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Working Paper — Digitized Version

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Seminar paper, Institute for International Economic Studies, University of Stockholm, No. 361

Provided in Cooperation with:
Kiel Institute for the World Economy (IfW)

Suggested Citation: Lindbeck, Assar; Snower, Dennis J. (1986) : Union activity and wage-employment movements, Seminar paper, Institute for International Economic Studies, University of Stockholm, No. 361, Institute for International Economic Studies, University of Stockholm, Stockholm

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Seminar Paper No. 361

UNION ACTIVITY AND WAGE-EMPLOYMENT MOVEMENTS

by

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Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.

September, 1986

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ABSTRACT

The paper analyzes how the influence of labor unions over wage contracts may make the labor market less "resilient". Loss of resilience is depicted in two conceptually independent ways: (i) a tendency for changes in unemployment caused by exogenous labor demand shock to persist, (ii) a long-run tendency of such shocks to generate a bounded wage-unemployment ratchet. In this manner, the analysis may help explain why unemployment rates in Europe and the United States have had an upward trend over the past one and a half decades and why the expansion of employment has been much greater in the U.S. than in most European economies after the recent deep recessions.
1. Introduction

The idea underlying this paper is quite simple. Consider a labor market containing firm-specific unions facing labor demand shocks which are transient (in the sense that the distribution of shocks has a constant mean and finite variance). After an adverse shock, firms reduce the size of their workforces (through dismissals or failure to replace retiring employees) and unemployment rises. The remaining incumbent employees are now in a better position than previously: since they are smaller in number but face the same distribution of shocks, their chances of retaining their jobs have risen. Now, acting through their unions, they respond to this enhanced job security by driving up their wages. However, the unemployed workers cannot underbid on account of labor turnover costs. At the new higher wages, the firms will employ fewer workers (for any given new labor demand shock) than they would otherwise have done. In this way, unions help perpetuate the initial rise in unemployment. We will explore how this unemployment persistence is related to the degree of union power and how unions may generate wage-unemployment ratchets.

A number of recent studies have presented various analytical frameworks in which unions (or, simply, workers with market power) may be responsible for unemployment persistence. Yet unlike the present paper, these studies do not show how unemployment persistence is related to the degree of union power; nor do they explore how union activity may give rise to wage-unemployment ratchets.

Our model of union behavior is an outgrowth of "insider-outsider analysis" (for a survey see Lindbeck and Snower 1986), which presumes that labor turnover costs give a group of privileged, employed workers ("insiders") more favorable

1. Gottfries and Horn (1986) and Blanchard and Summers (1986a) consider unemployment persistence in the context of a union whose members have uncertain employment prospects, whereas Lindbeck and Snower (1985) deal with persistence due to labor turnover costs arising from the discrimination of "outsiders" by "insiders" by way of non-cooperation and harassment activities of insiders (who may or may not be unionized). Horn (1983) shows how an expansion of the government sector leads to lower employment and higher wages in the private sector, but a government contraction leaves the private sector unable to absorb the layed-off workers in the public sector. Finally, Drazen (1985) considers how unemployment persistence can occur through the attrition of human capital in a recession.
conditions of work than the unprivileged, unemployed workers ("outsiders"). The insiders exercise market power on their own behalf in the process of wage determination.

Applying this approach to labor union activity, we distinguish between two types of insiders: union members and employed non-members. The former have some influence on wage determination, the latter do not. Our model has two salient, distinguishing features:

(i) firms must bear some labor turnover costs whenever they fire current employees and hire other workers instead, and

(ii) union members exert market power (generated by the above labor turnover costs) in wage determination, without taking other workers into account.

2. Wages, Employment, and Unemployment

For simplicity, we focus on a single firm whose insiders belong to a single union. Let work be a discrete activity, with each employee providing one unit of work. The firm's production function is $Q = \varepsilon \cdot f(L)$, $f' > 0$, $f'' < 0$, where $Q$ is output, $L$ is the number of employees (union members and employed non-members), and $\varepsilon$ is a random variable with a time invariant distribution ($G(\varepsilon)$) with zero mean and finite variance.

We assume that, in each period, labor market decisions are made in two stages. First, the wage ($W$) is set before the realized value of $\varepsilon$ is known (but with full information about $G(\varepsilon)$). Second, the employment decision is made after $\varepsilon$ is observed. We call a worker an "incumbent insider" if he is an employed union member in the current period before $\varepsilon$ is revealed. The wage is assumed to be the outcome of a Nash bargain between the firm and the union, which consists of incumbent insiders. The employment decision is assumed to be made unilaterally by the firm.
Consider the second stage of decision making first. Given the known values of \( W \) and \( \varepsilon \), the firm sets employment so as to maximize its profit: \( \varepsilon \cdot f(L) - W \cdot L \), which yields the labor demand equation

\[
L = \xi(W/\varepsilon), \quad \xi' < 0.
\]

Now turn to the first stage of decision making. Suppose that the incumbent insiders are risk neutral and that each of them faces the same probability of being retained by the firm. Moreover, suppose that the union is run by a majority voting rule and that the majority of union members are employed. Thus, in the first stage of decision making, the union's objective is to maximize an incumbent insider's expected utility.

For simplicity, suppose that if the worker is employed in the current period, he experiences utility of \( U(W) \), where \( U' > 0, U'' < 0 \); yet if he is fired, then his utility is zero. Let \( \sigma \) be the incumbent insider's expected probability of being retained. Then the union's objective is \( \sigma \cdot U(W) \) and its threat point is zero.

Let \( L^I \) be the number of incumbent insiders in the current period and let \( \xi(W/\varepsilon) = L^I \). Then \( \sigma \) may be defined as

\[
(2a) \quad \sigma = \frac{1}{\xi} \left[ \frac{L}{L^I} \right] \cdot G(\varepsilon) d\varepsilon + \int_\varepsilon^0 G(\varepsilon) d\varepsilon.
\]

Here we implicitly assume that the union is able to give its members an advantage over the non-members in retaining their jobs, so that when the firm fires incumbents, non-members are fired first.\(^3\) By implication,

\[
(2b) \quad \sigma = \sigma(L^I, W),
\]

\( (-) (-) \)

2. Note that the concavity of the utility function does not contradict our assumption of risk neutrality since \( W \) is not a random variable in our model.

3. At the opposite extreme, the union is unable to do so, and thus the retention probabilities of members and non-members are identical. In that case, \( L^I \) must be replaced by \( L^- \) (last period's total labor force) in equation (2a). This amendment does not affect our qualitative conclusions with regard to the unemployment persistence effect, but it does imply that there is no wage-employment ratchet.
for any density \( G(\varepsilon) \) which is strictly positive between a minimum and maximum value of \( \varepsilon \). (In other words, the more incumbents there are in the firm and the higher their wage, the lower is each incumbent's chance of being retained.)

The firm's objective in the wage bargain is to maximize its profit. If an agreement with the union is reached, whereby the wage \( W \) is accepted and no incumbent insiders are replaced by outsiders, the firm's expected profit is

\[
\pi(W) = \int_{-\infty}^{\infty} \left\{ \varepsilon \cdot f(\ell(W/\varepsilon)) - W \cdot f(\ell(W/\varepsilon)) \cdot G(\varepsilon) d\varepsilon \right\},
\]

where we assume that \( \pi', \pi'' < 0 \).

Let \( T \) be the firm's turnover cost (i.e. the cost of firing an incumbent insider and hiring an outsider instead), which we assume to be a constant. (For a microeconomic derivation of \( T \), see Lindbeck and Snower (1984 a and b, 1985)). Let \( R \) be the reservation wage (at which a worker is indifferent between employment and unemployment). Then, for simplicity, we specify the firm's threat-point profit as \( \pi = \pi(R + T) \). (The \( \pi(R+T) \) function may be interpreted as the profit which the firm earns upon replacing all its insiders by outsiders.)

Let \( B = \pi(W) - \pi(R + T) \) be the firm's objective in wage negotiation and let \( C = \sigma \cdot U(W) \) be the union's objective. Then the negotiated wage may be expressed as the solution to the following generalized Nash bargaining problem:

\[
\text{Max } \Omega = B^a \cdot C^{1-a}, \text{ subject to } W \geq R, \pi(W) \geq \pi(R+T), \pi(W) \geq 0,
\]

4. We do not exclude the possibility that these minimum and maximum values of \( \varepsilon \) may be \(-\infty\) and \(+\infty\), respectively.

5. In general, the firm's costs of altering its workforce may be divided into two categories: (i) the cost of replacing current employees by outsiders and (ii) the cost of expanding or contracting the workforce. Per employee, the former costs (which may involve litigation costs, severance pay, and a drop in the morale of the remaining employees) are usually much greater than the latter (pertaining to temporary layoff and subsequent recall). Our analysis forces attention on the former costs (as a threat which the union uses in wage negotiation) and ignores the latter.

6. We assume all workers to have the same utility function and thus the same reservation wage.
where \( R \) and \( T \) are exogenously given to both negotiating parties, and the constant \( a (0 < a < 1) \) measures the (exogenously given) bargaining strength of the firm relative to that of the union. Note that the turnover cost poses a threat to the firm, without which the union would have no bargaining power. The first constraint of problem (4a) ensures that union members prefer employment to unemployment; the second and third constraints ensure that the firm has no incentive to replace its insiders by outsiders or to close down its operations, respectively. By (3), it is evident that the second constraint implies that

\[
W < R + T.
\]

The first-order condition for an interior solution is

\[
A = C_w + \delta \cdot (C/B) \cdot B_w = 0
\]

where \( \delta = a/(1-a) \). From this condition, along with some restrictions on the density \( G \) and the production function \( f \) (see the appendix), we can show that the wage depends on the number of incumbent insiders \( L^I \) and on \( (R + T) \) in the following way:

\[
W = \phi (L^I, R + T)
\]

for values of \( W \) in the range

\[
R < W < \min[R+T, \pi^{-1}(0)] = W_{\text{max}}.
\]

(with \( \pi^{-1}(0) \) given by (3)). The larger the number of incumbent insiders, the lower the retention probability, and thus the lower the wage is set. Also, the greater \( (R + T) \), the lower the firm's threat-point profit, and the higher the wage.

Having analyzed wage formation, we now turn to the determinants of the firm's current incumbent workforce, \( L^I \). Let \( r \) be the retirement rate (a positive constant), so that \( r \cdot L_{-1}^I \) of last period's incumbent insiders retire. Let \( h[(1-r) \cdot (L_{-1} - L^I_{-1})] \) be the "entry-exit function", which describes how many of the firm's non-retired, employed non-members \((1-r) \cdot (L_{-1} - L^I_{-1}) \) when \( L_{-1} > L^I_{-1} \) become
union members or how many of the non-retired insiders who have been dismissed

\((1-\tau) \cdot (L_{-1} - L_{-1})\) when \(L_{-1} < L_{-1}^I\) exit from the union. Then the current
incumbent insider workforce is

\[
L^I = (1-\tau) \cdot L_{-1}^I + h[(1-\tau) \cdot (L_{-1} - L_{-1}^I)].
\]

The entry-exit function has the following properties:

\[ h = 0 \text{ if } L^I = L_{-1}^I; \]

\[ h = 1 \text{ if } L_{-1} < L^I; \]

(i.e. when incumbent insiders are dismissed, they lose their influence in wage
determination, since – as noted – union behavior is determined by majority vote, with the majority consisting of the employed members), and

\[ 0 < h < 1 \text{ if } L_{-1} > L^I \]

(i.e. a fraction of the non-retired, employed non-members enter the union). 7,8

In short, our model of the labor market consists of the labor demand function
(1) (pictured in Figure 1a), the wage determination conditions (5a) and (5b)
(pictured in Figure 1c), and entry-exit function (6) (pictured in Figure 1d), which
specifies the incumbent insider workforce. To characterize the labor market
equilibrium in a particularly simple way, we make the following assumptions. First,
we suppose that the labor market contains a fixed number of identical firms, union
members, employed non-members, and outsiders. Then the wage-employment activity
within an individual firm may be seen as a microcosm of that for the entire labor
market. Second, we assume that the parameters of our model are such that, for any
given value of \(\varepsilon\), there exists a unique, stable, stationary equilibrium \((W, \hat{L},
\hat{L}^I)\), where \(\hat{L}^I = \hat{L}_{+1}\) in the figure. Finally, we suppose that each realization

7. How many of such workers join the union in the real world depends on laws,
social norms, transactions costs, inertia in non-members' behavior - all of which
lie beyond the influence of the union itself (and beyond the scope of this paper).

8. Note two extreme cases. On the one hand, there is "free entry", where each
of last period's employed non-members becomes a union member in the current period
if he retains his job. Here, \(h = 1\) for all \((L_{-1} - L^I)\), so that \(L^I = (1-\tau) \cdot L_{-1}\).
On the other hand, there is the case of "no entry", where employed non-members have
no opportunity of joining the union. Here \(h = 0\) for \(L_{-1} - L^I > 0\), so that \(L^I =
(1-\tau) \cdot L_{-1}\) over this range.
FIGURE 1  Labor Market Equilibrium and Unemployment Persistence Effect
of $\varepsilon$ persists for long enough so that this equilibrium is reached. Such an equilibrium is illustrated by the points $E_1$ in Figures 1.

3. The Influence of Union Power on Economic Resilience

We now show how the exercise of union power in wage bargaining may make the labor market less "resilient" in the face of cyclical swings in employment. We specify this loss of resilience in terms of

(a) an "unemployment persistence effect", whereby the union influences the wage in such a way that any random variation in current labor demand tends to create a persistent change in unemployment (e.g. an adverse random variation means that the future employment will be lower than it otherwise would have been, ceteris paribus) and

(b) a "wage-unemployment ratchet effect", whereby random variations in labor demand through time lead to an upward trend in the wage and unemployment rates (because favorable variations lead to larger wage changes, per unit of employment, than unfavorable variations).

Consider the unemployment persistence effect first. Assume that given the level of $\varepsilon$, the initial equilibrium wage ($W$) lies strictly between the upper and lower bounds given in condition (5b). Now consider what happens when there is a transient adverse shock to labor demand - generated by a fall in $\varepsilon$ - after the current wage $\hat{W}$ has been negotiated. In other words, the labor demand curve in Figure 1a shifts downwards, so that for the current wage ($\hat{W}$), current employment ($\hat{L}$) is lower (as shown by point $E_1'$ in Figure 1a). Assuming that $h > 0$ (so that the right-hand segment of the entry-exit function in Figure 1c has a positive slope), the current incumbent insider workforce ($L_I$) falls (as shown by point $E_2'$ in Figure 1c). Since a fall in the incumbent workforce (under the same distribution of employment shocks, $\varepsilon$, as before) raises each incumbent's retention probability, the union negotiates a higher wage. Thus, the wage rises above $\hat{W}$, as shown by point $E_2$ in Figure 1d (where the new wage is less than $W_{\max}$). The wage increase discourages the firm from employing as many workers as it would otherwise have done.
Thus, for any given $\epsilon_{+1}$, current employment will be lower than it would otherwise have been.

Given that the economy has $n$ workers and $m$ firms and that the negotiated wage ($W$) exceeds the reservation wage ($R$), the level of involuntary unemployment is $(n - m L)$. Then the argument above implies that once an employment slump occurs, the wage-setting activity of unions tends to make it persist, provided that $h > 0$ and $W < W_{\text{max}}$.

Note that once the wage hits $W_{\text{max}}$ (given by (5b)), the unemployment persistence effect disappears, in the sense that no further adverse shock in employment leads to a rise in the wage. (The reason is, of course, that if the union would allow the wage exceed this maximum level, the firm would respond either by replacing the insiders by outsiders or by closing down.) In other words, the unemployment persistence effect is bounded from above.

We now consider the influence of union bargaining power on the magnitude of this unemployment persistence effect. In our analysis, a rise in such power is mirrored in (i) a fall in $a$ (and hence $\delta$) in Equation 4c (reflecting a fall in the bargaining strength of the firm relative to that of the union) and/or (ii) a rise

---

9. Of course, insider market power is not the only conceivable rationale for such an effect. Other rationales include the depreciation of human and non-human capital during prolonged periods of unemployment, changes in workers' tastes and job search behavior over such periods (in particular, an increased preference for leisure relative to work and a loss of self-confidence in job search).

10. Observe that this result is superficially similar to that of Blanchard and Summers (1986) in this volume. However, their unemployment persistence rests on a fundamentally different relation between wages and labor demand: in our model, unions may be responsible for "excessive" real wages and "deficient" employment due to production processes characterized by diminishing returns to labor; in their model, unions may give rise to excessive nominal wages, implying excessive product prices and thereby leading to deficient product demand and thus to a deficient derived demand for labor.

11. Observe furthermore that if $h < 1$ when $L_{-1} > L_{-1}^T$ and if adverse employment swings lead to shrinkage of the labor force while favorable swings lead to net hiring, then the unemployment persistence effect tends to be weaker in a boom than in a slump. Blanchard and Summers (1986a) find empirical confirmation of this phenomenon.
in the firm's labor turnover cost, $T$.  

It can be shown that each of these phenomena not only raises the wage ($W$), but makes the wage more responsive to changes in the incumbent insider workforce ($L^I$) (i.e. each increases the absolute value of $\frac{\partial W}{\partial L^I}$ in Equation 5) and thereby augments the unemployment persistence effect. This proposition is proved formally in the Appendix. Intuitively, when $\delta$ falls or $T$ rises in the bargaining solution (4c), a given drop in $L^I$ calls forth a larger rise in $W$, because now the firm's loss from a wage increase is weighted less heavily (relative to the union's associated gain) in the Nashian objective function. By implication, the $W-L^I$ schedule in Figure 1c becomes flatter. Thus, when there is an adverse employment shock, the resulting wage increase and the associated employment decrease are larger than they would otherwise have been. In this way a rise in union power strengthens the unemployment persistence effect.

Figures 1 also indicate how union influence on wage bargaining may generate a "wage-employment ratchet effect". In order for this effect to operate, it is necessary that there is less than "free entry" (i.e. $h < 1$) and that the random variations in labor demand are "large" relative to the incumbent insider workforce, i.e. adverse swings cause $L_{-1} < L_{-1}^I$ and favorable swings cause $L_{-1} > L_{-1}^I$.

To see this, observe that when $h < 1$, the left-hand branch of the entry-exit locus in Figure 1d is steeper (45 degrees) than the right-hand branch. In other words, the insiders who are dismissed all relinquish their influence on wage determination (since the union is assumed only to represent the interests of employed members), but the entrants who are hired do not all gain influence on wage determination (because when $h < 1$, some entrants do not promptly join the union). Consequently, the random variations in labor demand lead to a downward trend in the incumbent insider workforce and, by implication, an upward trend in the wage and unemployment rates. The greater union power (viz. the lower $\delta$ or the greater $T$) and the smaller

---

12. Note that the degree of union power, as defined here, does not necessarily have anything to do with the degree of centralization in bargaining.

13. As the Appendix shows, these are not the only channels whereby a fall in $\delta$ or a rise in $T$ may influence the relation between $W$ and $L^I$. Suffice it to say that the other channels pull in the same direction.
rates. The greater union power (viz. the lower $\delta$ or the greater $T$) and the smaller $h$, the larger this wage-employment ratchet effect, ceteris paribus. The ratchet is bounded from above by $W < W^{\max}$. Once the wage hits $W^{\max}$, adverse swings reduce employment while the wage remains rigid.

4. Concluding Remarks

Our analysis indicates that (a) unions' power over wages may, to some extent, hinder an economy in recovering from a recession and (b) the greater the unions' power (as reflected in their bargaining strength and the magnitude of labor turnover costs), the bleaker the economy's recovery prospects may become. In this light, the more widespread and intensive influence of unions in Europe than in the United States may help explain the drastically different product wage trajectories in these two parts of the world and Europe's comparative lack of success in reducing its unemployment after the recession of the early 1980's.\(^{14,15}\) Thus, our analysis may help explain why unemployment rates in Europe and the US have had an upward trend over the past one and a half decade. It also provides some microeconomic underpinning for the notion that European unemployment is more closely related to "excessive wages" than American unemployment.

\(^{14}\) The same may be said of sectors within these economies. For example, unions play a comparative important role in wage determination of the steel and automobile industries in the U.S: and it has been these industries that have witnessed relatively low employment rates.

\(^{15}\) It is worth noting that the unemployment persistence effect also works in reverse: union wage setting tends to perpetuate favorable random variations in employment, and the stronger the unions are, the more pronounced this effect will be. Thus, it may be argued that whereas a rise in union power generally leads to higher wages and lower employment (both in comparative static terms and via the bounded ratchet effect), union wage setting is more harmful in a recession than in a boom on account of the unemployment persistence effect.
REFERENCES


APPENDIX

The effect of the incumbent workforce on the wage:

Rewriting Equation (4c).

\[ A = \left[ \sigma \cdot U' + \sigma_L \cdot \ell' \cdot U \right] + \delta \cdot (\sigma \cdot U/B) \cdot \pi'(W) = 0. \]

In order for the second-order condition for optimality to be fulfilled, we need to assume that \([(\sigma_L \cdot \ell' / \sigma_L) - (\ell'' / \ell')]\) exceeds some negative critical value. For the sake of algebraic simplicity below, however, we suppose that \(\ell'' = 0\), \(\sigma_L < 0\), and \(\sigma_L L = 0\). These conditions may be derived by imposing the appropriate restrictions on the density \(G\) and the production function \(f\).

\[ \left( \frac{\partial W}{\partial L} \right)_{A=0} = \frac{-A_L}{A_W}, \quad \text{where} \]
\[ A_W = \left( \frac{\partial A}{\partial W} \right) \]
\[ = -\ell' \cdot [\sigma \cdot U' + \sigma_L \cdot \ell' \cdot U + \delta \cdot \pi'(W) \cdot \sigma \cdot U/B] \]
\[ + \sigma \cdot U'' + \sigma_L \cdot \ell' \cdot U' + (\delta \cdot \sigma/B) \cdot \left[ \pi'(W) \cdot U' - (\pi'(W) \cdot U/B) - U \cdot \pi''(W) \right] \]

which must be negative in order for the second-order condition for optimality to be fulfilled. We assume that \(\sigma_L < 0\) to ensure this; furthermore, assuming \(\sigma_L L = 0\) for simplicity,

\[ A_L = \left( \frac{\partial A}{\partial L} \right) = \sigma_L \cdot \left[ U' + (\delta \cdot U \cdot \pi'(W))/B \right] < 0 \]

Thus, \(\left( \frac{\partial W}{\partial L} \right)_{A=0} < 0.\)
The effect of \((R + T)\) on the wage:

\[
\left[ \frac{\partial W}{\partial (R + T)} \right]_{A=0} = - A_{R+T}/A_W, \text{ where}
\]

\[
A_{R+T} = \left[ \frac{\partial A}{\partial (R+T)} \right] = - \delta \cdot (\sigma \cdot Y/B^2) \cdot \pi'(W) \cdot \left[ \frac{\partial B}{\partial (R + T)} \right] > 0.
\]

Thus, \(\left[ \frac{\partial W}{\partial (R + T)} \right]_{A=0} > 0.\)

The effect of \(\delta\) on the wage:

\[
\left[ \frac{\partial W}{\partial \delta} \right]_{A=0} = - (\sigma \cdot U \cdot \pi'(W))/B \cdot A_W < 0.
\]

The effect of \(T\) on the responsiveness of \(W\) to \(L^1:\)

Note that

\[
\left( \frac{A_W}{A_L} \right) = - \xi' + (\xi/A_L) \equiv D, \text{ where}
\]

\[
\xi = \sigma \cdot U'' + \sigma' \cdot L' \cdot U' + (\delta \cdot \sigma / B) \cdot [\pi'(W) \cdot U' - (\pi'(W) \cdot U/B) - U \cdot \pi''(W)].
\]

Then

\[
\frac{\partial D}{\partial \delta} = \frac{\xi_{\delta} \cdot A_L - A_L \delta \cdot \xi}{(A_L)^2}, \text{ which is positive because}
\]

\[
A_L \delta = \sigma_L \cdot \pi'(W) \cdot U/B > 0,
\]

\[
\xi_{\delta} = (\pi'(W) \cdot \sigma / B^2) \cdot [U' \cdot B - \pi'(W) \cdot U] + (\sigma \cdot U \cdot \pi''(W)/B) < 0
\]

and thus

\[
\xi_{\delta} \cdot A_L - A_L \delta \cdot \xi = \xi_{\delta} \cdot \left[ \sigma_L \cdot U' - \sigma'' \cdot L' \cdot U \right] - A_L \delta \cdot \left[ \sigma \cdot U'' + \sigma' \cdot L' \cdot U' \right] > 0
\]

Hence \(\left[ \frac{\partial W}{\partial L^1} \right]_{A=0} > 0.\)