

Härdle, Wolfgang Karl; Sirotko-Sibirskaya, Natalia; Wang, Weining

Working Paper

TENET: Tail-Event driven NETwork risk

SFB 649 Discussion Paper, No. 2014-066

Provided in Cooperation with:

Collaborative Research Center 649: Economic Risk, Humboldt University Berlin

Suggested Citation: Härdle, Wolfgang Karl; Sirotko-Sibirskaya, Natalia; Wang, Weining (2014) : TENET: Tail-Event driven NETwork risk, SFB 649 Discussion Paper, No. 2014-066, Humboldt University of Berlin, Collaborative Research Center 649 - Economic Risk, Berlin

This Version is available at:

<https://hdl.handle.net/10419/107925>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

TENET: Tail-Event driven NETwork risk

Wolfgang Karl Härdle*
Natalia Sirotko-Sibirskaya*
Weining Wang*

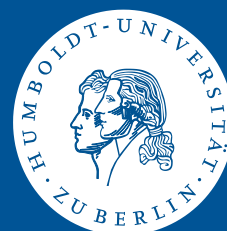


* Humboldt-Universität zu Berlin, Germany

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

<http://sfb649.wiwi.hu-berlin.de>
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin



TENET: Tail-Event driven NETWORK risk*

Wolfgang Karl Härdle[†] Natalia Sirotko-Sibirskaya[‡] Weining Wang[§]

November 1, 2014

Abstract

We propose a semiparametric measure to estimate systemic interconnectedness across financial institutions based on tail-driven spill-over effects in a ultra-high dimensional framework. Methodologically, we employ a variable selection technique in a time series setting in the context of a single-index model for a generalized quantile regression framework. We can thus include more financial institutions into the analysis, to measure their interdependencies in tails and, at the same time, to take into account non-linear relationships between them. A empirical application on a set of 200 publicly traded U. S. financial institutions provides useful rankings of systemic exposure and systemic contribution at various stages of financial crisis. Network analysis, its behaviour and dynamics, allows us to characterize a role of each sector in the financial crisis and yields a new perspective of the financial markets at the U. S. financial market 2007 - 2012.

Keywords: Systemic Risk, Systemic Risk Network, Generalized Quantile, Quantile Single-Index Regression, Value at Risk, CoVaR, Lasso

JEL: G01,G18,G32,G38, C21, C51, C63.

*Financial support from the Deutsche Forschungsgemeinschaft (DFG) via SFB 649 “Ökonomisches Risiko” and IRTG 1792 “High-Dimensional Non-Stationary Times Series” is gratefully acknowledged.

[†]Professor at Ladislaus von Bortkiewicz Chair of Statistics and Director of C.A.S.E. - Center for Applied Statistics and Econometrics, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany. Email: haerdle@wiwi.hu-berlin.de.

[‡]Research associate at Ladislaus von Bortkiewicz Chair of Statistics, C.A.S.E. - Center for Applied Statistics and Econometrics, IRTG 1792, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany. Email: natalia.sirotko-sibirskaya@wiwi.hu-berlin.de.

[§]Professor at Ladislaus von Bortkiewicz Chair of Statistics, C.A.S.E. - Center for Applied Statistics and Econometrics, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany. Email: wangwein@wiwi.hu-berlin.de.

1. Introduction

Systematic risk is often known as risk stemming from the aggregate fluctuations in the economy. The sources of risk are complex, as both exogenous and endogenous factors are involved. This usually calls for a study on a financial network which accounts for interaction between the agents in the financial market. Unlike idiosyncratic risk, the systemic risk is not diversifiable. Although the notion *systemic risk* is not novel in the academic literature (see, e.g, Minsky (1977)), it has been neglected both in the academia and in the financial risk industry until the outbreak of the financial crisis in 2007-2009. The magnitude of repercussions caused by the financial crisis in 2007-2009 and its complexity revealed a significant flaw in financial regulation which has been focused primarily on stability of a single financial institution and triggered several political initiatives across the world such as establishment of Financial Stability Board (FSB) after G-20 London summit in 2009, integration of systemic risk agenda into Basel III in 2010 prior to G-20 meeting in Seoul, enacting the Dodd Frank Wall Street Reform and Consumer Protection Act ('Dodd Frank Act') in U. S. in 2010 which is said to bring the most radical changes into the U. S. financial system since the Great Depression.

These initiatives created several challenges such as identifying *systemically important financial institutions* (SIFIs) whose failure can not only impair the functioning of the financial system but also have adverse effects on the real sector of the economy, studying the propagation mechanism of a shock in a system, or in a network formed by financial institutions, investigating the response of a system to a shock as a whole as well as revealing certain structural patterns in evolution and behavior of a network and establishing a theoretical framework for systemic risk as such.

Although systemic risk is a relatively straightforward concept aimed at measuring risk stemming from interaction between the agents, the variety of risk measures employed at estimating systemic risk and diversity of possible methods to model interaction effects leads to a fact that the literature on this topic is highly heterogenous. The relevant literature in this field can be broadly divided into two groups: economic modelling of systemic risk and financial intermediation including microeconomic (e.g., Beale et al. (2011) and macroeconomic approaches (e.g., Gertler and Kiyotaki (2010) with the emphasis on theoretical, structural framework, and quantitative modelling with the emphasis on empirical analysis. The quantitative literature can be further classified by statistical methodology into quantile regression based modelling such as linear bivariate model by Adrian and Brunnermeier (2011), Acharya et al. (2012), Brownlees and Engle (2012), high-dimensional linear model by Hautsch et al. (2014), partial quantile regression by Giglio et al. (2012) and partial linear model by Chao et al. (2014). Further approaches include principal-component-based analysis, e.g., by Bisias et al. (2012), Rodriguez-Moreno and

Peña (2013) and others; statistical modelling based on default probabilities by Lehar (2005), Huang et al. (2009), and others; graph theory and network topology, e.g., Boss et al. (2006), Chan-Lau et al. (2009).

Our paper belongs to the quantitative group of the aforementioned literature, namely, modelling the tail event driven network risk based on quantile regressions augmented with non-linearity and variable selection in ultra-high dimensional time series setting. As a starting point of our research we take co-Value-at-Risk, or CoVaR, model by Adrian and Brunnermeier (2011) (AB), where ‘co-’ stands for ‘conditional’, ‘contagion’, ‘co-movement’. To capture the tail interconnectedness between the financial institutions in the system AB evaluate bivariate linear quantile regressions for publicly traded financial companies in the U. S. The CoVaR concept builds upon the concept of VaR with the difference that CoVaR is not simply VaR of an institution itself but is augmented with weighted VaR of another financial institution.

Whereas AB focus on bivariate measurement of tail risk we aim at assessing the tail interconnectedness of a single financial institution with all other financial companies simultaneously. Thus, the primary challenge is selecting the set of relevant risk drivers for each financial institution. Statistically we address this issue by employing the a variable selection method in the context of single-index model for generalized quantile regressions, i.e. for quantiles and expectiles. We further extend it to a time series variable selection context in ultra-high dimensions. The semi-parametric framework due to the single-index model allows us to investigate possible non-linearities in tail interconnectedness. Based on identified relevant risk drivers we construct a financial network based on spill-over effects across financial institutions. Further we propose two indices, namely, systemic contribution index and systemic exposure index where we rank all the companies based on their degree of their contribution (or exposure) to systemic risk.

The assumption of non-linear relationship between returns of financial companies is motivated by previous work by Chao et al. (2014), who find that the dependency between any pair of financial assets is often non-linear, especially in periods of economic downturn. Moreover, non-linearity assumption is more flexible especially in a ultra-high dimensional setting where the system becomes too complex to support the belief of linear relationships.

The model is evaluated based on daily return data on 200 publicly traded U. S. financial institutions from January 1, 2006 till September 1, 2012. The financial institutions are grouped according to their SIC code by industry. The time period from January 1, 2006 to September 1, 2012 covers one recession (2007-2009) and several documented financial crises (2008, 2011). Dividing companies by sectors and including several market perturbations allows not only to select the key players for each time period, but also additionally to highlight the connections between financial industries, which can in turn

provide additional information on the nature of market dislocations.

The rest of the paper is organized as follows. In Section 2 our approach to systemic risk modelling is outlined. Section 3 presents the statistical methodology and the related theorems. Section 4 illustrates the empirical application. Section 5 concludes. Appendix A contains proofs and Appendix B contains estimation results.

2. Systemic Risk Modelling

Traditional measures assessing riskiness of a financial institution such as VaR, or expected shortfall (ES) are based either on company characteristics and/or integrate macroprudential variables which account for the general state of the economy. Thus, for example, VaR of a financial institution, a risk measure most commonly used in practice, is defined as

$$P(-X_{i,t} \geq VaR_{i,t}^\tau | D_{i,t}) = P(X_{i,t} \leq q_{i,t}^\tau | D_{i,t}) = \tau, \quad (1)$$

where $X_{i,t}$ is the return of a financial institution, $q_{i,t}^\tau$ is the conditional τ -quantile of $X_{i,t}$ at $\tau = (0, 1)$ and $D_{i,t}$ denotes the risk drivers relevant for company i , e.g., returns of a financial institution itself, and/or variables reflecting the general state of the economy.

However, recent perturbations at the financial market lead to wide recognition of the fact that in terms of risk what is optimal for a single company is not optimal for the economy as a whole. Therefore, in our model we extend this information set, $D_{i,t}$, and include asset returns of other financial institutions. This allows us to model interaction effects between financial institutions. By extending the information set with asset returns of other financial institutions we assume that the risks are transmitted through returns. This is a simplified assumption, since the risk transmission channels can be defined in a finer way, however, asset returns are known to reflect many aspects of the state of a financial institution simultaneously and, thus, we think it is justified to use them for the analysis of network effects.

The starting point of our research is the AB model proposed. Built upon the concept of VaR they proposed: CoVaR, estimated in two steps. In the first step linear quantile regressions are estimated under standard assumption of $F_{\varepsilon_{i,t}}^{-1}(\tau | M_{t-1}) = 0$ and $F_{\varepsilon_{j|i,t}}^{-1}(\tau | M_{t-1}, X_{i,t}) = 0$:

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, \quad (2)$$

$$X_{j,t} = \alpha_{j|i} + \gamma_{j|i} M_{t-1} + \beta_{j|i} X_{i,t} + \varepsilon_{j|i,t}, \quad (3)$$

where $X_{i,t}$ is the log return of institution i and M_{t-1} are lagged macroprudential variables describing the general state of the economy (See Section 4 for description of macroprudential variables). AB propose to determine VaR of an institution i by regressing log return of company i on macroprudential variables. The obtained $\beta_{j|i}$ in the equation (3) has standard linear regression interpretation, i.e. it determines the sensitivity of log return of an institution j to changes in log return of an institution i . In the second step the CoVaR is calculated by plugging in VaR of company i at level τ into the equation (3):

$$\widehat{\text{VaR}}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}, \quad (4)$$

$$\widehat{\text{CoVaR}}_{j|i,t}^{AB} = \hat{\alpha}_{j|i} + \hat{\gamma}_{j|i} M_{t-1} + \hat{\beta}_{j|i} \widehat{\text{VaR}}_{i,t}^{\tau}, \quad (5)$$

$$= \widehat{\text{VaR}}_{j|X_i=\widehat{\text{VaR}}_{i,t}^{\tau}}^{\tau} + \hat{\beta}_{j|i} \widehat{\text{VaR}}_{i,t}^{\tau}. \quad (6)$$

Thus, the risk of a financial institution j is estimated as the sum of its own value-at-risk conditional on the fact that the financial institution i is at its VaR level τ and a weighted VaR of an institution i where the magnitude of the weight is determined by the degree of interconnectedness between institutions i and j reflected in $\beta_{j|i}$, or $\beta_{j \leftarrow i}$. By setting j equal to the return on a system, e.g. value-weighted average return on a financial index, and i to the return on a financial company i , we obtain the *contribution* CoVaR which characterizes how a company i influences the rest of the financial system. By doing the reverse, i. e. by setting j equal to a financial institution and i to a financial system, one obtains *exposure* CoVaR, i. e. the extent to which a single institution is exposed to the overall risk of a system.

This approach allows to identify the key elements of systemic risk, namely, network effects, a single institution's contribution to systemic risk and a single institution's exposure to systemic risk, however, it has certain limitations. First of all, it is questionable how to define the return on a system: it has to be proxied by the return on publicly available financial institutions, which, in turn, can be problematic since as AB point out it can create mechanical correlation between a single financial institution and the value-weighted financial index. Although they state that no such correlation is detected, this approach has to be adopted with caution. Secondly, by performing only pairwise quantile regres-

sions one assumes that two companies are interacting in an isolated environment which is not a realistic assumption since all other interaction effects are suppressed.

This motivates us to extend this bivariate model to a (ultra)high dimensional setting by including more variables into the analysis and also allowing for non-linear relationship between the variables. The key element in identifying systemic interconnectedness between the financial institutions lies in precise measurement of the network effects. Based on estimated network effects we evaluate single institution's exposure and single institution's contribution to systemic risk.

To identify the spill-over effects we employ the methodology by Fan et al. (2014) extended to a time series setting. This requires estimation of a single-index model and performing variable selection to identify the key risk drivers for each financial institution simultaneously. More precisely, we estimate:

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, \quad (7)$$

$$X_{i,t} = g(\beta_{i|-i}^\top D_{-i,t}) + \varepsilon_{i,t}, \quad (8)$$

where $X_{i,t}$ is the log return, $D_{-i,t}$ contains the risk drivers relevant for institution i , $g(\cdot)$ is a link function allowing for the nonlinear relationships. Here $D_{-i,t}$ is equal to macroprudential variables as well as log returns of the financial institutions except for an institution i . We employ the same macroprudential variables as AB, and estimate the VaR by linear quantile regression (7) of log returns of an institution i on macroprudential variables. This is justified by the analysis of Chao et al. (2014), who found no nonlinear effects in regressing $X_{i,t}$ on M_t .

Estimation of equation (8) is performed in two steps: in the first step we perform variable selection to identify relevant risk drivers; in the second step, we estimate link function, $g(\cdot)$, which characterizes relationship between asset returns of an institution i and the rest of the financial system defined in our case as all financial institutions in a sample except for an institution i . Thus, the spill-over effects, for example, from institutions $-i$ to an institution i are determined by coefficients $\beta_{i|-i}$.

We then obtain VaR and CoVaR as follows:

$$\widehat{\text{VaR}}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}, \quad (9)$$

$$\widehat{\text{CoVaR}}_{i|-i,t}^{TENET} = \hat{g}(\hat{\beta}_{i|-i}^\top \hat{D}_{-i,t}^*) + \varepsilon_{i,t} \quad (10)$$

where $\hat{D}_{-i,t}^* = (\text{VaR}_{-i,t}^*, M_{t-1})$ and a star denotes that only the VAR of those financial

institutions are included which are chosen to be relevant by the variable selection procedure. As one sees in (10) CoVaR comprises not only the influences of financial institutions in a sample except for i , but also incorporates non-linearity reflected in the shape of a link function g . Therefore, we name it $CoVaR^{TENE}$ which stands for Tail-Event-driven NETWORK risk.

Non-zero $\hat{\beta}$ coefficients obtained as a result of estimation of a single-index model allow to measure spill-over effects across the financial institutions and characterize their evolution as a system represented by a *network*. The term *network* refers to a (directed) *graph*, formally written as $G = (V, E)$ where V is a set of vertices and E is a set of links, or edges. To classify financial institutions by their role in the systemic risk context we focus rather on matrix than on network representation of a system and summarize the estimation results in a form of a weighted and unweighted adjacency matrices. A weighted adjacency matrix contains $\hat{\beta}$ coefficients, see Table 1, whereas an unweighted adjacency matrix is constructed by replacing non-zero $\hat{\beta}$ coefficients with $d_i = \mathbf{1}\{\hat{\beta}_i \neq 0\}$.

| | $x_{s1,1}$ | \dots | x_{s1,n_1} | $x_{s2,1}$ | \dots | x_{s2,n_2} | \dots | $x_{s4,1}$ | \dots | x_{s4,n_4} | To |
|--------------|---|----------|-------------------------------|-----------------------------|---------|-------------------------------|----------|-----------------------------|----------|-------------------------------|---|
| $x_{s1,1}$ | 0 | \dots | $\hat{\beta}_{s1,n_1 s1,1}$ | $\hat{\beta}_{s2,1 s1,1}$ | \dots | $\hat{\beta}_{s2,n_2 s1,1}$ | \dots | $\hat{\beta}_{s2,1 s1,1}$ | \dots | $\hat{\beta}_{s4,1 s1,1}$ | $\sum_{j=1}^p \hat{\beta}_{j x_{s1,1}}$ |
| \vdots | \vdots | \vdots | \vdots | \vdots | \dots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots |
| x_{s1,n_1} | $\hat{\beta}_{s1,1 s1,n_1}$ | \dots | 0 | $\hat{\beta}_{s2,1 s1,n_1}$ | \dots | $\hat{\beta}_{s2,n_2 s1,n_1}$ | \dots | $\hat{\beta}_{s4,1 s1,n_1}$ | \dots | $\hat{\beta}_{s4,n_4 s1,n_1}$ | \vdots |
| $x_{s2,1}$ | $\hat{\beta}_{s1,1 s2,1}$ | \dots | $\hat{\beta}_{s1,n_1 s2,1}$ | 0 | \dots | $\hat{\beta}_{s2,n_2 s2,1}$ | \dots | $\hat{\beta}_{s4,1 s2,1}$ | \dots | $\hat{\beta}_{s4,n_4 s2,1}$ | \vdots |
| \vdots | \vdots | \vdots | \vdots | \vdots | \dots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots |
| x_{s2,n_2} | $\hat{\beta}_{s1,1 s2,n_2}$ | \dots | $\hat{\beta}_{s1,n_1 s2,n_2}$ | $\hat{\beta}_{s2,1 s2,n_2}$ | \dots | 0 | \dots | $\hat{\beta}_{s4,1 s2,n_2}$ | \dots | $\hat{\beta}_{s4,n_4 s2,n_2}$ | \vdots |
| \vdots | \vdots | \vdots | \vdots | \vdots | \dots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots |
| $x_{s4,1}$ | $\hat{\beta}_{s1,1 s4,1}$ | \dots | $\hat{\beta}_{s1,n_1 s4,1}$ | $\beta_{s1,n_1 s4,1}$ | \dots | $\hat{\beta}_{s2,n_2 s4,1}$ | \dots | 0 | \dots | $\hat{\beta}_{s4,n_4 s4,1}$ | \vdots |
| \vdots | \vdots | \vdots | \vdots | \vdots | \dots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots |
| x_{s4,n_4} | $\hat{\beta}_{s1,1 s4,n_4}$ | \dots | $\hat{\beta}_{s1,n_1 s4,n_4}$ | $\beta_{s2,1 s4,n_4}$ | \dots | $\hat{\beta}_{s2,n_2 s4,n_4}$ | \dots | $\hat{\beta}_{s4,1 s4,n_4}$ | \dots | 0 | \vdots |
| From | $\sum_{i=1}^p \hat{\beta}_{x_{s1,1} i}$ | \dots | \dots | \dots | \dots | \dots | \dots | \dots | \dots | \dots | $\sum_{i=1}^p \sum_{j=1}^p \hat{\beta}_{i j}$ |

Table 1: An adjacency matrix for financial institutions classified according sectors denoted as $s_{m,k}$ where $m = 1, \dots, 4$ are sectors and $k = 1, \dots, n_m$ are the corresponding samples in a sector.

The above $p \times p$ matrix in Table 1 where p is equal to the number of financial institutions represents total connectedness across variables at each time point $t = 1, \dots, n$. The adjacency matrix, or a total connectedness matrix, is sparse and off-diagonal since our model by construction does not allow for self-loop effects (namely one variable cannot be regressed on itself). The rows of this matrix correspond to outgoing edges for a variable in a respective row and the columns correspond to incoming edges for a variable in a

respective column. For example, row 1 contains $\widehat{\beta}$ coefficients of all the variables that are influenced by company 1 from sector 1, $x_{s_{1,1}}$. Thus, a cell $(x_{s_{1,1}}, x_{s_{1,n_1}})$ contains $\widehat{\beta}_{s_{1,n_1}|s_{1,1}}$ which is the magnitude of the influence of company 1 from sector 1, $x_{s_{1,1}}$, on company n from sector 1, $x_{s_{1,n_1}}$.

To classify institutions according to their systemic importance we construct two indices based on the weighted adjacency matrix: index of systemic contribution, $C_{i,t}$, and index of systemic exposure, $E_{i,t}$. This allows us to identify risk emitters and risk recipients. We define the index of systemic contribution, $C_{i,t}^{m,k}$, as a proportion of the sum of $\widehat{\beta}$ coefficients corresponding to outgoing links, or a sum of each row of a weighted adjacency matrix, to the total sum of $\widehat{\beta}$ coefficients at the particular $t = 1, \dots, n$:

$$C_{i,t}^{m,k} \stackrel{\text{def}}{=} \left(1 + \frac{\sum_{j \neq s_{m,k}} \widehat{\beta}_{j|s_{m,k}}}{\sum_{i=1}^p \sum_{j=1}^p \widehat{\beta}_{i|j}}\right) \widehat{\text{VaR}}_{i,t}^{m,k}, \quad (11)$$

where $m = 1, \dots, 4$ are sectors and $k = 1, \dots, n_m$ are the sample sizes of corresponding sectors. Index of systemic contribution weights VaR of a corresponding financial institution with proportion of its influence on the rest of the financial companies. In other words, VaR of a financial institution is proportional to its monopoly power at the market defined through its systemic connectedness.

Correspondingly, we define the index of systemic exposure, $E_{i,t}^{m,k}$, as a proportion of the sum of $\widehat{\beta}$ coefficients corresponding to incoming links, or sum of each column of a weighted adjacency matrix, to the total sum of $\widehat{\beta}$ coefficients at the particular $t = 1, \dots, T$:

$$E_{i,t}^{m,k} \stackrel{\text{def}}{=} \left(1 + \frac{\sum_{i \neq s_{m,k}} \widehat{\beta}_{s_{m,k}|i}}{\sum_{i=1}^p \sum_{j=1}^p \widehat{\beta}_{i|j}}\right) \widehat{\text{VaR}}_{i,t}^{m,k}, \quad (12)$$

The conceptual difference to AB is that whereas they assume that CoVaR of a financial institution increases if it is influenced by another institution, we assume that CoVaR increases if a financial institution is not only being influenced but also exerts significant influence on other financial institutions itself. This is motivated by the fact that not only heavily influenced institutions become riskier in the period of crisis, but also those with high degree of monopoly power in terms of systemic interconnectedness are potentially the first ones to fail. After constructing these indices we classify a financial institution either as a risk-emitter, or as a risk-recipient by computing $\max\{C_{i,t}^{m,k}, E_{i,t}^{m,k}\}$. As a example, the value-at-risk based on exposure and contribution indices for selected companies, Fairfield Greenwich Group (FGG, Broker-Dealers), FBL Financial Group (FFG, Insurance), which

are the largest risk-contributors and risk-receivers are presented in Figures 11 and 12. A more comprehensive empirical analysis can be found in Section 4.

3. Statistical Methodology

Let us denote $X_t \in \mathbf{R}^p$ as p as the variables $D_{-i,t}$ before, p can be very large, namely of exponential rate. We also suppress the subscripts of the coefficients $\beta_{i|j}$ s, as we focus on one regression. The SIM of (8) is defined to be:

$$Y_t = g(X_t^\top \beta^*) + \varepsilon_t, \quad (13)$$

where $\{X_t, \varepsilon_t\}$ are strong mixing processes. Regressors X_t s can be the lagged variables of Y_t .

Note that (13) can be formulated in a location model and identified in a quasi maximum likelihood framework: the direction β (for known $g(\cdot)$) is the solution of

$$\min_{\beta} \mathbb{E} \rho_{\tau}\{Y_t - g(X_t^\top \beta)\}, \quad (14)$$

with loss function

$$\rho_{\tau}(u) = \rho(u) = \tau u \mathbf{1}(u > 0) + (1 - \tau)u \mathbf{1}(u < 0), \quad (15)$$

$$\mathbb{E}(\psi_{\tau}\{Y_t - g(X_t^\top \beta)\} | X_t) = 0 \quad a.s.$$

(where $\psi_{\tau}(\cdot)$ is the derivative (a subgradient) of $\rho_{\tau}(\cdot)$). It can be reformulated as $F_{\varepsilon|X_t}^{-1}(\tau) = 0$.

The model is similar to the location scale model considered in Franke et al. (2014). Note that it is not hard to extend it to a quantile AR-ARCH type of single index model,

$$Y = g(X_t^\top \beta) + \sigma(X_t^\top \gamma) \varepsilon_t \quad (16)$$

To estimate the shape of a link function $g(\cdot)$ and selected β coefficients we adopt minimum average contrast estimation approach (MACE) outlined in Fan et al. (2014). The estimation of β and $g(\cdot)$ is as following:

$$\begin{aligned}
\hat{\beta}, \hat{g}(\cdot) &\stackrel{\text{def}}{=} \arg \min -L_n(\beta) \\
&= \arg \min n^{-1} \sum_{j=1}^m \sum_{t=1}^n \rho_{\tau} \{Y_t - g(\beta^{\top} X_t) - g'(\beta^{\top} X_t) \beta^{\top} (X_t - X_j)\} \\
&\quad K_h \{\beta^{\top} (X_t - X_j)\} / \sum_{t=1}^n K_h(X_{tj}^{\top} \beta),
\end{aligned} \tag{17}$$

where $K_h(\cdot) = h^{-1}K(\cdot/h)$, $K(\cdot)$ is a kernel e.g. Gaussian kernel and h is a bandwidth. Since the data is not equally spaced we choose a bandwidth h based on k-nearest neighbor procedure (See Härdle et al. (2004)). The optimal k , number of neighbors, are selected based on a cross-validation criterion. The implementation involves an iteration between estimating β and $g(\cdot)$, with a consistent initial estimate for β , see for example Wu et al. (2010). Further in our procedure, we argument the estimation problem in equation (17) with variable selection to estimate β :

$$\hat{\beta}_{\tau}, \hat{g}(\cdot) = \arg \min_{\beta, g(\cdot)} n^{-1} \sum_{j=1}^n \sum_{t=1}^n \rho_{\tau} \{X_t - g(\beta^{\top} X_j) - g'(\beta^{\top} X_j) X_{tj}^{\top} \beta\} \omega_{tj}(\beta) + \sum_{l=1}^p \gamma_{\lambda}(|\beta_l|^{\theta}), \tag{18}$$

where $X_{tj} = X_t - X_j$, $\omega_{tj}(\beta) \stackrel{\text{def}}{=} \frac{K_h(X_{tj}^{\top} \beta)}{\sum_{t=1}^n K_h(X_{tj}^{\top} \beta)}$, $\theta \geq 0$, and $\gamma_{\lambda}(t)$ is some non-decreasing function concave for $t \in [0, +\infty)$ with a continuous derivative on $(0, +\infty)$. We now discuss the choices of an optimal penalty function and an optimal penalization parameter. There are several approaches in the literature concerning choosing penalty function. These approaches can be classified based on the properties desirable for an optimal penalty function, namely, unbiasedness, sparsity and continuity (Fan and Li (2001)). The classical approach known as least absolute shrinkage and selection operator (LASSO) is proposed for mean regression by Tibshirani (1996), which is based on L_1 penalty for the coefficients. Numerous studies further adapt LASSO variable selection procedure to a quantile regression framework such as Yu et al. (2003), Li and Zhu (2008), Belloni and Chernozhukov (2011), etc. While achieving sparsity, L_1 -norm penalty tends to over-penalize the large coefficients as the LASSO penalty increases linearly in the magnitude of its argument, and, thus, may introduce large bias to estimation. As a remedy to this problem adaptive LASSO estimation procedure has been proposed (Zou (2006); Zheng et al. (2013)). Another approach to alleviate bias stemming from LASSO procedure is proposed by Fan and Li (2001) known as Smoothly Clipped Absolute Deviation (SCAD):

$$\gamma_\lambda(t) = \begin{cases} \lambda|t| & \text{for } |t| \leq \lambda, \\ -(t^2 - 2a\lambda|t| + \lambda^2)/2(a-1) & \text{for } \lambda < |t| \leq a\lambda, \\ (a+1)\lambda^2/2 & \text{for } |t| > a\lambda, \end{cases}$$

where $\lambda > 0$ and $a > 2$. Fan and Li (2001) recommend to use $a = 3.7$.

As for selecting λ , there are two common ways: data-driven generalized cross-validation criterion (GCV) (Fan and Li (2001)) and likelihood-based Schwartz, or Bayesian information criterion-type criteria (SIC, or BIC) (Schwarz (1978); Koenker et al. (1994)), and their further modifications. The most commonly used criterion is GCV (Fan and Li (2001), Tibshirani (1996)), however, it has been shown that it leads to an overfitted model (Wang et al. (2007)). Therefore, we primarily employ a modified BIC-type model selection criteria proposed by Wang et al. (2007) and use GCV criterion only to verify whether GCV and BIC diverge significantly.

Define $\hat{\beta}_\tau \stackrel{\text{def}}{=} (\hat{\beta}_{\tau(1)}^\top, \hat{\beta}_{\tau(2)}^\top)^\top$ as the estimator for $\beta^* \stackrel{\text{def}}{=} (\beta_{(1)}^{*\top}, \beta_{(2)}^{*\top})^\top$ attained by the loss in (18). Let $\hat{\beta}_{\tau(1)}$ and $\hat{\beta}_{\tau(2)}$ be the first q components and the remaining $p - q$ components of $\hat{\beta}_\tau$ respectively. If in the iterations, we have the initial estimator $\hat{\beta}_{(1)}^{(0)}$ as a $\sqrt{n/q}$ consistency one for $\beta_{(1)}^*$, we will obtain with a very high probability, an oracle estimator of the following type, say $\tilde{\beta}_\tau = (\tilde{\beta}_{\tau(1)}^\top, \mathbf{0}^\top)^\top$, since the oracle knows the true model $\mathcal{M}_* \stackrel{\text{def}}{=} \{l : \beta_l^* \neq 0\}$. The following theorem shows that the penalized estimator enjoys the oracle property. Define $\hat{\beta}^0$ as the minimizer with the same loss in (18) but within subspace $\{\beta \in \mathbb{R}^p : \beta_{\mathcal{M}_*^c} = \mathbf{0}\}$. We have the following conditions needed for theorems.

Condition 1. The kernel $K(\cdot)$ is a continuous symmetric function. The link function $g(\cdot) \in C^2$.

Condition 2. The loss function $\rho_\tau(x)$ is convex and $\psi_\tau(x)$, the derivative (or a subgradient) of $\rho_\tau(x)$, satisfies $\mathbb{E} \psi_\tau(\varepsilon_t) = 0$ and $\inf_{|v| \leq c} \partial \mathbb{E} \psi_\tau(\varepsilon_t - v) = C_1$ where $\partial \mathbb{E} \psi_\tau(\varepsilon_t - v)$ is the partial derivative with respect to v , and C_1 is a constant.

Condition 3. The density of $\beta^{*\top} X$ is bounded with bounded absolute continuous first-order derivatives on its support. Assume $\mathbb{E}\{\psi_\tau(\varepsilon|X)\} = 0$ a.s., which means for a quantile loss we have $F_{\varepsilon|X}^{-1}(\tau) = 0$. Let $X_{t(1)}$ denote the sub-vector of X_t consisting of its first q elements. Let $Z_t \stackrel{\text{def}}{=} X_t^\top \beta^*$ and $Z_{tj} \stackrel{\text{def}}{=} Z_t - Z_j$. Define $C_{0(1)} \stackrel{\text{def}}{=} \mathbb{E} \mathbb{E}\{\psi_\tau^2(\varepsilon_t)|Z_t\} \{[g'(Z_t)]^2 (\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)}) (\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)})\}^\top$, and $C_{0(1)} \stackrel{\text{def}}{=} \mathbb{E}\{\partial \mathbb{E} \psi_\tau(\varepsilon_t)|Z_t\} \{[g'(Z_t)]^2 (\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)}) (\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)})\}^\top$ and the matrix $C_{1(1)}$ satisfies $0 < L_1 \leq \lambda_{\min}(C_{0(1)}) \leq \lambda_{\max}(C_{0(1)}) \leq L_2 < \infty$ for positive constants L_1 and L_2 . There exists a constant $c_0 > 0$ such that $\sum_{t=1}^n \{\|X_{t(1)}\|/\sqrt{n}\}^{2+c_0} \rightarrow 0$, with $0 < c_0 < 1$. $v_{tj} \stackrel{\text{def}}{=} Y_t - a_j - b_j X_{tj}^\top \beta$. Also, exists a constant C_3 such that for all β close

to β^* ($\|\beta - \beta^*\| \leq C_3$)

$$\left\| \sum_t \sum_j X_{(0)tj} \omega_{tj} X_{(1)tj}^\top \partial \mathbf{E} \psi_\tau(v_{tj}) \right\|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1}).$$

Condition 4. The penalty parameter λ is chosen such that $\lambda = \mathcal{O}(n^{-1/2})$, with $D_n \stackrel{\text{def}}{=} \max\{d_l : l \in \mathcal{M}_*\} = \mathcal{O}(n^{\alpha_1 - \alpha_2/2} \lambda) = \mathcal{O}(n^{-1/2})$, $d_l \stackrel{\text{def}}{=} \gamma_\lambda(|\beta_l^*|)$, $\mathcal{M}_* = \{l : \beta_l^* \neq 0\}$ be the true model. Furthermore assume $qh \rightarrow 0$ and $h^{-1} \sqrt{q/n} = \mathcal{O}(1)$ as n goes to infinity, $q = \mathcal{O}(n^{\alpha_2})$, $p = \mathcal{O}\{\exp(n^\delta)\}$, $nh^3 \rightarrow \infty$ and $h \rightarrow 0$. Also, $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$.

Condition 5. The error term ε_t satisfies $\text{Var}(\varepsilon_t) < \infty$. Assume that

$$\begin{aligned} \sup_t \mathbf{E} |\psi_\tau^m(\varepsilon_t)/m!| &\leq s_0 M^m \\ \sup_t \mathbf{E} |\psi_\tau^m(x_{tj})/m!| &\leq s_0 M^m \end{aligned}$$

where s_0 and M are constants, and $\psi_\tau(\cdot)$ is the derivative (a subgradient) of $\rho_\tau(\cdot)$.

Condition 6. The conditional density function $f(\varepsilon|Z_t = u)$ is bounded and absolutely continuous differentiable.

Condition 7. The link function $g(\cdot)$ satisfies a Lipschitz condition in the support of $\beta^\top X$:

$$|g(z) - g(\tilde{z})| \leq C|z - \tilde{z}| \quad (19)$$

Conditions 8. $\{X_{tj}, \varepsilon_t\}_{t=-\infty}^{t=\infty}$ are strong mixing process for any j . Moreover, there exists positive constants c_{m1} and c_{m2} such that the α -mixing coefficient for every $j \in \{1, \dots, p\}$,

$$\alpha(l) \leq \exp(-c_{m1} l^{c_{m2}}), \quad (20)$$

where $c_{m2} > 2\alpha$.

With all the above definitions and conditions, we can derive the following theorems.

THEOREM 3.1. *Under Conditions 1-8, the estimators $\hat{\beta}^0$ and $\hat{\beta}_\tau$ exist and coincide on a set with probability tending to 1. Moreover,*

$$\mathbf{P}(\hat{\beta}^0 = \hat{\beta}_\tau) \geq 1 - (p - q) \exp(-C' n^\alpha) \quad (21)$$

for a positive constant C' .

THEOREM 3.2. *Under Conditions 1-8, we have*

$$\|\hat{\beta}_{\tau(1)} - \beta_{(1)}^*\| = \mathcal{O}_p\{(D_n + n^{-1/2})\sqrt{q}\} \quad (22)$$

For any unit vector \mathbf{b} in \mathbb{R}^q , we have

$$\mathbf{b}^\top C_{0(1)}^{1/2} C_{1(1)}^{-1/2} C_{0(1)}^{1/2} \sqrt{n}(\hat{\beta}_{\tau(1)} - \beta_{(1)}^*) \xrightarrow{\mathcal{L}} \mathbb{N}(0, 1) \quad (23)$$

where recall that $C_{1(1)} \stackrel{\text{def}}{=} \mathbb{E}\{\mathbb{E}\{\psi_\tau^2(\varepsilon_t)|Z_t\}[g'(Z_t)]^2[\mathbb{E}(X_{(1)}|Z_t) - X_{t(1)}][\mathbb{E}(X_{(1)}|Z_t) - X_{t(1)}]^\top\}$, and $C_{0(1)} \stackrel{\text{def}}{=} \mathbb{E}\{\partial \mathbb{E} \psi_\tau(\varepsilon_t)|Z_t\}\{[g'(Z_t)]^2(\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)})(\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)})^\top\}$. Note that $\mathbb{E}(X_{(1)}|Z_t)$ denotes a $p \times 1$ dimension vector with j th element $\mathbb{E}(X_{j(1)}|Z_t)$, $j = 1, \dots, q$, and $Z_t \stackrel{\text{def}}{=} X_t^\top \beta^*$, $\psi_\tau(\varepsilon)$ is a choice of the subgradient of $\rho_\tau(\varepsilon)$ and $\sigma_\tau^2 \stackrel{\text{def}}{=} \mathbb{E}[\psi_\tau(\varepsilon_t)]^2 / [\partial \mathbb{E} \psi_\tau(\varepsilon_t)]^2$, where

$$\partial \mathbb{E} \psi_\tau(\cdot)|Z_t = \frac{\partial \mathbb{E} \psi_\tau(\varepsilon_t - v)^2|Z_t}{\partial v^2} \Big|_{v=0}. \quad (24)$$

Let us now look at the distribution of $\hat{g}(\cdot)$ and $\hat{g}'(\cdot)$, the estimator of $g(\cdot)$, $g'(\cdot)$.

THEOREM 3.3. *Under Conditions 1-8, let $\mu_j \stackrel{\text{def}}{=} \int w^j K(u)du$ and $\nu_j \stackrel{\text{def}}{=} \int w^j K^2(u)du$, $j = 0, 1, 2$. For any interior point $z = x^\top \beta^*$, $f_Z(z)$ is the density of Z_t , $t = 1, \dots, n$, if $nh^3 \rightarrow \infty$ and $h \rightarrow 0$, we have*

$$\sqrt{nh} \sqrt{f_Z(z)/(\nu_0 \sigma_\tau^2)} \left\{ \hat{g}(x^\top \hat{\beta}) - g(x^\top \beta^*) - \frac{1}{2} h^2 g''(x^\top \beta^*) \mu_2 \partial \mathbb{E} \psi_\tau(\varepsilon) \right\} \xrightarrow{\mathcal{L}} \mathbb{N}(0, 1),$$

Also, we have

$$\sqrt{nh^3} \sqrt{\{f_Z(z) \mu_2^2\}/(\nu_2 \sigma_\tau^2)} \left\{ \hat{g}'(x^\top \hat{\beta}) - g'(x^\top \beta^*) \right\} \xrightarrow{\mathcal{L}} \mathbb{N}(0, 1),$$

The dependence doesn't have any impact on the rate of the convergence of our non-parametric link function. As the degree of the dependence is measured by the mixing coefficient α , is weak enough such that Condition 8 is satisfied. This is also in line with the results in Kong et al. (2010). In fact we assume exponential decaying rate here, which implies the (A.4) in Kong et al. (2010).

4. Empirical Analysis

4.1. Data

Our analysis focuses on the panel of 200 publicly traded U. S. financial institutions between January 1, 2006 and September 1, 2012 corresponding to SIC code from 6000 to 6799. SIC codes are used to divide companies into the following sectors: (1) depositories, (2) insurance companies, (3) broker-dealers, (4) others, see Table 2 in Appendix B for a complete list.

Due to its high-dimensionality this dataset approximates the aggregate fluctuations in the system fairly well. Thus we do not include companies operating in the real sector and do not investigate linkages between financial and real sectors of the economy. We also do not include any proxy for the shadow banking sector which may play a role in the 2007-2009 crisis. The time period from January 1, 2006 till September 1, 2012 covers one recession (2007-2009) and several financial crises (2008, 2011).

Our analysis is based on the daily returns of the above mentioned financial institutions (Table 2). Market returns are a rich and robust source of information reflecting the overall state of the company. Apart from the data on the financial companies we use daily observations of macroprudential variables which characterize the general state of the economy. These variables are defined as follows: (i) the implied volatility index, VIX, reported by the Chicago Board Options Exchange; (ii) short term liquidity spread denoted as the difference between the three-month repo rate and the three-month bill rate to measure short-term liquidity risk; (iii) the changes in the three-month Treasury bill rate from the Federal Reserve Board; (iv) the changes in the slope of the yield curve corresponding to the yield spread between the ten-year Treasury rate and the three-month bill rate from the Federal Reserve Board; (v) the changes in the credit spread between BAA-rated bonds and the Treasury rate; (vi) the weekly equity market returns from CRSP, and (vii) the returns on a real estate sector in excess of the market returns.

4.2. Estimation Results

Empirical analysis is performed at three levels: first of all, we characterize the behavior of a system as a whole, in the second step, we investigate fluctuations across the financial sectors, and at last, we analyze the systemic importance of particular companies. Figure 1 presents the risk network estimated for the window starting at 20070111, and Figure 2 considers the sparsified network with further thresholding.

To describe aggregate fluctuations in a system represented by financial institutions in a

sample we define total connectedness in terms of $\hat{\beta}$ coefficients, $TC^s \stackrel{\text{def}}{=} \sum_{i=1}^p \sum_{j=1}^p |\hat{\beta}_{i|j}|$, and number of links, $TC^n \stackrel{\text{def}}{=} \sum_{i=1}^p \sum_{j=1}^p d_{i|j} \stackrel{\text{def}}{=} \sum_{i=1}^p \sum_{j=1}^p \mathbf{1}\{\hat{\beta}_{i|j} \neq 0\}$. We distinguish between the aggregate magnitude of $\hat{\beta}_{i|j}$ coefficients and total number of links at each time point to account for the periods when there are large number of small links, and/or few significant links. Figure 3 shows that the beginning of 2007-2009 financial crisis is characterized by smaller magnitude of $\hat{\beta}_{i|j}$ coefficients and fewer links. As the crisis was unfolding, the system became more heavily interconnected and reached its peak in the second quarter of 2008 and lasted until beginning of 2009. Total connectedness across financial institutions started to decrease in the second quarter of 2009 and reached its minimum by the first half of 2010. Several negative shocks at the U. S. financial market contributed to a slow increase both in TC^s and TC^n from the second quarter of 2010 to through 2011. Examples are Flash Crash in May 2010 attributed to the U. S. reaction to the debt crisis in Greece and US debt-ceiling crisis in July 2011. The fluctuations in linkages in 2010-2012 were much smaller than during the financial crisis due to overall positive performance of the markets.

The average values for a penalization parameter $\hat{\lambda}$ for 200 financial companies are presented in Figure 3 by a dotted line. One sees that the estimation results of total connectivity measures, TC^s and TC^n have the same time trend as $\hat{\lambda}$. The penalization parameter $\hat{\lambda}$ determines the number of selected risk drivers in the whole procedure, and the fluctuations in $\hat{\lambda}$ can also document the noise level and interconnections of the whole system.

Figure 3 plots also $\hat{\lambda}$ empirically. While staying at a low level in the beginning of 2007, $\hat{\lambda}$ increases together with TC^s and TC^n . Both TC^s and average $\hat{\lambda}$ values reach its maximum approximately in the middle of 2009, which is during the period of subprime crisis. As the economy recovers the average $\hat{\lambda}$, TC^s and TC^n values all start to fall, which suggest a lessened comovement across the financial institutions. Market disturbances in 2011 corresponding to European debt crisis are reflected by another smaller bump TC^s and TC^n .

We now turn to analyzing the relative contribution and exposure of each sector as well as interconnectedness across sectors. To obtain a measure of linkages per company in each sector, we compute the following density measures for each sector in terms of exposure and contribution: $D_{exp}^m \stackrel{\text{def}}{=} \frac{1}{n_m} \sum_{i=1}^p \sum_{j=1}^{n_m} |\hat{\beta}_{s_m,j|-i}|$ and $D_{contr}^m \stackrel{\text{def}}{=} \frac{1}{n_m} \sum_{j=1}^p \sum_{i=1}^{n_m} |\hat{\beta}_{-j|s_m,i}|$ where m is fixed and n_m is the sample size of a corresponding sector. By dividing n_m we summarize variations of the relative intensity per link within each sector. The relative exposure D_{exp}^m does not exhibit great fluctuations across time as shown in Figure 4, except that one observes a slightly decreasing intensity for the depositories and a large increase in exposure intensity of the sector ‘‘Others’’ from 2010 to 2011. This significant increase

is possibly due to the heterogeneity in sector “Others” which contains companies such as, American Express, (a credit card company), Equifax, (a credit-rating agency), Sterling Financial Corporation, (a consumer and commercial bank acquired by PNC Financial Services Co. in 2007). In contrast to D_{exp}^m , the contribution density D_{contr}^m (displayed in Figure 5) is more volatile, see Figure 5. The contribution densities of depositories and insurance companies are relatively stable with the former dominating the latter in the given time period 2007-2012. However, one can observe an interesting interplay in contribution densities between depositories and insurance companies in 2007-2008 when insurance companies overtook leadership over the depositories for a short time during the crisis. Contribution density of broker-dealers exhibits seasonal-type fluctuations. Broker-dealers played a significant role in the financial crisis as suggested by their relatively higher contribution density. This is justifiable as they may be more susceptible to runs and bankruptcy due to their tendency to use short term credit markets to finance their operations. This was also the reason why during the crisis this particular sector needed extensive support from the government in order to prevent complete market break-down.

We further describe connections between different financial sectors by defining interconnectedness measures as follows: $IC_{exp} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n_{m1}} \sum_{j=1}^{n_{m2}} \hat{\beta}_{i|j}}{\sum_{i=1}^p \sum_{j=1}^p \hat{\beta}_{i|j}}$ and $IC_{contr} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n_{m1}} \sum_{j=1}^{n_{m2}} \hat{\beta}_{j|i}}{\sum_{i=1}^p \sum_{j=1}^p \hat{\beta}_{i|j}}$. In other words, to compute how depositories were influenced by, e.g., insurance sector, we sum up $\hat{\beta}_{i|j}$ in rows of the adjacency matrix corresponding to insurance sector and columns corresponding to depositories. The results for this network statistics are presented on Figure 6. One can see on Figure 6, rows 2, 3 and 4, that depositories dominate in the interconnectedness between the institutions for all sectors. Especially significant linkages both in terms of exposure and contribution exist between depositories and insurance sector. This pattern changes only around 2011 when broker-dealers influenced the banking sector more heavily than other sectors. While being the most exposed and most “contributing” sector among others broker-dealers unlike insurance sector has more balanced structure of interconnectedness with remaining three sectors.

Based on the above analysis we have the following conclusions: (1) the connections between institutions tend to increase both in terms of magnitude of coefficients and the number of links before the financial crisis, (2) the network is characterized by numerous heavy links at the peak of a crisis, (3) the connections between institutions reflected by magnitude of coefficients and the number of links get weaker as the financial system stabilized, (4) the in-going links, or exposure, is far less volatile than the out-going links, or contribution. Whereas banks and insurance exhibit relatively stable contribution pattern, the “Others” and, especially, broker-dealer sectors are highly volatile. While 2008 was dominated by the banks and/insurance, the second half and the aftermath of a financial crisis are characterized by increased contribution of broker-dealers. A significant increase

in the magnitude of coefficients due to broker-dealer sector is observed also around 2011. After having summarized network measures at the aggregate and sector level we turn to analysis of a network at a single companies level. We define in- and out- degrees of one variable as follows: $c_{d-}^{m,k} \stackrel{\text{def}}{=} \sum_{i=1}^p |\hat{\beta}_{s_{m,k}|-i}|$ and $c_{d+}^{m,k} \stackrel{\text{def}}{=} \sum_{j=1}^p |\hat{\beta}_{-j|s_{m,k}}|$. We further aggregate the two measure over all windows for each company. As for total connectedness we calculate both measures based on adjacency matrices.

The estimation results for in- and outdegree are presented in Figures 7 and 8. The most significant risk-recipients and risk-emitters are summarized in Tables 3 to 8. The distribution of incoming and outgoing links differ significantly: while judged by the number of incoming links almost all financial institutions are equally influenced by others except for Wisdom Tree Investments (WETF) having the most in-gong links (5584), the structure of out-going is more heterogenous with multiple companies dominating over the others. Based on the ratio of the total sum of β coefficients to the total number of links for each company, it is possible to identify the most influential financial institutions. In case for IN links these institutions are Fairfield Greenwich Group (FGG, Broker-Dealers), FBL Financial Group (FFG, Insurance), Radian Group (RDN, Insurance), Morgan Stanley (MS, Broker-Dealer), Pinnacle Financial Partners (PNFP, Depositories), and Hartford Financial Services Group (HIG, Insurance). Ratio ranking results suggests the general importance of broker-dealer industry in the whole financial system especially during financial crisis.

Whereas distribution of IN links allows to identify the key players obviously, it is not easy to identify dominating companies with OUT links. Based on the ranking of sum of $c_{d+}^{m,k}$ the five most influential companies are FBL Financial Group (FFG, Insurance), Janus Capital (JNS, Others), Lincoln National Corporation (LNC, Insurance), Morgan Stanley (MS, Broker-Dealer), and Fairfield Greenwich Group (FGG, Broker-Dealers). However, Janus Capital's links are so dispersed (10351) that it has very low ratio of sum to number of links. The same holds true for Lincoln National Corporation and Morgan Stanley. Thus, companies with the highest loads per OUT link are Fairfield Greenwich Group (FGG, Broker-Dealers) and FBL Financial Group (FFG, Insurance), Loews Corporation (L, Insurance) comes as the third one, followed by Arthur J Callagher & Co (AJG, Insurance) and BlackRock Inc. (BLK, Others). Unlike with IN links where both Fairfield Greenwich Group (FGG, Broker-Dealers) and FBL Financial Group (FFG, Insurance) share the leadership, for OUT links Fairfield Greenwich Group (FGG, Broker-Dealers) is clearly the only leader with 0.99 ratio.

Thus, in terms of systemic contribution the IN links are primarily dominated by broker-dealers and insurance sectors, whereas OUT links are primarily due to either broker-dealers or banks sectors. Other sectors have approximately equal share in terms of both

IN and OUT links. This is reasonable as it is likely that insurance companies were connected with the most of the companies to hedge against possible risk. However, insurance companies themselves were not the active players at the market, and were dominated by broker-dealers and/or banks in terms of OUT links.

Further, we examine the shape of the link function in the crisis period as well as in the period of relative financial stability. As one can observe in Figure 9 the link function tends to be almost linear in a financial crisis period. It exhibits non-linearity in a stable period as displayed in Figure 10.

Finally, we construct the indices of systemic exposure and systemic contribution, the results for the whole sample are presented in Table 9. (Cumulative Contribution is defined as summing up $\frac{\sum_{j \neq s_{m,k}} \hat{\beta}_{j|s_{m,k}}}{\sum_{i=1}^p \sum_{j=1}^p \hat{\beta}_{i|j}}$ over all windows and Exposure Indices is defined as summing up $\frac{\sum_{i \neq s_{m,k}} \hat{\beta}_{s_{m,k}|i}}{\sum_{i=1}^p \sum_{j=1}^p \hat{\beta}_{i|j}}$ over all windows.)

5. Conclusion

In this paper we propose a semiparametric framework to assess systemic importance of financial institutions based on their interconnectedness in tails. We use a semiparametric model to allow for more flexible modeling of relationship between the variables. This is especially justified in a ultra-high-dimensional setting when the assumption of linearity is not likely to hold. In order to face these challenges statistically we estimate a single-index model in a generalized quantile regression framework while simultaneously performing variable selection. Ultra-high dimensional setting allows us to include more variables into the analysis.

Our empirical results show that there is growing interconnectedness during the period of a financial crisis, and network-based measure reflecting the connectivity between companies can be used to forecast the market disturbances. Moreover, by including more variables into the analysis we can investigate the overall performance of different financial sectors, depositories, insurance, broker-dealers, and others. We base our analysis on the network measures. Estimations results show relatively high importance of broker-dealers industry in the financial crisis. We also observe strong non-linear relationship between the variables, especially, in the period of relative financial stability.

6. Appendix A: Proof

Condition 1. The kernel $K(\cdot)$ is a continuous symmetric function. The link function $g(\cdot) \in C^2$.

Condition 2. The loss function $\rho_\tau(x)$ is convex and $\psi_\tau(x)$, the derivative (or a subgradient) of $\rho_\tau(x)$, satisfies $\mathbb{E} \psi_\tau(\varepsilon_t) = 0$ and $\inf_{|v| \leq c} \partial \mathbb{E} \psi_\tau(\varepsilon_t - v) = C_1$ where $\partial \mathbb{E} \psi_\tau(\varepsilon_t - v)$ is the partial derivative with respect to v , and C_1 is a constant.

Condition 3. The density of $\beta^{*\top} X$ is bounded with bounded absolute continuous first-order derivatives on its support. Assume $\mathbb{E}\{\psi_\tau(\varepsilon|X)\} = 0$ a.s., which means for a quantile loss we have $F_{\varepsilon|X}^{-1}(\tau) = 0$. Let $X_{t(1)}$ denote the sub-vector of X_t consisting of its first q elements. Let $Z_t \stackrel{\text{def}}{=} X_t^\top \beta^*$ and $Z_{tj} \stackrel{\text{def}}{=} Z_t - Z_j$. Define $C_{0(1)} \stackrel{\text{def}}{=} \mathbb{E} \mathbb{E}\{\psi_\tau^2(\varepsilon_t)|Z_t\} \{[g'(Z_t)]^2 (\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)}) (\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)})^\top\}^\top$, and $C_{0(1)} \stackrel{\text{def}}{=} \mathbb{E}\{\partial \mathbb{E} \psi_\tau(\varepsilon_t)|Z_t\} \{[g'(Z_t)]^2 (\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)}) (\mathbb{E}(X_{t(1)}|Z_t) - X_{t(1)})^\top\}^\top$ and the matrix $C_{1(1)}$ satisfies $0 < L_1 \leq \lambda_{\min}(C_{0(1)}) \leq \lambda_{\max}(C_{0(1)}) \leq L_2 < \infty$ for positive constants L_1 and L_2 . There exists a constant $c_0 > 0$ such that $\sum_{t=1}^n \{\|X_{t(1)}\|/\sqrt{n}\}^{2+c_0} \rightarrow 0$, with $0 < c_0 < 1$. $v_{tj} \stackrel{\text{def}}{=} Y_t - a_j - b_j X_{tj}^\top \beta$. Also, exists a constant C_3 such that for all β close to β^* ($\|\beta - \beta^*\| \leq C_3$)

$$\left\| \sum_t \sum_j X_{(0)tj} \omega_{tj} X_{(1)tj}^\top \partial \mathbb{E} \psi_\tau(v_{tj}) \right\|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1}).$$

Condition 4. The penalty parameter λ is chosen such that $\lambda = \mathcal{O}(n^{-1/2})$, with $D_n \stackrel{\text{def}}{=} \max\{d_l : l \in \mathcal{M}_*\} = \mathcal{O}(n^{\alpha_1 - \alpha_2/2} \lambda) = \mathcal{O}(n^{-1/2})$, $d_l \stackrel{\text{def}}{=} \gamma_\lambda(|\beta_l^*|)$, $\mathcal{M}_* = \{l : \beta_l^* \neq 0\}$ be the true model. Furthermore assume $qh \rightarrow 0$ and $h^{-1} \sqrt{q/n} = \mathcal{O}(1)$ as n goes to infinity, $q = \mathcal{O}(n^{\alpha_2})$, $p = \mathcal{O}\{\exp(n^\delta)\}$, $nh^3 \rightarrow \infty$ and $h \rightarrow 0$. Also, $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$.

Condition 5. The error term ε_t satisfies $\text{Var}(\varepsilon_t) < \infty$. Assume that

$$\begin{aligned} \sup_t \mathbb{E} |\psi_\tau^m(\varepsilon_t)/m!| &\leq s_0 M^m \\ \sup_t \mathbb{E} |\psi_\tau^m(x_{tj})/m!| &\leq s_0 M^m \end{aligned}$$

where s_0 and M are constants, and $\psi_\tau(\cdot)$ is the derivative (a subgradient) of $\rho_\tau(\cdot)$.

Condition 6. The conditional density function $f(\varepsilon|Z_t = u)$ is bounded and absolutely continuous differentiable.

Condition 7. The link function $g(\cdot)$ satisfies a Lipschitz condition in the support of $\beta^\top X$:

$$|g(z) - g(\tilde{z})| \leq C|z - \tilde{z}| \quad (25)$$

Conditions 8. $\{X_{tj}, \varepsilon_t\}_{t=-\infty}^{t=\infty}$ are strong mixing process for any j . Moreover, there exists positive constants c_{m1} and c_{m2} such that the α -mixing coefficient for every

$j \in \{1, \dots, p\}$,

$$\alpha(l) \leq \exp(-c_{m1}l^{c_{m2}}), \quad (26)$$

where $c_{m2} > 2\alpha$.

Define $\hat{\beta}^0$ as the minimizer with the loss

$$\tilde{L}_n(\beta) \stackrel{\text{def}}{=} \sum_{j=1}^n \sum_{t=1}^n \rho_\tau(Y_t - a_j^* - b_j^* X_{tj}^\top \beta) \omega_{tj}(\beta^*) + n \sum_{l=1}^p d_l |\beta_l|,$$

but within the subspace $\{\beta \in \mathbb{R}^p : \beta_{\mathcal{M}_*^c} = \mathbf{0}\}$, and a_j^* , b_j^* are denoted as $g(\beta^{*\top} X)$ and $g'(\beta^{*\top} X)$. The following lemma assures the consistency of $\hat{\beta}^0$,

LEMMA 6.1. *Under Conditions 1-8, recall $d_j = \gamma_\lambda(|\beta_j^*|)$, we have that*

$$\|\hat{\beta}^0 - \beta^*\| = \mathcal{O}_p(\sqrt{q/n} + \|\mathbf{d}_{(1)}\|) \quad (27)$$

where $\mathbf{d}_{(1)}$ is the subvector of $\mathbf{d} = (d_1, \dots, d_p)^\top$ which contains q elements corresponding to the nonzero $\beta_{(1)}^*$.

PROOF. Note that the last $p - q$ elements of both $\hat{\beta}^0$ and β^* are zero, so it is sufficient to prove $\|\hat{\beta}_{(1)}^0 - \beta_{(1)}^*\| = \mathcal{O}_p(\sqrt{q/n} + \|\mathbf{d}_{(1)}\|)$.

We first show for $\gamma_n = o(1)$:

$$\mathbb{P} \left[\inf_{\|\mathbf{u}\|=1} \{ \tilde{L}_n(\beta_{(1)}^* + \gamma_n \mathbf{u}, \mathbf{0}) > \tilde{L}_n(\beta^*) \} \right] \rightarrow 1.$$

Then there exists a minimizer inside the ball $\{\beta_{(1)} : \|\beta_{(1)} - \beta_{(1)}^*\| \leq \gamma_n\}$. Construct $\gamma_n \rightarrow 0$ so that for a sufficiently large constant B_0 : $\gamma_n > B_0 \cdot (\sqrt{q/n} + \|\mathbf{d}_{(1)}\|)$. We will show that by the local convexity of $\tilde{L}_n(\beta_{(1)}, \mathbf{0})$ near $\beta_{(1)}^*$, there exists a unique minimizer inside the ball $\{\beta_{(1)} : \|\beta_{(1)} - \beta_{(1)}^*\| \leq \gamma_n\}$ with probability tending to 1.

Let $X_{(1)tj}$ denote the subvector of X_{tj} consisting of its first q components.

By the uniform Bahadur representation in Kong et al. (2010), there exist a compact set B with β^* a interior point, such that uniformly over $\mathbf{u} \in B$, we have

$$\begin{aligned}
& \tilde{L}_n(\beta_{(1)}^* + \gamma_n \mathbf{u}, \mathbf{0}) - \tilde{L}_n(\beta_{(1)}^*, \mathbf{0}) \\
&= -\gamma_n \sum_{t=1}^n \sum_{j=1}^n b_j^* \psi_\tau(Y_t - a_j^* - b_j^* X_{(1)tj}^\top \beta_{(1)}^*) \omega_{tj}(\beta^*) X_{(1)tj}^\top \mathbf{u} \\
&\quad + \frac{1}{2} \gamma_n^2 \sum_{t=1}^n \sum_{j=1}^n b_j^{*2} \partial \mathbb{E} \psi_\tau(Y_t - a_j^* - b_j^* X_{(1)tj}^\top \beta_{(1)}^* - b_j^* \bar{\gamma}_n X_{(1)tj}^\top \mathbf{u}) \omega_{tj}(\beta^*) (X_{(1)tj}^\top \mathbf{u})^2 \\
&\quad + n \sum_{l=1}^q d_l(|\beta_{(1)l}^* + \gamma_n u_l| - |\beta_{(1)l}^*|) + \mathcal{O}_p(\max\{n\gamma_n^2, n(n/q \log \log n)^{-1}\}) \\
&\stackrel{\text{def}}{=} P_1 + P_2 + P_3 + \mathcal{O}_p(\max\{n\gamma_n^2, n(n/q \log \log n)^{-1}\})
\end{aligned}$$

where $\bar{\gamma}_n \in [0, \gamma_n]$.

Define $\omega_{tj} \stackrel{\text{def}}{=} \omega_{tj}(\beta^*)$, it is not difficult to derive that $\omega_{tj} = \frac{K_h(Z_{tj})}{nf_Z(Z_j)} \{1 + \mathcal{O}_p(1)\}$ where $Z_t = X_t^\top \beta^*$, $Z_{tj} = Z_t - Z_j$ and $f_Z(\cdot)$ is the density of $Z = X^\top \beta^*$.

For P_1 , because $\|\mathbf{u}\| = 1$ and $Y_t = a_t^* + \varepsilon_t$, we get

$$\begin{aligned}
|P_1| &\leq \gamma_n \left\| \sum_{t=1}^n \sum_{j=1}^n b_j^* \psi_\tau(Y_t - a_j^* - b_j^* X_{(1)tj}^\top \beta_{(1)}^*) \omega_{tj} X_{(1)tj}^\top \right\| \{1 + \mathcal{O}_p(1)\} \\
&= \gamma_n \left\| \sum_{j=1}^n b_j^* \left\{ \frac{1}{n} \sum_{t=1}^n \psi_\tau(\varepsilon_t + a_t^* - a_j^* - b_j^* Z_{tj}) \frac{K_h(Z_{tj})}{f_Z(Z_j)} X_{(1)tj} \right\} \right\| \{1 + \mathcal{O}_p(1)\} \\
&= \gamma_n \left\| \sum_{j=1}^n b_j^* \mathbb{E}_{\varepsilon_t, X_t} \left\{ \psi_\tau(\varepsilon_t + a_t^* - a_j^* - b_j^* Z_{tj}) \frac{K_h(Z_{tj})}{f_Z(Z_j)} X_{(1)tj} \right\} \right\| \{1 + \mathcal{O}_p(1)\} \\
&= \gamma_n \left\| \sum_{j=1}^n b_j^* \mathbb{E}_{Z_t} \left\{ \mathbb{E}_{\varepsilon_t|Z_t} [\psi_\tau(\varepsilon_t + a_t^* - a_j^* - b_j^* Z_{tj})] \frac{K_h(Z_{tj})}{f_Z(Z_j)} \mathbb{E}(X_{(1)tj}|Z_t) \right\} \right\| \{1 + \mathcal{O}_p(1)\} \\
&= \gamma_n \left\| \sum_{j=1}^n b_j^* \mathbb{E}_{Z_j} [\psi_\tau(\varepsilon_j + a_j^* - a_j^*)] \{ \mathbb{E}(X_{(1)j}|Z_j) - X_{(1)j} \} \right\| \{1 + \mathcal{O}_p(1)\}
\end{aligned}$$

where $\mathbb{E}_{\varepsilon_t, X_t}$ means taking expectation with respect to the joint distribution of (ε_t, X_t) .

Furthermore we have

$$\begin{aligned}
&\mathbb{E} \left\| \sum_{j=1}^n b_j^* \mathbb{E} [\psi_\tau(\varepsilon_j + a_j^* - a_j^*)] \{ \mathbb{E}(X_{(1)j}|Z_j) - X_{(1)j} \} \right\| \\
&\leq \left\{ \mathbb{E} \psi_\tau^2(\varepsilon_j) \mathbb{E} \sum_{j=1}^n b_j^{*2} [\mathbb{E}(X_{(1)j}|Z_j) - X_{(1)j}]^\top [\mathbb{E}(X_{(1)j}|Z_j) - X_{(1)j}] \right\}^{1/2} \\
&= \sqrt{n} \{ \mathbb{E} \psi_\tau^2(\varepsilon_j) \text{tr}(C_{0(1)}) \}^{1/2},
\end{aligned}$$

recall $C_{0(1)} \stackrel{\text{def}}{=} \mathbf{E}\{[g'(Z_j)]^2(\mathbf{E}(X_{(1)j}|Z_j) - X_{(1)j})[\mathbf{E}(X_{(1)j}|Z_j) - X_{(1)j}]\}^\top$. We thus arrive at

$$P_1 = \mathcal{O}_p(\gamma_n \sqrt{nq}) \quad (28)$$

because $\text{tr}(C_{0(1)}) = \mathcal{O}(q)$ and $\mathbf{E} \psi_\tau^2(\varepsilon_j) < \infty$ by Condition 3.

For P_2 , according to the property of kernel estimation, it can be seen that

$$\begin{aligned} P_2 &= \frac{1}{2} \gamma_n^2 \sum_{t=1}^n \sum_{j=1}^n b_j^{*2} \partial \mathbf{E}_{\varepsilon_t, X_t} \psi_\tau(Y_t - a_j^* - b_j^* Z_{tj} - b_j^* \bar{\gamma}_n X_{(1)tj}^\top \mathbf{u}) \frac{K_h(Z_{tj})}{nf_Z(Z_j)} (X_{(1)tj}^\top \mathbf{u})^2 \{1 + \mathcal{O}_p(1)\} \\ &= \frac{1}{2} \gamma_n^2 \sum_{j=1}^n b_j^{*2} \partial \mathbf{E} \{ \psi_\tau(Y_t - a_j^* - b_j^* \bar{\gamma}_n X_{(1)tj}^\top \mathbf{u}) (X_{(1)tj}^\top \mathbf{u})^2 \} \{1 + \mathcal{O}_p(1)\} \end{aligned}$$

Let $H_t(c) = \inf_{|v| \leq c} \partial \mathbf{E} \psi(\varepsilon_t - v)$. By lemma 3.1 of Portnoy (1984), we have

$$P_2 \geq \frac{1}{2} \gamma_n^2 \sum_{t=1}^n \sum_{j=1}^n b_j^{*2} H(\gamma_n |X_{(1)tj}^\top \mathbf{u}|) (X_{(1)tj}^\top \mathbf{u})^2 \geq c \gamma_n^2 n \quad (29)$$

for some positive c .

For P_3 , it is clear that

$$|P_3| \leq n \gamma_n \sum_{l=1}^q d_l |u_l| \leq n \gamma_n \|\mathbf{d}_{(1)}\| \quad (30)$$

Combining (28), (29) and (30), the following inequality holds with probability tending to 1 that

$$\tilde{L}_n(\beta_{(1)}^* + \gamma_n \mathbf{u}, \mathbf{0}) - \tilde{L}_n(\beta_{(1)}^*, \mathbf{0}) \geq n \gamma_n \left(c \gamma_n - \sqrt{q/n} - \|\mathbf{d}_{(1)}\| \right) \quad (31)$$

$\gamma_n = B_0(\sqrt{q/n} + \|\mathbf{d}_{(1)}\|)$ and B_0 is a sufficiently large constant, so that the RHS of (31) is larger than 0. Owing to the local convexity of the objective function, there exists a unique minimizer $\hat{\beta}_{(1)}^0$ such that

$$\|\hat{\beta}^0 - \beta^*\| = \|\hat{\beta}_{(1)}^0 - \beta_{(1)}^*\| = \mathcal{O}_p(\sqrt{q/n} + \|\mathbf{d}_{(1)}\|)$$

Therefore, (27) is proved. \square

Recall that $X = (X_{(1)}, X_{(0)})$ and $\mathcal{M}_* = \{1, \dots, q\}$ is the set of indices at which β are nonzero.

Lemma 1 shows the consistency of $\hat{\beta}^0$, and we need to show further that $\hat{\beta}^0$ is the unique minimizer in \mathbb{R}^p on a set with probability tending to 1.

LEMMA 6.2. *Under conditions 1-8, minimizing the loss function $\tilde{L}_n(\beta)$ has a unique*

global minimizer $\hat{\beta} = (\hat{\beta}_1^\top, \mathbf{0}^\top)^\top$, if and only if on a set with probability tending to 1,

$$\sum_{j=1}^n \sum_{t=1}^n \psi_\tau(Y_t - \hat{a}_j - \hat{b}_j X_{tj}^\top \hat{\beta}_\tau) \hat{b}_j X_{(1)tj} \omega_{tj}(\beta^*) + n \mathbf{d}_{(1)} \circ \text{sign}(\hat{\beta}_\tau) = 0 \quad (32)$$

$$\|z(\hat{\beta}_\tau)\|_\infty \leq n, \quad (33)$$

where

$$z(\hat{\beta}_\tau) \stackrel{\text{def}}{=} \mathbf{d}_{(0)}^{-1} \circ \left\{ \sum_{j=1}^n \sum_{t=1}^n b_j^* \psi_\tau(Y_t - a_j^* - b_j^* X_{tj}^\top \hat{\beta}_\tau) X_{(0)tj} \omega_{tj}(\hat{\beta}_\tau) \right\} \quad (34)$$

where \circ stands for multiplication element-wise.

PROOF. According to the definition of $\hat{\beta}_\tau$, it is clear that $\hat{\beta}_{(1)}$ already satisfies condition (32). Therefore we only need to verify condition (33).

To prove (33), a bound for

$$\sum_{t=1}^n \sum_{j=1}^n b_j^* \psi_\tau(Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*) \omega_{tj} X_{(0)tj} \quad (35)$$

is needed. Define the following kernel function

$$\begin{aligned} & h_d(X_t, a_j^*, b_j^*, Y_t, X_j, a_t^*, b_t^*, Y_j) \\ &= \frac{n}{2} \left\{ b_j^* \psi_\tau(Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*) \omega_{tj} X_{(0)tj} + b_t^* \psi_\tau(Y_j - a_t^* - b_t^* X_{tj}^\top \beta^*) \omega_{jt} X_{(0)jt} \right\}_d, \end{aligned}$$

where $\{\cdot\}_d$ denotes the d th element of a vector, $d = 1, \dots, p - q$.

According to Borisov and Volodko (2009), based on Condition 5:

Define $U_{n,d} \stackrel{\text{def}}{=} \frac{1}{n(n-1)} \sum_{1 \leq i < j \leq n} h_d(X_t, a_j^*, b_j^*, Y_t, X_j, a_i^*, b_i^*, Y_j)$ as the U -statistics for (35). We have, with sufficient large c_{m2} in Condition 8.

$$\mathbb{P}\{|U_{n,d} - \mathbb{E}U_{n,d}| > \varepsilon\} \leq c_{m3} \exp(c_{m5}\varepsilon / (c_{m3} + c_{m4}\varepsilon^{1/2}n^{-1/2}))$$

where c_{m3}, c_{m4}, c_{m5} are constants. Moreover, let $\varepsilon = \mathcal{O}(n^{1/2+\alpha})$, as $\alpha < 1/2$, we can further have,

$$\mathbb{P}(\{|U_{n,d} - \mathbb{E}U_{n,d}| > \varepsilon\}) \leq c_{m3} \exp(-c_{m6}\varepsilon/2),$$

Define

$$F_{n,d} \stackrel{\text{def}}{=} (n)^{-1} \sum_{t=1}^n \sum_{j=1}^n b_j \psi_\tau(Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*) \omega_{tj} X_{(0)tj},$$

also it is not hard to derive that $U_{n,d} = F_{n,d}n/(n-1)$.

It then follows that

$$\begin{aligned} \mathbb{P}(|F_{n,d} - \mathbb{E}F_{n,d}| > \varepsilon) &= \mathbb{P}(|U_{n,d} - \mathbb{E}U_{n,d}|(n-1)/n > \varepsilon) \\ &\leq 2 \exp(-Cn^{\alpha+1/2}) \end{aligned}$$

Define $\mathcal{A}_n = \{\|F_n - \mathbb{E}F_n\|_\infty \leq \varepsilon\}$, thus

$$\mathbb{P}(\mathcal{A}_n) \geq 1 - \sum_{d=1}^{p-q} \mathbb{P}(|F_{n,d} - \mathbb{E}F_{n,d}| > \varepsilon) \geq 1 - 2(p-q) \exp(-Cn^{\alpha+1/2}).$$

Finally we get that on the set \mathcal{A}_n ,

$$\begin{aligned} \|z(\hat{\beta}^0)\|_\infty &\leq \|\mathbf{d}_{\mathcal{M}_*^c}^{-1} \circ F_n\|_\infty + \|\mathbf{d}_{\mathcal{M}_*^c}^{-1} \circ \sum_{t=1}^n \sum_{j=1}^n b_j [\psi_\tau(Y_t - a_j^* - b_j^* X_{tj}^\top \hat{\beta}^0) \\ &\quad - \psi_\tau(Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*)] \omega_{tj} X_{(0)tj}\|_\infty \\ &\leq \mathcal{O}(n^{1/2+\alpha}/\lambda + \|\mathbf{d}_{\mathcal{M}_*^c}^{-1} \circ \sum_{t=1}^n \sum_{j=1}^n \partial \mathbb{E} \psi_\tau(v_{tj}) b_j X_{(1)tj}^\top (\hat{\beta}_{(1)} - \beta_{(1)}^*) \omega_{tj} X_{(0)tj}\|_\infty), \end{aligned}$$

where v_{tj} is between $Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*$ and $Y_t - a_j^* - b_j^* X_{tj}^\top \hat{\beta}^0$. From Lemma 1,

$$\|\hat{\beta}^0 - \beta_{(1)}^*\|_2 = \mathcal{O}_p(\|\mathbf{d}_{(1)}\| + \sqrt{q}/\sqrt{n}).$$

Choosing $\|\sum_t \sum_j X_{(0)tj} \omega_{tj} X_{(1)tj}^\top \partial \mathbb{E} \psi_\tau(v_{tj})\|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1})$, $q = \mathcal{O}(n^{\alpha_2})$, $\lambda = \mathcal{O}(\sqrt{q/n}) = n^{-1/2+\alpha_2/2}$, $0 < \alpha_2 < 1$, $\|\mathbf{d}_{(1)}\| = \mathcal{O}(\sqrt{q}D_n) = \mathcal{O}(n^{\alpha_2/2}D_n)$

$$\begin{aligned} (n)^{-1} \|z(\hat{\beta}^0)\|_\infty &= \mathcal{O}\{n^{-1}\lambda^{-1}(n^{1/2+\alpha} + n^{1-\alpha_1}\sqrt{q}/\sqrt{n} + \|\mathbf{d}_{(1)}\|n^{1-\alpha_1})\} \\ &= \mathcal{O}(n^{-\alpha_2/2+\alpha} + n^{-\alpha_1} + n^{-\alpha_1+\alpha_2/2}D_n/\lambda), \end{aligned}$$

assume $D_n = \mathcal{O}(n^{\alpha_1-\alpha_2/2}\lambda)$, and let $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$, with rate $p = \mathcal{O}\{\exp(n^\delta)\}$, then $(n)^{-1} \|z(\hat{\beta}^0)\|_\infty = \mathcal{O}_p(1)$. \square

Proof of Theorem 1 . The results follows from Lemma 1 and 2. \square

Proof of Theorem 2. By Theorem 1, $\hat{\beta}_{\mathbf{w}(1)} = \beta_{(1)}$ almost surely. It then follows from Lemma 2 that

$$\|\hat{\beta}_{\tau(1)} - \beta_{(1)}^*\| = \mathcal{O}_p\{(D_n + n^{-1/2})\sqrt{q}\}.$$

This completes the first part of the theorem.

Given a_j^* , b_j^* , the estimate $\hat{\beta}_\tau$ is:

$$\hat{\beta}_\tau = \arg \min_{\beta} \sum_{j=1}^n \sum_{t=1}^n \rho_\tau(Y_t - a_j^* - b_j^* X_{tj}^\top \beta) \omega_{tj} + n \sum_{l=1}^p \gamma_\lambda(|\beta_l^*|) |\beta_l|$$

where $\omega_{tj} = \omega_{tj}(\beta^*)$ and $X_{tj} = X_t - X_j$.

Define $\hat{\xi} \stackrel{\text{def}}{=} \sqrt{n}(\hat{\beta}_\tau - \beta^*)$ and $Y_{tj} = Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*$. As $\omega_{tj}(\tilde{\beta}) = \frac{K_h(Z_{tj})}{nf_Z(Z_j)} \{1 + o_p(1)\}$, it follows that $\hat{\xi}$ is the minimizer of

$$\begin{aligned} H_n(\xi) &= \sum_{t=1}^n \sum_{j=1}^n \left\{ \rho_\tau(Y_{tj} - n^{-\frac{1}{2}} b_j^* X_{tj}^\top \xi) - \rho_\tau(Y_{tj}) \right\} \frac{K_h(Z_{tj})}{nf_Z(Z_j)} \{1 + o_p(1)\} \\ &\quad + n \sum_{l=1}^p \gamma_\lambda(|\beta_l^*|) (|\beta_l^* + n^{-1/2} \xi_l| - |\beta_l^*|) \\ &\stackrel{\text{def}}{=} Q_1(\xi) \{1 + o_p(1)\} + Q_2(\xi), \end{aligned}$$

recall $f_Z(z)$ is the density function of $Z = X_t^\top \beta^*$, $t = 1, \dots, n$. We study $Q_1(\xi)$ and $Q_2(\xi)$ respectively.

Let $\Delta_{ij}(\xi) \stackrel{\text{def}}{=} \rho_\tau\{Y_{tj} - n^{-\frac{1}{2}} g'(Z_j) X_{tj}^\top \xi\} - \rho_\tau(Y_{tj}) - n^{-\frac{1}{2}} \psi_\tau(Y_{tj}) g'(Z_j) X_{tj}^\top \xi$. It can be seen that

$$\begin{aligned} Q_1(\xi) &= \left\{ \sum_{t=1}^n \sum_{j=1}^n n^{-1/2} \psi_\tau(Y_{tj}) g'(Z_j) \frac{K_h(Z_{tj})}{nf_Z(Z_j)} X_{tj}^\top \xi \right. \\ &\quad \left. + \sum_{t=1}^n \sum_{j=1}^n \Delta_{ij}(\xi) \frac{K_h(Z_{tj})}{nf_Z(Z_j)} \right\} \{1 + o_p(1)\} \\ &\stackrel{\text{def}}{=} [A^\top \xi + I(\xi)] \{1 + o_p(1)\} \end{aligned}$$

Recall that $Y_{tj} = \varepsilon_t + a_t^* - a_j^* - b_j^* Z_{tj} + o_p(1)$.

Therefore we have

$$\begin{aligned} A &= \sum_{t=1}^n \sum_{j=1}^n \frac{1}{\sqrt{n}} \psi_\tau(\varepsilon_t + a_t^* - a_j^* - b_j^* Z_{tj}) g'(Z_j) \frac{K_h(Z_{tj})}{nf_Z(Z_j)} X_{tj} \{1 + o_p(1)\} \\ &= \sum_{t=1}^n \frac{1}{\sqrt{n}} \psi_\tau(\varepsilon_t) g'(Z_t) \{E(X|Z_t) - X_t\} \{1 + o_p(1)\}. \end{aligned}$$

Similarly we have, as before via the uniform Bahadur representation.

$$\begin{aligned}
I(\xi) &= \sum_{t=1}^n \sum_{j=1}^n \Delta_{ij}(\xi) \frac{K_h(Z_{tj})}{nf_Z(Z_j)} \\
&= \sum_{t=1}^n \sum_{j=1}^n \left\{ \rho_\tau(Y_{tj} - n^{-\frac{1}{2}}g'(Z_j)X_{tj}^\top \xi) - \rho_\tau(Y_{tj}) \right. \\
&\quad \left. - n^{-\frac{1}{2}}\psi_\tau(Y_{tj})g'(Z_j)X_{tj}^\top \xi \right\} \frac{K_h(Z_{tj})}{nf_Z(Z_j)} \\
&= \sum_{j=1}^n \mathbb{E}_{\varepsilon_t, X_t} \left[\left\{ \rho_\tau(Y_{tj} - n^{-\frac{1}{2}}g'(Z_j)X_{tj}^\top \xi) - \rho_\tau(Y_{tj}) \right. \right. \\
&\quad \left. \left. - n^{-\frac{1}{2}}\psi_\tau(\varepsilon_t)g'(Z_t)X_{tj}^\top \xi \right\} \frac{K_h(Z_{tj})}{f_Z(Z_j)} \right] \{1 + o(1)\} \\
&= \sum_{j=1}^n \mathbb{E}_{Z_t} \mathbb{E}_{\varepsilon_t, X_t|Z_t} \left[\left\{ \rho_\tau(Y_{tj} - n^{-\frac{1}{2}}g'(Z_j)X_{tj}^\top \xi) - \rho_\tau(Y_{tj}) \right. \right. \\
&\quad \left. \left. - n^{-\frac{1}{2}}\psi_\tau(\varepsilon_t)g'(Z_t)X_{tj}^\top \xi \right\} \frac{K_h(Z_{tj})}{f_Z(Z_j)} \right] \{1 + o(1)\} \\
&= \sum_{j=1}^n \left\{ \mathbb{E}_{\varepsilon, Z} \rho_\tau(\varepsilon - n^{-\frac{1}{2}}g'(Z_j)(X_j - \mathbb{E}(X_j|Z_j))^\top \xi) - \mathbb{E}_\varepsilon \rho_\tau(\varepsilon) \right. \\
&\quad \left. - n^{-\frac{1}{2}}\mathbb{E}_{\varepsilon, Z} [\psi_\tau(\varepsilon)]g'(Z_j)(X_j - \mathbb{E}(X_j|Z_j))^\top \xi \right\} \{1 + o(1)\} \\
&= (2n)^{-1} \sum_{j=1}^n \mathbb{E} \{ \partial \mathbb{E} \psi_\tau(\varepsilon_j) \} [g'(Z_j)]^2 \xi^\top \{X_j - \mathbb{E}(X_j|Z_j)\} \{X_j - \mathbb{E}(X_j|Z_j)\}^\top \xi \{1 + o(1)\} \\
&= \frac{1}{2} \xi^\top \mathbb{E} \{ \{ \partial \mathbb{E} \psi_\tau(\varepsilon_j) \} [g'(Z_j)]^2 \{X_j - \mathbb{E}(X_j|Z_j)\} \{X_j - \mathbb{E}(X_j|Z_j)\}^\top \} \xi \{1 + o(1)\} \\
&\stackrel{\text{def}}{=} \frac{1}{2} \xi^\top C_0 \xi \{1 + o(1)\},
\end{aligned}$$

where $C_0 \stackrel{\text{def}}{=} \mathbb{E} \{ \partial \mathbb{E} \psi_\tau(\varepsilon_t) | Z_t \} \{ [g'(Z_t)]^2 (\mathbb{E}(X_i|Z_t) - X_i)(\mathbb{E}(X_i|Z_t) - X_i) \}^\top$.

About $Q_2(\xi)$, we find that if $\beta_l^* = 0$, that is $q < l \leq p$, $\sqrt{n}(|\beta_l^* + n^{-1/2}\xi| - |\beta_l^*|) = |\xi|$, otherwise when $1 \leq l \leq q$, $\sqrt{n}(|\beta_l^* + n^{-1/2}\xi| - |\beta_l^*|) \rightarrow \xi \text{sign}(\beta_l^*)$. In addition, Theorem 1 implies that $\hat{\beta}_{\mathbf{w}(2)}$ will converge to zero with probability one. Thus

$$\begin{aligned}
Q_2(\xi) &= n \sum_{l=1}^p \gamma_\lambda(|\beta_l^*|) (|\beta_l^* + n^{-1/2}\xi| - |\beta_l^*|) \\
&= n \sum_{l=1}^q \gamma_\lambda(|\beta_l^*|) \xi_l \text{sign}(\beta_l^*) \{1 + o_p(1)\}.
\end{aligned}$$

As $\gamma_\lambda(|\beta_l^*|) = \mathcal{O}(D_n) = o(\sqrt{n})$, the loss function satisfies

$$H_n(\xi) = D_n(\xi)\{1 + o(1)\} \stackrel{\text{def}}{=} [-\xi_{(1)}^\top A_{(1)} + \frac{1}{2}\xi_{(1)}^\top C_{0(1)}\xi_{(1)}]\{1 + o(1)\}$$

where $A_{(1)}$ is the sub-vector of A consisting of its first q components, and $C_{0(1)}$ is the up-left $q \times q$ sub-matrix of C_0 . Applying the uniform Bahadur representation, we obtain $\hat{\xi}_{(1)} = \sqrt{n}(\hat{\beta}_{(1)} - \beta_{(1)}^*) = C_{0(1)}^{-1}A_{(1)} + o_p(1)$.

Recall that $A_{(1)} = \sum_{t=1}^n \frac{1}{\sqrt{n}} \psi_\tau(\varepsilon_t) g'(Z_t) \{E(X_{(1)}|Z_t) - X_{t(1)}\} \{1 + o_p(1)\}$, and \mathbf{b} is a unit vector in \mathbf{R}^q . Thus we have

$$\begin{aligned} \sqrt{n}\mathbf{b}^\top C_{0(1)}^{1/2} C_{1(1)}^{-1/2} C_{0(1)}^{1/2} (\hat{\beta}_{(1)} - \beta_{(1)}^*) &= \mathbf{b}^\top C_{0(1)}^{1/2} C_{1(1)}^{-1/2} C_{0(1)}^{1/2} C_{0(1)}^{-1} \\ &\quad \times \frac{1}{\sqrt{n}} \sum_{t=1}^n \psi_\tau(\varepsilon_t) g'(Z_t) \{E(X_{(1)}|Z_t) - X_{t(1)}\} \{1 + o_p(1)\} \\ &\xrightarrow{\mathcal{L}} \mathbf{N}(0, 1) \end{aligned}$$

where $C_{1(1)} \stackrel{\text{def}}{=} E\{E\{\psi_\tau^2(\varepsilon_t)|Z_t\}[g'(Z_t)]^2[E(X_{(1)}|Z_t) - X_{t(1)}][E(X_{(1)}|Z_t) - X_{t(1)}]^\top\}$, and $C_{0(1)} \stackrel{\text{def}}{=} E\{\partial E \psi_\tau(\varepsilon_t)|Z_t\} \{[g'(Z_t)]^2(E(X_{t(1)}|Z_t) - X_{t(1)})(E(X_{t(1)}|Z_t) - X_{t(1)})^\top\}$. The asymptotic normality of $\hat{\beta}_{(1)}$ is therefore proved. \square

Proof of Theorem 3. We note that

$$\sqrt{nh} \left\{ \hat{g}(x^\top \hat{\beta}_\tau) - g(x^\top \beta^*) \right\} = \sqrt{nh} \left\{ \hat{g}(x^\top \hat{\beta}_\tau) - \hat{g}(x^\top \beta^*) \right\} + \sqrt{nh} \left\{ \hat{g}(x^\top \beta^*) - g(x^\top \beta^*) \right\}$$

As ρ_τ is strictly convex, then $\hat{g}(\cdot)$ is continuous almost surely. As $qh \rightarrow 0$, the consistency of $\hat{\beta}$ in Theorem 2 implies $\sqrt{nh} \left\{ \hat{g}(x^\top \hat{\beta}_\tau) - \hat{g}(x^\top \beta^*) \right\} = o_p(1)$. Consequently it is sufficient to prove

$$\sqrt{nh^3} \sqrt{\{f_Z(z)\mu_2^2\}/(\nu_2\sigma_\tau^2)} \left\{ \hat{g}(x^\top \beta^*) - g(x^\top \beta^*) \right\} \xrightarrow{\mathcal{L}} \mathbf{N}(0, 1),$$

recall μ_2 and ν_2 are defined as $\mu_j \stackrel{\text{def}}{=} \int u^j K(u) du$ and $\nu_j \stackrel{\text{def}}{=} \int u^j K^2(u) du$.

We now prove equation (36). Let $Z_t = X_t^\top \beta^*$ and $z = x^\top \beta^*$. Recall that $\hat{g}(z) = \hat{a}$ and $\hat{g}'(z) = \hat{b}$ where

$$(\hat{a}, \hat{b}) \stackrel{\text{def}}{=} \arg \min_{(a,b)} \sum_{t=1}^n \rho_\tau \{Y_t - a - b(Z_t - u)\} \omega_t(\beta^*) \quad (36)$$

where $\omega_t(\beta^*) = K_h(Z_t - z) / \sum_{k=1}^n K_h(Z_k - z) = \{1 + o_p(1)\} K_h(Z_t - z_t) / \{nf_Z(z)\}$

Denote $\theta = [\sqrt{nh}\{a - g(z)\}, \sqrt{nh^3}\{b - g'(z)\}]^\top$ and $\hat{\theta} = [\sqrt{nh}\{\hat{g}(z) - g(z)\},$

$\sqrt{nh^3}\{\hat{g}'(z) - g'(z)\}^\top$. Also let $\mathbf{Z}_t \stackrel{\text{def}}{=} (1, (Z_t - z)/h)^\top$ and $\delta_t \stackrel{\text{def}}{=} g(Z_t) - g(z) - g'(z)(Z_t - z)$. Then $\hat{\theta}$ is the minimizer of

$$L_{g,n}(\theta) \stackrel{\text{def}}{=} \sum_{t=1}^n \{\rho_\tau(\varepsilon_t + \delta_t - (nh)^{-1/2} \mathbf{Z}_t^\top \theta) - \rho_\tau(\varepsilon_t + \delta_t)\} \frac{K_h(Z_t - z)}{f_Z(z)} \{1 + o(1)\}. \quad (37)$$

By similar expansion of $Q_1(\xi)$ in the proof Theorem 2, we have

$$\begin{aligned} \sqrt{nh} L_{g,n}(\theta) &= \frac{1}{\sqrt{nh}} \sum_{t=1}^n \psi_\tau(\varepsilon_t + \delta_t) \mathbf{Z}_t^\top \theta \frac{K\{(Z_t - z)/h\}}{f_Z(z)} \{1 + o(1)\} \\ &\quad + \frac{1}{2nh} \sum_{t=1}^n \{\partial \mathbf{E} \psi_\tau(\varepsilon_t + \delta_t)\} (\mathbf{Z}_t^\top \theta)^2 \frac{K\{(Z_t - z)/h\}}{f_Z(z)} \{1 + o(1)\} \\ &= A_n^\top \theta + \frac{1}{2} \theta^\top S_n \theta \{1 + o(1)\} \end{aligned}$$

where

$$\begin{aligned} A_n &= \frac{1}{\sqrt{nh}} \sum_{t=1}^n \psi_\tau(\varepsilon_t + \delta_t) \mathbf{Z}_t \frac{K\{(Z_t - z)/h\}}{f_Z(z)} \\ S_n &= \frac{1}{nh} \sum_{t=1}^n \{\partial \mathbf{E} \psi_\tau(\varepsilon_t + \delta_t)\} \mathbf{Z}_t \mathbf{Z}_t^\top \frac{K\{(Z_t - z)/h\}}{f_Z(z)}. \end{aligned}$$

The leading term is then

$$\begin{aligned} S_n &\stackrel{\text{def}}{=} \frac{1}{h} \mathbf{E} \left\{ \partial \mathbf{E} \psi_\tau(\varepsilon_t + \delta_t) \mathbf{Z}_t \mathbf{Z}_t^\top \frac{K\{(Z_t - z)/h\}}{f_Z(z)} \middle| Z_t \right\} \\ &= \frac{1}{h} \mathbf{E} \left\{ \partial \mathbf{E} \psi_\tau(\varepsilon_t + \delta_t) \middle| Z_t \right\} \mathbf{Z}_t \mathbf{Z}_t^\top \frac{K\{(Z_t - z)/h\}}{f_Z(z)} \\ &= \partial \mathbf{E} \psi_\tau(\varepsilon_t) \int \begin{pmatrix} 1 & (\xi - u)/h \\ (\xi - u)/h & (\xi - z)^2/h^2 \end{pmatrix} K\left(\frac{\xi - z}{h}\right) \frac{d\xi}{h} \\ &= [\partial \mathbf{E} \psi_\tau(\varepsilon_t)] \begin{pmatrix} 1 & 0 \\ 0 & \mu_2 \end{pmatrix} \{1 + o(1)\} \stackrel{\text{def}}{=} S \{1 + o(1)\} \end{aligned}$$

where $\mu_2 = \int t^2 K(t) dt$. Therefore it follows that $\hat{\theta} = -S^{-1} A_n \{1 + o_p(1)\}$.

It is clear that $\Sigma^{-1/2} A_n$ is asymptotically distributed as a bivariate normal distribution

$\mathbb{N}(0, 1)$. The asymptotic bias $m_A = o(1)$, in particular

$$\begin{aligned}
m_A &= \frac{1}{\sqrt{h}} \mathbb{E} \left\{ \psi_\tau(\varepsilon_t + \delta_t) \mathbf{Z}_t \frac{K\{(Z_t - z)/h\}}{f_Z(z)} \right\} \\
&= \frac{1}{\sqrt{h}} \mathbb{E} \left\{ \mathbb{E} [\psi_\tau(\varepsilon_t + \delta_t) | Z_t] \mathbf{Z}_t \frac{K\{(Z_t - z)/h\}}{f_Z(z)} \right\} \\
&= \frac{1}{\sqrt{h}} \iint \psi_\tau \{ \varepsilon + g(\xi) - g(z) - g'(z)(\xi - z) \} f_E(\varepsilon) d\varepsilon \\
&\quad \times \begin{pmatrix} 1 \\ (\xi - z)/h \end{pmatrix} K((\xi - z)/h) d\xi \\
&= \frac{1}{2} h^{5/2} g''(z) \mu_2 [\partial \mathbb{E} \psi_\tau(\varepsilon)] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \{1 + o(1)\}.
\end{aligned}$$

The scaling variance covariance matrix is then

$$\begin{aligned}
\Sigma &= \frac{1}{h} \mathbb{E} \left\{ \psi_\tau^2(\varepsilon_t + \delta_t) \mathbf{Z}_t \mathbf{Z}_t^\top \left[\frac{K\{(Z_t - z)/h\}}{f_Z(z)} \right]^2 \right\} \{1 + o(1)\} \\
&= \frac{1}{h} \int \int \psi_\tau^2 \{ \varepsilon + g(\xi) - g(z) - g'(z)(\xi - z) \} f_E(\varepsilon) d\varepsilon \\
&\quad \times \begin{pmatrix} 1 & (\xi - z)/h \\ (\xi - z)/h & (\xi - z)^2/h^2 \end{pmatrix} \frac{K^2\{(Z_t - z)/h\}}{f_Z(z)} d\xi \{1 + o(1)\} \\
&= \frac{1}{f_Z(z)} \mathbb{E}[\psi_\tau^2(\varepsilon)] \begin{pmatrix} \nu_0 & 0 \\ 0 & \nu_2 \end{pmatrix} \{1 + o(1)\},
\end{aligned}$$

where $\nu_k = \int t^k K^2(t) dt$.

Thus we finally obtain that as n tends to infinity, $\Sigma^{-1/2} S \hat{\theta} - \Sigma^{-1/2} S m_A$ converges in distribution to $\mathbb{N}(0, 1)$. Slight algebra gives

$$\begin{aligned}
\Sigma^{-1/2} S m_A &= \frac{1}{2} h^{5/2} g''(z) f_Z(z)^{1/2} \nu_0^{-1/2} \{ \partial \mathbb{E} \psi_\tau(\varepsilon) \}^2 \{ \mathbb{E} \psi_\tau^2(\varepsilon) \}^{-1/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
S^{-1} \Sigma S^{-1} &= \frac{\mathbb{E}[\psi_\tau^2(\varepsilon)]}{\{ [\partial \mathbb{E} \psi_\tau(\varepsilon)]^2 \}^2 f_Z(z)} \begin{pmatrix} 1 & 0 \\ 0 & \mu_2^{-1} \end{pmatrix} \begin{pmatrix} \nu_0 & 0 \\ 0 & \nu_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \mu_2^{-1} \end{pmatrix} \\
&= \sigma_{\mathbf{w}}^2 \frac{1}{f_Z(z)} \begin{pmatrix} \nu_0 & 0 \\ 0 & \nu_2/\mu_2^2 \end{pmatrix}.
\end{aligned}$$

This completes the proof. □

| SRCE | 1st Source Corporation | Y | Alleghany Corp | MKL | Markel Corp. |
|------|---------------------------------|------|--------------------------------------|------|-----------------------------------|
| STBA | S & T Bancorp, Inc. | ALL | Allstate Corp | MMC | Marsh & McLennan |
| SCBT | SCBT Financial Corporation | AEL | American Equity Inv Life Hldg Co. | MBI | MBIA Inc. |
| SBNY | Signature Bank | AFG | American Financial Group Inc | MCY | Mercury General Corporation |
| STT | State Street | AIG | American International Group, Inc. | MET | MetLife Inc |
| STI | SunTrust Banks, Inc. | ANAT | American National Insurance Co. | NFP | National Financial Partners Corp. |
| SUSQ | Susquehanna Bancshares, Inc. | AON | AON Corp | NWLI | National Western Life Ins Company |
| SIVB | SVB Financial Group | AJG | Arthur J Callagher & Co. | NAVG | Navigators Group, Inc. |
| SNV | Synovus Financial Corporation | AIZ | Assurant | ORI | Old Republic International Corp. |
| TCB | TCF Financial Corporation | BHLB | Berkshire Hills Bancorp Inc. | PFG | Principal Financial Group |
| TCBI | Texas Capital Bancshares Inc. | BRO | Brown & Brown Inc. | PRA | ProAssurance Corporation |
| TMP | Tompkins Financial Corporation | CB | Chubb Corp | PGR | Progressive |
| UMBF | UMB Financial Corporation | CINF | Cincinnati Financial Corp | PL | Protective Life Corporation |
| UMPQ | Umpqua Holdings Corporation | CIA | Citizens, Inc. | PRU | Prudential Financial, Inc. |
| UBSI | United Bancshares, Inc. | CNA | CNA Financial Corporation | RDN | Radian Group Inc. |
| USB | US Bancorp | NO | CNO Financial Group, Inc. | RLI | RLI Corp. |
| VLY | Valley National Bancorp | ERIE | Erie Indemnity Company | SAFT | Safety Insurance Group, Inc. |
| WAFD | Washington Federal Inc. | FFG | FBL Financial Group Inc. | SIGI | Selective Insurance Group Inc. |
| WBS | Webster Financial Corp. | FNF | Fidelity National Financial | SFG | StanCorp Financial Group Inc. |
| WFC | Wells Fargo & Co | HIG | Hartford Fin. Services Group, Inc. | STFC | State Auto Financial Corporation |
| WSBC | WesBanco, Inc. | HCC | HCC Insurance Holdings Inc. | THG | The Hannover Insurance Group |
| WABC | Westamerica Bancorp. | HCSG | Healthcare Services Group Inc. | TRV | The Travelers Companies, Inc. |
| WTFC | Wintrust Financial Corporation | HMN | Horacle Mann Educators Corp. | TMK | Torchmark |
| WRLD | World Acceptance Corp. | IPCC | Infinity Property and Casualty Corp. | TWGP | Tower Group, Inc. |
| UFCS | United Fire Group, Inc. | SF | Stifel Financial Corporation | SLM | SLM Corporation |
| UNM | Unum Group | TROW | T. Rowe Price Group, Inc. | AMTD | TD Ameritrade |
| WRB | W. R. Berkley Corporation | SCHW | The Charles Schwab Corporation | | |
| WTM | White Mountains Ins Group, Ltd. | TRMK | Trustmark Corporation | | |
| | Broker-Dealers (25) | WDR | Waddell & Reed Financial Inc. | | |
| AMG | Affiliated Managers Group Inc. | WETF | WisdomTree Investments, Inc | | |
| AB | AllianceBernstein Holding L.P. | | Others (19) | | |

| | | | |
|------|------------------------------------|------|------------------------------------|
| BGCP | BGC Partners, Inc. | ABM | ABM Industries, Inc. |
| BLW | BlackRock Lim. Duration Inc. Trust | AXP | American Express |
| EPHC | Epoch Investment Partners, Inc. | AMP | Ameriprise Financial |
| ETFC | E-Trade Financial | BLK | BlackRock, Inc. |
| FGG | Fairfield Greenwich group | COF | Capital One Financial |
| FII | Federal Investors, Inc. | CME | CME Group. |
| GBL | GAMCO Investors, Inc. | DNB | Dun & Bradstreet Corp. |
| GS | Goldman Sachs | EV | Eaton Vance |
| IVZ | Invesco Ltd. | EFX | Equifax Inc. |
| JEF | Jefferies Group, Inc. | FITB | Fifth Third Bancorp |
| KFN | KKR Financial Holdings LLC | BEN | Franklin Resources |
| MKTX | Marketaxess Holdings | JNS | Janus Capital |
| MS | Morgan Stanley | LM | Legg Mason |
| MORN | Morningstar Inc. | MCO | Moody's Corp. |
| NDAQ | Nasdaq OMX Group Inc. | PRAA | Portfolio Recovery Associates Inc. |
| RJF | Raymond James Financial Inc. | ROL | Rollins Inc. |
| STSA | Sterling Financial Corporation | SEIC | SEI Investments Company |

Table 2: Financial companies with tickers classified by industry: depositories (100), insurance (56), broker-dealers (25) and others (19).

2007-01-11

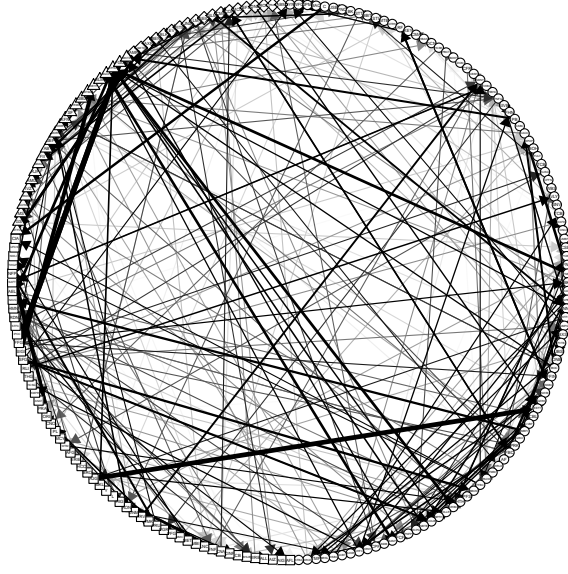


Figure 1: A risk network formed by 200 financial institutions without thresholding of coefficients, a circular representation of a weighted adjacency matrix, Depositories: a circle, Insurance: a square, Broker-Dealers: a triangle, Others: a diamond, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

2007-01-11

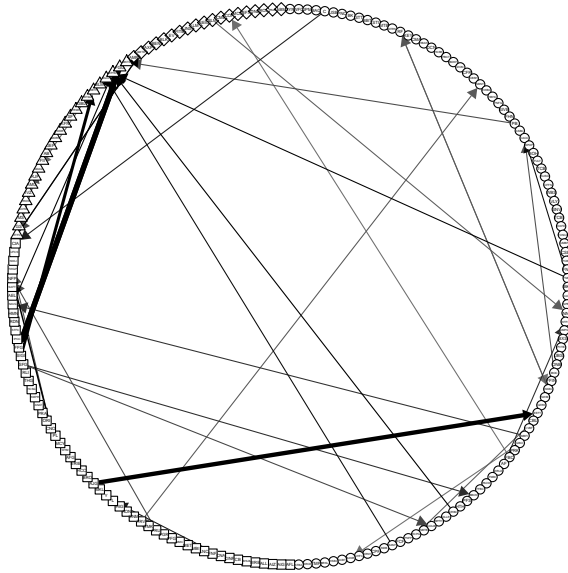


Figure 2: A risk network formed by 200 financial institutions with thresholding of coefficients (all links below the average value are set to 0); a circular representation of a weighted adjacency matrix, Depositories: a circle, Insurance: a square, Broker-Dealers: a triangle, Others: a diamond, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

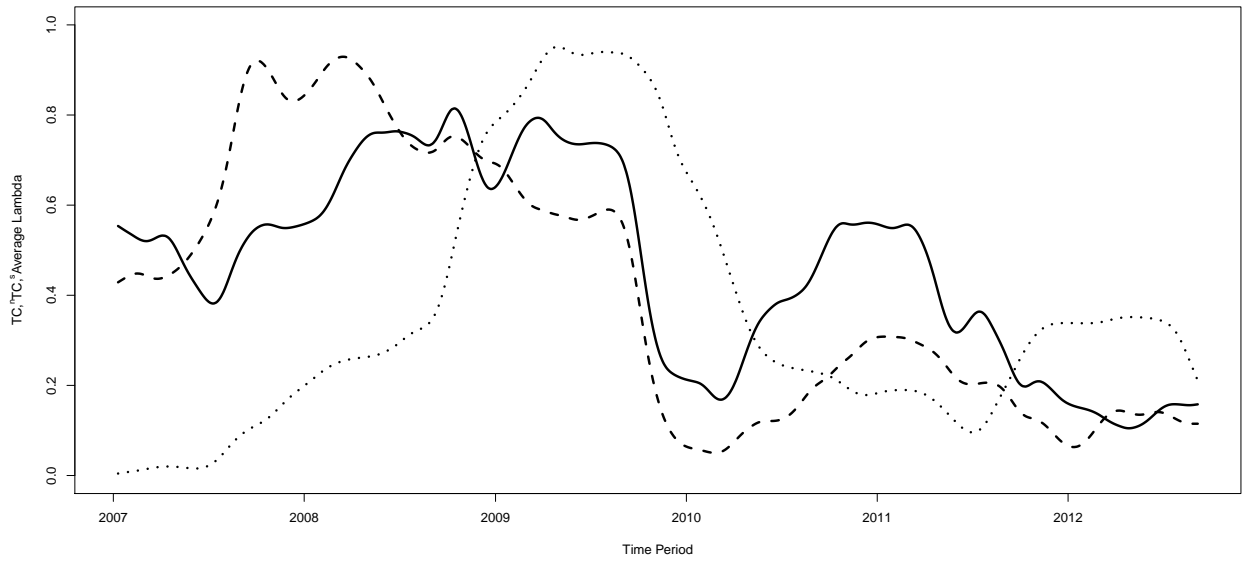


Figure 3: Total Connectedness standardized on $[0, 1]$ scale, TC^s : solid line, TC^s : dashed line, $\bar{\lambda}$: dotted line, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

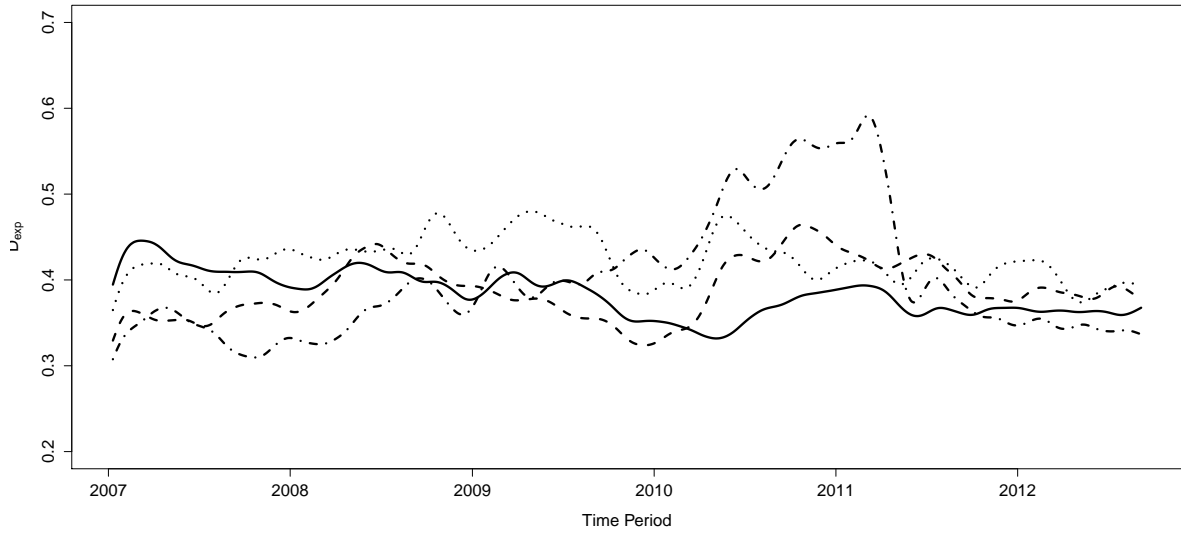


Figure 4: D_{exp}^m , Depositories: solid line, Insurance: dashed line, Broker-Dealers: dotted line, Others: dash-dot line, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

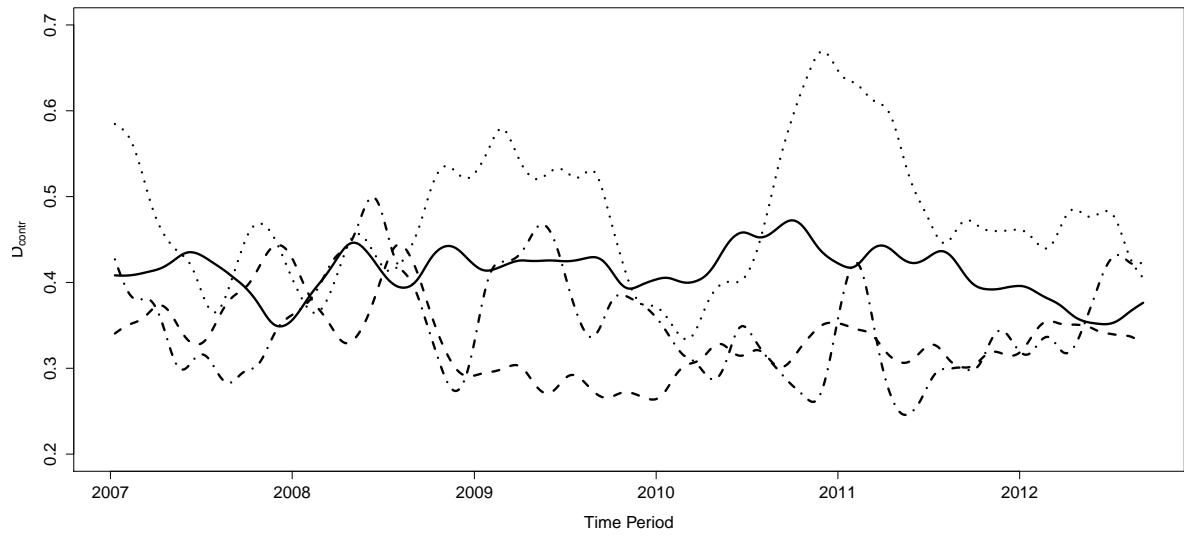


Figure 5: D_{contr}^m , Depositories: solid line, Insurance: dashed line, Broker-Dealers: dotted line, Others: dash-dot line, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

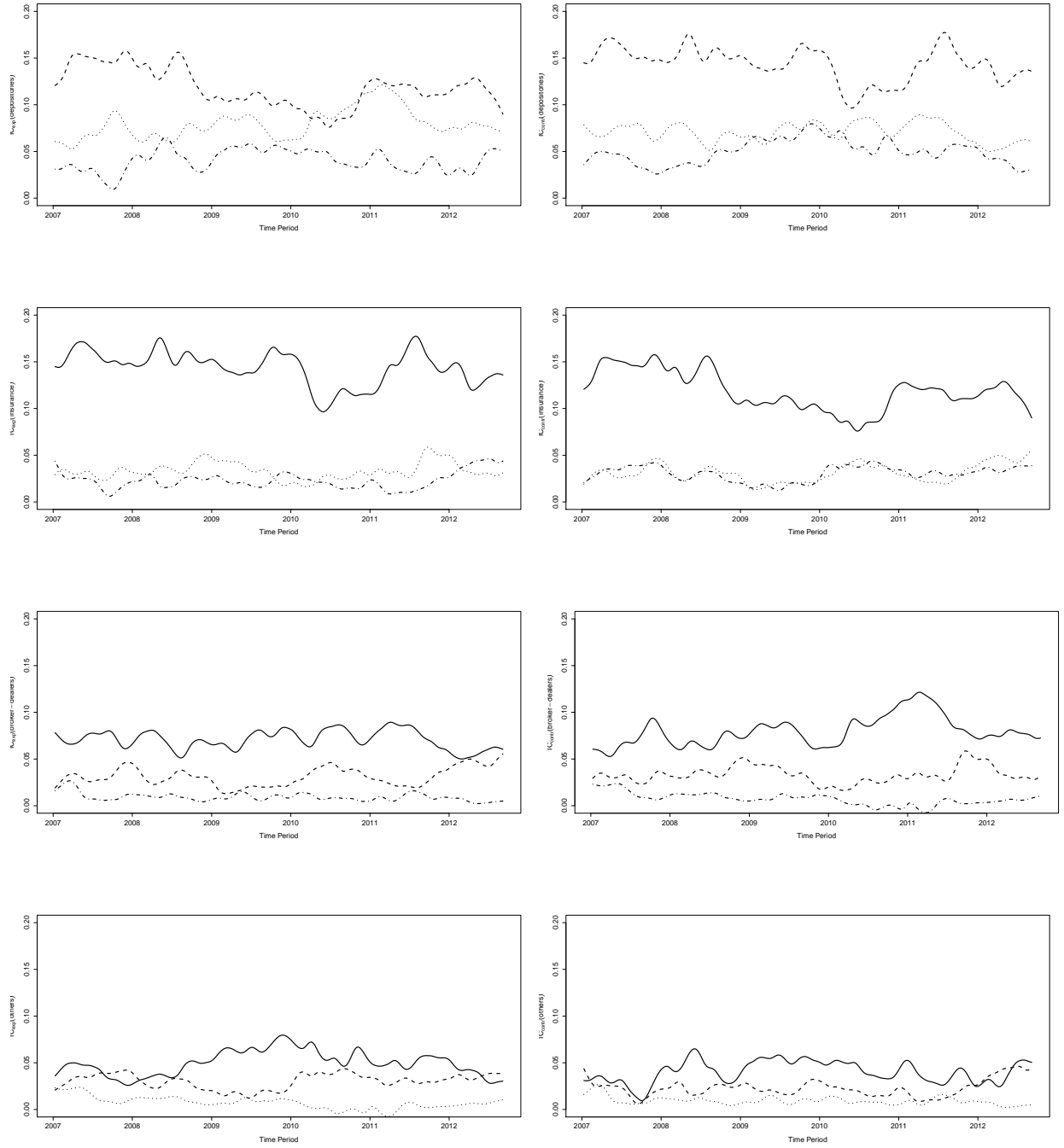


Figure 6: Connectivity between Sectors: IC_{exp} (left) and IC_{contr} (right); Depositories (row 1): solid line, Insurance (row 2): dashed line, Broker-Dealers (row 3): dotted line, Others (row 4): dash-dot line; $T = 1669$, $\tau = 0.05$, window size $n = 125$.

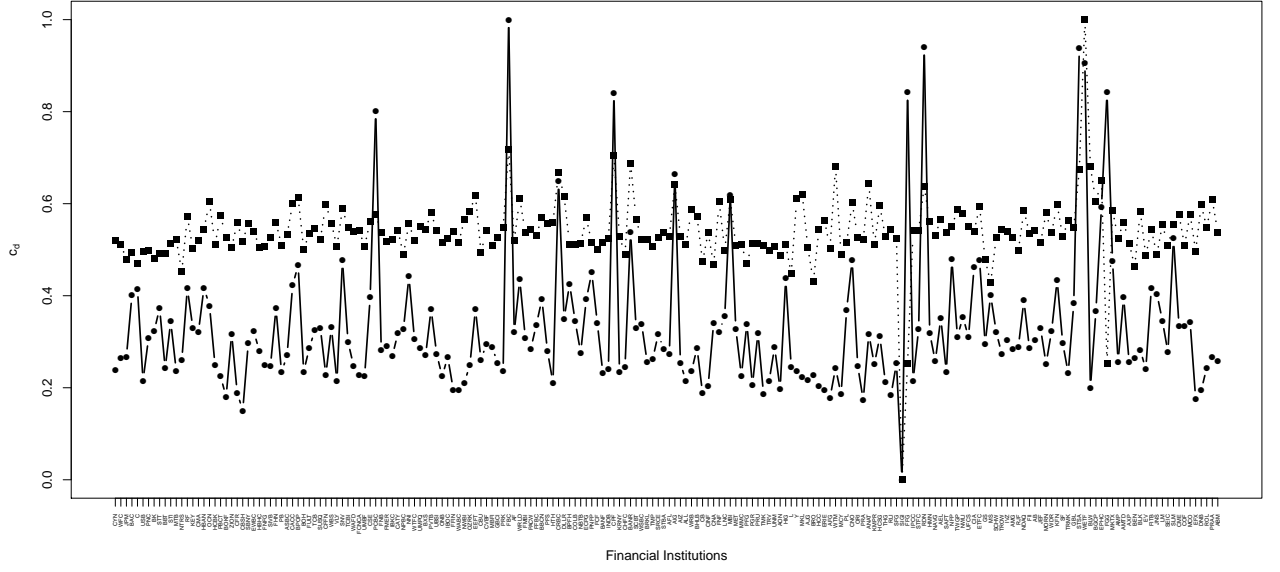


Figure 7: Indegree of 200 financial institutions, $c_{d-}^{m,k} = \sum_{i=1}^p |\hat{\beta}_{s_{m,k}|-i}|$ (sum over all windows): solid line, circles; $c_{d-}^{m,k} = \sum_{i=1}^p d_{s_{m,k}|-i}$ (sum over all windows): dotted line, squares; standardized on $[0, 1]$ scale, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

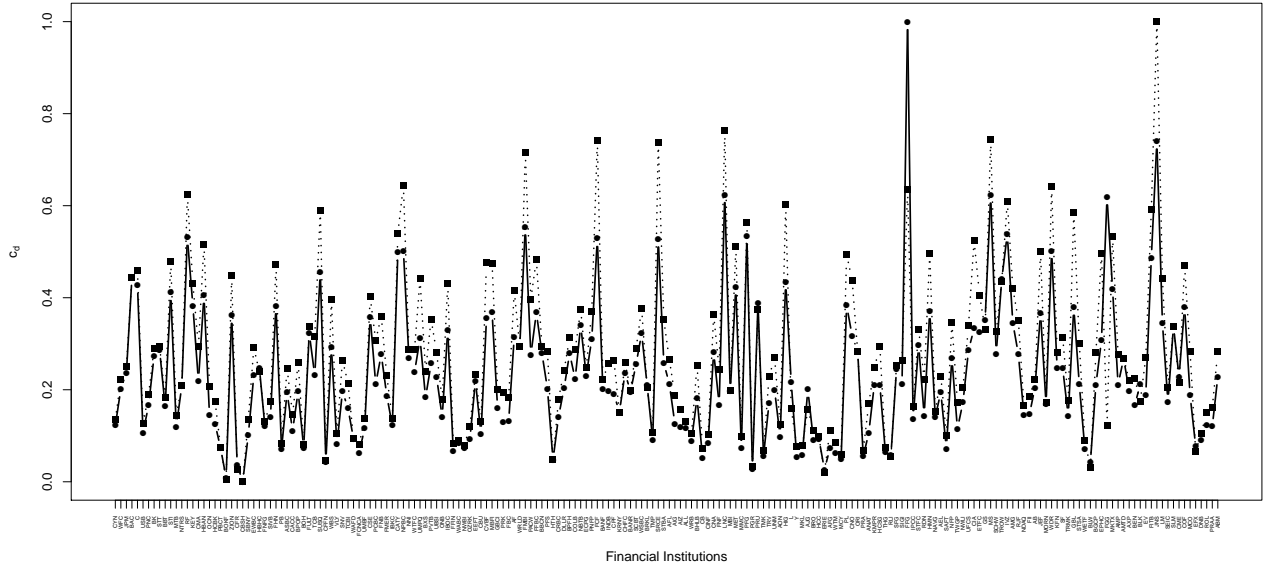


Figure 8: Outdegree of 200 financial institutions, $c_{d+}^{m,k} = \sum_{j=1}^p |\hat{\beta}_{-j|s_{m,k}}|$ (sum over all windows): solid line, circles; $c_{d+}^{m,k} = \sum_{j=1}^p d_{-j|s_{m,k}}$ (sum over all windows): dotted line, squares; standardized on $[0, 1]$ scale, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

| | Ticker | Ratio IN | Total Sum of IN Coefficients | Total Number of IN Links |
|-----|--------|----------|------------------------------|--------------------------|
| 180 | FGG | 0.99 | 1416.99 | 1417 |
| 144 | FFG | 0.99 | 1416.99 | 1417 |
| 147 | RDN | 0.28 | 1021.09 | 3562 |
| 159 | MS | 0.28 | 671.50 | 2390 |
| 87 | PNFP | 0.25 | 733.87 | 2875 |
| 122 | HIG | 0.25 | 728.37 | 2855 |
| 48 | PCBC | 0.24 | 786.25 | 3223 |
| 189 | JNS | 0.24 | 657.57 | 2730 |
| 192 | SLM | 0.23 | 733.99 | 3097 |
| 152 | NFP | 0.23 | 728.83 | 3076 |

Table 3: Top 10 financial institutions ranked according to the ratio of total sum of IN coefficients to total sum of number of IN links.

| | Ticker | Ratio IN | Total Sum of IN Coefficients | Total Number of IN Links |
|-----|--------|----------|------------------------------|--------------------------|
| 176 | WETF | 0.01 | 77.25 | 5584 |
| 72 | FBC | 0.18 | 726.54 | 4014 |
| 91 | CPF | 0.10 | 425.74 | 3935 |
| 94 | BANR | 0.09 | 349.72 | 3843 |
| 131 | WTM | 0.06 | 242.98 | 3801 |
| 177 | BLW | 0.04 | 187.27 | 3797 |
| 175 | STSA | 0.13 | 494.79 | 3763 |
| 81 | CRBC | 0.16 | 599.66 | 3727 |
| 179 | EPHC | 0.10 | 386.51 | 3628 |
| 137 | ANAT | 0.08 | 290.53 | 3590 |

Table 4: Top 10 financial institutions ranked according to the total sum of number of IN links.

| | Ticker | Ratio IN | Total Sum of IN Coefficients | Total Number of IN Links |
|-----|--------|----------|------------------------------|--------------------------|
| 180 | FGG | 0.99 | 1416.99 | 1417 |
| 144 | FFG | 0.99 | 1416.99 | 1417 |
| 147 | RDN | 0.28 | 1021.09 | 3562 |
| 48 | PCBC | 0.24 | 786.25 | 3223 |
| 192 | SLM | 0.23 | 733.99 | 3097 |
| 87 | PNFP | 0.25 | 733.87 | 2875 |
| 152 | NFP | 0.23 | 728.83 | 3076 |
| 122 | HIG | 0.25 | 728.37 | 2855 |
| 72 | FBC | 0.18 | 726.54 | 4014 |
| 102 | AIG | 0.19 | 687.36 | 3578 |

Table 5: Top 10 financial institutions ranked according to the total sum of IN coefficients.

| | Ticker | Ratio OUT | Total Sum of OUT Coefficients | Total Number of OUT Links |
|-----|--------|-----------|-------------------------------|---------------------------|
| 180 | FGG | 0.98 | 1421.37 | 1446 |
| 144 | FFG | 0.34 | 2278.29 | 6652 |
| 123 | L | 0.28 | 514.96 | 1815 |
| 126 | AJG | 0.26 | 479.85 | 1809 |
| 186 | BLK | 0.25 | 505.35 | 1988 |
| 177 | BLW | 0.23 | 122.82 | 526 |
| 196 | EFX | 0.23 | 202.69 | 872 |
| 23 | CFR | 0.22 | 110.98 | 483 |
| 158 | GS | 0.22 | 819.06 | 3567 |
| 193 | CME | 0.22 | 540.86 | 2389 |

Table 6: Top 10 financial institutions ranked according to the ratio of total sum of OUT coefficients to total sum of number of OUT links.

| | Ticker | Ratio OUT | Total Sum of OUT Coefficients | Total Number of OUT Links |
|-----|--------|-----------|-------------------------------|---------------------------|
| 189 | JNS | 0.16 | 1694.21 | 10351 |
| 111 | LNC | 0.18 | 1432.52 | 7945 |
| 159 | MS | 0.18 | 1429.95 | 7756 |
| 88 | FCF | 0.15 | 1217.92 | 7736 |
| 99 | SRCE | 0.15 | 1215.23 | 7690 |
| 75 | FMBI | 0.17 | 1274.22 | 7466 |
| 53 | NPBC | 0.17 | 1158.28 | 6744 |
| 170 | WDR | 0.17 | 1158.48 | 6708 |
| 144 | FFG | 0.34 | 2278.29 | 6652 |
| 14 | RF | 0.18 | 1223.46 | 6533 |

Table 7: Top 10 financial institutions classified according to total sum of number of OUT links.

| | Ticker | Ratio OUT | Total Sum of OUT Coefficients | Total Number of OUT Links |
|-----|--------|-----------|-------------------------------|---------------------------|
| 144 | FFG | 0.34 | 2278.29 | 6652 |
| 189 | JNS | 0.16 | 1694.21 | 10351 |
| 111 | LNC | 0.18 | 1432.52 | 7945 |
| 159 | MS | 0.18 | 1429.95 | 7756 |
| 180 | FGG | 0.98 | 1421.37 | 1446 |
| 75 | FMBI | 0.17 | 1274.22 | 7466 |
| 162 | IVZ | 0.19 | 1239.72 | 6396 |
| 115 | PFG | 0.20 | 1230.95 | 5917 |
| 14 | RF | 0.18 | 1223.46 | 6533 |
| 88 | FCF | 0.15 | 1217.92 | 7736 |

Table 8: Top 10 financial institutions classified according to total sum of OUT coefficients.

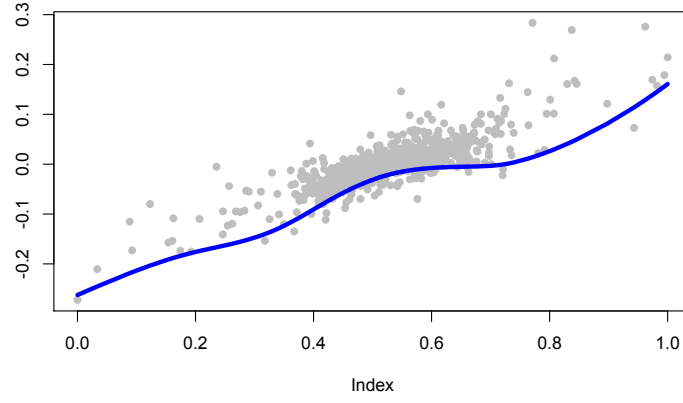


Figure 9: Average Link function vs Average Returns, September 1, 2008, Gaussian kernel, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

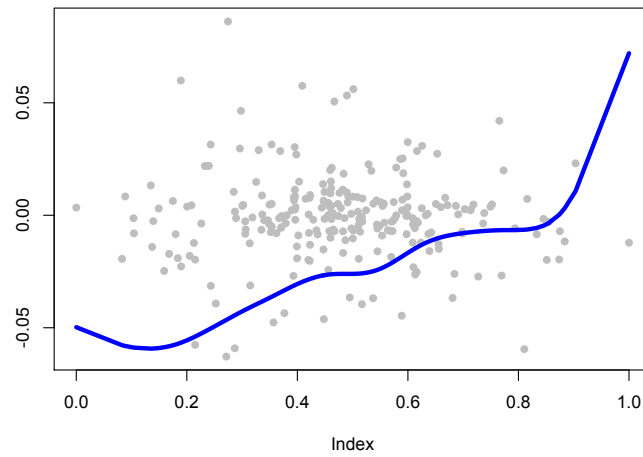


Figure 10: Average Link function vs Average Returns, June 1, 2010, Gaussian kernel, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

| | Ticker | Exposure | | Ticker | Contribution |
|-----|--------|----------|-----|--------|--------------|
| 180 | FGG | 21.65 | 144 | FFG | 33.01 |
| 144 | FFG | 21.65 | 189 | JNS | 24.83 |
| 147 | RDN | 15.40 | 180 | FGG | 21.72 |
| 48 | PCBC | 12.11 | 111 | LNC | 21.13 |
| 87 | PNFP | 11.25 | 75 | FMBI | 18.92 |
| 192 | SLM | 11.21 | 115 | PFG | 18.68 |
| 72 | FBC | 11.05 | 159 | MS | 18.16 |
| 122 | HIG | 11.03 | 14 | RF | 17.81 |
| 152 | NFP | 11.00 | 88 | FCF | 17.46 |
| 102 | AIG | 10.23 | 52 | CATY | 17.45 |
| 159 | MS | 10.13 | 53 | NPBC | 16.62 |
| 17 | HBAN | 10.12 | 99 | SRCE | 15.92 |
| 189 | JNS | 10.03 | 170 | WDR | 15.52 |
| 156 | CIA | 10.01 | 4 | BAC | 15.51 |
| 83 | BPFH | 9.97 | 188 | FITB | 15.49 |
| 4 | BAC | 9.94 | 38 | SUSQ | 15.45 |
| 14 | RF | 9.59 | 122 | HIG | 14.39 |
| 181 | MKTX | 9.51 | 113 | MET | 14.36 |
| 9 | STT | 9.34 | 5 | C | 14.06 |
| 5 | C | 9.30 | 181 | MKTX | 13.92 |
| 112 | MBI | 9.27 | 11 | STI | 13.13 |
| 188 | FITB | 9.24 | 117 | PRU | 13.08 |
| 133 | PL | 9.23 | 15 | KEY | 13.00 |
| 111 | LNC | 9.11 | 30 | FHN | 12.56 |
| 34 | BPOP | 9.09 | 69 | MBFI | 12.41 |

Table 9: Cumulative Contribution $\frac{\sum_{j \neq s_{m,k}} \widehat{\beta}_{j|s_{m,k}}}{\sum_{i=1}^p \sum_{j=1}^p \widehat{\beta}_{i|j}}$ (right) and Exposure Indices $\frac{\sum_{i \neq s_{m,k}} \widehat{\beta}_{s_{m,k}|i}}{\sum_{i=1}^p \sum_{j=1}^p \widehat{\beta}_{i|j}}$ (left), aggregated over windows, $T = 1669$, $\tau = 0.05$, window size $n = 125$, top 25 financial institutions.

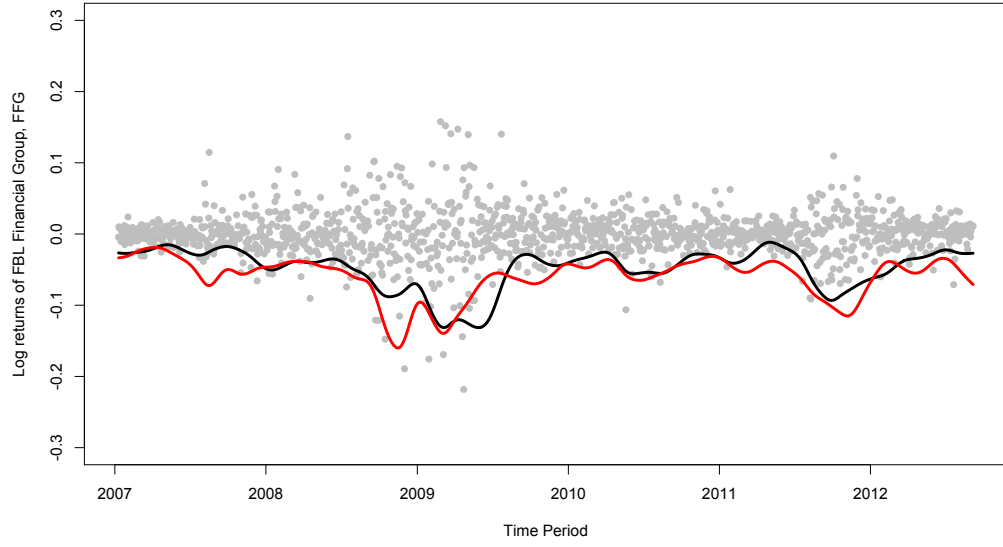


Figure 11: $\widehat{\text{VaR}}_{i,t}$ of FBL Financial Group (FFG) based on linear quantile regression on macroprudential variables (black) and on $\max\{C_{it}, E_{it}\}$ (red), $T = 1669$, $\tau = 0.05$, window size $n = 125$.

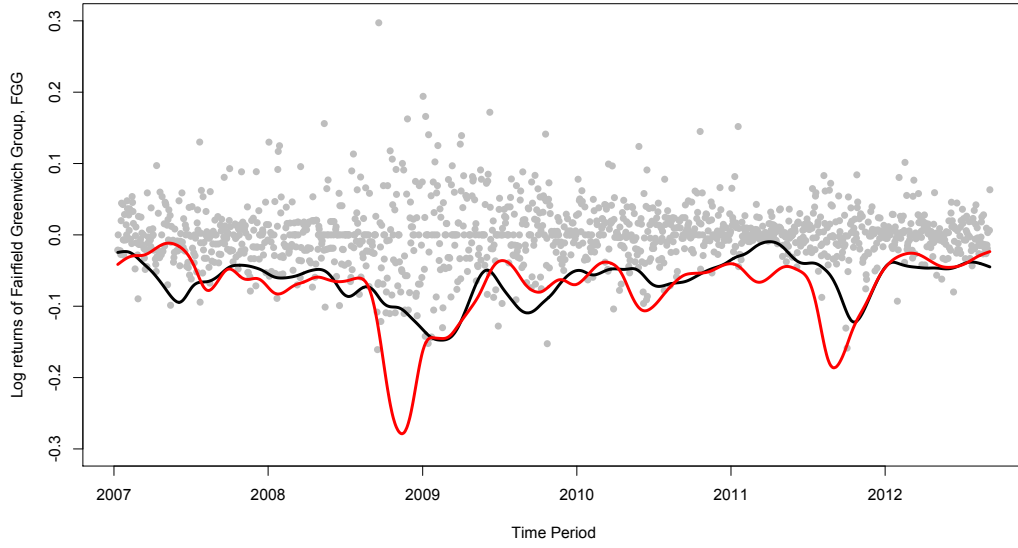


Figure 12: $\widehat{\text{VaR}}_{i,t}$ of Fairfield Greenwich Group (FGG) based on linear quantile regression on macroprudential variables (black) and on $\max\{C_{it}, E_{it}\}$ (red), $T = 1669$, $\tau = 0.05$, window size $n = 125$.

References

- Acharya, V., Engle, R., and Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulating systemic risks. *The American Economic Review*, 102(3):59–64
- Adrian, T. and Brunnermeier, M. K. (2011). CoVaR. Staff reports 348, Federal Reserve Bank of New York.
- Beale, N., Rand, D. G., Battey, H., Croxson, K., May, R. M., and Nowak, M. A. (2011). Individual versus systemic risk and the regulator’s dilemma. *Proceedings of the National Academy of Sciences*, 108(31):12647–12652
- Belloni, A. and Chernozhukov, V. (2011). L1-penalized quantile regression in high-dimensional sparse models. *The Annals of Statistics*, 39(1):82–130
- Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2012). A survey of systemic risk analytics. *Annu. Rev. Financ. Econ.*, 4(1):255–296
- Borisov, I. and Volodko, N. (2009). Exponential inequalities for the distributions of canonical u-and v-statistics of dependent observations. *Siberian Advances in Mathematics*, 19(1):1–12.
- Boss, M., Krenn, G., Pühr, C., and Summer, M. (2006). Systemic risk monitor: A model for systemic risk analysis and stress testing of banking systems. *Financial Stability Report*, 11:83–95.
- Brownlees, C. T. and Engle, R. F. (2012). Volatility, correlation and tails for systemic risk measurement.
- Chan-Lau, J., Espinosa, M., Giesecke, K., and Solé, J. (2009). Assessing the systemic implications of financial linkages. *IMF Global Financial Stability Report*, 2.
- Chao, S.-K., Härdle, W. K., and Wang, W. (2014). Quantile regression in risk calibration. In Lee, C.-F. and Lee, J. C., editors, *Handbook of Financial Econometric and Statistics*. Springer.
- Fan, J. and Li, R. (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. *Journal of the American Statistical Association*, 96(456):1348–1360
- Fan, Y., Härdle, W. K., Wang, W., and Zhu, L. (2014). CoVaR with very high-dimensional covariates. submitted.
- Franke, J., Mwita, P., and Wang, W. (2014). Nonparametric estimates for conditional quantiles of time series. *ASTA Advances in Statistical Analysis*, pages 1–24.

- Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of monetary economics*, 3(11):547–599.
- Giglio, S., Kelly, B., Pruitt, S., and Qiao, X. (2012). Systemic risk and the macroeconomy: An empirical evaluation. *Fama-Miller Working Paper*.
- Härdle, W. K., Müller, M., Sperlich, S., and Werwatz, A. (2004). *Nonparametric and semiparametric models*. Springer.
- Hautsch, N., Schaumburg, J., and Schienle, M. (2014). Financial network systemic risk contributions. *Review of Finance*.
- Huang, X., Zhou, H., and Zhu, H. (2009). A framework for assessing the systemic risk of major financial institutions. *Journal of Banking and Finance*, 33(11):2036–2049.
- Koenker, R., Ng, P., and Portnoy, S. (1994). Quantile smoothing splines. *Biometrika*, 81(4):673–680
- Kong, E., Linton, O., and Xia, Y. (2010). Uniform bahadur representation for local polynomial estimates of m-regression and its application to the additive model. *Econometric Theory*, 26(05):1529–1564.
- Lehar, A. (2005). Measuring systemic risk: A risk management approach. *Journal of Banking and Finance*, 29(10):2577–2603 0378–4266.
- Li, Y. and Zhu, J. (2008). L1-norm quantile regression. *Journal of Computational and Graphical Statistics*, 17(1).
- Minsky, H. P. (1977). A theory of systemic fragility. *Financial crises: Institutions and markets in a fragile environment*, pages 138–52.
- Portnoy, S. (1984). Asymptotic behavior of m-estimators of p regression parameters when p^2/n is large. i. consistency. *The Annals of Statistics*, pages 1298–1309.
- Rodriguez-Moreno, M. and Peña, J. I. (2013). Systemic risk measures: The simpler the better? *Journal of Banking and Finance*, 37(6):1817–1831
- Schwarz, G. (1978). Estimating the dimension of a model. *The annals of statistics*, 6(2):461–464
- Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, pages 267–288
- Wang, H., Li, R., and Tsai, C.-L. (2007). Tuning parameter selectors for the smoothly clipped absolute deviation method. *Biometrika*, 94(3):553–568

- Wu, T. Z., Yu, K., and Yu, Y. (2010). Single-index quantile regression. *Journal of Multivariate Analysis*, 101(7):1607–1621.
- Yu, K., Lu, Z., and Stander, J. (2003). Quantile regression: applications and current research areas. *Journal of the Royal Statistical Society: Series D (The Statistician)*, 52(3):331–350
- Zheng, Q., Gallagher, C., and Kulasekera, K. (2013). Adaptive penalized quantile regression for high dimensional data. *Journal of Statistical Planning and Inference*, 143(6):1029–1038
- Zou, H. (2006). The adaptive lasso and its oracle properties. *Journal of the American statistical association*, 101(476):1418–1429

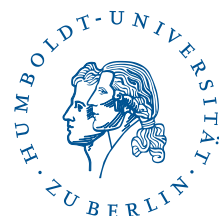
SFB 649 Discussion Paper Series 2014

For a complete list of Discussion Papers published by the SFB 649, please visit <http://sfb649.wiwi.hu-berlin.de>.

- 001 "Principal Component Analysis in an Asymmetric Norm" by Ngoc Mai Tran, Maria Osipenko and Wolfgang Karl Härdle, January 2014.
- 002 "A Simultaneous Confidence Corridor for Varying Coefficient Regression with Sparse Functional Data" by Lijie Gu, Li Wang, Wolfgang Karl Härdle and Lijian Yang, January 2014.
- 003 "An Extended Single Index Model with Missing Response at Random" by Qihua Wang, Tao Zhang, Wolfgang Karl Härdle, January 2014.
- 004 "Structural Vector Autoregressive Analysis in a Data Rich Environment: A Survey" by Helmut Lütkepohl, January 2014.
- 005 "Functional stable limit theorems for efficient spectral cointegration estimators" by Randolph Altmeyer and Markus Bibinger, January 2014.
- 006 "A consistent two-factor model for pricing temperature derivatives" by Andreas Groll, Brenda López-Cabrera and Thilo Meyer-Brandis, January 2014.
- 007 "Confidence Bands for Impulse Responses: Bonferroni versus Wald" by Helmut Lütkepohl, Anna Staszewska-Bystrova and Peter Winker, January 2014.
- 008 "Simultaneous Confidence Corridors and Variable Selection for Generalized Additive Models" by Shuzhuan Zheng, Rong Liu, Lijian Yang and Wolfgang Karl Härdle, January 2014.
- 009 "Structural Vector Autoregressions: Checking Identifying Long-run Restrictions via Heteroskedasticity" by Helmut Lütkepohl and Anton Velinov, January 2014.
- 010 "Efficient Iterative Maximum Likelihood Estimation of High-Parameterized Time Series Models" by Nikolaus Hautsch, Ostap Okhrin and Alexander Ristig, January 2014.
- 011 "Fiscal Devaluation in a Monetary Union" by Philipp Engler, Giovanni Ganelli, Juha Tervala and Simon Voigts, January 2014.
- 012 "Nonparametric Estimates for Conditional Quantiles of Time Series" by Jürgen Franke, Peter Mwita and Weining Wang, January 2014.
- 013 "Product Market Deregulation and Employment Outcomes: Evidence from the German Retail Sector" by Charlotte Senftleben-König, January 2014.
- 014 "Estimation procedures for exchangeable Marshall copulas with hydrological application" by Fabrizio Durante and Ostap Okhrin, January 2014.
- 015 "Ladislaus von Bortkiewicz - statistician, economist, and a European intellectual" by Wolfgang Karl Härdle and Annette B. Vogt, February 2014.
- 016 "An Application of Principal Component Analysis on Multivariate Time-Stationary Spatio-Temporal Data" by Stephan Stahlschmidt, Wolfgang Karl Härdle and Helmut Thome, February 2014.
- 017 "The composition of government spending and the multiplier at the Zero Lower Bound" by Julien Albertini, Arthur Poirier and Jordan Roulleau-Pasdeloup, February 2014.
- 018 "Interacting Product and Labor Market Regulation and the Impact of Immigration on Native Wages" by Susanne Prantl and Alexandra Spitz-Oener, February 2014.

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".



SFB 649 Discussion Paper Series 2014

For a complete list of Discussion Papers published by the SFB 649, please visit <http://sfb649.wiwi.hu-berlin.de>.

- 019 "Unemployment benefits extensions at the zero lower bound on nominal interest rate" by Julien Albertini and Arthur Poirier, February 2014.
- 020 "Modelling spatio-temporal variability of temperature" by Xiaofeng Cao, Ostap Okhrin, Martin Odening and Matthias Ritter, February 2014.
- 021 "Do Maternal Health Problems Influence Child's Worrying Status? Evidence from British Cohort Study" by Xianhua Dai, Wolfgang Karl Härdle and Keming Yu, February 2014.
- 022 "Nonparametric Test for a Constant Beta over a Fixed Time Interval" by Markus Reiß, Viktor Todorov and George Tauchen, February 2014.
- 023 "Inflation Expectations Spillovers between the United States and Euro Area" by Aleksei Netšunajev and Lars Winkelmann, March 2014.
- 024 "Peer Effects and Students' Self-Control" by Berno Buechel, Lydia Mechtenberg and Julia Petersen, April 2014.
- 025 "Is there a demand for multi-year crop insurance?" by Maria Osipenko, Zhiwei Shen and Martin Odening, April 2014.
- 026 "Credit Risk Calibration based on CDS Spreads" by Shih-Kang Chao, Wolfgang Karl Härdle and Hien Pham-Thu, May 2014.
- 027 "Stale Forward Guidance" by Gunda-Alexandra Detmers and Dieter Nautz, May 2014.
- 028 "Confidence Corridors for Multivariate Generalized Quantile Regression" by Shih-Kang Chao, Katharina Proksch, Holger Dette and Wolfgang Härdle, May 2014.
- 029 "Information Risk, Market Stress and Institutional Herding in Financial Markets: New Evidence Through the Lens of a Simulated Model" by Christopher Boortz, Stephanie Kremer, Simon Jurkatis and Dieter Nautz, May 2014.
- 030 "Forecasting Generalized Quantiles of Electricity Demand: A Functional Data Approach" by Brenda López Cabrera and Franziska Schulz, May 2014.
- 031 "Structural Vector Autoregressions with Smooth Transition in Variances – The Interaction Between U.S. Monetary Policy and the Stock Market" by Helmut Lutkepohl and Aleksei Netsunajev, June 2014.
- 032 "TEDAS - Tail Event Driven ASset Allocation" by Wolfgang Karl Härdle, Sergey Nasekin, David Lee Kuo Chuen and Phoon Kok Fai, June 2014.
- 033 "Discount Factor Shocks and Labor Market Dynamics" by Julien Albertini and Arthur Poirier, June 2014.
- 034 "Risky Linear Approximations" by Alexander Meyer-Gohde, July 2014
- 035 "Adaptive Order Flow Forecasting with Multiplicative Error Models" by Wolfgang Karl Härdle, Andrija Mihoci and Christopher Hian-Ann Ting, July 2014
- 036 "Portfolio Decisions and Brain Reactions via the CEAD method" by Piotr Majer, Peter N.C. Mohr, Hauke R. Heekeren and Wolfgang K. Härdle, July 2014
- 037 "Common price and volatility jumps in noisy high-frequency data" by Markus Bibinger and Lars Winkelmann, July 2014
- 038 "Spatial Wage Inequality and Technological Change" by Charlotte Senftleben-König and Hanna Wielandt, August 2014
- 039 "The integration of credit default swap markets in the pre and post-subprime crisis in common stochastic trends" by Cathy Yi-Hsuan Chen, Wolfgang Karl Härdle, Hien Pham-Thu, August 2014

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".



SFB 649 Discussion Paper Series 2014

For a complete list of Discussion Papers published by the SFB 649, please visit <http://sfb649.wiwi.hu-berlin.de>.

- 040 "Localising Forward Intensities for Multiperiod Corporate Default" by Dedy Dwi Prastyo and Wolfgang Karl Härdle, August 2014.
- 041 "Certification and Market Transparency" by Konrad Stahl and Roland Strausz, September 2014.
- 042 "Beyond dimension two: A test for higher-order tail risk" by Carsten Bormann, Melanie Schienle and Julia Schaumburg, September 2014.
- 043 "Semiparametric Estimation with Generated Covariates" by Enno Mammen, Christoph Rothe and Melanie Schienle, September 2014.
- 044 "On the Timing of Climate Agreements" by Robert C. Schmidt and Roland Strausz, September 2014.
- 045 "Optimal Sales Contracts with Withdrawal Rights" by Daniel Krähmer and Roland Strausz, September 2014.
- 046 "Ex post information rents in sequential screening" by Daniel Krähmer and Roland Strausz, September 2014.
- 047 "Similarities and Differences between U.S. and German Regulation of the Use of Derivatives and Leverage by Mutual Funds – What Can Regulators Learn from Each Other?" by Dominika Paula Gałkiewicz, September 2014.
- 048 "That's how we roll: an experiment on rollover risk" by Ciril Bosch-Rosa, September 2014.
- 049 "Comparing Solution Methods for DSGE Models with Labor Market Search" by Hong Lan, September 2014.
- 050 "Volatility Modelling of CO2 Emission Allowance Spot Prices with Regime-Switching GARCH Models" by Thijs Benschop, Brenda López Cabrera, September 2014.
- 051 "Corporate Cash Hoarding in a Model with Liquidity Constraints" by Falk Mazelis, September 2014.
- 052 "Designing an Index for Assessing Wind Energy Potential" by Matthias Ritter, Zhiwei Shen, Brenda López Cabrera, Martin Odening, Lars Deckert, September 2014.
- 053 "Improved Volatility Estimation Based On Limit Order Books" by Markus Bibinger, Moritz Jirak, Markus Reiss, September 2014.
- 054 "Strategic Complementarities and Nominal Rigidities" by Philipp König, Alexander Meyer-Gohde, October 2014.
- 055 "Estimating the Spot Covariation of Asset Prices – Statistical Theory and Empirical Evidence" by Markus Bibinger, Markus Reiss, Nikolaus Hautsch, Peter Malec, October 2014.
- 056 "Monetary Policy Effects on Financial Intermediation via the Regulated and the Shadow Banking Systems" by Falk Mazelis, October 2014.
- 057 "A Tale of Two Tails: Preferences of neutral third-parties in three-player ultimatum games" by Ciril Bosch-Rosa, October 2014.
- 058 "Boiling the frog optimally: an experiment on survivor curve shapes and internet revenue" by Christina Aperjis, Ciril Bosch-Rosa, Daniel Friedman, Bernardo A. Huberman, October 2014.
- 059 "Expectile Treatment Effects: An efficient alternative to compute the distribution of treatment effects" by Stephan Stahlschmidt, Matthias Eckardt, Wolfgang K. Härdle, October 2014.
- 060 "Are US Inflation Expectations Re-Anchored?" by Dieter Nautz, Till Strohsal, October 2014.

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".



SFB 649 Discussion Paper Series 2014

For a complete list of Discussion Papers published by the SFB 649, please visit <http://sfb649.wiwi.hu-berlin.de>.

- 061 "Why the split of payroll taxation between firms and workers matters for macroeconomic stability" by Simon Voigts, October 2014.
- 062 "Do Tax Cuts Increase Consumption? An Experimental Test of Ricardian Equivalence" by Thomas Meissner, Davud Rostam-Afschar, October 2014.
- 063 "The Influence of Oil Price Shocks on China's Macro-economy : A Perspective of International Trade" by Shiyi Chen, Dengke Chen, Wolfgang K. Härdle, October 2014.
- 064 "Whom are you talking with? An experiment on credibility and communication structure" by Gilles Grandjean, Marco Mantovani, Ana Mauleon, Vincent Vannetelbosch, October 2014.
- 065 "A Theory of Price Adjustment under Loss Aversion" by Steffen Ahrens, Inske Pirschel, Dennis J. Snower, November 2014.
- 066 "TENET: Tail-Event driven NETwork risk" by Wolfgang Karl Härdle, Natalia Sirotko-Sibirskaya, Weining Wang, November 2014.

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

