

Kessing, Sebastian G.; Schneider, Benny

**Working Paper**

## Regional investment and individual redistribution in a federation

Volkswirtschaftliche Diskussionsbeiträge, No. 168-14

**Provided in Cooperation with:**

Fakultät III: Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht, Universität Siegen

*Suggested Citation:* Kessing, Sebastian G.; Schneider, Benny (2014) : Regional investment and individual redistribution in a federation, Volkswirtschaftliche Diskussionsbeiträge, No. 168-14, Universität Siegen, Fakultät III, Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht, Siegen

This Version is available at:

<https://hdl.handle.net/10419/108827>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



Volkswirtschaftliche Diskussionsbeiträge  
Discussion Papers in Economics

No. 168-14

October 2014

Sebastian G. Kessing · Benny Schneider

**Regional Investment and Individual Redistribution in a  
Federation**

Universität Siegen  
Fakultät III  
Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht  
Fachgebiet Volkswirtschaftslehre  
Hölderlinstraße 3  
D-57068 Siegen  
Germany

<http://www.uni-siegen.de/fb5/vwl/>

ISSN 1869-0211

Available for free from the University of Siegen website at  
<http://www.uni-siegen.de/fb5/vwl/research/diskussionsbeitraege/>

Discussion Papers in Economics of the University of Siegen are indexed in RePEc  
and can be downloaded free of charge from the following website:  
<http://ideas.repec.org/s/sie/siegen.html>

# Regional Investment and Individual Redistribution in a Federation\*

Sebastian G. Kessing<sup>†</sup>      Benny Schneider<sup>‡</sup>

October 2014

## Abstract

We study the strategic incentives of regional governments to allocate their budget to public investment and to public consumption expenditures against the background of an incentive-compatible redistribution policy set by the central government. Regional investment changes the productivity distribution in the economy, which affects the design of the optimal tax-transfer system by the central government. The strategic incentives can differ between rich and poor regions depending on the nature of the investment. Rich and poor regions both have strategic incentives to reduce investment which increases the productivity of all individuals in a region. For investment which only increases the productivity of a part of the population, rich regions have reduced investment incentives, whereas poor regions have increased strategic incentives to invest. Our results hint at potential benefits of appropriate differentiation of matching grants.

*JEL classification:* H21, H72, H77, H54

*Keywords:* optimal income taxation; regional investment; fiscal federalism.

---

\*We would like to thank Craig Brett, Thiess Büttner, Thomas A. Gresik, Niels Johannesen, Karl-Josef Koch, and Christos Kotsogiannis for comments. We also acknowledge useful feedback from participants at presentations at the IIPF 2013 Congress, the APET 2013 Conference, the "The End of Federalism?" workshop at WZB, and the 2013 Public Economics Conference at the University of Exeter. The usual caveat applies.

<sup>†</sup>University of Siegen and CESifo. E-mail: kessing@vwl.wiwi.uni-siegen.de

<sup>‡</sup>University of Siegen. E-mail: schneider@vwl.wiwi.uni-siegen.de

Corresponding address: University of Siegen, Department of Economics, Hölderlinstrasse 3, 57076 Siegen, Germany. Phone: +49-271-740-3080

# 1 Introduction

Federal states largely assign the government's redistribution function to the central government. Regional governments play a minor role for redistribution, but are very active in the local provision of public goods and services and are largely responsible for public investment. A recent study by Allain-Dupré (2011) reports that more than two thirds of total public investment in OECD countries is carried out by sub-national governments. Accordingly, a key aspect of regional public spending is the decision how to allocate public funds on public goods and services that can be considered public consumption and on those that can be considered public investment (infrastructure investment, spending on education, etc.). Not surprisingly, the fiscal federalism has studied various facets of this decision and has identified a number of determinants of the efficiency of decentralized spending decisions. We add to this literature by considering the potential strategic interaction between regional spending and the centrally determined redistribution. Our analysis shows, how this interaction can generate strategic incentives for regional expenditure decisions. By focusing on the redistribution between individuals via the tax-transfer scheme set by the central government we draw attention to an aspect that has so far been neglected in the fiscal federalism literature.

The intuition of our argument is straightforward. If regions are sufficiently asymmetric, and their citizens are accordingly treated differently by the tax-transfer system, regions have different incentives to manipulate the redistributive system chosen by the central government. Regional governments' expenditure decisions affect the productivity distribution of the regional workforce. Since the productivity distribution is a key element of the central government's problem of designing an incentive-compatible redistribution scheme, the regional expenditure decisions can strategically influence the central redistribution policy. Our analysis reveals these strategic incentives, shows how they depend on the nature of regional public investment, and finds that they can potentially differ between high and low productivity regions.

We consider two different kinds of public investment, and show that the distinction between these can be conceptually important. First, we discuss investment that improves the composition of the workforce. Public spending on higher education, for example, increases the share of high-skilled individuals. Such investment increases the average productivity in the region, but leaves the productivity of individuals, who do not benefit directly, unaffected. We call these investments "specific". Our analysis finds that, for such investment, low productivity regions have a strategic incentive to increase investment, whereas high productivity regions have a strategic incentive to reduce it. Decreasing the number of low productivity individuals makes it less costly to distort their labor supply. This results in a more redistributive policy, which, on average, benefits low productivity regions but harms high productivity regions.

Alternatively, we study the case of public investment that increases wages of all individ-

uals in a region. Examples may be early childhood education for all individuals or improved public infrastructure. We call this kind of investment "general". In this case, we show that high and low productivity regions suffer from a strategic incentive to reduce their investment, in order to manipulate the central government's redistribution scheme in their favor. Low productivity regions trigger additional redistribution between individuals by under-investing, whereas high productivity regions reduce redistribution between individuals by under-investing.

Our findings have important implications for corrective policies by central governments. Besides the classic inter-regional externalities, the design of vertical intergovernmental grants may also take into account the potential strategic incentives of regional governments and internalize these appropriately. The design of matching grants could be tailored to the nature of different productivity-enhancing investments, and a differentiation of matching rates for regional investment expenditures, such as higher matching rates for particularly poor regions, may be optimal.<sup>1</sup>

Our argument is more relevant for federations of a few heterogeneous states or regions (think of Belgium, as an example), or for federations where there are a few large regions that can reasonably be thought to strategically influence the federal government (think of Canada and Ontario, as an example), but it is less relevant in large federations with relatively small sub-national jurisdictions which can hardly influence the central government's redistribution policy through a change in the overall productivity distribution in the population. The strategic incentives also disappear if regions are completely symmetric, since, in this case, all regions are affected alike by the centrally determined tax-transfer scheme.

Inefficient regional investment in federations has received substantial attention in the literature, and several factors causing sub-optimal regional investment levels have already been highlighted. First, regions may insufficiently internalize the positive spill-overs of investment on other regions, Oates (1972). Second, fiscal equalization schemes often leave little incentives for regional governments to invest in their own development and to increase tax revenues, see Smart (1998), Buettner (2006), Koethenbueger (2007), Koethenbueger (2008), Koethenbueger (2011), and Hindriks et al. (2008), among others. Koethenbueger (2007) is closest to our approach since he analyzes ex-post equalizing transfers between regions. Another reason for insufficient regional investment may be found in the problem of soft budget constraints of lower level governments, as studied by Wildasin (1997), Qian and Roland (1998), Pettersson-Lidbom (2010), inter alia.

Our approach also relates to contributions that have extended the analysis of optimal taxation to federal settings with more than a single government tier. Aronsson and Blomquist (2008) study optimal taxation with regional governments providing public goods. Their

---

<sup>1</sup>Similarly, the analysis may also explain the widespread existence of categorical block grants in federations. As discussed by Huber and Runkel (2006), such grants are widely used in many federations. Our analysis suggests that such categorical block grants may also be interpreted as a response to strategic under-investment by sub-national governments.

focus, however, is mostly with the co-occupation of the tax base by the central and regional governments, a point we abstract from in our analysis. Aronsson (2010) also considers optimal taxation in a federal set-up, but the non-linear income taxes are set by the regional governments, whereas the federal level decides on inter-regional transfers. Gordon and Cullen (2012) discuss the optimal allocation of the redistribution function within a federal hierarchy, but also do not consider strategic incentives. Strategic incentives in optimal taxation have been studied by Kessing and Konrad (2006), who show that unions have strategic incentives to restrict working time to achieve higher levels of redistribution, and Martimort (2001), who considers strategic incentives from a political economy perspective. To the best of our knowledge, no study to date has considered the strategic investment incentives of lower level governments to affect the optimal tax-transfer schedule set by the central government.

## 2 The framework

We consider a two-type optimal direct taxation model in the tradition of Stern (1982) and Stiglitz (1982) but introduce two regions  $i = 1, 2$  of equal size. The total population is normalized to one in each region, and there is no migration. There are two productivity types  $j = H, L$  with wage rates  $w_H > w_L$  for each unit of labor supplied, and we denote by  $m_{ij}$  the share of type  $j$  individuals in region  $i$ , and by  $m_j = \sum_{i=1,2} m_{ij}$  the total share of each productivity type. Neither the wage  $w_j$  nor labor supply  $l_j$  are observable by the government. It conditions the tax-transfer system on observable gross income  $y_j = w_j l_j$ . Individuals pay taxes  $T(y_j)$  so that private consumption equals  $x_j = y_j - T(y_j)$ . Preferences are quasi-linear in private consumption  $x_j$

$$U_{ij} = x_j - h(l_j) + v(z_i), \quad (1)$$

where  $z_i$  is a publicly provided good which is consumed locally, and  $v'(\cdot) > 0$ ,  $v''(\cdot) < 0$ . The marginal disutility of labor is positive, increasing and convex,  $h'(l_j) > 0$ ,  $h''(l_j) > 0$ . The utility from the publicly provided good  $z$  enters in an additive way in order to abstract from interactions between the provision level and incentive compatibility, as studied by Boadway and Keen (1993). We also use this simple formulation of preferences to be able to apply comparative static results, which are notoriously difficult to obtain in more general optimal taxation problems, see Weymark (1987), Brett and Weymark (2008a), Brett and Weymark (2008b), and Simula (2010). The central government also determines transfers to the regions  $B_i$ , and these are tax-financed. These transfers are the only source of revenue for the regions.<sup>2</sup> The regions allocate the available funds either to a publicly provided

---

<sup>2</sup>This is an extreme case of vertical fiscal imbalance. In practice, regional or state governments typically have some sources of own revenues. However, vertical fiscal imbalances are common in most federations and have actually increased on average in the recent past, see Eyraud and Lusinyan (2013).

consumption good  $z_i$  or to regional investment  $g_i$ . These investments may be interpreted either as physical investments in the public capital stock or as investments in regional human capital. The prices for  $g$  and  $z$  do not differ between regions and are normalized to one.

We first assume that regional investment increases the share of high productivity individuals in the region. In this case average regional productivity increases due to a change in the composition of the workforce. Higher education is an example of this type of investment, since it increases the number of high-skilled in the region. We call this type of investment "specific". It can be contrasted with "general investment", which directly benefits all individuals in a region, and which we discuss in Section 3. Regions differ in their productivity, which we model by the assumption that the initial share of high-skilled workers  $\bar{m}_{iH}$  is larger in Region 2 than in Region 1

$$m_{1H}(g_1) = \bar{m}_{1H} + m(g_1) \text{ and } m_{2H}(g_2) = \bar{m}_{2H} + m(g_2), \quad (2)$$

where  $\bar{m}_{2H} > \bar{m}_{1H}$  and  $m'(g_i) > 0$ ,  $m''(g_i) < 0$ . This assumption regarding the investment technology  $m(\cdot)$  is a useful benchmark since it equalizes regional marginal productivity at equal investment levels. Of course, if regions differed in their capability to turn public investment into a more productive workforce, our results would need to be adjusted accordingly.

## 2.1 Information-constrained second best

We first establish the information-constrained second best as our benchmark. This may be identified with the optimal policy in a situation in which all taxation and expenditure decisions are taken at the federal level by a benevolent government.<sup>3</sup> The government is restricted by its information limitations, such that the tax-transfer system needs to be incentive-compatible, and we focus on the case where incentive compatibility is binding downwards. Moreover, the government cannot regionally differentiate the tax-transfer system. Finally, we require the government to split the budget equally between the two regions ("equal splitting"). Our results do not qualitatively depend on this assumption but it allows us to single out more clearly the strategic effects under decentralized expenditure decisions. We discuss in Section 4 that the strategic effects also exist in a more general setting where the central government is free to choose budget shares for each region which are optimal from its encompassing benevolent perspective.

The government has a Utilitarian objective function with the welfare weight  $\mu$ ,  $0 < \mu < 1$ ,

---

<sup>3</sup>Accordingly, this may also be called the centralized regime. However, given that the allocation of the expenditure decision to lower level governments is typically motivated by information advantages of lower-level governments, which are outside the scope of our framework, we prefer to regard this more as a conceptual benchmark against which we contrast the outcome with decentralized spending and centralized redistribution.



for the high-skilled. The information-constrained second best is the solution to

$$\max_{l_L, l_H, x_L, x_H, g_1, g_2, z_1, z_2} W = m_{1L}U_{1L} + \mu m_{1H}U_{1H} + m_{2L}U_{2L} + \mu m_{2H}U_{2H}, \quad (3)$$

subject to

$$m_L(w_L l_L - x_L) + m_H(w_H l_H - x_H) \geq B, \quad (3A)$$

$$x_H - h(l_H) \geq x_L - h(\hat{l}), \quad (3B)$$

$$B/2 = B_1 \geq g_1 + z_1 \text{ and } B/2 = B_2 \geq g_2 + z_2, \quad (3C)$$

where  $B$  is the exogenous level of total government expenditure, and  $\hat{l} \equiv \frac{w_L}{w_H} l_L$  is the labor supply of a "mimicker".<sup>4</sup> While total expenditure  $B$  could also be determined endogenously, we abstract from this aspect, since we are interested in the composition of public spending under decentralized spending, and the results we derive below hold in general for any level of total spending, including the optimal one. The constraint (3A) is the government budget constraint, (3B) is the incentive compatibility constraint and (3C) are the regional budget constraints. Solving the constraints (3A) and (3B) for  $x_L$  and  $x_H$  and the constraints (3C) for  $z_1$  and  $z_2$ , and substituting, the first order conditions with respect to  $g_1$  and  $g_2$  are

$$\left[ \mu U_{1H} - U_{1L} + \frac{M}{2} (T_H - T_L) \right] m'(g_1) = M_1 v'(z_1), \quad (4)$$

$$\left[ \mu U_{2H} - U_{2L} + \frac{M}{2} (T_H - T_L) \right] m'(g_2) = M_2 v'(z_2), \quad (5)$$

where  $M \equiv m_{1L} + \mu m_{1H} + m_{2L} + \mu m_{2H}$ ,  $M_1 \equiv m_{1L} + \mu m_{1H}$ ,  $M_2 \equiv m_{2L} + \mu m_{2H}$ . Moreover, the optimal policy is characterized by the omitted first order conditions with respect to  $l_H$  and  $l_L$ , which render the standard usual "no distortion at the top" result, and also show that the labor supply of the low productivity type is distorted.<sup>5</sup> Equations (4) and (5) provide guidelines on how to allocate investment and consumptive public spending in each region. For both regions, the marginal utility of the publicly provided good evaluated at the average regional social welfare weight should equal the marginal increase of the share of high-skilled multiplied by the benefit of increasing their number in the region. The latter effect is evaluated by the difference in the utility level of high and low-skilled from a social perspective, plus a fiscal term, which takes into account that the high-skilled pay higher taxes. From these first order conditions, we derive our benchmark.

**Proposition 1** *The information-constrained second best with equal budget splitting is characterized by higher public consumption expenditures in the poor region,  $z_1 > z_2$ . The ratio of consumption expenditures over investment expenditures is higher in the poor region.*

<sup>4</sup>Except for the additional introduction of regions and their expenditure decisions, our framework thus corresponds to the standard textbook model discussed, for example, by Salanié (2012, p. 88 ff.).

<sup>5</sup>We only consider interior solutions throughout.

**Proof.** See Appendix A ■

Since the marginal productivity of investment is equal across regions for equal investment levels, the result is driven by the different opportunity costs. These are higher in the low productivity region, because public spending can be used as a targeted redistribution device, which complements redistribution via the tax system and does not affect incentive compatibility.

## 2.2 Decentralized expenditure decisions

In the decentralized case, we assume the following sequence of events. In STAGE 1, the central government splits the exogenous budget by assigning grants  $B_i$  to each region, and we again require  $B_1 = B_2 = B/2$ . In STAGE 2, regional governments decide how much to spend on consumption  $z_i$  and how much on investment  $g_i$ . Finally, in STAGE 3, the central government implements an incentive-compatible tax-transfer scheme and raises the necessary funds to balance the budget. This sequence of events reflects the fact that investment decisions, which affect the productivity distribution, are long-term decisions. Moreover, the up-front transfer by the federal government implies that it is able to commit towards regional governments with respect to such transfers. This allows us to abstract from potential additional strategic incentives caused by an ex-post inter-regional transfer scheme, as considered by Koethenbueger (2007).

Solving backwards, consider STAGE 3 first. The central government observes the outcome of the spending decisions at the regional level and its consequences for the distribution of low and high-skilled in the population. Its problem is to choose  $l_L, l_H, x_L, x_H$  to maximize (3) subject to (3A) and (3B), and taking the regional composition of the workforce as given. From the resulting first order conditions we calculate the comparative statics, see Appendix B.1,

$$\frac{\partial l_L^*}{\partial m_{1H}} = \frac{\partial l_L^*}{\partial m_{2H}} < 0. \quad (6)$$

Intuitively, the lower the share of low-skilled individuals the more these will be distorted by the optimal policy, since the efficiency costs associated with this distortion are decreasing in the low-skilled's share. Note that this reduction in labor supply and gross income of the low-skilled is accompanied by an increase in redistribution, such that the low-skilled are actually made better off under the resulting policy of the central government. Finally, there are no effects on high-skilled labor supply as a consequence of the no distortion at the top result.

Consider next STAGE 2, in which both regions non-cooperatively choose their levels of investment and consumption spending. The regional governments are assumed to be benevolent, but only towards their own citizens. Consider Region 1, the poor region, first.

Using the regional budget constraint  $B_1 = g_1 + z_1$ , the regional government's problem is to

$$\max_{g_1} W_1 = m_{1L} [x_L - h(l_L) + v(B_1 - g_1)] + \mu m_{1H} [x_H - h(l_H) + v(B_1 - g_1)]$$

with the first order condition

$$\left[ (\mu U_{1H} - U_{1L}) + \frac{M_1}{2} (T_H - T_L) + \Omega_1 \frac{\partial l_L^*}{\partial m_H} \right] m'(g_1) = M_1 v'(z_1), \quad (7)$$

where  $\Omega_1 \equiv m_{1L} \left[ \frac{\partial x_L}{\partial l_L^*} - h'(l_L^*) \right] + \mu m_{1H} \frac{\partial x_H}{\partial l_L^*}$ . This condition implicitly defines Region 1's best response to the choice of  $g_2$  by Region 2. The regional policy equates the social marginal utility of regional public consumption with the marginal benefits of increasing the share of high-skilled individuals in the region. The benefits consist of three elements represented by the terms in the square brackets on the LHS of (7). The first is the direct utility increase of turning an additional citizen into a high-skilled individual. The second is the fiscal effect. Comparison of (7) to (4) indicates that this beneficial effect of regional investment is smaller than in the benchmark case, since  $M_1 < M$ . It represents the classic externality of not sufficiently taking the positive spill-overs on the rest of the country into account. Finally, the term  $\Omega_1 \frac{\partial l_L^*}{\partial m_H}$  is the strategic investment effect of forcing the central government to adjust its redistribution policy. It is the product of the marginal effect on the center's policy and the effect of this marginal change on regional welfare. Since  $\frac{\partial l_L^*}{\partial m_H} < 0$  and  $\Omega_1 < 0$ , where the latter follows from central government's optimal policy, see Appendix B.1, we have  $\Omega_1 \frac{\partial l_L^*}{\partial m_H} > 0$ . Thus, for the low productivity region there is a strategic incentive to increase its investment. As the number of low-skilled individuals is reduced, it becomes less costly for the center to distort labor supply of the low skilled and to increase redistribution. Given the center's redistributive objective, the optimal policy becomes more redistributive. This benefits the low productivity region on average.

Region 2 solves an analogous problem with the resulting first order condition

$$\left[ (\mu U_{2H} - U_{2L}) + \frac{M_2}{2} (T_H - T_L) + \Omega_2 \frac{\partial l_L^*}{\partial m_H} \right] m'(g_2) = M_2 v'(z_2), \quad (8)$$

where  $\Omega_2 \equiv m_{2L} \left[ \frac{\partial x_L}{\partial l_L^*} - h'(l_L^*) \right] + \mu m_{2H} \frac{\partial x_H}{\partial l_L^*}$ . Again the region's marginal benefit of increasing its share of high skilled individuals consists of a direct effect of turning a low skilled into a high skilled, a fiscal effect, and a strategic effect. Comparing the fiscal effects in (8) to (5) indicates that, since  $M_2 < M$ , the high productivity region also suffers from the insufficient internalization of investment spill-overs, and even stronger so, since  $M_2 < M_1$ . For the strategic effect for Region 2 we have  $\Omega_2 \frac{\partial l_L^*}{\partial m_H} < 0$ , since  $\frac{\partial l_L^*}{\partial m_H} < 0$  and  $\Omega_2 > 0$ , where the latter follows from the central government's optimal policy, see Appendix B.1. Just as for the low productivity region, increasing the share of high skilled induces the government to distort the low skilled more strongly and to increase redistribution. However, this is to the

high productivity region's disadvantage, on average, creating a strategic incentive to reduce investment.

With regards to relative investment levels in the high and low productivity regions, comparing (7) with (8) shows that under decentralized spending there are the fiscal effects and the strategic effects that favor higher level of investment spending in the poorer region relative to the rich region. However, the comparison of the RHS of (7) and (8) also indicates that the opportunity costs of the low productivity region in terms of foregone public consumption are higher, since it has a higher share of low-income citizens whose marginal benefits from publicly provided consumption have a higher social welfare weight. Which effect dominates is, in general, ambiguous, such that investment can be higher in either region.

For the high productivity region, the comparison of (5) to (8) implies unambiguously lower investment under decentralization relative to the information-constrained second best. The region insufficiently internalizes the fiscal effect and additionally has a strategic incentive to reduce investment. For the low productivity region the outcome is less obvious. Comparison of (4) to (7) shows that its marginal investment benefits are lowered by the insufficient internalization of the fiscal effect, but are increased by the strategic effect. Accordingly, it depends on the relative size of the two effects, whether investment in the low productivity region increases or decreases relative to the information-constrained second best. We summarize these results in our next proposition.

**Proposition 2** *With decentralized public spending and equal budget splitting between the regions, (i) investment in Region 1 (the poor region) can be higher than in Region 2 (the rich region), (ii) in Region 2 (the rich region) the level of investment is lower than in the information-constrained second best, (iii) in Region 1 (the poor region) investment may be higher or lower than in the information-constrained second best.*

Strategic considerations regarding the design of the redistributive policy affect the composition of regional public spending. By reducing the number of low-skilled individuals, regions can strategically induce the central government to increase redistribution to the benefit of the low-skilled and to the disadvantage of the high-skilled. Since this affects asymmetric regions differently, these have diverging strategic incentives.

### 3 General public investment

Consider now the case in which public investment spending does not change the composition of the regional workforce, i.e., it is not specific, but it is general in nature, such that it increases the productivity of all individuals in the region. In order to keep the analysis tractable, we assume perfect correlation between individual productivity and the place of residence, and assume that the high productivity types reside in Region 2 and the low productivity types in Region 1,  $m_{1L} = m_{2H} = 1$  and  $m_{1H} = m_{2L} = 0$ , so that we drop

the subscripts  $H$  and  $L$  from now on.<sup>6</sup> Investment  $g_i$  increases all wages in Region  $i$ , and, analogous to our assumption in Section 2, we assume the following relationship between productive public spending and regional wages

$$w_1(g_1) = \bar{w}_1 + w(g_1) \text{ and } w_2(g_2) = \bar{w}_2 + w(g_2), \quad (9)$$

with  $w'(\cdot) > 0$ ,  $w''(\cdot) < 0$ , and  $\bar{w}_1 < \bar{w}_2$ .<sup>7</sup>

### 3.1 Information-constrained second best

The information-constrained second best benchmark is the solution to

$$\max_{l_1, l_2, x_1, x_2, g_1, g_2, z_1, z_2} W = U_1 + \mu U_2 \quad (10)$$

subject to (3A), (3B), (3C), and again assuming an exogenous level of total spending  $B$ . Solving the constraints (3A) and (3B) for  $x_1$  and  $x_2$  and the conditions (3C) for  $z_1$  and  $z_2$  and substituting, we derive the first order conditions with respect to  $g_1$  and  $g_2$

$$\left[ l_1 + h'(\hat{l}) \frac{l_1}{w_2} + \mu \left( l_1 - h'(\hat{l}) \frac{l_1}{w_2} \right) \right] \frac{w'(g_1)}{2} = v'(z_1), \quad (11)$$

$$\left[ l_2 - h'(\hat{l}) l_1 \frac{w_1}{w_2^2} + \mu \left( l_2 + h'(\hat{l}) l_1 \frac{w_1}{w_2^2} \right) \right] \frac{w'(g_2)}{2} = \mu v'(z_2), \quad (12)$$

along with the conditions for  $l_1$  and  $l_2$ . The RHS of (11) and (12) indicate that investment in the low productivity region again has higher opportunity costs, since the marginal social welfare of public consumption spending is higher. Moreover, on the LHS of equations (11) and (12) we see the effects of increased productivity on consumption. In both regions the latter consists of a direct effect of increased output from more productive workers, as well as of an indirect effect on incentive compatibility. Whereas the direct effect favors more investment in the high productivity region, at least as long as the labor supply from the high-skilled exceeds the labor supply of the low-skilled, which will be the case given our formulation of preferences, the indirect effect via incentive compatibility favors more investment in the low productivity region. We summarize these arguments in our next proposition.

**Proposition 3** *Investment can be higher in either region in the information-constrained second best in case of general investment with equal budget splitting.*

---

<sup>6</sup>We maintain the assumption that the government cannot condition the tax schedule on the region. If this was possible, the central government could implement the first-best by using the region as an additional tag in the design of its tax-transfer schedule. Allowing for different productivity types and different investment levels in each region would result in a more involved structure with more than two types. We discuss such issues further in Section 4.

<sup>7</sup>This formulation is similar to Krause (2006) who studies optimal education policies in an optimal taxation framework.

### 3.2 Decentralized expenditure decisions

With decentralized expenditure decisions, we assume the same sequence of events as in Section 2.2. Consider STAGE 3, where the central government observes the productivity distribution as a result of the regional spending decisions. Its problem is

$$\max_{l_1, l_2, x_1, x_2} W = U_1 + \mu U_2$$

subject to (3A), (3B) and (3C), taking regional productivity  $w_i$  as given. From the resulting first order conditions we calculate the comparative statics  $\frac{\partial l_1^*}{\partial w_1} > 0$  and  $\frac{\partial l_1^*}{\partial w_2} < 0$ , see Appendix B.2. Higher productivity of the low-skilled increases their optimal labor supply, which requires lower marginal tax rates and is accompanied by reduced transfers. Higher productivity of the high-skilled has opposite effects on the labor supply of the low-skilled and on transfers.

Region 1's problem in STAGE 2, given the regional budget constraint  $B_1 = g_1 + z_1$ , is to

$$\max_{g_1} W_1 = x_1 - h(l_1) + v(B_1 - g_1)$$

with the first order condition

$$\left[ l_1 + h'(\hat{l}) \frac{l_1}{w_2} + \left( w_1 + h'(\hat{l}) \frac{w_1}{w_2} - 2h'(l_1) \right) \frac{\partial l_1^*}{\partial w_1} \right] \frac{w'(g_1)}{2} = v'(z_1). \quad (13)$$

Region 2 solves an analogous problem with the first order condition

$$\left[ l_2 + h'(\hat{l}) l_1 \frac{w_1}{w_2^2} + \left( w_1 - h'(\hat{l}) \frac{w_1}{w_2} \right) \frac{\partial l_1^*}{\partial w_2} \right] \frac{w'(g_2)}{2} = v'(z_2). \quad (14)$$

Each region only considers its own marginal benefits of investing and trades them off against the marginal utility of additional consumption of publicly provided goods. In this case we have the following proposition.

**Proposition 4** (i) *Increasing regional investment in either region beyond the level under decentralized investment increases total welfare.* (ii) *Under decentralized spending investment is lower in both regions than in the information-constrained second best.*

**Proof.** Denote by  $\tilde{W} = \tilde{W}(l_1^*(\tilde{g}_1, \tilde{g}_2), l_2^*(\tilde{g}_1, \tilde{g}_2))$  the value function of the welfare maximization problem of the central government at STAGE 3, where  $\tilde{g}_1$  and  $\tilde{g}_2$  are the optimal investment levels corresponding to the regions' maximization problems. Using the Envelope Theorem, the effect of a marginal increase in regional investment beyond the level chosen by

the low productivity region (Region 1) is

$$\frac{\partial \tilde{W}}{\partial \tilde{g}_1} = \left[ \mu l_1 \left( 1 - \frac{h'(\hat{l})}{h'(l_2)} \right) - \left( w_1 + h'(\hat{l}) \frac{w_1}{w_2} - 2h'(l_1) \right) \frac{\partial l_1^*}{\partial w_1} \right] \frac{w'(\tilde{g}_1)}{2} > 0, \quad (15)$$

where we have used (13) to substitute for  $v'(z_1)$ , and since  $\frac{h'(\hat{l})}{h'(l_2)} < 1$ ,  $w_1 + h'(\hat{l}) \frac{w_1}{w_2} - 2h'(l_1) < 0$ , and  $\frac{\partial l_1^*}{\partial w_1} > 0$ . The effect of an increase in regional investment in the high productivity region beyond the level under decentralized spending is

$$\frac{\partial \tilde{W}}{\partial \tilde{g}_2} = \left[ l_2 - h'(\hat{l}) l_1 \frac{w_1}{w_2^2} - \mu \left( w_1 - h'(\hat{l}) \frac{w_1}{w_2} \right) \frac{\partial l_1^*}{\partial w_2} \right] \frac{w'(\tilde{g}_2)}{2} > 0, \quad (16)$$

where we have used (14) in place of  $v'(z_2)$ , and since  $l_1 < l_2$ ,  $\frac{h'(\hat{l})w_1}{w_2^2} < 1$ ,  $w_1 - h'(\hat{l}) \frac{w_1}{w_2} > 0$ , and  $\frac{\partial l_1^*}{\partial w_2} < 0$ . These two effects prove (i). Given that the objective function  $W$  is strictly concave, this also implies that  $\tilde{g}_1 < g_1^*$  and  $\tilde{g}_2 < g_2^*$ , where  $g_1^*$  and  $g_2^*$  are the optimal values in the information-constrained second best, proving (ii). ■

Not surprisingly, the lack of internalization of benefits for the other region results in reduced investment in each region. Moreover, for the rich region, opportunity costs are increased, such that investment is further reduced. Our main interest, however, are the strategic effects caused by the redistribution between individuals carried out by the central government. Contrary to the case of specific investment, both regions suffer from a strategic incentive to reduce their investment as is evident from (13) and (14). Productivity-enhancing investment by the low productivity region changes the equity-efficiency trade-off the central government faces. A more productive workforce in the low productivity region optimally requires lower marginal tax rates for the low productivity workers and reduces the transfers to the low skilled residing in the low productivity region. The regional government therefore has an incentive to reduce productivity enhancing investment relative to the information-constrained second best. Similarly, investment by the high productivity region triggers higher marginal tax rates for the low skilled and consequently reduces their labor supply but increases redistribution, which hurts the high productivity region's citizens. Its government therefore also has a strategic incentive to reduce its investment.

## 4 Discussion and conclusion

Our analysis has highlighted the strategic incentives of regions to manipulate the mix of public consumption and public investment expenditures to influence the tax-transfer scheme set by the central government. Increasing general public investment in the high productivity region implies higher contributions from the local population to the federal tax-transfer scheme. In the low productivity region, higher general public investment reduces the redis-

tributive transfers to the local population. Therefore, both regions have a strategic incentive to reduce general investment. For specific investment that improves the productivity composition of the local workforce, the strategic investment incentives are asymmetric. A high productivity region has a strategic incentive to reduce such investment, since a higher share of high productivity workers results in more redistribution, which, on average, makes the region worse off. A low productivity region, however, has an incentive to increase investment, since a higher share of high productivity individuals increases redistribution, which, on average, benefits the region.

To identify the strategic effects most clearly, we have used the equal splitting assumption for the benchmark cases as well as for the cases with decentralized expenditure decisions. If the center was allowed to adjust expenditure shares freely across the regions, this policy would not be optimal. Typically, it would allocate a larger budget share to the poor region on redistributive grounds, using the region as an expenditure side tagging device. However, also in this case, the strategic effects are present for the regions. Moreover, they cannot be internalized sufficiently by an appropriate design of budget shares. To see this, consider first the case of general investments. With equal splitting both regions under-invest relative to the benchmark case. To increase investment the center needs to increase the budget of both regions, which is obviously not possible for a given overall budget. The same argument holds for the case of specific investment with under-investment in both regions. In the case of specific investment with under-investment in the high productivity region and over-investment in the low-productivity region, the appropriate increase in the budget share of the high productivity region would have to match exactly the necessary budget reduction in the low productivity region. This will not be the case in general, but only happen by coincidence.

We have stated our argument in the simple framework of a standard two-type optimal taxation model. The central government always chooses "no distortion at the top", such that there is no possibility for either region to strategically influence the redistribution through the labor supply of the high-skilled. In a more general model with more than two types, regions have incentives to strategically influence marginal tax rates of all types, except of the most productive one, to influence the tax-transfer scheme in the interest of their own population. Similarly, with a continuum of types as in Mirrlees (1971), the strategic regional incentives also exist. With differences between the regional productivity distribution and the nationwide productivity distribution, incentives to manipulate the productivity distribution are present, and these incentives will differ between investment that improves the productivity of all workers in a region and investment that only improves the productivity at specific productivity levels.

Empirically, it may be rather difficult to separate specific from general investments given that, in a world with more than two productivity types or a continuous productivity distribution, it is rather difficult to identify specific investments that only have a positive impact on



a single or a subset of productivity types. However, the typical government investment data on public physical capital formation probably correspond relatively closely to the general investment case, for which our analysis delivers clear empirical implications. In particular, regional heterogeneity is a necessary condition for the strategic effects to arise. For homogeneous regions the interests regarding the optimal design of the tax-transfer-system are completely aligned, such that no region can gain from strategic manipulation of the redistribution between individuals. Moreover, regions must be sufficiently large to affect the nationwide productivity distribution. Thus, the strategic negative effects on investment should be the more pronounced the more regions differ from the nationwide average and should be increasing in the relative size of a region.<sup>8</sup>

Finally, from a policy perspective, the challenge of inefficient regional investment originating from strategic concerns may be addressed by appropriately tailored federal grants. Differentiated matching grants, which realign the regions' marginal investment costs and benefits with the corresponding trade-off in the information-constrained second best, can restore efficiency. Such a policy requires higher matching rates for states or regions that are substantially poorer or richer than the country average. In practice, richer regions tend to have higher own tax revenues, a point we abstracted from in our analysis. This typically tends to increase their investment, such that higher matching rates only for poor regions may be the optimal policy response.

## References

- Allain-Dupré, D. (2011). Multi-level governance of public investment: Lessons from the crisis. *OECD Regional Development Working Papers*, (2011/5).
- Aronsson, T. (2010). Optimal income taxation and decentralized fiscal federalism. *Regional Science and Urban Economics*, 40(4):187–195.
- Aronsson, T. and Blomquist, S. (2008). Redistribution and provision of public goods in an economic federation. *Journal of Public Economic Theory*, 10(1):125–143.
- Boadway, R. and Keen, M. (1993). Public goods, self-selection and optimal income taxation. *International Economic Review*, 34(3):463–478.
- Brett, C. and Weymark, J. A. (2008a). The impact of changing skill levels on optimal nonlinear income taxes. *Journal of Public Economics*, 92(7):1765–1771.

---

<sup>8</sup>Of course, the size of regions also impacts on the internalization of spill-overs with larger states suffering less from this problem. Accordingly, in a regression with regional investment as the dependent variable, one should use regional size and regional size interacted with the absolute difference in productivity from the country average as explanatory variables, together with an appropriate set of further controls.

- Brett, C. and Weymark, J. A. (2008b). Public good provision and the comparative statics of optimal nonlinear income taxation. *International Economic Review*, 49(1):255–290.
- Buettner, T. (2006). The incentive effect of fiscal equalization transfers on tax policy. *Journal of Public Economics*, 90(3):477–497.
- Eyraud, L. and Lusinyan, L. (2013). Vertical fiscal imbalances and fiscal performance in advanced economies. *Journal of Monetary Economics*, 60(5):571–587.
- Gordon, R. H. and Cullen, J. B. (2012). Income redistribution in a federal system of governments. *Journal of Public Economics*, 96(11-12):1100–1109.
- Hindriks, J., Peralta, S., and Weber, S. (2008). Competing in taxes and investment under fiscal equalization: New directions in fiscal federalism. *Journal of Public Economics*, 92(12):2392–2402.
- Huber, B. and Runkel, M. (2006). Optimal design of intergovernmental grants under asymmetric information. *International Tax and Public Finance*, 13(1):25–41.
- Kessing, S. G. and Konrad, K. A. (2006). Union strategy and optimal direct taxation. *Journal of Public Economics*, 90(1–2):393–402.
- Koethenbueger, M. (2007). Ex-post redistribution in a federation: Implications for corrective policy. *Journal of Public Economics*, 91(3–4):481–496.
- Koethenbueger, M. (2008). Federal tax-transfer policy and intergovernmental pre-commitment. *Regional Science and Urban Economics*, 38(1):16–31.
- Koethenbueger, M. (2011). How do local governments decide on public policy in fiscal federalism? Tax vs. expenditure optimization. *Journal of Public Economics*, 95(11–12):1516–1522.
- Krause, A. (2006). Redistributive taxation and public education. *Journal of Public Economic Theory*, 8(5):807–819.
- Martimort, D. (2001). Optimal taxation and strategic budget deficit under political regime switching. *Review of Economic Studies*, 68(3):573–592.
- Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *The Review of Economic Studies*, 38(2):175–208.
- Oates, W. E. (1972). *Fiscal federalism*. Harcourt Brace Jovanovich, New York.
- Petterson-Lidbom, P. (2010). Dynamic commitment and the soft budget constraint: An empirical test. *American Economic Journal: Economic Policy*, 2(3):154–179.

- Qian, Y. and Roland, G. (1998). Federalism and the soft budget constraint. *The American Economic Review*, 88(5):1143–1162.
- Salanié, B. (2012). *The economics of taxation*. MIT Press, Cambridge (MA), 2 edition.
- Simula, L. (2010). Optimal nonlinear income tax and nonlinear pricing: Optimality conditions and comparative static properties. *Social Choice and Welfare*, 35(2):199–220.
- Smart, M. (1998). Taxation and deadweight loss in a system of intergovernmental transfers. *Canadian Journal of Economics*, 31(1):189–206.
- Stern, N. (1982). Optimum taxation with errors in administration. *Journal of Public Economics*, 17(2):181–211.
- Stiglitz, J. E. (1982). Self-selection and pareto efficient taxation. *Journal of Public Economics*, 17(2):213–240.
- Weymark, J. A. (1987). Comparative static properties of optimal nonlinear income taxes. *Econometrica*, 55(5):1165–1185.
- Wildasin, D. E. (1997). Externalities and bailouts: Hard and soft budget constraints in intergovernmental fiscal relations. *Policy Research Working Paper No. 1843*, World Bank, Washington.

## A Appendix: Proof of proposition 1

**Proof.** The left-hand sides (LHS) of (4) and (5) are *identical* functions of  $z_1$  and  $z_2$ . The right-hand sides (RHS) of (4) and (5) are also functions of  $z_1$  and  $z_2$ , respectively, but are not identical functions. The RHS are falling in  $z_i$ , but for  $z_1 = z_2$ , the RHS of (4) is larger than the RHS of (5),  $M_1 v'(z) > M_2 v'(z)$ . From the necessary second order condition for  $g_2$  of the government's maximization problem we find that

$$\Theta_2'(z_2) m'(g_2) - \Theta_2 m''(g_2) > M_2 v''(z_2) + M_2'(z_2) v'(z_2), \quad (\text{A.1})$$

where  $\Theta_i \equiv [\mu U_{iH} - U_{iL} + \frac{M}{2} (T_H - T_L)]$ . Accordingly, the LHS intersects the RHS of (5) from below, such that, given that the RHS of (4) is bigger than the RHS of (5) for any given  $z$ , the LHS must intersect the RHS of (4) at a higher level of  $z$  than the RHS of (5), such that at the optimum we must have  $z_1^* > z_2^*$ . With equal budget splitting, this implies  $z_1^*/g_1^* > z_2^*/g_2^*$ . ■

## B Appendix: Comparative statics of the central government's optimal tax problem

### B.1 Specific investments

Solving (3A) and (3B) for  $x_L$  and  $x_H$ , and substituting, the first order conditions with respect to  $l_L$  and  $l_H$  of the central government's problem at STAGE 3 are

$$0 = (m_{1L} + m_{2L}) \left[ \frac{\partial x_L}{\partial l_L} - h'(l_L) \right] + \mu (m_{1H} + m_{2H}) \frac{\partial x_H}{\partial l_L} \equiv \Phi^{l_L} \quad (\text{B.1})$$

$$0 = (m_{1L} + m_{2L}) \frac{\partial x_L}{\partial l_H} + \mu (m_{1H} + m_{2H}) \left[ \frac{\partial x_H}{\partial l_H} - h'(l_H) \right] \equiv \Phi^{l_H}. \quad (\text{B.2})$$

We linearize this system denoting the partial derivatives by  $\Phi_t^s$ ,  $s = l_L, l_H$  and  $t = l_L, l_H, g_1, g_2$ ,

$$\begin{aligned} \Phi_{l_L}^{l_L} dl_L + \Phi_{l_H}^{l_L} dl_H + \Phi_{g_1}^{l_L} dg_1 + \Phi_{g_2}^{l_L} dg_2 &= 0 \\ \Phi_{l_L}^{l_H} dl_L + \Phi_{l_H}^{l_H} dl_H + \Phi_{g_1}^{l_H} dg_1 + \Phi_{g_2}^{l_H} dg_2 &= 0 \end{aligned}$$

The comparative statics can be calculated as

$$\frac{\partial l_L^*}{\partial g_1} = - \frac{\begin{vmatrix} \Phi_{g_1}^{l_L} & \Phi_{l_H}^{l_L} \\ \Phi_{g_1}^{l_H} & \Phi_{l_H}^{l_H} \end{vmatrix}}{\begin{vmatrix} \Phi_{l_L}^{l_L} & \Phi_{l_H}^{l_L} \\ \Phi_{l_L}^{l_H} & \Phi_{l_H}^{l_H} \end{vmatrix}} \quad \text{and} \quad \frac{\partial l_L^*}{\partial g_2} = - \frac{\begin{vmatrix} \Phi_{g_2}^{l_L} & \Phi_{l_H}^{l_L} \\ \Phi_{g_2}^{l_H} & \Phi_{l_H}^{l_H} \end{vmatrix}}{\begin{vmatrix} \Phi_{l_L}^{l_L} & \Phi_{l_H}^{l_L} \\ \Phi_{l_L}^{l_H} & \Phi_{l_H}^{l_H} \end{vmatrix}}.$$

By the second order conditions the denominators of both these expressions are positive. First, we calculate the sign of  $\frac{\partial l_L^*}{\partial g_1}$  by

$$\text{sign} \left( \frac{\partial l_L^*}{\partial g_1} \right) = -\text{sign} \begin{vmatrix} \Phi_{g_1}^{l_L} & \Phi_{l_H}^{l_L} \\ 0 & \Phi_{l_H}^{l_H} \end{vmatrix},$$

where the zero entry  $\Phi_{g_1}^{l_H} = 0$  follows from  $w_H = h'(l_H)$ . Since  $\Phi_{g_1}^{l_L} < 0$  and  $\Phi_{l_H}^{l_H} < 0$ , we have

$$\begin{vmatrix} \Phi_{g_1}^{l_L} & \Phi_{l_H}^{l_L} \\ 0 & \Phi_{l_H}^{l_H} \end{vmatrix} = \Phi_{g_1}^{l_L} \Phi_{l_H}^{l_H} > 0, \Rightarrow \frac{\partial l_L^*}{\partial g_1} < 0.$$

Next we calculate the sign of  $\frac{\partial l_L^*}{\partial g_2}$  by

$$\text{sign} \left( \frac{\partial l_L^*}{\partial g_2} \right) = -\text{sign} \begin{vmatrix} \Phi_{g_2}^{l_L} & \Phi_{l_H}^{l_L} \\ \Phi_{g_2}^{l_H} & \Phi_{l_H}^{l_H} \end{vmatrix} = -\text{sign} \begin{vmatrix} \Phi_{g_2}^{l_L} & \Phi_{l_H}^{l_L} \\ 0 & \Phi_{l_H}^{l_H} \end{vmatrix},$$

where the zero entry  $\Phi_{g_2}^{l_H} = 0$  follows from  $w_H = h'(l_H)$ . Since  $\Phi_{g_2}^{l_L} < 0$  and  $\Phi_{l_H}^{l_H} < 0$ , we have

$$\begin{vmatrix} \Phi_{g_2}^{l_L} & \Phi_{l_H}^{l_L} \\ 0 & \Phi_{l_H}^{l_H} \end{vmatrix} = \Phi_{g_2}^{l_L} \Phi_{l_H}^{l_H} > 0 \Rightarrow \frac{\partial l_L^*}{\partial g_2} < 0.$$

## B.2 General investment

Solving (3A) and (3B) for  $x_L$  and  $x_H$ , and substituting, the first order conditions with respect to  $l_1$  and  $l_2$  of the central government's problem at STAGE 3 are

$$0 = \frac{1}{2} \left[ w_1 + \frac{w_1}{w_2} h'(\hat{l}) \right] - h'(l_1) + \mu \frac{1}{2} \left[ w_1 - \frac{w_1}{w_2} h'(\hat{l}) \right] \equiv \Phi^{l_1} \quad (\text{B.3})$$

$$0 = \frac{1}{2} [w_2 - h'(l_2)] + \mu \left[ \frac{1}{2} [w_2 + h'(l_2)] - h'(l_2) \right] \equiv \Phi^{l_2}. \quad (\text{B.4})$$

We are interested in the sign of  $\frac{\partial l_1^*}{\partial g_1}$  and  $\frac{\partial l_1^*}{\partial g_2}$ . We linearize the system of equation (B.3) and (B.4) using the partial derivatives of  $\Phi_t^s$ ,  $s = l_1, l_2$  and  $t = l_1, l_2, g_1, g_2$ .

$$\begin{aligned} \Phi_{l_1}^{l_1} dl_1 + \Phi_{l_2}^{l_1} dl_2 + \Phi_{g_1}^{l_1} dg_1 + \Phi_{g_2}^{l_1} dg_2 &= 0 \\ \Phi_{l_1}^{l_2} dl_1 + \Phi_{l_2}^{l_2} dl_2 + \Phi_{g_1}^{l_2} dg_1 + \Phi_{g_2}^{l_2} dg_2 &= 0 \end{aligned}$$

The comparative statics can be calculated as

$$\frac{\partial l_1^*}{\partial g_1} = - \frac{\begin{vmatrix} \Phi_{g_1}^{l_1} & \Phi_{l_2}^{l_1} \\ \Phi_{g_1}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix}}{\begin{vmatrix} \Phi_{l_1}^{l_1} & \Phi_{l_2}^{l_1} \\ \Phi_{l_1}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix}} \quad \text{and} \quad \frac{\partial l_1^*}{\partial g_2} = - \frac{\begin{vmatrix} \Phi_{g_2}^{l_1} & \Phi_{l_2}^{l_1} \\ \Phi_{g_2}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix}}{\begin{vmatrix} \Phi_{l_1}^{l_1} & \Phi_{l_2}^{l_1} \\ \Phi_{l_1}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix}}. \quad (\text{B.5})$$

By the second order conditions the denominators of both equations in (B.5) are positive. First, we calculate the sign of  $\frac{\partial l_1^*}{\partial g_1}$  by

$$\text{sign} \left( \frac{\partial l_1^*}{\partial g_1} \right) = -\text{sign} \begin{vmatrix} \Phi_{g_1}^{l_1} & \Phi_{l_2}^{l_1} \\ \Phi_{g_1}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix} = -\text{sign} \begin{vmatrix} \Phi_{g_1}^{l_1} & 0 \\ \Phi_{g_1}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix}, \text{ since } \Phi_{l_2}^{l_1} = 0.$$

Since  $\Phi_{g_1}^{l_1} > 0$  and  $\Phi_{l_2}^{l_2} < 0$ , we can sign the determinant

$$\begin{vmatrix} \Phi_{g_1}^{l_1} & 0 \\ \Phi_{g_1}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix} = \Phi_{g_1}^{l_1} \Phi_{l_2}^{l_2} < 0 \Rightarrow \frac{\partial l_1^*}{\partial g_1} > 0.$$

Next, we calculate the sign of  $\frac{\partial l_1^*}{\partial g_2}$  by

$$\text{sign} \left( \frac{\partial l_1^*}{\partial g_2} \right) = -\text{sign} \begin{vmatrix} \Phi_{g_2}^{l_1} & \Phi_{l_2}^{l_1} \\ \Phi_{g_2}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix} = -\text{sign} \begin{vmatrix} \Phi_{g_2}^{l_1} & 0 \\ \Phi_{g_2}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix},$$

where  $\Phi_{l_2}^{l_1} = 0$ . Since  $\Phi_{g_2}^{l_1} < 0$  and  $\Phi_{l_2}^{l_2} < 0$ , we can sign the determinant

$$\begin{vmatrix} \Phi_{g_2}^{l_1} & 0 \\ \Phi_{g_2}^{l_2} & \Phi_{l_2}^{l_2} \end{vmatrix} = \Phi_{g_2}^{l_1} \Phi_{l_2}^{l_2} > 0 \Rightarrow \frac{\partial l_1^*}{\partial g_2} < 0.$$