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Miles, speed and technology: traffic safety under oligopolistic insurance

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Abstract:

We study road safety when insurance companies have market power, and can influence drivers' behavior via insurance premiums. We obtain first- and second-best premiums different market structures in insurance. The insurance program consists of an insurance premium, and marginal dependencies of that premium on speed and own safety technology choice. A private monopolist internalizes accident externalities up to the point where compensations to users' benefit matches the full (immaterial) costs; in oligopolistic markets, insurers do not fully internalize accident externalities. Analytical results demonstrate how insurance firms' incentives to influence traffic safety coincide with or deviate from socially optimal incentives.

Keywords: Road safety, accident externalities, traffic regulation, congestion externalities

JEL codes: D43, D62, R41, R42, R48

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1. Introduction

Accident externalities are among the most important external costs of road transport, see for example [Parry et al. \(2007\)](#). The social cost of road accidents is a multiple of that of congestion externalities ([Steimetz \(2008\)](#) provides an overview of the literature on accident externalities). Both drivers' behaviour (such as speeding, distance to the next car, attention paid towards the other road users) and technical characteristics (such as safety belts, advanced breaking systems, window shields, lights, weight, etc.) of vehicles heavily influence the safety of the car driver and passengers, as well as of others on the road. This conclusion has been drawn from both empirical (see, for example, [Lave \(1985\)](#), [Cohen and Einav \(2003\)](#), [Aarts and van Schagen \(2006\)](#), [Steimetz \(2008\)](#), [Hultkrantz and Lindberg \(2011\)](#), [Hultkrantz et al. \(2012\)](#)) and theoretical works (e.g., [Jansson \(1994\)](#), [Verhoef and Rouwendal \(2004\)](#), [Nitzsche and Tucharaktschiew \(2013\)](#), [Wang \(2013\)](#)).²

²Reanalysis ([Aarts and van Schagen \(2006\)](#)) of the data from [Kloeden et al. \(2001\)](#) revealed an exponential function between individual speed and the risk of being involved into an accident on urban roads. Also, on urban roads the accident rate increases more with increasing speed than on rural roads. In [Cohen and Einav \(2003\)](#), authors state that seat belt usage enforcement severely reduced traffic fatalities: "We estimate that a 1-percentage-point increase in usage saves 136 lives (using a linear specification), and a 1% increase in usage reduces occupant fatalities by about 0.13% (using a log-log specification)".

Furthermore, [Delhaye \(2007\)](#), [Rizzi \(2008\)](#), and [Hultkrantz et al. \(2012\)](#), suggest that incentives stemming from insurances can change drivers' strategic behaviour. However, the effect of insurance companies, efforts and incentives to affect driver behaviour remain under-investigated in the economic literature. For instance, based on [Steimetz \(2004\)](#), and [Gossner and Picard \(2005\)](#), [Rizzi \(2008\)](#) considers a rational driver who optimally chooses risk-reducing efforts (care), such as speed, distance between the cars etc., in a model where car insurance is available. Rizzi clearly shows that insurance influences driver's efforts to drive safely. However, in his work insurance agents do not play an active role controlling drivers' choice, and only drivers' utility functions are maximized.

We study the regulation of road safety, when insurance companies have market power, and can influence road users' choices in terms of aggregate mileage, investments in private car safety for drivers, and speed. The latter may benefit the driver as well as a possible "partner" in a collision. A social regulator, in turn, has instruments to affect both insurance providers and thus indirectly the drivers.

Our model describes a two-stage game between car insurance providers and road users. First, insurance companies maximize their profit by optimizing the level of insurance premiums, and how these depend on speed and technology, subject to equilibrium constraints. Then, each atomistic road

user opts for a safety technology and speed, in order to minimize its generalized price. Next, an aggregate kilometrage results from the inverse demand function for trips, given this minimized generalized price. This price includes time costs, investments into own safety technology, insurance premiums, and a (possibly immaterial) part of the expected accident costs not covered by the insurance. We assume that an individual's speed choice does affect both one's own and other road users' safety, while the technology affects only the former. The technology chosen by a driver could in reality also influence the safety level of other road users, but distinguishing between a strictly internal safety measures (technology investment) and a combined internal-external safety measure (speed) is helpful for a clear interpretation of our results.³

Following and extending the reasoning provided in papers [Verhoef and Rouwendal \(2004\)](#) and [Dementyeva et al. \(2015\)](#), we obtain marginal conditions for the first- and second-best premiums. In our model we assume that companies can influence drivers' behavior via insurance programs. A social regulator can then impose taxes or subsidies on companies and/or road users,

³In our terminology, 'own safety technologies' include, for example, air bags, interior head-impact protection, seat belts, child car seats, flammability of interior materials, etc. Advanced breaking systems, tire-pressure monitoring system, high intensity lamps would rather be included into the other characteristics of driving, affecting also the safety of others. We refer to such characteristics as 'speed'.

finer for speeding over a certain speed limit, and other regulations. We consider social welfare-maximizing and private profit-maximizing monopolies, and oligopolistic markets of firms playing Nash in a Cournot fashion.

A number of conclusions stand out: For each type of market structure, the insurance premium (function) drivers face is defined by an insurance premium level in the equilibrium point, and marginal dependence of that premium function on speed and technology, given by what we will call optimal “slopes” of the premium function with respect to the individual driver’s choice of speed, and choice of technology (the latter is also modeled as a continuous variable). Although such a sophisticated, continuous design of insurance premiums may appear unrealistic, we model it in such a way in order to identify whether and how the company would prefer to affect speed and technology choices of its insurees, in an analytical setting that does not introduce additional second-best distortions arising from imperfect instruments.

The insurance premiums we derive reflect that monopolists fully internalize the accident externalities imposed by their drivers upon one another, while competing firms provide only partial internalization. The same is true for the optimal “slopes” of the premium with respect to the speed and technology choices of the road users. A safer technology may only influence driver’s own accident costs, and does not directly affect the chance to be

guilty of an accident. The speed choice, on the other hand, decreases the risk to cause a collision and therewith other drivers' risks, and this fact is reflected in the control over the marginal change of the premium.

Policy implications of the results can be of various nature. Both public and private insurers' objectives depend on the safety level on the road; however, the distinction of between their objective functions causes different insurance prices, as well as different marginal premiums. In order to compensate for non-optimal pricing (and, correspondingly, to fill in the gap between the first- and second-best aggregate kilometrage), a social regulator can introduce subsidies and/or taxes imposed on the firms, [Dementyeva et al. \(2015\)](#). Correcting the speed and safety technology choices, the social regulator might have to address mispricing by introducing appropriate limits and/or supplementary fines or bonuses. We derive analytical expressions describing these.

The paper is organized as follows: We introduce the model in [Sec. 2](#). Then we start the analysis by finding the first-best social optimum in [Sec. 3](#). We continue with the analysis of an oligopolistic market of insurance firms, competing in Nash–Cournot manner. [Sec. 4](#) provides us with the profit-optimizing insurance premiums, as well as the optimal (from insurers' point of view) regulations of speed and technology choices. Firms that do not have perfect control over all choices of their clients are considered in [Sec. 5](#). [Sec. 6](#)

concludes.

2. Model description

There are three types of actors in this model: Road users consume kilometers driven, and choose the speed at which to drive and vehicle technology. Insurance companies provide auto insurance to (partially) cover accident costs of the drivers. In doing so, they choose the premium per kilometer, which depends on speed and technology chosen by the insuree. We will refer to this dependence as the “slopes” of the premium, with respect to speed and technology. Finally, the regulator aims to maximize social welfare, for example, by setting taxes or subsidies for drivers or insurers.

One of the main assumptions of the model is that road users are homogeneous in costs, and are individually infinitely small. Due to the atomistic nature of the drivers, we may assume that individual kilometrage driven by each of them is infinitesimal compared to the aggregate kilometrage driven in the network, so that there is no self-imposed accidents, and that each unit of consumption corresponds to a different driver.⁴ The terms “driver” and “kilometer driven”, thus, may and will be used interchangeably, unless

⁴Infinitely small individual kilometrage lets us avoid drivers making decision on speeding every unit of distance driven. Infinitesimal individual kilometrage also does not affect accident costs of other drivers.

it causes confusion. In order to emphasize the atomistic nature of units of consumption, we will also speak of “particles” in the continuum of kilometers driven. This aspect is important in the context of our paper, as it defines which part of marginal costs are internal and which are external to a driver.

We denote with \bar{K}_m the set of the drivers/road particles insured by firm $m = 1, \dots, N$, where N is the total number of firms on the market. The cardinal number $K_m = |\bar{K}_m|$ is the total kilometrage of drivers insured by firm m . $K = |\bigcup_{m=1}^N \bar{K}_m| = \sum_{m=1}^N K_m$ is the aggregate kilometrage of all drivers in the network.

Road user k faces the following costs of driving: expected accident cost C_{A_k} , travel time cost C_{T_k} , and cost of investment into own safety technology C_{M_k} , all per kilometer driven. Accident cost of driving a road distance particle k is an additive function $C_{A_k}(\cdot) = \sum_{m=1}^N K_m c_k^m(S_k, \mu_k; \bar{S}_m)$. The function $C_{A_k}(\cdot)$ represents the accident cost per kilometer driven, and, thus, functions $c_k^m(\cdot)$, for all $m = 1, \dots, N$, are accident costs “per kilometer, per other drivers’ kilometer”. Here, speed S_k and own safety technology μ_k (both scalar) are chosen by the driver consuming particle k . The notation \bar{S}_m stands for a scalar function of generalized speed of all drivers insured by firm m taking the continuum of all particles’ speeds from the set \bar{K}_m as argument.⁵ Because drivers are symmetric by assumption, in equilibrium,

⁵This implies that marginal reaction of driver k ’s generalized costs C_{A_k} when individual

generalized speed \bar{S}_m is equal to the equilibrium speed S_m^* of every driver insured by the firm m . Time cost C_{T_k} is a function of individual speed S_k of driver particle k . The safety investment C_{M_k} has the own safety technology μ_k as its argument. Economic life-time of a car is assumed to be proportional to distance driven, therefore, the total spending on car is considered to be variable costs proportional to kilometrage.

Insurance firms cover an exogenous (legally determined) share α of the drivers' accident costs. The assumption $0 < \alpha < 1$ reflects that insurance companies may fully cover material/monetary costs of drivers, but leave other immaterial costs, such as emotional costs of drivers, at least partly unreimbursed.

A driver k insured by firm l , $k \in \bar{K}_l$, minimizes his generalized price of driving when choosing S_k and μ_k :

$$\min_{S_k, \mu_k} p_k(S_k, \mu_k, \cdot) = \pi_l(\cdot) + (1 - \alpha)C_{A_k}(\cdot) + C_{T_k}(S_k) + C_{M_k}(\mu_k), \quad (1)$$

where $\pi_l(\cdot)$ is the insurance premium charged by the firm. The premium function depends on many parameters and variables including individual speed S_k , speed functions \bar{S}_m , for every insurance company m , and own technology μ_k , and some of those the firm might be able to control via the premium

speed S_n of driver n insured by firm l changes is $\left. \frac{\partial C_{A_k}}{\partial S_n} \right|_{k \neq n} = K_l \frac{\partial c_k^l}{\partial \bar{S}_l} \cdot \frac{\partial \bar{S}_l}{\partial S_n}$, for n and k being insured by the same firm l .

slopes. We first assume that firms can affect both individual speed and technology, and then in Sec. 5 and App. [Appendix A](#) insurers have imperfect control. The latter cases allow us to analyse how firms use available instruments in order to compensate for the lack of control. The premium slopes $\frac{\partial \pi_l}{\partial S_k}$ and $\frac{\partial \pi_l}{\partial \mu_k}$ do not directly affect the level of driving costs, as these can be set independent of the premium level, but they are present in the individual's minimization problem (1).

We assume that, for a given accident, the probabilities of guilt of both parties depend on their speed choices. This could be reflected by introducing a function $\gamma_k^m(\cdot) = \gamma(S_k, \bar{S}_m)$, which assigns the probability of guilt of driver k for a given accident between driver k and any other driver insured by a firm m .⁶ Again, because drivers are assumed to be symmetric, we can use a firm-specific superscript in γ_k^m , as γ_k^m will be equal for all drivers of firm m . In order to present the insurance firm's objective function, we assume that the drivers' cost functions are continuous on \bar{K} , which is satisfied for identical drivers, as we assume is the case, but also if distributions of preferences over drivers or kilometers is continuous. Insurer's profit is the difference between

⁶Hence, $\gamma_m^k(\cdot) = 1 - \gamma_k^m(\cdot)$ is the probability of individual k not being guilty of an accident with a driver insured by firm m . We choose the risk to cause an accident to be independent of safety technology choice μ_k in order to emphasize that μ_k only influences own expected accident costs of driver k .

the firm's revenue, i.e. insurance premiums collected from all drivers $k \in \bar{K}_l$, and its payments of two sorts. One type of payments is coverage of the accident costs of own customers when they are guilty of causing accidents; the other type of payments is coverage of the accident costs of non-guilty parties suffered from drivers of that firm. Firm's l profit function Π_l is then as follows:

$$\begin{aligned} \Pi_l(\cdot) = & \int_{\bar{K}_l} \pi_l(S_k, \mu_k) dk \\ & - \alpha \int_{\bar{K}_l} \sum_{m=1}^N \gamma_k^m(S_k, \bar{S}_m) K_m c_k^m(S_k, \mu_k, \bar{S}_m) dk \\ & - \alpha \sum_{m=1}^N \int_{\bar{K}_m} (1 - \gamma_x^l(S_x, \bar{S}_l)) K_l c_x^l(S_x, \mu_x, \bar{S}_l) dx. \end{aligned} \quad (2)$$

Let us go through equation (2) in more detail. The first integral in (2) sums up insurance premiums collected from every driver $k \in \bar{K}_l$, and, thus, gives the firm's revenue. The other terms reflect the firm's expenses. An insurance firm covers accident costs if its client is the guilty party. The term on the second line represents the compensation paid to firms l 's customers k when having caused an accident. For each customer/road particle dk driven, $\alpha \sum_{m=1}^N \gamma_k^m(\cdot) c_k^m(\cdot)$ is the expected payment from firm l to driver k if an accident occurs and the driver is guilty (otherwise it is covered by the insurer of the "accident partner"). The first integral represents the total expenses of this kind, and it thus sums over the particles k insured by the firm.

The final term in (2) represents the aggregate payment of the firm to non-

guilty parties of accidents. The term under the integral sign is the expected payment to the second, non-guilty driver involved into each accident caused by an insuree of firm l . The non-guilty driver might be a client of firm l , or from any other firm m on the market. Hence, we integrate over the sets \bar{K}_m to sum up the expected accident costs of the victims of customers of firm l . For $m = l$, the integral covers the same particles as the integral in the second term; for $m \neq l$, other particles are covered, and we emphasize it by using notation x for the particles in the last integral. The values of the integrals in (2) depend on the domain of integration, and, hence, differ for \bar{K}_m , for all $m = 1, \dots, N$, even though the functional form of the expected accident costs is the same due to symmetry of the drivers.

Let $\mathcal{B}(K_1, \dots, K_N)$ be the social benefit function giving the user benefits of trips. Firm l maximizes the profit (2) with respect to its total kilometrage K_l , speed S_k and technology μ_k choices of its customers $k \in \bar{K}_l$, subject to equilibrium conditions where driver's willingness to pay $\mathcal{D}_l(K_1, \dots, K_N) = \frac{\partial \mathcal{B}(\cdot)}{\partial K_l}$, is equal to the generalized price of road use:

$$\pi_l^* = \mathcal{D}_l(\cdot) - (1 - \alpha)C_A(S_l^*, \mu_l^*, S_l^*) - C_T(S_l^*) - C_M(\mu_l^*), \quad (3)$$

and from (1):

$$\frac{\partial p_k}{\partial S_k} = \frac{\partial p_k}{\partial \mu_k} = 0, \quad \forall k \in \bar{K}_l. \quad (4)$$

The F.O.C. $\frac{\partial \Pi_l}{\partial K_l} = 0$, subject to the equilibrium condition (3), gives the

profit-maximizing insurance premium level; $\frac{\partial \Pi_l}{\partial S_k} = 0$, and $\frac{\partial \Pi_l}{\partial \mu_k} = 0$, for all $k \in \bar{K}_l$, subject to the individual particle conditions (4), defines the optimal marginal reaction of the premium to the speed and technology choices for every road user insured by firm l . Together, the optimal premium level and the optimal slopes $\frac{\partial \pi_l}{\partial S_k}$ and $\frac{\partial \pi_l}{\partial \mu_k}$, for all $k \in \bar{K}_l$, determine the optimal insurance premium for firm l .

3. Public welfare-maximizing monopoly

A natural reference and benchmark is the first-best social optimum. We find this by solving the maximization problem for what we call the “social monopolist”, who maximizes social surplus, taken as the measure for social welfare:

$$\max \mathcal{W}(\cdot) = \mathcal{B}(K) - \int_{\bar{K}} (C_{A_k}(\cdot) + C_{T_k}(S_k) + C_{M_k}(\mu_k)) dk. \quad (5)$$

The equilibrium condition (3) has the following form:

$$\pi^{fb}(\cdot) = \mathcal{D}(K^{fb}) - (1 - \alpha)C_{A_n}(S^{fb}, \mu^{fb}, S^{fb}) - C_{T_n}(S^{fb}) - C_{M_n}(\mu^{fb}), \quad \forall n. \quad (6)$$

In addition, every driver $n \in \bar{K}$ chooses own safety technology μ_n and individual speed S_n such that the conditions (4) hold:

$$\frac{\partial \pi}{\partial \mu_n} = -(1 - \alpha)K^{fb} \frac{\partial c_n^1(\mu_n, \cdot)}{\partial \mu_n} - \frac{\partial C_{M_n}(\mu_n)}{\partial \mu_n}, \quad (7)$$

$$\frac{\partial \pi}{\partial S_n} = -(1 - \alpha) \frac{\partial C_{A_n}(S_n, \cdot)}{\partial S_n} - \frac{\partial C_{T_n}(S_n)}{\partial S_n}. \quad (8)$$

In case of a monopolistic market, the accident cost function can be rewritten as follows: $C_{A_n}(\cdot) = Kc_n^1(S_n, \mu_n, \bar{S})$. Then, given first-best choice of speed and technology S^{fb}, μ^{fb} , the F.O.C. is

$$\frac{\partial \mathcal{W}}{\partial K} = \mathcal{D}(K^{fb}) - (C_{A_n}(\cdot) + C_{T_n}(S^{fb}) + C_{M_n}(\mu^{fb})) - \int_{\bar{K}} \frac{\partial C_{A_k}(\cdot)}{\partial K} dk = 0, \quad \forall n.$$

Using equilibrium constrain (6), we can rewrite this F.O.C. and get the solution of the maximization problem of the public welfare-maximizing monopolist as the first-best premium:

$$\pi^{fb} = (\alpha + 1)C_{A_n}(S^{fb}, \mu^{fb}, \cdot), \quad (9)$$

which essentially implies full internalization of the accident externalities that the drivers impose on each other plus a correction for the moral hazard problem of not considering all accident costs when a part is insured and, thus, borne by an insurer. Insurance premium level guaranties the optimal aggregate kilometrage, but does not directly motivate drivers to optimally care about safe speed and technology.

Let us now consider the socially optimal regulation towards speed and technology choices. The technology-related first-order condition for particle

n is:

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \mu_n} &= -\frac{\partial}{\partial \mu_n} \int_{\bar{K}} (C_{A_k}(\mu, \cdot) + C_{T_k}(\cdot) + C_{M_k}(\mu_k)) dk \\ &= -\frac{\partial C_{A_n}(\mu_n, \cdot)}{\partial \mu_n} - \frac{\partial C_{M_n}}{\partial \mu_n} = 0.\end{aligned}\quad (10)$$

Substitution of (7) into (10) gives:

$$\frac{\partial \pi}{\partial \mu_n} = \alpha \frac{\partial C_{A_n}(\mu_n, \cdot)}{\partial \mu_n}.\quad (11)$$

The first-best slope captures the driver's insured responsibility for accident costs he is involved in.

The first-order condition w.r.t. the driver's speed choice is:

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial S_n} &= -\frac{\partial}{\partial S_n} \int_{\bar{K}} (K c_k^1(S_k, \cdot) + C_{T_k}(S_k) + C_{M_k}(\cdot)) dk \\ &= -K \frac{\partial c_n^1(S_n, \cdot)}{\partial S_n} - \frac{\partial C_{T_n}(S_n)}{\partial S_n} - \int_{\bar{K} \setminus \{n\}} K \frac{\partial c_k^1(\cdot, \bar{S})}{\partial S_n} dk = 0.\end{aligned}\quad (12)$$

Combining conditions (8) and (12), we have:

$$\frac{\partial \pi}{\partial S_n} = \alpha K \frac{\partial c_n^1(S_n, \cdot)}{\partial S_n} + \int_{\bar{K} \setminus \{n\}} K \frac{\partial c_k^1(\cdot, \bar{S})}{\partial S_n} dk.\quad (13)$$

It is intuitive that a social regulator internalizes full accident externality: the first term in (13) reflects the part of expected accident costs covered by insurance and, thus, would not be taken into account by driver himself, and the second term counts for the entire accident externality imposed by driver upon all other drivers. The reason why this second term is not weighted by α , to make it correspond to the firm's compensation to other drivers, is that all

drivers are insured with the same firm, and the uninsured accident cost that driver n imposes on other drivers fully translated into a reduced willingness to pay premium, so that the firm would also fully face that uninsured part of the externality. Adding up the fractions α (compensation to be paid by the firm) and $1 - \alpha$ (reduced willingness to pay for other insurees, a term unity remains); hence, the second term.

4. Firms playing Nash–Cournot fashion and controlling drivers' choices of technology and speed

4.1. Profit-maximizing insurance premium

We now turn to the market form of interest: oligopolistic supply of insurance.

In order to define a profit-maximizing insurance premium (per kilometer driven), an insurance firm l solves the F.O.C.:

$$\begin{aligned}
\frac{\partial \Pi_l}{\partial K_l} &= \frac{\partial}{\partial K_l} \int_{\bar{K}_l} \left(\mathcal{D}_l(\cdot) - (1 - \alpha)C_{A_k}(\cdot) - C_{T_k}(\cdot) - C_{M_k}(\cdot) \right) dk & \} 1st \\
&- \alpha \frac{\partial}{\partial K_l} \int_{\bar{K}_l} \sum_{m=1}^N \gamma_k^m(\cdot) K_m c_k^m(\cdot) dk & \} 2nd \\
&- \alpha \sum_{m=1}^N \frac{\partial}{\partial K_l} \int_{\bar{K}_m} (1 - \gamma_x^l(\cdot)) K_l c_x^l(\cdot) dx = 0, & \} 3rd \quad (14)
\end{aligned}$$

subject to the equilibrium condition, i.e. for individual n 's choice S_i^{sb} of speed

and μ_l^{sb} of technology, for all $n \in \bar{K}_l$:

$$\pi_l^{sb} = \mathcal{D}_l(\cdot) - (1 - \alpha)C_{A_n}(\cdot) - C_{T_n}(S_l^{sb}) - C_{M_n}(\mu_l^{sb}). \quad (15)$$

We mark the lines in (14) in order to make it easier to follow the derivations and to interpret the terms in the final expression. Let us first do the derivations:

$$\begin{aligned} \frac{\partial \Pi_l}{\partial K_l} &= (\mathcal{D}_l - (1 - \alpha)C_{A_n} - C_{T_n} - C_{M_n}) && \} 1st \\ &+ \int_{\bar{K}_l} \left(\frac{\partial \mathcal{D}_l}{\partial K_l} - (1 - \alpha) \frac{\partial C_{A_k}}{\partial K_l} \right) dk && \} 1st \\ &- \alpha \sum_{m=1}^N \gamma_n^m K_m c_n^m - \alpha \int_{\bar{K}_l} \gamma_k^l c_k^l dk && \} 2nd \\ &- \alpha \sum_{m=1}^N \int_{\bar{K}_m} (1 - \gamma_x^l) c_x^l dx - \alpha (1 - \gamma_n^l) K_l c_n^l = 0, && \} 3rd \end{aligned}$$

therefore, given second-best equilibrium (S_l^{sb}, μ_l^{sb}) , optimal premium level is:

$$\pi_l^{sb} = -K_l^{sb} \frac{\partial \mathcal{D}_l}{\partial K_l} + (1 - \alpha) K_l^{sb} \frac{\partial C_{A_n}}{\partial K_l} \quad \} 1st \quad (16)$$

$$+ \alpha \sum_{m=1}^N \gamma_n^m(\cdot) K_m c_n^m(\cdot) + \alpha K_l^{sb} \gamma_n^l(\cdot) c_n^l(\cdot) \quad \} 2nd \quad (17)$$

$$+ \alpha \sum_{m=1}^N K_m^{sb} (1 - \gamma_m^l) c_m^l(\cdot) + \alpha (1 - \gamma_n^l) K_l c_n^l(\cdot). \quad \} 3rd \quad (18)$$

In (16), the insurance premium (per kilometer driven) is followed by the demand related mark-up, and the compensation for the appearance of an extra driver on the road that decreases willingness to pay of other drivers of the same firm. The terms in the second line refer to additional payments from

firm l to its own customers when guilty. The first of these is the (expected) payment to the additional particle when guilty. The second term represents the increase in payments to firm l 's inframarginal particles, which have an increased probability of causing an accident due to the marginal increase in K_l . The terms on the third line refer to additional payments from firm l to non-guilty drivers. The first term is such that payments when the marginal particle insured is guilty; it sums over all other particles on the road, including those insured by firm l . The second term represents the additional payments on the firms' inframarginal insurances, when these drivers cause an accident and the marginal driver is the non-guilty partner and must be compensated by firm l .

Let us note that for private profit-maximizing monopoly case, $N = 1$, the insurance premium (16)–(18) reduces to:

$$\pi_1^{sb} = -K_1^{sb} \frac{\partial \mathcal{D}}{\partial K_1} + C_{A_n}(\cdot) + \alpha C_{A_n}(\cdot). \quad (19)$$

The private monopolist makes drivers to compensate fully for their expected accident costs, as well as for their impact on the other drivers on the road due to negative externality. Quite intuitively, and as will be continued below, on top of the market mark-up, a term is added which is equal to the socially optimal price rule.

4.2. Optimal speed and technology choice

In addition to insurance premiums, insurance programs include “rules” to control drivers’ choices of speed and own safety technology.

Let us first analyze how insurance providers can influence individual drivers’ choice of safety technology level. We assume that each driver insured with firm l ($n \in \bar{K}_l$) looks for a technology μ_n to balance the investments and the safety, therefore, minimizing the own generalized price (1):

$$\frac{\partial p_n}{\partial \mu_n} = \frac{\partial \pi_l(\cdot)}{\partial \mu_n} + (1 - \alpha) \frac{\partial C_{A_n}(\cdot, \mu_n)}{\partial \mu_n} + \frac{\partial C_{M_n}(\mu_n)}{\partial \mu_n} = 0,$$

which is equivalent to

$$\frac{\partial \pi_l(\cdot)}{\partial \mu_n} = -(1 - \alpha) K_l \frac{\partial c_n^l(\mu_n, \cdot)}{\partial \mu_n} - \frac{\partial C_{M_n}(\mu_n)}{\partial \mu_n}, \quad \forall n \in \bar{K}_l. \quad (20)$$

From firm l ’s perspective, individual’s choice must optimize firm’s expected profit (2):

$$\begin{aligned} \frac{\partial \Pi_l}{\partial \mu_n} = & \frac{\partial}{\partial \mu_n} \int_{\bar{K}_l} \left(\underbrace{\mathcal{D}_l(\cdot) - (1 - \alpha) \sum_{m=1}^N K_m c_k^m(\mu_k, \cdot) - C_{T_k}(\cdot) - C_{M_k}(\mu_k)}_{\pi_l} \right) dk \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial \mu_n}} \right\} 1st \\ & - \alpha \frac{\partial}{\partial \mu_n} \int_{\bar{K}_l} \sum_{m=1}^N \gamma_k^m(\cdot) K_m c_k^m(\mu_k, \cdot) dk \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial \mu_n}} \right\} 2nd \\ & - \alpha \sum_{m=1}^N \frac{\partial}{\partial \mu_n} \int_{\bar{K}_m} (1 - \gamma_x^l(\cdot)) K_l c_x^l(\mu_x, \cdot) dx = 0, \quad \forall n \in \bar{K}_l. \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial \mu_n}} \right\} 3rd \end{aligned} \quad (21)$$

In order to make the derivations more transparent, let us point out that among all integrals in (21), only those taken over the domain \bar{K}_l have terms

depending on choice of technology μ_n made by an individual $n \in \bar{K}_l$, and that each driver's choice appears among μ_k in the second line and among μ_x in the third. Let us do the derivations, and take into account drivers' reasoning (20):

$$\begin{aligned}
\frac{\partial \Pi_l}{\partial \mu_n} &= - (1 - \alpha) \sum_{m=1}^N K_m \frac{\partial c_n^m(\mu_n, \cdot)}{\partial \mu_n} - \frac{\partial C_{M_n}(\mu_n)}{\partial \mu_n} \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial \mu_n}} \right\} 1st \\
&\quad - \underbrace{\alpha \sum_{m=1}^N \gamma_n^m(\cdot) K_m \frac{\partial c_n^m}{\partial \mu_n}}_{2nd} - \underbrace{\alpha(1 - \gamma_n^l) K_l \frac{\partial c_n^l}{\partial \mu_n}}_{3rd} \\
&= \frac{\partial \pi_l}{\partial \mu_n} - \alpha \sum_{m=1}^N \gamma_n^m K_m \frac{\partial c_n^m(\mu_n, \cdot)}{\partial \mu_n} - \alpha(1 - \gamma_n^l) K_l \frac{\partial c_n^l(\mu_n, \cdot)}{\partial \mu_n} = 0.
\end{aligned}$$

Hence, insurance firm's control for the premium slope w.r.t. personal technology is as follows:

$$\frac{\partial \pi_l}{\partial \mu_n} = \alpha \sum_{m=1}^N \gamma_n^m(\cdot) K_m \frac{\partial c_n^m}{\partial \mu_n} + \alpha(1 - \gamma_n^l) K_l \frac{\partial c_n^l}{\partial \mu_n}. \quad (22)$$

To interpret this slope condition, let us go back to equation (21). The terms relating to the first line of (21) represent the impact of μ_n on particle n 's private cost. Because this translates directly into willingness to pay premium, the firm takes this effect fully into account. The term stemming from the second line can be seen as a moral hazard term: it is that part of damage incurred by driver n himself when guilty that he would ignore in setting μ_n because it is insured. Hence, it is α times the marginal impact of μ_n upon self-inflicted accident costs. The third line reflects the marginal savings for

firm l on compensation to particle n when other drivers insured by firm l cause an accident with particle n . There is no further firm-internal externalities involved for this particular firm, as the safety technology choice only influences own safety and by assumption does not affect any other driver directly. Of course, for cross-firm effects there are externalities, but those are not taken into account by firm l .

Comparing condition (22) with formula (11) for the slope of a monopolist maximizing social surplus, we can see that the second-best slope with respect to technology lacks the part of driver's marginal accident costs covered by the firm-insurer $m \neq l$ of the driver guilty of an accident (in case when driver n is the injured party). However, a private monopolist's optimal slope coincides with the first-best one, and fully internalized driver's responsibility.

Let us now turn to individual speed choice analysis. From driver's $n \in \bar{K}_l$ perspective, in marginal terms the costs of driving have to be compensated by benefits from speeding up:

$$\frac{\partial p_n}{\partial S_n} = \frac{\partial \pi_l}{\partial S_n} + (1 - \alpha) \frac{\partial C_{A_n}(S_n, \cdot)}{\partial S_n} + \frac{\partial C_{T_n}(S_n)}{\partial S_n} = 0, \quad \forall n \in \bar{K}_l.$$

Equivalently,

$$\frac{\partial \pi_l}{\partial S_n} = -(1 - \alpha) \frac{\partial C_{A_n}(S_n, \cdot)}{\partial S_n} - \frac{\partial C_{T_n}(S_n)}{\partial S_n}, \quad \forall n \in \bar{K}_l. \quad (23)$$

And insurer maximizes its profit for each driver $n \in \bar{K}_l$ by taking the partial

derivative with respect to S_n :

$$\begin{aligned}
\frac{\partial \Pi_l}{\partial S_n} &= \frac{\partial}{\partial S_n} \int_{\bar{K}_l} \left(\mathcal{D}_l(\cdot) - (1 - \alpha) C_{A_k}(S_k, \bar{S}, \cdot) - C_{T_k}(S_k) - C_{M_k}(\cdot) \right) dk & \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 1st \\
&\quad - \alpha \frac{\partial}{\partial S_n} \int_{\bar{K}_l} \sum_{m=1}^N \gamma_k^m(S_k, \bar{S}_m) K_m c_k^m(S_k, \bar{S}_m, \cdot) dk & \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 2nd \\
&\quad - \alpha \sum_{m=1}^N \frac{\partial}{\partial S_n} \int_{\bar{K}_m} (1 - \gamma_x^l(S_x, \bar{S}_l)) K_l c_x^l(S_x, \mu_x, \bar{S}_l) dx = 0. & \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 3rd
\end{aligned}
\tag{24}$$

Let us do the derivations:

$$\begin{aligned}
\frac{\partial \Pi_l}{\partial S_n} &= - (1 - \alpha) \frac{\partial C_{A_n}(S_n, \cdot)}{\partial S_n} - \frac{\partial C_{T_n}(S_n)}{\partial S_n} - \int_{\bar{K}_l \setminus \{n\}} (1 - \alpha) \frac{\partial C_{A_k}(S_k, \bar{S}_l, \cdot)}{\partial S_n} dk \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 1st \\
&- \alpha \sum_{m=1}^N \left(\frac{\partial \gamma_n^m}{\partial S_n} K_m c_n^m + \gamma_n^m K_m \frac{\partial c_n^m}{\partial S_n} \right) \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 2nd \\
&- \alpha \int_{\bar{K}_l \setminus \{n\}} \left(\frac{\partial \gamma_k^l}{\partial S_n} K_l c_k^l + \gamma_k^l K_l \frac{\partial c_k^l}{\partial S_n} \right) dk \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 2nd \\
&- \alpha \sum_{m=1}^N \int_{K_m \setminus \{n\}} \left((1 - \frac{\partial \gamma_x^l}{\partial S_n}) K_l c_x^l + (1 - \gamma_x^l) K_l \frac{\partial c_x^l}{\partial S_n} \right) dx \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 3rd \\
&- \alpha \left((1 - \frac{\partial \gamma_n^l}{\partial S_n}) K_l c_n^l + (1 - \gamma_n^l) K_l \frac{\partial c_n^l}{\partial S_n} \right) \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 3rd \\
&= \frac{\partial \pi_l}{\partial S_n} - \int_{\bar{K}_l \setminus \{n\}} (1 - \alpha) K_l \cdot \frac{\partial c_k^l(S_k, \mu_k, \bar{S}_l)}{\partial S_n} dk \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 1st \\
&- \alpha \sum_{m=1}^N \left(\frac{\partial \gamma_n^m}{\partial S_n} K_m c_n^m + \gamma_n^m K_m \frac{\partial c_n^m}{\partial S_n} \right) \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 2nd \\
&- \alpha \int_{\bar{K}_l \setminus \{n\}} \left(\frac{\partial \gamma_k^l}{\partial S_n} K_l c_k^l + \gamma_k^l K_l \frac{\partial c_k^l}{\partial S_n} \right) dk \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 2nd \\
&- \alpha \sum_{m=1}^N \int_{K_m \setminus \{n\}} \left((1 - \frac{\partial \gamma_x^l}{\partial S_n}) K_l c_x^l + (1 - \gamma_x^l) K_l \frac{\partial c_x^l}{\partial S_n} \right) dx \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 3rd \\
&- \alpha \left((1 - \frac{\partial \gamma_n^l}{\partial S_n}) K_l c_n^l + (1 - \gamma_n^l) K_l \frac{\partial c_n^l}{\partial S_n} \right) = 0. \quad \left. \vphantom{\frac{\partial \Pi_l}{\partial S_n}} \right\} 3rd
\end{aligned}$$

$$\frac{\partial C_{A_k}(S_k, \bar{S}_l, \cdot)}{\partial S_n} \Big|_{k \neq n, n \in \bar{K}_l} = \sum_{m=1}^N K_m \frac{\partial c_k^m(S_k, \mu_k, \bar{S}_m)}{\partial S_n} \Big|_{k \neq n, n \in \bar{K}_l} = K_l \cdot \frac{\partial c_k^l(S_k, \mu_k, \bar{S}_l)}{\partial S_n} \Big|_{k \neq n} \quad (25)$$

Equivalently:

$$\begin{aligned}
\frac{\partial \pi_l}{\partial S_n} = & \underbrace{\int_{\bar{K}_l \setminus \{n\}} (1 - \alpha) K_l \cdot \frac{\partial c_k^l(S_k, \mu_k, \bar{S}_l)}{\partial S_n} dk}_{1.1} \\
& + \underbrace{\alpha \sum_{m=1}^N \frac{\partial \gamma_n^m}{\partial S_n} K_m c_n^m}_{2.1} + \underbrace{\alpha \sum_{m=1}^N \gamma_n^m K_m \frac{\partial c_n^m}{\partial S_n}}_{2.2} \\
& + \underbrace{\alpha \int_{\bar{K}_l \setminus \{n\}} \frac{\partial \gamma_k^l}{\partial S_n} K_l c_k^l dk}_{2.3} + \underbrace{\alpha \int_{\bar{K}_l \setminus \{n\}} \gamma_k^l K_l \frac{\partial c_k^l}{\partial S_n} dk}_{2.4} \\
& + \underbrace{\alpha \sum_{m=1}^N \int_{\bar{K}_m \setminus \{n\}} \left(1 - \frac{\partial \gamma_x^l}{\partial S_n}\right) K_l c_x^l dx}_{3.1} \\
& + \underbrace{\alpha \sum_{m=1}^N \int_{\bar{K}_m \setminus \{n\}} (1 - \gamma_x^l) K_l \frac{\partial c_x^l}{\partial S_n} dx}_{3.2} \\
& + \underbrace{\alpha \left(1 - \frac{\partial \gamma_n^l}{\partial \bar{S}_l} \frac{\partial \bar{S}_l}{\partial S_n}\right) K_l c_n^l}_{3.3} + \underbrace{\alpha (1 - \gamma_n^l) K_l \frac{\partial c_n^l}{\partial \bar{S}_l} \frac{\partial \bar{S}_l}{\partial S_n}}_{3.4}.
\end{aligned} \tag{26}$$

Here we have three channels of n 's speed choice influence: S_n directly affects the probability of inflicting self-damage via the cost incurred given being guilty, and via the probability of being guilty, and indirectly via the generalized speed \bar{S}_l as an argument of accident cost functions as well as the probabilities of being involved into an accident when not being guilty. The term 1.1 reflects changes of other drivers of firm l willingness to pay for insurance, as speeding of n increases accident costs of other drivers through the generalized speed \bar{S}_l . Comparing the other terms with those obtained for

μ , we spot some similarities. As such, the term 2.2 mirrors the first term from (22), and being now accompanied by terms 2.1, 2.3, and 2.4, it represents the moral hazard, namely, self-induced accident cost covered by insurance and, thus, not taken into account by the driver. The last term 3.4 of (26) mirrors the last term in (22), and now together with 3.1,⁷ 3.2, and 3.3, it is equal to a compensation to particle n when another driver of firm l is guilty. Because, in contrast to what we assumed for the technology, speed not only affects one's own expected accident costs but also those of fellow drivers, we therefore get additional terms in (26) compared to (22). In particular, terms 2.1 and 3.3 reflect that individual speed S_n affects the probability γ_n^m to cause an accident and to be involved into one as a non-guilty party. When the guilty party is insured by firm l , the cost are firm-internal, and hence firm l fines it, optimal to adjust the slope accordingly. Terms 2.3, 2.4, 3.1, and 3.2 reflect indirect effect of individual speed choice on accident costs and probabilities of guilt via generalized speed \bar{S}_l .

The second-best monopolistic slope is represented as follows:

$$\frac{\partial \pi_1}{\partial S_n} = \alpha K \frac{\partial c_n^1}{\partial S_n} + \int_{\bar{K} \setminus \{n\}} K \frac{\partial c_k^1}{\partial S_n} dk + \alpha \int_{\bar{K} \setminus \{n\}} K c_x^1 dx + \alpha K c_n^1. \quad (27)$$

It is easy to notice (cf. (13)) that, unlike with safety technology choice, a social regulator has to introduce a policy in order to keep drivers' speed

⁷In this term, $\bar{K}_m \setminus \{n\} = \bar{K}_m$, for all $m \neq l$, as $n \in \bar{K}_l$.

choice optimal even if there is only one insurance firm present on the market. The difference is not only in the two last terms of (27) but also stems in the difference between the first and second-best aggregate kilometrage. This conclusion is in line with what we observe in reality, where speed limits are one of the widely spread road safety regulations.

5. Insurance market with imperfect control of speed or technology choice

Let us now consider the case where firms do not have perfect control over all choices. We assume now that firms charge drivers insurance premiums that do not depend on speed choices. Firms therefore cannot influence drivers' speed directly, but can take into account drivers' incentives to balance marginal accident costs and time spent on the road, which is given by:

$$(1 - \alpha) \frac{\partial C_{A_n}}{\partial S_n} + \frac{\partial C_{T_n}}{\partial S_n} = 0, \quad \forall n \in \bar{K}_l. \quad (28)$$

This is equivalent to

$$(1 - \alpha) \sum_{m=1}^N K_m \frac{\partial c_n^m}{\partial S_n} + \frac{\partial C_{T_n}}{\partial S_n} = 0, \quad \forall n \in \bar{K}_l.$$

Furthermore, the equilibrium condition for aggregate kilometrage becomes:

$$\tilde{\pi}_l = \mathcal{D}_l(\cdot) - (1 - \alpha)C_{A_k}(\cdot) - C_{T_k}(S_k) - C_{M_k}(\mu_k). \quad (29)$$

Firm l maximizes its profit (2), where the premium π_l is now substituted by a new premium $\tilde{\pi}_l$, which is not regulated by speed restrictions:

$$\begin{aligned}\tilde{\Pi}_l(\cdot) &= \int_{\bar{K}_l} \tilde{\pi}_l(\cdot) dk \\ &\quad - \alpha \int_{\bar{K}_l} \sum_{m=1}^N \gamma_k^m(S_k, \bar{S}_m) K_m c_k^m(S_k, \mu_k, \bar{S}_m) dk \\ &\quad - \alpha \sum_{m=1}^N \int_{\bar{K}_m} (1 - \gamma_x^l(S_x, \bar{S}_l)) K_l c_x^l(S_x, \mu_x, \bar{S}_l) dx,\end{aligned}\tag{30}$$

with respect to the equilibrium conditions (28) and (29). The Lagrangian of this maximization problem is

$$L_l = \tilde{\Pi}_l + \lambda_S \left((1 - \alpha) \sum_{m=1}^N K_m \frac{\partial c_n^m}{\partial S_n} + \frac{\partial C_{T_n}}{\partial S_n} \right),\tag{31}$$

where the Lagrangian multiplier λ_S is the shadow price reflecting the marginal impact of condition (28) on optimized profits. The higher the shadow price, the stronger the inability of insurance company to control drivers' speed influences its profit, and so is the adjustment for the remaining instruments to imperfectly compensate for this. Analysis of this Lagrangian will provide us with a new, second-best insurance premium, as well as a second-best slope with respect to driver's own safety technology. We discuss this and the analytical representation of λ_S below, but first present the second-best optimum premium and slope for technology under this constraint, leaving λ_S as a variable in our analytical expressions.

The F.O.C. for the Lagrangian with respect to aggregate kilometrage K_l

is:

$$\begin{aligned}\frac{\partial L_l}{\partial K_l} &= \frac{\partial \tilde{\Pi}_l}{\partial K_l} + \lambda_S \frac{\partial}{\partial K_l} \left((1 - \alpha) \sum_{m=1}^N K_m \frac{\partial c_n^m}{\partial S_n} + \frac{\partial C_{T_n}}{\partial S_n} \right) \\ &= \frac{\partial \tilde{\Pi}_l}{\partial K_l} + \lambda_S (1 - \alpha) \frac{\partial c_n^l}{\partial S_n} = 0.\end{aligned}\quad (32)$$

Because $\frac{\partial \tilde{\Pi}_l}{\partial K_l}$ in (32) takes on the same form as $\frac{\partial \Pi_l}{\partial K_l}$ in the original firm problem (14), the new premium per kilometer driven is:

$$\tilde{\pi}_l = \pi_l - \lambda_S (1 - \alpha) \frac{\partial c_n^l}{\partial S_n}, \quad (33)$$

where π_l represents the analytical expression in (16)–(18). The last term in (33) corrects for the inability to directly affect speed choice, and only takes care of the uninsured part of accident costs within the firm. The fact that the per kilometer accident costs are different (typically, higher) when speed cannot be affected will be reflected already in the different equilibrium values for the variables in π_l .

The F.O.C. with respect to individual safety technology choice μ_n is:

$$\begin{aligned}\frac{\partial L_l}{\partial \mu_n} &= \frac{\partial \tilde{\Pi}_l}{\partial \mu_n} + \lambda_S \frac{\partial}{\partial \mu_n} \left((1 - \alpha) \sum_{m=1}^N K_m \frac{\partial c_n^m}{\partial S_n} + \frac{\partial C_{T_n}}{\partial S_n} \right) \\ &= \frac{\partial \tilde{\Pi}_l}{\partial \mu_n} + \lambda_S (1 - \alpha) K_l \frac{\partial^2 c_n^l}{\partial S_n \partial \mu_n} = 0.\end{aligned}\quad (34)$$

Comparing (34) to (21), the last term of (34) shows that the stronger interrelation between driver's speed and technology choices is, the higher the indirect control of the firm is, and the more strongly it will adjust the slope

with respect to μ_n in order to also affect S_n . Since drivers only react to the change of uninsured part of the costs, assuming the rest covered by the insurer, this cross-effect is multiplied by $(1 - \alpha)$. In this extra term, the insurance firm internalizes only internal accident costs, and ignores those accidents where other firms' clients involved. The latter implies that a larger firm has more powerful instruments to influence its drivers' behavior as it fines drivers for a larger share of the accident externality they impose on others.

Finally, the F.O.C. with respect to individual speed S_n allows us to find an analytical representation of the Lagrangian multiplier λ :

$$\begin{aligned} \frac{\partial L_l}{\partial S_n} &= \frac{\partial \tilde{\Pi}_l}{\partial S_n} + \lambda_S \frac{\partial}{\partial S_n} \left((1 - \alpha) \frac{\partial C_{A_n}}{\partial S_n} + \frac{\partial C_{T_n}}{\partial S_n} \right) \\ &= \frac{\partial \tilde{\Pi}_l}{\partial S_n} + \lambda_S \left((1 - \alpha) \frac{\partial^2 C_{A_n}}{\partial S_n^2} + \frac{\partial^2 C_{T_n}}{\partial S_n^2} \right) = 0. \end{aligned} \quad (35)$$

From (35) we obtain the Lagrangian multiplier λ , equal to the marginal firm's profit over the marginal relaxation of the condition (28):

$$\lambda_S = - \frac{\partial \tilde{\Pi}_l / \partial S_n}{(1 - \alpha) \frac{\partial^2 C_{A_n}}{\partial S_n^2} + \frac{\partial^2 C_{T_n}}{\partial S_n^2}}. \quad (36)$$

The magnitude of the shadow price λ_S (and so an incentive of insurance firm to control its drivers' speed choice via premiums and optimal "slopes" for technology) is proportional to firm's marginal profit with respect to individual speed choice, in the numerator of (36). The higher the losses a firm faces due to speeding, the stronger will be its effort to use accessible instruments

to control the speed choice. On the other hand, λ_S is in inverse proportion to the marginal change of the equilibrium condition (28), which reflects the sensitivity of the equilibrium perceived price with respect to speed. The closer the denominator to zero, the higher the speed change induced in reaction to the firm's manipulation of speed via the instruments available to the firm (μ_n and K_l , here), because drivers will have to make a larger adjustment in speed to achieve equilibrium again. The role of the denominator is thus to reduce λ when perceived price reacts strongly to S_n , as this implies that a change in that price will induce only a relatively small adjustment in S_n .

Lack of firm's control over drivers' choice of own safety technology in the presence of (firm's) speed regulation, as well as an inability to control either of the variables S_n and μ_n , are not conceptually different from the case we have just considered and will therefore not be presented analytically. The corresponding Lagrangian multipliers of the cross effects are the following:

$$\lambda_M = - \frac{\partial \bar{\Pi}_l / \partial \mu_n}{(1 - \alpha) \frac{\partial^2 C_{A_n}}{\partial \mu_n^2} + \frac{\partial^2 C_{M_n}}{\partial \mu_n^2}}, \quad (37)$$

where $\bar{\Pi}_l$ stands for the profit of firm whilst insurance premium is considered to be dependent on speed but not on technology.

Appendix A presents the case where the firm can only set a premium, independent of individual's speed or technology choice. The firm hence has to compensate for lack over optimal slopes control by adapting its insurance premium level.

6. Conclusion

In this paper we analyzed car insurance schedules that allow insurance firms and a social regulator to influence safety on the road via multiple controls over individual drivers' behavior. We compare the first-best social optimum with an insurance market of private profit-maximizing firms, given that insurance is obligatory. The insurance premium schedules that we consider consist of the insurance premium, along with what we call the "slopes" of the premium function with respect to the individual driver's choices of speed and own safety technology. Speed may be considered to represent all the individual characteristics of driving that influence both the drivers' own and other drivers' safety. Unlike speed, safety technology by assumption only decreases drivers' own accident costs, without influencing other drivers' safety. Congestion was not considered, meaning that individual speed does not directly depend on the number of drivers entering the road.

Our main framework assumes that insurance firms can control both speed and technology choices of the drivers. In that case, companies offer insurance for a certain premium per kilometer driven, and use the optimal slopes in order to motivate drivers to choose a certain equilibrium speed and technology. The insurance premium is thus characterized by three choices on behalf of the insurance company: the level of the premium and the two slopes. The level is used to affect kilometrage. Similar to what was found in [Dementyeva et al.](#)

(2015), the profit maximizing premium internalizes part externality, but not all as long as there are more tax are also similar to those in [Dementyeva et al. \(2015\)](#).

The internalization of externalities by an insurance firm is also reflected in the marginal dependence of the premium functions on individual's speed and technology level (see formulae (11), (13), (22), and (26)). When setting the optimal profit-maximizing slopes, the firm only partly internalizes marginal externalities imposed by their insurees upon other drivers. That is, it takes care only of that part of the total accident costs it has to cover, and ignores the costs covered by other firms, as well as other firms' drivers' own risks. For instance, in case of technology control, this implies that an insurance firm's optimal slope is less steep than the one of a social regulator; that is, a social regulator is stricter to an individual driver's choice of own safety technology, than a private firm, and a larger firm is stricter, than a firm with lower market power. And only a private profit-maximizing monopolist's slope with respect to technology coincides with the first-best one. On the contrary, the analysis of the first-best and second-best slopes with respect to individual speed choice shows that any private insurance fails to motivate drivers well enough, and extra regulation is required in order to keep speed choices on the socially optimal level.

We also considered the case of imperfect control over drivers' behavior.

When only one of the two “slope” variables (e.g., technology choice) can be controlled by the insurance firm directly, the other one (here, speed) can still be controlled indirectly via the insurance premium, and by the slope of the first one. The corresponding optimization problem represents a second-best problem. The associated solutions for the insurance premium and the remaining optimal slope include, in the Lagrangian multiplier, terms correcting for the lack of control over the third margin. The stronger the mutual dependence between driver’s speed and technology is, the more effective indirect control of insurance firm is. Also, a firm with larger market power implies a higher degree of accident externalities internalization. Thus, a larger firm has better instruments of indirect control. The higher the losses a firm faces due to speeding, the stronger will be its effort to use accessible instruments to control the speed choice. The role of the corrective term is reduced when perceived price reacts strongly to individual speed, as this means that a change in that price will induce a relatively small adjustment in speed.

The results achieved in this paper can be used for evaluation of road safety policies. Adjustment of the insurance premium level was thoroughly studied in [Dementyeva et al. \(2015\)](#). Depending on the number of insurance firms in the market and their size, a social regulator can introduce subsidies or taxes in order to correct for the accident externalities uninternalized by insurance firms. A no-claim policy can be used as an instrument to implement the

optimal slopes by the firms. According to our analysis, such means as (upper) speed and (lowest) technology limits as well as other regulative actions are needed even when (oligopolistic) firms manage to influence drivers' behaviour directly, and only a private profit-maximizing monopolist would give exactly the same incentive for drivers to choose their own safety technology as a social regulator does.

This paper also offers some perspective on future research. Our model includes a number of important assumptions. The first to mention is that the model is deterministic. While in reality many processes and variables of the road traffic are better described as stochastic, for the moment, we exclude any of it from our consideration. Furthermore, we excluded possible congestion (and so its influence on speed choice) from consideration, the drivers were assumed to be symmetric, information about expected accident costs as well as about the dependance of insurance premiums on speed and technology was full and available equally to all actors on the market. Relaxation of these assumptions such as including drivers' diversity (they can have different value of time or safety, level of income, be more or less dangerous and/or risk-averse), and considering asymmetric information and stochastic elements (notably, accidents), gives possible directions to further develop the model.

Appendix A. Insurance market with no control over speed neither technology choices

Once a private profit-maximizing firm does not have control of speed nor technology choice of its drivers, it solves the following constrain maximization problem: $\max \hat{\Pi}_l$ with premiums that do not directly reflect individual speed and safety technology. The equilibrium constraints are:

$$\hat{\pi}_l(\cdot) = D_l - (1 - \alpha)C_{A_n} - C_{T_n} - C_{S_n}, \quad (\text{A.1})$$

$$(1 - \alpha) \frac{\partial C_{A_n}}{\partial \mu_n} + \frac{\partial C_{M_n}}{\partial \mu_n} = 0, \quad (\text{A.2})$$

$$(1 - \alpha) \frac{\partial C_{A_n}}{\partial S_n} + \frac{\partial C_{T_n}}{\partial S_n} = 0. \quad (\text{A.3})$$

We will use notations $condM$ and $condS$ for the expressions on the left-hand side (A.2) and (A.3). We work with the corresponding Lagrangian that includes $\hat{\lambda}_M$ and $\hat{\lambda}_S$, the Lagrangian multipliers of $condM$ and $condS$, respectively:

$$\hat{L} = \hat{\Pi}_l + \hat{\lambda}_M \cdot condM + \hat{\lambda}_S \cdot condS.$$

From FOCs we conclude that the Lagrangian multipliers are a solution of the following linear system:

$$\frac{\partial \hat{\Pi}_l}{\partial S_n} + \hat{\lambda}_M \cdot \frac{\partial condM}{\partial S_n} + \hat{\lambda}_S \cdot \frac{\partial condS}{\partial S_n} = 0, \quad (\text{A.4})$$

$$\frac{\partial \hat{\Pi}_l}{\partial \mu_n} + \hat{\lambda}_M \cdot \frac{\partial condM}{\partial \mu_n} + \hat{\lambda}_S \cdot \frac{\partial condS}{\partial \mu_n} = 0. \quad (\text{A.5})$$

Solving the system, we get:

$$\hat{\lambda}_M = - \left| \begin{array}{cc} \frac{\partial \hat{\Pi}_l}{\partial S_n} & \frac{\partial \text{cond}S}{\partial S_n} \\ \frac{\partial \hat{\Pi}_l}{\partial \mu_n} & \text{cross} \end{array} \right| / \left| \begin{array}{cc} \text{cross} & \frac{\partial \text{cond}S}{\partial S_n} \\ \frac{\partial \text{cond}M}{\partial \mu_n} & \text{cross} \end{array} \right|, \quad (\text{A.6})$$

$$\hat{\lambda}_S = - \left| \begin{array}{cc} \text{cross} & \frac{\partial \hat{\Pi}_l}{\partial S_n} \\ \frac{\partial \text{cond}M}{\partial \mu_n} & \frac{\partial \hat{\Pi}_l}{\partial \mu_n} \end{array} \right| / \left| \begin{array}{cc} \text{cross} & \frac{\partial \text{cond}S}{\partial S_n} \\ \frac{\partial \text{cond}M}{\partial \mu_n} & \text{cross} \end{array} \right|, \quad (\text{A.7})$$

where *cross* stands for the cross-effects $\frac{\partial \text{cond}M}{\partial S_n}$ and $\frac{\partial \text{cond}S}{\partial \mu_n}$, both being equal to $\frac{\partial^2 C_n}{\partial S_n \partial \mu_n}$.

The resulting Lagrangian multipliers $\hat{\lambda}_M$ and $\hat{\lambda}_S$ are not (easily) representable via the analytical forms of λ_S from (36) and λ_M from (37), and do not allow clear intuitive interpretation. We therefore leave it out of the main paper.

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