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LITIGATION AND SETTLEMENT UNDER JUDICIAL AGENCY

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Litigation and Settlement under Judicial Agency*

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Abstract

We model the settlement of a legal dispute when the trial outcome depends on the behavior of a strategically motivated judge. A defendant, who is uninformed about the level of harm that he has caused, makes a take-it-or-leave-it offer to an informed plaintiff. If the parties cannot agree on a settlement and the case goes to trial, the judge decides how much effort to exert in discovering the actual damages. We show that under very general assumptions this model exhibits multiple equilibria. In some equilibria, the judge exerts less effort and more cases settle out of court, whereas in others the opposite occurs. We also show that the judge prefers the low effort equilibria with high settlement rate and argue that a "managerial judge" could easily steer the parties towards low effort equilibria. This may be deemed undesirable since in low-effort equilibria, the terms of the settlement heavily favor the informed plaintiff, and this in turn induces over-investment in ex ante preventive care by the defendant.

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Keywords: Litigation, Settlement, Managerial Judges.

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1 Introduction

There is a substantial literature on pretrial settlement, but only a few papers explicitly model the trial stage and fewer yet examine the role of the trial judge. A typical game-theoretic analysis assumes that there is asymmetric information about damages (or liability, or both) at the settlement stage, but the truth will come out at the trial, perhaps with some exogenously specified probability.\(^1\) In reality, how much is learned during the trial depends on many factors and the trial judge is one of the most important. Our main contribution is to model the behavior of the trial judge and its effect on pretrial settlement. More precisely, we assume that the probability of discovering the truth depends on how much effort the judge expends during trial and that she chooses her effort strategically.

In our model, a plaintiff who knows the actual damages that he suffered sues a defendant. During the pretrial settlement stage, the defendant, who is uninformed about the level of damages, makes a take-it-or-leave-it settlement offer to the plaintiff. If the plaintiff rejects this offer, the case goes to trial and the judge awards damages that she deems appropriate. The judge is initially uninformed about the actual damages but she learns the true value with some probability, which increases in her (costly) effort during trial. We assume that she cares about accuracy so that if she discovers the true value, she awards that amount, while if she remains uninformed, she awards the expected damages (after updating her prior beliefs using the fact that the case has come to trial).\(^2\) The basic tradeoff that the judge faces is simple: effort is costly but it increases the accuracy of her decisions.

We first establish that the equilibrium settlement offer by the uninformed defendant is higher, and hence more cases settle, when the judge is expected to exert lower effort at trial. This simply follows from the fact that the parties would fail to settle only if they are asymmetrically informed about the trial outcome. Since only one of the parties knows the true value of the damages, the degree of this asymmetric information increases as the judge's decision becomes more accurate. In fact, as we show later on, what really matters is not how close the judge's award to the true value of damages, but rather how sensitive it is to changes in that value.

We fully characterize the set of equilibria and show that the model has generically multiple equilibria. This is a novel feature and the intuition is as follows: If the litigants expect high effort from the judge, then more cases go to trial. This means that the variation in damages among the tried cases is large and hence the uninformed judge is likely to make a large error in assigning damages. Therefore, she will indeed have an incentive to exert high effort. Similarly, low effort becomes self-sustaining because in that case fewer cases go to trial and the judge can not err too much, and this diminishes her incentives for effort.

We also show that the judge and the informed plaintiff are better off in the low effort (and high settlement rate) equilibria whereas the uninformed defendant is better off in the high effort (and low settlement rate) equilibria. In low effort equilibria, the variance of damages among the tried cases is small, which implies that the judge will make small mistakes in awarding damages. Therefore, she avoids exerting too much effort and obtains a high payoff. In contrast, in high effort equilibria fewer

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\(^1\)The best-known models are the screening model of Bebchuk (1984) and the signaling model of Reinganum and Wilde (1986). Both models study ultimatum bargaining games in which either the plaintiff or the defendant has private information about damages or liability. For models with two-sided asymmetric information, see Friedman and Wittman (2006) and Daughety and Reinganum (1994).

\(^2\)In order to simplify the analysis, we assume that the litigants have a passive role at the trial. Friedman and Wickelgren (2010) investigate a settlement problem similar to ours where the plaintiff can exert effort at the trial to increase his probability of winning.
cases settle and hence the judicial errors can be large. Therefore, she exerts high effort, but if she still remains uninformed she also ends up making larger errors. This implies that she obtains a lower payoff.

We argue that these results are quite relevant for the ongoing debate on the so-called ‘managerial judges,’ i.e., judges who get actively involved in the pretrial stage, presumably to promote and encourage settlement over trial. Schrag (1999) observes that the “proponents of managerial judging identify abuse of the pretrial discovery privilege as a main cause of both high litigation costs and the slow resolution of disputes. Judges can improve outcomes by intervening in the earliest stages of legal disputes.” But there is also a drawback. As Resnik (1982) pointed out, the “judges are acting more forcefully. [...] Some warn the parties that the judge would take a dim, and possibly hostile, view of either side’s insistence on going to trial.” Our results suggest that the judge would indeed like to get involved in the pretrial settlement stage and signal that she will select the low equilibrium effort. Even if this signal takes the form of cheap-talk, i.e., simple announcements or hints, it will succeed in selecting the low effort equilibrium. Furthermore, such an announcement is “self-signaling” and “self-committing,” i.e., the judge would make this statement if and only if it is true and she would indeed want to choose low effort if it is believed. Therefore, there is a very strong reason for the litigants to believe this announcement and behave according to the low effort equilibrium.

Even a small degree of self interest on the part of the judges, therefore, can have an enormous impact on settlement rates above and beyond the more commonly considered factors, such as the degree of asymmetric information, the size of the trial awards, the magnitude of trial costs, discovery rules, etc. Furthermore, as we indicated above, higher settlement rates come with significant distributional implications, which is precisely the concern raised in Fiss (1984) and Resnik (1982). It might be true that legal disputes are resolved at lower cost when they are settled out of court, but the terms of the settlement usually favor the party with the informational advantage. Therefore, there may be good reasons to try and limit judicial involvement in the pretrial bargaining stage.

There are few other papers in which the trial outcome is dictated by an imperfectly informed judge who rationally updates her beliefs when the case comes to court. But there is no model in which she herself chooses how much information to have. Daughety and Reinganum (1995) model the settlement stage as an ultimatum bargaining game in which the informed party is the proposer. If the case goes to trial, the judge learns the true damages with an exogenously specified probability, if not, she must infer it from observable actions of the plaintiff and the defendant. Kim and Ryu (2000) study a similar problem using a screening model (the uninformed party is the proposer) where the judge is assumed to receive a noisy signal about damages. Finally, Rasmusen (1995) studies a plaintiff’s decision to bring suit when the court assesses true damages with an exogenous error. Anticipating that such errors will influence the pool of plaintiffs who go to trial, the court adjusts the award accordingly, the direction of which depends on whether the error is predictable by the plaintiff.

\footnotesize
\begin{itemize}
  \item Resnik (1982) and Fiss (1984) are the two widely cited papers against this type of “managerialism.”
  \item See Farrell and Rabin (1996) on the credibility of pregame cheap-talk messages and a discussion of “self-signaling” and “self-committing.”
  \item For models in which the judge makes systematic errors, see Hylton (2002) and Landeo, Nikitin and Baker (2006).
  \item They show that if the judge can observe the plaintiff’s settlement demand, then she uses that information, and this feeds back into the settlement process, resulting in the plaintiff making demands to influence the judge. As the judge’s dependence on such information increases (i.e., as the probability of learning the truth decreases), more and more types of the plaintiff pool by making a high demand.
  \item They find that when the judge observes the defendant’s offer, the plaintiff rejects a larger set of offers in order to influence the judge’s subsequent beliefs to his advantage.
\end{itemize}
The rest of the paper is structured as follows. Section 2 lays out the model. Section 3 analyzes the case with exogenous judicial error. In Section 4 we endogenize judicial error by allowing the judge to become better informed by exerting a costly effort. In Section 5 we discuss the policy implications of our analysis. Section 6 contains some discussion about the possible generalizations and extensions of our model and Section 7 concludes. All the proofs omitted in the main text are in Section 8.

2 The Model

We present a simple model of litigation under strict liability in which a risk-neutral plaintiff claims to be harmed by a risk-neutral defendant.\(^8\) We let \(\theta \in [\theta, \theta]\) denote the actual damages suffered by the plaintiff and assume that only the plaintiff knows \(\theta\), whereas the defendant has probabilistic beliefs about it. We represent his beliefs by a probability distribution function \(F\), with density \(f > 0\) and full support on \([\theta, \theta]\). The parties have the option of settling the issue among themselves, but if they fail to do so, the case goes to trial where the court decides on a settlement.

A trial is costly for both parties. We let \(c_p > 0\) and \(c_d > 0\) denote these costs for the plaintiff and the defendant, respectively.\(^9\) Due to trial costs, the parties have some interest in settling the issue through private negotiations. These negotiations can take many forms and the outcome may depend on the bargaining protocol assumed, especially when the parties have asymmetric information. We employ a very simple and commonly used model and assume that the ( uninformed) defendant makes a take-it-or-leave-it settlement offer \(s \geq 0\) to the (informed) plaintiff, who either accepts or rejects it.\(^10\) If the offer is accepted, then the payoffs of the plaintiff and the defendant are \(s\) and \(-s\), respectively. If it is rejected, then the case goes to trial and the court decides on the amount that the defendant must pay the plaintiff.

The court is represented by a judge who has the same prior belief about \(\theta\) as the defendant and cannot observe its true value unless she expends costly effort. In particular, if she exerts an effort level \(e \in [0, \pi]\) at trial, she learns the true value of \(\theta\) with probability \(p(e)\), but incurs a cost of \(c(e)\). We assume that \(p(e)\) and \(c(e)\) are twice continuously differentiable and strictly increasing functions such that \(p'(e)\) is finite for all \(e > 0\), \(p'' \leq 0\), and \(c'' > 0\).

We interpret effort as the amount of deliberation and mental energy the judge devotes to the case and \(c(e)\) as the opportunity cost of this deliberation. The amount of deliberation that goes to a given case is an important judicial decision and this view of judicial behavior is in line with the findings from a number of recent studies. For example, Bainbridge and Gulati (2002) argue that the judges “commonly rely on rules of thumb–decision making heuristics and shortcuts,” rather than applying “the complex modes of legal reasoning.” The study of trial judges by Guthrie, Rachlinski and Wistrich (2007) also emphasizes the costly deliberation vs. intuitive decision making trade-off. In their view, judges are “predominantly intuitive decision makers,” and we need to recognize “both the important role of the judicial hunch, and the importance of deliberation in constraining the influence of intuition which is generally more likely than deliberation to lead judges astray.”

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\(^8\)Risk neutrality assumption is made only for the ease of exposition. Our main results would go through with minor modifications if the parties were instead risk-averse.

\(^9\)These costs are incurred regardless of the trial’s outcome.

\(^10\)Spier (1992) considers a finitely repeated version of this model and shows that if all costs are borne at trial, then the equilibrium outcome is equivalent to the single-offer model. In Section 6 we comment on how our results change under different bargaining environments.
We assume that the judge is impartial and suffers a disutility if the amount she awards to the plaintiff diverges from the true damages.\footnote{This is also similar to Usman (2002) in which the judge needs to exert a costly effort to verify whether the defendant in a contractual dispute has fulfilled his obligations.} This disutility is given by $-\alpha(\theta - a)^2$, where $a$ is the amount awarded to the plaintiff and $\alpha > 0$. Her payoff function therefore is given by

$$u_j(a, e, \theta) = -\alpha(\theta - a)^2 - c(e)$$

This specification is fairly common and there are several ways to motivate it (cf. Ryu and Kim (2000) and Daughety and Reinganum (1995)). For example, it could be that with some probability the court’s decision will be appealed and the true value of $\theta$ will be revealed at a higher court. If the judge’s reputation depends on the frequency with which her decisions are overturned, she would indeed try to minimize the distance between her award and the true value of $\theta$. (See Shavell (1995) for more on this.)

The optimal award for the judge is the expected value of $\theta$ given her information about $\theta$. If she observes $\theta$, then this is equal to $\theta$ itself, whereas if she remains uninformed, it is the conditional expectation calculated from her prior beliefs and the equilibrium strategies of the plaintiff and the defendant in the bargaining game. In any case, she never awards an amount smaller than $\underline{\theta}$, and in order to rule out the possibility that the plaintiff may drop the case if settlement fails, we assume that $c_p < \underline{\theta}$.\footnote{See Nalebuff (1987) for an analysis of a model in which this condition does not hold.} Finally, we assume that the judge cannot observe the defendant’s offer in the pretrial settlement bargaining game and all of the above is common knowledge.\footnote{In Section 6 we comment on what would happen if the judge could observe the offer.}

The equilibrium concept that we use is perfect Bayesian equilibrium, which we will sometimes refer to simply as the equilibrium. In a perfect Bayesian equilibrium, each player’s strategy must be optimal given the information she has and the equilibrium behavior of the other players, and each player’s beliefs must be updated using Bayes rule, whenever possible.

To facilitate a better understanding of our results, we decompose our analysis into two different sections. We will first analyze the pretrial bargaining assuming that there is a fixed probability with which the judge learns the true value of damages. Afterwards, we will endogenize this probability by making it the outcome of the judge’s costly effort choice.

## 3 Pretrial Bargaining under Exogenous Judicial Accuracy

In this section we assume that the judge learns the true value of $\theta$ with some exogenously given probability $q \in [0, 1]$, known to the litigants, whereas with probability $1 - q$, she has the prior beliefs given by $F$, which she updates by taking into account the fact that the case has come to trial. Therefore, with probability $q$ she awards $\theta$ and with probability $1 - q$ she awards some amount $w$ that is optimal given her beliefs. We assume that the judge updates her beliefs using Bayes’ rule whenever possible.

Therefore, if trial probability is positive in equilibrium, then $w$ is equal to the conditional expectation of $\theta$ using the prior and the fact that the plaintiff has rejected the defendant’s offer. If, on the other hand, trial does not occur in equilibrium, then Bayes’ rule does not apply and hence the judge’s beliefs on $\theta$ can be specified arbitrarily, i.e., $w$ can be specified as an arbitrary number in $[\underline{\theta}, \overline{\theta}]$.

Suppose that the uninformed defendant makes a take-it-or-leave-it settlement offer $s \geq 0$. If the
plaintiff has damages \( \theta \) and accepts this offer, his payoff is \( s \), whereas if he rejects it, his expected payoff is \(-c_p + q\theta + (1 - q)w\). Therefore, he accepts an offer \( s \) if \( s \geq -c_p + q\theta + (1 - q)w \) and rejects otherwise.\(^{14}\)

If \( q = 0 \), then the trial outcome is common knowledge and the parties have an interest in avoiding trial costs. Indeed, if \( q = 0 \), then there is a continuum of equilibria in which all cases are settled without a trial (see Proposition 1). More interesting cases are with \( q > 0 \). So let \( q > 0 \), and define

\[
t(s) = \begin{cases} 
\theta, & s < -c_p + q\theta + (1 - q)w \\
\frac{s + c_p - (1 - q)w}{q}, & -c_p + q\theta + (1 - q)w \leq s \leq -c_p + q\theta + (1 - q)w \\
\bar{\theta}, & s > -c_p + q\theta + (1 - q)w 
\end{cases}
\] (2)

Given the offer \( s \), \( t(s) \) is the marginal (or the threshold) plaintiff type such that all plaintiffs with damages \( \theta > t(s) \) reject the offer \( s \). Therefore, the case is settled with probability \( F(t(s)) \) and the defendant pays \( s \). With probability \( 1 - F(t(s)) \), the case goes to trial and the defendant pays his share of the trial cost as well as the expected award by the judge. The latter is equal to \( \theta \) with probability \( q \) and \( w \) with probability \( 1 - q \). Therefore, the defendant's expected payoff from making an offer \( s \) is given by

\[
V(s) = -F(t(s))s - [1 - F(t(s))\left(c_d + q\int_{t(s)}^{\theta} yf(y)dy + (1 - q)w\right)]
\] (3)

In equilibrium, the defendant chooses \( s \) to maximize (3). In general, he may choose an offer so that the probability of trial is equal to zero. This would, in turn, render the beliefs of the judge and hence the value of \( w \) indeterminate. Proposition 1 shows that this happens if and only if \( q \leq c_t f(\overline{\theta}) \), where \( c_t = c_d + c_p \) denotes the sum of the litigants’ trial costs. It is also easy to show that the equilibrium with positive trial probability is unique. The comparative static properties of the equilibrium are easily proved if \( F \) exhibits the monotone hazard rate property (MHRP), which we will from now on assume to be the case.

**Assumption 1** (Monotone Hazard Rate Property). \( x > x' \) implies

\[
\frac{F(x)}{f(x)} > \frac{F(x')}{f(x')}
\]

We can now prove

**Proposition 1.** (a) If \( q > c_t f(\overline{\theta}) \), then there is a unique equilibrium offer \( s^* \) such that \( \theta < t(s^*) < \overline{\theta} \),

\[
\frac{c_t}{q} = \frac{F(t(s^*))}{f(t(s^*))}
\] (4)

and

\[
w = \frac{\int_{t(s^*)}^{\overline{\theta}} yf(y)dy}{1 - F(t(s^*))}
\] (5)

\(^{14}\)We assume, without loss of generality, that he accepts when indifferent.
(b) If \( q \leq c_t f(\theta) \), then there is an equilibrium for each \( w \in [\underline{\theta}, \overline{\theta}] \) such that the equilibrium offer is \( s^* = -c_p + q\overline{\theta} + (1 - q)w \) and the trial probability is zero.

This result states that if the court is accurate enough so that \( q > c_t f(\theta) \), then in equilibrium the settlement offer is such that a positive, but less than one, fraction of the plaintiffs go to trial. This fraction is given by \( \theta(c_t/q) \in (\underline{\theta}, \overline{\theta}) \), defined as the solution to the following equation:

\[
\frac{c_t}{q} = \frac{F(\theta(c_t/q))}{f(\theta(c_t/q))}
\]  

(6)

In equilibrium, plaintiffs with \( \theta > \theta(c_t/q) \) go to trial whereas those with \( \theta \leq \theta(c_t/q) \) settle, which implies that the settlement rate is equal to \( F(\theta(c_t/q)) \in (0, 1) \). (See Figure 1).

We should note that (4) is the well-known condition in Bebchuk (1984), except that the left hand side here is multiplied by \( 1/q \). The intuition is also similar (See also Spier (2007)): The optimal offer is found by equating the marginal benefit to marginal cost. The benefit of a small decrease in \( s \) is equal to \( F(t(s)) \), which is the probability that the offer will be accepted. But a decrease in \( s \) also raises the likelihood of trial by an amount equal to \( dF(t(s))/d\theta \). Therefore, the marginal cost of reducing the offer slightly is \( c_p + c_d \) multiplied with \( f(t(s))/q \).

The marginal plaintiff type \( t(s) \) is indifferent between accepting the offer \( s \) and going to trial, which implies that at trial he expects to receive \( s + c_p \). By lowering \( s \) slightly, the defendant makes that type go to trial and ends up paying \( c_p \) more than what he would have paid if he were to offer \( s \). In addition, the defendant ends up incurring trial costs himself, which is equal to \( c_d \). Therefore, the marginal cost of reducing the offer slightly is \( c_p + c_d \) multiplied with \( f(t(s))/q \).

![Figure 1: Equilibrium under Exogenous Judicial Accuracy](image)

When the case goes to trial and the judge learns the true value of damages, the defendant pays \( \theta \). If the judge does not learn \( \theta \), then the defendant pays \( w(c_t/q) \), which is equal to the expected damages, given that all tried cases have damages greater than \( \theta(c_t/q) \), i.e.,

\[
w(c_t/q) = \frac{\bar{\theta} f(\theta(c_t/q)) y f(y) dy}{1 - F(\theta(c_t/q))}
\]  

(7)
The equilibrium offer is given by

\[ s^*(c_t/q) = -c_p + q\theta(c_t/q) + (1-q)w(c_t/q) \]  

(8)

If, on the other hand, \( q \leq c_t f(\theta) \), then there is a continuum of equilibria, one for each \( w \in [\theta, \bar{\theta}] \), in which the offer is \(-c_p + q\bar{\theta} + (1-q)w\) and the case is settled with probability one.

An interesting question is the effect of a decrease in the accuracy of the court and an increase in the cost of trial on the likelihood of pre-trial settlement. If \( q \leq c_t f(\theta) \), a decrease in the accuracy of the court, \( q \), leaves that likelihood unchanged, i.e., all cases are settled. A more interesting comparative statics result is obtained when \( q > c_t f(\theta) \).

**Proposition 2.** If \( q > c_t f(\theta) \), then a decrease in \( q \) (a increase in \( c_t \)) leads to an increase in settlement rate, the equilibrium offer, the expected award to the plaintiff at trial, (ex ante) expected payoff of the plaintiff, and a decrease in the expected payoff of the defendant.

Lower accuracy promotes settlement simply by reducing the “asymmetric information” between the defendant and the plaintiff. Reinganum and Wilde (1986) and Daughety and Reinganum (1995) obtain a similar result, albeit in different models. In these papers, the proposer is the informed party in the settlement stage, whereas in our model it is the uninformed party. In spite of this difference, it turns out that, in the separating equilibrium that they focus on, the trial probability is increasing in the accuracy of the court.\(^{15}\) There are two other papers, Hylton (2002) and Landeo, Nikitin, and Baker (2006), that consider the impact of court errors on the probability of settlement. They are “life-cycle litigation” models hence not comparable to ours. Nevertheless, we should note that they reach different conclusions. Hylton finds that court errors have an ambiguous effect on settlement rates whereas Landeo et al. find that errors increase settlement rates, which is in line with our result.\(^{16}\)

**Remark 1.** We have modeled the process that allows the court to award more accurate damages as “all or nothing”. An alternative, and more general, specification would allow the judge to receive a noisy signal about the actual damages and use this together with her prior beliefs and the fact that the case has come to court in choosing the award amount. In order to capture such cases let \( \nu(\theta) \) denote the optimal award by the judge if the true value of damages is \( \theta \). In the model we use in the paper, this is given by \( \nu(\theta) = q\theta + (1-q)w \). Then, for any offer \( s \) such that \(-c_p + \nu(\theta) \leq s \leq -c_p + \nu(\bar{\theta})\) we can define the threshold type \( t(s) \) implicitly as

\[ s = -c_p + \nu(t(s)) \]

and the defendant’s expected payoff would be

\[ V(s) = -F(t(s))s - [1 - F(t(s))] (c_d + E[\nu(\theta) | \theta \geq t(s)]). \]

The first order condition for maximizing this with respect to \( s \) is

\[ \frac{c_t}{\nu'(t(s^*))} = \frac{F(t(s^*))}{f(t(s^*))} \]

\(^{15}\)As we mentioned in Introduction, the important difference between these papers and ours is that we later endogenize the accuracy by making it the outcome of the judge’s effort choice.

\(^{16}\)There is a large literature on accuracy in adjudication (See Kaplow 1994). For a recent contribution to the debate on the efficacy of the negligence rule vs. strict liability under court errors, see Ackermann (2010).
which is the counterpart of the condition (4). This also makes it clear that what is important is not accuracy of the award, i.e., how close the award is to $\theta$ per se, but rather how responsive the award is to changes in actual damages. In other words, the fraction of cases that settle depends not on how much the parties think they will be ordered to pay (or receive) in court, but rather on how sensitive they think the award is to actual damages.

4 Equilibria with Judicial Agency

In this section we assume that if the judge expends effort $e$ she observes $\theta$ with probability $p(e)$, in which case she chooses to award $\theta$, and obtain a payoff of $-c(e)$. If she remains uninformed, which happens with probability $1 - p(e)$, she sets the award equal to the expected value of $\theta$, using her prior beliefs given by $F$ and the equilibrium strategies of the plaintiff and the defendant in the bargaining game. Therefore, her expected payoff to effort level $e$ is given by

$$U_j(e) = -\alpha [1 - p(e)] \text{var}(\theta) - c(e)$$

where $\text{var}(\theta)$ is the conditional variance of $\theta$, calculated using the judge’s posterior beliefs.

The previous section has analyzed the equilibrium of the bargaining game for exogenously given accuracy, $q$. Alternatively, we could think of $q$, as the common belief of the defendant and the plaintiff about judicial accuracy, i.e., their belief about the probability that the true damages will be established at trial. In this section, this probability is determined endogenously in equilibrium. If $e^*$ is the effort level that the judge expends in equilibrium, then $e^*$ must maximize $U_j(e)$ given $q$, and $q$ must be equal to the equilibrium level of accuracy, i.e., $q = p(e^*)$.

4.1 Equilibria with no Trial

Our first observation is that there is a continuum of equilibria in which the effort choice $e^*$ of the judge is such that $p(e^*) \leq f(\bar{\theta})c_t$, or equivalently $e^* \leq \underline{e}$, where

$$\underline{e} = p^{-1}(f(\bar{\theta})c_t).$$

(9)

In any such equilibrium the probability of trial is zero (see Proposition 1). Therefore, the information set of the judge is off-the-equilibrium path and her beliefs cannot be determined by Bayes’ rule. In other words, in these perfect Bayesian equilibria, we are free to choose any distribution of $\theta$ as the judge’s beliefs. In particular, we could choose the distribution so that the variance derived from that distribution satisfies the following condition

$$\alpha(p(e) - p(e^*)) \text{var}(\theta) \leq c(e) - c(e^*), \quad \text{for all } e \in [0, \bar{e}],$$

which makes it optimal for the judge to choose $e^* \leq \underline{e}$.

We should note that such equilibria exist only if

$$p(0) < f(\bar{\theta})c_t,$$

(10)

i.e., if the judge does not show any effort, probability of discovering the truth is small enough. If this
condition does not hold, \( e \) is either not well-defined (or equal to zero) and the presentation of our results becomes cumbersome. Therefore, from now on we assume that (10) holds.

### 4.2 Equilibria with Trial

More interesting are equilibria in which \( e^* > e \_\text{crit} \), so that a positive fraction of the cases go to trial (see Proposition 1). Letting the accuracy of the judge be \( q \), plaintiffs with \( \theta \leq \theta(c_t/q) \) settle (where \( \theta(c_t/q) \) is as defined in equation (6)) and plaintiffs with \( \theta > \theta(c_t/q) \) go to trial. For each value of \( q \), the conditional variance derived from the judge’s posterior beliefs is given by

\[
v(c_t/q) = \frac{\int_{\theta(c_t/q)}^{\overline{\theta}} (y - w(c_t/q))^2 f(y)dy}{1 - F(\theta(c_t/q))}
\]

where \( w(c_t/q) \) is the expected damages for the pool of cases that go to trial, as defined in equation (7).

The nature of the equilibria can be easily understood by studying the following function:

\[
\Phi(e) \equiv \alpha \frac{p'(e)v(c_t/p(e))}{p(e)} - c'(e),
\]

which is defined for all \( e > e \_\text{crit} \). By exerting effort, the judge increases the probability of being informed, which, in turn, increases her payoff. For an arbitrarily given conjecture \( q \) about the judicial accuracy, the marginal benefit of this is equal to \( \alpha p'(e)v(c_t/q) \). The marginal cost, in turn, is given by \( c'(e) \).

In equilibrium, we have \( q = p(e) \). Therefore, \( \Phi(e) \) measures the net marginal benefit of an arbitrary effort level \( e > e \_\text{crit} \), assuming that the litigants believe that the judge indeed exerts this level of effort.

If \( \Phi(e^*) < 0 \), then \( e^* \) cannot be an equilibrium, since the judge would be strictly better off by choosing a smaller effort level.\(^{17}\) If, on the other hand, \( \Phi(e^*) > 0 \), then \( e^* \) cannot be an equilibrium unless \( e^* = e \_\text{crit} \). It is easy to see that any \( e^* \) such that \( \Phi(e^*) = 0 \) is an equilibrium. More formally, we have:

**Proposition 3.** An effort level \( e^* > e \_\text{crit} \) is an equilibrium if and only if \( \Phi(e^*) = 0 \) or \( \Phi(e^*) > 0 \) and \( e^* = e \_\text{crit} \).

Note that as effort level \( e \) converges to \( e \_\text{crit} \), the threshold level of damages \( \theta(c_t/p(e)) \) converges to \( \overline{\theta} \), i.e., fewer and fewer cases go to trial. Therefore, the conditional variance \( v(c_t/p(e)) \) converges to zero as \( e \) converges \( e \_\text{crit} \). Together with our assumptions on \( p(e) \) and \( c(e) \), it easily follows that

\[
\lim_{e \to e \_\text{crit}} \Phi(e) < 0.
\]

We further make the following assumption

**Assumption 2.** There exist \( e \) such that \( \Phi(e) > 0 \).

Then, the following result easily follows from the continuity of \( \Phi \) and the intermediate value theorem.

**Corollary 1.** There exist at least two equilibria with effort level greater than \( e \_\text{crit} \).

\(^{17}\)A smaller effort level exists since \( e \_\text{crit} > 0 \) by assumption (10).
This is an interesting result and has important implications. Due to judicial agency, whenever there is an equilibrium in which some cases go to trial and the judge exerts some effort to learn the true value of damages, then, regardless of the legal costs, degree of asymmetric information, the distribution function, etc., there always exists a second equilibrium in which less effort is exerted by the judge. This is illustrated in Figures 2 and 3.

![Figure 2: Effort Choice](image1.png)

![Figure 3: Bargaining Under Judicial Agency](image2.png)

An interesting question is which one of these equilibria do the players prefer ex ante? The ranking for the plaintiff and the defendant directly follows from Proposition 2: The informed plaintiff prefers the low effort and the uninformed defendant prefers the high effort equilibrium. The following result states that the judge is better off in the low effort equilibrium.

**Proposition 4.** The judge and the plaintiff prefer the low effort equilibria whereas the defendant prefers the high effort equilibria.

In the Introduction we have provided an intuitive explanation of this result. The proof is simple and uses a revealed preference argument. Consider two equilibria, one with low and the other with high effort. First, low effort must be a better response than high effort when the variance of the tried cases is calculated using low effort, since otherwise low effort would not be an equilibrium choice. Second, convexity of the cost function and concavity of the probability function imply that at the high effort equilibrium, the variance of the tried cases must be larger, since otherwise higher effort would not be optimal. Since for a fixed effort level the payoff of the judge is decreasing in the variance, these two facts establish that she prefers the low effort equilibrium.

The following example provides a closed form solution and illustrates many of the points made so far. In fact, Figures 2 and 3 correspond to this example.

**Example: Uniform Distribution and Quadratic Costs**

Let \( \theta \) be uniformly distributed over \([0, 1]\), \( p(e) = e \) and \( \bar{e} = 1 \), and the cost of effort be \( c(e) = e^2/2 \). For a given common conjecture held by the litigants about the trial accuracy level \( q > c_t \), equilibrium
threshold $\theta$ above which plaintiff goes to trial is given by

$$\theta(c_t/q) = \frac{c_t}{q}.$$ 

Equilibrium award by an uninformed judge is

$$w(c_t/q) = \frac{1 + c_t/q}{2}$$

and the conditional variance

$$v(c_t/q) = \frac{1}{12} \left(1 - \frac{c_t}{q}\right)^2$$

Payoff function of the judge is

$$U_j(e) = -\alpha(1 - e) \frac{1}{12} \left(1 - \frac{c_t}{e}\right)^2 - \frac{e^2}{2}$$

which is maximized at

$$e^* = \alpha \frac{1}{12} \left(1 - \frac{c_t}{e}\right)^2$$

Equilibria are characterized by the behavior of the following function which is obtained from the preceding equation when we substitute $p(e) \equiv e$ for $q$, that is, when we impose the condition that in equilibrium the conjectures of the litigants must be correct.

$$\Phi(e) = \frac{\alpha}{12} \left(1 - \frac{c_t}{e}\right)^2 - e$$

Note that $e = c_t$ and that $\lim_{e \to c_t} \Phi(e) = -c_t < 0$. Therefore, by Proposition 3, an equilibrium $e^* > e$ exists only if $\Phi(e) = 0$ has a solution $c_t < e \leq 1$. This will be the case if and only if $\alpha/c_t \geq 81$. If this inequality holds strictly, then there exist two equilibria, one of which has a lower effort level and a higher settlement rate.

Figure 2 has been drawn for $\alpha = 12$ and $c_t = 0.1$, in which case the two equilibrium effort levels are given by $e_1^* \approx 0.17$ and $e_2^* \approx 0.75$ with corresponding settlement rates of 0.59 and 0.13, respectively.

### 4.2.1 Comparative Statics

Figures 2 and 3 also help us understand the effect of changes in various aspects of the environment that surrounds settlement negotiations and judicial behavior. Any change that increases the judge’s marginal benefit of effort or decreases the marginal cost leads to an increase in equilibrium effort, as long as the marginal benefit curve intersects the marginal cost curve from above at equilibrium, or $\Phi'(e^*) < 0$. If, on the other hand, $\Phi'(e^*) > 0$, then we obtain exactly the opposite result. Therefore, depending upon the sign of $\Phi'(e^*)$, we can unambiguously sign the effect of exogenous changes in $\alpha$, $p'$, and $c'$. Effects of an exogenous change in $c_t$ or the function $p$ depend on whether the conditional (truncated) variance function $v(c_t/p(e))$ is monotonic or not. In general, the monotonicity properties of truncated variances are well-known only for log-concave densities: If $\theta$ has a log-concave density,
then \( v(c_t/p(e)) \) is decreasing in \( c_t/p(e) \).\(^{18}\) We obtain the following result, whose proof is obvious and hence skipped.

**Proposition 5.** If the equilibrium effort choice \( e^* \) is such that \( \Phi'(e^*) < 0 \) (\( \Phi'(e^*) > 0 \)), then the following changes lead to an increase (decrease) in equilibrium effort level:

1. An increase in \( \alpha \)
2. An exogenous increase in \( p' \)
3. An exogenous decrease in \( c' \)

If \( f \) is log-concave, then we can add the following to the above list:

1. A decrease in \( c_t \)
2. An exogenous increase in \( p \)

**Remark 2.** As Samuelson (1947) observed, and called it the “correspondence principle”, there is an intimate relationship between the stability properties of an equilibrium and its comparative statics. Although our model is static, the following is a plausible dynamic model

\[
\dot{e} = \alpha p'(e) v(c_t/e) - c'(e),
\]

since one would expect the judge to increase his effort if marginal benefit of doing so is greater than the marginal cost, and conversely. A steady state of this differential equation is an equilibrium of our model and is asymptotically stable only if

\[
\Phi'(e) \leq 0,
\]

which is to say that the slope of the marginal benefit curve is smaller than the slope of the marginal cost curve. Therefore, one may conclude that comparative static results reported in Proposition 5 are reliable only for the case in which \( \Phi'(e^*) < 0 \).

## 5 Policy Implications and Welfare Properties

In this section we will discuss a few of the potentially important policy implications of our results. We will first revisit the debate on the managerial judges (who promote settlement over trial) and then discuss the effects of limiting judicial discretion over how much effort to exert. In the last subsection, we will discuss some of the welfare implications of our results.

### 5.1 Managerial Judges and the Social Desirability of Settlement

There is an ongoing debate about the social desirability of settlement over trial. One view is that settlement replaces a costly trial and therefore saves valuable resources and improves the aggregate welfare. The opponents of this view, Galanter (2004) among others, emphasize the negative implications

\(^{18}\)If, instead, the density is log-convex and has support \([a, \infty)\), then the truncated variance is increasing. See, for example, Heckman and Honoré (1990). Many commonly used distributions, such as the uniform, normal, exponential distributions, are log-concave.
of high settlement rates. One such implication is that out of court settlements further disadvantage the weak party in the dispute by forcing them to accept a settlement at unfavorable terms (see Resnik (1982) and Hadfield (2004)). Indeed, our results show that equilibria with higher settlement rates have both high settlement offers and trial awards, which hurts the uninformed defendant. In that sense, high settlement rates have some negative distributional consequences.

Furthermore, our results show that the judge is better off in equilibria with high settlement rates. This is all the more important in the light of the recent reports that more and more judges actively pursue strategies that promote settlement over a full trial. In her classic paper, Resnik (1982) pointed out that “many federal judges have departed from their relatively disinterested pose to adopt a more active, 'managerial' stance. In growing numbers, judges are [...] meeting with parties in chambers to encourage settlement of disputes and to supervise case preparation.” Our results suggest a bleaker picture: In the presence of multiple equilibria the judge may act as a coordination device and direct the litigants towards the equilibrium that she prefers the most, i.e., towards settlement. This will save resources by more quickly resolving disputes but at the cost of resolving them in a more inaccurate and perhaps unfair way.

5.2 Limiting the Effort Choices of the Judge

As we have just discussed, one negative implication of judicial agency is that if the judge exerts low effort, then more cases settle at unequal terms. In other words, in the equilibrium with low judicial effort, the settlement effectively robs the uninformed party of his right to a fair trial. Therefore, the policy makers might want to take measures to prevent judges from choosing low effort at the trial stage. Suppose that such measures make it impossible for the judge to choose an effort level less than \( e_m \).

In order to understand the possible effects of such a constraint, consider an example with linear cost of effort and uniform distribution of \( \theta \), which is illustrated in Figure 4. As can be seen from this figure, when there are no constraints on effort choice, there are two equilibria, one in which the judge exerts the maximum effort level, \( e \), and another one in which she chooses effort level \( e^* \). Suppose now that the judge has to choose an effort level of at least \( e_m \), where \( e < e_m < e^* \). Note that this minimum effort requirement is lower than both levels that would be equilibrium effort levels, and hence one may conjecture that it will have no effect. However, and in contrast to this intuition, this lower bound emerges as a new (and stable) equilibrium effort level. Therefore, in the presence of judicial agency, a naive attempt to enforce higher effort by limiting judicial discretion, creates exactly the opposite effect unless the standard imposed is sufficiently severe.

5.3 Welfare

In Proposition 4 we established that the judge and the plaintiff prefer the low effort equilibria whereas the defendant prefers the high effort equilibria. If we define social welfare as the sum of the payoffs

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20One may also wonder why such a lower limit is imposed in the first place, if the rationale is to eliminate the low effort equilibrium \( e^* \). A possible explanation is that the environment is more complicated than we assumed; For instance, the exact value of \( e^* \) could be uncertain.
21To see this note that at effort level \( e_m \), the marginal benefit is less than the marginal cost of effort. Therefore, the judge would want to lower her effort, but since she is already at the lowest permissible effort level, she cannot do so.
of the three parties involved, then this result and the fact that the total payoff of the plaintiff and the
defendant decreases in trial probability, imply that low effort equilibrium is better for social welfare.

The analysis is slightly more involved if the social welfare function puts some weight on how ac-
curately the dispute is resolved (either by settlement or trial). For example, consider the following
social welfare function:

$$-\gamma \left[ F(t(s)) E_{\theta \leq t(s)}((s-\theta)^2) + (1-F(t(s))(1-p(e)) E_{\theta > t(s)}((\theta - w)^2) \right] - (1-\gamma) (1-F(t(s))) [c_t + c(e)],$$

where, as before, $s$ is the settlement offer, $t(s)$ the marginal plaintiff type that accepts the offer, $F(t(s))$
the probability of settlement, and $\gamma \in [0, 1]$ represents the weight put on by the society on accuracy.
Settlement will be valued by such a society because it lowers the cost of trial $c_t + c(e)$ but will be costly
because it has lower accuracy. Therefore, if $\gamma$ is high enough, then the society will prefer the high
effort equilibria, whereas if $\gamma$ is low, low effort equilibria will be preferred.

There are also incentive effects that need to be taken into account in a welfare analysis. For exam-
ple, suppose that accidents can be prevented if the defendant takes sufficient care at an earlier stage.
Let $x$ denote the level of ex ante care, $\phi(x)$ the cost of care, and $r(x)$ the probability that an accident
will occur given the level of preventive care. Then we can write the social welfare function as\footnote{Here we follow the standard approach in formulating a social welfare function with three components: the cost of care, the expected harm, i.e., the unconditional expectation of $\theta$, and the expected trial costs, which includes the trial costs of the litigants as well as the effort cost of the judge. See Polinsky and Rubinfeld (1988) and Shavell (1999).}

$$-\phi(x) - r(x) [E(\theta) + (1-F(t(s)))(c_t + c(e))].$$

If the judge exerts too little effort at the trial, then most cases settle but the settlement amounts are
very large. The low trial probability improves the social welfare. However, high settlement awards
make the defendant overinvest in preventive care and this reduces social welfare. Therefore, if pre-
ventive care is too sensitive to the expected payments at the litigation stage, and trial costs are not
too large, then high effort equilibria are better. Otherwise low effort equilibria are better for social welfare.

6 Extensions

Nearly all the models in the literature, including ours, model the pretrial settlement stage as a “take-it-or-leave-it” (or ultimatum) bargaining game. As Friedman and Wittman (2006), and many others, have observed “[t]his creates vastly different outcomes, depending on which side makes the (first) offer.” Another issue that has been discussed in the literature is the observability of the settlement offers by the judge. In this section we discuss if and how our results change under alternative specifications of the model.

**Plaintiff-Offer Game**

Assume that the plaintiff is the uninformed party and makes a “take-it-or-leave-it” settlement demand from the informed defendant. In such a model our main results would carry over with slight modifications. In particular, the settlement rates would still be inversely related to the trial accuracy (and hence the amount of judicial effort at trial), and there would still be multiple equilibria. Also, the uninformed party, who is the plaintiff this time, would still be worse off in the low effort equilibria. The difference would be that the equilibrium settlement demands by the uninformed plaintiff would become smaller as the judge exerts less effort.

**Signaling Game**

Another possible variation is to switch to a signalling game in which the informed party makes the settlement offer. If we assume that the plaintiff is the informed party, then we obtain a model similar to the one in Reinganum and Wilde (1986) and Daughety and Reinganum (1995). In the separating equilibrium of such a model, the trial probability increases in judicial effort, just like in our model. Furthermore, the probability that the offer made by a plaintiff of type \( \theta \) is rejected increases in \( \theta \), which implies that if the case comes to court, then a Bayesian judge should interpret this as \( \theta \) being higher (compared with her prior beliefs). This is also in line with our results. The only difference is that in such a signaling model, if the case comes to court, entire belief distribution of the judge shifts so that the posterior dominates (in a first order stochastic sense) the prior, whereas in our model the posterior is just a truncated (from left) version of the prior distribution. This changes how the posterior variance of \( \theta \) is calculated and the judge’s equilibrium effort, but we conjecture that our qualitative results would not change.\(^{23}\)

**Observable Offers**

Especially with the increasing involvement of judges at earlier stages of litigation, it is highly likely that the judges will learn about settlement demands and offers. Furthermore, anecdotal evidence suggests that in some cases the judge herself suggests the settlement terms. We can show that our main results continue to hold but the equilibrium trial probability is higher when the settlement offers are observable. This is in line with the results of earlier papers that studied this issue, albeit in different settings, namely Daughety and Reinganum (1995), and Kim and Ryu (2000).

\(^{23}\)If the informed party is the defendant, who also makes the offer, then the trial probability would still increase in judicial effort but if the case comes to court the judge would interpret this as \( \theta \) being small.
7 Concluding Remarks

Surveying the pretrial settlement bargaining literature, Daughety and Reinganum (2008) note that “most models [...] either ignore the role of the attorneys, the experts to the litigants, and the court, which is usually taken to be a judge or a jury, or relegate them to the background.” In most models there is asymmetric information about damages or liability (or both) between the litigants, which will then be resolved at the trial stage, possibly with some errors. There are models where the trial outcome depends on the actions of the litigants but no model allows for the judge to take a private and costly action that becomes a key factor in determining the outcome. Our work focuses on this aspect of the litigation process.

We have used a very standard (and basic) bargaining protocol and added a minimal element of strategic behavior on the part of the judge: She needs to exert a costly and discretionary effort to correctly resolve the dispute but otherwise is neither corrupt nor has any bias for either party. This has led to significant new results concerning the workings of the pretrial bargaining system. First, we discovered that the model has multiple equilibria, with different welfare implications. Second, we argued that a judge who does not care about the fairness of pretrial settlement, may lead the others to coordinate on the equilibrium in which she exerts less effort at trial. This leads to high settlement rates at unequal terms to the disadvantage of the uninformed party.

We have left out many other aspects of the litigation process, such as the discovery stage, that we believe would be better understood by allowing judges to be strategic actors. We hope to study these issues in the future.

8 Proofs

Proof of Proposition 1. (a) We first show that in any equilibrium \( \theta < t(s^*) < \overline{\theta} \). Suppose, for contradiction, that \( t(s^*) = \overline{\theta} \). Then, \( s^* = -c_p + q\overline{\theta} + (1-q)w \), since otherwise the defendant can decrease the offer and increase his payoff. Since \( V \) is continuous and its left-side derivative at \(-c_p + q\overline{\theta} + (1-q)w\) is negative,

\[
V'(s^*) = \frac{c_I}{q} f(\overline{\theta}) - 1 < 0
\]

the defendant can increase his payoff by reducing the offer by some amount. This proves that \( t(s^*) < \overline{\theta} \). Now suppose that \( t(s^*) = \underline{\theta} \). Then, \(-c_p + q\underline{\theta} + (1-q)w\) is an equilibrium offer as well. Again, since \( V \) is continuous and its right-side derivative is positive at \(-c_p + q\underline{\theta} + (1-q)w\),

\[
V'(s^*) = \frac{c_I}{q} f(\underline{\theta}) > 0
\]

the defendant can increase his payoff by increasing his offer by some amount.

Therefore, \( 0 < t(s^*) < 1 \), which in turn implies that the solution to the maximization problem of the defendant, \( s^* \), is in the interior of an interval:

\[-c_p + q\overline{\theta} + (1-q)w < s^* < -c_p + q\underline{\theta} + (1-q)w.\]
Thus, the first order condition for maximizing (3) must hold:

\[ V'(s^*) = \frac{c_t}{q} f(t(s^*)) - F(t(s^*)) = 0, \]

which is equivalent to (4).

To prove existence, note that

\[ \frac{F(\theta)}{f(\theta)} = \frac{1}{f(\theta)} > 0 \quad \text{and} \quad \frac{F(\theta)}{f(\theta)} = 0 = \frac{F(\theta)}{f(\theta)}. \]

Continuity of the function \( F/f \) and the intermediate value theorem then implies that there exists a \( \theta^* \in (0, 1) \) such that

\[ \frac{c_t}{q} = \frac{F(\theta^*)}{f(\theta^*)}. \]

Uniqueness easily follows from the monotone hazard rate property.

(b) First, assume that \( q = 0 \) and suppose, for contradiction, \( s^* < -c_p + w \). Then, this offer is rejected with probability one and the defendant’s equilibrium payoff is \(-c_d - w\). But, the defendant can instead offer \(-c_p + w + c_2/2\), which would be accepted with probability one and lead to a strictly greater payoff. Similarly, \( s^* > -c_p + w \) cannot be an equilibrium offer either, since otherwise the defendant can decrease the offer by some small amount and improve his payoff. Therefore, in equilibrium \( s^* = -c_p + w \). Furthermore, this offer must be accepted with probability one. To see this suppose that it is rejected with probability \( \sigma > 0 \). This leads to a contradiction, since the defendant can offer \(-c_p + w + \sigma c_2/2\), which would be accepted with probability one and lead to a payoff that is strictly greater than the equilibrium payoff of \(-(1-\sigma)(-c_p + w) - \sigma(c_d + w)\).

Now, assume that \( 0 < q \leq c_t f(\theta) \). We can establish that \( t(s^*) > \theta \) as in part (a). If \( t(s^*) < \theta \), then the solution to the defendant’s maximization problem is in the interior and the first order condition must hold, i.e.,

\[ \frac{c_t}{q} = \frac{F(t(s^*))}{f(t(s^*))} = \frac{F(\theta)}{f(\theta)} = \frac{1}{f(\theta)}. \]

which contradicts that \( q \leq c_t f(\theta) \). Therefore, \( t(s^*) = \theta \) and \( s^* \geq -c_p + q\theta + (1-q)w \). But, \( s^* > -c_p + q\theta + (1-q)w \) leads to a contradiction since the defendant can decrease the offer a little and increase his payoff. This establishes that \( s^* = -c_p + q\theta + (1-q)w \) and the trial probability is zero.

In both cases, \( q > 0 \) and \( q = 0 \), there is an equilibrium for any \( w \in [\theta, \theta] \), in which the equilibrium offer is \(-c_p + q\theta + (1-q)w \), the trial probability is zero, and the judge believes that \( \theta = w \) with probability one if the case comes to trial.

**Proof of Proposition 2.** Take any \( q > c_t f(\theta) \) and consider \( q' < q \). Denote the equilibrium offers at \( q' \) and \( q \), by \( s' \) and \( s \), respectively, and note that, by Proposition 1, equilibrium settlement probability at \( q \) is \( F(t(s)) \in (0, 1) \). Suppose first that \( q' \leq c_t f(\theta) \). Proposition 1 implies that the equilibrium settlement probability at \( q' \) is one and we are done. Therefore assume that \( q' > c_t f(\theta) \) and suppose, for contradiction, that \( F(t(s')) \leq F(t(s)) \). This implies that \( t(s') \leq t(s) \), and hence, by Proposition 1

\[ \frac{F(t(s'))}{f(t(s'))} = \frac{c_t}{q'} > \frac{c_t}{q} = \frac{F(t(s))}{f(t(s))}, \]

contradicting the monotone hazard rate property. Therefore, we conclude that \( F(t(s')) > F(t(s)) \).
Since $\theta(c_t/q) = t(s(c_t/q))$, this also implies that $\theta(c_t/q)$ is decreasing in $q$.

Define

$$\omega(\theta) = \int_{\theta}^{\bar{\theta}} yf(y)dy \quad 1 - F(\theta)$$

(14)

and observe that this function is strictly increasing

$$\omega'(\theta) = \frac{f(\theta)}{1 - F(\theta)} [\omega(\theta) - \theta] > 0$$

because $\omega(\theta) > \theta$ for any $\theta < \bar{\theta}$. Since the expected award $w(c_t/q)$ is equal to $\omega(\theta(c_t/q))$ and $\theta(c_t/q)$ is increasing in $c_t/q$, $w$ is increasing in $c_t/q$, and hence decreasing in $q$. From this it also follows that

$$\frac{\partial s^*}{\partial q} = -(w(c_t/q) - \theta(c_t/q)) - \frac{c_t}{q^2} [q\theta'(c_t/q) + (1 - q)w'(c_t/q)] < 0.$$

We can write the expected payoff of the defendant in terms of an arbitrary settlement probability $t \in (0, 1)$ as follows:

$$U_d(t, q) = -c_d + c_t F(t) - q t F(t) - q \int_{\theta}^{\bar{\theta}} yf(y)dy - (1 - q)w(c_t/q)$$

(15)

where we take into account of the fact that in equilibrium $w$ is a function of $c_t/q$. The defendant maximizes this function with respect to $s$, and hence indirectly with respect to $t$. Since we showed that the solution to this maximization problem is in the interior, the derivative of this function with respect to $t$, when evaluated at the equilibrium, is identically equal to zero. Applying the envelope theorem, the derivative of the equilibrium expected payoff of the defendant with respect to $q$ is given by

$$(w(c_t/q) - \theta(c_t/q))F(\theta(c_t/q)) + \frac{c_t}{q^2}(1 - q)w'(c_t/q) > 0.$$

Expected payoff of the plaintiff is equal to

$$-U_d(t, q) - (1 - F(t))c_t$$

Since equilibrium $t$ is decreasing in $q$ and $F$ is increasing, derivative of this function with respect to $q$ is negative.

Proof of Proposition 3. “Only if” part has already been proved in the paragraph preceding Proposition 3. To prove the “If” part let $\Phi(e^*) = 0$ and consider the following strategies and beliefs. Let the judge’s effort choice be $e^*$, her beliefs be given by conditional distribution of $\theta$ given that it is greater than $\theta(c_t/p(e^*))$, and her award when uninformed be $w(c_t/p(e^*))$. Given her beliefs, this award is optimal and her payoff function is given by

$$U_j(e) = -\alpha(1 - p(e))v(c_t/p(e^*)) - c(e)$$
Since this function is strictly concave and
\[ U_j'(e^*) = \alpha p'(e^*) v(c_i/p(e^*)) - c'(e^*) = \Phi(e^*) = 0 \]
by hypothesis, \( e^* \) is the unique maximizer of \( U_j(e) \). The case where \( \Phi(e^*) > 0 \) and \( e^* = \bar{e} \) is proved similarly.

**Proof of Proposition 4.** Lower effort equilibria have lower accuracy and hence for the plaintiff and the defendant the result follows from Proposition 2. To prove the result for the judge let \( e_H > e_L \) be two equilibrium effort levels and \( v_H \) and \( v_L \) be the corresponding equilibrium variances, i.e.,
\[ v_H = v(c_i/p(e_H)), \quad v_L = v(c_i/p(e_L)). \]

We first claim that \( v_H \geq v_L \). To see this, suppose, for contradiction, that \( v_H < v_L \). Proposition 3 implies that
\[ \alpha p'(e_L) v_L - c'(e_L) = 0 \]
\[ \alpha p'(e_H) v_H - c'(e_H) \geq 0. \]
Therefore
\[ c'(e_L) = \alpha p'(e_L) v_L > \alpha p'(e_H) v_H \geq c'(e_H), \]
which contradicts \( e_H > e_L \).

Now define
\[ u_j(e, v) = -\alpha \left[ 1 - p(e) \right] v - c(e) \]
as the judge’s payoff for an arbitrary effort level \( e \) and variance \( v \). Then, the following is true
\[ u_j(e_L, v_L) > u_j(e_H, v_L) \geq u_j(e_H, v_H) \]
where the first inequality follows from the fact \( u_j \) is strictly concave and \( e_L \) is an equilibrium effort level. Therefore, \( e_L \) constitutes a unique best response by the judge to \( v_L \). The second inequality follows from the fact that \( u_j \) is decreasing in its second argument.

**References**


