

Zhenxi, Chen; Lux, Thomas

**Working Paper**

## Estimation of sentiment effects in financial markets: A simulated method of moments approach

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# FinMaP-Working Paper No.37



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FINMAP –

**FINANCIAL DISTORTIONS AND MACROECONOMIC  
PERFORMANCE: EXPECTATIONS, CONSTRAINTS AND  
INTERACTION OF AGENTS**

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**TITLE**

**Estimation of Sentiment Effects in Financial  
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**by: Zhenxi Chen and Thomas Lux**

## ABSTRACT

We take the model of Alfarano et al. (Journal of Economic Dynamics & Control 32, 2008, 101-136) as a prototype agent-based model that allows reproducing the main stylized facts of financial returns. The model does so by combining fundamental news driven by Brownian motion with a minimalistic mechanism for generating boundedly rational sentiment dynamics. Since we can approximate the herding component among an ensemble of agents in the aggregate by a Langevin equation, we can either simulate the model in full at the micro level, or investigate the impact of sentiment formation in an aggregate asset pricing equation. In the simplest version of our model, only three parameters need to be estimated. We estimate this model using a simulated method of moments (SMM) approach. As it turns out, sensible parameter estimates can only be obtained if one first provides a rough "mapping" of the objective function via an extensive grid search.

Due to the high correlations of the estimated parameters, uninformed choices will often lead to a convergence to any one of a large number of local minima. We also find that even for large data sets and simulated samples, the efficiency of SMM remains distinctly inferior to that of GMM based on the same set of moments. We believe that this feature is due to the limited range of moments available in univariate asset pricing models, and that the sensitivity of the present model to the specification of the SMM estimator could carry over to many related agent-based models of financial markets as well as to similar diffusion processes in mathematical finance.

*Keywords: simulation-based estimation; herding; Agent-based model; model validation*

*JEL: C14; C15; F31*

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# Estimation of Sentiment Effects in Financial Markets: A Simulated Method of Moments Approach\*

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## Abstract

We take the model of Alfarano et al. (Journal of Economic Dynamics & Control 32, 2008, 101 – 136) as a prototype agent-based model that allows reproducing the main stylized facts of financial returns. The model does so by combining fundamental news driven by Brownian motion with a minimalistic mechanism for generating boundedly rational sentiment dynamics. Since we can approximate the herding component among an ensemble of agents in the aggregate by a Langevin equation, we can either simulate the model in full at the micro level, or investigate the impact of sentiment formation in an aggregate asset pricing equation. In the simplest version of our model, only three parameters need to be estimated. We estimate this model using a simulated method of moments (SMM) approach. As it turns out, sensible parameter estimates can only be obtained if one first provides a rough "mapping" of the objective function via an extensive grid search. Due to the high correlations of the estimated parameters, uninformed choices will often lead to a convergence to any one of a large number of local minima. We also find that even for large data sets and simulated samples, the efficiency of SMM remains distinctly inferior to that of GMM based on the same set of moments. We believe that this feature is due to the limited range of moments available in univariate asset pricing models, and that the sensitivity of the present model to the specification of the SMM estimator could carry over to many related agent-based models of financial markets as well as to similar diffusion processes in mathematical finance.

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# 1 Introduction

By taking into account features such as the existence of heterogeneous agents with different trading strategies, bounded rationality or interactions among agents, behaviorally motivated models of financial markets have undergone a burgeoning development over the past two decades. Quite a number of these models are able to replicate and, therefore, explain the documented stylized facts of financial markets, including fat tails and temporal dependence of volatility. Recent surveys of this literature can be found in Hommes (2006), LeBaron (2006) and Lux (2009b) among others.

This literature got started by Day and Huang (1990) who model a market populated by fundamentalists and chartists to study randomly alternating bullish and bearish market episodes. Kirman (1993), De Grauwe et al. (1995) and Lux (1995) add further aspects of traders' interactions such as herd behavior and switching of strategies and already make first attempts to explain selected stylized facts. Brock and Hommes (1998) initiated a related strand of literature based upon a discrete choice framework for agents' choice of strategies. Chiarella and He (2002) additionally allow agents to have different risk attitudes within such a setting.

Most of this literature mainly uses simulation studies to explain some of the stylized facts based on complex nonlinear models. Alfarano and Lux (2007) and Alfarano et al. (2008) are among the few exceptions. They derive analytical solutions for the time-variation of higher moments and related measures of the stylized facts, enabling them to determine the conditions under which these particular features arise. Alfarano and Lux (2007) derive closed form solutions of variance and kurtosis of the return distribution of their model based upon the unconditional distribution of an index of the average expectations of their boundedly rational agents. In their setting, expectation formation is formalized via Kirman's (1993) seminal "ant" model for the herding interactions among agents. Alfarano et al. (2008) further incorporate autonomous changes of sentiment into the dynamics of agents' interactions, in addition to the herding mechanism. They also derive approximate closed form solutions of autocorrelation functions. In a companion paper to ours, Ghonghadze and Lux (2015) derive higher-order approximations for the same moments and explore their applicability in a generalized method of moments (GMM) setting. This paper uses the model of Alfarano et al. (2008) to explore the issue of efficiency of estimation of such a model via the simulated method of moments (SMM).

The robustness of the theoretical models in generating empirical "stylized facts" inspires the empirical application and validation of agent based models. Since much of this literature is based on simula-

tion, the use of simulated moments seems tailor-made for bringing these models to the data. Simulated method of moments has been proposed initially by McFadden (1989), Pakes and Pollard (1989), Lee and Ingram (1991) and Duffie and Singleton (1993). SMM or closely related approaches have been applied in a variety of settings. Molina et al. (2005) use SMM to estimate behavioral parameters in a model of a transportation network. Rahmandad and Sabounchi (2012) apply SMM to investigate population obesity dynamics. Ruge-Murcia (2007) estimates a dynamic stochastic general equilibrium model (DSGE) using SMM. Grammig and Schaub (2014) combine GMM and SMM in estimating an asset pricing model, combined with a DSGE model of the underlying fundamental dynamics.

In a context closely related to ours, Gilli and Winker (2003) already developed a nonlinear optimization technique combining the Nelder-Mead and a threshold acceptance algorithm to estimate Kirman's "ant" model. Winker et al. (2007) propose a criterion function based on moments related to the stylized facts to assess the empirical application of agent based models. Franke (2009) applies such an objective function within an SMM framework to estimate the model of Manzan and Westerhoff (2005). Franke and Westerhoff (2011) also adopt SMM to estimate a structural stochastic volatility model, but instead of the standard SMM setting of Duffie and Singleton (1993) use a bootstrap method for generating the weighting matrix of their moments. Franke and Westerhoff (2012) and Franke and Westerhoff (2014) continue this line of research by estimating different "reduced" types of simple agent-based models and comparing their capability to explain the stylized facts. Jang (2013) uses SMM to estimate the parameters of the model of Alfarano and Lux (2007). He reports a variety of hurdles in the SMM approach such as a rugged and possibly very flat surface of the objective function and difficulty to obtain unique parameter estimates from different initial conditions. Similar problems are also highlighted in a different context by Grammig and Schaub (2014) who point to principal limitations of an SMM approach in the presence of small datasets used to estimate complex theoretical models.

Most of the other applications, however, confine themselves to estimate the agent based models, without systematic exploration of the performance of their estimation algorithm. We here attempt to close this particular gap in the literature by more systematically exploring the performance of various SMM specifications using the model of Alfarano et al. (2008) for an exemplary case study. The model of Alfarano et al. (2008) allows for two different scenarios: a bimodal and unimodal distribution of the sentiment index. It is shown that strong interpersonal communication corresponding to the bimodal distribution typically matches best the empirical data. As this model matches the most prevalent stylized facts, it should be observationally equivalent to a number of alternative specifications. As an added

advantage and in contrast to many related models, analytical moment conditions (at least approximate ones) exist for this model so that we can compare the performance of SMM to a generalized method of moment (GMM) estimator developed in a companion paper (Ghoshadze and Lux, 2015). The availability of GMM results for the same setting provides the opportunity to capture the performance of SMM and GMM for the very same setting, and, therefore, allows an assessment of the relevance of the additional noise induced by simulations. As we will see, this comparison turns indeed out revealing a much lower efficiency of SMM relative to GMM than expected on theoretical grounds. Another contribution of this paper is the development of a systematic approach to handle estimation problems with multiple minima and discontinuous gradients of the objective function. It is found that a systematic "mapping" of the objective function via an extensive grid search is indispensable for determination of sensible initial conditions to start an optimization algorithm for local fine-tuning of parameter estimates. As concerns the details of the SMM estimator, we find that many other variations of our setting (using different or more moments, adopting different weighting matrices) have virtually no influence on the quality of our estimates. We also find that the variability of our estimates decreases much more slowly with sample size than expected, and that the estimation errors remain much higher than those of the GMM estimates even with extremely large simulated samples.

The rest of the paper is organized as follows. Section 2 introduces the theoretical model that basically boils down to a diffusion process of an asset price that combines fundamental factors and sentiment dynamics. Section 3 develops the methodology. After illustrating the surface properties of the objective function, we conduct Monte Carlo simulations to compare the performance of different weighting matrices. Section 4 applies the developed methodology to estimate the parameters of this behavioral model using a broad selection of empirical high-frequency data. Lastly, Section 5 concludes the paper.

## 2 Theoretical Model

In our financial market model, the log price and log fundamental value at time  $t$  are denoted by  $p_t$  and  $F_t$ . There are two groups of investors: fundamentalists and noise traders. The fundamentalist group has  $N_f$  members with average trading volume  $V_f$ . Fundamentalists invest based on the price deviation from the fundamental value. They purchase under-valued assets and sell over-valued ones. Their excess

demand, then, amounts to

$$D^f = N_f V_f (F_t - p_t). \quad (1)$$

The log fundamental value,  $F_t$  is assumed to follow Brownian motions without a drift, so that over finite time intervals, the fundamental value at time  $t + \Delta t$  can be obtained by a Normal distribution ( $F_{t-1}, \sigma_f \Delta t$ ). For a unit time change (i.e., daily data),  $\Delta t = 1$ , this setting is equivalent to a random walk process characterizing the fundamentals:

$$F_{t+1} = F_t + \sigma_f \cdot e_t, \quad (2)$$

where  $\sigma_f$  is the standard deviation of the innovations of the fundamental value and  $e_t \sim iidN(0, 1)$ .

The group of noise traders consists of a finite number of agents  $N_c$ , each of them being in either state 1 or 2, the optimistic or pessimistic state. Agents will buy or sell  $V_c$  units of the asset if they are in state 1 or 2, respectively. The number of agents in the optimistic state is denoted by  $n_t$ . Define the population configuration:  $x_t = 2n_t/N_c - 1$ , which signals a balanced disposition if equal to zero and amounts to optimistic or pessimistic majorities if positive or negative. The process of agents' change of opinion is, in general, also formulated in continuous time. A noise trader can switch between the optimistic and pessimistic states. The herding dynamics is characterized by time-varying transition rates for a pessimistic trader to become an optimistic one ( $\pi_{x,t}^+$ ) and vice versa ( $\pi_{x,t}^-$ ). The transition rates are given by combinations of autonomous switches of opinion (happening with a Poisson intensity  $a$ ) and switches brought about by pair-wise communication (happening with a rate  $b$  that has to be multiplied with the probability of being paired with an agent of the opposite opinion). Formally, this amounts to:

$$\begin{cases} \pi_{x,t}^+ = \frac{N_c - n_t}{N_c} \left( a + b \frac{n_t}{N_c} \right) = (1 - x_t) [2a/N_c + b(1 + x_t)] N_c^2, \\ \pi_{x,t}^- = \frac{n_t}{N_c} \left( a + b \frac{N_c - n_t}{N_c} \right) = (1 + x_t) [2a/N_c + b(1 - x_t)] N_c^2. \end{cases} \quad (3)$$

As shown by Alfarano et al. (2008), the temporal development of the probability density of  $x_t$ ,  $\omega(x)$ , is characterized by the Fokker-Planck or forward Kolmogorov equation:

$$\frac{\partial \omega(x, t)}{\partial t} = \frac{\partial}{\partial t} A(x) \omega(x, t) + \frac{1}{2} \frac{\partial^2}{\partial t^2} (D(x) \omega(x, t)) \quad (4)$$

with drift and diffusion terms:  $A(x) = -2ax$ ,  $D(x) = 2b(1 - x^2) + \frac{4a}{N}$ . As a consequence, the macroscopic dynamics of the agent-based model can be approximated by a continuous-time diffusion



process  $\tilde{x}_{N_c}$ :

$$d\tilde{x}_{N_c}^2 = A(\tilde{x}_{N_c}) dt + \sqrt{D(\tilde{x}_{N_c})} dB_t \quad (5)$$

with  $B_t$  standard Brownian motion, and  $A(\cdot)$  and  $D(\cdot)$  the drift and diffusion defined above. Ethier and Kurtz (1986) show that this approximation obeys:

$$\sup_{t \leq T} |x_{N_c}(t) - \tilde{x}_{N_c}(t)| \leq \Gamma_{N_c}^T \frac{\log N_c}{N_c}$$

with  $\Gamma_{N_c}^T$  a random variable which decays asymptotically as  $N_c^{-2}$ . Since the simulation of the "true" agent-based model would impose too high a computational burden for Monte Carlo simulations, we use this approximation for the numerical implementation of the sentiment part of our model. Note that the quality of the approximation as derived by the above formula indicates that with the high number of agents we typically see interacting in financial market, the discrepancy between  $x_t$  and  $\tilde{x}_{N_c}$  should be negligible. The diffusion approximation will also be discretized with sufficiently small increment  $\Delta t_0$  to not jeopardise its proximity to the true agent based model (ABM)  $x_t$ .

The time varying  $x_t$  affects the excess demand of noise traders according to

$$D^c = N_c V_c x_t. \quad (6)$$

With  $V_c$  the constant trading volume of each noise trader. Under a standard Walrasian price adjustment mechanism, price changes depend on the overall excess demands

$$\begin{aligned} \frac{dp}{dt} &= \beta (D^f + D^c) \\ &= \beta [N_f V_f (F_t - p_t) + N_c V_c x_t]. \end{aligned} \quad (7)$$

Assuming instantaneous market clearing with  $\beta \rightarrow \infty$ , we derive the equilibrium market price driven by the fundamental price and the population configuration of noise traders at time  $t$ ,

$$p_t = F_t + \frac{N_c V_c}{N_f V_f} x_t. \quad (8)$$

Over discrete time intervals, returns can be defined as log price changes according to

$$\begin{aligned}
r_{t+1} &= p_{t+1} - p_t \\
&= F_{t+1} - F_t + \frac{N_c V_c}{N_f V_f} (x_{t+1} - x_t) \\
&= \sigma_f \cdot e_t + \frac{N_c V_c}{N_f V_f} (x_{t+1} - x_t).
\end{aligned} \tag{9}$$

This means that returns are determined by the changes of the fundamental value and the change in noise traders' behavior. Note that this equation could also be seen as a generalization of previous time series models that have introduced sentiment as a linear risk factor in otherwise standard asset pricing models (e.g. Brown and Cliff, 2004). Here, we estimate a model with a nonlinear sentiment component.

In continuous-time, the complete asset pricing model under investigation can be written as

$$\begin{cases}
dF_t = \sigma_f dB_{1,t}, \\
dx_t = -2ax_t dt + \sqrt{2b(1-x_t^2) + \frac{4a}{N_c}} dB_{2,t}, \\
p_t = F_t + \frac{N_c V_c}{N_f V_f} x_t.
\end{cases} \tag{10}$$

where  $B_{1,t}$  and  $B_{2,t}$  are independent standard Wiener processes and the difference between  $x_t$  and  $\tilde{x}_{N_c,t}$  has been suppressed to simplify notation.

The herding process has a mean-reverting drift just because the opinion index is a bounded stochastic process so that any wave of optimistic or pessimistic sentiment will eventually find its end. The nonlinear diffusion shows that for  $b$  relatively high compared to  $a$  the random variation would slow down in the presence of a strong positive or negative majority,  $x \rightarrow \pm 1$ . This shows the persistence of majority opinions that is caused by interpersonal communication reinforcing an existing dominance of one opinion over the other. For  $b > a$ , the agent-based system and its diffusion approximation are characterized by a bimodal distribution of the opinion index  $x_t$ . For weaker interpersonal influence,  $a > b$ , the distribution is unimodal and centered at  $x_t = 0$ .

### 3 Simulated Method of Moments Estimation

For the quantitative analysis of a return time series  $r_t$  with length  $T$ , several elementary statistics or moments are typically used. These moment statistics include the second and fourth moment of

returns ( $E(r^2)$  and  $E(r^4)$ ) as well as the auto-covariance of both raw returns and certain higher powers, e.g., squared returns, at various lags. For univariate asset prices, these conditional and unconditional moments provide the range of inputs one could use in any estimation based on moment matching. At the same time, these moments are quantitative measurements of the well-known stylized facts of fat tails and volatility clustering so that they should be in the center of interest for any model that attempts to explain these salient features. In our estimation approach, we stack the moments of interest in a vector  $m = (m_1, \dots, m_n)'$ . For an L-sample of returns  $r_t$ , define the compact variable  $z_t = (r_t, r_{t-1}, \dots, r_{t-L})$ . Moments for (pseudo-)empirical data are computed as the time average

$$m^{emp} = (1/T_{emp}) \sum_{t=1}^{T_{emp}} m(z_t^{emp}). \quad (11)$$

Similarly, moments of the simulated data can be computed as

$$m^{sim} = (1/T_{sim}) \sum_{t=1}^{T_{sim}} m(z_t^{sim}(\theta, \varepsilon_t)). \quad (12)$$

with  $\theta = (a, b, \sigma_f)$  the vector of parameters that we attempt to estimate, and  $\varepsilon_t$  the joint influence of the two independent Wiener processes of eq. (10). These processes do, in fact, not only enter via the time  $t$  realization, but also over the time discretization steps of the Euler approximation of the two diffusions in eq. (10), but we summarize all stochastic factors that play a role between time  $t - 1$  and time  $t$  by  $\varepsilon_t$ . The length of the simulated data  $T_{sim}$  can be arbitrarily long depending on the budget of computing time. Usually, it should in any case be longer than  $T_{emp}$  such that the ratio of the simulated length to the empirical one,  $R = T_{sim}/T_{emp}$  with  $R \geq 1$ .

The simulated moments  $m^{sim}$  are a function of  $\theta$ . Moment matching requires that  $h_T = m^{emp} - m^{sim}$  should be minimized and as a result, parameter estimates  $\hat{\theta}$  can be obtained as

$$\hat{\theta}_T = \arg \min_{\theta} h_T(\theta)' W_T h_T(\theta), \quad (13)$$

where  $W_T$  is a positive definite and possibly random weighting matrix which should reflect the different degrees of precision in the measurement of the different moments. Lee and Ingram (1991) and Duffie and Singleton (1993) show that under general regularity conditions, the SMM estimator for  $\theta$  is asymptotically consistent.

The most efficient estimator is obtained with the choice of  $W_T$  as the inverse of an unbiased and

efficient estimator of the variance-covariance matrix of the moment conditions  $h_T(\theta)$ . A standard estimator would, for instance, be the Newey-West estimator

$$\left\{ \begin{array}{l} \widehat{\Omega} = \Gamma_0 + \sum_{j=1}^p \left(1 - \frac{j}{p+1}\right) (\Gamma_j + \Gamma_j'), \\ \text{with } \Gamma_j = \frac{1}{T} \sum_{t=j+1}^T h_{T,t}(\theta) h_{T,t-j}(\theta)', \\ h_{T,t}(\theta) = m(z_t^{emp}) - \frac{1}{T^{sim}} \sum_{t=1}^{T^{sim}} m(z_t^{sim}(\theta, \varepsilon_t)). \end{array} \right. \quad (14)$$

The weighting matrix  $W_T$  is obtained as the inverse of the Newey-West estimator

$$W_T = \left( \left(1 + \frac{1}{R}\right) \widehat{\Omega} \right)^{-1}, \quad (15)$$

where the factor  $\frac{1}{R}$  captures the additional variability for the simulation of moments compared to the use of analytical moments in a GMM estimation.

Franke and Westerhoff (2014) propose as an alternative that the weighting matrix  $W_T$  can be obtained through bootstrapping of the moments in the empirical data. For a given return time series, the bootstrap gives a collection of  $b = 1, \dots, B$ ,  $B$  new time series and the corresponding moments  $m^b$ . The vector of the mean values of the moments in the bootstrapped samples is  $\bar{m} = \frac{1}{B} \sum_{b=1}^B m^b$ . The covariance matrix of the bootstrapped moments is then given by

$$\widehat{\Sigma} = \frac{1}{B} \sum_{b=1}^B (m^b - \bar{m})(m^b - \bar{m})'. \quad (16)$$

The weighting matrix  $W_T$  is again the inverse of the covariance matrix. Franke and Westerhoff's (2014) bootstrap approach does not amount to efficient estimation of the covariance matrix of the moment estimates and, thus, care has to be taken with the asymptotic distribution of the resulting parameter estimates.

Note that our model, eq. (10), should obey standard "regularity conditions" for GMM and SMM estimation. First, the agent-based process is Markovian, strictly stationary and ergodic. We can, in fact, derive its asymptotic distribution and at least compute approximations to various moments that can be used to also arrive at approximate moment conditions for the process governing returns. Note, however, that our process has a limiting case that would not obey standard regularity conditions. If we let  $N_c \rightarrow \infty$ , in the bimodal case,  $b > a$ , all probability mass would become concentrated at the boundaries of the support of  $x_t$ , -1 or +1, and hence, positive recurrence of the Markov chain would cease

to prevail in this limit. This can also be seen in the diffusion approximation where the diffusion term would converge to  $2b(1-x_t^2) = 0$  at  $x_t = \pm 1$  so that any trajectory would either get stuck at  $x_t = -1$  or  $x_t = 1$  as absorbing boundaries. It is actually known that the stochastic differential equation that represents the "reduced form" of our ABM in the limit of an infinite number of agents is non-ergodic for  $b > a$  (Larsen and Sørensen, 2007), but assuming any finite number of agents version of the model saves us from this inconvenient scenario. Of course, with a very large number of agents, we might get very close to the limiting case in the sense that the process might for very long time be trapped in the vicinity of one of its outer borders. We, therefore, choose a "moderate" number of agents,  $N_c = 100$ , in our further exploration.<sup>1</sup> Fig. 1 shows that in this case, the agent-based process explores evenly the whole support of  $x_t$  and switches repeatedly between optimistic and pessimistic majorities. The figure also illustrates that these switches are responsible for phases of high and persistent volatility, i.e., the emergence of the stylized facts.

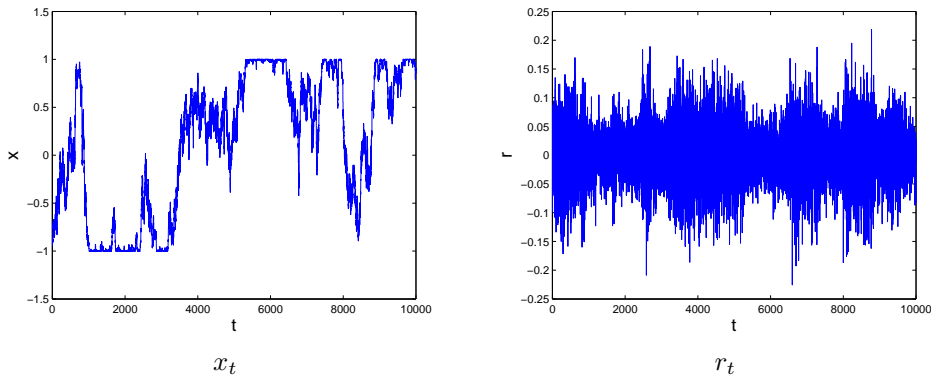


Figure 1: Trajectories of  $x_t$  and  $r_t$ . The underlying parameters of this simulation are those of the bimodal setting, i.e.,  $a = 0.0003, b = 0.0014, \sigma_f = 0.03$ .

### 3.1 Smoothness of the objective function

Before we proceed to develop the estimation methodology, we first conduct a preliminary check of the sensitivity of the moments with respect to the model parameters and investigate the properties of the surface of the objective function. Based on Alfarano et al. (2008),  $a > b$  implies the dominance of

<sup>1</sup>Lux (2009a) estimates a related model directly for a sentiment index of the German economy, and finds that the behavior of the index can be well reproduced once one allows for an "effective" number of agents that is smaller than the notional one. He justifies this adjustment by agents being not as autonomous as the basic model assumes, but actually belonging to a limited number of groups with relatively uniform behavior and interaction in the sense of eq. (3) occurring between these groups.

autonomous sentiment changes of noise traders over their herding tendency, leading to unimodality of the unconditional distribution of the index  $x$  with a peak at  $x = 0$ . On the other hand,  $a < b$  leads to a bimodal distribution of  $x$ . We distinguish between these two settings in checking the smoothness of the objective function. In the following, we choose  $(a, b) = (0.0003, 0.0014)$  and flipped values  $(a', b') = (0.0014, 0.0003)$  to represent the bimodal and unimodal setting, respectively. Table 1 lists the default parameter setting for our pseudo-empirical model to generate price trajectories. The default moment conditions adopted in our evaluations are: second moment of returns  $E(r^2)$  ( $m_1$ ), auto-covariance of returns at lag 1 ( $m_2$ ), auto-covariance of squared returns at lag 1 ( $m_3$ ), and fourth moment of returns  $E(r^4)$  ( $m_4$ ). Note that in this way we capture both excess kurtosis and dependency in higher moments, and a small (negative) autoregressive dependency of raw returns is also in harmony with most empirical finding for high frequency financial data. Also note that for all the trajectories of this paper, by default the first 5,000 periods are discarded to remove the transient phase. The iteration length  $T_{emp}$  is the effective length used in the analysis. We generate the moments  $m^{emp}$  and the weighting matrix  $W$  based on the simulated price trajectories. Set  $\theta = (a, b, \sigma_f)$  as parameters to be estimated. Suppressing the influence of the factor  $\frac{N_c V_c}{N_f V_f}$  in eq. (9) is a consequence of our experience (detailed below) of the simple setting with three parameters only,  $\theta = (a, b, \sigma_f)$ . As it turns out, even in this case we do have to cope with the issue of weak identification due to high correlations of our parameters. Adding  $\frac{N_c V_c}{N_f V_f}$  as a fourth parameter would deteriorate results by so much that the outcomes of our estimation would become almost useless. However, we can interpret  $\frac{N_c V_c}{N_f V_f}$  as a scale factor that can be approximately factored out (in fact, this holds exactly only for the fundamental dynamics).

Table 1: Parameters for price trajectories simulation

	$N_c$	$T_{emp}$	$T_{sim}$	$\Delta t$	$\frac{N_c V_c}{N_f V_f}$	$\sigma_f$	$a$	$b$
bimodal	100	5,000	20,000	0.025	1	0.03	0.0003	0.0014
unimodal	100	5,000	20,000	0.025	1	0.03	0.0014	0.0003

As a starting step to our subsequent exploration of parameter estimation, we investigate how sensitive the chosen moments are to variation of the model parameters. To assess the effect of changing parameters, we have conducted in each case 100 Monte Carlo runs with 20,000 observations (after discarding a transient of 5,000 data points) with identical initial conditions, but changes by 25 percent of each one of the parameters, respectively. We can see from Table 2 that all changes of parameters lead to changes of moments of various magnitudes. Hence, our moment conditions are sensitive with respect to the underlying parameters. All moments are most sensitive to  $\sigma_f$  and least so with respect to  $a$

which might indicate that we should expect different degrees of precision in their estimation. Indeed,  $a$  seems to exert very little effect on most moments in the unimodal regime, and we will find that its estimation comes with a particularly high variance in this case. Other critical points could be high simulation variance of the autocovariance of raw returns and the largely similar reaction of  $m_3$  and  $m_4$  to variations of the parameters.

Table 2: Sensitivity of Moments to Parameter Changes with 100 Monte Carlo runs. Values in parentheses are the standard deviations.

Bimodal setting			
	$a$	$b$	$\sigma_f$
$m_1 = E(r_t^2)$	0.08 (0.069)	0.034 (0.082)	0.291 (0.024)
$m_2 = E(r_t r_{t-1})$	0.138 (3.424)	-0.212 (5.367)	0.317 (2.028)
$m_3 = E(r_t^2 r_{t-1}^2)$	0.149 (0.142)	0.166 (0.176)	0.509 (0.047)
$m_4 = E(r_t^4)$	0.148 (0.142)	0.167 (0.176)	0.509 (0.048)
Unimodal setting			
	$a$	$b$	$\sigma_f$
$m_1 = E(r_t^2)$	0.016 (0.002)	0.079 (0.002)	0.339 (0.003)
$m_2 = E(r_t r_{t-1})$	0.019 (0.123)	0.057 (0.568)	0.39 (1.551)
$m_3 = E(r_t^2 r_{t-1}^2)$	0.032 (0.003)	0.167 (0.005)	0.791 (0.008)
$m_4 = E(r_t^4)$	0.032 (0.003)	0.167 (0.005)	0.792 (0.008)

Next, to reveal the surface properties of the objective function, three-dimensional contour plots of the objective value with respect to pairs of the parameters are plotted. Table 3 lists the grid points of each set of varied parameters. Parameters not varied in plots are set equal to their default values of Table 1. In total, there are three combinations of the parameter pairs for each case.

Table 3: Grid points in three-dimensional contour plot

	label	grid points	step size
bimodal	$\sigma_0$	$1.50 * 10^{-2} + i * h$	$h = 7.50 * 10^{-4}, i = 0, \dots, 40$
	$a$	$1.50 * 10^{-4} + i * h$	$h = 7.50 * 10^{-6}, i = 0, \dots, 40$
	$b$	$7.00 * 10^{-4} + i * h$	$h = 3.50 * 10^{-5}, i = 0, \dots, 40$
unimodal	$\sigma_0$	$1.50 * 10^{-2} + i * h$	$h = 7.50 * 10^{-4}, i = 0, \dots, 40$
	$a$	$7.00 * 10^{-4} + i * h$	$h = 3.50 * 10^{-5}, i = 0, \dots, 40$
	$b$	$1.50 * 10^{-4} + i * h$	$h = 7.50 * 10^{-6}, i = 0, \dots, 40$

Fig. 2 plots the contour figures using the identity matrix as weighting matrix. The surface of the

objective function exhibits differences for the bimodal and unimodal setting. For the bimodal case, the surface exhibits more abrupt changes with pronounced discontinuity of the surface, especially for the plot of  $a$  vs  $b$ . In addition, there are relatively flat regions around the minimum in the plots of  $\sigma_0$  vs  $a$  and  $\sigma_0$  vs  $b$ , suggesting a potential problem of identifying the global minimum of the objective function. For the unimodal case, the three contour plots have relatively more smooth surfaces although the plot of  $a$  vs  $b$  still shows a certain roughness of the objective function. There are again long valleys in the plots of  $\sigma_0$  vs  $a$  and  $\sigma_0$  vs  $b$ , implying strong correlation between these parameters in certain parts of their admissible space. Note that since analytical moments have been derived in Alfarano et al. (2008) and Ghonghadze and Lux (2015), we can exclude strict colinearity. Nevertheless, the apparent proximity to colinear behavior (or weak identification) will be a major concern in our subsequent explorations. Fig. 3 plots the same contours using the inverse of the Newey-West estimator of the covariance matrix as the weighting matrix. The bimodal setting still shows a non-smooth surface very much like in the case of the identity weighting matrix. For the unimodal case, the surface now becomes smoother than in the previous case for the plot of  $a$  vs  $b$ . This indicates that the usual weighting of the moment conditions with an efficient estimate of their respective degrees of precision should already be helpful in overcoming some of the hurdles present in the properties of the simulated data. However, the surface also still exhibits flat valleys of the objective function. We also note that when increasing  $T_{sim}$ , the surface becomes increasingly smooth. Nevertheless, the degree of "roughness" we obtain with as many as 20,000 simulated data points appears remarkable.

The irregular and non-smooth surface of the objective function creates challenges for the estimation. Normal optimization techniques utilizing derivatives are not suitable for such problems as the derivative of the objective function is discontinuous in this case. In the following, we adopt the Nelder–Mead simplex method which is well known for being able to cope in a robust way with nonlinear optimization problems for objective function without a smooth gradient.



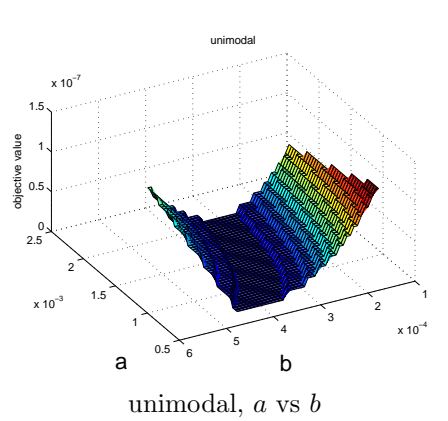
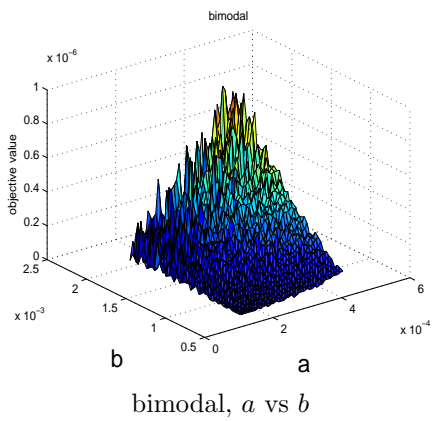
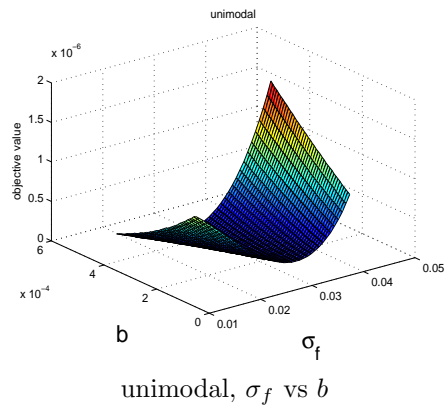
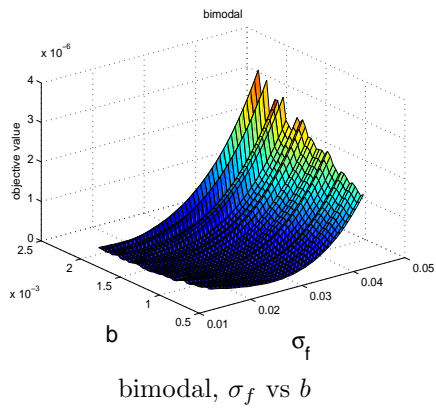
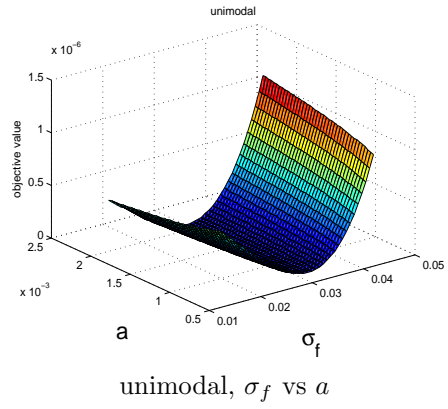
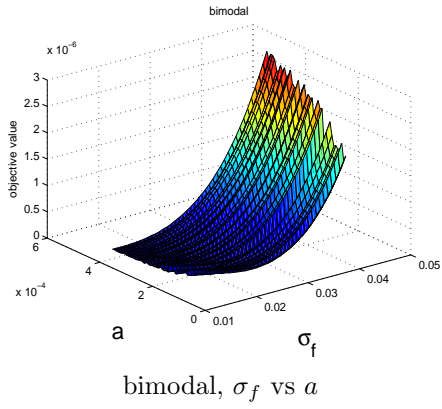


Figure 2: 3-D contour plots of the objective function for the identity matrix used as weighting matrix.

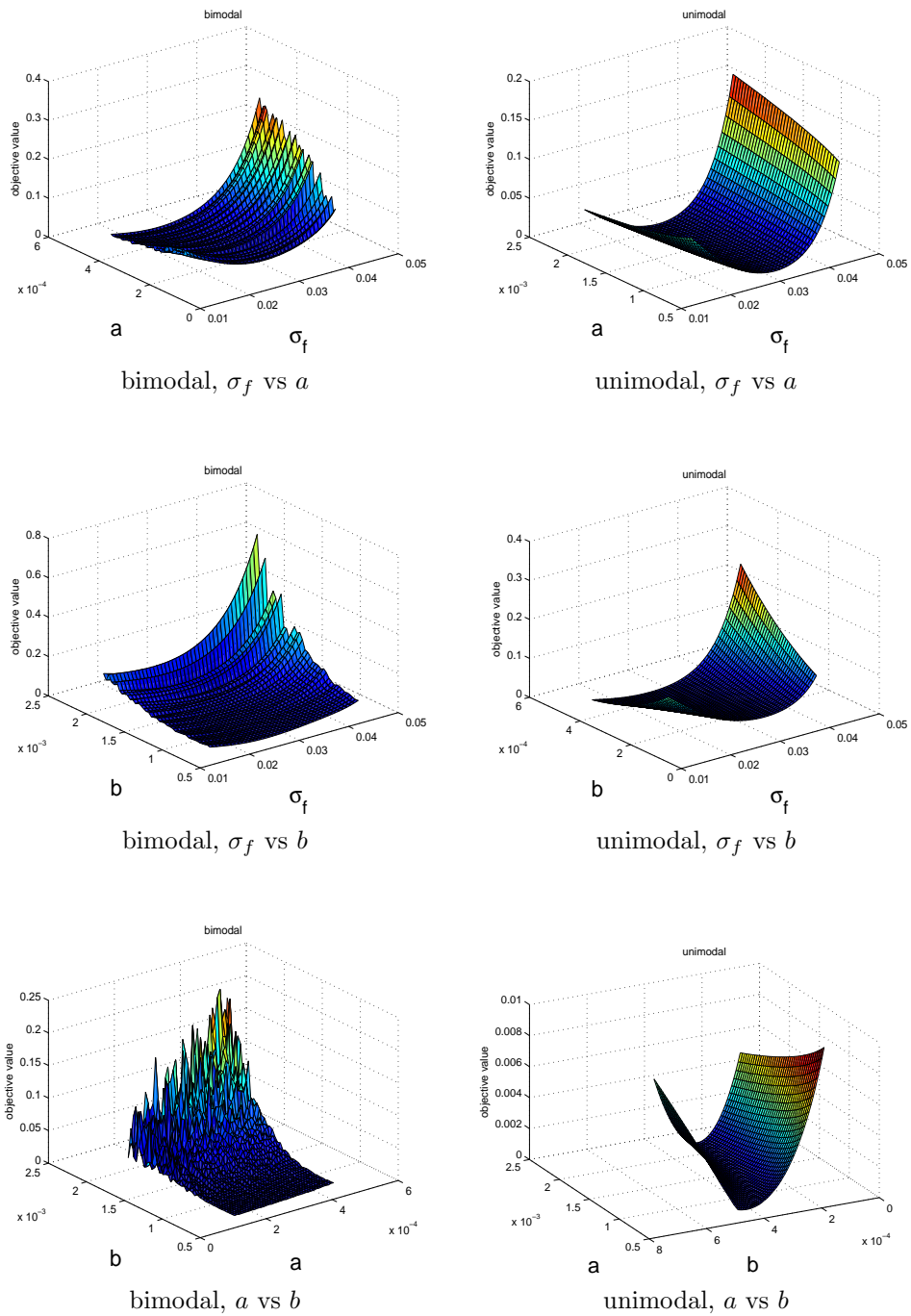


Figure 3: 3-D contour plots of the objective function for the inverse of the Newey-West estimator of the covariance of the moment conditions used as weighting matrix.

## 3.2 Monte Carlo Simulations

When implementing an SMM estimator for the present model, we can choose from a large range of possibilities for the moment conditions and other details of the setting. Note, that for a simulated method of moments approach, an estimator for the covariance matrix might be, for instance, based on both the simulated as well as the empirical data. Indeed, estimating the covariance matrix on the base of simulated data might be preferable since the empirical observations are limited in size whereas no such limit applies to simulated data. Hence, the covariance matrix can be estimated at a higher precision from the simulations that need to be performed anyway (cf. Carrasco and Florens, 2002). To explore the performance of SMM for our simple model, we first proceeded along the classical lines of generalized moment estimators: (1) perform a first round of parameter estimation using the identity matrix as weighting matrix of the moments, (2) estimate a better weighting matrix using the inverse of the estimated covariance matrix of the moments that one obtains as a by-product of step (1), and (3) iterate the estimation process with this new weighting matrix to obtain more efficient parameter estimates.

Unfortunately, our implementation of such a relatively straightforward SMM algorithm showed very limited success initially with pertinent estimators being characterized by huge biases and standard deviations. Inspection showed that due to the lack of smoothness of the simulated objective function as shown in the previous contour plots, we have to cope with multiple local minima as well as with relatively flat surfaces in certain regions of the parameter space. Any standard optimization algorithm could, thus, not be expected to converge to a unique solution from different initial conditions. Our results were by and large similar to those reported by Grammig and Schaub (2014) in a similar baseline SMM approach. Different initial conditions, often led to different parameter estimates that could vary quite sharply across the parameter space. It, thus, seems indispensable to "guide" the optimization phase toward a sensible subset of the parameter space by some kind of preliminary exploration of various combinations of the parameters, and overall, the resulting behavior of the objective function. We, therefore, modified our approach by first conducting a grid search and subsequently initiating a systematic optimization using the Nelder-Mead algorithm with the ten best grid points as initial conditions. The 'empirical' estimate for each such run on a test data set is, then, the parameter set with the lowest value of the objective function from the ten runs of the Nelder-Mead algorithm in phase two of the estimation. The grid points were chosen equidistantly along all three dimensions of the

parameter space. We have anchored the grid around combinations of the parameters  $a$ ,  $b$  and  $\sigma_f$  that together would yield a variance for returns equal to the one of the "pseudo-empirical" test data based on the (slightly generalized) closed-form solution derived by Alfarano et al. (2008):

$$\sigma_r^2 = \sigma_f^2 + \frac{4a}{2\frac{a}{b} + 1}.$$

The fundamental variation has been set equal to  $\sigma_f^2 = k\sigma_r^2$  with  $k = 0.1, 0.2, \dots, 0.9$  to allow for different outcomes in terms of the contribution of fundamental factors and sentiment noise to overall asset price fluctuations. In addition,  $a$  and  $b$  have been varied each over nine equidistant grid points as well, centered at the bifurcation value  $\varepsilon_0 = a/b$  at which the sentiment dynamics switches from unimodal to bimodal. With setting of  $R = T_{sim}/T_{emp} = 4$ , we run Monte Carlo simulation for  $T_{emp} = 5,000$ , 10,000 and 20,000 and corresponding  $T_{sim} = 20,000$ , 40,000 and 80,000, respectively using this design for 100 replications. Results are shown in Table 4. With increasing  $T_{emp}$ , the finite sample standard deviations (FSSE) and root-mean squared errors (RMSE) continue to improve for both bimodal and unimodal settings. However, while the variation declines, it also seems that the biases of all parameters seem to increase slightly.

Table 4: Monte Carlo results for our baseline GMM algorithm with initial grid search,  $R = 4$  and varying  $T_{emp}$ .

	Bimodal			Unimodal		
	$a$	$b$	$\sigma_f$	$a$	$b$	$\sigma_f$
True	0.3	1.4	30	1.4	0.3	30
$T_{emp} = 5,000$						
Mean	0.477	1.542	27.82	1.307	0.657	31.583
FSSE	0.362	0.841	10.167	3.107	1.248	9.798
RMSE	0.402	0.849	10.348	3.092	1.293	9.876
$T_{emp} = 10,000$						
Mean	0.484	1.45	25.238	1.29	0.363	31.353
FSSE	0.271	0.563	9.078	2.758	0.6	9.193
RMSE	0.327	0.563	10.211	2.746	0.6	9.246
$T_{emp} = 20,000$						
Mean	0.495	1.521	24.827	0.869	0.219	32.925
FSSE	0.246	0.453	7.89	1.684	0.208	6.933
RMSE	0.313	0.467	9.402	1.757	0.222	7.493

Notes: the table shows the means, finite sample standard errors (FSSE) and root-mean squared errors (RMSE) of 100 replications of each scenario. Estimated parameters are multiplied by  $10^3$  for better readability.

Inspecting the details, even with such an initial grid search, we have often encountered estimates that were still far off the ‘true’ values in Monte Carlo runs. As we will see below, there is sizable correlation between the three parameters and far-off combinations of parameter values might generate moments that are close to the ones we obtain for a different parameter set. The adjustment of the weighting function might not always be sufficient to leave such ‘traps’ or local valleys of the objective function. In particular, we find that there is a high negative correlation between parameters  $a$  and  $\sigma_f$  (i.e., sentiment and fundamental variance), and since shifting between both will lead to changes of conditional moments, (i.e., volatility persistence), additional changes in  $b$  might bring different combinations of  $a$  and  $\sigma_f$  in line with similar values of measures of volatility persistence. Hence, even in this very simple model, the three parameters might allow for sufficient sensitivity to get close to a sample of moments with different parameter combinations.

Due to the not-fully satisfactory performance of the estimates obtained so far, we also tried various different initializations of the SMM algorithm as well as other choices of moment conditions. For instance, rather than using the identity matrix in the first round and computing the estimate of the covariance matrix of the moment conditions based upon the first-round estimates, we attempted to add a data-driven initial weighting matrix in order to direct the search process toward combinations of estimates and subsequent weighting matrices that do not wander too far away from the ‘true’ values. This follows a proposal by Ghonghadze and Lux (2015) who in a companion paper on GMM estimation showed that a data-driven initial weighting matrix might strongly reduce overall biases and variability of the estimates.

Our approach, thus, consists in first computing an initial weighting matrix as the inverse of the estimated covariance matrix of the moment conditions for the (pseudo-)empirical or test data, and using this weighting matrix for the evaluation of the objective function during the grid search. In the second step, we activate the Nelder-Mead algorithm from the best ten grid points now using an estimated covariance matrix based upon the simulated data as we do in our baseline approach (because of the larger number of available simulated data points). We have also tested the performance of different sets of moment conditions where the default setting uses the second moment, fourth moment and autocovariance of raw returns and squared returns at lag 1 as outlined above. Altogether, we have used four different implementations of the SMM algorithm depending on the estimator of the covariance matrix in step 1 and the moment conditions:

- SMM I uses the default moments statistics. The initial weighting matrix is obtained by the

bootstrap method as proposed by Franke and Westerhoff (2014),

- SMM II uses the same moments, but replaces the bootstrap estimated weighting matrix by a more standard Newey-West heteroskedasticity consistent estimator using the empirical data in the initial grid search,
- SMM III also uses the Newey-West estimate of the covariance matrix as in SMM II, but uses moments of lower order in computing the moment statistics. In particular, the lower moments entering this estimator are: the first absolute moment (absolute returns), the auto-covariances of raw returns and absolute returns, and the second moment. The idea is that replacing squared returns by absolute returns might reduce estimation noise using less variable moment conditions.
- SMM IV uses the same setting of SMM II but differs in the way of calculating the weighting matrix in the optimization phase, namely, computing the covariance matrix from the difference between empirical to simulated moments, not simulated ones only. This corresponds to the standard way of estimating the covariance matrix in GMM with simulated moments replacing the analytical ones.

We evaluate the performance of these four algorithms using again a setting of  $R = 4$ , and  $T_{emp} = 5,000, 10,000$  and  $20,000$ . Based on the summarized statistics of Table 5, among our four configurations, SMM II appears slightly preferable. Note that SMM IV is also supreme for the estimation of parameter  $b$ , especially for the bimodal setting. Particularly hard seems the estimation of the parameter  $a$  in the unimodal scenario with high sentiment noise ( $a = 0.0014$ ) which had also been observed in the baseline GMM set-up of Table 4 and which is in agreement with the low sensitivity of our moments with respect to  $a$ . All in all, however, the difference between both the baseline setting and the four modifications explored in Table 5 seems very small and given the variability of the estimates, probably hardly any of the slight numerical differences could be considered significant.

Table 6, Fig. 4 and Fig. 5 provide additional illustrations of the distortive effects present in all settings. Results here are taken from setting II, but are pretty identical in all other variations of our SMM estimator. Table 6 shows the population correlation matrices for the first parameter set  $\theta$ , i.e., the bimodal setting. Particularly high is the correlation between  $a$  and  $\sigma_f$  which we also find to increase with sample size. Since estimates become more accurate with larger sample size, this indicates that more and more of the residual variation is due to the near-colinearity between these two parameters. In contrast to the GMM implementation, also the correlations between  $a$  and  $b$  and  $b$  and  $\sigma_f$ , respectively, are

relatively large which is in line with the overall finding of less precise estimates in our SMM framework. Figs. 4 and 5 show that the estimates of  $a$  and  $b$  suffer from a pronounced right-hand skewness while  $\sigma_f$  is characterized by left-hand skewness. As we can see, the range and the number of outliers decreases with sample size for all parameters, so that convergence to asymptotic normality seems to assert itself, albeit very slowly. The usual goodness-of-fit statistics, however, seems to be well approximated by its asymptotic  $\chi^2$  distribution at all sample sizes so that the test of the underlying process being an appropriate data-generating process for the selected moments appears to be unbiased. We might note that in all these exercises we did never apply any censoring to our estimates as it has often been done in similar Monte Carlo simulations in the literature. Hence, if the estimate of  $\sigma_f$  turned out to be literally 0, for instance, we did not discard it as non-sensible. The reason is that we do not have any clear prior for the parameters, and attempted to document the complete range of possibilities.

Further experiments also led to results that were virtually identical to those reported in Table 5: First, replacing the noisy auto-correlation of raw returns by another auto-correlation of squared returns (e.g.  $r_t^2 r_{t-5}^2$ ) had no discernible effect. Neither did we have improvements by using sums of autocovariance such as  $\sum_{i=1}^{50} r_t^2 r_{t-i}^2$  to better cover the curvature of the entire dependence structure of volatility. While such moments had dramatically improved the estimations in a GMM setting (Ghonghadze and Lux, 2015), here they also appear ineffective. Finally, using more than four moments rather led to deterioration of results. We have also used antithetic random variables that are known to reduce the variance of simulated samples, but also this modification had no discernible effect on the quality of our estimates. It, thus, appears that the variability of moment conditions in simulations imposes certain restrictions to the quality of our estimation that cannot be overcome by any combination of the typical information we can extract from a univariate series.

Table 5: SMM Monte Carlo result for  $R = 4$  and varying  $T_{emp}$ .

	SMM I			SMM II			SMM III			SMM IV		
	$a$	$b$	$\sigma_f$	$a$	$b$	$\sigma_f$	$a$	$b$	$\sigma_f$	$a$	$b$	$\sigma_f$
True	0.3	1.4	30	0.3	1.4	30	0.3	1.4	30	0.3	1.4	30
$T_{emp} = 5,000$												
Mean	0.378	2.021	28.652	0.357	2.059	29.356	0.427	1.941	28.64	0.376	1.799	28.919
FSSE	0.406	1.299	11.888	0.395	1.287	11.882	0.416	1.053	10.308	0.371	0.949	9.94
RMSE	0.412	1.434	11.905	0.397	1.441	11.84	0.433	1.179	10.346	0.377	1.025	9.949
$T_{emp} = 10,000$												
Mean	0.334	1.894	29.858	0.34	1.942	29.698	0.387	1.84	29.052	0.395	1.702	27.124
FSSE	0.264	0.94	8.341	0.281	1.057	8.811	0.343	0.763	8.539	0.31	0.854	10.372
RMSE	0.265	1.058	8.301	0.283	1.183	8.772	0.352	0.877	8.549	0.322	0.902	10.713
$T_{emp} = 20,000$												
Mean	0.357	1.737	28.87	0.356	1.655	28.842	0.318	1.872	30.575	0.387	1.578	27.512
FSSE	0.255	0.804	7.854	0.225	0.675	7.32	0.278	0.618	7.068	0.269	0.645	8.662
RMSE	0.26	0.869	7.896	0.231	0.718	7.374	0.277	0.776	7.056	0.282	0.666	8.971
True	1.4	0.3	30	1.4	0.3	30	1.4	0.3	30	1.4	0.3	30
$T_{emp} = 5,000$												
Mean	2.298	0.399	27.643	2.465	0.385	26.787	2.607	0.456	27.548	3.903	0.29	28.479
FSSE	4.139	0.344	12.144	4.071	0.281	12.238	4.628	0.416	11.342	6.291	0.26	10.633
RMSE	4.215	0.357	12.311	4.188	0.292	12.594	4.76	0.442	11.548	6.741	0.259	10.688
$T_{emp} = 10,000$												
Mean	2.815	0.294	28.526	2.76	0.291	28.976	2.332	0.413	29.048	2.816	0.33	28.854
FSSE	4.387	0.226	9.874	4.532	0.222	9.252	3.525	0.473	9.407	4.652	0.228	8.906
RMSE	4.588	0.225	9.934	4.709	0.221	9.263	3.629	0.484	9.408	4.841	0.229	8.935
$T_{emp} = 20,000$												
Mean	2.682	0.301	28.345	2.523	0.305	29.217	1.973	0.336	29.972	2.545	0.289	29.328
FSSE	3.266	0.204	8.473	3.173	0.215	8.095	2.446	0.271	7.598	3.458	0.216	8.462
RMSE	3.494	0.203	8.592	3.35	0.214	8.092	2.501	0.272	7.56	3.626	0.216	8.446

Notes: Estimated parameters are multiplied by  $10^3$  for better readability. SMM I through SMM IV stand for the different specifications of the simulated method of moments estimator using different moment sets, weighting matrices and designs of the initial grid search as explained in the text. 100 replications have been conducted for each scenario, with the same random numbers used for each setting.



Table 6: Population correlation matrices for bimodal setting with  $R = 4$  and varying  $T_{emp}$ .

	SMM I			SMM II			SMM III		
	$a$	$b$	$\sigma_f$	$a$	$b$	$\sigma_f$	$a$	$b$	$\sigma_f$
$T_{emp} = 5,000$									
$a$	1	-0.421	-0.821	1	-0.431	-0.852	1	-0.449	-0.85
$b$	-0.421	1	0.543	-0.431	1	0.637	-0.449	1	0.558
$\sigma_f$	-0.821	0.543	1	-0.852	0.637	1	-0.85	0.558	1
$T_{emp} = 10,000$									
$a$	1	-0.486	-0.915	1	-0.544	-0.907	1	-0.492	-0.931
$b$	-0.486	1	0.491	-0.544	1	0.553	-0.492	1	0.605
$\sigma_f$	-0.915	0.491	1	-0.907	0.553	1	-0.931	0.605	1
$T_{emp} = 20,000$									
$a$	1	-0.413	-0.96	1	-0.49	-0.94	1	-0.509	-0.952
$b$	-0.413	1	0.443	-0.49	1	0.514	-0.509	1	0.642
$\sigma_f$	-0.96	0.443	1	-0.94	0.514	1	-0.952	0.642	1

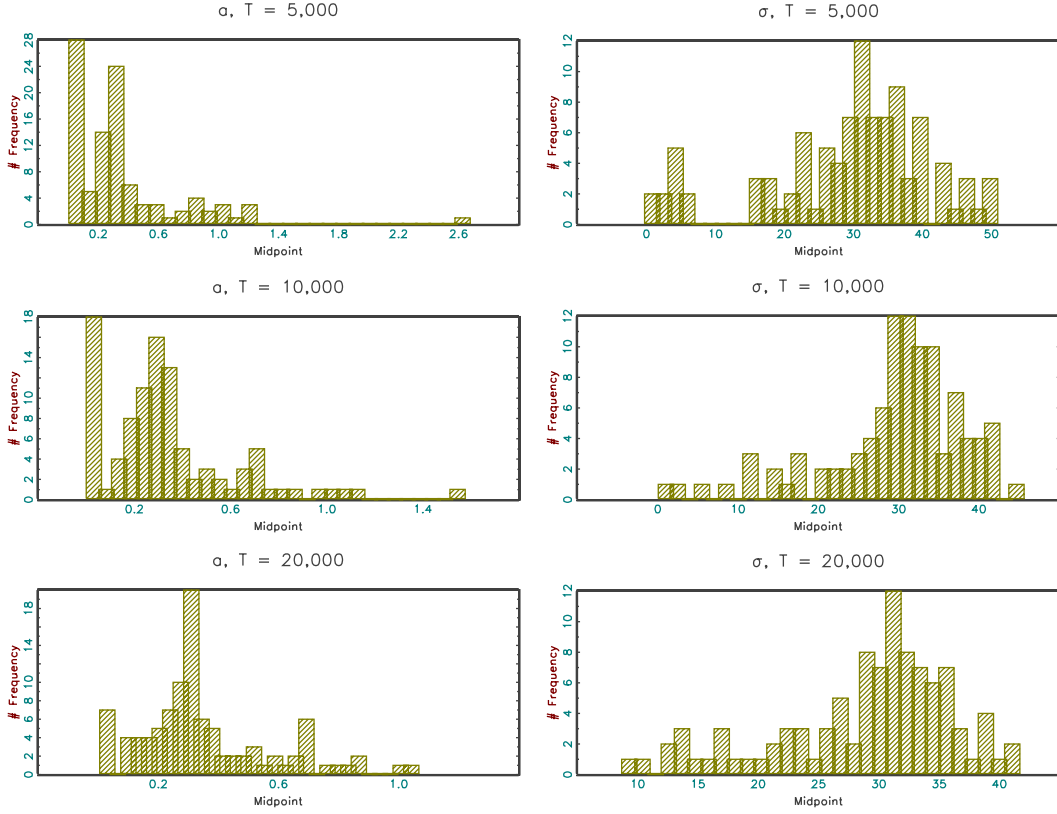


Figure 4: SMM Monte Carlo distribution for parameter set  $\theta$  of bimodal setting,  $R = 4$  and varying  $T_{emp}$  using SMM II. The left and right panels exhibit  $a$  and  $\sigma_f$ , respectively, Parameters are multiplied by  $10^3$  for better readability. The "true" values are  $a = 0.0003$  and  $\sigma_f = 0.03$ .

Note that settings SMM I and SMM II are broadly equivalent to the GMM estimator explored by Ghonghadze and Lux (2015) so that a comparison of the results could help us assess the influence of using simulated rather than analytical moments. Asymptotically, the differences between both estimators are known (cf. Duffie and Singleton, 1993; Carrasco and Florens, 2002) and they should, of course, vanish for simulated sample sizes going to infinity. If  $R \rightarrow \infty$ , the SMM estimator should, thus, converge to the GMM estimator in terms of the distribution of the estimator and the value of the objective function. By fixing  $T_{emp} = 5,000$  and varying  $R = 4, 8$  and  $16$ , we evaluate the effect of  $R$  on the performance of the estimation for SMM II. The results are shown in Table 7.

For the covariance estimator, the simulation noise accounts for a factor  $1/R$  "more" variation compared to the GMM covariance of moment conditions. Choosing factors  $R = 4, 8$ , and  $16$ , our results

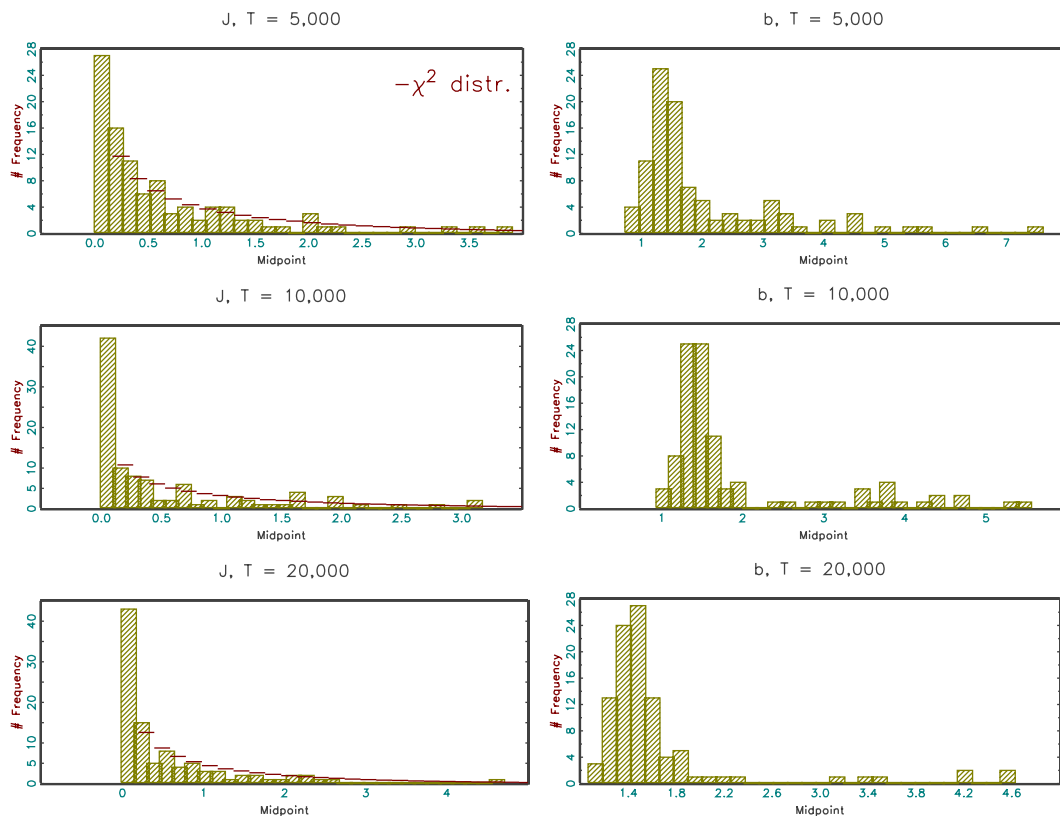


Figure 5: SMM Monte Carlo distribution for parameter set  $\theta$  of bimodal setting,  $R = 4$  and varying  $T_{emp}$  using SMM II. The left and right panels exhibit the J-test statistics and  $b$ , respectively. Also shown is the asymptotic  $\chi^2$  distribution with one degree of freedom to which the J-statistics should converge to. The parameter  $b$  is multiplied by  $10^3$  for better readability. The "true" value of  $b$  is 0.014.

should not be too far from those reported in Ghonghadze and Lux (2015). Hence, we would expect a relative efficiency of our SMM compared to a GMM estimator with the same moments of  $R/(1+R) = 0.8, 0.89, \text{ and } 0.94$  for  $R = 4, 8, \text{ and } 16$ . It, then, seems somewhat unexpected that the FSSEs and RMSEs from Table 7 are mostly at least twice as high compared to the GMM results. Thus, while we see a convergence towards the "true" underlying parameter values, the additional simulation noise leads to still strong distortions compared to analytical moments even for our large sample and simulation sizes. Overall, the influence of simulation noise in our setting is, thus, much stronger than expected from asymptotic theory.

Table 7: SMM Monte Carlo result for fixed  $T_{emp} = 5,000$  and varying  $R$ .

	SMM II		
	$a$	$b$	$\sigma_f$
True	0.3	1.4	30
$R = 4$			
Mean	0.326	2.156	30.917
FSSE	0.365	1.274	9.575
RMSE	0.364	1.476	9.571
$R = 8$			
Mean	0.337	2.096	30.306
FSSE	0.38	1.26	10.327
RMSE	0.38	1.434	10.28
$R = 16$			
Mean	0.26	2.176	32.377
FSSE	0.245	1.457	7.745
RMSE	0.247	1.644	8.064

Notes: the table shows the means, finite sample standard errors (FSSE) and root-mean squared errors (RMSE) of 100 replications of each scenario. Estimated parameters are multiplied by  $10^3$  for better readability. SMM II stands for the specification of the simulated method of moments estimator as explained in the text.

## 4 Empirical Application

In this section, we apply the approach developed in the previous parts of this paper to estimate our model for a number of financial asset indices and prices. The selected empirical datasets include three stock market indices, three foreign exchange rates and the price of gold, all extracted from Datastream. The stock market indices are: the German DAX, the S&P 500, and the Japanese Nikkei. The foreign exchange rates are: U.S. dollar to euro (USD/Euro), U.S. dollar to Japanese yen (USD/YEN) and Swiss franc to euro (CHF/Euro). The three stock indices and the gold price use daily data with sample period from 01/01/1980 to 12/31/2005. Data for the foreign exchange rates include periods 01/01/1999 to 12/31/2010 (USD/Euro), 01/02/1986 to 12/31/2005 (USD/YEN) and 07/15/2003 to 12/31/2010 (Euro/CHF). The approach used in the estimation is SMM IV. We initially used setting II but found a solely simulation-based estimator of the covariance matrix to be problematic in a number of our applications. The reason is that it ended up more often in extreme scenarios. Imagine all three parameters assume very small values. Then the simulation produces a series that is almost flat so that all (co)variances of moments go to zero. Such pathological developments of the estimator were found a number of times in the empirical data, but not in the previous Monte Carlo simulation. Using the standard Newey-West estimate based on empirical and simulated data avoids this effect. We also use another variant of algorithm SMM IV by adding one more round of Nelder-Mead optimization with a new weighting matrix evaluated at the parameters from the first round of SMM optimization. These two algorithms are labeled as 1-step and 2-step estimator, respectively. For the simulations, we set  $R = 4$ , i.e.,  $T_{sim} = 4 \cdot T_{emp}$ . Table 8 shows the estimated results.

Overall, the two algorithms lead to mostly comparable results. The  $J$  statistics accepts the model as possible data generating process for the three stock markets and the exchange rate CHF/EUR. However, among these four markets, only for the DAX and S&P 500 do we find mostly significant parameter values. The  $J$  statistics rejects the model for the USD/EUR at 5% significance level as well as USD/YEN and gold at the 10% significance level while for all other assets, the Alfarano et al. (2008) model cannot be rejected as the "true" moment-generating process. In the case of the Nikkei index, our estimates indicate a purely sentiment-driven process whereas for the USD/EUR rate, we find a pure fundamental determination of the exchange rate. If we compare the estimated values of  $a$  and  $b$ ,  $b$  is always found to exceed  $a$  except for the case of USD/EUR where both estimates are equal to zero. Recall that  $b > a$  indicates a bimodal distribution of the noise sentiment, suggesting a

relatively large herding effect in the interpersonal communication in these markets. Table 9 compares empirical moments with the simulated moments from the estimations. The overall impression is quite mixed, despite non-rejection of the model by the  $J$  test. Results appear worst for the auto-covariance of returns at lag 1 and the exchange rate CHF/EUR which has no significant parameters in the estimation. The best match is typically obtained for the squared returns which is plausible as this should be the least noisy of our moment conditions. The two algorithms have comparable performance in matching the empirical moments for the majority of the empirical datasets and moment conditions. Only a small number of estimates are, however, close to their GMM counterparts in Ghonghadze and Lux (2015), e.g., those for the DAX and for gold. While in the case of GMM estimation different designs resulted in mostly very similar estimates, we find those of our SMM estimates to exhibit larger variation compared to their GMM counterparts. This, presumably, indicates that even with our large simulated samples, simulated moments still display quite some variation around their analytical benchmarks.

## 5 Conclusion

We have investigated simulated method of moment estimation of the model of Alfarano et al. (2008), a simple equilibrium asset pricing model based on the interplay of fundamental factors and a nonlinear sentiment process among the noise traders.

Due to the stochastic nature of simulated moments, the objective function has discontinuous derivatives and a partially very flat surface, implying multiple local minima of the objective function and identification problems. These challenges render the classical simulated moment estimator unsatisfactory as even with long data sets we observe strong sensitivity of estimation results on the starting values for the optimization. To tackle these challenges, we develop a systematic approach embedding a grid search phase for initialization followed by systematic SMM estimation. In our SMM estimation, we compare different weighting matrices generated by Newey-West or bootstrap estimators. It turns out that various specifications of the weighting matrix and moment conditions show by and large the same performance.

As it turns out, many different scenarios basically lead to practically indistinguishable results of Monte Carlo simulations. This seems to suggest that the inherent simulation noise provides some principal limitations to the precision of parameter estimates even of a simple model like the present one. Since applicability of asymptotic theory is guaranteed in the present setting, we conclude that

Table 8: J-test and estimated parameters. 1-step and 2-step denote how many rounds of SMM estimation have been used. Values in parentheses are the p-value for the J-test and standard errors for the three parameters. \*, \*\*, and \*\*\* stand for 10%, 5% and 1% significance level.

		J	$a * 10^3$	$b * 10^3$	$\sigma_f * 10^3$
DAX	1-step	0.702 (0.402)	0.024*** (0.004)	0.341*** (0.067)	4.314 (16.354)
	2-step	0.559 (0.455)	0.025 (0.028)	0.41** (0.173)	6.744*** (2.150)
S&P 500	1-step	1.446 (0.229)	0.001 (0.001)	0.229*** (0.038)	8.658*** (3.133)
	2-step	1.296 (0.255)	0.001 (0.002)	0.157*** (0.047)	6.547*** (1.594)
Nikkei	1-step	1.637 (0.201)	0.036 (0.073)	0.215 (0.443)	0.000
	2-step	1.644 (0.2)	0.036 (0.227)	0.215 (1.018)	0.000
USD/EUR	1-step	3.981** (0.046)	0.000	0.000	5.824*** (1.979)
	2-step	3.983** (0.046)	0.022 (179.204)	0.000	5.809*** (2.033)
YEN/USD	1-step	2.984* (0.084)	0.008 (0.1)	0.094*** (0.017)	2.577 (5.371)
	2-step	2.97* (0.085)	0.009 (0.103)	0.093*** (0.017)	2.565 (5.454)
CHF/EUR	1-step	2.531 (0.112)	0.000	0.003 (23.027)	0.108 (17.137)
	2-step	2.531 (0.112)	0.000	0.003 (23.336)	0.137 (17.659)
Gold	1-step	3.049* (0.081)	0.002*** (0.000)	0.333*** (0.076)	5.266 (7.894)
	2-step	3.056* (0.080)	0.002*** (0.000)	0.333*** (0.076)	5.266 (7.832)

much more data will be required than typically available for financial markets to reach the realm of, for instance,  $\sqrt{T}$  scaling of errors or relative efficiency of SMM against GMM as predicted by theory. This is in line with recent findings by Grammig and Schaub (2014) for a different type of asset-pricing model. The reason for this slow improvement of the quality of an estimator with both increasing sample size and increasing size of simulation might be the very nature of the time series and their stylized facts. It is well-known, for example, that also the confidence intervals of tail index estimates are much wider in the presence of heteroskedasticity than predicted by asymptotic theory (Kearns and Pagan, 1997). Since the tail index averages over the extremal part of a density (e.g., the highest 10 percent of observations), it might already be expected to have smaller errors than the single moments used in our setting. Hence, the widening of error bounds compared to GMM or slowing down of convergence rates as we observed

Table 9: Comparison of empirical moments and their counterparts in the simulated series. 1-step and 2-step denote how many rounds of SMM estimation have been used.

	$m_1 = E(r^2) * 10^4$	$m_2 = E(r_t r_{t-1}) * 10^7$	$m_3 = E(r_t^2 r_{t-1}^2) * 10^8$	$m_4 = E(r^4) * 10^8$
DAX	1.751	-2.36	8.649	30.185
1-step	1.68	-14.443	8.666	25.839
2-step	2.608	-11.029	14.827	45.669
S&P 500	1.089	20.103	6.74	50.19
1-step	1.679	4.782	5.804	16.754
2-step	1.733	-0.554	4.249	13.783
Nikkei	1.593	-15.437	8.172	30.611
1-step	1.349	-10.978	4.191	13.199
2-step	1.349	-10.978	4.191	13.199
USD/EUR	0.403	8.572	0.189	0.986
1-step	0.34	-3.734	0.112	0.35
2-step	0.339	-3.721	0.111	0.349
USD/YEN	0.516	-9.084	0.665	2.856
1-step	1.58	-36.334	2.574	8.088
2-step	1.569	-36.054	2.535	7.965
CHF/EUR	0.096	4.178	0.034	0.114
1-step	0.061	-1.801	0.004	0.012
2-step	0.061	-1.811	0.004	0.012
Gold	1.376	-64.243	8.954	45.983
1-step	1.538	-6.27	8.283	23.453
2-step	1.538	-6.27	8.283	23.453

it for our various SMM estimators might have its source in the very nature of the stylized facts that the present model and similar ones attempt to capture. Precision of univariate asset pricing models via (simulated) moment matching might, thus, face some principal limitations.



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