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**ON THE IMPORTANCE OF FERTILITY BEHAVIOR
IN SCHOOL FINANCE POLICY DESIGN**

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On the Importance of Fertility Behavior in School Finance Policy Design*

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February 6, 2014

Abstract

To design an optimal education policy, it is essential to account for the fertility differential between the poor and the rich because it affects the human capital investment through the child quantity-quality tradeoff of children. We develop a dynamic general equilibrium in which parents choose the quantity of children, transfer a preschool ability to their children, determine the quality of children by choosing private expenditures on basic education in addition to public expenditures on basic education, leave a bequest that could be used to finance college education. Moreover, there is an uncertainty in college completion depending on ability and endogenous wage determination based on the amount of schooling in the economy. It is very important to consider general equilibrium effects because the change in either fertility behavior or college outcomes as a result of policy changes leads to a large change in aggregate skill distribution. We find that ignoring fertility behavior, especially differential fertility substantially underestimates the role of credit constraints in the economy. We also analyze the impact of basic education subsidies and college subsidies on welfare, inequality, and intergenerational mobility. Strikingly, the choice between these two policies is found to be dependent on the magnitude of differential fertility rate.

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1 Introduction

Regardless of whether it is a basic education system (K-12) or colleges, governments around the world subsidize schools heavily. Not surprisingly, U.S. is not an exception. In 2007, U.S. had public expenditures for public elementary and secondary education (K-12) of approximately 4.9 % of GDP, out of which 1.3 % was spent on colleges. Given the very large magnitude of education subsidies, government intervention in education has recently received a growing research attention because it is not clear neither why nor how government should get involved in education.

Considering the objective of redistributive motives, Hanushek et al. (2003) finds a limited general support for college subsidies. An alternative justification for college subsidies revolves around credit constraints for human capital investment (e.g. Becker 1993[1964] or Garratt and Marshall 1994) and has been further studied in Fender and Wang (2003), Caucutt and Kumar (2003) and Hanushek et al. (2004). In a separate stream of literature, some studies (e.g. Hanushek and Yilmaz (2012) or Epple and Platt (1998)) have developed models to analyze a series of both the past and present school finance policies for K-12. The separation of those two literature lines seems to be artificial and hence, it is natural to think the education system is as a whole while designing school finance policies. After all, a change in K-12 would have substantial impacts on college outcomes or vice versa. Our paper attempts to fill in this gap in the literature.

A broad consensus exists that children in families with more resources enjoy considerable advantages in their development and long-term prospects (e.g. Mayer (1997), Duncan et al. (2001), and Duncan and Magnuson (2005)). Wealthy parents invest more resources than ever before in their children (in weekend sports, ballet, music lessons, math tutors, and in overall involvement in their children's schools), while lower-income families, which are more likely to have more children, are highly stretched for resources (e.g. Kornrich and Furstenberg (2012)). As a result, an achievement gap between rich and poor children exists (for example, see Hanushek (1992)). The empirical evidence is consistent with what is known as the "quantity-quality tradeoff theory", advanced by Becker (1960), Becker and Lewis (1973), and Willis (1973), that predicts poor (rich) parents are likely to have many (few) children and provide a little (good) education.

Our paper finds that to design an optimal education policy, it is essential to account for the quantity-quality tradeoff between the poor and the rich. Moreover, the choice between K-12 subsidies and college subsidies depends upon the degree of quantity-quality tradeoff. As a matter of fact, the degree of quantity-quality tradeoff is determined by fertility behavior. Over the last 200 years, U.S. has gone through a major demographic transition. A long term reduction in overall fertility has occurred. The decline was substantial, starting with about 5.5 children per woman born in 1828, falling to about two children per woman in 1958 (Jones and Tertilt (2008)). Besides, a negative relationship between fertility and income is found to exist. More importantly, the fertility gap between the poor and the rich has been continuously narrowing, and fertility has become less income sensitive over time (known as the compression of fertility). Our paper concludes that the optimal school finance policy depends on fertility behavior and since fertility behavior changes over time, what is optimal school finance policy in the past may not be the optimal school finance policy today.

Our objective is to develop a general equilibrium model in which both fertility and basic education decisions are interdependent. Our work is based on Hanushek et al. (2012). We use a similar overlapping generations framework, but model endogenous fertility decisions along the lines of Becker and Barro (1988), and De la Croix and Doephe (2003). We begin in a world where ability determines labor productivity, and is produced through a basic education production function that uses preschool ability and expenditures (both public and private) on basic education as inputs. The preschool ability of a child is transmitted from her parents and assumed to be correlated with her parent's preschool ability. Parents face a quantity-quality tradeoff in their decisions on children. Poor families tend to have more children and provide a little basic education. Later, the child becomes an adult and as an adult, she makes a college attendance decision depending on whether she

is financially constrained and whether attending college provides her with a higher level of utility. After all, the college attendance decision is a risky decision. The adult could fail and become an unskilled (uneducated) worker at the end. The probability of success is determined by the ability of the adult. If the adult chooses not to attend college, she is employed as an unskilled worker during her college years. Once the college outcome is realized, she provides skilled or unskilled labor in the labor market for the rest of her life, depending on her college outcome. Moreover, she makes a decision so as to the quantity and quality of children, savings and her consumption. As she moves to the last period, she becomes old and decides on her consumption as well as the size of the bequest she leaves for her children to finance their college tuition in case of a college attendance. If a child chooses not to attend college, she consumes all the bequest she inherits. In the paper, we consider the impact of two education subsidy schemes, namely basic education subsidies and college subsidies, on welfare, inequality and intergenerational mobility. We find that the impact of each policy seems to depend on fertility behavior and without fertility behavior, any model with ability formation would get the impact of credit constraints biased.

This paper is organized as follows: In section 2, we develop the model. Section 3 calibrates the model to the U.S. data. We describe the base outcomes in section 4. In section 5, we show the bias in the role of credit constraints by ignoring differential fertility behavior. We perform a sensitivity analysis under different fertility parameters in section 6. In section 7, we conduct two policy experiments: basic education subsidies and college subsidies. And we study their impact on welfare, inequality and intergenerational mobility under different differential fertility parameters. Finally, section 8 concludes.

2 An Overlapping Generations Model

Our economy is populated by individuals who live for three periods: childhood, adulthood, and old age. Individuals in the second period of their lives, with newborn children, form a household unit that lasts for two periods. An important aspect of the paper is endogenous fertility. The poor has more children and invests less on the basic education of their children than the rich. The individual dies at the end of third period of her life, and the child becomes an adult. As a result of endogenous fertility, the population changes over time. At any time t , three successive generations of a dynasty co-exist: grandparents in old age period, the children of grandparents in adulthood period (parents) and the children of parents in childhood period (grandchildren).

Individuals differ by their human capital, that determines their earnings in the second and third period of their lives. At the beginning of second period, a decision so as to whether to attend college or not is made. Clearly, individuals without sufficient resources (i.e. bequest) to pay off college tuition cannot attend college. Moreover, attending college is a risky decision. Depending on an individual's human capital, the individual attending college either succeeds (skilled worker) or fails (unskilled worker). The individual who chooses not to attend college is employed as unskilled worker during her college years and consumes all of the bequests left by her parents. In the rest of the second period, individuals are employed as either skilled or unskilled workers depending on their college decision outcomes. Besides, individuals decide on the amount of consumption and saving, the number of children, and the amount of resources devoted to their children's basic education in addition to public education (K-12). In the third period, individuals make consumption and bequest decisions. Bequests are then used by their children either (i) to pay for the college, or (ii) to be consumed.

Children are born with preschool abilities, that is also correlated with their parent's preschool abilities. Our model allows siblings to differ by preschool ability. In the first period (childhood), their human capital is formed and depends on three things: preschool ability, private and public expenditures on basic education.

The output is produced through a constant returns to scale production function, that uses skilled

and unskilled workers as inputs. Our model abstracts from physical capital, and assumes interest rate is constant. The role of the government is solely to tax the labor and use tax revenues to provide basic public education and subsidize colleges. More importantly, the quality of education in basic education is not a function of the expenditures, but real resources (i.e. the quality of teachers). Our focus is on stationary equilibrium in which population growth, all prices and aggregate variables are constant over time.

Old Age: An old individual¹ is uniquely characterized by a state vector, $\vec{x}_o=(I^e, \hat{z})$, where I^e is an indicator function that takes the value one if the individual successfully completes the college, and \hat{z} is the ability. An old individual with n children cares about her old age consumption, c_o and bequest left to each child, b' . She receives her savings, s in the previous period back with an interest and a wage income. Formally, her problem could be stated as:²

$$V_o(\vec{x}_o; s, n) = \max_{c_o \geq 0, b' \geq 0} \frac{[\frac{c_o^\alpha (nb')^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}]^{1-\sigma} - 1}{1-\sigma} \quad s.t. \quad \begin{aligned} c_o + nb' &= y_o + r_3 s \\ y_o &= (1 - \tau) h_o w_o \\ h_o &= \zeta h_p = \zeta (k_0 + k_1 \hat{z}) \\ w_o &= w_u (1 - I^e) + w_e I^e \end{aligned}$$

Where $0 < \alpha < 1$, $\sigma > 0$, $k_o > 0$, $k_1 > 0$, and $\zeta > 0$ are constants, w_o is the wage and h_o is the human capital of the old individual. Clearly, the solution is the consumption and bequest per child are a fixed fraction of after tax income:³ $c_o = \alpha(y_o + r_3 s)$, and $b' = \frac{(1-\alpha)(y_o + r_3 s)}{n}$. Note that human capital, h_o is measured in efficiency units that depend on ability, \hat{z} and wages are a function of whether an individual is college educated ($I^e = 1$) or not. If an individual is college educated, then $w_o = w_e$. To capture life cycle profile of earnings, human capital for an individual grows over time. In other words, human capital when old, h_o is as $\zeta > 1$ times as human capital when young, h_p . Savings, s and the number of children, n are not state variables because they are determined in the second period and can be expressed as a function of adulthood state vector, \vec{x}_p . Clearly, bequest is a function of the old age state variable of an individual (i.e. $b' = b'(\vec{x}_o; s, n)$).

Adulthood: Adulthood consists of two subperiods based on college attendance decisions and college outcomes. The first φ fraction of adulthood is assumed to be the first subperiod. In the first subperiod (ex-ante), the adult makes a college decision. If the individual chooses not to attend college, she is employed as unskilled worker and consumes all the parental bequest in the first subperiod. Also she provides unskilled labor in second subperiod and old age. If she decides to attend college, she pays the college tuition, t with her parental bequest, b and consumes the remaining bequest, $b - t$ in the first subperiod. The opportunity cost of attending college is the unskilled worker income foregone during the college years. The college attendance is a risky decision. There is a possibility that an individual with ability $\hat{z} \in [0, 1]$, if attends, might fail (succeed) the college with a probability of $1 - \hat{z}$ (\hat{z}). Clearly, the odds of success depends on the ability, \hat{z} of the individual. The college outcome is realized at the beginning of the second subperiod (ex-post). Successful individuals end up with the wages of a skilled workers and failure leads to the wages of unskilled workers in the labor market. In the second subperiod, an adult is best described by a state vector, $\vec{x}_p=(I^e, \hat{z}, z)$. From the perspective of an adult, the preschool ability of an adult, z is also a state variable due to the fact that it affects her children's preschool abilities. To capture the fact

¹Without loss of generality, we analyze the problem of the cohort of parents. For simplicity, a variable without prime refers a parent while a variable with prime refers to her children in the cohort of grandchildren.

²The Cobb-Douglas function is rescaled to generate an indirect utility function in terms of third period income.

³We do have no interest in the intrafamily pattern of bequests. Our interest here is nonetheless on the level of bequests. A variety of models highlight differences in bequests among siblings, something that Behrman and Rosenzweig (2004) question: see Becker and Tomes (1979, 1986), Behrman, Schleifer, and Summers (1985), Behrman and Severinov (2003), and Kolm(2006). Based on the estimates of Canon (2010), we simulated a modified version of our model that allowed bequests to be conditional on child's ability. The extended model, however, does not produce any significantly different outcome from our original model.

that the preschool ability of the parent, z and child, z' are correlated, it is assumed that $z'|z$ follows a first order Markov process in which :

$$z' = \lambda_0 + \lambda z + \epsilon \quad \epsilon \sim N(0, \sigma_\epsilon^2) \quad (1)$$

where $\lambda_0 < 0$ is a constant, λ is the correlation between the preschool abilities of parent and child, and ϵ is the white noise. Moreover, our model is flexible in the sense that for any parent, a different preschool ability for each child is drawn from this stochastic process. Put differently, any individual's children are allowed to differ in their preschool abilities. Notice that ability, \hat{z} rather than preschool ability, z determines the human capital of an individual as explained below.

At the beginning of the second subperiod (ex-post), the problem of a individual is described by:

$$\begin{aligned} V_p(\vec{x}_p) = \max_{c_p \geq 0, s, e \geq 0, n \geq 0} & \quad \frac{c_p^{1-\sigma} - 1}{1-\sigma} + \beta_3 V_o(\vec{x}_o; s, n) + \delta \frac{[n(\frac{e+\theta}{w_e})^\eta]^{1-\sigma} - 1}{1-\sigma} \\ \text{s.t.} & \\ c_p + s + ne = & \quad (1 - \phi n)y_p \\ y_p = (1 - \tau)(1 - \varphi)h_p w_p & \\ h_p = (k_0 + k_1 \hat{z}) & \\ w_p = w_u(1 - I^e) + w_e I^e & \end{aligned}$$

where $\delta > 0$ and $\eta \in [0, 1]$ are constants, c_p is the consumption, s is the saving, n is the number of children, e denotes the basic education (K-12) expenditure per child, θ is the amount of public spending per child in the basic education. Note that the second subperiod income is adjusted by the length of the second subperiod, $(1 - \varphi)$. Besides, it is assumed that raising a child takes fraction $\phi \in (0, 1)$ of parent's time.

A basic assumption in this formulation is that individuals are altruistic towards their children. An individual's utility depends on the number of children as well as the amount of resources devoted to each children's basic education.⁴ We assume each children within a family get the same amount of resources.⁵ Moreover, the crucial idea behind this specification is that individuals care about the real resources they can invest on their children, as far as they are concerned, all children are equal. One can interpret the real resource assumption in the utility function as the amount of teacher efficiency units devoted to each child's basic education. Real resources are calculated in terms of skilled efficiency units because teachers are assumed to be skilled workers. The concavity assumption in the real resources per child devoted to basic education implies that individuals realize the marginal returns to basic education, keeping other things constant, diminishes with higher real expenditures on basic education.

To see the quantity-quality trade-off of children faced by individuals, it worths analyzing the special case of iso-elastic utility function when $\sigma = 1$. It can be easily shown that the solution is given by:

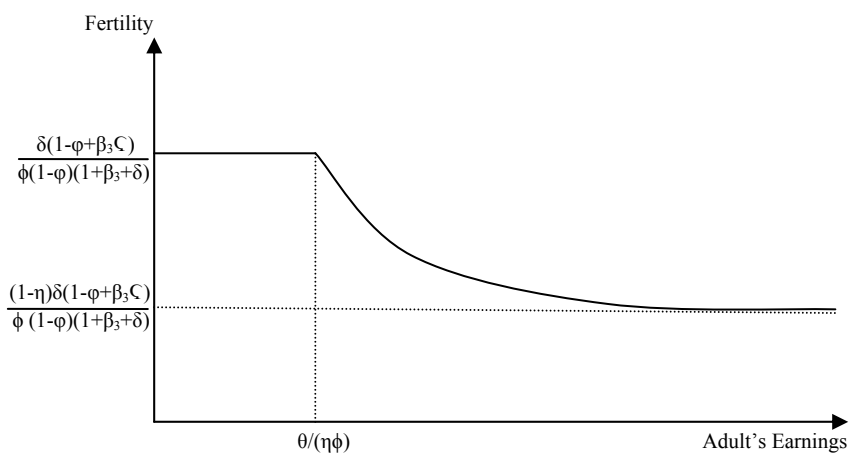
$$\begin{aligned} n &= \min \left[\frac{\delta(1-\varphi+\beta_3\zeta)}{\phi(1-\varphi)(1+\beta_3+\delta)}, \frac{(1-\eta)\delta(1-\varphi+\beta_3\zeta)y_p}{(1-\varphi)(1+\beta_3+\delta)(\phi y_p - \theta)} \right] \\ e &= \max \left[0, \frac{\eta\phi y_p - \theta}{1-\eta} \right] \\ s &= \frac{y_p \beta_3 (1-\varphi - (1+\delta)\zeta)}{(1-\varphi)(1+\beta_3+\delta)} \\ c_p &= \frac{y_p (1-\varphi+\beta_3\zeta)}{(1-\varphi)(1+\beta_3+\delta)} \end{aligned}$$

<< FIGURE 1 ABOUT HERE >>

⁴Along the lines of Becker and Barro (1988), and de la Croix and Doepke (2003), both fertility and expenditures on basic education are chosen endogenously and simultaneously in the model. Thus, parents face a quality-quantity trade-off in their decision on children. We also abstract from the spacing of children.

⁵Hanushek (1992) finds that parents appear to act in a compensatory manner, favoring lower-ability children within the family, or in a neutral manner.

Figure 1: Fertility by Adult's Earnings



As shown in Figure 1, the fertility rate is constant and the highest for poor individuals with $y_p < \frac{\theta}{\eta\phi}$. Those individuals completely rely on public education and spend no additional money on basic education (i.e. $e = 0$). For individuals with $y_p > \frac{\theta}{\eta\phi}$, fertility decreases with income while private expenditures on basic education increases with income. It implies that people with high levels of income (human capital) invest relatively more on the quality of children than their quantity. The reason is that while the public contribution on basic education, θ is fixed, the cost of raising children, ϕy_p is rising with income (human capital). As seen in the figure, there is a lower bound on the fertility rate and thus, the upper bound on the fertility differential is given by $\frac{1}{1-\eta}$.

Recall that $V_p(I^e = 1, \hat{z}, z)$ and $V_p(I^e = 0, \hat{z}, z)$ denotes the ex-post value functions at the beginning of second subperiod of adulthood for an individual (parent) who completes the college successfully and fails, respectively. At the beginning of adulthood, the individual (parent) receives a bequest, b from her parent (grandparent). The bequest she receives is a function of her parent's education status and her human capital (ability) (i.e. $b = b(\bar{x}_o^g) = b(I^{eg}, \hat{z}^g)$). By assuming individuals can finance their college education by their bequest, we introduce a capital market imperfection into the model. In the first subperiod, an individual is best described by a (ex-ante) state vector, $\bar{x}_a^g = \{\hat{z}, I^{eg}, \hat{z}^g, z^g\}$. At this point, it is important to emphasize the fact that given ex-ante state vector, \bar{x}_a^g , both the bequest left to the individual by her parent, b and her ability, \hat{z} are also known. It is because the ex-ante state vector, \bar{x}_a^g contains all the necessary information about the parent of individual (i.e. $b = b(I^{eg}, \hat{z}^g)$). Individuals with a bequest less than the tuition, $b < t$, have no choice but to remain uneducated. Let I^r be an indicator function that takes the value 1 if an individual attends college. Then, the expected lifetime utility of an individual not attending college, $I^r = 0$ is given by:

$$EV(I^r = 0, \bar{x}_a^g) = U_0 + \frac{[(b + \varphi(1 - \tau)h_p w_u)^\nu]^{1-\sigma} - 1}{1 - \sigma} + \beta_2 V_p(I^e = 0, \hat{z}, z) \quad (2)$$

Where $U_0 > 0$ and $\nu > 0$ are constants. The value of ν determines the marginal utility of consumption in the first subperiod, and $U_0 > 0$ captures all nonmonetary benefits associated with not attending college. The higher U_0 is, the less pleasant attending college for an individual is. A high value of parameter ν would increase the number of rich but less able individuals attending college and decrease the number of poor but smart not attending college. For an individual who attends college, there is an opportunity cost of attending college. An individual works as unskilled worker during college years, if she does not attend college. For an individual who attends college, the outcome is risky and her lifetime expected utility is given by:

$$EV(I^r = 1, \bar{x}_a^g) = \frac{[(b - t)^\nu]^{1-\sigma} - 1}{1 - \sigma} + \beta_2 [\hat{z} V_p(I^e = 1, \hat{z}, z) + (1 - \hat{z}) V_p(I^e = 0, \hat{z}, z)] \quad (3)$$

For those with enough bequest, $b \geq t$ it is optimal to attend college (i.e. $I^r = 1$) if $EV(I^r = 1, \bar{x}_a^g) \geq EV(I^r = 0, \bar{x}_a^g)$. Formally, the problem of an individual at the beginning of adulthood is described by:

$$EV_a(\bar{x}_a^g) = \begin{cases} \max_{I^r \in \{0,1\}} \{EV(I^r = 0, \bar{x}_a^g), EV(I^r = 1, \bar{x}_a^g)\} & \text{if } b(\bar{x}_o^g) \geq t \\ EV(I^r = 0, \bar{x}_a^g) & \text{if } b(\bar{x}_o^g) < t \end{cases} \quad (4)$$

Note that an individual with an ability, $\hat{z} = 0$ never attends college since the probability of success for her is, for sure, zero.

Childhood: As pointed out before, a child (parent) is born with a preschool ability, z that is correlated with her parent's (grandparent's) preschool ability, z^g . A child's ability, \hat{z} that determines the human capital of the child, depends on the preschool ability of the child, z . Besides, it also depends on resources devoted to her basic education in the childhood⁶ (i.e. θ and $e^g = e^g(\bar{x}_p^g)$).

⁶In a separate stream of literature, numerous studies have emphasized the role of early childhood education in determining future outcomes (e.g. Curie and Tomas (1999), Neal and Johnson (1996), or Keane and Wolphin (1997)).

Explicitly, the production function for ability,⁷ $G(z, e^g, \theta)$ is given by:

$$G(z, e^g, \theta) = z + c_0 + c_1 \exp(c_2 z) \left(\frac{e^g + \theta}{w_e}\right)^\eta, \quad 0 < \eta < 1 \quad (5)$$

Where $c_0 > 0$, $c_1 > 0$ and $c_2 > 0$ are constants. By obtaining a basic education, any child increases her ability production function by c_0 . Note that the private and public expenditures on basic education are perfect substitutes. One striking feature of this production function is that it does not depend on expenditures, but real resources devoted to basic education. To educate children, we need skilled workers as teachers. The term, $\frac{e^g + \theta}{w_e}$ shows the amount of teacher efficiency units employed in basic education per child. A second feature is the concavity assumption. More importantly, we do have a value added notation in which a child's ability production function increases by real resources at a diminishing rate. Lastly, another feature of this functional form is the interaction between preschool ability and expenditures. As a matter of fact, it implies that marginal returns to investment in basic education are higher for higher preschool ability children. In other words, the value added is higher for children with higher preschool abilities, holding real resources constant. With the value added notation, it is easy to justify real resources in the second subperiod utility function. Parent acting in neutral manner toward her children, care about the value added by school resources in their utility function.

In our model, ability, \hat{z} also reflects college completion probabilities as in Ben-Porath (1970). If attends, an individual successfully completes the college with a probability of \hat{z} , and fails with a probability of $1 - \hat{z}$. In theory, the ability production function, $G(z, e^g, \theta)$ takes always a value on $(-\infty, \infty)$. We need a mapping from the ability production function, $G(z, e^g, \theta)$ to ability that determines human capital, $\hat{z} \in [0, 1]$. Apparently, for our purposes, any cumulative distribution function suffices. Therefore, we assume

$$\hat{z} = \Phi(G(z, e^g, \theta)) \quad (6)$$

Where Φ is the cumulative distribution function for standard normal distribution.

Aggregate Dynamics: By using the method introduced by Tauchen (1986), we can construct a mapping from continuous AR(1) process for the transmission of preschool abilities between parents and grandchildren to a first order Markov chain with a discrete state space for $z'|z$. More importantly, the aggregate dynamics of the model economy can be described by a first-order Markov Chain.

Let $f_t(\vec{x}_p)$ be the ex-post probability distribution function (pdf) for parents when they are adults at time t , over the state space, Ω , where $\vec{x}_p \in \Omega = \{(z, \hat{z}, I^e) | \hat{z} \in [0, 1], I^e = \{0, 1\}\}$ is the vector of the state variables for a parent in the second subperiod of adulthood. Without loss of generality, the population of adults (parents) at time t is normalized to be one. Moreover, let $F_t(\Omega)$ be a vector representation of the probability distribution over the state variables of parents in the second subperiod of adulthood at time t (i.e. $F_t(\Omega) = f_t(\Omega)$). Put it differently, $F_t(\Omega)$ contains all the information for the probability distribution function of parents when they are in the second subperiod of adulthood at time t , $f_t(\vec{x}_p)$ over the state space Ω . Then, as in Hanushek et al. (2012), given wages and public expenditures on basic education, the evolution of the economy can be captured by a first order chain:

$$F_t(\Omega) = \Pi \left[\frac{1}{\gamma_g} F_{t-1}(\Omega) \right],$$

and the transition to next generation is given by:

In the paper, we do not explicitly consider investment and ability formation in early childhood. See, for instance, Cunha, Heckman and Schennach (2010) for the optimality of investing relatively more in the early stages of childhood than in later stages.

⁷In general, test scores are measured in standardized test scores as in our calibration data. Needless to say, the preschool ability, z is implicitly assumed to be measured in the same standardized test scores that also measures the ability in standardized test scores, $G(z, e^g, \theta)$.

$$\gamma_g F_{t+1}(\Omega) = \Pi F_t(\Omega),$$

where γ_g -1 is the population growth rate and Π is the transition matrix that depends on the transition probabilities of preschool abilities, public and private expenditures on basic education, and more importantly, the participation and profitability constraints that arises from college decision making.⁸ Note that the transition matrix seems to be time-invariant. As a matter of fact, it is time-variant. Since our focus is on the stationary equilibrium, it is thought of being a time invariant matrix in our model. The first transition maps the pdf of grandparents in adulthood, $F_{t-1}(\Omega)$ to the next generation, namely the pdf of parents in adulthood, $F_t(\Omega)$. Besides, the second transition maps the pdf of parents in adulthood, $F_t(\Omega)$ to the pdf of grandchildren in adulthood, $F_{t+1}(\Omega)$. In equilibrium, clearly, we expect to see the pdf of any generation to be time invariant (i.e. $F(\Omega) = F_{t-1}(\Omega) = F_t(\Omega) = F_{t+1}(\Omega)$) and hence, $f(\vec{x}_p) = f_{t-1}(\vec{x}_p) = f_t(\vec{x}_p) = f_{t+1}(\vec{x}_p) \forall \vec{x}_p \in \Omega$. At this point, it is tempting to think that the population remains constant. It is not the case. Since we are dealing with pdfs, their sum is normalized to be one although the population grows at a steady rate, γ_g -1 (i.e. the population of grandparents is $\frac{1}{\gamma_g}$ while the population of grandchildren is γ_g). Then, given the pdf of parents in adulthood, $f(\vec{x}_p)$ it is easy to calculate the population growth rate parameter, γ_g , the number of adults enrolled in the college, succeeded or failed as:

$$\begin{aligned} \gamma_g &= \int n(\vec{x}_p) f(\vec{x}_p) d\vec{x}_p \\ N^e &= \int_{I^e=1} f(\vec{x}_p) d\vec{x}_p \\ N^r &= \int_{I^e=1} \frac{f(\vec{x}_p)}{\hat{z}} d\vec{x}_p \\ N^f &= N^r - N^e \end{aligned} \quad (7)$$

where $[I^e = 1]$ implies an integration over skilled workers. Enrollment calculation is based on the fact that attending college is a risky decision and an individual with ability, \hat{z} succeeds college with probability \hat{z} , if attends.

Moreover, it is possible to calculate the proportion of parents in adulthood at state, \vec{x}_p^g with their parents (i.e. grandparents) in adulthood at state, $\vec{x}_p^{g^g}$ by $f(\vec{x}_p, \vec{x}_p^{g^g}) = Prob(\vec{x}_p, \vec{x}_p^{g^g}) = Prob(\vec{x}_p | \vec{x}_p^{g^g}) f(\vec{x}_p^{g^g})$, where $Prob(\vec{x}_p | \vec{x}_p^{g^g})$ is the entry in the transition matrix, Π that corresponds to the transition from state $\vec{x}_p^{g^g}$ to state \vec{x}_p^g .

Production Technology: A CES production function that uses skilled, \mathbb{E}^{s_p} and unskilled efficiency, \mathbb{E}^{u_p} units as inputs is assumed and given by:

$$Y = A[\xi(\mathbb{E}^{s_p})^{(\rho)} + (1 - \xi)(\mathbb{E}^{u_p})^{(\rho)}]^{1/\rho}, \quad (8)$$

where $A > 0$, $0 < \xi < 1$, and the elasticity of substitution is $1/(1 - \rho)$. In a competitive market, the representative firm maximizes its profit and hence, its profit is given by:

$$\max_{\mathbb{E}^{s_p} > 0, \mathbb{E}^{u_p} > 0} \{Y - w_s \mathbb{E}^{s_p} - w_u \mathbb{E}^{u_p}\} \quad (9)$$

Schools: All schools that provide the basic education and colleges uses skilled workers as teachers. Given that the college enrollment is N^r , we assume rN^r skilled labor efficiency units are needed to provide a college education with those individuals. For early childhood education, recall that $n(\vec{x}_p)(\frac{\theta + e(\vec{x}_p)}{w_e})$ is the skilled labor efficiency units invested by a parent on her children at state \vec{x}_p . The total amount of skilled efficiency units needed at schools is, therefore;

$$\mathbb{E}^{s_s} = rN^r + \int n(\vec{x}_p) \left(\frac{\theta + e(\vec{x}_p)}{w_e} \right) f(\vec{x}_p) d\vec{x}_p \quad (10)$$

⁸Details are available from the authors.

Parameter	Value	Parameter	Value	Parameter	Value
σ	1	ϕ	0.15	ρ	0.31
ζ	1.12	φ	0.16	η	0.6
ξ	0.68	k_0	1	k_1	9
r	0.16	δ	0.2	α	0.85
ν	0.01	λ_0	-0.68	λ	0.28
σ_ϵ	0.67	c_0	0.25	c_1	0.9
c_2	0.2	A	1.5	τ	4%

Table 1: Some Calibration Parameters

Market: For both skilled and unskilled workers, labor markets clear.

$$\begin{aligned}
\mathbb{E}^{s_p} + \mathbb{E}^{s_s} &= \mathbb{E}^s = \int_{I^{e_g}=1} h_o^g \frac{1}{\gamma_g} f(\vec{x}_p^g) d\vec{x}_p^g + (1 - \varphi) \int_{I^e=1} (1 - \phi n(\vec{x}_p)) h_p f(\vec{x}_p) d\vec{x}_p \\
\mathbb{E}^{u_p} &= \mathbb{E}^u = \int_{I^{e_g}=0} h_o^g \frac{1}{\gamma_g} f(\vec{x}_p^g) d\vec{x}_p^g + (1 - \varphi) \int_{I^e=0} (1 - \phi n(\vec{x}_p)) h_p f(\vec{x}_p) d\vec{x}_p \\
&\quad + \varphi \int \int_{I^{r'}=0} h_p \gamma_g f(\vec{x}_p', \vec{x}_p) d\vec{x}_p' d\vec{x}_p
\end{aligned} \tag{11}$$

Recall that grandparents, parents and grandchildren co-exist anytime. Skilled grandparents and parents provide the skilled labor while unskilled grandparents and parents as well as grandchildren in their college years not attending college provide unskilled labor. Note that as we calculate the unskilled efficiency units provided by grandchildren not attending college, we used the joint probability distribution. As for the government, it raises income taxes to finance basic education and college subsidies. The tax rate, τ and the ratio of public expenditures on basic education to college subsidies, $\frac{\theta \int n(\vec{x}_p) f(\vec{x}_p) d\vec{x}_p}{(r w_e - t) N^r}$ are exogenously set by the government. The government runs a balanced budget anytime, which is given by:

$$r N^r w_e + \theta \int n(\vec{x}_p) f(\vec{x}_p) d\vec{x}_p = t N^r + \tau [w_e \mathbb{E}^s + w_u \mathbb{E}^u] \tag{12}$$

Definition. A stationary equilibrium in this economy is a population growth rate parameter, γ_g , a set of policy and value functions such as $c_p(\vec{x}_p)$, $s(\vec{x}_p)$, $e(\vec{x}_p)$, $n(\vec{x}_p)$, $c_o(\vec{x}_o; s, n)$, $b(\vec{x}_o; s, n)$, $V_o(\vec{x}_o; s, n)$, $V_p(\vec{x}_p)$, $EV(I^r, \vec{x}_a)$, $EV_a(\vec{x}_a)$, a distribution of total human capitals across economy such as \mathbb{E}^{u_p} , \mathbb{E}^{s_p} , \mathbb{E}^{s_s} , wage rates (w_u and w_s), a government expenditure per child in basic education, θ , a probability distribution vector, namely $F(\Omega)$ such that:

- (i) Given wages, taxes, college tuition, public expenditure per child on education, adult and old individuals' policy functions solve their optimization problems.
- (ii) Given wages, the representative firm maximizes its profits.
- (iii) Government always runs a balanced budget. Labor markets clear.
- (iv) The probability distribution vector for children when they are adult, $F_{t+1}(\Omega)$ is related to that of their parents' when they are adult, $F_t(\Omega)$ by a first order Markov chain in which the transition matrix takes the evolution of exogenous states, child's and parents' optimal decisions into account.
- (v) The population growth, γ_g is constant over time. Also, all other variables, functions, and probability distribution functions are time invariant (i.e. fixed point).

3 Calibration

The model is calibrated and its solution is computed numerically. We choose the parameters of the model such that our benchmark economy resembles empirical features of the U.S. economy

around 2008. We provide a list of the key parameter values in Table 1. A sufficient number of evenly spaced points are chosen to replicate AR(1) process for the transmission of preschool ability, $z'|z$. We take a model period to be 25 years and a college period to be 4 years if an individual attends college. At this point, it is important to emphasize the fact that the statistics for the United States are reported annually, and needs to be adjusted accordingly by the corresponding period definitions of our model.

There is no easy way about the appropriate choice of parameter values. Fortunately, a few parameters can be taken directly from the literature.

(i) The evidence on the value for intertemporal preference, σ is mixed. Based on a micro model that explicitly allows for borrowing constraints, Keane and Wolpin (2001) reports a value about 0.5. On the other extreme, the estimates based on consumption Euler equations find a value of 3 (Hubbard et al. (1994)). Nothing is applicable to our model because there is infinite consumption substitution possibilities within a period. To be on the safe side, we choose $\sigma = 1$, somewhere in the middle.

(ii) The interest rate, r_3 and the discount factors, β_2 and β_3 are chosen to reflect a 4 % interest rate, as in Kydland and Prescott (1982).

(iii) The evidence on the cost parameter, ϕ for having a child (Haveman and Wolfe (1995) and Knowles (1999)) suggests a value equivalent to about 15% of parent's time endowment as long as the child lives with the parent. Accordingly, we choose a ϕ value about $0.15 * \frac{18}{21}$ by assuming that children live with their parents for 18 years.

(iv) As for the elasticity of substitution between skilled and unskilled efficiency units, Katz and Murphy (1992) reports a value of 1.41. Therefore, we set ρ so that it is 1.45 in the model.

(v) Obtained for the year 2008 from 2009 Current Population Survey (CPS), the ratio of the average income of a household with a 45-64 year old head of household to that of a household with a 25-44 year old head of household defines the value of life cycle profile of earnings parameter, ζ to be about 1.12.

(vi) The opportunity cost of attending college, φ is set to be $\frac{4}{25}$, based on the assumption that an individual provides unskilled labor during college years if she does not attend college.

(vii) The parameter, η determines the upper bound on fertility differential in the economy (See Figure 1). In the U.S. data, fertility declines with income. Moreover, the fertility differential has decreased over time (Jones and Tertilt (2007)). Hence, we set $\eta = 0.6$, that yields an upper bound of 2.5 on the fertility differential.

For the other parameters, we take an indirect approach. We assign the values of endogenous variables (calibration targets) in the model to match several important observations for the United States. The remaining parameters are simultaneously calibrated to be consistent with the values of those endogenous variables described in detail below:

(viii) At the dawn of the twentieth century, the college wage premium was exceptionally high. And then, it decreased in several stages over the next eight decades. But starting in the early 1980s the labor market premium to college degree rose sharply and by 2005, the college wage premium was back at its 1915 level (Goldin and Katz (2007)). Hence, the wage ratio of a skilled worker to an unskilled worker with the same human capital is targeted to be $\frac{w_u}{w_e} \approx 0.55$. Put it differently, it implies a wage premium of about 80 percent for college attendance. This fact defines ξ as 0.68. As for the other production function parameter, A is set to be 1.5 so that the wage for skilled worker, w_e is normalized to be 1.

(ix) The parameters for the human capital function are set to be $k_0 = 1$ and $k_1 = 9$ so that the ratio of maximum income to minimum income in the model is about 18.

(x) In 2008, the average tuition for a 4-year public college is \$13,429. Eighty percent of all 4-year public college students received some type of financial aid, and the average amount of federal student aid is \$6,600 (NCES). In addition to that, the median income of households with a college graduate head is \$78,290 (CPS). For an individual with average human capital, these numbers suggests that $\frac{t}{w_e} = \frac{0.96 * 1.12 * 5.5 * 4 * 6,829}{25 * 78,290} \approx 0.08$. Given the subsidy scheme, the ratio of college tuition to college

costs is given by $\frac{t}{rw_e} = \frac{4*6,829}{4*13,429}$. Therefore, r is set to be $r = 0.16$.

(xi) In 2008, the expenditure per child in K-12 is \$12,028, of which \$6,778 is received from federal and state governments.⁹ Then, $\frac{\theta \int n(x_p) f_{X_p}(x_p) dx_p}{(rw_e - t)N^r} = \frac{\theta \gamma_g}{(rw_e - t)N^r} = \frac{13*6,778*1.02}{4*6,600*0.65} = 5.24$. This ratio defines how government allocates its revenues: the ratio of public expenditures on basic education to college subsidies is exogenously set by the government as 5.24 in the budget. Besides, the income tax rate is chosen to be $\tau = 4\%$ so that the government budget balances.

(xi) The parameter δ determines the average level of fertility. The total fertility rate has been around 2 children per woman in the U.S. since early 70s. Accordingly, we choose $\delta = 0.2$ so that the population growth rate is close to zero.

(xii) Of all the youth who graduates from high school in 2008, 68.6 % are attending college in 2008 (CPS). Among high school graduates enrolled in college, 93.2 percent are full-time students. Approximately, 57 percent completed a bachelors degree (NCES). Those statistics allow us to define the calibration targets for the enrollment ratio and the ratio of individuals who successfully complete the college.

(xiii) The third period utility function parameter α determines the fraction of income that is left as a bequest to children. Clearly, it determines the ratio of individuals without enough funds to pay for the college. To get a ratio about 6.3 percent, α is set to be $\alpha = 0.85$. Also, as suggested by Carneiro and Heckman (2002), the ratio of financially constrained agents is in between 4 percent and 8 percent. Both the nonmonetary benefit of not attending college, U_0 and the taste parameter for the first period consumption, ν determines the college enrollment and the ratio of financially constrained agents in the model. We set U_0 and ν to be 3.12 and 0.01, respectively so that the college enrollment is about 64 percent and the ratio of financially constrained agents, calculated as in Carneiro and Heckman (2002), is about 4.2 percent.

(xiv) As in Hanushek and Yilmaz (2004), a Galtonian regression is estimated by using standardized test scores for children and their mothers. The regression yields a slope of 0.4, and zero intercept as expected. In the transmission of preschool abilities, the slope is calibrated to be $\lambda = 0.28$ so that the correlation of parent and child's abilities in standardized test scores is about 0.4 in the model. Moreover, the standard error, σ is chosen to be about 0.7 so that the standard deviation of children's abilities in standardized test scores is unity. The intercept, λ_0 is assigned to a value about -0.68 so that children's preschool abilities have a mean of -0.75. The calibration of intercept allows us to match the fact that an individual with a degree less than 9th grade (i.e. no high school degree in the model) earns approximately as half as the earnings of an individual with a high school degree (CPS).

(xv) The evidence on the impact of resources on human capital formation is mixed. In his well known work, Hanushek (1996) reports that the impact of resources on student achievement is not clear, and there are even a significant number of studies who find statistically significant negative impacts. For instance, Heckman, Layne-Farmer and Todd (1996) conclude that the school resources (K-12) have little effect on earnings. On the other hand, Card and Krueger (1996) find a value of 0.2 for the wage elasticity of expenditures on education. Our job is clearly not easy. The parameters in the early education production function are set to be $c_0 = 0.25$, $c_1 = 0.9$ and $c_1 = 0.2$ to produce the following facts: (i) the average of children's abilities in standardized test scores is zero, (ii) the wage elasticity, for an individual with an average preschool ability (i.e. $z = -0.68$)

⁹The U.S. K-12 school system is a highly decentralized system that relies heavily on local property taxes. Due to a high degree of segregation by income, schools in richer school districts are capable of generating more funding. As in Restuccia and Urrutia (2004), one can see local expenditures in K-12 as private expenditures in early education and state and federal government expenditures as public expenditures in early education. In an attempt to improve the inequality in educational opportunity, we also see a growing involvement by the states to reduce the spending disparities across school districts. Both the method and the degree of school finance equalization differ by state. See Hanushek and Yilmaz (2007, 2010) for a study of some prominent school finance equalization policies in the U.S. With a school finance scheme that relies more heavily on state and federal government expenditures (i.e. public expenditures) and less on local expenditures (i.e. private expenditures), we re-simulated our model and the policy experiments. The findings are quite similar to the ones reported in the paper.

and an average expenditure on basic education, with respect to expenditures on basic education is about 0.12, and (iii) for an individual with an average expenditure on basic education, the value added gap between an individual at +1 standard deviation preschool ability and an individual at -1 standard deviation preschool ability is about +0.17 standard deviation ability. Put it differently, while holding education status constant, an individual at average preschool ability, $z = -0.68$ with average expenditure on basic education would make about 7 percent more if she was endowed with one more standard deviation of preschool ability.

4 Base Outcomes

Schooling Outcomes and Borrowing Constraints: The base outcomes for the model are summarized in Figures 2 through 4 and Table 3. Figure 2 shows the average college attendance and completion rates by parent earnings quartiles. Consistent with the data¹⁰, 64.9 percent of children are enrolled in the college, but only 43.2 percent successfully completes the college. Besides, the college attendance and completion rates are positively correlated with parent's earnings and children with richer families are more likely to both attend and successfully succeed in the college.

<< FIGURES 2-4 ABOUT HERE >>

For our model, it is impossible to directly calculate the ratio of financially constrained children since the expected utility of attending college is not defined for financially constrained children. To get over this problem, we follow the strategy of Carneiro and Heckman (2002). What they do is to divide their sample into twelve groups by parent earnings quartiles and children's ability terciles by using Armed Forces Qualification Test (AFQT) scores as a proxy for ability. For each ability tercile, they assume that children in the highest income quartile are not financially constrained and then, they define the percent credit constrained as the gap between the percentage enrollment in the highest income quartile and the percentage in other income quartiles. By multiplying the percent credit constrained by the population weight of each group, they find the number of financially constrained children at each group. We find that 4.2 percent of children are financially constrained. Needless to say, this number is consistent with the estimate of Carneiro and Heckman (2002) that finds, after controlling for the long run factors, between 4 and 8 percent of children in the United States are financially constrained. Figure 2 also traces out the ratio of financially constrained children by their parent's earnings. By definition, there are no financially constrained children for the top earnings quartile (the rich). The ratio of financially constrained children increases as income decreases and reaches a level about 13.2 percent at the bottom earnings quartile (the poor).

Quality and Quantity of Children: The economic forces behind the college enrollment and completion rates, and borrowing constraints can be best seen in Figure 3. In the benchmark, the population growth rate is about 2 percent. Since a period is 25 years in the model, the population growth rate should be interpreted over 25 years. As in the data, it is almost zero. The average fertility rate, as expected, decreases with earnings. To be able to compare the outcomes of the model on fertility to the relevant empirical evidence, the fertility measure in the model needs to be multiplied by two to yield the corresponding fertility measure in the empirical literature, namely the number of children per woman. In the benchmark economy, the average fertility is about 2.04 children per women. As for the differential fertility, it is determined by the average fertility gap between the top earnings quartile and the bottom earnings quartile. Consistent with Jones and Tertilt (2008), the average fertility rates are 1.36 and 2.5 children per woman for the bottom and top earnings quartiles, respectively.

¹⁰Recalling that our model is calibrated to the year 2008, it is hard to directly compare our benchmark attendance and completes rates to the U.S. Current Population Survey reported in Manski (2002). However, the college attendance and completion patterns of our model resembles to his.

Figure 2: Distribution of Choices (Benchmark)

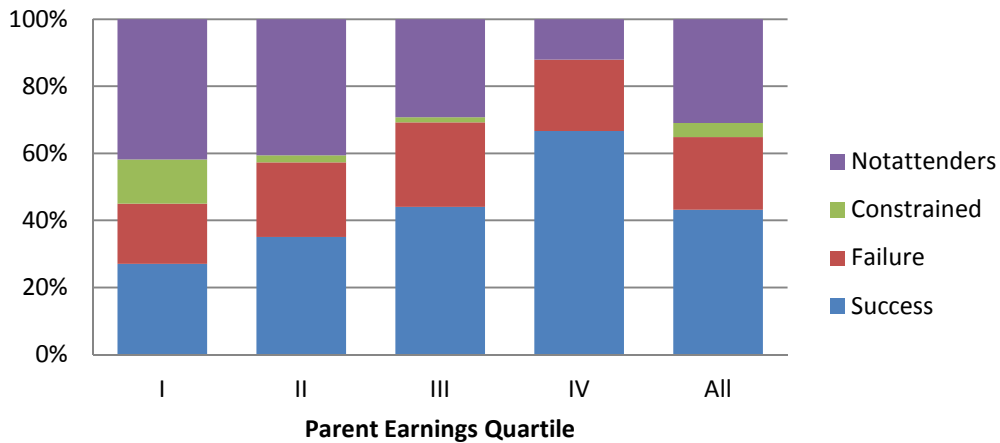


Figure 3: Quantity and Quality of Children

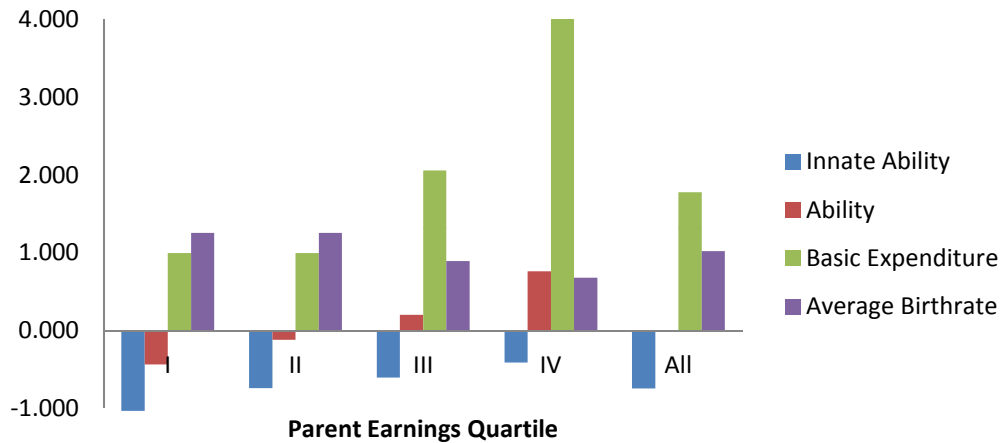
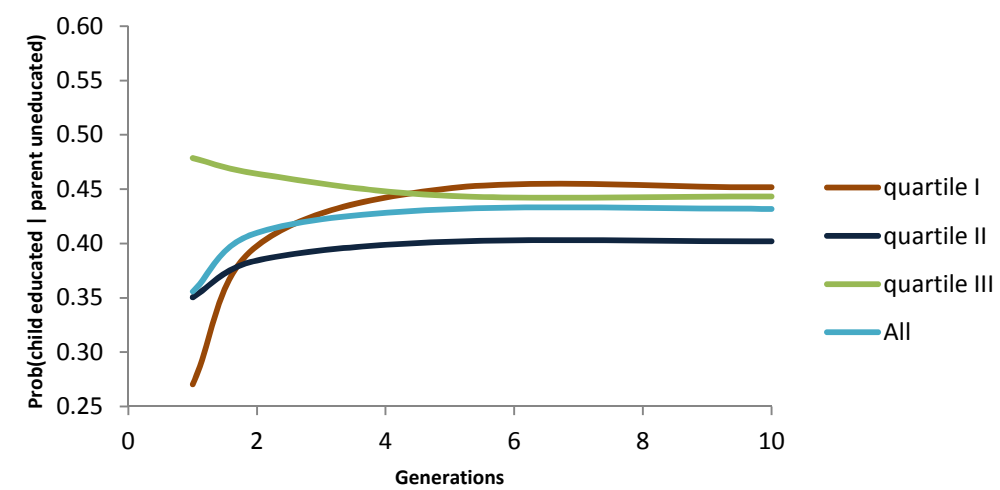


Figure 4: Parent-offspring Evolution of Education



Quartile	I	II	III	IV	All
Corr.	0.32	0.12	-0.07	0.17	0.4

Table 2: Parent-Child Earnings Correlation by Parent Earnings Quartiles

For each parent earnings quartile, we also plot average preschool ability, ability, and expenditures (per child) on basic education for comparison purposes. Recall that basic expenditures on education have both private and public components. We normalize total expenditures on basic education by public expenditure on basic education. The first (bottom) and second quartiles purely rely on public basic education and have no supplementary private spending on basic education. The top quartile (rich) spends as 3.3 times public expenditure on basic education as private expenditure on basic education. Not surprisingly, the average child preschool abilities increase with parent earnings because parent abilities as well as preschool abilities are correlated with parent earnings, and a fraction of parent preschool ability is inherited. In case of child abilities, the relationship with parent earnings is much steeper, with a value added of about 0.6 standard deviation for the bottom parent earnings quartile while about 1.2 standard deviation for the top parent earnings quartile. Clearly, the differences in expenditures on basic education between poor parents and rich parents are also behind this amplification. To sum up, preschool ability that is correlated with parent preschool ability, and expenditure on basic education that is correlated with parent earnings interact to amplify the initial difference in preschool ability. What we see is nothing but a tradeoff between the quantity and the quality of children.

Inequality and Intergenerational Mobility: Table 2 shows the earnings correlation between a child and her parent by parent earnings quartile. In line with the estimates of Solon (1992, 2002), we find an earnings correlation of 0.4. Not surprisingly, the earnings correlation is smaller at the mid-quartiles. The persistence is larger at the bottom quartile (poor). This is due to the fact that the poor (rich) has too many (few) children and less (more) resources to invest on each of them. As a matter of fact, most financially constrained children are at the bottom parent earnings quartile, aside from the fact that the value added after basic education is smaller for them as a result of overstretched parent resources.

Ever since the beginning of the United States' war on poverty, policymakers and academics are interested in the persistence of educational status because obtaining higher education levels are seen as a key step to reach higher income levels (Hanushek, Leung, and Yilmaz (2003)). In Figure 4, we plot the probability that a child is college educated given that her parent is college uneducated over successive generations. We do this for each parent earnings quartile. As can be noticed, the top parent earnings quartile is missing because there are no children with college uneducated parents at the top parent earnings quartile. The pattern, as expected, shows a regression to mean. The most significant curve is the one for the bottom parent earnings quartile. The probability rises from 27 percent to 40 percent in the second generation, and reaches its steady level of 45 percent after 5 successive generations.

We are also interested in the cross-sectional income inequality. To measure the cross-sectional inequality, we use the gini coefficient. In the U.S., it was steady about 38 percent during 1960 through 1990s. After 90s, it started increasing and reached a level of 42 percent in 2010. At any period, we have both adult and old workers in the model. Based on their earnings, we calculate a gini coefficient of 39.2 percent for the benchmark economy.

5 An Economy without Fertility Differential

Throughout the paper, we argue that differential fertility matters. This is because especially parents with low earnings do not have enough resources to invest on their children. Not only because they are poor, they also have more children. In other words, these parents can spare less resources on

	Benchmark	No Differential Fertility
Schooling outcomes		
Enrollment	64.9%	69.6%
Success	43.2%	47%
Constrained	4.2%	0.9%
Correlation		
Ability	0.4	0.39
Income	0.4	0.36
Other		
Avg. Ability	0	0.1

Table 3: Benchmark versus an Economy without Differential Fertility

each child per se. To show the importance of differential fertility, we perform a partial equilibrium experiment. In our experiment, we assume that each parent has one child only. All other remaining parameters are the same as in the benchmark economy, but only the tax rate is adjusted to balance the government’s budget. We analyze the new economic environment as a result of altered individual decision makings, and report the experiment results along with the benchmark in Table 3. Compared to the benchmark, more children attend and succeed in college as we shut down differential fertility. Clearly, the reason is that now the poor has more resources to invest on her children, and the rich has more smart children. As a matter of fact, the average ability in the economy is higher.

We can see the importance of explicitly modeling differential fertility, especially when we calculate the ratio of financially constrained children. The experiment underestimates it substantially and finds a ratio of 0.9 percent, all of which comes from the bottom parent earnings quartile. This number corresponds to 3.5 percent of the bottom parent earnings quartile, which is much less than the corresponding benchmark value of 13.2 percent. When we look at parent-child earnings correlation, the fertility differential accounts for a 10 percent of the correlation. This underestimation is also valid for the correlation of parent-child abilities, although the degree is smaller. The bottom line is that a researcher cannot ignore the fact that the poor has more children and has to split up already scarce resources among her children. Hence, eliminating fertility differential would underestimate the presence of financially constrained children. This underestimation becomes a much more serious concern when the aim is to study financially constrained children.

6 Sensitivity Analysis

For our model, the key parameters have not been estimated with much precision. More importantly, both the fertility and differential fertility rates have been declining over time. We therefore perform a sensitivity analysis.

In Table 4, we analyze the outcomes by assuming different altruism parameter, δ and differential fertility rate, η values, and report schooling outcomes, welfare and inequality measures in ex-post expected utilities,¹¹ cross-sectional inequality and intergenerational mobility of earnings, and the population growth rate. For comparison purposes, the benchmark outcomes are described in the middle column. As in Hanushek et. al. (2003), the welfare is measured by Aggregate Expected Utility (AEU), which is the mean value of expected utilities, and inequalities are measured by Gini coefficient. The benchmark value of AEU is normalized to be one. Notice that the bottom row shows population growth rate. As we alter the value of altruism parameter, δ by 25 percent, all the reported benchmark outcomes are in between the corresponding new outcomes. Compared to the benchmark, a decrease in the value of altruism parameter, δ moves the economy into a better state. In other words, more children attend and successfully complete the college, less children

¹¹The same pattern arises for the calculations based on ex-ante expected utilities

	δ		Benchmark	η	
	25 % lower	25 % higher		25 % lower	25 % higher
Schooling outcomes					
Enrollment	67.3%	58.7%	64.9%	65.2%	64.8%
Success Rate	67.2%	59.9%	66.6%	67.2%	68.1%
Constrained	0.4%	25.2%	4.2%	2.9%	5.8%
Expected Utility					
AEU	1.09	0.73	1	1.04	1.03
1-GINI	0.6	0.27	0.54	0.57	0.55
Earnings					
Correlation	0.32	0.49	0.4	0.33	0.48
1-GINI	0.62	0.53	0.61	0.60	0.60
Other					
Pop. growth rate	-15%	21%	2%	12%	-9%

Table 4: Some Aggregate Statistics under Alternative Model Parameters

are financially constrained, and the welfare, is higher. Besides, we have more equality in terms of expected utilities, higher intergenerational mobility, and less cross sectional inequality of earnings. On the other hand, an increase in the value of altruism parameter, δ leads the economy to an opposite, worse state than the benchmark. As for the story behind this pattern, a higher altruism parameter, δ leads to a higher fertility for every individual in the economy. Since the population growth rate is larger, both private and public expenditures per child on basic education, θ is lower and college tuition, t is higher. Put it differently, the society has less resources for children and ends up with children of lower quality. As a result, we have worse schooling outcomes and a lower welfare. A higher altruism parameter, δ also leads to a higher fertility gap between top and bottom parent earnings quartiles. As a result, the child quality gap between a rich parent and a poor parent becomes higher. Therefore, we have worse inequality measures, a more persistent intergenerational mobility of earnings, and more financially constrained children.

Altering the value of differential fertility parameter, η by 25 percent yields a similar pattern with the exception of success rate and the statistics based on expected utilities. As the value of differential fertility parameter, η decreases, the economy moves into a better state. We see more enrollment, a higher success rate, less financially constrained children, a better cross sectional and intergenerational mobility measures. The story behind this pattern is that a lower fertility differential, (η) means more smart and well educated children by rich parents, leading to a lower return to college in equilibrium. The welfare is higher because we have more high quality children with rich parents. The inequality of expected utilities is lower because we have less financially constrained children and the wage ratio of skilled worker to unskilled worker is lower. Clearly, more higher quality children for the rich results in a lower persistence of the nexus between parent and child abilities. It leads to a lower parent-child earnings correlation. A higher fertility differential, (η) leads to less enrollment, a higher success rate and more financially constrained children. In addition to that, both welfare and inequality in terms expected utilities improve, and intergenerational mobility of earnings becomes more persistent. Although it looks like a puzzle, it is not the case. The reason is as follows: Due to less population growth rate, however, the public expenditure on basic education, θ is substantially higher and college tuition, t is lower. After all, the government runs a balanced budget. It is true that we have less high quality children from rich parents. On the other hand, a substantial increase in public expenditure on basic education, θ and more private expenditure on basic education by the rich leads to a significant increase in children's abilities. The latter dominates the first and the return to college becomes lower. The net effect is a decrease in enrollment rates and increase in success rate. Not surprisingly, the welfare is higher and the inequality in expected utilities is lower. We see a more persistent intergenerational mobility of earnings because rich has less children that causes

	η		
	<i>25 % lower</i>	<i>no change</i>	<i>25 % higher</i>
θ higher	1.037 (0.567)	1.012 (0.545)	1.049 (0.562)
no change	1.039 (0.568)	1 (0.538)	1.029 (0.550)
t lower	1.055 (0.586)	1.029 (0.565)	1.045 (0.566)

Table 5: Aggregate expected utility (1-Gini of expected utilities) after the policy change

	η		
	<i>25 % lower</i>	<i>no change</i>	<i>25 % higher</i>
θ higher	0.323 (2.9%)	0.362 (2.7%)	0.469 (2.7%)
no change	0.329 (2.9%)	0.395(4.2%)	0.482 (5.8%)
t lower	0.313 (0.1%)	0.360 (0%)	0.472 (3.1%)

Table 6: Parent-child earnings correlations (ratios of financially constrained children) after the policy change

a persistence in the intergenerational mobility of ability. As for the reason behind the increase in the ratio of financially constrained children, the total expenditure on basic education, hence ability increases more sharply by parent earnings level. As a result, the enrollment gap with the top parent earnings quartile is higher at each parent earnings quartile.

7 A New Finance Policy

Recently, early childhood education policies have regained its popularity (e.g. Heckman (2006)). In order to provide an understanding of the general equilibrium implications of differential fertility on the early childhood education finance and college subsidy policies, we analyze the effect of a 10 percent increase in total government expenditures for education accompanied by an increase in income tax rate, τ so that the government budget clears. We consider two cases: The additional revenues are allocated to (i) public expenditures on basic education or (ii) subsidies to college education. To highlight the importance of differential fertility, we simulate the experiment under different differential fertility parameter, η values.

We report the welfare (AEU) and inequality of expected utilities in Table 5 as well as the parent-child earnings correlation and the ratio of financially constrained children in Table 6. For the benchmark parameter values, the experiment results are reported in the middle columns. As the additional government revenues are allocated to the public expenditures on basic education, the welfare and the equality in expected utilities increases. The economy ends up with less financially constrained children and less persistent intergenerational mobility of earnings. We see the same pattern as the additional government revenues are allocated to the college subsidies. At the benchmark, the public expenditure on basic education, θ is much higher than college tuition, t and everybody gets a basic education while a subpopulation obtains a college degree. Given a 10 percent increase in government expenditures on education, the change in public expenditure in public education, θ , hence, is relatively much smaller than the change in college tuition, t . As a matter of fact, the latter policy is more effective and gains are larger.

As the differential fertility parameter, η changes, so does the population growth rate. More precisely, the population growth rate decreases as η increases. As a result, public expenditure on education, θ increases more. Besides, recall that the differential fertility parameter, η also shows up in the education production function. The higher the η is, the more productive resources are, and the higher the value added in ability is. Hence, subsidizing public expenditures on basic education becomes a better policy option than subsidizing colleges. On the contrary, subsidizing college becomes

a better policy option than subsidizing public expenditure on basic education as η decreases.

8 Conclusion

We develop a general equilibrium model that can provide various insights into the implications of basic education (K-12) and college subsidies for welfare, cross sectional inequality, and intergenerational mobility. A major contribution of this paper is the explicit modeling of fertility behavior and ability formation by incorporating an education production function. The quantity-quality tradeoff of children leads to an underinvestment in basic education and borrowing constraints becoming a much more serious problem for the poor. Our model provides an environment to study issues related to the role of government in the finance of schools and colleges. As a matter of fact, we use the model to compare effectiveness of basic education and college subsidies under different fertility patterns.

The paper has some interesting findings and provides new insights. First, it shows the importance of differential fertility while studying borrowing constraints. We learn that the poor has already limited resources and has to split it up among her children. As fertility behavior changes, its effect on borrowing constraints changes as well. Ignoring altruism, δ and differential fertility, η can lead us into wrong territories. The sensitivity analysis provides support for our basic findings as well. In fact, it steers us to the design of an education finance policy that depends on fertility behavior.

Second, we analyze two different education finance policies: A 10 percent increase in government revenues are allocated to (i) public expenditures on basic education or (ii) college subsidies. Both policies improves the welfare, inequality and intergenerational mobility. As expected, the optimal policy depends on the value of differential fertility parameter. As the differential fertility parameter, η increases (decreases), the basic education subsidies (college subsidies) becomes a better policy.

Clearly, there is a role for college subsidies, depending on the value of differential fertility parameter, η . Our paper considers only uniform subsidies for both basic education and college subsidies. Fortunately, Hanushek et al. (2004) compares different college subsidies in their paper. In the quest for the optimal education finance policy, a more targeted basic education subsidies might be a better option than these two policies we have in the paper. But, what we have in the paper is a common policy practice all over the world: the government provides a basic education with uniform subsidies for all.

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