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Household Bargaining and the Design of Couples’ Income Taxation

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ABSTRACT

Household Bargaining and the Design of Couples’ Income Taxation*

This paper studies the design of couples’ income taxation when consumption and labor supply decisions within the couple are made by maximizing a weighted sum of the spouses’ utilities; bargaining weights are given but specific to each couple. Information structure and labor supply decisions follow the Mirrleesian tradition. However, while the household’s total consumption is publicly observable, the consumption levels of the individual spouses are not observable. With a utilitarian social welfare function we show that the expression for a spouses’ marginal income tax rate includes a “Pigouvian” (paternalistic) and an incentive term. The Pigouvian term favors a marginal subsidy (tax) for the high-weight (low-weight) spouse, whose labor supply otherwise tends to be too low (high). The sign and the magnitude of the incentive term depend on the weight structure across couples. In some cases both terms have the same sign and imply a positive marginal tax for the low-weight spouse (who may be female) and a negative one for the high-weight spouse (possibly the male). This is at odds with the traditional Boskin and Sheshinski results. Our conclusions can easily be generalized to more egalitarian welfare functions. Finally, we present numerical simulations based on a calibrated specification of our model. The calculations confirm that the male spouse may well have the lower (and possibly even negative) marginal tax rate.

JEL Classification: H21, H31, D10

Keywords: couples’ income taxation, household bargaining, optimal income taxation, household labor supply

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1 Introduction

Family taxation rules continue to differ significantly across countries, and their design remains a widely debated issue. While there appears to be a trend towards more “individualized” tax systems in some countries, like France, the systems remain to a substantial part family-based.

Accounting for the family dimension when studying optimal income taxation thus appears to be highly important. Following the seminal papers by Boskin (1975) and Boskin and Sheshinski (1983) the analysis of optimal family income taxation has traditionally been restricted to the framework of only linear instruments.\(^1\) Nonlinear taxation has been studied by some authors like Schroyen (2003), Brett (2007), Kleven \textit{et al.} (2009), Cremer \textit{et al.} (2012) and more recently by Frankel (2014). One of the major issues underlying all these studies is the appropriate choice of the tax unit. Specifically, it has been examined under which conditions “extreme” solutions like pure \textit{joint taxation} (the tax function depends only on the sum of family income), or \textit{individual taxation} (the tax paid by the family is the addition of two tax functions each depending only upon one spouse’s income) arise. While the results are often quite complex (due to the multi-dimensional nature of the problem), it appears fair to say that the extreme solutions arise only under very restrictive conditions. In most realistic settings, one can expect the solution to be in between these extreme cases, and one obtains a general tax function with the two spouses’ incomes as separate arguments. Marginal income tax rates typically differ between spouses, and one spouse’s marginal tax \textit{does} depend on the other spouse’s income.\(^2\)

The more challenging task is to characterize this general tax schedule and to study the factors which affect the spouses’ relative marginal tax rates. Cremer \textit{et al.} (2012), for instance, take a step in that direction and show that depending on the pattern of mating, the celebrated result according to which the spouse with the more elastic labor supply faces the lower marginal tax rate may or may not hold.

\(^1\)Examples are papers by Apps and Rees (1988; 1999), Kleven (2004), or more recently by Alesina, Ichino and Karabarbounis (2011), and Bastani (2013).

\(^2\)More formally, the second order cross derivative of the tax function is not zero.
While each of these studies has its specific features they are all based on a so called “unitary” view of the couple. In reality, however, household decision making is likely to result from some more or less complex bargaining process between spouses. This is often referred to as the collective approach to couples’ decision making; see Chiappori (1988). Integrating this feature into the tax design is not just an intellectual challenge but also has very practical policy implications. For instance, an argument often used in favor of individual taxation schemes is that joint taxation discourages female labor force participation (as it implies a high marginal tax rate for the secondary earner). Such an argument is clearly at odds with the unitary view of the couple. Underlying this claim is the idea that the induced impact on female labor supply (though compatible with a unitary couple’s utility maximization) is in some way not “desirable”.

A first step of integrating a more complex approach to household decision making into the tax design problem is undertaken by Immervoll, Kleven, Kreiner and Verdelin (2014). The authors study how a couple’s bargaining process affects, as compared to the unitary approach, the optimal income tax scheme when labor supply decision are made at the extensive margin. They show that when the government’s weights differ from the spouses’ bargaining weights, the optimal income tax scheme also includes a Pigouvian term which corrects for “suboptimal” participation choices.

This paper extends the study by Immervoll et al. (2014) in two main aspects. First, spouses’ weights differ across couples. This is in line with the empirical literature which shows that income sharing rules within couples depend on wages. This source of heterogeneity turns out to have a significant impact on the sign and magnitude of the optimal marginal tax rate. Second, we do not restrict labor supply decisions to the extensive margin. Instead, we consider a standard Mirrleesian type specification of the spouses’ labor supply decisions which allows for both intensive and extensive margins of adjustments.

More precisely, we consider an economy consisting of $n$ types of couples which are characterized by the wage and bargaining weight of the male and the female spouse. The mating pattern is such that wages of the female and the male spouses are positively (though not perfectly) correlated.\textsuperscript{3} Both spouses have identical preferences over (their

\textsuperscript{3}Assortative mating is commonly observed and has been increasing over the last decades; see Schwartz
individual) consumption and labor supply. Consumption and labor supply decisions within the couple are made in a cooperative way according to some bargaining scheme. Specifically, the couple maximizes a weighted sum of the spouses’ utilities. In this paper, we assume that the spouses bargaining weights (specific to each couple) are exogenously given.4

The information structure is the traditional one in Mirrleesian nonlinear income tax models. Individuals’ wages, bargaining weights and labor supplies are not publicly observable, but before tax income of each spouse is observable. Consequently, the tax schedule can depend on the income levels of the two spouses which are treated as separate arguments. This includes individual taxation (separable tax function) and joint taxation (the tax function depends only total household income) as special cases. While the household’s total consumption is publicly observable, the consumption levels of the individual spouses are not observable.5 The household’s problem is modeled as a two stage optimization process. In the first stage the couple chooses the spouses’ levels of labor supply (gross incomes). In the second stage the allocation of net income (gross income minus tax payments) to the consumption of the two spouses is determined. The same (couple specific) weights are used in both stages.6

We determine the incentive compatible allocation that maximizes utilitarian welfare (the sum of individual utilities) under the information structure described above and study its implementation via a nonlinear tax function based on the income levels of a couples’ spouses. Our utilitarian specification introduces a paternalistic dimension into the optimal tax problem whenever the female and male spouses have different weights in the couple’s bargaining process. In other words, the social objective puts equal weights on all individuals. This is different from most of the existing literature where the couple is treated as a “black box” and where social welfare is defined over the utility functions of unitary households; see for instance Cremer et al. (2012).

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4 This is similar to the objective function used by Immervoll et al. (2011) what they call the collective approach.
5 Total consumption is simply equal to gross income minus the tax, both being observable.
6 Two recent papers, Alesina, Ichino and Karabarbounis (2011), and Bastani (2013) deal with very similar issues, but restrict all instruments to be linear.
The expressions for the optimal marginal tax rates of the spouses include Pigouvian (paternalistic) terms in addition to the more traditional optimal tax (incentive) terms. These Pigouvian terms tend to decrease the marginal tax of the spouse with the higher bargaining weight in the considered couple. If, say, the male spouse has a higher bargaining power the *laissez-faire* solution implies that the husband does not work enough (compared to the utilitarian optimum). This can be corrected through a marginal subsidy on male labor. We show that while the “no distortion at the top result” does not hold, marginal tax rates for the “top” couple (the one with the highest wages to which no incentive constraint is binding) have the same sign as the Pigouvian rates. Consequently, a higher bargaining weight for the male translates into a negative marginal tax rate for the male and a positive one for the female. This is of course at odds with the conventional results à la Boskin and Sheshinski (1983) which are based solely on labor supply elasticity.\(^7\)

For all the other (non “top”) couples optimal tax (incentive) terms reappear which may mitigate or even outweigh the Pigouvian terms. As usual, these incentive terms depend on the relative slope of the mimicking and the mimicked couple’s indifference curves in the space of male and female labor supplies. It is the distortion in this space which will reflect the couple’s relative marginal tax rates. In Cremer *et al.* (2012) results were mainly driven by relative (male and female) labor supply elasticities and the wage gap (ratio between wages). In the current paper elasticity differences are assumed away. Wage gaps continue to be relevant but the relative bargaining weights now also enter the picture. For instance, we show that when the bargaining weight of the low-weight spouse decreases with wages, Pigouvian and incentive terms go in the same direction.\(^8\) Consequently, when the male spouse has the higher bargaining weight his marginal tax rate will be negative in all couples while all female spouses face a positive marginal tax rate. The property that the sign of the incentive terms can be reversed depending on the pattern of bargaining weights represents probably the most dramatic departure of our results both from the unitary couple setting and from the specification with equal

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\(^7\) A Pigouvian term with a similar interpretation is also identified by Immervoll *et al.* (2014)

\(^8\) As long as weight differentials (between couples) dominate wage differentials for the high-weight spouses.
weights of Immervoll et al. (2014). These authors argue that apart from the Pigouvian term, their results with bargaining are similar to those with unitary couples. We show that this is no longer true when bargaining weights differ across couples and the empirical relevance of a possible reversal of the sign of the incentive term is illustrated by our calibrations presented in Section 5.

Our paper is organized as follows. In Section 2 we first analyze how a couple with given bargaining weights allocates disposable income across spouses. Then, we study the couple’s labor supply decisions for a given tax system. The optimization problem of a utilitarian government is examined in Section 3. In Section 4 we consider a social welfare function which applies a concave transformation to individual utilities. This generalizes the utilitarian case to allow for more egalitarian social objectives. Numerical simulations based on a model calibrated on US data are presented in Section 5. Section 6 offers some concluding remarks, while more technical material is relegated to the Appendix.

2 The couple

Consider a population with \( i = 1, \ldots, n \) couples. The proportion of couple \( i \) is \( \pi^i \). Members of the couple are indexed by the subscript \( j = f, m \). Each spouse in couple \( i \) supplies \( \ell^i_j \) units of labor earning a wage rate \( w^i_j \). Gross earnings are given by \( y^i_j = w^i_j \ell^i_j \). The mating pattern is such that spouses’ wages are positively correlated and couples are ordered such that \( w^i_j < w^{i+1}_j \). In other words, a higher index refers to a couple in which both spouses have a higher wage. Consequently, there is a single level of \( w_f \) associated with each level of \( w_m \). The difference in wages between spouses may differ across couples. The utility of a spouse \( j \) in a couple of type \( i \) is given by

\[
U^i_j = u \left( c^i_j \right) - v \left( \ell^i_j \right),
\]

where \( c^i_j \) is the consumption of a numeraire (private) good. Assume \( u' > 0 \) and \( u'' < 0 \) while labor disutility, \( v \), satisfies \( v' > 0 \) and \( v'' > 0 \). Couples act cooperatively, that is, they maximize the weighted sum of spouses’ utilities. The weights attached to the female and male member in couple \( i \) denoted by \( \alpha^i_f \) and \( \alpha^i_m \) sum up to two, \( \alpha^i_f + \alpha^i_m = 2 \).
These weights which reflect the bargaining power of each spouse are *exogenously* given, and a single vector of weights is associated with every vector of wages.\(^9\)

Let \(I^i\) denote the household’s disposable (after tax) income. For any bundle \(\left(I^i, y^i_m, y^i_f\right)\) couple \(i\) solves

\[
\begin{align*}
\max_{c^i_m, c^i_f} & \quad W^i = \sum_{j=m,f} \alpha^i_j \left[u\left(c^i_j\right) - v\left(\frac{y^i_j}{w^i_j}\right)\right] \\
\text{s.t.} & \quad \sum_{j=m,f} c^i_j \leq I^i.
\end{align*}
\]

Maximization of the above problem leads to the following first order condition (FOC)

\[-\alpha^i_m u'\left(c^i_m\right) + \alpha^i_f u'\left(c^i_f\right) = 0.\]  

This equation, along with the budget constraint (3) defines the male’s and female’s consumption levels as functions of their family income, \(c^i_m\left(I^i\right)\) and \(c^i_f\left(I^i\right)\). We have

\[
\begin{align*}
\frac{\partial c^i_m\left(I^i\right)}{\partial I^i} &= \alpha^i_f u''\left(c^i_f\right) > 0, \\
\frac{\partial c^i_f\left(I^i\right)}{\partial I^i} &= \alpha^i_m u''\left(c^i_m\right) > 0,
\end{align*}
\]

since the second order condition (SOC) is negative

\[SOC = \alpha^i_m u''\left(c^i_m\right) + \alpha^i_f u''\left(c^i_f\right) < 0.\]

To simplify notation let us define

\[\hat{u}^i_j\left(I^i\right) \equiv u\left(c^i_j\left(I^i\right)\right)\]

as the indirect subutility of disposable household income for household member \(j\).

Three properties of the couple’s optimal allocation of consumption will be useful for our analysis. First, given \(\left(I^i, y^i_m, y^i_f\right)\), the optimal allocation of consumption depends only on overall income \(I^i\) and on the weights \(\left(\alpha^i_f, \alpha^i_m\right)\) but not on each spouse’s labor

\(^9\)The expressions would remain valid for a more general distribution of weights across couples. However, the determination of the pattern of binding incentive constraints would then be more delicate.
supply and gross income \( (y_m^i, y_f^i) \). This is due to the separability of utility. Second, note that

\[
\sum_{j=f,m} \frac{\partial c_j^i(I^i)}{\partial I^i} = 1. \tag{8}
\]

In words, when a couple’s income increases by one dollar so does the sum of their consumption. Third, the welfare change of an income increase for couple \( i \) is given by

\[
\frac{\partial W^i}{\partial I^i} = \sum_{j=f,m} \alpha_j^i u'(c_j^i(I^i)) \frac{\partial c_j^i(I^i)}{\partial I^i},
\]

which using (4) and (8) yields

\[
\frac{\partial W^i}{\partial I^i} = \alpha_m^i u'(c_m^i(I^i)) = \alpha_f^i u'(c_f^i(I^i)). \tag{9}
\]

## 3 Government’s optimization

Throughout the paper we take a paternalistic approach and consider the utilitarian optimum based on equal weights between husband and wife, \( \alpha_f^i = \alpha_m^i \forall i \). The objective function of the government is thus given by

\[
W^1 = \sum_{i=1}^{n} \pi^i \sum_{j=f,m} \left[ \hat{u}_j^i(I^i) - v \left( \frac{y_j^i}{w_j^i} \right) \right]. \tag{10}
\]

While the government observes each spouse’s (before tax) income \( y_j^i \) (and the distribution of types), it does not observe productivities, labor supplies nor the spouses’ individual consumption levels. Under the considered information structure the government’s instrument consists of a possibly nonlinear income tax scheme \( T^i = T(y_f^i, y_m^i) \) which can be positive or negative. This specification includes joint taxation, \( T^i = T(y_f^i + y_m^i) \), or individual taxation, \( T^i = T(y_f^i) + T(y_m^i) \), as special cases. Observe that while the tax administration knows \( I^i \) (which is by definition equal to gross income minus tax payment) it cannot observe how this consumption budget is allocated between two spouses.
3.1 Couple’s problem and public policy

To study the implementation of the optimal allocation and its implications for the spouses’ respective tax treatments, we first have to revisit the problem of the couple when it faces an income tax schedule \( T(y^i_f, y^i_m) \). Using the indirect utility function \( \hat{u}^i_j(I^i) \) defined by (7) which accounts for the way the couple allocates its disposable income between the spouses, this problem can be stated as follows

\[
\max_{I^i, y^i_j} \quad W^i = \sum_{j=f,m} \alpha^i_j \left[ \hat{u}^i_j(I^i) - v \left( \frac{y^i_j}{w^i_j} \right) \right] \quad (11)
\]

s.t. \[ \sum_{j=f,m} y^i_j - T(y^i_m, y^i_f) - I^i \geq 0. \quad (12) \]

The FOCs of the above problem are given by

\[
\sum_{g=f,m} \alpha^i_g \hat{u}^i_g'(I^i) = \sigma^i, \quad (13)
\]

\[
\frac{\alpha^i_j v_f}{w^i_j} \left( \frac{y^i_j}{w^i_j} \right) = \sigma^i \left( 1 - \frac{\partial T^i}{\partial y^i_j} \right), \quad (14)
\]

where \( \sigma^i \) denotes the Lagrangean multiplier associated with the couple’s budget constraint (12). Making use of (9), equations (13) and (14) can be rewritten as

\[
MRS^i_{y_j} = \frac{1}{w^i_j v^i \left( \frac{\phi^i_j}{c^i_j} \right)} = 1 - T^i_{y_j} \quad \forall \ i, \quad (15)
\]

\[
MRS^i_{y_m} = \frac{\alpha^i_m w^i_j v^i \left( \frac{\phi^i_m}{c^i_j} \right)}{\alpha^i_f w^i_m v^i \left( \frac{\phi^i_f}{c^i_f} \right)} = \frac{1 - T^i_{y_m}}{1 - T^i_{y_f}} \quad \forall \ i, \quad (16)
\]

where \( T^i_{y_j} = \partial T^i/\partial y^i_j \) denotes the marginal tax rate faced by spouse \( j \) in couple \( i \). As usual in optimal tax models this characteristic of the tax function tells us in which direction a spouse’s labor supply is distorted (for a given indifference curve). This distortion is considered in consumption–labor supply space, that is in \((I, y)\)-space. Distortions between the spouses’ respective labor supplies are assessed in \((y_f, y_m)\)-space. When \( MRS^i_{y_f y_m} = 1 \), labor supplies are chosen in an efficient way, that is to minimize
the couple’s disutility of labor for a given total (before tax) income. Expression (16) shows that this distortion is determined by the ratio of one minus the marginal income tax rates. Specifically, when \( T^i_{yf} > T^i_{ym} \) we have \( MRS^i_{yf,ym} > 1 \) and the tax system encourages the male’s labor supply, \( \ell_m \), at the expense of the female’s labor supply, \( \ell_f \). Interestingly, identical marginal tax rates (even if different from zero) imply that the tradeoff between male and female labor supply is not distorted. This would be, for instance, the case under a joint income tax schedule, \( T(y_f + y_m) \).

### 3.2 General solution

We now turn to the determination of the optimal incentive compatible allocation. With the considered information structure feasible allocations must satisfy the following incentive constraint

\[
\sum_{j=f,m} \alpha^i_j \left[ \hat{a}^i_j(I^i) - v \left( \frac{y^i_j}{w^i_j} \right) \right] \geq \sum_{j=f,m} \alpha^i_j \left[ \hat{a}^i_j(I^k) - v \left( \frac{y^i_k}{w^i_k} \right) \right] \quad \forall \ i \neq k. \tag{17}
\]

That is any type-\( i \) couple must be prevented from mimicking any type-\( k \) couple. In addition, the resource constraint

\[
\sum_{i=1}^{n} \pi^i \left( \sum_{j=f,m} y^i_j - I^i \right) \geq 0 \tag{18}
\]

must hold.

The government maximizes (10) subject to the constraints (17) and (18). The La-

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\[^{10}\text{Formally this means that } (y^j_f, y^m_m) \text{ solves}
\]

\[
\min_{y^j_f, y^m_m} \alpha^i_f v \left( \frac{y^i_f}{w^i_f} \right) + \alpha^i_m v \left( \frac{y^i_m}{w^i_m} \right)
\]

\[
\text{s.t. } y^i_f + y^i_m = I^i,
\]

where the total family income \( I^i \) is given.
grangian $\mathcal{L}^1 \equiv \mathcal{L}(I^i, y_j^i, y_m^i)$ can be written as

$$
\mathcal{L}^1 = \frac{\pi^i}{\pi^i} \sum_{j=f,m} \left[ \hat{u}_j^i(I^i) - v \left( \frac{y_j^i}{w_j^i} \right) \right] + \mu \sum_{j=f,m} \left[ \sum_{i=1}^n \alpha_j^i \left[ \hat{u}_j^i(I^i) - v \left( \frac{y_j^i}{w_j^i} \right) \right] \right] - \sum_{i=1}^n \sum_{k=1,k\neq i} \lambda_{ik} \left[ \hat{u}_j^i(I^k) - v \left( \frac{y_j^k}{w_j^k} \right) \right],
$$

where $\mu > 0$ is the Lagrange multiplier of the resource constraint while $\lambda_{ik} \geq 0$ is the Lagrange multiplier associated with the self-selection constraint from a type-$i$ to a type-$k$ couple. The FOCs with respect to $I^i$, $y_j^i$ are given by

$$
\frac{\partial \mathcal{L}^1}{\partial I^i} = \pi^i \sum_{j=f,m} \hat{u}_j^i(I^i)
+ \sum_{k=1,k\neq i} \lambda_{ik} \sum_{g=f,m} \alpha_{gj}^i \hat{u}_g^i(I^i) - \sum_{k=1,k\neq i} \lambda_{ki} \sum_{g=f,m} \alpha_{gj}^k \hat{u}_g^k(I^i) - \mu \pi^i = 0, \tag{20}
$$

$$
\frac{\partial \mathcal{L}^1}{\partial y_j^i} = - \frac{\pi^i}{w_j^i} \left( \ell_j^i \right) - \sum_{k=1,k\neq i} \lambda_{ik} \alpha_{kj}^i \frac{1}{w_j^i} \left( \ell_j^k \right) + \sum_{k=1,k\neq i} \lambda_{ki} \alpha_{kj}^k \frac{1}{w_j^k} \left( \ell_j^k \right) + \mu \pi^i = 0, \tag{21}
$$

where $\hat{u}_j^k(I^i)$ and $\ell_j^k$ denote marginal utility and labor supply of spouse $j$ in couple $k$ when mimicking spouse $j$ in couple $i$.

In Appendix A we show that by combining the two FOCs, the marginal rate of substitution between labor and consumption is given by

$$
MRS_{Iy_j}^i = \frac{\pi^i}{\alpha_j^i} \sum_{g=f,m} \frac{u'(c_{gj}^i)}{u'(c_j^i)} \frac{\partial c_j^i}{\partial I^i} + \sum_{k=1,k\neq i} \lambda_{ik} - \sum_{k=1,k\neq i} \lambda_{ki} \frac{\alpha_j^k \alpha_{kj}^i \frac{u''(c_{kj}^i)}{u'(c_j^i)}}{\alpha_j^i}, \tag{22}
$$

where $c_{kj}^i$ denotes consumption of couple $k$ when mimicking couple $i$. Recall that $c_{gj}^i$, $c_j^i$ and $c_{kj}^i$ are functions of $I^i$; see Section 2.
3.3 Optimal tax policy

From the marginal rate of substitution given by equation (22) we can determine the marginal tax rate of the implementing tax function, which from equation (15), is given by

\[ T_{ij} = 1 - MRS_{ij} \]

As usual in optimal income tax models this expression shows how the marginal tax rate must be set to induce couples to choose the consumption bundle and labor supplies intended for them. This expression is very general and is valid whatever the pattern of binding incentive constraints (i.e., which, if any, of the \( \lambda_{ik} \)’s and \( \lambda_{ki} \)’s are strictly positive). However, the \( \lambda_{ik} \)’s and \( \lambda_{ki} \)’s play a crucial role since the marginal tax rate faced by a specific type depends on the incentive constraints binding to this type and from this type. We will now successively study different configurations starting with the benchmark case where wages are observable. Then, we return to the Mirrleesian information structure with unobservable wages.

3.3.1 Benchmark: observable wages

Let us first study the case where wages are publicly observable implying that the self-selection constraints can be neglected so that \( \lambda_{ik} = \lambda_{ki} = 0 \) \( \forall \ i, j \). Expression (22) then yields the “Pigouvian” tax (subsidy)\(^{11} \)

\[ T_{ij} = T_{ij}^{P} = 1 - \sum_{g=f,m} \frac{u'(c_{ij}^{g})}{u'(c_{ij}^{f})} \frac{\partial c_{ij}^{f}}{\partial \Pi} \]

\(^{11}\)We use the term “Pigouvian” throughout the paper. The tax arises to correct for a difference in social and private welfare. Alternatively, it could be referred to as paternalistic.
Using (8) and (9) the Pigouvian tax (subsidy) for the female and male in couple $i$ can be expressed as follows

$$
T_{yi}^{P_i} = 1 - \frac{\alpha^i_f}{\alpha^i_m} \frac{\partial c^i_m}{\partial I^i} - \frac{\partial c^i_f}{\partial I^i} = \left( 1 - \frac{\alpha^i_f}{\alpha^i_m} \right) \frac{\partial c^i_m}{\partial I^i},
$$

(25)

$$
T_{ym}^{P_i} = 1 - \frac{\alpha^i_m}{\alpha^i_f} \frac{\partial c^i_f}{\partial I^i} - \frac{\partial c^i_m}{\partial I^i} = \left( 1 - \frac{\alpha^i_m}{\alpha^i_f} \right) \frac{\partial c^i_f}{\partial I^i},
$$

(26)

Since $\partial c^i_f / \partial I^i \geq 0$ we have

$$
T_{yi}^{P_i} \leq 0 \iff \alpha^i_f \leq \alpha^i_m,
$$

(27)

$$
T_{ym}^{P_i} \leq 0 \iff \alpha^i_m \leq \alpha^i_f.
$$

(28)

In words, the spouse with the lower bargaining weight faces a marginal tax on labor supply while the spouse with the higher bargaining weight faces a marginal subsidy on labor supply. The intuition behind this result is as follows. The spouse with the lower bargaining power receives a smaller share of the cake (the common income $I^i$) implying a lower consumption (and thus a higher marginal utility of consumption). In the laissez-faire, this negative income effect in turn increases this spouse’s labor supply (above the optimal level that would arise for equal weights). The opposite holds for the spouse with the higher bargaining weight. Roughly speaking, the spouse with the low bargaining weight works too much while the one with the high bargaining weight does not work enough. The Pigouvian tax (subsidy) corrects for these non-optimal labor supply decisions. Observe that when the male spouse has a higher weight, his marginal tax rate will be negative (his labor supply is subsidized). This may be surprising at first since he has a lower weight in the welfare function than in the couple’s utility. So we do not expect him to be subsidized. However, the subsidy applies only to the marginal tax rate; it will increase his labor supply and thus his contribution to the common household budget.

Note that the Pigouvian tax does not, in general, yield the first-best utilitarian optimum because spouses’ individual consumption levels remain unobservable. This is because, unless weights are equal, marginal utilities of consumption are not equalized between spouses; see equation (9). However, with $\lambda_{ik} = \lambda_{ki} = 0 \ \forall \ i, j$, equation (21)
implies

\[ v'(\frac{y_i}{w_i})/w_i = v'(\frac{y_j}{w_j})/w_j, \]

which corresponds to the first-best condition under full information. In words, labor supplies are set according to the first-best rule.

### 3.3.2 Second-best tax policy

We now return to the original second-best problem where wages are not publicly observable. Consider first a “top” couple, that is a couple whom nobody mimics. Formally, we consider a couple with index \( i \) so that \( \lambda_{ki} = 0 \ \forall \ k \), but where \( \lambda_{ik} > 0 \) for at least one \( k \). Since couples are indexed according to increasing wage levels and the welfare function is utilitarian, we can expect this to be the couple with the highest wages. Using the definition of the Pigouvian tax (24) and setting all \( \lambda_{ki} = 0 \), equation (22) can be rewritten as

\[
T_{y_j}^i = 1 - \frac{\pi^i}{\alpha^i} \left( 1 - T_{y_j}^{P_i} \right) + \sum_{k=1,k\neq i}^{n} \lambda_{ik} \frac{\pi^i T_{y_j}^{P_i}}{\alpha^i} + \sum_{k=1,k\neq i}^{n} \lambda_{ik} \left( \frac{\pi^i}{\alpha^i} \right) = 0.
\]

If in this couple the bargaining power is equally divided so that \( \lambda^i_{yj} = \lambda^i_{yj} = 0 \) both spouses face no distortion since then \( T_{y_j}^i = 0 \). If, however, the bargaining power is unequally distributed, we have \( T_{y_j}^i < T_{y_j}^{P_i} \) implying a lower tax (subsidy) than the Pigouvian one. In other words, the “no distortion at the top” result no longer holds. A similar result is obtained by Cremer and Roeder (2013) when individuals are myopic.

Myopia justifies a social objective different from the private one which gives rise to a Pigouvian subsidy. While the current setting is more complicated in that the Pigouvian tax can be positive or negative, the intuition behind the result remains essentially the same. A distortion is optimal even for the top couple because the consumption and labor supplies are weighted differently in the incentive constraint than in the social objective. This opens the door for relaxing incentive constraints by not restoring a first-best tradeoff for this couple.

Now consider a spouse who is not part of the “top” couple. For such a couple the
optimal tax rate is

\[
T_{ij}^i = 1 - \frac{\pi_i^j}{\alpha_j^i} \left(1 - T_{ij}^{P_i}\right) + \sum_{k=1, k \neq i}^n \lambda_{ik} - \sum_{k=1, k \neq i}^n \lambda_{ki} \frac{\alpha_k^i u'(c_k^i)}{\alpha_j^i u'(c_j^i)} \frac{MRS_{ij}^{ki}}{MRS_{ij}^{kj}}.
\]  

(30)

This expression includes a Pigouvian term and “incentive” terms, that is terms which reflect the impact of the tax policy on the incentive constraints. To get a more precise understanding of the structure of these incentive terms and their interaction with the Pigouvian term, we can rearrange equation (30) to obtain the following condition

\[
T_{ij}^i \leq 0 \iff \sum_{k=1, k \neq i}^n \lambda_{ki} \frac{\alpha_k^i u'(c_k^i)}{\alpha_j^i u'(c_j^i)} \frac{MRS_{ij}^{ki}}{MRS_{ij}^{kj}} - 1 \leq \frac{\pi_i^j}{\alpha_j^i} T_{ij}^{P_i}.
\]  

(31)

The sign of the marginal income tax rate depends on the relative magnitude of the Pigouvian term (RHS of the second inequality) and the incentive term (LHS of the second inequality). In standard optimal taxation models the marginal rate of substitution of the mimicker (couple \(ki\)) is always smaller than that of the mimicked (couple \(i\)), as long as only downward incentive constraints are binding.\(^{12}\) In our setting, things are more complicated because marginal rates of substitution depend on (unobservable) consumption levels which, in turn, depend on the spouses’ weights.

To interpret condition (31) assume first that weights are the same across couples. In other words, all females have the same weight and so do all males. If these weights are equal to one the Pigouvian term, \(T_{ij}^{P_i}\), is equal to zero, and the sign of the marginal tax, \(T_{ij}\), depends only on the way it affects binding incentive constraints. We have \(T_{ij}^i > 0\) if binding incentive constraints are from high-wage to low-wage couples since with \(\alpha_j^i = \alpha_k^i = 1\), we have

\[
\frac{MRS_{ij}^{ki}}{MRS_{ij}^{kj}} < 1 \iff w_j^i < w_j^k.
\]  

(32)

This is pretty much the standard result obtained in Mirrleesian models.

Now assume that weights differ between gender, but that the weight of a given gender is the same in all couples \(\alpha_j^i = \alpha_j \forall i\). Then, we effectively have both Pigouvian

\(^{12}\)Which is typically the case with a utilitarian social welfare function.
and incentive terms. The incentive term (LHS of the second inequality in 31) calls for positive taxation while the Pigouvian term (RHS of the second inequality in 31) calls for a subsidization (taxation) of labor for the gender who has the higher (lower) bargaining power; see Section 3.3.1. Consequently, for the gender with the lower bargaining power both effects go in the same direction implying $T^i_{y_j} > T^{Pi}_{y_j} > 0$, while for the gender with the higher bargaining power, the two effects go in opposite direction implying $T^i_{y_j} < T^{Pi}_{y_j} < 0$.

If additionally weights differ across couples, we have $c^i_j > c^{ki}_j$ iff $\alpha^i_j > \alpha^k_j$ so that for a couple with $w^k_j = w^i_j$, we have (by concavity of $u$)

$$\frac{MRS^k_{1y_j}}{MRS^i_{1y_j}} \leq 1 \iff \alpha^k_j \leq \alpha^i_j.$$  \hspace{1cm} (33)

Combining (33) with (32) implies that

$$\frac{MRS^k_{1y_j}}{MRS^i_{1y_j}} < 1 \text{ if } w^i_j < w^k_j \text{ and } \alpha^k_j \leq \alpha^i_j.$$  \hspace{1cm} (34)

In words, when spouse $j$ has a lower bargaining weight in the high-wage couple, then the couples’ indifference curves cross in the “usual way”; the low-wage spouse has a steeper indifference curve and the LHS in the second inequality of (31) is negative. The above condition is not an if and only if condition. To see this note that

$$\frac{MRS^k_{1y_j}}{MRS^i_{1y_j}} = \frac{w^i_j}{w^k_j} \frac{u'(\frac{w^i_j}{w^j})}{u'(\frac{w^k_j}{w^j})} \frac{u'(c^i_j)}{u'(c^{ki}_j)}.$$

When weights are increasing with wages we have $c^{ki}_j > c^i_j$, and the last term on the RHS is larger than 1. However, the full expression can nevertheless be smaller than one. This will be the case, when the heterogeneity in weights is small compared to the heterogeneity in wages. On the other hand, when the heterogeneity in weights is sufficiently large, expression (35) can be larger than one.
To illustrate this idea of relative heterogeneity consider the case where utility of consumption $u$ is logarithmic, while disutility of labor $v$ is quadratic. Then, we have

$$\frac{MRS_{ki}}{MRS_{ij}} \leq 1 \iff \left( \frac{w_i^j}{w_j^j} \right)^2 \leq \frac{\alpha_j}{\alpha_k},$$

so that the comparison hinges on the ratio of wages and the ratio of weights between the mimicked and the mimicking couple.

To understand the full implications of the inequalities stated in equation (31), consider a couple $i$ for which only $\lambda_{ki} > 0$. Assume that downward incentive constraints are binding, so that $w_j^k > w_i^j$.

First, examine the spouse with a low weight $\alpha_j^i \leq 1$. The Pigouvian term of this spouse is always positive. The incentive term also calls for a tax if $\alpha_j^k \leq \alpha_j^i$, that is if the bargaining power of the low-weight spouse decreases with wages. Then, the overall effect is clear-cut and this spouse faces a positive marginal income tax rate. If, however, the weight of this spouse increases with wages, $\alpha_j^k > \alpha_j^i$, the incentive term is positive (and so is the overall marginal tax rate) only if the difference in wages dominates the difference in weights. Otherwise, the incentive and Pigouvian term go in opposite directions and we are not able to sign the marginal tax rate.

Next analyze the high-weight spouse $\alpha_j^i > 1$. Now, we have a negative Pigouvian tax rate, while the incentive term again calls for a positive tax if $\alpha_j^k \leq \alpha_j^i$ (decreasing weights of the high-weight spouse). In this case, the effects go in opposite directions. When the bargaining weights increase with wages, $\alpha_j^k \geq \alpha_j^i$, the sign of the incentive term again depends on the relative difference in wages and weights. Specifically, the incentive term calls for a negative marginal tax rate if the difference in weights is the dominating one and a positive marginal tax rate if the difference in wages is the dominating one. The overall effect in the former case is thus clear-cut, implying a negative marginal tax rate when the difference in weights dominates.

Finally, observe that when the weight of the low-weight spouse increases with wages, it follows of course that the weight of the high-weight spouse decreases. Consequently, in this case results are unambiguous for both spouses if for the high-weight spouse the dif-

\[13\] More precisely, we have $\lambda_{hi} = 0$ when $h \neq k$. This does not rule out $\lambda_{ih} > 0$ for some $h$. 

The main results of this section are summarized in the following proposition.

**Proposition 1** (i) When couples’ types are observable, income will be subject to a Pigouvian tax or subsidy to correct the misallocation of labor supply by couples. The member of the household who has the lower bargaining weight will face a marginal tax on labor income and the member with the higher weight will face a marginal subsidy on labor income. Specifically, we have that $T_{y_i}^{P_i} \geq 0$ iff $\alpha_j^i \leq \frac{1}{2}$ $\forall$ $i, j$. Consequently, when the male spouse has the higher bargaining weight his marginal tax rate will be negative while that of the female spouse will be positive.

---

<table>
<thead>
<tr>
<th>$w_j^i &lt; w_j^k$</th>
<th>weight profile</th>
<th>$T_{y_i}^{P_i}$</th>
<th>overall effect $T_{y_i}^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_j^i \leq 1 \lor (i) \alpha_j^i \leq \alpha_j^k$</td>
<td>$\geq 0$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>(ii) $\alpha_j^i \geq \alpha_j^k$</td>
<td>$\geq 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$\alpha_j^i &gt; 1 \lor (i) \alpha_j^i \geq \alpha_j^k$</td>
<td>$&lt; 0$</td>
<td>$&gt; 0$</td>
<td>$\triangleright 0$</td>
</tr>
<tr>
<td>(ii) $\alpha_j^i \leq \alpha_j^k$</td>
<td>$&lt; 0$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

Table 1: Marginal tax rates for the low- and high-weight spouse and for (i) increasing and (ii) decreasing bargaining weights of the low-weight spouse.
(ii) When couples’ types are not observable:

(a) A spouse’s marginal income tax rate is defined by expression (30), which shows that it depends on a paternalistic (Pigouvian) and on a redistributive (incentive related) term. The paternalistic term has the same sign as the Pigouvian tax described in item (i). The sign of the incentive term depends on the ratio of the marginal rate of substitution of the considered spouse in the mimicking and in the mimicked couple. This, in turn, depends on the distribution of weights and wages.

(b) We depart from the “no distortion at the top” result. The absolute value of the marginal tax or subsidy will be smaller than its Pigouvian counterpart. Formally, if

\[ \bar{T} \]

we continue to have that

\[ T_{ij}^i > 0 \]

but with

\[ |T_{ij}^i| < |T_{ij}^P| \].

(c) Assume that the bargaining weight of the low-weight spouse is decreasing in wages (which automatically implies that the bargaining weight of the high-weight spouse increases in wages) and that the weight heterogeneity is dominant for the high-weight spouse. Then, for both spouses, the Pigouvian and incentive term go in the same direction and we have

\[ T_{ij}^i > 0 \]

for the low-weight spouse and

\[ T_{ij}^i < 0 \]

for the high-weight spouse. Consequently, the Pigouvian results stated in item (i) are reinforced. Specifically, when the male spouse has the higher bargaining weight his marginal tax rate will be negative while that of the female spouse will be positive.

4 Extension: concave transformation of spouses’ utilities

So far, we have considered a simple utilitarian social welfare function, which does account for social benefits from redistribution. It is well known that with concave individual preferences redistribution from high-income to low-income couples increases utilitarian welfare. However, it is also interesting to examine if and how our results are affected when a more egalitarian social welfare function is considered.\textsuperscript{14} To do so redefine welfare as

\[
W^2 = \sum_{i=1}^{n} \pi^i \sum_{j=f,m} \Psi \left( \hat{u}_j^i(I^i) - \nu \left( \frac{y_j^i}{w_j^i} \right) \right),
\]

\textsuperscript{14}A more egalitarian welfare function is also considered by Kleven et al. (2009).
where $\Psi$ is a concave and strictly increasing function of spouses’ utility levels: $\Psi_{ij}^0 > 0$ and $\Psi_{ij}'' \leq 0$. When $\Psi_{ij}'' = 0$ we return to the utilitarian specification defined by (10). The welfare function becomes increasingly redistributive as the degree of concavity of $\Psi$ increases.

The welfare function becomes increasingly redistributive as the degree of concavity of $\Psi$ increases.

The optimal allocation is now determined by maximizing (36) subject to (17) and (18); the FOCs are stated in Appendix B. Proceeding exactly as in the previous section, the optimal marginal tax rate of spouse $j$ in couple $i$ can be written as

$$T^i_{yj} = 1 - \frac{\pi^i}{\alpha^i_j} \sum_{g=f,m} \Psi'_{ij} u'(c_i^g(P)) \frac{\partial c_j^g}{\partial I} + \sum_{k=1,k\neq i}^n \lambda_{ik} - \sum_{k=1,k\neq i}^n \lambda_{ki} \frac{\alpha^k_i \alpha^k_j \Psi_{ij} w'(c_i^k(P))}{MRS_{ij}},$$

$$\frac{\pi^i}{\alpha^i_j} \Psi'_{ij} + \sum_{k=1,k\neq i}^n \lambda_{ik} - \sum_{k=1,k\neq i}^n \lambda_{ki} \frac{\alpha^k_i \alpha^k_j \Psi_{ij} w'(c_i^k(P))}{MRS_{ij}}$$

which generalizes expression (37).

The interesting changes appear in the Pigouvian term. For $\lambda_{ik} = \lambda_{ki} = 0 \forall i, j$, expression (37) yields the Pigouvian tax (subsidy)

$$T^i_{yj} = T^{pi}_{yj} = 1 - \sum_{g=f,m} \frac{\Psi'_{ij} u'(c_i^g)}{\Psi'_{ij} u'(c_i^j)} \frac{\partial c_j^g}{\partial I}. \tag{38}$$

Using (8) and (9) the Pigouvian tax (subsidy) for the female and male spouse in couple $i$ can now be expressed as follows

$$T^{pi}_{yf} = 1 - \frac{\Psi'_{im} \alpha^i_m \frac{\partial c_i^j}{\partial I}}{\Psi'_{im} \alpha^i_m} - \frac{\partial c_i^j}{\partial I} = 1 - \frac{\Psi'_{im} \alpha^i_m}{\Psi'_{im} \alpha^i_m} \frac{\partial c_i^j}{\partial I}, \tag{39}$$

$$T^{pi}_{ym} = 1 - \frac{\Psi'_{if} \alpha^i_m \frac{\partial c_i^j}{\partial I}}{\Psi'_{if} \alpha^i_m} - \frac{\partial c_i^j}{\partial I} = 1 - \frac{\Psi'_{if} \alpha^i_m}{\Psi'_{if} \alpha^i_m} \frac{\partial c_i^j}{\partial I}. \tag{40}$$

Since $\frac{\partial c_i^j}{\partial I} \geq 0$ we have

$$T^{pi}_{ym} \leq 0 \iff \frac{\alpha^i_m}{\alpha^i_f} > \frac{\Psi'_{im}}{\Psi'_{if}}, \tag{41}$$

$$T^{pi}_{yf} \leq 0 \iff \frac{\alpha^i_f}{\alpha^i_m} > \frac{\Psi'_{if}}{\Psi'_{im}}. \tag{42}$$

These expressions are straightforward generalizations of (27) and (28) and their interpretation is similar. The spouse whose relative bargaining weight (within the couple)
is lower than the relative social weight (again within the couple) faces a positive marginal tax on labor supply. Conversely, the spouse whose relative bargaining weight is larger than the relative social weight faces a marginal subsidy on labor supply. Roughly speaking, the spouse with the low relative bargaining weight works too much while the one with the high relative bargaining weight does not work enough. The Pigouvian tax (subsidy) corrects for these non-optimal labor supply decisions. Observe that when the male spouse’s bargaining weight exceeds his social weight, his marginal income tax rate will be negative (his labor supply is subsidized). In Section 3.3.1, the relative social weights were always equal to one, whereas with the concave transformation, they will in general differ from one. The weight in social welfare $\Psi'_{ij}$ depends on a spouse’s total utility which, in turn, depends both on the bargaining weight and on the wage rate. Consequently, Pigouvian tax rates will now differ from zero, even when spouses’ weights are equal, as long as wages are different.

At this level of generality the impact of the welfare weights on the size, and even on the sign of Pigouvian tax rates does not appear to be unambiguous. Assume for instance that the male spouse has both the higher weight and the higher wage. Then, it is not clear if his total utility will be larger than hers; the higher weight tends to increase utility, but the higher wage may go in the other direction (as a larger labor supply implies a larger disutility of labor). Consequently, we cannot compare $\Psi'_{im}$ and $\Psi'_{jf}$.

Except for these extra complications concerning the Pigouvian term, the remainder of the analysis presented in Section 3 remains essentially unaffected. Rearranging (37) shows that expression (31) now has to be replaced by the following condition

$$T^i_{y_j} \leq 0 \iff \sum_{k=1, k \neq i}^n \lambda_k \frac{\alpha^k_j w'(c^k_j)}{\alpha^k_j u'(c^k_j)} \left( \frac{\text{MRS}^k_{y_j}}{\text{MRS}^i_{y_j}} - 1 \right) \geq \frac{\Psi'_{ij}}{\alpha^i_j} T^i_{y_j}.$$

Note that the incentive term (as a rule) is unaffected. In the last term $\Psi'_{ij}/\alpha^i_j$ replaces $1/\alpha^i_j$, to express the ratio between a spouse’s social weight and his or her bargaining weight in the couple.
5 Numerical results

Our analysis has shown that the solution crucially depends on the distribution of bargaining weights and wages both within and across couples. To obtain more specific results we now provide numerical simulations based on a calibrated version of our model.

We make the following parametric assumptions

\[ u(c) = \ln c \quad \text{and} \quad v(\ell) = \frac{\varepsilon}{1 + \varepsilon} \ell^{1 + \varepsilon} \] (44)

so that the Frisch labor supply elasticity \( \varepsilon \) is constant. Following the empirical labor supply literature we assume \( \varepsilon = 0.5 \); see, e.g., Blundell and Macurdy (1999). The social welfare function is CRRA, \( \Psi = x^{1-\sigma}/(1-\sigma) \), where \( \sigma \geq 0 \) measures the preferences for equity. We assume \( \sigma = 0 \) in our benchmark case and also consider \( \sigma = 1 \) and \( \sigma = 5 \).

We calibrate the (hourly) wage distribution for the year 2011 using the PSID. We restrict the sample to married couples with no children. Couples are ranked with respect to the wife’s wage rate and then divided into quintiles. For each quintile we compute the average wage rate of both male and female. The corresponding average wage rates \( w^i_j \) and \( w^i_m \) in quintile \( i = 1, \ldots, 5 \) are reported in Table 2. The female’s wage rate is always lower than the male’s wage rate. And while the wage gap in the lowest quintile is rather high, it is decreasing in wages and becomes almost zero in the highest quintile.

The bargaining weights which, with the logarithmic utility function specified in (44) also correspond to the spouses’ consumption shares, are taken from Couprie (2007) model (I) p. 301 Table 4. For quintile \( i = 1, \ldots, 5 \) we use Couprie’s calculated consumption shares at the 10th, 30th, 50th, 70th and 90th percentile respectively to get

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<td>12.93</td>
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<td>26.82</td>
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<tr>
<td>( \alpha^i_j )</td>
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<td>0.94</td>
<td>1.08</td>
<td>1.22</td>
</tr>
<tr>
<td>( \alpha^i_m )</td>
<td>1.36</td>
<td>1.20</td>
<td>1.06</td>
<td>0.92</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Table 2: Parameter values.
the “average weight” for our quintiles. Since the values for the 30th and 70th percentiles are not reported, we used a simple interpolation. The calculated values are also shown in Table 2. The bargaining power of the female spouse is increasing in her wage rate. While it is lower in the first three quintiles it becomes higher than her husband’s bargaining power in the last two quintiles.

As in the theoretical part, we start with the utilitarian case obtained when \( \sigma = 0 \). Table 3 presents the optimal Pigouvian, \( T^{P_i}_{yf} \), and marginal, \( T^{i}_{yf} \), income tax rates for both spouses in quintile \( i = 1, \ldots, 5 \). As shown in Section 3.3.1 the Pigouvian tax rate is positive for the spouse with the lower bargaining weight, and negative for the one with the higher bargaining weight. Since the spouse with the low bargaining weight receives a smaller share of gross income, she works too much (as compared to the equal weight solution) and thus her labor is taxed at the margin. The opposite is true for the high-weight spouse. The numerical results merely illustrate these findings. In particular Table 3 shows that the Pigouvian tax of the female spouse switches sign from being positive for the first three quintile to being negative in the last two quintiles. The opposite pattern emerges of course for the male spouse.\(^{15}\)

We know from expression (31) that a spouse’s marginal income tax rate, \( T^{i}_{yj} \), depends on the relative strength and sign of the incentive term and the Pigouvian term; see Table 1 and the discussion provided there. For \( i = 1, 2 \) and 3 case (i) in Table 1 applies since \(^{15}\)The fact that the Pigouvian taxes are of equal magnitude, i.e., \( |T^{i}_{yf}| = |T^{i}_{ym}| \), is due to the in-specification of utility. More precisely, the Pigouvian tax is given by \( T^{P_i}_{yf} = 1 - \alpha_j \).
the low-weight female spouse of the mimicking couple has a higher bargaining weight than the low-weight female spouse in the mimicked couple. The incentive term of the female spouse reinforces the Pigouvian term if the difference in wages dominates. Our numerical results in Table 3 indicate that this is the case for all $i \in \{1, 2, 3\}$. The incentive term for the male spouse, however, always goes against the Pigouvian term. This is revealed by the fact that $T_{ym}^i > T_y^{Pi}$ in the three first quintiles. And while the marginal tax rate $T_{ym}^i$ for $i = 1$ and 2 remains negative it becomes positive for $i = 3$; in that case the incentive term not only opposes but actually outweighs the Pigouvian term.

For couples of type $i = 4$ case (ii) in Table 1 applies. Now the low-weight spouse is male, and his weight is lower in the mimicking than in the mimicked couple. In this case the theory tells us that the marginal tax rate of the low-weight spouse is always positive while its sign is ambiguous for the high-weight spouse. Table 3 shows that with our calibration the high-weight female spouse faces a positive marginal tax rate, that is, the incentive term offsets the Pigouvian term. Finally, $i = 5$ is special in the sense that it represents the top couple, for which the incentive term is zero. Consequently, the sign of the marginal tax rate is that of the Pigouvian rate. However, we also know from Section 3.3.2 that the absolute value of the marginal tax rate is always smaller than that of the Pigouvian tax rate. The numerical results for $i = 5$ simply illustrate this result.

The numerical simulations also allow us to compare our optimal income tax system with that obtained under joint income taxation, that is when a couple's tax solely depends on the couple's total income. Let $\bar{T}(y_f + y_m)$ denote the optimal joint income tax schedule. By definition the two spouses in any given couple then face the same marginal income tax rates, $\bar{T}_{y_f}^i = \bar{T}_{ym}^i$. The marginal tax rates under joint taxation

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16 Our simulation results confirm that incentive constraints are only binding from high-wage to low-wage couples.
17 Joint taxation continues to be used in many countries including the Netherlands and Germany; income splitting notwithstanding, couples tax liability depend on their total income.
18 To determine this solution we solve the same problem as before (maximizing welfare subject to resource and incentive constraints) with the additional constraint that $MRS_{1y_f}^i = MRS_{1ym}^i$. This constraint ensures that the solution can be implemented by a joint income tax schedule.
Table 4: Optimal Tax Rates: Concave Transformation of Spouses’ Utilities.

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Table 4: Optimal Tax Rates: Concave Transformation of Spouses’ Utilities.

are reported in the last line of Table 3. Not surprisingly the joint marginal income tax rate is always between the individual (unrestricted) levels, i.e., $\min \{T_{yi}, T_{ym}\} < \bar{T}_{yi} = \bar{T}_{ym} < \max \{T_{yi}, T_{ym}\}$. The last column of the table also reports the welfare difference between the two solutions. It shows that when the optimal tax is replaced by a joint tax, the loss in (utilitarian) welfare is 0.0189. To express this in monetary terms we have also calculated the compensating transfer $r^0 = 0.4267$, that could be given to each couple to bring back welfare to the optimal level. This number, represents a monetary estimation of welfare cost incurred if a joint tax schedule is used rather than our optimal policy. It is less spectacular than the pattern of marginal tax rates might suggest. Indeed, this pattern is far from uniform within couples. Still, it is not negligible since it is slightly less than 1% of total output, $\sum_i \pi_i I_i$.19

Table 7 reports the results with more egalitarian social welfare functions as considered in Section 4. Recall that the utilitarian case was obtained by setting $\sigma = 0$.

19So in the US case used for our calibrations we are talking about a welfare loss of about 1% of 16.8 trillion USD.
We now consider two alternative levels, namely $\sigma = 1$ and $\sigma = 5$ which imply higher degrees of concavity. The striking feature is that while the actual numbers differ from the utilitarian case, the overall pattern of marginal tax rates is remarkably unaffected. The only changes that appear significant concern the Pigouvian tax rates of the low income couples. Specifically, the male spouse now receives a larger (Pigouvian) subsidy, particularly in the case where $\sigma = 5$. However, this has little impact on the overall marginal tax rates. This suggests that as the degree of concavity increases the incentive term becomes relatively more important than the Pigouvian term, at least for the male spouse in the low-wage couple. This property obviously depends on the considered calibrations. It can be understood by comparing equations (31) and (43). As discussed in Section 4 the two expressions differ in that the Pigouvian term is premultiplied by $\Psi'_{im}/\alpha^i_m$ rather than by $1/\alpha^i_m$. Now, the Pigouvian term already depends on $\Psi'_{im}/\alpha^i_m$ and the very fact that the Pigouvian subsidy of the male spouse becomes so large shows that the private bargaining weight of the male spouse is “much” larger than his social weight ($i.e., \Psi'_{im}/\alpha^i_m$ is very small). In other words, while the Pigouvian subsidy increase the premultiplying term decreases. As our results show these two effects pretty much cancel out when it comes to the determination of this spouse’s marginal tax rate.

The last column again compares welfare between the unrestricted optimal solution and the joint taxation solution. Clearly, welfare changes cannot be compared across the $\sigma$’s. Consequently, we have also calculated the compensating transfers $\tau$, which are now given by $\tau^1 = 0.6933$ and $\tau^5 = 1.860$. In other words, the monetary valuation of the welfare loss implied by using joint taxation rises as inequality aversion increases. Not surprisingly, the value of an additional redistributive instrument is enhanced when the redistributive concern increases.

So far, we have only reported the marginal tax rates that implement our solutions. This illustrates the solution for the variable on which our analytical part has concentrated. To provide a more complete picture of the underlying solution, we present a full characterization of the underlying allocations in Appendix C. We start with the laissez-faire and the solution with observable (wages) and then report the second-best allocation for $\sigma = 0$ (the utilitarian case) as well as for $\sigma = 1$ and $\sigma = 5$. These solutions
present a few interesting features which are worth pointing out.

First, the pattern of redistribution which emerges is always the same: the high-wage couples pay taxes which finance transfers to all other types of couples. This appears to be rather extreme, but it is simply due to the fact that we consider a purely redistributive tax. It is plain that with a positive revenue requirement, more types of couples (and possibly all) would face a positive tax bill.

As usual in Mirrleesian optimal tax models, the extent of redistribution between couples is limited by the incentive constraints and it depends on the social welfare function. In particular, transfers between couples are larger when types are observable than in any of the second-best allocations. Not surprisingly, the second best transfers also increase with the value of $\sigma$; recall that a larger value corresponds to a more egalitarian welfare function. More interestingly, the results also illustrate the “conflict” that may arise between inter-couple and intra-couple redistribution. Concerning the latter, the income tax is of limited effectiveness. While it may “correct” labor supply through the Pigouvian part of the marginal tax rates, it has no leverage on the allocation of consumption between spouses. This is most striking in Tables 6 where the consumption of the male spouses in couple 3, 4 and 5 is effectively lower than that of their counterpart in couple 1. The same is true for couples 3 and 4 in all the considered second-best allocations. If spouses’ consumption levels were observable, the utilitarian solution would imply equal consumption within and across couples. In our setting, however, the transfer to the low-wage couples benefits mostly the male spouse, so much that he will have a much larger consumption than the high-wage male spouse (who has both a lower weight and is taxed more heavily on his larger pre-tax income).

6 Concluding comments

This paper has studied the design of couples’ income taxation in a household bargaining setting. A couple’s consumption levels and labor supplies are chosen to maximize a weighted sum of spouses’ utilities. The weights represent the spouses’ respective bargaining powers. The main lesson that emerges from our paper is that the traditional Boskin and Sheshinski (1983) result calling for a lower marginal tax on the female
spouse appears to be seriously challenged by the departure from a unitary couple model towards a bargaining setting. While the results are often ambiguous, it is clear that it takes rather rigorous conditions to obtain a lower marginal tax rate for the female spouse. Numerical simulations based on a calibrated specification confirm that the marginal tax rate of the male spouse in some couples may effectively be negative while that of the female spouse is positive. The traditional results are typically driven by differences in labor supply elasticity which are neglected in our setting; spouses have the same individual preferences, including disutility of labor. Differences in labor supply elasticities can be expected to mitigate our results. However, since our results are rather spectacular, with spouses’ marginal tax rates differing in their sign, it is not clear that the elasticity effect could reverse them. At the very least this would require rather significant differences in the gender specific labor supply elasticities.

Appendix

A Derivation of $MRS_{i \cdot y_j}^i$

Dividing each term in equations (20) and (21) by $\alpha_j u'(c_j^i(I^i))$, using the couple’s FOC for the optimal distribution of consumption (4) and rearranging, we have

$$\frac{\mu \pi^i}{\alpha_j u'(c_j^i(I^i))} = \pi^i \sum_{g=f,m} \frac{\hat{u}^i_g (I^i)}{\alpha_j u'(c_j^i(I^i))} + \sum_{k=1,k \neq i}^n \lambda_{ik} - \sum_{k=1,k \neq i}^n \lambda_{kj} \frac{\alpha_j^k u'(c_j^k(I^i))}{\alpha_j u'(c_j^i(I^i))},$$  \hspace{1cm} (A1)

$$\frac{\mu \pi^i}{\alpha_j u'(c_j^i(I^i))} = \pi^i 1 \frac{v'((\ell_j^i))}{\alpha_j w_j u'(c_j^i(I^i))} + \sum_{k=1,k \neq i}^n \lambda_{ik} 1 \frac{v'((\ell_j^i))}{\alpha_j w_j u'(c_j^i(I^i))} - \sum_{k=1,k \neq i}^n \lambda_{kj} \frac{\alpha_j^k}{\alpha_j} 1 \frac{v'((\ell_j^k))}{\alpha_j w_j u'(c_j^i(I^i))}.$$  \hspace{1cm} (A2)

With the definition of the marginal rate of substitution (equation 15)

$$MRS_{i \cdot y_j} = \frac{1}{w_j} \frac{v'((\ell_j^i))}{u'(c_j^i(I^i))},$$
we can rewrite equation (A2) as

\[
\frac{\mu \pi^i}{\alpha^i_j u'(c^i_j(I^i))} = MRS^i_{I^i y_j} \left[ \frac{\pi^i}{\alpha^i_j} + \sum_{k=1,k \neq i}^n \lambda_{ik} - \sum_{k=1,k \neq i}^n \lambda_{ki} \frac{\alpha^k_j u'(c^k_j(I^i))}{\alpha^i_j u'(c^i_j(I^i))} MRS^{k i}_{I^i y_j} \right].
\] (A3)

Equalizing equations (A1) and (A3) and solving for \( MRS^i_{I^i y_j} \), we get

\[
MRS^i_{I^i y_j} = \frac{\pi^i}{\alpha^i_j} \sum_{g=f,m} \frac{u'(c^i_j(I^i))}{u'(c^i_j(I^i))} \frac{\partial c^i_j(I^i)}{\partial I^i} + \sum_{k=1,k \neq i}^n \lambda_{ik} - \sum_{k=1,k \neq i}^n \lambda_{ki} \frac{\alpha^k_j u'(c^k_j(I^i))}{\alpha^i_j u'(c^i_j(I^i))} MRS^{k i}_{I^i y_j}
\]

which can be rewritten as

\[
MRS^i_{I^i y_j} = \frac{\pi^i}{\alpha^i_j} \sum_{g=f,m} \frac{u'(c^i_j(I^i))}{u'(c^i_j(I^i))} \frac{\partial c^i_j(I^i)}{\partial I^i} + \sum_{k=1,k \neq i}^n \lambda_{ik} - \sum_{k=1,k \neq i}^n \lambda_{ki} \frac{\alpha^k_j u'(c^k_j(I^i))}{\alpha^i_j u'(c^i_j(I^i))} MRS^{k i}_{I^i y_j}
\]

\[= \frac{\pi^i}{\alpha^i_j} \sum_{g=f,m} \frac{u'(c^i_j(I^i))}{u'(c^i_j(I^i))} \frac{\partial c^i_j(I^i)}{\partial I^i} + \sum_{k=1,k \neq i}^n \lambda_{ik} - \sum_{k=1,k \neq i}^n \lambda_{ki} \frac{\alpha^k_j u'(c^k_j(I^i))}{\alpha^i_j u'(c^i_j(I^i))} MRS^{k i}_{I^i y_j}.
\]

**B  FOCs for concave transformation of spouses’ utilities**

The first-order conditions with respect to \( I^i, y_j^i \) are given by

\[
\frac{\partial L}{\partial I^i} = \pi^i \sum_{j=f,m} \Psi^i_{I^i, y_j^i} \left( I^i \right)
\]

\[+ \sum_{k=1,k \neq i}^n \lambda_{ik} \sum_{g=f,m} \alpha^g_j \hat{u}'(c^g_j(I^i)) - \sum_{k=1,k \neq i}^n \lambda_{ki} \sum_{g=f,m} \alpha^k_j \hat{u}'(c^k_j(I^i)) - \mu \pi^i = 0,
\]

\[
\frac{\partial L}{\partial y_j^i} = -\pi^i \Psi^i_{I^i, y_j^i} \left( \hat{y}_j^i \right) - \sum_{k=1,k \neq i}^n \lambda_{ik} \alpha^k_j \frac{1}{w_j^i} \hat{u}'(c^k_j(I^i)) + \sum_{k=1,k \neq i}^n \lambda_{ki} \alpha^k_j \frac{1}{w_j^i} \hat{u}'(c^k_j(I^i)) + \mu \pi^i = 0.
\]

**C  Optimal allocations**
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Table 5: Laissez-faire allocation.

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Table 7: Optimal allocation for $\sigma = 0$. 
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Table 8: Optimal allocation for $\sigma = 1$ and $\sigma = 5$. 
References


