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A Theory of Price Adjustment under Loss Aversion

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A Theory of Price Adjustment under Loss Aversion

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Abstract

We present a new partial equilibrium theory of price adjustment, based on consumer loss aversion. In line with prospect theory, the consumers’ perceived utility losses from price increases are weighted more heavily than the perceived utility gains from price decreases of equal magnitude. Price changes are evaluated relative to an endogenous reference price, which depends on the consumers’ rational price expectations from the recent past. By implication, demand responses are more elastic for price increases than for price decreases and thus firms face a downward-sloping demand curve that is kinked at the consumers’ reference price. Firms adjust their prices flexibly in response to variations in this demand curve, in the context of an otherwise standard dynamic neoclassical model of monopolistic competition. The resulting theory of price adjustment is starkly at variance with past theories. We find that - in line with the empirical evidence - prices are more sluggish upwards than downwards in response to temporary demand shocks, while they are more sluggish downwards than upwards in response to permanent demand shocks.

JEL classification: D03, D21, E31, E50.
Keywords: price sluggishness, loss aversion, state-dependent pricing

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1 Introduction

This paper presents a theory of price sluggishness based on consumer loss aversion, along the lines of prospect theory (Kahneman and Tversky, 1979). The theory has distinctive implications, which are starkly at variance with major existing theories of price adjustment. In particular, the theory implies that prices are more sluggish upwards than downwards in response to temporary demand shocks, while they are more sluggish downwards than upwards in response to permanent demand shocks.

These implications turn out to be consonant with recent empirical evidence. Though this evidence has not thus far attracted much explicit attention, it is clearly implicit in a range of influential empirical results. For instance, Hall et al. (2000) document that firms mostly accommodate negative temporary demand shifts by temporary price cuts, yet they are reluctant to temporarily increase their prices in response to positive temporary demand shifts. Furthermore, the empirical evidence provided by Kehoe and Midrigan (2008) indicates that temporary price reductions are - on average - larger and much more frequent than temporary price increases, implying that prices are relatively downward responsive.


While current theories of price adjustment (e.g. Taylor, 1979; Rotemberg, 1982; Calvo, 1983; among many others) fail to account for these empirical regularities, this paper offers a possible theoretical rationale.

The basic idea underlying our theory is simple. Price increases are associated with utility losses for consumers, whereas price decreases are associated with utility gains. In the spirit of prospect theory, losses are weighted more heavily than gains of equal magnitude. Consequently, demand responses are more elastic to price increases than to price decreases. The result is a kinked demand curve, for which the kink depends on the consumers’ reference price. In the spirit of Kőszegi and Rabin (2006), we model the reference price as the consumers’ rational price expectations. We assume that consumers know, with a one period lag, whether any given demand shock is temporary or permanent. Permanent shocks induce changes in the consumers’ rational price expectations and thereby in their reference price, while temporary shocks do not.

Given the demand shock is temporary, the kink of the demand curve implies that sufficiently small shocks do not affect the firm’s price. This is the case of price rigidity. For larger shocks, the firm’s price will respond temporarily, but the size of the response will be asymmetric for positive and negative shifts of equal magnitude. Since negative shocks move the firm along the relatively steep portion of the demand curve, prices decline stronger to negative shocks than they increase to equiproportionate positive shocks.

By contrast, given the demand shock is permanent, the firm can foresee not only the change in demand following its immediate pricing decision, but also the resulting change in the consumers’ reference price. A rise in the reference price raises the firms’ long-run profits (since the reference price is located at the kink of the demand curve),

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1 Modeling price sluggishness by means of a kinked demand curve is of course a well-trodden path. Sweezy (1939) and Hall and Hitch (1939) modeled price rigidity in an oligopolistic framework along these lines. In these models, oligopolistic firms do not change their prices flexibly because of their expected asymmetric competitor’s reactions to their pricing decisions. A game theoretic foundation of such model is presented by Maskin and Tirole (1988).
whereas a fall in the reference price lowers long-run profits, a phenomenon which we term the reference-price updating effect. On this account, firms are averse to initiating permanent price reductions. By implication, prices are more sluggish downwards than upwards for permanent demand shocks.

These results are extremely important for the conduct of monetary policy, since they imply that the sign of the asymmetry of price adjustment depends on the persistence of the underlying demand shock. In particular, if temporary demand shocks are interpreted as non-persistent and permanent demand shocks as fully persistent, our analysis implies that there exists a balance point (i.e., an intermediate degree of persistence of the shock) at which the asymmetry reverses. For shocks less persistent than the balance point prices are more sluggish upwards than downwards, while they are more sluggish downwards than upwards for more persistent shocks. Whether the degree of persistence at the balance point is relatively high or low depends on the adjustment speed of the reference price and on the firm’s discount factor. An increase in the adjustment speed of the reference price, as well as in the firm’s discount factor, strengthens the role of the reference-price updating effect, increasing upward flexibility and downward sluggishness at any given positive persistence of the shock. Therefore, the balance point will be associated with a lower level of persistence. To the best of our knowledge, there is no other paper studying the ramifications of the persistence of the demand shock for asymmetric price adjustment.

The paper is structured as follows. Section 2 reviews the relevant literature. Section 3 presents our general model setup and in Section 4 we analyze the effects of various demand shocks on prices, both analytically and numerically. Section 5 concludes.

2 Relation to the Literature

We now consider the empirical evidence suggesting that prices respond imperfectly and asymmetrically to exogenous positive and negative shocks of equal magnitude, and that the implied asymmetry depends on whether the shock is permanent or temporary.

There is much empirical evidence for the proposition that, with regard to permanent demand shocks, prices are generally more responsive to positive shocks than to negative ones. For example, in the context of monetary policy shocks, Kandil (1996, 2002b), Kandil (1995), and Weise (1999) find support for the United States over a large range of different samples. Moreover, Kandil (1995) and Karras and Stokes (1999) supply evidence for large panels of industrialized OECD countries, while Karras (1996) provides evidence for developing countries. In the case of the United States, Kandil (2001, 2002a) shows that the asymmetry also prevails in response to permanent government spending shocks. Kandil (1999, 2006, 2010), on the other hand, looks directly at permanent aggregate demand shocks and also confirms the asymmetry for a large set of industrialized countries as well as for a sample of disaggregated industries in the United States. Comparing a large set of industrialized and developing countries, Kandil (1998) finds that the asymmetry is even stronger for many developing countries compared to industrialized ones.

In addition to the asymmetric price reaction in response to permanent demand shocks, the above studies also find an asymmetric reaction of output. They show that output responds significantly less to permanent positive demand shocks relative to negative ones. This asymmetry—which is also predicted by our model (as shown below)—is further documented by a large body of empirical literature that explicitly focuses on output. For example, DeLong and Summers (1988), Cover (1992), Thoma (1994), and
Ravn and Sola (2004) show for the United States that positive changes in the rate of money growth induce much weaker output reductions than negative changes in the rate of money supply. Morgan (1993) and Ravn and Sola (2004) confirm this asymmetry, when monetary policy is conducted via changes in the federal funds rate. Additional evidence is provided by Tan et al. (2010) for Indonesia, Malaysia, the Philippines, and Thailand and by Mehrara and Karsalari (2011) for Iran.

There is also significant empirical evidence for the proposition that, with regard to temporary demand shocks, prices are generally less responsive to positive shocks than to negative ones. For example, the survey by Hall et al. (2000) indicates that firms regard price increases as response to temporary increases in demand to be among the least favorable options. Instead, firms rather employ more workers, extend overtime work, or increase capacities. By contrast, managers of firms state that a temporary fall in demand is much more likely to lead to a price cut. Further evidence for the asymmetry in response to temporary demand shocks is provided by Kehoe and Midrigan (2008), who analyze temporary price movements at Dominick’s Finer Foods retail chain with weekly store-level data from 86 stores in the Chicago area. They find that temporary price reductions are much more frequent than temporary price increases and that, on average, temporary price cuts are larger (by a factor of almost two) than temporary price increases. However, neither of these studies empirically analyzes the asymmetry characteristics of the output reaction in the face of temporary demand shocks.

Despite this broad evidence, asymmetric reactions to demand shocks have been unexplored by current theories of price adjustment. Neither time-dependent pricing models (Taylor, 1979; Calvo, 1983), nor state-dependent adjustment cost models of (S,s) type (e.g., Sheshinski and Weiss, 1977; Rotemberg, 1982; Caplin and Spulber, 1987; Caballero and Engel, 1993, 2007; Golosov and Lucas, 2007; Gertler and Leahy, 2008; Dotsey et al., 2009; Midrigan, 2011) are able to account for the asymmetry properties in price dynamics in response to positive and negative exogenous temporary and permanent shifts in demand.\(^2\)

In this paper, we offer a new theory of firm price setting resting on consumer loss aversion in an otherwise standard model of monopolistic competition. The resulting theory provides a novel rationale for the above empirical evidence on asymmetric price sluggishness. Although there is no hard evidence for a direct link from consumer loss aversion to price sluggishness, to the best of our knowledge, there is ample evidence that firms do not adjust their prices flexibly in order to avoid harming their customer relationships (see, e.g., Fabiani et al. (2006) for a survey of euro area countries, Blinder et al. (1998) for the United States\(^3\), and Hall et al. (2000) for the United Kingdom).\(^4\)

Furthermore, there is extensive empirical evidence that customers are indeed loss adverse in prices. Kalwani et al. (1990), Mayhew and Winer (1992), Krishnamurthi et al. (1992), Putler (1992), Hardie et al. (1993), Kalyanaram and Little (1994), Raman and

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2Once trend inflation is considered, menu costs can generally explain that prices are more downward sluggish than upwards (Ball and Mankiw, 1994). By contrast, our model does not rely on the assumption of trend inflation.

3In their survey, Blinder et al. (1998) additionally find clear evidence that the pricing of those firms for which the fear of antagonizing their customers through price changes plays an important role is relatively upward sluggish. Unfortunately, the authors do no distinguish between temporary and permanent shifts in demand in their survey questions.

Bass (2002), Dossche et al. (2010), and many others find evidence for consumer loss aversion with respect to many different product categories available in supermarkets. Furthermore, loss aversion in prices is also well documented in diverse activities such as restaurant visits (Morgan, 2008), vacation trips (Nicolau, 2008), real estate trade (Genesove and Mayer, 2001), phone calls (Bidwell et al., 1995), and energy use (Griffin and Schulman, 2005; Adeyemi and Hunt, 2007; Ryan and Plourde, 2007).

In our model, loss-averse consumers evaluate prices relative to a reference price. K˝oszegi and Rabin (2006, 2007, 2009) and Heidhues and K˝oszegi (2005, 2008, 2014) argue that reference points are determined by agents’ rational expectations about outcomes from the recent past. There is much empirical evidence suggesting that reference points are determined by expectations, in concrete situations such as in police performance after final offer arbitration (Mas, 2006), in the United States TV show “Deal or no Deal” (Post et al., 2008), with respect to domestic violence (Card and Dahl, 2011), in cab drivers’ labor supply decisions (Crawford and Meng, 2011), or in the effort choices of professional golf players (Pope and Schweitzer, 2011). In the context of laboratory experiments, Knetsch and Wong (2009) and Marzilli Ericson and Fuster (2011) find supporting evidence from exchange experiments and Abeler et al. (2011) do so through an effort provision experiment. Endogenizing consumers’ reference prices in this way allows our model to capture that current price changes influence the consumers’ future reference price and thereby affect the demand functions via what we call the “reference-price updating effect.” This effect rests on the observation that firms tend to increase the demand for their product by raising their consumers’ reference price through, for example, setting a “suggested retail price” that is higher than the price actually charged (Thaler, 1985; Putler, 1992). These pieces of evidence are consistent with the assumptions underlying our analysis. Our analysis works out the implications of these assumptions for state-dependent price sluggishness in the form of asymmetric price adjustment for temporary and permanent demand shocks.

There are only a few other papers that study the implications of consumer loss aversion on firms’ pricing decisions. In an early account of price rigidity in response to demand and cost shocks has been presented by Sibly (2002, 2007). In a static environment, Sibly (2002, 2007) shows that a monopolist may not change prices if she faces loss averse consumers with fixed, exogenously given reference prices. In their particularly insightful contributions, Heidhues and K˝oszegi (2008) and Spiegler (2012) analyze static monopolistic pricing decisions to cost and demand shocks under the assumption that the reference price is determined as a consumer’s recent rational expectations personal equilibrium in the spirit of K˝oszegi and Rabin (2006). Spiegler (2012) shows that incentives for price rigidity are even stronger for demand shocks compared to cost shocks. We follow Heidhues and K˝oszegi (2008) and Spiegler (2012) and assume endogenous rational expectations reference price formation, but, by contrast, consider a dynamic approach to the pricing decision of a monopolistically competitive firm facing loss averse consumers. Our dynamic approach confirms earlier findings that consumer loss aversion engenders price rigidity and allows us to study the asymmetry characteristics of pricing reactions to temporary and permanent demand shocks of different sign. Another study close to ours is Popescu and Wu (2007); although they analyze optimal pricing strategies in repeated market interactions with loss averse consumers and endogenous reference prices, they do not analyze the model’s reaction to demand shocks.

Finally, this paper offers a new microfounded rationale for state-dependent pricing. The importance of state-dependence for firms’ pricing decisions is well documented. For instance, in the countries of the euro area (Fabiani et al., 2006; Nicolitsas, 2013),
Scandinavia (Apel et al., 2005; Langbraaten et al., 2008; Ólafsson et al., 2011), the
United States (Blinder et al., 1998), and Turkey (Şahinoğlu and Saraçoğlu, 2008), ap-
proximately two third of the firms’ pricing decisions are indeed driven by the current
state of the environment.\(^5\) Menu costs, by contrast, are clearly rejected as a significant
driver for deferred price adjustments in each of the empirical studies above.

3 Model

We incorporate reference-dependent preferences and loss aversion into an otherwise
standard model of monopolistic competition. Consumers are price takers and loss averse with respect to prices. They evaluate prices relative to their reference prices, which depend on their rational price expectations. Prices higher than the reference price are associated with utility losses, while prices lower than the reference price are associated with utility gains. Losses are weighted more heavily than gains of equal magnitude. Firms are monopolistic competitors, supplying non-durable differentiated goods. Firms can change their prices freely in each period to maximize their profits.

3.1 Consumers

We follow Sibly (2007) and assume that the representative consumer’s period-utility
\(U_t\) depends positively on the consumption of \(n\) imperfectly substitutable nondurable goods \(q_{i,t}\) with \(i \in \{1, \ldots, n\}\) and negatively on the "loss-aversion ratio" \((p_{i,t}/r_{i,t})\), i.e. the ratio of the price \(p_{i,t}\) of good \(i\) to the consumer’s respective reference price \(r_{i,t}\) of the good. The loss-aversion ratio, which describes how the phenomenon of loss aversion enters the utility function, may be rationalized in terms of (i) Thaler’s transaction utility (whereby the total utility that the consumer derives from a good is in part determined by how the consumer evaluates the quality of the financial terms of the acquisition of the good (Thaler, 1991)), (ii) Okun’s implicit firm-customer contracts (whereby firms and customers implicitly agree on fair and stable prices despite fluctuations in demand (Okun, 1981)), or (iii) Rotemberg’s customer anger or regret (Rotemberg 2005, 2010).

Further approaches that describe reference-dependence in the consumer’s utility function in terms of a ratio of actual prices to references prices are McDonald and Sibly (2001, 2005) in the context of loss aversion with respect to wages and Sibly (1996, 2002) in the context of loss aversion with respect to prices and quality.\(^6\)

The consumer’s preferences in period \(t\) are represented by the following utility function:

\[
U_t(q_{1,t}, \ldots, q_{n,t}) = \left\lfloor \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu} q_{i,t} \right\rfloor^\rho, \tag{1}
\]

where \(0 < \rho < 1\) denotes the degree of substitutability between the different goods. The parameter \(\mu\) is an indicator function of the form

\[
\mu = \begin{cases} 
\Gamma & \text{for } p_{i,t} < r_{i,t}, \text{ i.e. gain domain} \\
\Delta & \text{for } p_{i,t} > r_{i,t}, \text{ i.e. loss domain}
\end{cases}, \tag{2}
\]

\(^5\) However in the United Kingdom (Hall et al., 2000) and Canada (Amirault et al., 2004) state-dependence seems to be somewhat less important for firms’ pricing decision.

\(^6\) Other examples in which prices directly enter the utility function are, for instance, Rosenkranz (2003) and Rosenkranz and Schmitz (2007) in the context of auctions and Popescu and Wu (2007), Nasiry and Popescu (2011), and Zhou (2011) in the context of customer loss aversion.
which describes the degree of the consumer’s loss aversion. For loss averse consumers, $\Delta > \Gamma$, i.e. the utility losses from price increases are larger than the utility gains from price decreases of equal magnitude. The consumer’s reference price $r_{i,t}$ is formed at the beginning of each period. In the spirit of Kőszegi and Rabin (2006), we assume that the consumer’s reference price depends on her lagged rational price expectation. Demand shocks, which may or may not trigger price adjustment, materialize unexpectedly in the course of the period and therefore do not enter the information set used by the consumer at the beginning of the period to form the reference price. Therefore, there is no instantaneous reaction of the reference price in the shock period even if the firm immediately adjusts its price in response to the shock. At the beginning of the next period, however, consumers update their information set and adjust their price expectation accordingly (since they can now infer about the nature of the demand shock and the corresponding price change). While temporary price changes do not provoke a change in the consumer’s reference price\(^7\), the reference price changes in the period after the occurrence of a permanent shock. Thus the consumer’s reference price is given by $r_{i,t} = E[p_{i,t} | I_{t-1}]$. The consumer’s budget constraint is given by

$$\sum_{i=1}^{n} p_{i,t} q_{i,t} = P_{t} Y_{t},$$  \hspace{1cm} (3)$$

where $Y_{t}$ denotes the consumer’s real income in period $t$ which is assumed to be constant and $P_{t}$ is the aggregate price index. For simplicity, we abstract from saving. This implies that consumers are completely myopic.\(^8\) In each period the consumer maximizes her period-utility function (1) with respect to her budget constraint (3). The result is the consumer’s period $t$ demand for the differentiated good $i$ which is given by

$$q_{i,t}(p_{i,t}, r_{i,t}, \mu) = r_{i,t} \eta - \mu \left( r_{i,t} \eta - \mu \right) Y_{t},$$  \hspace{1cm} (4)$$

where $\eta = \frac{1}{1 + \rho}$ denotes the elasticity of substitution between the different product varieties. The aggregate price index $P_{t}$ is given by

$$P_{t} = \left[ \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{\eta} \right]^{\frac{1}{1 - \eta}}.$$

(5)

We assume that the number of firms $n$ is sufficiently large so that the pricing decision of a single firm does not affect the aggregate price index. Defining $\lambda = \eta (1 + \mu) - \mu$, we can simplify equation (4) to

$$q_{i,t}(p_{i,t}, r_{i,t}, \mu) = r_{i,t} \eta - \mu \left( r_{i,t} \eta - \mu \right) P_{t} Y_{t},$$  \hspace{1cm} (6)$$

where the parameter $\lambda$ denotes the price elasticity of demand, which depends on $\mu$ and therefore takes different values for losses and gains. To simplify notation, we define

$$\lambda = \begin{cases} \gamma & \text{for } p_{i,t} < r_{i,t} \\ \delta & \text{for } p_{i,t} > r_{i,t} \end{cases},$$  \hspace{1cm} (7)$$

\(^7\)Support for this assumption can be found in the example of sales, i.e. promotions, characterized by non-permanent price decreases, used by firms to temporarily increase consumers’ demand for their product (see e.g. Eichenbaum et al., 2011). Sales do not affect the consumers’ reference price. Otherwise firms would not conduct sales because any downward adjustment of the consumer’s reference price reduces long-run profits for the firm.

\(^8\)Evidence to support this assumption is provided by Elmaghraby and Keskinocak (2003) who show that many purchase decisions of non-durable goods take place in economic environments which are characterized by myopic consumers.
with $\delta = \eta (1 + \Delta) - \Delta > \gamma = \eta (1 + \Gamma) - \Gamma$. Equation (6) indicates that the consumer’s demand function for good $i$ is kinked at the reference price $r_{i,t}$. The kink, lying at the intersection of the two demand curves $q_{i,t}(p_{i,t}, r_{i,t}, \gamma)$ and $q_{i,t}(p_{i,t}, r_{i,t}, \delta)$, is given by the price-quantity combination

$$ (p_{i,t}, q_{i,t}) = \left( r_{i,t}, \frac{r_t - \eta_t \frac{p_{i,t}^\rho Y_t}{r_t}}{\lambda_t - \eta_t \frac{p_{i,t}^\rho Y_t}{r_t}} \right), $$

(8)

where “$\hat{}$” denotes the value of a variable at the kink. Changes in the reference price $r_{i,t}$ give rise to a change of the position of the kink and also shift the demand curve as a whole. The direction of this shift depends on the sign of the difference $\lambda_t - \eta_t$. We restrict our analysis to $\lambda_t \geq \eta_t$, i.e. we assume that an increase in the reference price shifts the demand curve outwards and vice versa.9

Needless to say, abstracting from reference-dependence and loss aversion in the consumer’s preferences represented by utility function (1), restores the standard textbook consumer demand function for a differentiated good $i$, given by

$$ q_{i,t}(p_{i,t}) = p_{i,t}^\eta \frac{p_{i,t}^\rho Y_t}{r_t}. $$

(9)

In what follows, we will use this standard model as a benchmark case, against which we compare the pricing decisions of a monopolistic competitive firm facing loss averse consumers.

### 3.2 Monopolistic Firms

Firms seek to maximize the discounted stream of current and future profits, taking into account the implications of their current pricing decision for the customers’ reference price. For simplicity, we assume a two period time horizon. (This can serve as a rough approximation for forms of short-sightedness, such as hyperbolic discounting, when the first-period discount rate exceeds the second-period one.10)

All $n$ firms are identical, enabling us to drop the subscript $i$. In what follows we assume that the firm’s total costs are given by $C_t(q_t) = c q_t^2$, where $c$ is a constant, implying that marginal costs are linear in output: $MC_t(q_t) = c q_t$. In the presence of loss aversion ($\delta > \gamma$), the downward-sloping demand curve has a concave kink at the current reference price: $\hat{p}_t = r_t$. Thus the firm’s marginal revenue curve is discontinuous at the kink:

$$ MR_t(q_t, r_t, \lambda) = \left( 1 - \frac{1}{\lambda} \frac{q_t}{r_t^{\lambda - \eta} \frac{p_{i,t}^\rho Y_t}{r_t}} \right)^{-\frac{1}{\lambda}}, $$

(10)

9The positive relationship between reference price and demand has become a common feature in the marketing sciences (e.g., Thaler, 1985; Putler, 1992; Greenleaf, 1995). It manifests itself, e.g., through the “suggested retail price,” by which raising the consumers’ reference price causes increases in demand (Thaler, 1985). Furthermore, Putler (1992) provides evidence that an extensive use of promotional pricing in the late 80’s had lead to an erosion in demand by lowering consumers’ reference prices.

10Many authors have shown that consumers’ discount rates are generally much higher in the short run than in the long run (e.g. Loewenstein and Thaler, 1989; Ainslie, 1992; Loewenstein and Prelec, 1992; Laibson, 1996, 1997). Firm behavior is also often found to be short-sighted for the same reason. The theory of managerial myopia argues that managers often almost exclusively focus on short-term earnings (either because they have to meet certain goals or because their career advancement and compensation structure depends on the firm’s current performance), even if this has adverse long-run effects (Jacobson and Aaker, 1993; Graham et al., 2005; Mizik and Jacobson, 2007; Mizik, 2010). For a review of the early literature refer to Grant et al. (1996).
with $\lambda = \gamma$ for the gain domain and $\lambda = \delta$ for the loss domain, respectively. The interval $[MR_t(\hat{q}_t, r_t, \gamma), MR_t(\hat{q}_t, r_t, \delta)]$, where $MR_t(\hat{q}_t, r_t, \gamma) < MR_t(\hat{q}_t, r_t, \delta)$, we call "marginal revenue discontinuity" $MRD_t(\hat{q}_t, r_t, \gamma, \delta)$.

We assume that in the initial steady state, the exogenously given reference price is $r_{ss}$. Furthermore, in the steady state the firm’s marginal cost curve intersects the marginal revenue discontinuity, as depicted in Figure 1. To fix ideas, we assume that initially the marginal cost curve crosses the midpoint of the discontinuity in the marginal revenue curve.\(^{11}\) This assumption permits us to derive the symmetry characteristics of the responses to positive and negative demand shocks. This implies that the firm’s optimal price in the initial steady state $p^*_{ss}$ is equal to $r_{ss}$.\(^{12}\)

### 4 Demand Shocks

The demand for each product $i$ is subject to exogenous shocks, which may be temporary or permanent. These demand shocks, represented by $\varepsilon_t$, are unexpected and enter the demand function multiplicatively:

$$q_t(p_t, r_t, \lambda, \varepsilon_t) = r_t^{(\lambda - \eta)} p_t^{-\lambda} P_\eta t Y_t \varepsilon_t. \tag{11}$$

The corresponding marginal revenue functions of the firm are

$$MR_t(q_t, r_t, \lambda, \varepsilon_t) = \left(1 - \frac{1}{\lambda}\right) \left(\frac{q_t}{r_t^{(\lambda - \eta)} P_\eta t Y_t \varepsilon_t}\right)^{-\frac{1}{\lambda}}. \tag{12}$$

We consider the effects of a demand shock that hits the economy in period $t = 0$. The demand shock shifts the marginal revenue curve, along with the marginal revenue

\(^{11}\)To satisfy this condition, the slope parameter $c$ of the marginal cost curve has to take the value $c = \frac{1}{2} \left[\frac{1}{MR_t(\hat{q}_t, r_t, \gamma)} + MR_t(\hat{q}_t, r_t, \delta)\right]$.

\(^{12}\)The proof is straightforward: Let $\varepsilon$ be an arbitrarily small number. Then for prices equal to $r_{ss} + \varepsilon$ the firm faces a situation in which marginal revenue is higher than marginal costs and decreasing the price would raise the firm’s profit, while for prices equal to $r_{ss} - \varepsilon$ the firm faces a situation in which marginal revenue is lower than marginal costs and increasing the price would raise the firm’s profit. Thus $p^*_{ss} = r_{ss}$ has to be the profit maximizing price in the initial steady state.
discontinuity \( MRD_t(\hat{q}_t, r_t, \gamma, \delta, \varepsilon_t) \). We define a "small" shock as one that leaves the marginal cost curve passing through the marginal revenue discontinuity, and a "large" shock as one that shifts the marginal revenue curve sufficiently so that the marginal cost curve no longer passes through the marginal revenue discontinuity.

The maximum size of a small shock for the demand function (11) is

\[
\varepsilon_t(\lambda) = \left(1 - \frac{1}{\lambda}\right) \frac{1+\eta}{cP}\eta Y_t, \tag{13}
\]
i.e. \( \varepsilon_t(\lambda) \) is the shock size for which the marginal cost curve lies exactly on the boundaries of the shifted marginal revenue discontinuity \( MRD_t(\hat{q}_t, r_t, \gamma, \delta, \varepsilon_t(\lambda)) \).\(^{13}\) In the analysis that follows, we will distinguish both between small and large demand shocks and between temporary and permanent demand shocks.

### 4.1 Temporary Demand Shocks

For a temporary (one-period) demand shock, the consumers’ reference price is not affected (since information reaches them with a one-period lag and they have rational expectations). Thus the firm’s price response to the shock is the same as that of a myopic firm (which maximizes its current period profit).

**Proposition 1:** In response to a small temporary shock, prices remain rigid.

As noted, for a sufficiently small demand shock \( \varepsilon_0^s \leq \varepsilon_0(\lambda) \) the marginal cost curve still intersects the marginal revenue discontinuity, i.e. \( MC_0(\hat{q}_0) \in MRD_0(\hat{q}_0, r_{ss}, \gamma, \delta, \varepsilon_0^s) \). Therefore, the prevailing steady state price remains the firm’s profit-maximizing price,\(^{14}\) i.e. \( p_0^* = p_{ss}^* \), and we have complete price rigidity. By contrast, the profit-maximizing quantity changes to \( q_0^* = r_{ss}^\gamma P_0^\eta_0 Y_0 \varepsilon_0^s \), thus the change of quantity is given by

\[
\Delta q_0^* = \frac{q_0^*}{q_{ss}^*} = \frac{\varepsilon_0^s}{\varepsilon_{ss}^s} \neq 1. \tag{14}
\]
This holds true irrespective of the sign of the small temporary demand shock.

**Proposition 2:** In response to a large temporary shock, prices are more sluggish upwards than downwards.

For a large shock, i.e. \( \varepsilon_0^l > \varepsilon_0(\lambda) \), the marginal cost curve intersects the marginal revenue curve outside the discontinuity of the latter. Consequently both, a price and a quantity reaction are induced. The new profit-maximizing price of the firm is

\[
p_0^* = \left(\frac{r_{ss}^\gamma P_0^\eta_0 Y_0 \varepsilon_0^l}{q_0^*}\right)^{\frac{1}{\lambda}}, \tag{15}
\]
while its corresponding profit-maximizing quantity is

\[
q_0^* = \left(\frac{1}{\varepsilon_0^l} \left(1 - \frac{1}{\lambda}\right) \frac{1+\eta}{cP}\gamma \frac{1}{\lambda} P_0^\eta_0 Y_0 \varepsilon_0^l \right)^{\frac{\lambda+1}{\lambda}}, \tag{16}
\]

\(^{13}\)For \( \eta(\delta) \), the marginal cost curve intersects the marginal revenue gap on the upper bound, whereas for \( \eta(\gamma) \) it intersects it on the lower bound.

\(^{14}\)Compare the proof from Section 3.2.
where $\lambda = \delta$ for positive and $\lambda = \gamma$ for negative shocks, respectively.

In comparison to the standard firm the price reaction of the firm facing loss-averse consumers in response to a large temporary demand shock is always smaller, whereas the quantity reaction is always larger. Additionally, prices and quantities are less responsive to positive than to negative shocks. The intuition is obvious once we decompose the demand shock into the maximum small shock and the remainder:

$$\epsilon_l^0 = \epsilon_0(\lambda) + \epsilon_{rem}^0.$$  \hspace{1cm} (17)

From our theoretical analysis above, the maximum small shock $\epsilon_0(\lambda)$ has no price effects, but feeds one-to-one into demand. This holds true irrespective of the sign of the shock. By contrast, the remaining shock $\epsilon_{rem}^0$ has asymmetric effects. Let $q_0$ be the quantity corresponding to $\epsilon_0(\lambda)$. Then the change in quantity in response to $\epsilon_{rem}^0$ is given by

$$\Delta q_{rem}^0 = \frac{q_0}{q_0} = \left( \frac{\epsilon_{rem}^0}{\epsilon_0(\lambda)} \right)^{1/\gamma}.$$ \hspace{1cm} (18)

As can be seen from equation 18, the change of quantity in response to $\epsilon_{rem}^0$ depends negatively on $\lambda$, the price elasticity of demand. Since by definition $\delta > \gamma$, the quantity reaction of the firm facing loss-averse consumers is smaller in response to large positive temporary demand shocks than to large negative ones. This however implies that prices are also less responsive to positive than to negative large temporary demand shocks, because the former move the firm along the relatively flat portion of the demand curve, whereas the latter move it along the relatively steep portion of the demand curve. This asymmetric sluggishness in the reaction to positive and negative large temporary demand shocks is a distinct feature of consumer loss aversion and stands in obvious contrast to the standard textbook case of monopoly pricing.

### 4.2 Permanent Demand Shocks

Now consider a permanent, demand shock that occurs in period $t = 0$. Whereas the firm is assumed to change its price immediately in response to this shock, consumers update their reference price in the following period $t = 1$, i.e. $r_1 = E_0[p_1]$. Consequently, for price increases (decreases) the demand curve shifts outwards (inwards) and the kink moves to

$$(\hat{p}_1, \hat{q}_1) = (r_1, (P_1/r_1)^\eta Y_1 \epsilon_1).$$ \hspace{1cm} (19)

An outward shift of the demand curve (initiated by an upward adjustment in the reference price) increases the firm’s long-run profits, whereas an inward shift (initiated by a downward adjustment of the reference price) lowers them. We term this phenomenon the “reference-price updating effect.” The firm can anticipate this. Thus, it may have an incentive to set its price above the level that maximizes its profits in the shock period $p_0' > p_0$, therewith exploiting (dampening) the outward (inward) shift of the demand curve resulting from the upward (downward) adjustment of the consumers’ reference price for positive (negative) permanent shocks.\hspace{1cm} (15) Whether this occurs depends on whether the firm’s gain from a price rise relative to $p_0'$ in terms of future

\hspace{1cm}

\hspace{1cm}Needless to say, setting a price lower than optimal in the shock period with the aim to decrease the reference price permanently is not a preferable option for the firm.
Table 1: Base calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Discount rate</td>
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<td>implying substitutability</td>
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<td>12</td>
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<tr>
<td>Loss aversion</td>
<td>$\kappa$</td>
<td>2</td>
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<tr>
<td>Exogenous nominal income</td>
<td>$Y$</td>
<td>1</td>
</tr>
<tr>
<td>Exogenous price index</td>
<td>$P_t$</td>
<td>1</td>
</tr>
</tbody>
</table>

profits ($\Pi_1(r_1 = p'_0) > \Pi_1(r_1 = p_0)$), due to the relative rise in the reference price) exceeds the firm’s loss in terms of present profits ($\Pi_0(p'_0) < \Pi_0(p_0)$; since the price $p'_0$ is not appropriate for maximizing current profit).

To analyze which effect dominates, we calibrate the model and solve it numerically.

4.3 Calibration

We calibrate the model for a quarterly frequency in accordance with standard values in the literature. We assume an annual interest rate of 4 percent, which yields a discount factor $\beta = 0.99$. We follow Schmitt-Grohé and Uribe (2007) and set the monopolistic markup to 25 percent, i.e. $\eta = 5$, which is also close to the value supported by Erceg et al. (2000) and which implies that goods are only little substitutable, i.e. $\rho = 0.8$. Since we impose $\lambda \geq \eta$, we set $\gamma = 6$ in our base calibration. Loss aversion is measured by the relative slopes of the demand curves in the gain and loss domain, i.e. $\kappa = \frac{\delta}{\gamma}$. The empirical literature on loss aversion in prices finds that losses induce demand reactions approximately twice as large as gains (Tversky and Kahnemann, 1991; Putler, 1992; Hardie et al., 1993; Griffin and Schulman, 2005; Adeyemi and Hunt, 2007). Therefore, we set $\kappa = 2$. The exogenous nominal income $Y$ and price index $P_t$ are normalized to unity.\textsuperscript{16} The base calibration is summarized in Table 1.

4.4 Numerical Simulation

Tables 2 and 3 present the numerical results of our base calibration in the two-period model. In the tables we report the shock-arc-elasticities of price ($\overline{\eta}_{e, p} = \frac{\% \Delta p}{\% \Delta e}$) and output ($\overline{\eta}_{e, q} = \frac{\% \Delta q}{\% \Delta e}$) in the period of the shock $t = 0$ for positive and negative temporary and permanent shocks for the firm facing loss averse consumers.

The results in Tables 2 and 3 confirm the theoretical analysis above for the temporary shock, summarized in Propositions 1 and 2. However, not all of these results carry over in the case of permanent demand shocks.

**Proposition 3:** For all permanent shocks, prices are less sluggish upwards than downwards.

In line with the theoretical analysis above, our numerical results in table 2 and 3 indicate that in the case of a permanent shock the firm exploits the “reference-price up-

\textsuperscript{16}All results are completely robust to variations of these numerical values.
temporary shock permanent shock

<table>
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<tr>
<th>( e_{0}^{i} )</th>
<th>( \eta_{p,p} )</th>
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Table 2: Shock elasticities of price and output in \( t = 0 \) to positive permanent demand shocks, \( \bar{\epsilon}_{0}(\gamma) = 1.0476 \)

dating effect" and generally sets a price that is higher than the price it would optimally set in response to a temporary shock, i.e. \( p_{0}^{t} > p_{0}^{*} \). For positive permanent demand shocks this implies that the pricing reaction of the firm is always stronger than for positive temporary demand shocks for both, small\(^{17}\) and large shocks\(^{18}\). By contrast, for negative permanent demand shocks firms either do not adjust their prices at all for sufficiently small shocks or to a considerably lower extent than for negative temporary shocks.

As a consequence, price sluggishness is considerably less pronounced for positive than for negative permanent demand shocks. The asymmetry of the price reaction to positive and negative shocks therefore reverses, when moving from temporary to permanent shocks. While this result may seem surprising at first glance, it is straightforward intuitively: As noted, for temporary shocks, consumers abstract from updating their reference price. Therefore, the firm does not risk to suffer from a downward adjustment of the consumers’ reference price, when encountering a temporary drop in demand with a price reduction. On the other hand, for positive temporary shocks, the firm cannot generate permanent increases in demand due to upward-adjustments of the reference price. Since consumers react more sensitive to price increases relative to price decreases, the price and quantity reactions are smaller for positive temporary shocks compared to negative ones. By contrast, for permanent demand shocks, the firm exploits the positive “reference-price updating effect” which follows from price increases in response to positive shocks, whereas it tries to avoid the negative “reference-price updating effect” which follows from price decrease in response to negative shocks.\(^{19}\)

\(^{17}\)Of course, one can find a range of shocks, which are small enough to induce full price rigidity for permanent positive shocks. Due to the reference-price updating effect, this threshold is, however, very small. Given the base calibration, the threshold value for a sufficiently small positive shock is \( \bar{\epsilon}_{0}(\delta) = 1.0087 \).

\(^{18}\)Our numerical analysis indicates, however, that the positive reference-price updating effect is never strong enough to invalidate the general result that the pricing reaction of the firm facing loss averse consumers is more sluggish compared to the standard firm.

\(^{19}\)Since the firm avoids price reductions, which lead to downward-adjustments in the reference price, but
5 Conclusion

In contrast to the standard time-dependent and state-dependent models of price sluggishness, our theory of price adjustment is able to account for asymmetric price and quantity responses to positive and negative temporary and permanent shocks of equal magnitude. In contrast to the New Keynesian literature, our explanation of price adjustment is thoroughly microfounded, without recourse to ad hoc assumptions concerning the frequency of price changes or physical costs of price adjustments.

There are many avenues of future research. Consideration of heterogeneous firms and multi-product firms will enable this model to generate asynchronous price changes, as well as the simultaneous occurrence of large and small price changes, and heterogeneous frequency of price changes across products. Extending the model to a stochastic environment will generate testable implications concerning the variability of individual prices. Furthermore, our model needs to be incorporated into a general equilibrium setting to validate the predictions of our theory.

6 References


conducts price reductions, which do not influence the reference price, loss aversion offers a simple rationale for the firm’s practice of “sales” (see e.g. Eichenbaum et al., 2011).


Appendix

1. Demand Curve of Loss Averse Consumers

The loss averse consumer maximizes her utility function (1) subject to her budget constraint (3). The corresponding Lagrangian problem reads:

$$\max_{q_{i,t}} L = \left[ \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu} q_{i,t} \right]^\frac{1}{\rho} - \phi \left[ \sum_{i=1}^{n} p_{i,t} q_{i,t} - P Y_t \right], \quad (20)$$

where $\phi$ is the Lagrangian multiplier. The first-order condition of the Lagrangian function (20) is

$$\frac{\partial L}{\partial q_{i,t}} = \frac{1}{\rho} \left[ \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu} q_{i,t} \right]^{\frac{1}{\rho}-1} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu \rho} q_{i,t}^{\rho-1} \rho - \phi p_{i,t} = 0. \quad (21)$$

We collect all terms including demand components on the left hand side

$$q_{i,t}^{\rho-1} \left[ \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu} q_{i,t} \right]^{\frac{1}{\rho}} = \phi p_{i,t} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{\mu \rho}, \quad (22)$$

and simplify the exponentials

$$q_{i,t} \left[ \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu} q_{i,t} \right]^{-\frac{1}{\rho}} = \left( \phi p_{i,t} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{\mu \rho} \right)^{\frac{1}{\rho \cdot \eta}}. \quad (23)$$

We define overall demand according to a Dixit and Stiglitz (1977) aggregate, which reads

$$q_t = \left[ \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu} q_{i,t} \right]^{\frac{1}{\rho}}. \quad (24)$$

Applying (24) as well as the definition of the elasticity of substitution (i.e. $\eta = \frac{1}{1-\rho}$), we can simplify (23) to

$$q_{i,t} = (\phi p_{i,t})^{-\eta} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu (\eta-1)} q_t. \quad (25)$$

To determine the Lagrangian multiplier $\phi$, we plug (25) into (24)

$$q_t = \left[ \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu} (\phi p_{i,t})^{-\eta} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu (\eta-1)} q_t \right]^{\frac{1}{\rho \cdot \eta}}. \quad (26)$$
which after some simple manipulations yields

\[ \varphi = \left[ \sum_{i=1}^{n} \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu} \right]^{\frac{1}{1-\eta}} \equiv P_t^{-1}. \tag{27} \]

We define the inverse of the Lagrangian multiplier \( \varphi \) as the overall price index \( P_t \). Plugging (27) back into (25) yields

\[ q_{i,t} = P_t^\eta \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu(\eta^{-1})} p_{i,t}^\eta q_t. \tag{28} \]

Applying the budget constraint (3) yields

\[ q_{i,t} = P_t^\eta \left( \frac{p_{i,t}}{r_{i,t}} \right)^{-\mu(\eta^{-1})} p_{i,t}^\eta Y_t. \tag{29} \]

Finally, we simplify (29) using the definition \( \lambda = \eta(1 + \mu) - \mu \), which yields the demand curve for the differentiated good \( i \)

\[ q_{i,t} = r_{i,t}^{\lambda-\eta} p_{i,t}^{\lambda} P_t^\eta Y_t. \tag{30} \]

Including the shock term, equation (30) reads

\[ q_{i,t} = r_{i,t}^{\lambda-\eta} p_{i,t}^{\lambda} P_t^\eta Y_t \varepsilon_t. \tag{31} \]

2. Price and Quantity at the Kink

The kink is given by the particular price at which the two demand curves intersect, i.e. \( q_{i,t}(p_{i,t}, r_{i,t}, \gamma) = q_{i,t}(p_{i,t}, r_{i,t}, \delta) \). Given (31) and the definition of \( \lambda \) from equation (7) it must hold that

\[ r_{i,t}^{\gamma-\eta} p_{i,t}^{\gamma} P_t^\eta Y_t \varepsilon_t = r_{i,t}^{\delta-\eta} p_{i,t}^{\delta} P_t^\eta Y_t \varepsilon_t, \tag{32} \]

which simplifies to

\[ r_{i,t}^{\gamma-\eta} p_{i,t}^{\gamma} = r_{i,t}^{\delta-\eta} p_{i,t}^{\delta}. \tag{33} \]

Sorting terms yields

\[ p_{i,t}^{\delta-\gamma} = r_{i,t}^{\delta-\eta-\gamma+\eta}. \tag{34} \]

From (34) it is obvious that \( p_{i,t} = r_{i,t} \) at the kink. Plugging (34) back into (31) gives the quantity at the kink

\[ q_{i,t} = r_{i,t}^{\gamma-\eta} P_t^\eta Y_t \varepsilon_t. \tag{35} \]
6.1 3. Demand Curve of Standard Consumers

The standard consumer (i.e. the non-loss averse consumer) maximizes her utility function

\[ U_t(q_{1,t}, \ldots, q_{n,t}) = \left[ \sum_{i=1}^{n} q_{i,t}^\rho \right]^\frac{1}{\rho}, \tag{36} \]

subject to her budget constraint (3). The corresponding Lagrangian problem reads:

\[
\max_{q_{i,t}} \left[ \sum_{i=1}^{n} q_{i,t}^\rho \right]^\frac{1}{\rho} - \phi \left[ \sum_{i=1}^{n} p_{i,t} q_{i,t} - P_t Y_t \right], \tag{37} \]

where \( \phi \) is the Lagrangian multiplier for the standard textbook problem. The first-order condition of the Lagrangian problem (37) is

\[
\frac{\partial L}{\partial q_{i,t}} = \frac{1}{\rho} \left[ \sum_{i=1}^{n} q_{i,t}^\rho \right]^{\frac{1}{\rho}-1} q_{i,t}^{\rho-1} \rho - \phi p_{i,t} = 0. \tag{38} \]

We collect all terms including demand components on the left hand side and simplify the exponentials

\[
q_{i,t} \left[ \sum_{i=1}^{n} q_{i,t}^\rho \right]^{-\frac{1}{\rho}} = (\phi p_{i,t})^{\frac{1}{\rho}}. \tag{39} \]

We define overall demand for the standard consumer by a Dixit and Stiglitz (1977) aggregate of the form

\[
q_t = \left[ \sum_{i=1}^{n} q_{i,t}^\rho \right]^\frac{1}{\rho}. \tag{40} \]

Applying (40) as well as the definition of the elasticity of substitution, we can simplify (39) to

\[
q_{i,t} = (\phi p_{i,t})^{-\eta} q_t. \tag{41} \]

To determine the Lagrangian multiplier \( \phi \), we plug (41) into (40)

\[
q_t = \left[ \sum_{i=1}^{n} ((\phi p_{i,t})^{-\eta} q_t)^{\frac{\eta}{1-\eta}} \right]^{\frac{1}{\eta}}, \tag{42} \]

which after some simple manipulations yields

\[
\phi = \left[ \sum_{i=1}^{n} p_{i,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \equiv P_t^{1-\eta}. \tag{43} \]

We define the inverse of the Lagrangian multiplier \( \phi \) as the overall price index \( \bar{P}_t \) for the standard textbook problem. Plugging (43) back into (41) yields

\[
q_{i,t} = p_{i,t}^{1-\eta} \bar{P}_t q_t. \tag{44} \]

Applying the budget constraint (3) yields the demand curve for the differentiated good \( i \) for the standard consumer

\[
q_{i,t} = p_{i,t}^{-\eta} p_{i,t}^{\eta} Y_t \tag{45} \]
Including the shock term, equation (45) reads
\[ q_{t,i} = p_{i}^{-\eta} P_{t}^\eta Y_{t} \varepsilon_{t}. \] (46)

4. Marginal Revenue Curve

Since all firms are assumed to be identical, we drop the subscript \( i \) for the firm derivations. Revenue is given by
\[ R_{t} = p_{t}(q_{t}, r_{t}, \lambda_{t}, \varepsilon_{t}) q_{t} = \left( \frac{q_{t}}{r_{t}^{\lambda-\eta} P_{t}^\eta Y_{t} \varepsilon_{t}} \right)^{-\frac{1}{\lambda}} q_{t}, \] (47)
or in short
\[ R_{t} = \left( \frac{r_{t}^{\lambda-\eta} P_{t}^\eta Y_{t} \varepsilon_{t}}{r_{t}} \right)^{\frac{1}{\lambda}} q_{t}^{-\frac{1}{\lambda}}. \] (48)
The first-order condition with respect to \( q_{t} \) yields the marginal revenue curve
\[ MR_{t} = \frac{\partial R_{t}}{\partial q_{t}} = \left( 1 - \frac{1}{\lambda} \right) \left( \frac{q_{t}}{r_{t}^{\lambda-\eta} P_{t}^\eta Y_{t} \varepsilon_{t}} \right)^{-\frac{1}{\lambda}}. \] (49)

5. Critical Shock Size

The critical value for the small shock is given by the particular shock \( \varepsilon_{t}(\lambda) \), for which the marginal cost curve exactly intersects the critical bounds of the shifted marginal revenue discontinuity, i.e.
\[ MC_{i}(q_{t}^*) = MR_{t}(q_{t}^*, r_{t}, \lambda, \varepsilon_{t}(\lambda)), \] (50)
where \( MC_{i}(q_{t}) = \frac{\partial C_{i}(q_{t})}{\partial q_{t}} = c q_{t} \), with \( C_{i}(q_{t}) = \frac{\varepsilon_{t} q_{t}^{2}}{2} \). Evaluating the marginal revenue curve (49) and the marginal cost curve at the post-shock optimum yields
\[ c q_{t}^* = \left( 1 - \frac{1}{\lambda} \right) \left( \frac{q_{t}^*}{r_{t}^{\lambda-\eta} P_{t}^\eta Y_{t}} \varepsilon_{t}(\lambda) r_{t}^{\lambda-\eta} P_{t}^\eta Y_{t} \right)^{-\frac{1}{\lambda}}. \] (51)
From the analysis of small shocks we know that the new quantity of the maximum small shock is \( q_{t}^* = \varepsilon_{t}(\lambda) r_{t}^{-\eta} P_{t}^\eta Y_{t} \). Applying this, we obtain
\[ c \varepsilon_{t}(\lambda) r_{t}^{-\eta} P_{t}^\eta Y_{t} = \left( 1 - \frac{1}{\lambda} \right) \left( \frac{\varepsilon_{t}(\lambda) r_{t}^{-\eta} P_{t}^\eta Y_{t}}{\varepsilon_{t}(\lambda) r_{t}^{\lambda-\eta} P_{t}^\eta Y_{t}} \right)^{-\frac{1}{\lambda}}. \] (52)
Solving for \( \varepsilon_{t}(\lambda) \) yields the critical shock size
\[ \varepsilon_{t}(\lambda) = \left( 1 - \frac{1}{\lambda} \right) \left( \frac{1}{c P_{t}^\eta Y_{t}} \right)^{\frac{1+\eta}{\lambda}}. \] (53)
6. Optimal Price and Quantity in Reaction to a Large Shock for the Myopic Firm

The new optimal price lies at the intersection of the marginal cost curve with the shifted marginal revenue curve, which by definition is outside the marginal revenue discontinuity

\[ MC_t(q^*_t, \varepsilon_t) = MR_t(q^*_t, r_t, \lambda_t, \varepsilon_t). \]  

Applying the respective functions yields

\[ cq^*_t = \left(1 - \frac{1}{\lambda} \right) \frac{q^*_t}{r_t^{(\lambda-\eta)} p_t^{\eta} Y_t^{\eta} \varepsilon_t}. \]  

Solving this equation for \( q \), we obtain

\[ q^*_t = \left(1 - \frac{1}{\lambda} \right)^{\frac{1}{\lambda+1}} \frac{r_t^{(\lambda-\eta)} p_t^{\eta} Y_t^{\eta} \varepsilon_t}{c q^*_t}. \]  

The optimal price can be calculated by plugging \( q^*_t \) into the inverse demand curve, given by

\[ p^*_t = \left(\frac{q^*_t}{r_t^{(\lambda-\eta)} p_t^{\eta} Y_t^{\eta} \varepsilon_t}\right)^{-\frac{1}{\lambda}}. \]
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