integrated bank could perform equally well. Taken all these observations together, we now ask whether integration arises endogenously in stage \( t = 0 \) of our model. For this deposit insurance - or the absence of it - play a key role.

Besides affecting the interest rates that prevail in the market, the absence of deposit insurance makes the following key qualitative difference. Without deposit insurance, interest rates positively react to the extent to which depositors’ claims are coinsured by investments in both markets A and B. When banks are separated, this is only the case for the depositors of bank A and only when subsequently an interbank loan is made. Likewise, in cases where integration leads to greater risk taking, this will be equally anticipated by depositors and lead to higher funding costs. Such a feedback channel between funding costs and the decision to integrate is fully absent when depositors are insured. Instead, when deposits are insured, then only the immediate coinsurance externality remains, so that integration is never beneficial for shareholders.

**Proposition 9** Consider the case where deposits insured. Then banks will remain separate as integration would reduce shareholders’ joint profits.

A key prediction of Proposition 9 is that banks financed by insured deposits are likely to remain small and to resist mergers.\(^{27}\) This should thus apply particularly to smaller,

\(^{27}\)Note that one reason why we have in the main text abstracted from possible competition for deposits without integration is that then integration of banks would trivially lead to benefits, namely by lowering funding costs. We conjecture that the non-profitability result survives as long as competition is not too intense without integration - or, likewise, in case integration would not sufficiently reduce deposit rates as the two considered banks face competition also from other financial institutions or other investments.
traditional savings and loans banks. Our argument is that this is the case as these banks can reap the benefits from reallocating resources also through interbank loans, to the extent that they wish to do so, but without providing at the same time an immediate coinsurance benefit (to depositors or the deposit insurance fund) through integration. Such an immediate coinsurance benefit also exists without deposit insurance, but in this case shareholders internalize the benefit through a lower interest rate.

**Proposition 10** Consider the case where deposits are uninsured. Then there exists a unique threshold $\tilde{\rho}_I \leq \rho^* \leq \tilde{\rho}_h$ such that for $\rho \leq \tilde{\rho}^*$ banks will remain separate as integration would reduce shareholders’ joint profits, and for $\rho \geq \tilde{\rho}^*$ banks will integrate as this increases shareholders’ joint profits.

As we noted, the interest rate for uninsured deposits internalizes the expected coinsurance benefits. We also noted repeatedly that in our model there is no built disadvantage of non-integration in terms of additional frictions. Why then does the choice between integration and non-integration make a difference, as predicted by Proposition 10. Their role is that of a commitment device vis-a-vis the providers of uninsured financing, given that once financing is obtained, the choice of the allocation of funding is made in the interest of shareholders alone. As shareholders are the residual claimants, from an ex-ante perspective they choose the "boundaries of the bank" so that the subsequent allocation of funds across loan markets is as efficient as possible. The "boundaries of the bank" are thereby derived from a single inefficiency that, as noted in the introduction, is also at the heart of the vast majority of contributions to the theory of banking: shareholders’ risk-shifting incentives.

7 **Collection of Implications**

**Empirical Implications.** In this Section we first provide in a more descriptive way a collection of the main positive implications that we have derived from our stylized model of locally segmented funding and lending markets. In our model, as (local) banks have an advantage in making loans, to achieve a more efficient allocation when there are differences in local funding, it is necessary that funds are either reallocated through interbank lending or within an integrated bank that operates across markets.\(^{28}\) We derive implications both

\(^{28}\)Notably, also retail competition alone is insufficient as long as a local bank still enjoys an advantage also on the funding side, e.g., due to switching costs of depositors; cf. Appendix B.
for loans made between banks and for whether and when we should observe integration that could facilitate the reallocation of funds.

**Implication 1.** *The size of an interbank exposure should increase both with the difference in banks’ local funding base and with the correlation between local lending markets.*

As in much of the theoretical literature on banking, recall that our results are driven by a risk-shifting motive of shareholders. In our model this expresses itself in an insufficient realization of efficiency gains from reallocating resources as the ensuing diversification would benefit depositors. To recall this is again the rationale for why interbank lending increases with the correlation between local lending markets. It should also be noted that this result is not driven by banks speculating on a "joint" bail-out and that the increase in interbank lending increases efficiency. This should be born in mind when considering our next implication.

**Implication 2.** *We should expect to see a tendency towards, on the one hand, either low (or zero) interbank exposure, notably when lending markets have relatively low correlation, and, on the other hand, relatively high interbank exposure that could have a "contagious" effect, notably when lending markets have relatively high correlation.*

Recall that the potential "clustering" of (empirical) observations (at low or high interconnectedness) follows from the fact that the described contagious effect, which decreases the positive externality of higher interbank lending on depositors and which only kicks in when the interbank exposure is sufficiently large. With regards to Implications 1 and 2 it should be noted that the respective results were derived both with and without deposit insurance.

**Implication 3.** *An integrated bank that operates in different (funding and lending) markets can have both a more and a less efficient allocation of funds across the different markets when compared to the operations of non-integrated banks that rely on interbank lending to reallocate funds across markets. The allocation of the integrated bank is more diversified and more efficient when the correlation between the loans across markets is relatively high, and it is less diversified and less efficient when the correlation is relatively low.*

Recall that the key insight for Implication 3, where one compares allocative efficiency and diversification across markets, is the following: In an integrated bank that secures funding from various markets all deposits represent claims to all assets, i.e., to all loans
made in different markets, whereas for non-integrated banks the respective deposits are only secured by local loans, unless there is interbank lending as well. It should be noted, however, that Implication 3 does not yet take into account that integration is itself endogenous. Still, as in practice there could be other obstacles to integration, such as regulatory or cultural constraint, as well as other conducive factors, such as managerial hubris, Implication 3 may also lend itself to the derivation of empirical prediction. The next implications focus, instead, on the equilibrium choice of integration.

**Implication 4.** Banks that rely (mainly) on insured deposits have lower (or even no) incentives to integrate, even when this leads to inefficiently low reallocation of funds through interbank lending.

When deposits are insured, shareholders can not benefit through lower funding costs from higher coinsurance of deposits when integration would lead to greater diversification of loan-making. Unless integration leads to other gains, such as reduced competition in the deposit market, it will thus not materialize when banks rely to a large extent or even exclusively on insured deposits. This could apply to, for instance, to cooperative or savings and loan banks that have a strong retail presence and thus typically a large retail deposit funding base, while when they are small they may lack large-scale investment opportunities. Implication 4 predicts that this segment of the banking industry should remain heavily fragmented. This is different for banks that rely more on uninsured funding.

**Implication 5.** Integration is more likely between banks with a more correlated lending market.

The correlation between two lending markets could itself be the outcome of smaller or greater economic integration between the two regional or national economies. When this is taken as given, Implication 5 predicts that also banking mergers between these two already more integrated economies become more likely. Recall that Implication 1 obtains an analogous prediction for interbank lending. Taken together, economic integration through real activity, such as trade, and financial integration through interbank lending or bank merger are thus complementary, rather than one being a (perfect) substitute for the other. This observation has also some direct normative implications with respect to policy and regulation that we explore further below. Note finally that rather than applying only "cross-sectionally", Implication 5 applies also when other forces, such as increasing trade or joint economic policy as witnessed in the European Union, lead to increasing economic
integration between different regions and countries. Then the integration of banks should follow suit, beyond what the removal of legal and regulatory obstacles would suggest. That however the level of integration through the "banking channel" remains still insufficient is stated in the Implication 7 below. Over time, the economic integration between different regions or countries may also decrease, or there may be other reasons for why lending markets become less correlated. Though this may admittedly be a far shot, given that our model is on purpose as parsimonious as possible, the currently witnessed disintegration (or "de-synchronization") of the European economies, notably the different development of those on its southern periphery, may be a case in place. Our model would predict that this should also reduce interbank lending beyond what can be accounted for by a worsening of economic prospects or financial fragility of debtor banks.

Implication 6. As economic integration between two regions or countries deepens, also financial integration through the "banking channel", that is both through interbank lending and the reallocation of funds through integrated banks, should increase. Instead, when the correlation between two markets decreases, also financial integration through banks and notably interbank lending should decrease.

Normative Implications. From an efficiency perspective, the following implication is key.

Implication 7. When the reallocation of funding across two (regional or national) funding and lending markets relies crucially on banks and their specific ability to collect funds from households and to invest in local business, then there is a strong tendency for too little financial integration, i.e., both through (inefficiently low) interbank lending and through (inefficiently rare) integration and the subsequent reallocation within the integrated bank.

Rather than excessive interconnectedness or excessive integration to form "too-big-too-fail" international banks, our parsimonious model of banking predicts the opposite: Too little exposure through interbank lending and too little financial integration through mergers and acquisitions among banks. As noted in the Introduction, we clearly abstract from other reasons for why banks may want to become "too-big-too-fail" or "too-interconnected-to-fail", namely if the expectation of a bail-out will lower their funding costs. What is however key, in our view, is the prediction that in the absence of such additional considerations the outcome will not be first-best efficient, but that it may involve a considerable gap in financial integration through interbank lending and mergers. The first-order effect
of regulatory activities that further curb these activities may then be non-negligible and negative by further reducing allocative efficiency. It is now straightforward to derive the following implication formally (e.g., by introducing a "tax" \( \tau > 0 \) on interbank lending).\(^{29}\)

**Implication 8.** *Suppose through regulatory intervention banks’ reliance on insured (retail) deposits becomes larger. Then rather than increasing financial integration, this makes financial integration through mergers and a reallocation of funds within integrated banks less likely, thereby reducing efficiency.*

Before further commenting on this result, note again that this is derived within the constraints of the chosen model. While we show in the Appendix how some core results survive when we allow also for competition for retail deposits, we notably do not compare the operation of retail deposit markets with and without deposit insurance - and notably not so when banks had different levels of riskiness, which would become irrelevant for insured depositors. Such an analysis would clearly exceed the scope of this paper. Given these restrictions, however, Implication 8 points to an unintended and likely ignored consequence of extended deposit insurance. Then, integration does no longer benefit banks through a commitment to more diversified lending, which then leads to lower funding costs. Instead, as we showed only the drain on profits through the positive coinsurance effect on depositors would remain, making integration unprofitable. Note that banks may also choose to rely more on insured retail deposits when regulation makes funding through other (wholesale) sources more expensive (e.g., through liquidity requirements that are, however, outside our model; cf. the introduction). Again, less financial integration may then be an unintended and negative consequence, according to Implication 8.

**References**


\(^{29}\) Clearly, the precise analysis depends on how this tax is levied. An immediate way to incorporate this in our analysis is to assume that \( \tau W \) must be paid out of shareholders’ proceeds, where it does not matter qualitatively whether this applies to shareholders of the debtor bank \( B \) or of the creditor bank \( A \).


Appendix A: Proofs

Proof of Proposition 1. Recall that implicit in expression (3) there are four different cases: Case 1 with $L(F_A) \geq R_A (1 + r_A)$ and $w < R_A (1 + r_A)$, Case 2 with $L(F_A) < R_A (1 + r_A)$ and $w < R_A (1 + r_A)$, Case 3 with $L(F_A) \geq R_A (1 + r_A)$ and $w \geq R_A (1 + r_A)$, and Case 4 with $L(F_A) < R_A (1 + r_A)$ and $w \geq R_A (1 + r_A)$. We treat these cases in turn and show that only Case 1 and Case 2 will be relevant for our subsequent analysis.

Consider first Case 1 where, after substituting $w = L(R_B + W) - L(R_B)$, it follows from (3) that the profits of bank A’s shareholders are given by

$$\pi_{A1} = p [L(R_A - W_1*) - R_A(1 + r_A)] + [p^2 + pp (1 - p)] [L(R_B + W_1*) - L(R_B)].$$

Note that the program is strictly concave in this case. From inspection of expression (6) in Proposition 1, note next that $W_1^*$ strictly increases in $\rho$. Using strict concavity, we can define for given $z > 0$ and $p$ a value $\rho_0$ so that $W_1^* > 0$ only if $\rho > \rho_0$:

$$\rho_0 := \frac{1}{1 - p} \left( \frac{L'(M + z)}{L'(M - z)} - p \right),$$

where further

$$\frac{d\rho_0}{dz} = \frac{1}{1 - p} \left( \frac{L''(M + z)L'(M - z) + L'(M + z)L''(M - z)}{L'(M - z)^2} \right) < 0.\quad (11)$$

In Case 2 shareholder profits are from (3) equal to

$$\pi_{A2} = [p^2 + pp (1 - p)] [L(R_A - W_2^*) - R_A(1 + r_A) + L(R_B + W_2^*) - L(R_B)],$$

and the first-order condition yields (7). Note that also in this case the program is strictly concave.

Now consider Case 3, where (3) becomes

$$\pi_{A3} = p [L(R_A - W_3^*) - R_A(1 + r_A) + L(R_B + W_3^*) - L(R_B)],$$

and the first order condition would imply that $R_A - W_3^* = R_B + W_3^* = M$. We now argue that if the interbank loan is sufficiently low so that repayment from its own loans is sufficient to repay A’s depositors, the repayment from the interbank loan $w$ can not at the same time be sufficiently high to repay depositors of bank A, i.e.,

$$L(R_B + W) - L(R_B) - R_A (1 + r_A) < 0.\quad (12)$$
Condition (12) is for $R_B = 0$ implied by assumption (2). It also holds for $R_B \in (0, M)$ since the partial derivative of (12) with respect to $R_B$ is, using $R_A = 2M - R_B$, given by

$$1 + r_A - L'(R_B) \leq \frac{1}{p} - L'(R_B),$$

which is strictly negative for $R_B \in [0, M]$ due to concavity of $L$ and (1). (Recall also that $r_A \leq \frac{1}{p} - 1$.)

Finally, in Case 4 shareholders’ profits in (3) are given by

$$\pi_{A4} = p[L(R_B + W_{1*}^{*}) - L(R_B) - R_A(1 + r_A)]$$

$$+ [p^2 + \rho p (1 - p)] L(R_A - W_{1*}^{*})$$

with first order condition

$$[p^2 + \rho p (1 - p)] L'(R_A - W_{1*}^{*}) = pL'(R_B + W_{1*}^{*}).$$

Now consider two subcases. If $[p^2 + \rho p (1 - p)] L'(R_B) > pL'(R_A)$, we have $W_{1*}^{*} > 0$, implying that Case 4 is always (weakly) inferior to Case 1 as

$$\pi_{A1} - \pi_{A4} = p (1 - p) (1 - \rho) L(R_B).$$

When instead $[p^2 + \rho p (1 - p)] L'(R_B) \leq pL'(R_A)$, then in Case 1 $W = 0$ so that $A$’s profits are given by $\pi_{A0} = p [L(R_A) - R_A(1 + r_A)]$ and the difference

$$\pi_{A0} - \pi_{A4} = p[L(R_A + L(R_B) - L(R_B + W_{1*}^{*})]$$

$$- [p^2 + \rho p (1 - p)] L(R_A - W_{1*}^{*})$$

is strictly positive as well. To see this note that when $[p^2 + \rho p (1 - p)] L'(R_B) = pL'(R_A)$, it holds that $\pi_{A0} = \pi_{A1}$ and thus (13) is equal to $p (1 - p) (1 - \rho) L(R_B)$. Differentiating (13) with respect to $R_B$, again using $R_A = 2M - R_B$, yields

$$\frac{d}{dR_B} (\pi_{A0} - \pi_{A4}) = p[L'(R_B) - L'(R_A)] > 0.$$
which leads to first order condition
\[ [p^2 + \rho p (1 - p)] L'(R_A + W) = pL' (R_B - W) . \]
Clearly, \( W = 0 \) unless
\[ [p^2 + \rho p (1 - p)] L'(R_A) > pL' (R_B) , \]
which is ruled out by \( R_A \geq R_B \). Q.E.D.

Proof of Proposition 2. Recall first that when a bank is not able to repay, then the deposit insurance agency covers the full repayment obligation. Hence, bank \( A \)'s depositors break even with \( r_A = 0 \). Thus, starting from \( W = 0 \) we have from (3) the following derivative of \( \pi_A \):
\[
\frac{d\pi_A}{dW} = \begin{cases} 
[p^2 + \rho p (1 - p)] L'(R_B + W) - pL'(R_A - W) & \text{if } L(R_A - W) \geq R_A \\
[p^2 + \rho p (1 - p)] L'(R_B + W) - L'(R_A - W) & \text{if } L(R_A - W) < R_A 
\end{cases}
\]
where \( R_A = M + z \) and \( R_B = M - z \). Suppose now first that even when \( W^* = z \), Case 2 does not apply as\[
L(M) \geq M + z,
\]Clearly, (14) holds when \( z = 0 \), while it does not hold for \( z = M \) due to (2). Hence, there is a cutoff \( \bar{z} \) defined by
\[
L(M) = M + \bar{z},
\]so that we can altogether rule out Case 2 if and only if \( z \leq \bar{z} \). Suppose now that Case 2 is feasible as \( z > \bar{z} \). Clearly, in Case 2 \( W^* = z \) no longer depends on \( \rho \). Also, it holds in Case 1 that \( W^* < z \) (unless \( \rho = 1 \), so that there is perfect positive correlation). The crux is now that the objective function for the maximization problem with respect to \( W \) is now altogether no longer quasiconcave as we shift between different cases. We now compare bank \( A \)'s shareholders' profits across the different cases evaluated at the respective optimal interbank loan. Consider first
\[
\frac{d}{d\rho} (\pi_{A2} - \pi_{A0}) = p (1 - p) \left[ 2L(M) - L(M - z) - (M + z) \right],
\]
which is strictly positive. This is surely the case for \( z = 0 \) (cf. the much stronger condition (1)). Next, differentiating the expression with respective to \( z \), it is strictly increasing when \( L'(M - z) > 1 \), which is also implied by (1). Next, we also show that \( \pi_{A2} - \pi_{A1} \) is increasing in \( \rho \). Making use of the first-order condition (6), we have
\[
\frac{d}{d\rho} (\pi_{A2} - \pi_{A1}) = p (1 - p) \left[ 2L(M) - L(M - z + W^*) - (M + z) \right].
\]
To confirm that this is strictly positive, it is sufficient to do so at the highest value
\( L (M - z + W^*) \) that is still compatible with Case 1, which in turn is the lowest value
at which still \( L(M + z - W^*) = M + z \). But then the sign of the derivative is determined by
\[
2L(M) - L(M - z + W^*) - L(M + z - W^*) > 0,
\]
where we used strict concavity of \( L \). \textbf{Q.E.D.}

\textbf{Proof of Proposition 3.} We are now rather brief as the analysis is largely analogous to
the comparative analysis in \( \rho \) of Proposition 2. Taking first Case 1, note that from (10) we
can now define, for given \( \rho \), a cutoff \( z_l \) so that indeed \( W^* > 0 \) when \( z > z_l \), where \( z_l < M \)
when
\[
\rho > \frac{1}{1-p} \left( \frac{L'(2M)}{L'(0)} - p \right).
\]
When \( W^* > 0 \), it is also strictly increasing in \( z \). Next, note that \( W^* = z \) will arise indeed
only if \( z \) is sufficiently high, as
\[
\frac{d}{dz} (\pi_{A2} - \pi_{A0}) = \left[ p^2 + p(1-p) \rho \right] L' (M - z) - pL' (M + z) + p(1-p)(1-\rho)(17)
\geq p (1-p) (1-\rho),
\]
which follows from \( [p^2 + p(1-p) \rho] L' (M - z) > pL' (M + z) \) for \( z > z_l \) and
\[
\frac{d}{dz} (\pi_{A2} - \pi_{A1}) = p (1-p) (1-\rho).
\]
Denote the critical level where \( \pi_{A2} = \pi_{A1} \) by \( z_h \). \textbf{Q.E.D.}

\textbf{Proof of Proposition 4.} First, recall that for a given interest rate \( r_A \), the interbank
loan set by bank \( A \) in \( t = 2 \) is either given by \( W^* = z \) or \( W^* \) which solves (6), where
the latter contains also the boundary solution with \( W^* = 0 \). We will now construct the
optimal choice of bank \( A \) for a given interest rate \( r_A \). If Case 2 applies and \( W^* = z \), bank
\( A \)'s shareholder’s profits are given by
\[
\pi_{A2} (r_A) = \left( p^2 + \rho p (1-p) \right) [2L(M) - L(M - z) - (M + z) (1 + r_A)].
\]
If Case 1 applies and \( W^* \) solves instead (6), profits are given by
\[
\pi_{A1} (r_A) = p \left[ L(M + z - W^*) - (M + z) (1 + r_A) \right]
\geq p \left[ L(M + z - W^*) - L(M + z) \right] + \left( p^2 + \rho p (1-p) \right) \left[ L(M - z + W^*) - L(M - z) \right].
\]
Let $q$ denote the probability with which $W^* = z$, such that shareholders’ expected profits are given by

$$\pi_A(q, r_A) := q \pi_{A2}(r_A) + (1 - q) \pi_{A1}(r_A).$$

Note that Case 2 can only arise for $r_A \in [r_{Al}, r_{Ah}]$, where $r_{A2} := \frac{[2L(M) - L(M - z)]}{(M + z)} - 1$ and $r_{Ah} := \frac{L(M)}{(M + z)} - 1$. Next, Case 1 where $W^*$ solves (6) can only arise for $r_A < r_{Ah} := \frac{L(M + z - W^*)}{(M + z)} - 1$. Now consider the difference in profits for a given interest rate $r_A$,

$$\Delta (r_A) := \frac{\partial \pi_A(q, r_A)}{\partial q} = \pi_{A2}(r_A) - \pi_{A1}(r_A),$$

and observe that

$$\Delta (r_{Ah}) = (p^2 + pp(1 - p)) [2L(M) - L(M - z + W^*) - L(M + z - W^*)]$$

$$= 0,$$

which follows immediately from strict concavity of $L$. Next,

$$\Delta (r_{Al}) = (p^2 + pp(1 - p)) [L(M) - L(M - z + W^*)] - pL(M + z - W^*) - L(M)$$

$$< [(p^2 + pp(1 - p)) L'(M - z + W^*) - pL'(M + z - W^*)] (z - W^*)$$

$$= 0,$$

where the inequality follows from concavity of $L$ and the last equality from (6). Furthermore, since $\partial \Delta/\partial r_A = p (1-p) (1-\rho) (M+z) > 0$, there exists a unique cutoff $\hat{r}_A \in (r_{Al}, r_{Ah})$ such that $\Delta (\hat{r}_A) = 0$ and the best response to $r_A$ is given by

$$q(r_A) = \begin{cases} 
q = 0 & \text{for } r_A \in [0, \hat{r}_A) \\
q \in [0, 1] & \text{for } r_A = \hat{r}_A \\
q = 1 & \text{for } r_A \in (\hat{r}_A, r_{Ah}] 
\end{cases}.$$

The interest rate $r_A(q)$ at which depositors of bank $A$ break even, given an anticipated probability $q$, is determined by

$$R(q, r_A) := q (p^2 + pp(1 - p)) + (1 - q) p \frac{q2L(M) + (1 - q) L(M - z + W^*) - L(M - z)}{(M + z)}$$

$$= 0,$$

where

$$r_A(0) = \frac{1 - p}{p} - \frac{p (1-p) (1-\rho) L(M - z + W^*) - L(M - z)}{M + z}.$$
and
\[ r_A(1) = \frac{1 - (p^2 + \rho p (1 - p))}{p^2 + \rho p (1 - p)} - \frac{p (1 - p) (1 - \rho)}{(p^2 + \rho p (1 - p))} \frac{2L(M) - L(M - z)}{M + z}. \]  
\( (21) \)

An equilibrium is therefore given by a fixed point \((q^*, r_A^*)\) of the correspondence \((q, r_A) : [0, \bar{r}_A] \times [0, 1] \rightarrow [0, 1] \times [r_A(0), r_A(1)]\). Note that \((q, r_A)\) is non empty since the sign of \(r_A\) is determined by the following expression
\[ p(2L(M) - L(M - z)) - (p^2 + \rho p (1 - p))(M + z), \]
which is positive by concavity of \(L\), and \(r_A(0) > 0\), so that there must indeed exist a fixed point with \(r_A^* = r_A(q^*)\) and \(q^* = q(r_A^*)\).

To show uniqueness, it is helpful to consider two cases. First, if there exists a \(\tilde{q} < 1\) such that \(r_A(\tilde{q}) \leq r_{Ah}\), then there will be a unique fixed point with \(q^* \in [\tilde{q}, 1]\) as \(r_A(q)\) is strictly decreasing in \(q\) for \(q \geq \tilde{q}\), i.e.
\[ \frac{\partial r_A(q)}{\partial q} = -\frac{\partial R(q)}{\partial q} / \partial r_A < 0, \]
which follows from
\[ \frac{\partial R(q, r_A)}{\partial q} = p(1 - p)(1 - \rho) [2L(M) - L(R_B + W_1^*) - (M + z)(1 + r_A)] \]
which is strictly positive for \(r_A \leq r_{Ah}\) and
\[ \frac{\partial R(q, r_A)}{\partial r_A} = [q(p^2 + \rho p (1 - p)) + (1 - q)p](M + z) > 0. \]

Second, if such a \(\tilde{q}\) does not exist, then \(r_A(q) > r_{Ah}\) for all \(q \leq 1\), so that there can only be a fixed point with \(q^* = \tilde{q}\), which must therefore be unique.

**Comparative Analysis in \(\rho\):** Consider the smallest \(\hat{\rho}_h\), such that \(q^* = 1\). Then \(q^* = 1\) also for \(\rho \in [\hat{\rho}_h, 1]\). This follows as the critical interest rate \(\hat{r}_A\) decreases in \(\rho\) as
\[ \frac{\partial \hat{r}_A}{\partial \rho} = -\frac{\partial \Delta / \partial \rho}{\partial \Delta / \partial r_A} < 0, \]  
\( (22) \)

where
\[ \frac{\partial \Delta (r_A)}{\partial \rho} = p(1 - p)2L(M) - L(M - z + W^*) - (M + z)(1 + r_A) > 0, \]  
\( (23) \)
and
\[ \frac{\partial \Delta (r_A)}{\partial r_A} = p(1 - p)(1 - \rho)(M + z) > 0. \]  
\( (24) \)
Furthermore, the interest rate that is required when \( q = 1 \) increases in which follows from differentiating the break even condition (19):

\[
\frac{\partial r_A(1)}{\partial \rho} = -\frac{\partial R(1, r_A)}{\partial \rho} / \partial r_A \\
> 0,
\]

where

\[
\frac{\partial R(1, r_A)}{\partial r_A} = (p^2 + \rho p (1 - p)) (M + z) \\
> 0
\]

and

\[
\frac{\partial R(1, r_A)}{\partial \rho} = -p (1 - p) [2L(M) - (M + z) (1 + r_A) - L(M - z)] \\
< 0.
\]

Since \( \partial r_A(q) / \partial q < 0 \), the only equilibrium that can be supported for \( \rho \in (\hat{\rho}_h, 1] \) is therefore that where \( W^* = z \) is chosen with probability one.

Now consider \( \hat{\rho}_t \), the largest value where \( q^* = 0 \) can be supported and where thus \( \Delta (r(0)) = 0 \). Next, note that at \( \hat{\rho}_h \) (which is the lowest \( \rho \) at which \( q^* = 1 \) can be supported) it holds that \( \Delta (r(1)) = 0 \). Since \( \partial r_A(q) / \partial q < 0 \) we must have that \( r_A(1) < r_A(0) \). Together with (24) and (23), this implies that \( \hat{\rho}_h > \hat{\rho}_t \). Hence, for \( \rho \in (\hat{\rho}_t, \hat{\rho}_h) \) there does not exist a pure strategy equilibrium and we will now show that \( q^* \) is strictly increasing in \( \rho \) on that interval. Note that the differential of (19), evaluated at the equilibrium levels \( q^* \) and \( \hat{r}_A \), can be written more explicitly as

\[
dR(q^*, r_A^* = \hat{r}_A) \frac{d\rho}{d\rho} = \frac{\partial q^*}{\partial \rho} p (1 - p) (1 - \rho) [2L(M) - L(M - z + W^*) - (M + z) (1 + \hat{r}_A)] \\
+ q^* p (1 - p) (M + z) (1 + r_A) \\
+ [q^* (p^2 + \rho p (1 - p)) + (1 - q^*) p] (M + z) \frac{\partial \hat{r}_A}{\partial \rho} \\
- p (1 - p) [q^* 2L(M) + (1 - q^*) L(M - z + W^*) - L(M - z)] \\
+ p (1 - p) (1 - \rho) (1 - q^*) L(M - z + W^*) \frac{\partial W^*}{\partial \rho}
\]

\[
= 0.
\]

Next, the difference in shareholder profits, \( \Delta (\hat{r}_A) \), has to stay equal to zero for all \( \rho \in \)
(\hat{\rho}_1, \hat{\rho}_h) and, thus,
\[
\frac{d\pi_{A1}(\hat{r}_A)}{d\rho} = p (1 - p) [L(M - z + W^*) - L(M - z)]
- p \left[ L'(M + z - W^*) \frac{\partial W^*}{\partial \rho} + (M + z) \frac{\partial \hat{r}_A}{\partial \rho} \right]
= p (1 - p) [2L(M) - L(M - z) - (M + z) (1 + \hat{r}_A)]
- (p^2 + \rho p (1 - p)) (M + z) \frac{\partial \hat{r}_A}{\partial \rho}
= \frac{d\pi_{A2}(\hat{r}_A)}{d\rho}.
\]

This implies, together with (6), that
\[
L'(M - z + W^*) \frac{\partial W^*}{\partial \rho} = - \frac{p^2 (1 - p)}{p^2 + \rho p (1 - p)} \left[ \frac{2L(M) - L(M - z + W^*)}{(M + z) (1 + \hat{r}_A)} - (M + z) (1 + \hat{r}_A) \right],
\]
(27)

Substituting (27) and (22) into (26) finally yields
\[
\frac{dR(q^*, r_A^* = \hat{r}_A)}{d\rho} = \frac{\partial q^*}{\partial \rho} p (1 - p) (1 - \rho) [2L(M) - L(M - z + W^*) - (M + z) (1 + \hat{r}_A)]
- \frac{p}{1 - \rho} [2L(M) - L(M - z + W^*) - (M + z) (1 + \hat{r}_A)]
- p (1 - p) [L(M - z + W^*) - L(M - z)]
= 0.
\]

Hence, as \( \hat{r}_A < r_{Ah} \), the expressions in square brackets in line 1 and 2 are positive and thus \( \frac{\partial q^*}{\partial \rho} > 0 \). Q.E.D.

**Proof of Proposition 5.** Consider the smallest value \( z_h \) for which \( q^* = 1 \) can be supported. Then, only \( q^* = 1 \) can be supported for \( z \in [z_h, 1] \). To see this, note first that the profit difference \( \Delta (r_A) = \pi_{A2} - \pi_{A1} \) is strictly increasing in the outstanding repayment obligation, \( (1 + r_A)(M + z) \), as
\[
\frac{d\Delta (r_A(1))}{dz} = p (1 - p) (1 - \rho) \left[ 1 + r_A(1) + (M + z) \frac{\partial r_A(1)}{\partial z} \right],
\]
(29)

where
\[
1 + r_A(1) + (M + z) \frac{\partial r_A(1)}{\partial z} = \frac{1 - p (1 - p)(1 - \rho) L'(M - z)}{p^2 + \rho p (1 - p)}.
\]

Note that Case 2 can only arise if \( L(M) < (M + z)(1 + r_A(1)) \) or, after substituting \( r_A(1) \) from (21), if
\[
L(M) < \frac{1}{p^2 + \rho p (1 - p)} [M + z - p (1 - p)(1 - \rho) (2L(M) - L(M - z))].
\]
(30)
Note further that (30) is satisfied for \( z = M \) due to Assumption (2) and it is violated for \( z = 0 \) due to Assumption (1). Furthermore Assumption (2) implies that for \( z = M \) it also holds that
\[
L (2M - W^*) < \frac{1}{p^2 + \rho p (1 - p)} [2M - p (1 - p) (1 - \rho) (2L (M) - L (0))],
\]
that is, the repayment obligation \((1 + r_A (1)) (M + z)\) exceeds the repayment from bank A’s corporate loans in Case 1 where \( W^* \) satisfies (6). Hence, as the right hand side of (30) is strictly concave in \( z \), it must be strictly increasing in \( z \), i.e.,
\[
1 > p (1 - p) (1 - \rho) L' (M - z),
\]
for \( z \leq \bar{z} \) which is defined as the smallest value for which (31) holds with equality. As for \( z > \bar{z} \) only \( q^* = 1 \) can be supported and for \( z \leq \bar{z} \) we get that \( d\Delta/dz > 0 \), \( q^* = 0 \) for \( z > \bar{z}_h \). Finally, since \( 1 + r_A (0) + (M + z) \frac{\partial r_A (0)}{\partial z} < 1 + r_A (1) + (M + z) \frac{\partial r_A (1)}{\partial z} \), it follows immediately that \( \bar{z}_l < \bar{z}_h \) and bank A mixes for \( z \in (\bar{z}_l, \bar{z}_h) \). Differentiating (19) with respect to \( z \) yields
\[
\frac{dR (q^*, r_A^* = \bar{r}_A)}{dz} = \frac{\partial q^*}{\partial z} p (1 - p) (1 - \rho) [2L (M) - L (M - z + W^*) - (M + z) (1 + r_A)]
+ [q^* (p^2 + \rho p (1 - p)) + (1 - q^*) p] \left[ (1 + r_A) + (M + z) \frac{\partial \bar{r}_A}{\partial z} \right]
- [1 - p (1 - p) (1 - \rho) L' (M - z)]
= 0.
\]
It then follows from differentiating the indifference condition \( \Delta (\bar{r}_A) = 0 \), that
\[
(1 + \bar{r}_A) + (M + z) \frac{\partial \bar{r}_A}{\partial z} = 0,
\]
such that due to (32), \( \frac{\partial q^*}{\partial z} > 0 \). Q.E.D.

**Proof of Proposition 6.** Note first that the profits of the integrated bank’s shareholders (8) are equal to the profits of bank A’s shareholders (3) once we substitute (5) and set \( R_A = 2M \) and \( R_B = 0 \) in (3). The required interest rate when deposits are insured is given by \( r_{AB} = 0 \). Hence, the equilibrium characterization and comparative statics can be inferred from extending the analysis of separate banks in Propositions 1 and 2 to the case with \( z = M \). More explicitly, when \( \rho \leq \tilde{\rho}' \), the equilibrium allocation of funds by an integrated bank, \( F_n^* \), solve (9) which mirrors Case 1 in Proposition 1 (the corner solution
of Case 1 with $F_B^* = 0$ and $F_A^* = 2M$ applies if $\rho \leq \rho^l$, given by (10) for $z = M$. We will refer to the $F_n^*$ that solves (9) – analogously to $W_1^*$ as $F_{n1}^*$, for $n = A, B$. When $\rho > \rho^l$, the efficient allocation is achieved with $F_n^* = M$, which mirrors Case 2 in Proposition 1.

From (11) it follows then immediately that $\rho^l_0 > \rho^l_0$ and, thus, the integrated bank achieves a strictly less efficient allocation than separate banks, as $F_B^* < R_B + W^*$ for $\rho < \rho^l_0$. Next, from combining (16) and (17) it follows immediately that $\tilde{\rho}_B^l > \tilde{\rho}_B^l$. Hence, for $\tilde{\rho}_B^l < \rho < \tilde{\rho}_S$, we get $R_B + W^* < F_B^* = M$, while $R_B + W^* = F_B^*$ for $\rho < \rho^l_0$ (provided that $\rho^l_0 < \tilde{\rho}^l_0$). Finally, and $R_B + W^* = F_B^* = M$ for $\rho \geq \tilde{\rho}^S$. Q.E.D.

**Proof of Proposition 8.** As in Proposition 6, the equilibrium and comparative analysis can be inferred from extending the analysis of separate banks in Propositions 1 and 4 to the case with $z = M$. Note that now the required interest rate $r_{AB}$ is given by (19) for $z = M$. It then follows immediately from (33) that $\tilde{\rho}_l^l \neq \tilde{\rho}_l^S$ and $\tilde{\rho}_h^l < \tilde{\rho}_h^S$.

Now consider the expected amount allocated to market $B$, which, in case of an integrated bank, is given by

$$T_B^I := q^l (M - F_{B1}^*) + F_{B1}^*,$$  
(35)

and in case of separate banks, it is given by

$$T_B^S := q^S (z - W_1^*) + (M - z + W_1^*).$$  
(36)

Note first that for $\rho \leq \tilde{\rho}_l^l$,

$$T_B^I - T_B^S = F_{B1}^* - (M - z + W_1^*) \leq 0$$

with strict inequality for $\rho < \rho^l_0$. (Recall that this thresholds denotes the value from which on there is positive interbank lending.) Next, for $\tilde{\rho}_l^l < \rho < \tilde{\rho}_h^l$, we show that $T_B^I - T_B^S$ is strictly increasing in $\rho$ and eventually turns positive. Note that we have to consider various cases, depending on whether one or both thresholds $\rho^l_0 < \rho^S_0$ fall into this interval. Differentiating (35) yields

$$\frac{dT_B^I}{d\rho} = \frac{dq^l}{d\rho} (M - F_{B1}^*) + (1 - q^l) \frac{dF_{B1}^*}{d\rho},$$

where $\frac{dF_{B1}^*}{d\rho} \geq 0$ (with strict inequality for $\rho > \rho^l_0$). Differentiating (36) yields

$$\frac{dT_B^S}{d\rho} = \frac{dq^S}{d\rho} (z - W_1^*) + (1 - q^S) \frac{dW_1^*}{d\rho},$$

39
where \( \frac{dW_t}{dp} \geq 0 \) (with strict inequality for \( \rho > \rho_0^S > \rho_0^I \)). Recall next that for \( \rho < \rho_0^S \), it holds that \( M - F_{B1}^* > z - W_1^* \). Next, from (28),

\[
\begin{align*}
\frac{dq^I}{dp} &= \frac{1}{(1-p) (1-\rho)^2} + \frac{1}{(1-\rho)} 2L(M) - L(F_{B1}^*) - 2M (1 + \hat{r}_A) \\
\frac{dq^S}{dp} &= \frac{1}{(1-p) (1-\rho)^2} + \frac{1}{(1-\rho)} 2L(M - z + W_1^*) - L(M - z) - (M + z) (1 + \hat{r}_A),
\end{align*}
\]

implying that \( \frac{dq^I}{dp} \geq \frac{dq^S}{dp} \) for \( \rho < \rho_0^S \) (with strict inequality for \( \rho_0^S > \rho > \rho_0^I \)). Taken together, we thus have \( \frac{\partial T^I_B}{\partial p} \geq \frac{\partial T^S_B}{\partial p} \) for \( \rho < \rho_0^S \). For \( \rho \geq \rho_0^S \), we have \( F_{B1}^* = M - z + W_1^* \) and, thus,

\[
T^I_B - T^S_B = (q^I - q^S) (M - F_{B1}^*) > 0.
\]

Finally, for \( \rho \geq \hat{\rho}_h^I \),

\[
T^I_B - T^S_B = (1 - q^S) (z - W_1^*) \geq 0
\]

with strict inequality \( (q^S < 1) \) for \( \hat{\rho}_h^I \leq \rho < \hat{\rho}_h^S \). Hence, we have shown that there exists a unique \( \hat{\rho}_h^I \leq \rho \leq \hat{\rho}_h^S \) such that \( T^I_B - T^S_B \geq 0 \) for \( \rho \geq \hat{\rho} \) and \( T^I_B - T^S_B \leq 0 \) for \( \rho \leq \hat{\rho} \). **Q.E.D.**

**Proof of Proposition 9.** Consider first \( \rho \geq \hat{\rho}_h^S \), where where we have

\[
\pi_{AB2} - (\pi_{A2} + \pi_B) = -p (1-p) (1-\rho) [L(M - z) - (M - z)] < 0.
\]

Next, take \( \hat{\rho}_h^I \leq \rho < \hat{\rho}_h^S \), for which \( \pi_{A1} > \pi_{A2} \) and, thus, \( \pi_{AB2} - (\pi_{A1} + \pi_B) < \pi_{AB2} - (\pi_{A2} + \pi_B) \). Finally, for \( \rho < \hat{\rho}_h^I \), we have

\[
\pi_{AB1} - (\pi_{A1} + \pi_B) = p L(F_{A1}^*) + (p^2 + pp (1-p)) L(F_{B1}^*) - p L(M + z - W_1^*) - (p^2 + pp (1-p)) L(M - z + W_1^*) - p (1-p) (1-\rho) L(M - z) \leq -p (1-p) (1-\rho) L(M - z) < 0,
\]

where the first inequality follows from concavity of \( L \) (it holds strictly for \( \rho < \rho_0^S \) where \( F_{A1}^* > M + z - W_1^* \) and \( F_{B1}^* < M - z + W_1^* \)). **Q.E.D.**

**Proof of Proposition 10.** Note that at \( t = 1 \) when banks decide whether or not to integrate, expected profits of an integrated bank are given by

\[
\pi_{AB} = p [q^I 2L(M) + (1 - q^I) (L(F_{A1}^*) + L(F_{B1}^*))] - 2M
\]

40
and joint expected profits of bank $A$ and $B$ are given by

$$\pi_A + \pi_B = p \left( q^S 2L(M) + (1 - q^S) (L(M+z - W_1^*) + L(M-z + W_1^*)) \right) - 2M.$$

Note first that for $\rho \leq \rho_l^I$,

$$\pi_{AB} - (\pi_A + \pi_B) = p \left( (L(F_{A1}\_1) + L(F_{B1}\_1)) - (L(M+z - W_1^*) + L(M-z + W_1^*)) \right) \leq 0$$

with strict inequality for $\rho < \rho_0^S$.

Next, for $\rho_l^I < \rho < \rho_h^I$, we show that $\pi_{AB} - (\pi_A + \pi_B)$ is strictly increasing in $\rho$ and eventually turns positive. First, for $\rho < \rho_0^I$, where $F_{B1}^* = W_1^* = 0$,

$$\frac{\partial}{\partial \rho} [\pi_{AB} - (\pi_A + \pi_B)] = p \left[ \begin{array}{c} \frac{\partial q^I}{\partial \rho} (2L(M) - L(2M)) \\ -\frac{\partial q^S}{\partial \rho} (2L(M) - L(M+z) - L(M-z)) \end{array} \right] > 0,$$

which follows from concavity of $L$ and the observation that by (37) $\frac{\partial q^S}{\partial \rho} = 0$ for $\rho \leq \rho_l^I$ and $\frac{\partial q^I}{\partial \rho} = \frac{\partial q^I}{\partial \rho}$ for $\rho_l^I < \rho < \rho_h^I$ (in case $\rho_l^I < \rho_h^I$). Second, for $\rho_0^I \leq \rho < \rho_0^S$,

$$\frac{\partial}{\partial \rho} [\pi_{AB} - (\pi_A + \pi_B)] = p \left[ \begin{array}{c} \frac{\partial q^I}{\partial \rho} (2L(M) - L(F_{A1}^*) - L(F_{B1}^*)) \\ -\frac{\partial q^S}{\partial \rho} (2L(M) - L(M+z) - L(M-z)) \\ + (1 - q^I) (L'(F_{B1}^*) - L'(F_{A1}^*)) \frac{\partial F_{B1}^*}{\partial \rho} \end{array} \right] > 0,$$

which follows from concavity of $L$ and the observation that by (37) $\frac{\partial q^S}{\partial \rho} = 0$ for $\rho \leq \rho_l^I$ and $\frac{\partial q^I}{\partial \rho} < \frac{\partial q^I}{\partial \rho}$ for $\rho_l^I < \rho < \rho_h^I$ (in case $\rho_l^I < \rho_h^I$). Next, for $\rho \geq \rho_0^S$, we have $F_{B1}^* = M - z + W_1^*$ and, thus,

$$\pi_{AB} - (\pi_A + \pi_B) = p (q^I - q^S) [2L(M) - (L(F_{A1}^*) + L(F_{B1}^*))] > 0.$$

Finally, for $\rho \geq \rho_h^I$,

$$\pi_{AB} - (\pi_A + \pi_B) = p (1 - q^S) [2L(M) - L(M+z - W_1^*) + L(M-z + W_1^*)] \geq 0,$$

with strict inequality for $\rho_h^I \leq \rho < \rho_h^S$. Thus, there exists a unique cut off $\rho_l^I \leq \rho^* \leq \rho_h^I$, such that $\pi_{AB} - (\pi_A + \pi_B) \leq 0$ for $\rho \leq \rho^*$ and $\pi_{AB} - (\pi_A + \pi_B) \geq 0$ for $\rho \geq \rho^*$. Q.E.D.
Appendix B: Market for Retail Deposits

In this Appendix we consider the case with non-integrated banks but allow for a reallocation of funds through retail competition, next to interbank lending. We show that still our main comparative insights for the size of interbank lending survive. Note that clearly the analysis with an integrated bank would not be affected. As noted in the main text, the fact that integration then eliminates competition on the retail deposit market would generate an advantage for integration, which would blur our analysis.

Model of the Retail Deposit Market. In this Appendix we allow for retail competition in stage \( t = 1 \) and show that key results are robust to a potential reallocation of funds via the retail deposit market. It is convenient, however, to first consider the case where only the retail market is active as there is no subsequent interbank lending market: \( W = 0 \) and thus \( F_n = R_n \). This analysis allows us to isolate some key features of how the retail deposit market works in our model. We then solve the model where both retail competition and interbank lending interact.

If a household residing in the local market of bank \( n \) deposits not with the local bank but instead with bank \( n' \), it incurs a switching cost \( s \geq 0 \). For each depositor the respective value of \( s \) represents an independent random draw from the cumulative distribution function \( G(s) \). The assumption of switching costs that are non-negligible relative to their savings reflects the small granularity of retail deposit financing. Given \( M_A \geq M_B \), it is intuitive that in equilibrium switching will occur at most out of market \( A \). For depositors in market \( A \) we can then determine a critical switching cost level, \( s^* \), so that only depositors with draws \( s \leq s^* \) take up the offer of the rival bank \( B \). If interior, with deposit insurance this yields \( s^* = r_B - r_A \) and without deposit insurance \( s^* = p(r_B - r_A) \). The respective attracted deposit volumes are then given by \( R_A = M_A[1 - G(s^*)] \) and \( R_B = M_B + M_A G(s^*) \).

If both banks choose an interest rate above the participation constraint of depositors, \( r_n = 0 \) or \( r_n = 1/p - 1 \) for the cases with and without deposit insurance, then these are pinned down by the respective first-order conditions

\[
\frac{dR_n}{dr_n} [L'(R_n) - (1 + r_n)] - R_n = 0. \tag{38}
\]

Here, we have \( \frac{dR_n}{dr_n} = pg(s^*)M_A \) without deposit insurance and \( \frac{dR_n}{dr_n} = g(s^*)M_A \) with deposit insurance. We restrict attention to cases where \( g(0) \) is sufficiently large to ensure that the
equilibrium deposit rates are characterized by the respective conditions (38).\textsuperscript{30} In order to facilitate the exposition, we stipulate a uniform distribution $s \in [0, \bar{s}]$ with $g(s) = 1/\bar{s}$ and thus suppose that $\bar{s}$ is not too large.

As is immediate, competition will not fully bridge the differences in the deposit base. Formally, this is most immediately seen from the first-order conditions (38): Given that $\frac{dR_A}{dr_A} = \frac{dR_B}{dr_B}$ and given that at $r_A = r_B$ it holds that $R_A = M_A > R_B = M_B$, these can indeed only jointly hold for bank $A$ and bank $B$ when $r_B > r_A$ though still $R_A > R_B$. The reason for this is the low granularity of retail deposits combined with inertia, as modeled by switching costs. There is, however, an interesting twist to this observation when we now briefly compare the outcomes with and without deposit insurance. We find that the resulting allocation remains more asymmetric when there is no deposit insurance than when there is deposit insurance. Essentially, deposit insurance leads to a decrease in the cost of attracting deposits. This intensifies competition and ensures that the outcome more closely reflects the different marginal profitability in loan making at the two banks, depending on the attracted and invested funds.

**Proposition A1.** Suppose (only for now) that interbank loans are not possible, but that there is competition for retail deposits across local markets, albeit hampered by frictions due to switching costs. Then, a difference in the size of the deposit base, $M_A > M_B$, still leads to different volumes of attracted deposits, $R_A > R_B$, and there is less reallocation of funds across markets (larger $R_A - R_B$) under deposit insurance than without deposit insurance.

**Proof.** It is now convenient to set up the banks’ problem slightly differently for the proof. We suppose that banks compete for depositors by promising some value $v_n$, so that $v_n = p_n(1 + r_n)$ without deposit insurance and $v_n = (1 + r_n)$ with insurance. Note that we can then always express the cutoff as $s^* = v_B - v_A$, provided that we still restrict wlog the analysis to the case where $M_A \geq M_B$. Without deposit insurance the first-order condition wrt $v_n$ is obtained from maximizing

$$
\pi_n = pL(R_n) - v_nR_n
$$

and thus equal to

$$
pL'(R_n)\frac{dR_n}{dv_n} - R_n - v_n\frac{dR_n}{dv_n} = 0.
$$

\textsuperscript{30}A corner solution may arise when, starting from the monopoly interest rates, an increase will induce instead too little switching.
With deposit insurance we have, instead,
\[ \pi_n = pL(R_n) - pv_n R_n \]
and thus
\[ pL'(R_n) \frac{dR_n}{dv_n} - pR_n - pv_n \frac{dR_n}{dv_n} = 0. \]

Note next that
\[ \frac{dR_n}{dv_n} = MAg(s) = MA \frac{1}{s}, \]
making use also of the uniform distribution of switching costs. If we now subtract the first-order condition for \( A \) from that for \( B \), we obtain without deposit insurance
\[ pMA \frac{1}{s} [L'(R_B) - L'(R_A)] = (R_B - R_A) + MA \frac{1}{s} (v_B - v_A), \quad (39) \]
where, as a function of \( v_B - v_A \), the left-hand side is strictly decreasing and the right-hand side strictly increasing, noting that \( R_B - R_A \) is strictly increasing in \( v_B - v_A \) and that \( L'' < 0 \).

The only change when there is deposit insurance is that the right-hand side of (39) is multiplied by \( p \), which yields
\[ pMA \frac{1}{s} [L'(R_B) - L'(R_A)] = \left[ (R_B - R_A) + MA \frac{1}{s} (v_B - v_A) \right] p. \quad (40) \]
Starting from the equilibrium without deposit insurance, where \( R_B < R_A \) so that both sides are strictly positive, and multiplying the right-hand side by \( p < 0 \), to restore equality so as to obtain (40), we must increase \( v_B - v_A \) and thus increase \( R_B - R_A \), as asserted in the Proposition.\textsuperscript{31} Q.E.D.

**Reallocation of Funds through Both the Deposit Market and Interbank Lending.** Suppose now again that interbank lending is feasible, though in contrast to the baseline analysis, funds can now also be reallocated through the retail deposit markets. We presently stipulate that there is full deposit insurance (covering, for concreteness, also the promised interest rate \( r_n \)). Also, we focus again on the case where \( \bar{s} \) is not too large, so that there is indeed competition in equilibrium.

How does the operation of a retail deposit market and interbank lending interact? Suppose first that in equilibrium \( W = W_1^* > 0 \). Hence, if this case prevails, a reallocation
\[ \text{[Footnote: This result can also be established by using (40) and implicitly differentiating \((v_B - v_A)\), obtaining thus that } d(v_B - v_A)/dp < 0.]\]
of funds is indeed obtained both through the retail and through the interbank channel. The first thing to note is that when $W^* > 0$, the final allocation of funds, as given by $F_A$ and $F_B$, does not depend on the outcome of retail competition. Still, even though this does not affect the final allocation in this case, banks have an incentive to acquire a larger fraction of the total retail deposit market. We obtain from the respective first-order conditions for $t = 1$ the requirements

$$\frac{d\pi_A}{dr_A} \left( \frac{1}{p} \right) = \left[ (L' (R_A) - (1 + r_A)) \frac{dR_A}{dr_A} - R_A \right]$$

$$+ \frac{1}{p} \left[ (p^2 + p(1 - p)) L' (R_B) - pL' (R_A) \right] \frac{dR_A}{dr_A}$$

$$= 0,$$

$$\frac{d\pi_B}{dr_B} \left( \frac{1}{p} \right) = \left[ (L' (R_B) - (1 + r_B)) \frac{dR_B}{dr_B} - R_B \right]$$

$$= 0.$$

We stipulate that the respective problems are strictly quasiconcave so that the best responses are uniquely determined. In both expressions in (41) the respective (first) term in rectangular brackets describes the first-order condition when there is no subsequent interbank lending. The second term in bank $A$’s first-order condition captures the profits that bank $A$ will extract from an interbank loan. Notably, using the first-order condition for $W = W_1^*$ this term is indeed strictly positive. This makes bank $A$ relatively more aggressive in the retail deposit market, compared to the benchmark situation where an interbank loan was exogenously ruled out.\(^{32}\) As a consequence, the anticipation of interbank lending reduces the reallocation of funds through retail deposit competition. Interestingly, this will be even more pronounced when the efficient allocation applies. Then, by rearranging the respective first-order condition for bank $A$ we obtain

$$\frac{d\pi_A}{dr_A} \left( \frac{1}{p^2 + p(1 - p)} \right) = \frac{((L' (R_A) - (1 + r_A)) \frac{dR_A}{dr_A} - R_A}{L' (R_B) - L' (R_A)} \frac{dR_A}{dr_A}$$

$$= 0.$$

The first-order condition for bank $B$ remains the same. The difference between (41) and (42) is that the repayment of an interbank loan from bank $B$ is now no longer weighted by

\(^{32}\)We can show that when the terms of the interbank loan are determined by symmetric Nash bargaining (cf. Appendix C), then this observation still holds for bank $A$, while then, in addition, bank $B$’s incentives to attract funds through the retail deposit market are muted. Taken together, the subsequently reported results then still hold.
in the first-order condition. This reflects the fact that bank A’s shareholders benefit more from a marginal increase in the interbank loan when this leads to contagion as then there is no coinsurance externality on its depositors. Consequently, compared to Case 1, the incentives for bank A to attract deposits - relative to the incentives of bank B - further increase in Case 2. As in Case 2 the overall reallocation of funds increases compared to Case 1, as then \( F_A = F_B \) prevails, and as we have noted that there will be less reallocation of funds through retail deposit competition, the size of interbank lending is then higher.

**Proposition A2.** The insights on interbank lending from the main analysis still apply when funds can be reallocated also through competition in the retail deposit market, as long as switching costs are not too low. Precisely, the size of interbank lending \( W^* \) is still increasing in both the difference in the deposit base \((z)\) and the correlation of lending markets \((\rho)\). Furthermore, \( W^* \) again jumps upwards when (at the respective levels of \( z \) or \( \rho \)) the interbank loan becomes sufficiently large to be contagious.

**Proof.** Recall from the preceding analysis that the reallocation via retail competition becomes (weakly) smaller if the possibility of interbank lending is introduced. Observe now that if switching costs (precisely, the upper support \( \bar{s} \)) are sufficiently high satisfying

\[
\bar{s} \geq \frac{M + z}{M - z} [L'(M - z) - 1],
\]

neither bank A nor bank B offers a deposit rate that exceeds the monopoly rate of zero even when interbank lending is possible. Then reallocation will take place only via interbank lending.

Next, recall the benchmark case where interbank lending is not possible and note that for intermediate values of \( \bar{s} \), namely

\[
L'(M + z) - 1 \leq \bar{s} < \frac{M + z}{M - z} [L'(M - z) - 1],
\]

it will become optimal for bank B to attract some of bank A’s depositors by offering \( r_B > 0 \), while still \( r_A = 0 \). Then Case 1 may arise whenever the allocation that is achieved in the benchmark setting without interbank lending is such that

\[
(p^2 + p\rho (1 - p)) L'(R_B) > pL'(R_A) .
\]

We denote the resulting profits for A when it abstains from interbank lending by \( \pi_{A0} \). Using that for \( r_A = 0 \) we have that \( r_{B1} = r_{B0} = r_B \), we have

\[
\frac{d}{dz} (\pi_{A1} - \pi_{A0}) = p ((p + \rho (1 - p)) L'(R_B) - L'(R_A)) \left[ (1 - G(r_B)) - R^{dr_B} \frac{dz}{dr} \right] > 0,
\]

46
which follows from implicit differentiation of the first order condition for $B$:

\[
\left( 1 - G(r_B) \right) - R^d r_B \frac{d r_B}{d z} = \frac{(1 - G(r_B)) - [L'(R_B) - (1 + r_B)] / \bar{s}}{2 - L''(R_B) R'} \\
= \frac{(1 - G(r_B)) - R_B / (\bar{s} R')} {2 - L''(R_B) R'} \\
= \frac{2 (R_A - M)}{ (M + z) [2 - L''(R_B) R']} \\
> 0. \tag{44}
\]

Likewise, differentiating $\pi_{A1} - \pi_{A0}$ with respect to $\rho$ and using the first order condition for $W$ in Case 1 yields

\[
d\frac{\pi_{A1}}{d \rho} = p (1 - p) w - \left[ (p^2 + p p (1 - p)) L'(R_B) R' - p R' \right] \frac{\partial r_B}{\partial \rho} \\
= p (1 - p) w \\
> 0,
\]

which follows from the observation that $\frac{\partial r_B}{\partial \rho} = 0$ when $r_A = 0$. To pin down the transition to Case 2, consider again

\[
d\frac{\pi_{A2} - \pi_{A0}}{d \rho} = p (1 - p) \left[ 2 L (M) - L (R_B) - R_A \right].
\]

Clearly, for $z = 0$ we have $r_B = 0$ such that $R_A = R_B = M$, implying that this expression is positive due to assumption (1). Differentiating with respect to $z$ yields

\[
p (1 - p) (L'(R_B) - 1) (1 - G(r_B)),
\]

which is strictly positive due to assumption (1). Differentiating $\pi_{A2} - \pi_{A1}$ and noting again that $r_{B2} = r_{B1} = r_B$ for $r_A = 0$ and using the first order condition for $W_1^*$ yields

\[
d\frac{\pi_{A2} - \pi_{A1}}{d \rho} = p (1 - p) \left[ 2 L (M) - L (F_{B1}) - R_A \right].
\]

As in the setting without retail competition, it suffices to show that this is positive for the highest $F_{B1}$ that is still compatible with Case 1. The sign of the derivative is then determined by

\[
2 L (M) - L (F_{A1}) - L (F_{B1}) > 0.
\]

With respect to variations in $z$, we have

\[
d\frac{\pi_{A2} - \pi_{A0}}{d z} = \left[ [p^2 + p p (1 - p)] L'(R_B) - p L'(R_A) \right] \frac{(1 - G(r_B)) - R' \frac{\partial r_B}{\partial z}}{2\bar{s}}.
\]
which is strictly greater than zero for \([p^2 + \rho p (1 - p)] L' (R_B) > p L' (R_A)\) due to (44).

Now consider
\[ \frac{d}{dz} (\pi_{A2} - \pi_{A1}) = p (1 - \rho) (1 - p) \left[ (1 - G (r_B)) - R' \frac{\partial r_B}{\partial z} \right], \]
which is strictly positive due to (44).

Finally, for \(s < L' (M + z) - 1\), both \(r_B\) and \(r_A\) are positive and interbank lending will never arise, even if \(p L' (R_{A0}) - (p^2 + \rho_1 p (1 - p)) L' (R_{B0}) < 0\). To see this, consider the point where
\[ (p^2 + \rho p (1 - p)) L' (R_{B0}) = p L' (R_{A0}), \]
and thus \(\pi_{A1} - \pi_{A0} = 0\). Since \(\frac{d \pi_{A0}}{dp} = 0\), it suffices to consider
\[ \frac{d \pi_{A1}}{dp} = p (1 - p) \left[ L (F_{B1}) - L (R_{B1}) \right] - p R_{A1} \frac{\partial r_{B1}}{\partial \rho} \]
\[ + \left[ (p^2 + \rho p (1 - p)) L' (F_{B1}) - p L' (F_{A1}) \right] \frac{\partial F_{B1}}{\partial \rho} \]
\[ - \left[ (p^2 + \rho p (1 - p)) L' (R_{B1}) - p (1 + r_{A1}) - R_{A1} \right] R' \frac{\partial s^*}{\partial \rho} \]
\[ = p (1 - p) w - p R_{A1} \frac{\partial r_{B1}}{\partial \rho}, \]
which is strictly negative at the point where \(w = 0\), since \(\frac{\partial r_{B1}}{\partial \rho} > 0\). The latter follows from implicit differentiation of the FOC for \(B\) which yields
\[ \frac{\partial r_{B1}}{\partial \rho} = [L'' (R_{B1}) R' - 1] \frac{\partial s^*}{\partial \rho}. \]

The derivative \(\frac{\partial s^*}{\partial \rho}\) is strictly negative, which follows from adding up the two first order conditions (41) giving
\[ y = R' s^* - (R_A - R_B) + R' [L' (R_A) - L' (R_B)] \]
\[ + R' \frac{1}{p} \left[ (p^2 + \rho p (1 - p)) L' (R_B) - p L' (R_A) \right] \]
\[ = 0, \]
and implicit differentiation, which yields
\[ \frac{\partial s^*}{\partial \rho} = \frac{L' (R_{B1})}{R' (1 - \rho) p (1 - p) L'' (R_{B1}) - 3p} < 0. \]
Q.E.D.

Proposition A2 thus confirms that our previous comparative analysis for the size of interbank lending is robust also to the operation of a market for retail deposits. This is intuitive given that the market for retail deposit does not provide an adequate substitute due to a combination of switching costs and the low granularity of individual deposits. Finally, also with regard to its regulatory implications, it is worthwhile to stress the following result:

**Proposition A3.** When interbank lending is feasible, then even though there is retail deposit competition it will always (weakly) increase the extent to which funds are reallocated between the two local markets and thereby improve efficiency. That is, when \( z > 0 \) and thus \( M_A > M_B \), interbank lending will reduce or even fully close the gap \( F_A - F_B > 0 \) that persists also under retail deposit competition.

**Proof of Corollary ??**. First, for

\[
\bar{s} \geq \frac{M + z}{M - z} [L'(M - z) - 1],
\]

there is no reallocation on the retail deposit market but interbank lending when \([p + \rho (1 - p)] > \frac{L'(M + z)}{L'(M - z)}\). Next, for

\[
L'(M + z) - 1 \leq \bar{s} < \frac{M + z}{M - z} [L'(M - z) - 1],
\]

there is there will be interbank lending when retail deposit competition without subsequent interbank lending leads to an allocation where \([p + \rho (1 - p)] > \frac{L'(R_A)}{L'(R_B)}\). Finally, for

\[
\bar{s} < L'(M + z) - 1,
\]

interbank lending is never optimal from the bank’s perspective and the allocation is not changed by introducing the potential for interbank lending. Q.E.D.

---

The preceding discussion as well as the proof of Proposition A2 entail in addition implications that relate directly to the operation of the retail deposit market, which we presently do not stress. For a given difference in the local deposit base, as captured by \( M_A - M_B = 2z > 0 \), the respective difference in attracted retail deposits, \( R_A - R_B < M_A - M_B \), is strictly higher when banks’ loan portfolios are less correlated (lower \( \rho \)). Interestingly, however, as the difference in the deposit base \( z \) increases, while interbank lending is always strictly increasing, this may not hold for the volume of retail deposits that bank \( B \) attracts in market \( A \).
Appendix C: Nash Bargaining over Interbank Lending

We will now relax the assumption that the creditor bank $A$ can make a take-it-or-leave-it offer to the debtor bank $B$ and suppose instead that the surplus generated by interbank lending is shared according to axiomatic Nash bargaining.

For this recall first that there is no asymmetric information, so that bargaining proceeds under common knowledge. When banks do not come to an agreement, we denote their respective outside options by $\pi_{n0}$, which are obtained by substituting $W = 0$ and $w = 0$ into (3). We next derive the bargaining (or Pareto) frontier. For some given (feasible) value of profits $\bar{\pi}_n'$ for bank $n'$ this entails finding the interbank loan that maximizes the other bank’s profits, $\pi_n$, subject to the constraint that $\pi_{n'} \geq \bar{\pi}_{n'}$. It is inessential which banks we choose as $n$ or $n'$. For specificity, suppose we maximize $\pi_A$. As is immediate, the constraint $\pi_B \geq \bar{\pi}_B$ will be binding so that the maximization problem gives rise to a function $\pi_A = \psi(\pi_B = \bar{\pi}_B).$ There are two cases to distinguish. In the first case, interbank lending is not optimal from the banks’ shareholders’ perspective, so that there does not exist a pair $(\pi_A, \pi_B)$ with $\pi_A \geq \pi_{A0}$ and $\pi_B \geq \pi_{B0}$, where at least one holds strictly. In the second case, such a pair exists. Then, if $\psi$ is concave (it is, in fact, linear, as we show below), the symmetric Nash solution is characterized as follows: The uniquely obtained solution $(\pi_A, \pi_B)$ maximizes the (symmetric) Nash product $[\psi(\pi_B - \pi_{A0})][\bar{\pi}_B - \pi_{B0}]$, which from the first-order condition is the case if

$$\frac{\psi(\pi_B) - \pi_{A0}}{\pi_B - \pi_{B0}} = -\psi'(\pi_B).$$  \hspace{1cm} (46)

The derivation of the Nash bargaining solution - or, more precisely, the derivation of the frontier $\pi_A = \psi(\pi_B)$ - is complicated by the following feature. The bargaining frontier does not have slope of minus one, as in the most standard case where risk-neutral players can simply make a fixed transfer ("transferable utility"). This results from the fact that a debtor bank can make its contractual repayment only if its own corporate loans perform. And even when a creditor bank receives such payment, it may go straight to depositors rather than banks’ shareholders when the bank becomes insolvent. It is now convenient to solve explicitly for $w$ from the binding constraint $\pi_B \geq \bar{\pi}_B$, so that in this case

$$w = L(R_B + W) - R_B(1 + r_B) - \bar{\pi}_B \frac{1}{p}. \hspace{1cm} (47)$$

Hence, in case there is a loan of size $W$ from bank $A$ to bank $B$, then the repayment $w$ as specified in (47) ensures that bank $B$’s profits are just equal to $\bar{\pi}_B$. Substituting for $w$,
$W \geq 0$ is then chosen so as to maximize

$$
\pi_A = \left[ p^2 + \rho p (1-p) \right] \left[ L(R_A - W) - R_A(1+r_A) + L(R_B + W) - R_B(1+r_B) - \frac{1}{p} \tilde{\pi}_B \right] 
+ p (1-p) (1-\rho) \left[ \max \left\{ 0, L(R_A - W) - R_A(1+r_A) \right\} 
+ \max \left\{ 0, L(R_B + W) - R_B(1+r_B) - \frac{1}{p} \tilde{\pi}_B - R_A(1+r_A) \right\} \right].
$$

Importantly, this implies that $\tilde{\pi}_B$ does not affect the optimal choice of $W$.

As a consequence, we have that under Nash bargaining the same optimal $W$ obtains as when one bank can make a take-it-or-leave-it offer. That the corresponding repayment level $w$, as given by (47), is different does not affect our results qualitatively (albeit it affects the thresholds for $z$ and $\rho$ got which Cases 1 and 2 apply in equilibrium).
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<thead>
<tr>
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<th>Recent Issues</th>
</tr>
</thead>
</table>
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<thead>
<tr>
<th>Year</th>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
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<tr>
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<tr>
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</tr>
</tbody>
</table>