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Yong Jin Kim and Jong-Wha Lee

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Abstract

This paper presents a theoretical model to analyze the effects of technology change on growth rates of income and human capital in the uncertain environments of technology. The uncertainty comes from two sources; the possibility of a technology advance and the characteristics of new technologies. We set up an overlapping generations model in which young agents invest in both width and depth of human capital in order to adopt new technologies. The model develops explicitly the micro-mechanism of the role of human capital in adopting new technologies as well as that of the process of human capital production in uncertain environments. In our model, a higher level for width of human capital relative to the level of depth leads one country to a higher growth path. We also show that an economy can have different growth paths depending on the initial structure of human capital and the uncertainty about the nature of new technologies. In particular, new technologies with more uncertain characteristics may adversely affect human capital accumulation and income growth, leading the economy to a low growth trap.

I. Introduction

In this paper, we investigate how the arrival of new technology shocks influences investments in human capital, and how long-term growth is determined by adopting new technologies in the uncertain environments of technology. The uncertainty comes from two sources; the possibility of a technology advance and the characteristics of new technologies. We explore these questions by explicitly modeling the micro-mechanism of human capital accumulation and technology adoption in the framework of the two dimensions of width and depth of human capital in the uncertain environments.¹

Several theoretical papers have described the mechanism of adoption of new technologies focusing on its certain features. For example, in Chari and Hopenhayn (1991), agents working with technology that is one level lower than a new technology can automatically adopt a new technology one period later. In a similar context, in Lloyd-Ellis (1999), the absorptive capacity increases continuously at an exogenously given constant rate. In other words, agents can adopt an increasingly broader spectrum of technologies automatically over time. Also in Caselli (1999), the adoption cost is a function of agents' ability distributed following a particular distribution function given exogenously. In Lucas (1993) and Parente and Prescott (1994), adoption of new technologies depreciates a constant fraction of agents' specific skills. And in Galor and Moav (2000), to lower the adoption cost and to increase the effect of ability, agents should spend an exogenously given constant fraction of their endowed time for schooling.

While in most of the papers cited above the adoption cost is virtually determined exogenously without its specific mechanism, this paper explicitly models the micro-mechanism of the role of human capital in technology adoption utilizing the framework

¹ The basic idea on this role of human capital in technology adoption comes from Schultz (1963, 71); Nelson and Phelps (1966); and Welch (1970). They argue that education increases agents' adaptability or flexibility to changing and uncertain socioeconomic environments. In other words, education can lower the cost related to information gathering and processing, labor mobility, technology adoption, and others. And, in a slightly different context, focusing on characterizing different mechanisms of economic growth depending on technology adoption or on creation, Vandenbussche, Aghion, and Meghir (2006) and Aghion and Howitt (2006) distinguish the role of human capital in the dimension of primary/secondary and tertiary education. They specifically assume that primary/secondary education tends to produce imitators, while tertiary (especially graduate) education tends to produce innovators. However, mainly to focus on a micro-mechanism of technology adoption, we distinguish human capital in the dimension of width and depth (quantity and quality of schooling).

of width and depth of human capital.² In the model, width of human capital determines the cost of technology adoption. More specifically, width of human capital represents the number of various specific knowledge points that human capital contains. Because each knowledge point helps to decipher and adopt a new technology at a lower cost if they are more closely related, wider human capital structure lowers the expected cost of adopting future technologies. The key idea of the micro-mechanism is that the more closely one agent's acquired knowledge is related to the knowledge needed to adopt a new technology (or to the new knowledge to be created), the less time the agent spends in adopting the technology (or in creating the knowledge). If the number of knowledge points acquired increase proportionally in the length of schooling period, then width of human capital can be proxied by "average years of schooling".

In contrast, depth of human capital determines the level of specific skill that helps operate the new technology to be adopted. In other words, higher quality of knowledge, accumulated when young, enables old agents to adopt higher level of specific skill (or blueprints) to run the adopted new technology. When an agent holds a higher level of depth of human capital regarding a specific type of knowledge, he/she acquires an ability to understand and operate that specific type of a new technology to a higher level. The adoption cost includes the cost of specific skill formation. Depth of human capital can be proxied by "annual expenditures on education for a student as a fraction of gross domestic product (GDP)".

The model is an overlapping generations model where human capital plays an essential role in adopting new technologies. We assume that agents, when young, make investments in width and depth of human capital, and old agents adopt new technologies by using these two dimensions of human capital that they have accumulated when young. In the model, technology adoption is endogenously determined by the expected cost of technology adoption and the uncertainty related to technology shocks. For example, if the adoption cost is low, or if the level of uncertainty related to future technology shocks is small (thus, they can make focused investment on narrow spectrum of knowledge points to adopt a future technology), agents make more investment in human capital when young, leading to adopting all the new technologies when old. In contrast, if not, then young agents make smaller investment in human capital, resulting in lower equilibrium growth rates of human capital and income.³

² Recent empirical studies including Benhabib and Spiegel (1994), Borensztein et al. (1998), and Caselli and Coleman (2001) show the importance of the technology-adopting role of education in the framework of cross-country regressions. There are also empirical studies that show technology-skill complementarity; that is, technology progress changes the relative demand for skill toward skilled and educated workers, and hence increases investments for human capital. See Bartel and Lichtenberg (1987); Autor, Katz, and Krueger (1998); Bartel and Sicherman (1998); and Acemoglu and Zilibotti (2001).

³ Some earlier studies have explored the effects of uncertainty on the investment for education, considering that higher rates of technological changes are in general associated with greater uncertainty about the characteristics of future technologies. In this context, Lavhari and Weiss (1974), Paroush (1976), and Eaton and Rosen (1980) show that uncertainty has an ambiguous effect on human capital investments. Increased uncertainty in future earnings will decrease demand for education under the standard assumption of risk aversion of workers. But agents can also increase investment in human capital that can help facilitate adjustments to future shocks.

The model presents several interesting implications about the interactions among technological change, human capital accumulation, and income growth. We show that an economy with a higher ratio of width to depth of human capital exhibits a higher growth path. It is because greater width decreases the technology adoption cost, resulting in more frequent technology adoptions, higher investment in human capital, and higher growth. We also show that higher probability of technology shocks increases investment in human capital and thus growth rates of income and human capital in the economy.

The economy can have different growth paths depending on the initial structure of human capital stock. Specifically, the condition that one country's ratio of width to depth of human capital is above a certain threshold ratio, which increases in the uncertainty and speed of new technology shocks, leads to the following equilibrium. In this equilibrium, the agents always increase investment in human capital and adopt the new technology, resulting in higher growth rates of income and human capital due to the lower adoption cost. The economy follows a sustained balanced long-run growth path. In contrast, if the ratio is below the threshold ratio, leading to the higher adoption cost, the economy shows decelerating growth rates of human capital and income over time, resulting in a poverty trap with no human capital accumulation and no technology adoption.⁴

This implies that increased inflows of uncertain new technologies whose uncertainty increases expected adoption cost can have an adverse effect on human capital accumulation and income growth, leading the economy to a low growth trap. In this context, our model implies that the current technological progress in the information and communication sector may not necessarily reduce the gap in income among countries. It is because global technology advances would not provide benefits for the countries that lack adequate human capabilities, and because the human capital structure differs substantially across countries. This can happen because the information and communication technology development increases not only the probability of having an advanced technology but also the level of the uncertainty about the characteristics of new technologies.

This paper is organized as follows. Section II describes and solves the basic model of certain technology adoption. In this economy, agents adopt all the new technologies whenever they occur. In Section III, a more general model of technology adoption, in

⁴ Redding (1996) also shows the possibility of multiple growth paths in a framework similar to ours. He assumes that both forms of investments in human capital and technology (research and development [R&D]) exhibit pecuniary externalities and are strategic complements. In his model, the incentives to invest in each are interdependent, and thus multiple equilibria exist: an economy can be trapped in either a "low-education, low-technology" or a "high-education, high-technology" trap. Some studies also point out that technology advance can cause temporary economic recession, although contributing to growth in the long run. For example, Helpman and Rangel (1998) show that a recession is unavoidable with technology change when human capital of experienced workers becomes obsolete as they move from the old sector to the new sector. Galor and Weil (2000) also show that multiple steady-states can arise from the positive interaction between education and technology. Azarizdis and Drazen (1990) and Galor and Tsiddon (1991) also show that the multiple balanced-growth paths can exist. But the presence of multiple balanced-growth paths is derived based on ad-hoc specification of human capital investment technology, for example, the presence of "threshold externalities".

which technology adoption is endogenous, is presented. In this economy agents adopt new technologies only when it is profitable. The possible existence of multiple equilibria of the model is also analyzed. Section IV concludes.

II. The Basic Model

This section describes technology and environments of the economy, and the formal maximization problem of a representative agent living for two periods in an overlapping generations setting. In this model, we assume that agents adopt new technologies whenever they are available to simplify the problem. This basic model serves as a stepping stone to the extended model, presented in the next section, with a more realistic assumption that agents can determine whether to adopt new technologies or not.

A. Technology and Environments of the Economy

The economy is described by an overlapping generations model with a linear technology with only one input of specific skill producing one type of final good. The model economy consists of identical agents living for two periods, young and old. To maximize their utility, they decide how to allocate their endowed two units of time (one unit for each period) between work and education when young, and between work and technology adoption when old. Old agents adopt a new technology when a technology shock occurs.⁵ They spend a certain fraction of the endowed one unit of time to adopt the new technology and also a specific skill to run it, utilizing human capital stock they have accumulated through education when young. The rest of the time will be used to earn wage income. However, without a technology shock, old agents use all the endowment of time and human capital stock only for production and wage income.

Human capital structure (or knowledge structure) is assumed to consist of two dimensions: width and depth of human capital. The width of human capital represents flexibility and adaptability, and the feature of the allocative role of general human capital in adopting a new technology. In contrast, the depth of human capital, accumulated when agents are young, measures the level of each specific knowledge point that human capital stock contains. Thus, it will determine the level of old agents' specific skill to be formed for the operation of a new technology, when they adopt it.

⁵ In this basic model, all the new technologies are adopted whenever they occur irrespective of their profitability. And to make the model time-consistent, we will impose certain adoption restrictions on the values of parameters of the model such that the second period utility with the adoption is higher than without it with these parameter values satisfying this restriction. In Section III, we present a more general model in which agents maximize their utility by considering that they have the option of adopting new technologies depending on their profitability in the second period.

1. Technology and Adoption Cost

In this economy, a new and advanced technology with a specific knowledge point is assumed to occur with a probability of P in each period.⁶ The characteristics of a new technology are described by a point on a continuous knowledge space of the real line $[0, S]$. Each point in the knowledge space represents a specific knowledge to understand and run the specific new technology with this characteristic. We assume that the characteristic of the future technology follows a uniform distribution over the support of $[0, S]$ to simplify the problem. We also assume that the level of a new technology improves at a rate of g_T compared to the previous technology as $A_t = A_{t-1} (1 + g_T)$. Thus, with the adoption of a new technology $A_t = A_{t-1} (1 + g_T)$, and without it $A_t = A_{t-1}$.

We denote the level of width of education as N_t representing a set of N units of different knowledge points in the knowledge space $[0, S]$. To adopt a new technology, an agent uses the knowledge that is most closely related to the characteristics of this technology. The larger the width of the accumulated knowledge, the more easily this new technology can be understood and adopted (i.e., specific skill to run the new technology is easily adopted). The wider human capital implies to a lower adoption cost because the characteristics of a new technology is more likely located at a point closer to the knowledge points already accumulated.

Adopting the specific skill (H) to operate a new technology whose knowledge point is located at x , old agents spend the adoption time of

$$I_A = a|x - s| \cdot H, \quad (1)$$

where s denotes the location of the knowledge that an agent uses to adopt a technology with a knowledge point $x \in [0, S]$, and is located closest to the point x among the agent's N_t number of invested knowledge points.

Because the depth of human capital (Q) determines the level of specific skill (H) to operate the new technology $A (1 + g_T)$ to be adopted, equation (1) can be expressed in Q , instead of in H :⁷

$$I_A = a|x - s| \cdot Q. \quad (1)'$$

⁶ The probability P is given exogenously in the model. However, the model endogenizes the magnitude of a technology shock, represented by the depth of human capital, instead of its probability. P can represent the probability of success in adopting technologies due to limited information. In the case of the R&D model, P represents the probability of success of R&D investments.

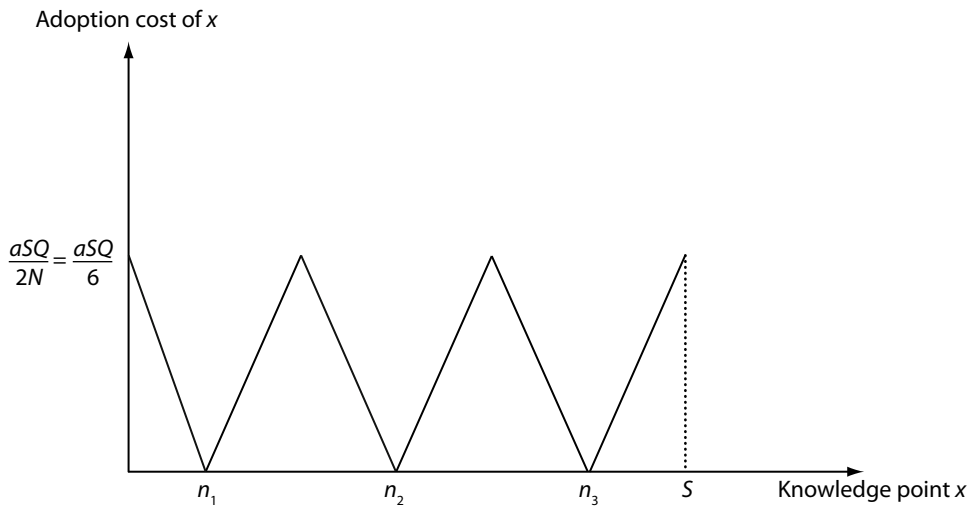
⁷ For this to hold, we need the following two assumptions. First, the level of old agents' adoption of a specific skill for a new technology is restricted by the level of depth of human capital, accumulated when young, as $H \leq Q$. Second, old agents adopt specific skills to the maximum level that the depth of human capital allows when they decide to adopt the new technology. Thus, $H = Q$.

This specification of adoption cost implies that to form the higher level of specific skill to operate a new technology, agents should pay the higher adoption cost. And the adoption time cost increases proportionally to the distance between two knowledge points (x and s). Here, this occurs because this distance represents the degree of similarity between these two pieces of knowledge.

To minimize the expected adoption cost of forming a specific skill for a new technology, the N knowledge points must be equally distributed over the knowledge space (technology space) as in Figure 1. Figure 1 depicts the relationship between the adoption cost and the location of the characteristics (knowledge point) of a new technology represented by x , when $N = 3$. $N = 3$ implies that agents invested in three knowledge points at n_1 , n_2 , and n_3 on the knowledge space $[0, S]$, which are

located at $\frac{S}{6}$, $\frac{3S}{6}$ and $\frac{5S}{6}$, respectively.⁸ More generally, $n_i = \frac{(2i-1)S}{2N}$, where $i = 1, 2, \dots, N$.

Figure 1: Relationship between Adoption Cost and Knowledge Point



2. Human Capital Accumulation

The level of specific skill of the old agents of generation $t-1$ at time t (equal to the depth of human capital accumulated when young at time $t-1$; Q_{t-1}) is given by

$H_{ot} = Q_{t-1}$ with the adoption, and

$= \delta \bar{Q}$ without it.

(2)

⁸ This modeling technique for technology adoption is similar to the one in Eaton and Schmitt (1994).

where the bar on the variable denotes belonging to “old agents of the previous generation” (i.e., of generation $t-2$). This equation implies that the old agent forms a specific skill of the level Q by adopting a new technology, and uses it for production.

We also assume that a certain fraction of the specific skill of the old in generation $t-1$ can also be learned by young agents of generation t without cost due to the spillover effect.⁹ Thus, the level of specific skill of the young of generation t at time t becomes:

$$H_{yt} = \delta H_{ot}, \quad (3)$$

where H_{ot} denotes the level of specific skill of the old of generation $t-1$ at time t , and δ measures the spillover effect, $0 < \delta < 1$.

A young agent, by investing a certain fraction of time in education, accumulates human capital for the second period. As already emphasized above, human capital has two dimensions, width (N) and depth (Q). We assume that, after observing whether the old agents of generation $t-1$ adopt a new technology at time t or not, the young agent of generation t invests an amount of time l_{Et} in education and accumulates human capital of $N_t Q_t$ by

$$N_t Q_t = b \cdot l_{Et} \cdot \bar{N} \bar{Q}, \quad (4)$$

where the bar on the variable denotes belonging to “old agents of the previous generation” (i.e., of generation $t-1$), and the parameter b , decreasing in S , measures the efficiency of human capital formation.¹⁰ And $b > 1$ implies that human capital stock can increase over time if the agent invests a certain fraction of her time in education such that $b \cdot l_{Et} > 1$.

This equation of human capital production implies that the more human capital old agents of the previous generation have, the more human capital young agents of the current generation can accumulate with any additional input of time investment in education. It also implies that because the agent cannot increase both N and Q simultaneously with a given time investment in education, there exists a tradeoff between N and Q .¹¹ Without

⁹ We assume that young and old agents work together in the workplace in which the technology spillover occurs.

¹⁰ See Appendix 1 for a micro-mechanism behind this equation of human capital formation through education.

Following Appendix 1, $b \cong \frac{2\delta}{k(1+e+e\delta)S}$. Note that the parameter b measuring the efficiency of human capital formation decreases in S .

¹¹ We can relate NQ to the two components of educational stock—quantity and quality of schooling (see Barro and Lee 1996). Let us take an example of the elementary, high school, and college educational system. Elementary school teaches students, for example, that knowledge is located at $1/2$ on the knowledge space $[0,1]$, while high school teaches the knowledge at $1/4$ and at $3/4$ on $[0,1]$. College teaches the knowledge at $1/8$, at $3/8$, at $5/8$, and at $7/8$. In other words, as the level of education increases, the distance between adjacent knowledge points that students acquired becomes smaller and smaller (i.e., the knowledge points becomes more finely distributed on $[0,1]$). In this context, an increase in the education level (years of schooling) increases N (the number of knowledge points). In this model, Q represents the depth of each piece of knowledge. Therefore, Q can be interpreted as

time investment in education, human capital stock cannot grow over generations in this economy.

3. Production Technology

The old agent is endowed with one unit of time and allocates it among technology adoption (l_{At}), and work ($1-l_{At}$) at time t . Young agents invest l_{Et} in education to accumulate human capital stock for their second period and work for the rest of the time ($1-l_{Et}$) at time t .

The representative firm employs young and old workers together, with the production function of

$$Y_t = A_t(H_{yt} \cdot (1-l_{Et}) + H_{ot} \cdot (1-l_{At})), \quad (5)$$

where $A_t = A_{t-1}(1+g_T)$ and $l_{At} = \min_{x \in \{N\text{-elements}\}} a|x-s| \cdot Q_{t-1}$ when the old of generation $t-1$ adopt a new technology at time t , while $A_t = A_{t-1}$ and $l_{At} = 0$ when they do not. Note, however, that in this basic model, we assume that old agents adopt new technologies whenever technology shocks arrive.

Here, we assume a linear production technology, which uses specific skills as the only input. We also assume that the competitive wage rate per one unit of labor supply adjusted by the level of specific skill is determined at the level of technology A_t . Note that with the adoption of a new technology $A_t = A_{t-1}(1+g_T)$, and without it $A_t = A_{t-1}$. This production function also implies that young and old agents' specific skills are perfect substitutes for each other. This simple assumption enables us to focus on main features of the model without excessive complexities.

B. Equilibrium

This subsection characterizes the equilibrium of the model economy. Recall that we assume in this basic model that once a new technology shock occurs in the second period, the agents will always adopt the new technology (i.e., form the new specific skill to operate the new technology). We assume that the utility of the second period with the technology adoption is always higher than that without by imposing time-consistent restrictions on the parameter values of the model so that the new technology is always adopted.¹²

the time investment per each piece of knowledge. In this context, Q can be proxied by annual expenditures on schooling for each student as a fraction of GDP. And NQ represents total expenditures on schooling for each

student. In this context, an increase in the length of years of schooling increases the ratio of $\frac{N}{Q}$, if the public plus private expenditures per one year of schooling for each student increase less than proportionally.

¹² We will relax this assumption in Section III. This assumption implies that in equation (6),

1. Maximization Problem of a Representative Agent

A representative agent maximizes the two-period discounted utility by allocating time between education and work while young and between technology adoption and work with a new technology shock when old, with wage rates per efficiency unit of specific skill equal to A_t given exogenously.

A representative young agent of generation t optimally decides the width (N) and depth (Q) of human capital to maximize the log utility preference facing uncertainties about new technology, given the current technology level A and the human capital structure of the old of generation $t-1$ of $(\bar{N}\bar{Q})$, as¹³

$$\begin{aligned}
 \max_{N_t, Q_t} U(c_{yt}, c_{ot+1}) &= \log c_{yt} + \frac{1}{1+\rho} E_{s \in S} [\log c_{ot+1}] \\
 \text{s.t.} \quad c_{yt} &= A\delta\bar{Q}(1 - I_{Et}) = A\delta\bar{Q}\left(1 - \frac{N_t Q_t}{b\bar{N}\bar{Q}}\right), \\
 c_{ot+1} &= A(1 + g_T)(1 - I_{At+1})Q_t \quad \text{with adoption of new technology,} \\
 c_{ot+1} &= A\delta\bar{Q} \quad \text{without it, and} \\
 I_{At+1} &= \min_{x \in \{N\text{-elements}\}} a|x - s| \cdot Q_t.
 \end{aligned} \tag{P1}$$

$A(1 + g_T)Q_t(1 - aQ_t \frac{S}{2N_t}) > A\delta\bar{Q} \Rightarrow 1 - \frac{\delta\bar{Q}}{(1 + g_T)Q_t} > \frac{aS}{2} \frac{Q_t}{N_t}$. Then, in the second period agents always adopt every new technology whenever it occurs, because the second period utility with the adoption of new technologies always exceeds that without the adoption. Note that this condition can be satisfied more likely when either δ or S is smaller. Note also that if δ is sufficiently large, then it is better for agents to keep an old technology rather than to adopt a new technology even when a technology shock occurs.

¹³ The solutions of the model with a constant relative risk aversion utility function are not qualitatively different from those with a logarithmic case assumed here. Even though the risk aversion parameter interacts with decision variables in a very complicated nonlinear way, this does not alter the qualitative nature of the implications we explore. In addition, the specification of (P1) does not change when we consider the old agent's allocation of time between creating knowledge and delivering it to young agents as in Appendix 1.

where $\frac{N_t Q_t}{b \bar{N} \bar{Q}}$ is the time investment for education (l_{Et})¹⁴, and l_{At+1} the time cost of technology adoption at time $t+1$. With this setup of the model, we solve the utility maximization problem recursively from the second period as in the below.

a. Second Period Expected Utility

Note that with a technology shock occurring with the probability P , agents can adopt the new advanced technology with the quality level of Q . Note also that without a technology shock agents must keep their previously inherited technology, working the whole one unit of time without incurring any technology adoption cost. From (1)' to (5), the expected utility of an agent of generation t in the second period is

$$\begin{aligned} \frac{P}{1+\rho} E[\log c_{ot+1}] &= \frac{P}{1+\rho} 2 \frac{N_t}{S} \int_0^{\frac{S}{2N_t}} \log(A_t(1+g_T)Q_t(1-aQ_t x)) dx + \frac{1-P}{1+\rho} \log(A_t \delta \bar{Q}) \\ &= \frac{P}{1+\rho} \frac{1}{S} \int_0^S \log(Q_t(1-aQ_t \frac{y}{2N_t})) dy + \frac{1-P}{1+\rho} \log(\delta \bar{Q}) + const \end{aligned} \quad (6)$$

b. First Period Maximization

Using equation (6), the first period maximization problem with a logarithmic utility can be described by

$$\max_{N_t, Q_t} \log c_{yt} + \frac{1}{1+\rho} E[\log c_{ot+1}] = \log(1 - \frac{N_t Q_t}{b \bar{N} \bar{Q}}) + \frac{P}{1+\rho} \frac{1}{S} \int_0^S \log[Q_t(1 - \frac{aQ_t y}{2N_t})] dy + const \quad (7)$$

¹⁴ With the more general description of the human capital accumulation equation of (4) as $N_t Q_t = \delta \bar{N} \bar{Q} + b l_{Et} \bar{N} \bar{Q}$ ($l_{Et} = \frac{N_t Q_t}{b \bar{N} \bar{Q}} - \frac{\delta_H}{b}$), where δ_H represents the fraction of old agents' human capital that young agents inherit through

spillover. However, this more general representation will not change the above maximization problem (P1) if we

assume that young agents are endowed with one plus $\frac{\delta_H}{b}$ units of time instead of one unit as assumed in the case of equation (4). Thus, we can easily see that the results will not change qualitatively with a slightly different functional form of (4).

2. Equilibrium Balanced Growth

We derive the FOCs with respect to N_t and Q_t , respectively, as

$$\begin{aligned} & -\frac{Q_t}{b\bar{N}\bar{Q} - N_t Q_t} - \frac{P}{1+\rho} \frac{1}{N_t} + \frac{P}{1+\rho} \frac{1}{S} \int_0^S \frac{1}{N_t - \frac{aQ_t y}{2}} dy \\ & = -\frac{Q_t}{b\bar{N}\bar{Q} - N_t Q_t} - \frac{P}{1+\rho} \frac{1}{N_t} - \frac{2P}{(1+\rho)aQ_t S} (\log(N_t - \frac{aSQ_t}{2}) - \log N_t) = 0, \end{aligned} \quad (8)$$

$$\begin{aligned} & -\frac{N_t}{b\bar{N}\bar{Q} - N_t Q_t} + \frac{P}{1+\rho} \frac{1}{Q_t} + \frac{P}{1+\rho} \frac{1}{S} \int_0^S \frac{\frac{-ay}{2}}{N_t - \frac{aQ_t y}{2}} dy \\ & = -\frac{N_t}{b\bar{N}\bar{Q} - N_t Q_t} + \frac{2P}{1+\rho} \frac{1}{Q_t} + \frac{2PN_t}{(1+\rho)aQ_t^2 S} (\log(N_t - \frac{aSQ_t}{2}) - \log N_t) = 0 \end{aligned} \quad (9)$$

Subtracting equation (8), multiplied on both sides by N_t , by equation (9) multiplied on both sides by Q_t , and additional algebra yield

$$\frac{3}{2}(z-1) = \log z, \quad (10)$$

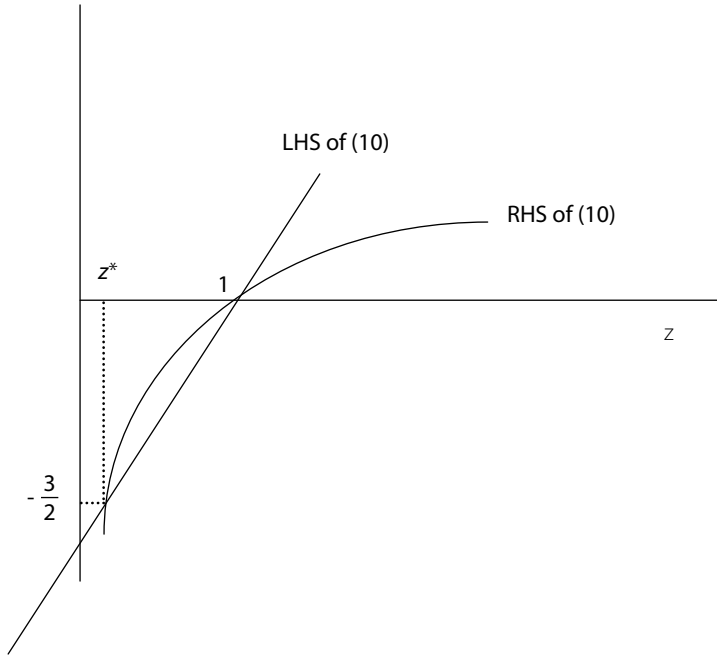
$$\text{where } z = 1 - \frac{aS Q_t}{2 N_t}.$$

The relationship of equation (10) is very simple. We can easily prove that there exists a unique solution for z satisfying (10), as depicted in Figure 2. Let us call this solution z^* . Then we can easily find z^* to be a constant of about 0.417 through computer simulation.

Utilizing the relationship of $z = 1 - \frac{aS Q_t}{2 N_t}$, the ratio of depth to width of human capital can be represented by

$$\frac{Q_t}{N_t} = \frac{2(1-z^*)}{aS}. \quad (11)$$

It is easy to solve for N_t and Q_t in the equilibrium. In the equilibrium, with a technological shock, N_t and Q_t grow at the same rate of g_H as (11) implies. We can derive easily that the equilibrium growth rate of income is equal to the growth rate of Q_t . Thus, the equilibrium is a balanced growth path.

Figure 2: Existence of the Solution for z 

By utilizing the relationship of $\frac{N_t}{N} = \frac{Q_t}{Q} = 1 + g_{Ht}$ derived from (11),¹⁵ the education time of young agents can be denoted as $l_{Et} = \frac{N_t Q_t}{b N Q} \Rightarrow l_{Et} = \frac{(1 + g_{Ht})^2}{b}$.

Using this and substituting equation (11) into (8) multiplied by N_t yields the growth rate of g_{Ht} as

¹⁵ From equation (11), we can see that this steady state relationship does not hold on the date when the parameter value of a or S changes, and then holds from one period after this date. However, even with any change of the other parameter values, this relationship still holds. From equations (8), (10), and (11), we can easily solve for Q_t and

N_t as: $Q_t = \sqrt{\frac{2(1-z^*)bPN\bar{Q}}{(2+2\rho+P)aS}}$ and $N_t = \sqrt{\frac{abPSN\bar{Q}}{2(1-z^*)(2+2\rho+P)}}$. From this we can infer that even on the date when S changes, the growth rate of N_t does not change in S , while that of Q_t decreases in S , considering that bS does not change in S (see footnote 10). Thus, we can infer that when S changes once and for all, the growth rate of N_t does not change now and decreases a little in the next period, then stays at this rate afterward. On the other hand, the growth rate of Q_t decreases on the date of change, then rises in the next period to a growth rate that is a little lower than the previous steady state growth rate, and stays at this rate afterward. We can also easily see that an increase in P raises N_t and Q_t by the same proportion, and that the growth rate jumps once and for all on the date of change, staying at this rate afterward, as equation (12) implies.

$$\begin{aligned}
& -\frac{N_t Q_t}{b\bar{N}\bar{Q} - N_t Q_t} - \frac{P}{1+\rho} - \frac{2PN_t}{(1+\rho)aQ_t S} \left(\log(N_t - \frac{aSQ_t}{2}) - \log N_t \right) = 0. \\
\Rightarrow & -\frac{(1+g_{Ht})^2}{b - (1+g_{Ht})^2} - \frac{P}{1+\rho} - \frac{2PN_t}{(1+\rho)aQ_t S} \log z^* = 0. \\
\Rightarrow & -\frac{(1+g_{Ht})^2}{b - (1+g_{Ht})^2} + \frac{P}{2(1+\rho)} = 0. \\
\Rightarrow & 1+g_{Ht} = \sqrt{\frac{Pb}{2(1+\rho)+P}}.
\end{aligned} \tag{12}$$

Intuitions and implications behind these two equations (11) and (12) will be explored in detail in the following subsection. In the case of the steady state, time subscripts will be omitted in the below.

Then, equation (12) yields the expected growth rate (g_I^E) of income of

$$1+g_I^E = P(1+g_T)(1+g_H) + (1-P)\delta. \tag{13}$$

Equations (12) and (13) imply that the expected growth rate increases in P , but does not change in S . Equation (13) implies that the expected growth rate of income increases in the growth rate of human capital (g_H), the probability of a technological change (P), and the spillover effect (δ).

Equation (11) yields the expected technology adoption time of

$$E[I_{At+1}] = \frac{P}{S} \int_0^S \frac{aQ_t y}{2N_t} dy = \frac{P}{2} (1 - z^*). \tag{14}$$

C. Technology Change and Human Capital Investment

The characterization of the equilibrium described above provides several interesting implications on human capital accumulation. In particular, the effects of uncertainty about new technologies on human capital accumulation are analyzed below.

1. Probability of Having a Technology Shock (P)

Enhancing knowledge spillovers among different countries through globalization or open trade policies, securing intellectual property rights, or discovering general purpose technologies increase the probability of having a technology shock. An increase in the probability of having a technology shock increases human capital and thus income growth rates. Therefore, a more certain occurrence of future technology shocks contributes to more rapid human capital accumulation.

From equations (11) and (12), we know that an increase in P increases growth rates of income and education level by offering more opportunities to upgrade technology by forming its specific skill. However, note that it does not change the relative investment size of width to depth of education.¹⁶

2. Uncertainty about the Characteristics of Future Technology (S)

An increase in S ¹⁷ represents an increase in the uncertainty about the characteristics of future technologies that agents will adopt in the next period. In other words, an increase in S implies that the knowledge space to which knowledge points of the future technology belong increases. Thus, with an increase in S , agents must increase their ratio of width to depth of human capital to lower the expected technology adoption cost.

An increase in S decreases growth rates of human capital and income. This occurs because it decreases the efficiency of human capital formation b , as stated in footnote 10

and Appendix 1. Appendix 1 shows that $b \cong \frac{2\delta}{k(1+e+e\delta)S}$. Note that the parameter b measuring the efficiency of human capital formation decreases in S . Also an increase in S raises the relative investment size of width to depth of human capital, as we can see from equation (11). It is very intuitive that agents will increase their adaptability or flexibility by investing relatively more resources in width of education in the face of increased uncertainty about future technology.

3. Improvements in the Efficiency of Human Capital Production and Technology Adoption

From equations (11) and (12), an increase in the efficiency of education (b) by raising the efficiency of creating knowledge (by lowering k in [A1] in Appendix 1), or a decrease in the adoption cost of technology by lowering a in equation (4) leads to higher growth rates of income and human capital, and to a higher relative investment size of depth to width of human capital. The following lemma summarizes these findings.

Lemma 1: In the basic model, an increase in the probability of having a technology shock (P) increases the growth rates of both width and depth of human capital and income, not affecting the relative investment size of width to depth of human capital. However, an increase in the uncertainty about the characteristics of future technologies (S) decreases the growth rates of both width and depth of human capital, income, and the relative investment size of depth to width of human capital.

¹⁶ See footnote 15 for the explanation of why both width and depth increase proportionally in P .

¹⁷ An increase in S shows that every interval between any two adjacent knowledge points increases by an equal amount. Another comment on the exercises of changing S is: A change in S has the identical effects with that of a throughout this paper, where a represents the inefficiency of adoption or “barriers to technology adoption” as in Parente and Prescott (1994). This occurs because S appears always with a in every equation of this paper as in the form of aS . Exercises on S (or a) provide identical implications with those of Parente and Prescott.

III. An Extended Model with a Choice of Technology Adoption

The basic model presented in the previous section shows the equilibrium dynamics of human capital and income with the assumption that once the technology shock occurs in the second period, the old agents always adopt it. The basic model assumes that the second period utility with technology adoption is always higher than that without it.

This section characterizes an extended model in which agents maximize their utility considering that agents will decide whether to adopt the new technology depending on the size of the adoption cost whenever the new technology shock occurs.¹⁸ If the adoption cost is too large, the agents will use the old technology rather than adopt a new technology. This can happen when the knowledge of the new technology is remotely located from any of the knowledge points invested. We can calculate the threshold level of the distance (s^*) between the knowledge of the new technology and its closest knowledge point, such that if the distance is above this level, then adopting the new technology becomes less profitable than sticking to the old technology.

This threshold value of s^* satisfies

$$A_t(1+g_T)Q_t(1-aQ_t s) = A\delta\bar{Q} \Rightarrow \frac{Q_t - \delta'\bar{Q}}{aQ_t^2} = s^* \in (0, \frac{S}{2N_t}), \text{ where } \delta' = \frac{\delta}{1+g_T},^{19} \quad (15)$$

which implies that the second period utility with technology adoption equals that without it at the threshold value of s^* .

¹⁸ Given a set of initial parameter values, the utility of this extended model is always higher than or equal to that of the basic model. This occurs because while the extended model does not have any market failing features, the basic model has a restriction on decision variables such that agents should "adopt every new technology irrespective of its profitability."

¹⁹ In the previous section, we solved the case in which $\frac{Q_t - \delta'\bar{Q}}{aQ_t^2} = s^* > \frac{S}{2N_t} \Rightarrow 1 - \frac{\delta'\bar{Q}}{Q_t} > \frac{aS}{2} \frac{Q_t}{N_t}$. Equations (11) and (12) transform this relationship into $\sqrt{\frac{Pb}{2(1+\rho)+P}} > \frac{\delta'}{z^*} \Rightarrow P > \frac{2\delta'^2(1+\rho)}{bz^{*2} - \delta'^2}$. In other words, under this condition in the basic model of Section II, agents in their second period will also adopt every new technology, even if they are not constrained to do so. This behavior is time-consistent because their second period utility with the adoption is greater than that without it.

A. A Brief Description of the Model

Before solving the complicated model, we will describe the basic structure of the model briefly as follows. The aggregate production function is described by

$$Y_t = A_t(H_{yt} \cdot (1 - l_{Et}) + H_{ot} \cdot (1 - l_{At})) = A_t(\delta \bar{Q} \cdot (1 - l_{Et}) + \bar{Q} \cdot (1 - l_{At})) \quad (5)'$$

where $H_{yt} = \delta H_{ot}$, $A_t = A_{t-1}(1 + g_T)$ with the adoption of a new technology, and $A_t = A_{t-1}$ without it.

Moreover, since there is no saving or lending between different cohorts and over time except through human capital investment, each agent's consumption equals his or her income in each period. Thus, for example, an old agent's consumption is described by $c_{ot+1} = A_t(1 + g_T)(1 - l_{At+1})H_{ot+1}$ with the adoption of a new technology and by $c_{ot+1} = \delta A_t(1 - l_{At+1})H_{ot+1}$ without it. Here we note that $H_{ot+1} = Q_t$ with the adoption and $H_{ot+1} = \delta \bar{Q}$ without it.

In the steady state (balanced growth path) where l_{Et} , l_{At} and the expected young (and old) agent's consumption share in aggregate output are constant, the growth rate of income is described by

$$\frac{Y_{t+1}}{Y_t} = \frac{A_t(H_{yt+1} \cdot (1 - l_E) + H_{ot+1} \cdot (1 - l_A))}{A_{t-1}(H_{yt} \cdot (1 - l_E) + H_{ot} \cdot (1 - l_A))} = \frac{A_t(\delta Q_t \cdot (1 - l_E) + Q_t \cdot (1 - l_A))}{A_{t-1}(\delta \bar{Q} \cdot (1 - l_E) + \bar{Q} \cdot (1 - l_A))} = \frac{(1 + g_T)Q_t}{\bar{Q}}$$

with the adoption of a new technology, and

$$\frac{Y_{t+1}}{Y_t} = \frac{A_{t-1}(H_{yt+1} \cdot (1 - l_E) + H_{ot+1})}{A_{t-1}(H_{yt} \cdot (1 - l_E) + H_{ot} \cdot (1 - l_A))} = \frac{A_{t-1}(\delta^2 \bar{Q} \cdot (1 - l_E) + \delta \bar{Q})}{A_{t-1}(\delta \bar{Q} \cdot (1 - l_E) + \bar{Q} \cdot (1 - l_A))} = \frac{\delta^2 \cdot (1 - l_E) + \delta}{\delta \cdot (1 - l_E) + (1 - l_A)}$$

without it. (Consumption growth rates are similarly described.)

From these, we can see that the expected growth rate of income depends not only on the probability of technology adoption that increases in the width of human capital (N), and

also on the growth rate of depth of human capital ($\frac{Q_t}{\bar{Q}}$). These are the key mechanisms behind the dynamics of the model that we solve and focus on below.

B. The Maximization Problem

A representative young agent with a logarithmic utility function solves the following maximization problem.²⁰

$$\begin{aligned} \max_{N_t, Q_t} \quad & \log\left(1 - \frac{N_t Q_t}{b \bar{N} \bar{Q}}\right) + \frac{2N_t P}{S(1+\rho)} \left\{ \int_0^{s^*} \log[Q_t(1 - a Q_t x)] dx + \int_{s^*}^{\frac{S}{2N_t}} \log \delta' \bar{Q} dx \right\} \\ & + \log \delta' \bar{Q} + \frac{1-P}{1+\rho} \log \delta' \bar{Q} + \log A_t + \frac{1}{1+\rho} \log(A_t(1 + g_T)) \Rightarrow \end{aligned} \quad (P2)$$

$$\begin{aligned} \max_{N_t, Q_t} \quad & \log\left(1 - \frac{N_t Q_t}{b \bar{N} \bar{Q}}\right) + \frac{2PN_t}{(1+\rho)S} \left\{ \left(s^* - \frac{1}{aQ_t}\right) \log(1 - aQ_t s^*) - s^* + s^* \log Q_t \right\} \\ & - \frac{P}{1+\rho} \frac{2N_t}{S} s^* \log \delta' \bar{Q} + \log \delta' \bar{Q} + \frac{1}{1+\rho} \log \delta' \bar{Q} + \text{const} \end{aligned}$$

(P2) is solved in the below. Equation (15) simplifies this maximization problem as

$$\max_{N_t, Q_t} \quad \log\left(1 - \frac{N_t Q_t}{b \bar{N} \bar{Q}}\right) + \frac{2PN_t}{(1+\rho)S} \left\{ \frac{1}{aQ_t} \left(\log \frac{Q_t}{\delta' \bar{Q}} + \frac{\delta' \bar{Q}}{Q_t} - 1 \right) \right\} + \log \delta' \bar{Q} + \frac{1}{1+\rho} \log \delta' \bar{Q} + \text{const}$$

The first order conditions with respect to N and Q are as follows:

$$-\frac{Q_t}{b \bar{N} \bar{Q} - N_t Q_t} + \frac{2P}{(1+\rho)S} \left\{ \frac{1}{aQ_t} \left(\log \frac{Q_t}{\delta' \bar{Q}} + \frac{\delta' \bar{Q}}{Q_t} - 1 \right) \right\} = 0 \quad (16)$$

$$-\frac{N_t}{b \bar{N} \bar{Q} - N_t Q_t} + \frac{2PN_t}{(1+\rho)S} \left\{ -\frac{1}{aQ_t^2} \left(\log \frac{Q_t}{\delta' \bar{Q}} + \frac{2\delta' \bar{Q}}{Q_t} - 2 \right) \right\} = 0 \quad (17)$$

Subtracting equation (17) times Q_t from equation (16) times N_t , and simple algebra lead to

$$\log \frac{\delta' \bar{Q}}{Q_t} = \frac{3}{2} \left(\frac{\delta' \bar{Q}}{Q_t} - 1 \right). \quad (18)$$

²⁰ This maximization problem needs the restriction that $s^* \leq \frac{S}{2N_t}$. However, to simplify the solution process, we first solve the model without this restriction. And then if the decision variable S^* is found to be bigger than the boundary value (i.e., $s^* > \frac{S}{2N_t}$) without this restriction, we have only to solve the basic model of Section II after setting S^* to be $\frac{S}{2N_t}$. The theoretical justification for this solution process is given in Appendix 2. We call the condition to produce the relationship of $s^* < \frac{S}{2N_t}$ as the uncertain adoption condition of the extended model.

Note that the structure of the above equation is exactly identical to equation (10). Therefore,

by replacing $\frac{\delta' \bar{Q}}{Q_t}$ by z^* and using the results of equation (10), we obtain

$$Q_t = \frac{\delta'}{z^*} \bar{Q}. \quad (19)$$

where z^* is a constant with the value of about 0.417. Equations (6) and (19) show that Q_t and s^* are functions only in $\delta' \bar{Q}$, and also that Q_t is always greater than $\delta' \bar{Q}$, leading to $s^* > 0$.

1. Human Capital Investment

Substituting equation (18) into (16) and simple algebra yield

$$1 + g_{Ht} = \frac{Pb \frac{2N_t}{S} s^*}{2(1 + \rho) + P \frac{2N_t}{S} s^*} \quad (20)$$

$$\text{where } g_{Ht} \equiv \frac{N_t Q_t}{N \bar{Q}} - 1 \text{ and } s^* = \frac{Q_t - \delta' \bar{Q}}{a Q_t^2}.$$

Equation (20) implies that the growth rate of human capital stock (g_H) in the case of the uncertain adoption case of the extended model (i.e., $s^* < \frac{S}{2N_t} \Rightarrow \frac{2N_t}{S} s^* < 1$) is always smaller than that in the certain adoption case (i.e., $s^* = \frac{S}{2N_t} \Rightarrow \frac{2N_t}{S} s^* = 1$) of the basic model.

Equation (19) shows the equilibrium growth rate of the depth of human capital stock (g_Q), which in this extended model is given by

$$(1 + g_Q) = \frac{Q_t}{\bar{Q}} = \frac{\delta'}{z^*}. \quad (21)$$

Therefore, the depth of human capital may increase or decrease over time depending only on the value of δ' . If there exists a rather strong spillover of technology over generations (i.e., $\delta' > z^* \cong 0.417$), the depth of human capital always increases over time. Even though an increase in δ decreases the probability of adopting new technologies by increasing the opportunity cost of the old technology, (21) says that it will increase the growth rate of the depth of human capital due to the following two reasons. One is the consumption smoothing between the two different states of technology adoption and no technology adoption. The other one is that by decreasing the width sufficiently, the marginal cost of increasing the depth decreases.

We can compare the equilibrium growth rate of $\frac{Q_t}{\bar{Q}}$ in this extended model with that in the certain technology adoption case of the basic model. In Section II, we solved the certain

technology adoption model in which $\frac{Q_t - \delta' \bar{Q}}{a Q_t^2} = s^* > \frac{S}{2N_t} \Rightarrow 1 - \frac{\delta' \bar{Q}}{Q_t} > \frac{aS}{2} \frac{Q_t}{N_t}$. By equations

(11) and (12), this relationship leads to $\sqrt{\frac{Pb}{2(1+\rho)+P}} > \frac{\delta'}{z^*}$. From this and equations (12) and (19), we can easily see that the growth rate of Q_t with the certain adoption of technology is always higher than that with the uncertain adoption of the extended model. By the same logic, the growth rate of income is also always higher with the certain adoption of technology than that with the uncertain adoption.

Substituting equations (18) and (19) into equation (16) yields

$$\begin{aligned} (1 + g_{N_t}) &= \frac{N_t}{\bar{N}} = \frac{bz^*}{\delta'} - \frac{(1+\rho)S}{P} \frac{1}{s^*} \\ &= \frac{bz^*}{\delta'} - \frac{a\delta'(1+\rho)S}{z^*(1-z^*)P} \frac{\bar{Q}}{\bar{N}}, \quad \text{if } \frac{2N_t}{S} s^* < 1 \\ &= \sqrt{\frac{Pb}{2(1+\rho)+P}}, \quad \text{if } \frac{2N_t}{S} s^* = 1. \end{aligned} \quad (22)$$

Equation (22) implies that the growth rate of the width of human capital ($\frac{N_t}{\bar{N}}$) is a function of several parameters, while that of the depth is a function only of δ' . This also implies that while an increase in P increases N_t , an increase in S decreases N_t .²¹ Equation (22) also

implies that the growth rate of width of human capital increases in $\frac{\bar{N}}{\bar{Q}} = \frac{\delta' \bar{N}}{z^* Q_t}$. That is, an increase in \bar{N} decreases the adoption cost of new knowledge points in the human capital formation process, while an increase in Q_t raises it, based on our interpretation of equation (4) presented in Appendix 1. We can summarize the results related to the two types of uncertainty in the following lemma.

Lemma 2: In the uncertain adoption case ($s^* < \frac{S}{2N_t}$), an increase in the probability of having a technology shock (P) increases the width of human capital but does not affect the depth, resulting in a higher level of human capital stock and higher growth rates of human capital stock and the expected income. A decrease in uncertainty about the characteristics of new technologies (S) shows the identical effects on these variables.

²¹ This result is quite different from the result in Section II in which an increase in S raises N_t as well as Q_t .

2. Growth Rate of Income

The expected growth rate of income can be calculated as²²

$$\begin{aligned}
 1 + g_t^E &= P(1 + g_T) \left(1 - \frac{\left(\frac{S}{2N_t} - s^* \right) \frac{Q_t}{Q}}{\frac{S}{2N_t}} \right) + \left(1 - P + \frac{\frac{S}{2N_t} - s^*}{\frac{S}{2N_t}} P \right) \delta \\
 &= P(1 + g_T) \left(\frac{2N_t s^*}{S} \right) \frac{Q_t}{Q} + \left(1 - \frac{2N_t s^*}{S} P \right) \delta
 \end{aligned} \tag{23}$$

$$\text{where } s^* = \frac{Q_t - \delta' \bar{Q}}{a Q_t^2} = \frac{z^* (1 - z^*)}{a \delta' \bar{Q}}.$$

Using equations (20), (22), and $s^* = \frac{z^* (1 - z^*)}{a \delta' \bar{Q}}$, we can solve $\frac{2N_t}{S} s^*$ as

$$\begin{aligned}
 \frac{2N_t}{S} s^* &= \frac{2}{S} \left\{ \frac{b z^{*2} (1 - z^*)}{a \delta'^2} \frac{\bar{N}}{\bar{Q}} - \frac{(1 + \rho) S}{P} \right\} < 1 \\
 \Rightarrow \frac{\bar{N}}{\bar{Q}} &< \left(\frac{S}{2} + \frac{(1 + \rho) S}{P} \right) \frac{a \delta'^2}{b z^{*2} (1 - z^*)} \equiv \Gamma
 \end{aligned} \tag{24}$$

This is the condition under which the extended model belongs to the uncertain adoption case. In other words, if this condition is not satisfied, the basic model instead of (P2) should be solved.

3. Dynamics of Human Capital Accumulation and Income Growth

Equations (20), (22), (23), and (24) show that growth rates of human capital stock (g_{Ht}), width of human capital (g_{Nt}), and income (g_{It}^E) change over time depending on the dynamics of the ratio of width to depth of human capital ($\frac{\bar{N}}{\bar{Q}}$).²³ If $\frac{\bar{N}}{\bar{Q}}$ increases over time, the growth rates of N_t , $N_t Q_t$ and income increase unambiguously over time, and vice versa.

²² The variable $\frac{2N_t s^*}{S}$, increasing in $\frac{\bar{N}}{\bar{Q}}$, represents the proportion of the band of specialized knowledge points in the whole knowledge space. The band of specialized knowledge points denotes that agents adopt only the new technologies whose characteristics fall on this band. Thus, it is quite intuitive that an increase in this band increases the growth rate of income since the broader band implies more frequent technology adoption.

²³ Recall that Q_t always grows at a constant growth rate of $\frac{\delta'}{z^*}$.

Using equations (18) and (22), the dynamics of $\frac{N_t}{Q_t}$ over time can be described by a difference equation of

$$\frac{N_t}{Q_t} = \frac{bz^{*2}}{\delta'^2} \frac{\bar{N}}{\bar{Q}} - \frac{a(1+\rho)S}{(1-z^*)P}. \quad (25)$$

Equation (25) describes the dynamics of width and depth of human capital over time.

At this point we must prove that the economic system converges to the certain adoption economy. In other words, we must prove that when $s^* > \frac{S}{2N_t}$ without the restriction of $s \leq \frac{S}{2N_t}$ in the extended model, the optimal s with this restriction will be $s^* = \frac{S}{2N_t}$. Then this will imply that if $s^* > \frac{S}{2N_t}$ when solving (P2) without the restriction, the basic model instead of (P2) should be solved again after setting $s^* = \frac{S}{2N_t}$. In other words, we should solve the certain adoption model in this case. For this, we will first prove that if the solution of s with the above restriction is strictly less than $\frac{S}{2N_t}$, this will not satisfy the conditions for maximization by proving that the utility increases in s if its value is less than $\frac{S}{2N_t}$. We will prove this below.

To prove that the utility increases in s , firstly we prove that an increase in s implies a decrease in Q_t by utilizing the fact that z^* is approximately 0.417 less than 0.5. Then the question whether the utility increases in s boils down to discerning the sign of the left-hand side (LHS) of equation (17) with the appropriate value of Q_t corresponding to the value of $s < s^*$, utilizing the relationship of equation (16), which holds due to the envelop theorem. The sign of the LHS of (17) represents the change of the utility with respect to a decrease in s in the uncertain adoption case of the extended model. This occurs because s is a decreasing function only in Q_t . It is easy to prove that the sign is negative when Q_t is larger than the optimal value of Q_t (in other words, when s is smaller than s^*).

This means that a decrease in s decreases the utility. Thus, we can infer that if $s^* > \frac{S}{2N_t}$ without the restriction, $s^* = \frac{S}{2N_t}$ will be the optimal solution for s in the extended model. The rest of the proof is easy. Thus, we can say that the high growth equilibrium converges to the certain adoption case.

In contrast, if $\frac{\bar{N}}{\bar{Q}} < \Phi$ ²⁴, $\frac{N_t}{Q_t}$ decreases over time. Then, these variables will move in the

²⁴ This inequality together with equation (24) implies $\sqrt{\frac{Pb}{2(1+\rho)+P}} > \frac{\delta'}{z^*} \Rightarrow P > \frac{2\delta'^2(1+\rho)}{bz^{*2}-\delta'^2}$. In other words, these two inequalities together satisfy the condition in footnote 19 under which agents will adopt all the new technologies even with the option of no adoption in the second period, in the basic model of Section II.

opposite direction. The proof is basically identical to the above. Lastly, if this condition holds with equality, then equation (25) says that $\frac{N_t}{Q_t}$ does not change over time.

The above results yield the following Lemmas 3 and 4, and Proposition 1.

Lemma 3: In the uncertain adoption case ($s^* < \frac{S}{2N_t}$), an increase in the probability of

having a technology shock (P) increases the width of human capital but does not affect the depth, resulting in a higher level of human capital and higher growth rates of human capital and the expected income. A decrease in uncertainty about the characteristics of new technologies (a decrease in S) shows the identical effects on these variables.

Lemma 4: If $b < \frac{\delta'^2}{z^{*2}}$, the ratio of width to depth of human capital, and the growth rates of human capital and expected income continuously decline over time, resulting in no technology adoption in the extended model.

(Proof) The dynamics of $\frac{N_t}{Q_t}$ in equation (25) implies that with the above condition in this proposition, $\frac{N_t}{Q_t}$ decreases continuously over time, forcing the economy to remain in the uncertain adoption mode. Additionally, (24) implies that the band of the specialized knowledge (technology) represented by $\frac{2N_t}{S}s^*$, increasing in $\frac{\bar{N}}{\bar{Q}}$, will decline over time. Then, equations (23) and (24) imply that the expected income growth rate also declines.

Lemma 4 implies that the economy can undoubtedly result in a poverty trap if the economy has a low efficiency of human capital production (b) caused by an increase in S , or a large spillover of the existing technology (δ'). In this economy, investment in the width of human capital becomes smaller and smaller over time, and fewer and fewer new technologies are adopted. Eventually, N_t goes to zero and the economy will be trapped in the old technology.

Lemma 5: If $b < \frac{\delta'^2}{z^{*2}}$, and if $\frac{(PS + 2(1+\rho)S)a\delta'^2}{2Pbz^{*2}(1-z^*)} \equiv \Gamma > \Phi \equiv \frac{a\delta'^2(1+\rho)S}{(1-z^*)(bz^{*2} - \delta'^2)P}$

$\Rightarrow P > \frac{2\delta'^2(1+\rho)}{bz^{*2} - \delta'^2}$, then the economic system has three different equilibria (one stable and two unstable ones) depending on the initial value of the ratio of width to depth of

human capital (denoted simply by $\frac{\bar{N}}{\bar{Q}}$ in the below). If $\frac{\bar{N}}{\bar{Q}}$ exactly equals a critical value

$\frac{1}{b}, \frac{S}{P}$, then the economy follows the steady-state balanced

growth path.²⁵ In this steady-state equilibrium, the ratio of width to depth of human capital ($\frac{\bar{N}}{\bar{Q}}$), human capital growth rate (g_H), and the expected income growth rate (g_I^E)

remain constant over time. If $\frac{\bar{N}}{\bar{Q}} > \Phi$, $\frac{\bar{N}}{\bar{Q}}$, $\frac{N_t}{Q_t}$, g_{Ht} , and g_{It}^E increase over time. Hence, the economy will eventually move into the certain technology adoption economy. In contrast,

if $\frac{\bar{N}}{\bar{Q}} < \Phi$, then $\frac{\bar{N}}{\bar{Q}}$, $\frac{N_t}{Q_t}$, g_{Ht} , and g_{It}^E decline over time, eventually leading to a poverty trap with no technology adoption.²⁶

(See Appendix 2 for the proof).

The inequality of $b < \frac{\delta'^2}{z^{*2}}$ implies that the possibility of new technology adoption is more profitable than resting on the inherited old technology, resulting in positive investment in human capital. Note that b denotes the efficiency of human capital formation to help

adopt future technologies, while $\delta' = \frac{\delta}{1+g_T}$ where δ denotes the fraction of the old technology inherited from the old generation through spillover. In this sense the case with this inequality is more interesting and realistic than that with the reversed inequality with which Lemma 4 deals. These characterizations of the equilibrium growth paths are summarized in the following proposition.

Proposition 1: If the efficiency of human capital formation is very low ($b \leq \frac{\delta'^2}{z^{*2}}$), the economy will be trapped in poverty with no investment in human capital. If $b > \frac{\delta'^2}{z^{*2}}$, various equilibrium paths are characterized as follows depending on the value of $\frac{\bar{N}}{\bar{Q}}$:²⁷

²⁵ In the case with an additional condition of equation (24) holding with equality, the solution equals that of the certain adoption case of the basic model. This is intuitively obvious because this condition implies that agents adopt all technologies that occur.

²⁶ For the difference equation of (25), there exists one unstable equilibrium. However, for the economic system, there exist two long-run, stable, steady-state equilibria—poverty trap and certain adoption equilibrium as described in this proposition. The case of the unstable equilibrium of the difference equation system represents the long-run,

unstable, steady state equilibrium of the economic system. (However, if $s^* = \frac{S}{2N_t}$ in equation (24), this equilibrium is identical to the certain adoption equilibrium, as stated in footnote 20.)

²⁷ The ratio of width to depth of human capital plays an important role here. With a given depth of human capital, the higher width (\bar{N}) lowers the technology adoption cost and thus leads to more frequent technology adoptions, higher investment in human capital and higher income growth. And with a given \bar{N} , the lower level of the current technology (\bar{Q}) decreases the opportunity cost of substituting the current technology with the new one, and thus enables technology adoptions to occur more frequently. From (20), (23), and (24), we can see that this ratio is the key variable determining the dynamics of the economy. The reasoning goes as follows: equations (20) and (23)

imply that an increase in the variable $\frac{2N_t s^*}{S}$ representing the band of the specialized knowledge points increases growth rates of income as well as human capital. This is because the broader band implies the more frequent technology adoption. Equation (24) in turn implies that the band of the specialized knowledge points increases in $\frac{\bar{N}}{\bar{Q}}$. This ratio is the key variable determining the dynamics of the economy.

Case 1:²⁸ $\Phi < \Gamma \Rightarrow P > \frac{2\delta'^2(1+\rho)}{bz'^2 - \delta'^2}$

(a) $\Phi < \Gamma \leq \frac{\bar{N}}{\bar{Q}}$: Certain Adoption Equilibrium Path

(b) $\Phi < \frac{\bar{N}}{\bar{Q}} < \Gamma$: Higher Growth Equilibrium Path in Uncertain Adoption Case

It converges to (a).

(c) $\frac{\bar{N}}{\bar{Q}} = \Phi < \Gamma$: Steady State Equilibrium Path in Uncertain Adoption Case

(d) $\frac{\bar{N}}{\bar{Q}} < \Phi < \Gamma$: Lower Growth Equilibrium Path in Uncertain Adoption Case

It converges to Poverty Trap.

Case 2: $\Phi \geq \Gamma \Rightarrow P \leq \frac{2\delta'^2(1+\rho)}{bz'^2 - \delta'^2}$

(e) $\Gamma \leq \Phi \leq \frac{\bar{N}}{\bar{Q}}$: Certain Adoption Equilibrium Path

(f) $\frac{\bar{N}}{\bar{Q}} < \Phi$: Lower Growth Equilibrium Path in Uncertain Adoption Case

It converges to Poverty Trap.

We can easily verify the local stability of each of the equilibria characterized in Proposition 2 as follows. For this, we note that the dynamics of the model depends on

the ratio of $\frac{\bar{N}}{\bar{Q}}$, as previously mentioned, and also that in the certain adoption equilibrium, the economy will be in the stable steady state at least after one period since it enters this equilibrium, which can be easily derived from equation (11) and footnote 15. Also we can easily see that only (c) of Case 1 is unstable in the sense that even a very small

perturbation of $\frac{\bar{N}}{\bar{Q}}$ will make this equilibrium to move either to (b) of Case 1 or to (d) of

Case 1. As for the other cases of equilibrium, a small perturbation of $\frac{\bar{N}}{\bar{Q}}$ will not move the equilibrium to other cases, still belonging to the same case in which each equilibrium reaches a long-run steady state, certain adoption or poverty trap equilibrium.

Proposition 1 implies that the log of “average years of schooling” (N_t) minus the log of “annual education expenditures for each student as a fraction of GDP” (Q_t) has a positive effect on economic growth. This implication is consistent with the findings of Barro and Lee (1996), Hanushek and Kimko (2000), Hanushek (2003), and others in the following sense. They, using an international data set, present empirical evidence that

²⁸ In the following subsection, we will focus on this case, which is more realistic and has more interesting aspects.

annual expenditures on education (Q_i) for each student as a fraction of GDP does not significantly affect students' performance measured by international test scores.

Following Proposition 1, as P or $\frac{1}{S}$ increases (or the efficiency of human capital

formation (b) or that of technology adoption ($\frac{1}{a}$) improves), the equilibrium growth path moves up from (d) to (c), from (c) to (b), and from (b) to (a) in Case 1; and from (f) to (e) in Case 2, because Γ and Φ decrease in b , $\frac{1}{a}$, P , or $\frac{1}{S}$. In other words, as P or $\frac{1}{S}$ (or b or $\frac{1}{a}$) increases, the equilibrium path will move to the upper and upper growth equilibrium path.²⁹

Proposition 2: An increase in P or $\frac{1}{S}$ moves the equilibrium path up to the upper growth equilibrium path. An increase in b or $\frac{1}{a}$ caused by an improvement in the efficiency of human capital formation or of technology adoption has the same effect.

Implications of these results are discussed in the next subsection.

C. Multiple Growth Paths

The model presents several implications related to the existence of multiple equilibria of an economy with the uncertain technology adoption.

In the economy of the certain technology adoption of the basic model in Section II, agents maximize their utilities with the restriction that they must always adopt new technology once it occurs irrespective of the profitability. In this basic model, agents make more investment in human capital leading to higher growth than the uncertain adoption case of the extended model. The economy in the basic model follows a sustained balanced growth path as described in Lemma 1 in Section II.

In contrast, in the model of the uncertain adoption of the extended model, the agents are not sure in advance of whether they will adopt the new technology in the next period or not. If the adoption cost turns out to be too high and thus the technology adoption is not profitable in the second period, the agents will not adopt the new technology even when

²⁹ Note that the certain adoption condition of the basic model described in footnote 19 is identical to that of Case 1 ($\Phi < \Gamma$). However, Case 1 includes various equilibrium paths of the certain adoption as well as the uncertain adoption of the extended model. That is, a more strict condition is necessary for the certain adoption of technologies in the extended model than in the basic model. Agents tend to invest more in human capital with the forced adoption restriction in the basic model, resulting in adopting more new technologies and leading to higher growth rates of income and human capital than those in the extended model. Thus, even with identical initial conditions, the equilibrium paths can be quite different depending on whether the model is basic or extended.

it occurs. With this uncertain adoption in the second period, the first-period human capital investment becomes smaller, and thus equilibrium growth rates of human capital and income are lower than those in the certain adoption case. The economy may eventually join the club of economies with the certain technology adoption as human capital increases over time, but can also lead to a poverty trap as human capital continues to decrease, as described in Proposition 1 of Section III, depending on its initial conditions.

The possibility of multiple equilibria in the economy with an uncertain technology adoption is intuitive. With the given level of income (or human capital; $\delta'Q$) of a young agent, this young agent can follow any one of two equilibrium paths. Along the high growth path, the agent makes high investment in the width of human capital (N_t), which allows more chances to adopt new technologies by lowering the technology adoption cost, and thus increases growth rates of income and human capital over time. The other path is low investment in human capital due to the expected high adoption cost. Along this lower growth path, the adoption cost continues to rise, lowering the possibility of technology adoption, which in turn lowers human capital investment and thus growth rates of income and human capital over time.

As described in Proposition 1, the growth path is determined by the ratio of width to depth of human capital ($\frac{\bar{N}}{Q}$), in the uncertain technology adoption model. If $\frac{\bar{N}}{Q}$ is larger than a critical value (Φ), which decreases in P , b , $\frac{1}{a}$, or $\frac{1}{S}$, the economy follows a higher growth path, and vice versa. Hence, if P or b is large, or if S is small, the economy is likely to follow the higher growth path and eventually becomes the certain technology adoption economy. Then technology change can lower growth rates of human capital and income. If it raises the uncertainty related to the characteristics of new technologies (S) sufficiently more than the possibility of having a new technology shock P , it can lower the growth rate of income, as we see in Proposition 2.

The model also shows that with fixed values of b , a , S and P , the country with more investment in width of human capital relative to depth can more likely show higher growth rates of human capital and income. This may be consistent with the empirical fact that rapidly growing developing countries such as East Asian countries invest relatively more in the quantity side (N_t) of human capital than in the quality side (Q_t) of human capital (see Barro and Lee 1996, and Lee 2000). Due to the presence of multiple equilibria, government intervention can be effective in promoting human capital accumulation and economic growth.

The model suggests that when the probability of having new technologies (P) or the efficiency of human capital formation (b) is higher, the more probably the economy will follow the higher growth path. Therefore, any public policy that increases either P or b will contribute to the higher growth of human capital and income.

The government can raise the probability of having a technology shock (P) also by increasing public investment in R&D or by providing subsidies to private R&D investment. The government can also increase growth rates of human capital and income by adopting technology policies that can lower the uncertainty about the characteristics of future technology shocks (S). Subsidies on education can also move the economy from a lower to a higher growth path by raising b .

It is reasonable to assume that an increase in trade openness and foreign investment raises the parameter value of P . A more open trade policy or more FDI inflows will increase the probability of having a technological change (P). The increased openness enables agents to have more access to new technologies and to make more accurate decisions about adopting new technologies because they can have more information about these new technologies.

But increased openness will also increase the uncertainty about the characteristics of new technologies (S) in addition to P . An increase in S , unlike P , has unfavorable effects on growth. A higher value of S makes agents invest less in human capital by lowering the profitability of human capital investment through raising the adoption cost, and thus more likely lead the economy to the lower growth path of the uncertain adoption case. Moreover, an increase in S decreases the efficiency of human capital formation (b), leading to lower growth rates of income and human capital. Therefore, the net effects of increased openness on human capital accumulation and income growth are ambiguous, depending on the relative magnitude of changes in P and S .

Finally, let us briefly comment on how the results of Proposition 1 change depending on the relative magnitude of changes in P and S . As a reference point, consider the case in which P and S increase at the same rate. In this case, we can easily see that Φ does not change while Γ increases. Then, as both parameters capturing the uncertainty about new technologies increase at the same rate, (a) of Case 1 will move to (b) of Case 1, while (c) and (d) of Case 1 remain identical. Also, (e) of Case 2 will move to (a) or (b) of Case 1. And (f) of Case 2 will move to (c) or (d) of Case 1. From this we easily infer that when P increases faster than S , the increased uncertainties about new technologies have a favorable effect on the economy.

IV. Conclusion

In this paper we have explored the effects of technology change on growth rates of income and human capital, using an overlapping generations model in which identical agents invest in both width and depth of human capital under uncertain environments. For this, we model a micro-mechanism of the role of human capital in adopting new technologies as well as that of the process of human capital production.

An interesting result of the model is that depending on the initial structure of human capital and the nature of technology uncertainties, an economy can have different growth paths. Hence, the lower width of human capital or increased inflows of new technologies with more uncertain characteristics may adversely affect human capital accumulation and income growth, leading the economy to a low growth trap. Hence, our model offers some plausible explanations for the observed failure of development or lower growth performance in less developed economies, which lack adequate human capital. Despite the wide spread of new technologies, particularly with the advance of the information and communication technology in recent decades, some less developed countries have fallen behind and remained in poverty. The model emphasizes the important role of public policies in the area of education and technology in making the economy adjust to uncertain technology change.

Our model can be extended in several avenues. First, we can derive implications about the optimal structure of R&D investment from the model. For this, we can interpret the human capital formation process as basic research activities carried out in the two dimensions of width and depth, and the technology adoption process as development research activities. In a similar context, for example, Vandenbussche, Aghion, and Meghir (2006) empirically explore the relationship between technology adoption levels (imitation or creation) and human capital levels.

Second, we can introduce a more flexible functional form of probability distribution of the characteristics of new technologies on knowledge space instead of the assumption of uniform distribution. A change in the distribution functional form can better represent a certain aspect of an increase in the uncertainty of new technologies. Third, we can model the technology adoption as well as the human capital formation process in a more delicate way. For example, we can introduce a mechanism of interactions among the possibility of technological change and investments in width and depth of human capital more realistically.

Fourth, we can introduce a more general form of utility function in the model, instead of log-utility. As we see that the main implications of the model do not depend on the degree of risk averseness, the results will not change much. However, we can conjecture that with too much uncertainty and with a high degree of risk averseness in the preference, it might be too costly to adopt technologies frequently, and utility might be higher when more resources are spent on consumption and less on adoption.

Finally, considering the externalities from old agents' adoption decision to young agents' level of specific skill, we can fully solve the social planner's problem, and explore the policy implications based on this solution. It is because without government intervention, all equilibrium will be characterized by a lower level of adoption than optimal. However, main implications will not change much in this case, either.

Appendix 1: A Micro-Mechanism of the Education Process

In this appendix, we will provide a simple micro-mechanism of human capital formation through education compactly described by equation (4).

We assume that young agents accumulate their human capital by spending education time (t_s) in schools, and by paying tuition to old agents, compensating old agents' opportunity cost of teaching efforts. Further assume that old agents' teaching efforts consist of two parts: creating the human capital structure of NQ that young agents want to have by spending time (t_c) and through using the old agents' human capital of \bar{NQ} ; and delivering this created human capital to young agents by spending education time (t_s) in schools.

Now, we will describe the process of old agents' creation of human capital structure NQ in the discussion below. This knowledge creation process is similar to the technology adoption process of (1)' but with several differences. We assume that to create a piece of knowledge located at x with depth of Q , old agents use the two neighboring pieces of knowledge points by spending the knowledge creation time of

$$t_c = \frac{k}{2} \{ |x - s_1| + |x - s_2| \} \frac{Q}{\bar{Q}}, \quad (A1)$$

where s_i 's denote the locations of the two neighboring knowledge points that are most closely located to x . Here, we further assume that if x happens to be identical to s_i , then s_2 can be either one of the two neighboring points.

The relationship of $|x - s_1| + |x - s_2| = \frac{S}{\bar{N} + 1}$ simplifies (A1) as

$$t_c = \frac{k}{2} \frac{S}{\bar{N} + 1} \frac{Q}{\bar{Q}}, \quad (A2)$$

where $\frac{S}{\bar{N} + 1}$ represents the distance between any two adjacent knowledge points of the old agents' human capital structure.

To calculate the equilibrium amount of tuition, we need two more assumptions. The education time in schools (t_s) needed to deliver the created NQ to young agents is proportional to the old agents' total knowledge creation time ($t_c N$) as

$$t_s = e t_c N \quad (A3)$$

and the old agents' opportunity cost per unit of time is the old agents' wage rate per unit of time of $A\bar{Q}$. The old agents' wage rate will be $A\bar{Q}$ due to the assumptions about the production technology described by equations (2) and (5).

Considering that tuition equals the total opportunity cost of the education activity of old agents, including knowledge creation and education activity in schools, in the competitive market, the tuition will be

$$\begin{aligned}
 c_E &= (t_s + t_c N) A \bar{Q} \\
 &= (1 + e) t_c N A \bar{Q} \\
 &= (1 + e) \left(\frac{k}{2} \frac{S}{\bar{N} + 1} A Q N \right)
 \end{aligned} \tag{A4}$$

Here, we assume that young agents pay the tuition by providing a fraction of their labor, such as apprenticeship, for the teacher. This assumption makes the aggregate amount of labor supply in equation (5) intact, because old agents' time spent teaching young agents is completely compensated by young agents' time devoted to teachers as tuition.

We can easily infer that this tuition is the market equilibrium price. This is so because if tuition is higher than this equilibrium price (i.e., the wage rate in schools is higher than $A \bar{Q}$), then all old agents will specialize in education, resulting in no production in this economy. And if it is lower than the equilibrium price, no old agents will supply education services.

Considering (A4), the fact that young agents must spend education time in schools (t_s) to learn and accumulate human capital, and that equations (3) and (5) say that young agents' wage rate is $A \delta \bar{Q}$, then the total time cost for young agents to accumulate human capital of NQ is

$$\begin{aligned}
 I_E &= t_s + \frac{c_E}{A \delta \bar{Q}} \\
 &= \frac{k}{2} \frac{S}{\bar{N} + 1} \frac{QN}{\bar{Q}} \left(e + \frac{1 + e}{\delta} \right) \\
 &\cong \frac{kS(1 + e + e\delta)}{2\delta} \frac{NQ}{\bar{N}\bar{Q}},
 \end{aligned} \tag{A5}$$

where if N and \bar{N} are large enough, the approximation in the third line of (A5) holds.

Comparing (A5) and equation (4), the parameter b in equation (4) can be described by

$$b \cong \frac{2\delta}{kS(1 + e + e\delta)}, \tag{A6}$$

which implies that the efficiency of human capital formation decreases in S . This condition exists because an increase in S increases the old agents' knowledge creation time.

Appendix 2: Proof of Lemma 5

Equation (25) implies that with $\frac{\delta'^2}{z^{*2}} < b$ and $\frac{\bar{N}}{\bar{Q}} > \Phi = \frac{a(1+\rho)\delta'^2}{(1-z^*)(bz^{*2}-\delta'^2)} \frac{S}{P}$, $\frac{N_t}{Q_t}$ increases over time to the point where equation (24) is violated. Thus, in this economy, the amount of investment in the width of human capital, the expected income growth rate, and the band of the specialized knowledge (technology) increase over time. We can also easily see that at the moment when

equation (24) is violated first by the continuous increase in $\frac{N_t}{Q_t}$, the economy with a choice of adoption will move into the higher growth path of the certain adoption mode. The income growth rate will also move as described above.

At this point we must prove that the economic system converges to the certain adoption economy.

In other words, we must prove that when $s^* > \frac{S}{2N_t}$ without the restriction of $s \leq \frac{S}{2N_t}$ in the extended model, the optimal s with this restriction will be $s^* = \frac{S}{2N_t}$. Then this will imply that if

$s^* > \frac{S}{2N_t}$ when solving (P2) without the restriction, the basic model instead of (P2) should be solved again after setting $s^* = \frac{S}{2N_t}$. In other words, we should solve the certain adoption model

in this case. For this, we will first prove that if the solution of s with the above restriction is strictly

less than $\frac{S}{2N_t}$, this will not satisfy the conditions for maximization by proving that the utility increases in s if its value is less than $\frac{S}{2N_t}$. We will prove this below.

To prove that the utility increases in s , firstly we prove that an increase in s implies a decrease in Q_t by utilizing the fact that z^* is approximately 0.417 less than 0.5. Then the question whether the utility increases in s boils down to discerning the sign of the left-hand side of equation (17) with the appropriate value of Q_t corresponding to the value of $s < s^*$, utilizing the relationship of equation (16), which holds due to the envelop theorem. The sign of the left-hand side of equation (17) represents the change of the utility with respect to a decrease in s in the uncertain adoption case of the extended model. This occurs because s is a decreasing function only in Q_t . It is easy to prove that the sign is negative when Q_t is larger than the optimal value of Q_t (in other words, when s is smaller than s^*). This means that a decrease in s decreases the utility. Thus, we can infer that

if $s^* > \frac{S}{2N_t}$ without the restriction, $s^* = \frac{S}{2N_t}$ will be the optimal solution for s in the extended model.

The rest of the proof is easy. Thus, we can say that the high growth equilibrium converges to the certain adoption case.

In contrast, if $\frac{\bar{N}}{\bar{Q}} < \Phi$ ³⁰, $\frac{N_t}{Q_t}$ decreases over time and these variables will move in the opposite direction. The proof is basically identical to the above. Lastly, if this condition holds with equality, then equation (25) says that $\frac{N_t}{Q_t}$ does not change over time.

³⁰ This inequality together with (24) implies $\sqrt{\frac{Pb}{2(1+\rho)+P}} > \frac{\delta'}{z^*} \Rightarrow P > \frac{2\delta'^2(1+\rho)}{bz^{*2}-\delta'^2}$. In other words, these two inequalities together satisfy the condition in footnote 19 under which agents will adopt all the new technologies even with the option of no adoption in the second period, in the basic model of Section II.

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About the Paper

Yong Jin Kim and Jong-Wha Lee explore the effects of technology change on growth rates of human capital and income, using an overlapping generations model in which identical agents invest in both width and depth of human capital under uncertain environments. This study argues that new technologies with more uncertain characteristics can adversely affect human capital accumulation and income growth, leading an economy to a low growth trap. It emphasizes the important role of public policies in the area of education and technology in making the economy adjust to uncertain technology change.

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