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**Incentives for Price  
Manipulation in Emission  
Permit Markets with  
Stackelberg Competition**

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### Summary

It has been shown in prior research that cost effectiveness in the competitive emissions permit market could be affected by tacit collusion or price manipulation when the corresponding polluting product market is oligopolistic. We analyze these cross market links using a Stackelberg model to show that under reasonable assumptions, there are no incentives to collude for lobbying prices up. However, incentives for manipulating the price of permits up appear if there is an initial free allocation of permits, which is a policy argument against grandfathering and in favor of auctioning. This effect is increasing with the amount of permits allocated to the leader. Moreover, the changes for price manipulation increase with those changes that tend to undermine the leader's advantage in output production or to reduce the leader's abatement cost.

**Keywords:** Emissions Permits, Collusion, Market Power, Duopoly, Stackelberg Model

**JEL Classification:** D43, L13, Q58

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# **Incentives for Price Manipulation in Emission Permit Markets with Stackelberg Competition**

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## **ABSTRACT**

It has been shown in prior research that cost effectiveness in the competitive emissions permit market could be affected by tacit collusion or price manipulation when the corresponding polluting product market is oligopolistic. We analyze these cross market links using a Stackelberg model to show that under reasonable assumptions, there are no incentives to collude for lobbying prices up. However, incentives for manipulating the price of permits up appear if there is an initial free allocation of permits, which is a policy argument against grandfathering and in favor of auctioning. This effect is increasing with the amount of permits allocated to the leader. Moreover, the changes for price manipulation increase with those changes that tend to undermine the leader's advantage in output production or to reduce the leader's abatement cost.

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## 1. Introduction

In this paper we check the existence of incentives for oligopolistic firms to collude in order to inflate the price of emission permits when there is a leader-follower relationship in the output market.

The use of cap-and trade (CAT) systems has become increasingly popular as a policy approach to incentivize firms to curb polluting emissions. Important examples include the US SO<sub>2</sub> trading system under the framework of the Acid Rain Program of the 1990 Clean Air Act as an early application or, more recently, the European Union Emission Trading System (EU ETS). The main reason why CAT programs are so attractive and popular among economists is that, theoretically, they allow reducing emissions in a cost-effective way by means of a price system. As long as marginal abatement costs differ across firms, incentives for trade exist and the market can play a positive role in achieving a pre-specified environmental target at a minimum cost.

Regardless of the initial allocation rule chosen for the permits, the cost-effectiveness property is well documented in the literature under the assumption of perfect competition (see Montgomery (1972) in a static setting and Rubin (1996) in a dynamic framework). Unfortunately, the perfect-market assumption rarely holds in practice and it is the case that the cost-effectiveness property is challenged if there is market power in either the permit market, in the associated product market or in both. The literature analyzing the relationship between imperfect competition and emission permits can be divided in three different branches, whether market power is introduced in the permits market, in the good market or in both simultaneously.

Regarding market power in the permit market, the pioneering work is Hahn (1984). Based on a static model a la Stackelberg, he stated that the efficiency loss due to market power depends on the initial allocation of permits, and the permit price is an increasing function of the leader's allocation. The dominant firm will manipulate the price (upwards if it is a seller and downwards if it is a buyer), unless the initial allocation equals the cost-effective one, which requires a perfectly informed regulator. Hagen & Westskog (1998) extended the Hahn setting in a dynamic two-period model and found a non-optimal distribution of abatement in an imperfect competitive market with banking and borrowing.

A second line of research addresses the concurrent existence of market power in both permit and output markets. This topic has received attention, among other authors,

by Misiolek & Elder (1989), who extended Hahn's setting to the product market and concluded that a single dominant firm can manipulate the permit market to drive up the fringe firm's cost in the product market. Hinterman (2011) found that the threshold of free allocation above which a dominant firm will set the permit price above its marginal abatement costs is below its optimal emissions in a competitive market, and that overall efficiency cannot be achieved by means of permit allocation alone

This paper fits within a third branch that considers imperfect competition in the product market but not in the permit market. The reason to choose this line is twofold. First, as noted by Montero (2009) and Muller *et al.* (2002), while market power among firms is very common in output markets, the existence of market power in emission permits is more likely to appear when the relevant players are countries rather than firms or facilities.<sup>1</sup> In the latter case, typically there are a very large number of them, which makes it very difficult that market power arises. It can be argued that this is the case in the EU ETS, with around 11,000 facilities involved and the latest steps of the European Commission seem to be in the direction of increasing even more the degree of competition (for example, by enlarging the number or involved sectors, centralizing the allocation of permits or moving from grandfathering to auctioning).

As a second reason, The EU-ETS price shock in 2005 generated a great deal of interest in issues related to market power. Initially, the price of allowances was far in excess of expectations, but in April 2006 the price suddenly fell and reached zero in mid-2007. Empirical studies have not been able to perfectly explain those too high price levels when the number of permits exceeded emissions in every year of the first phase. Then, it is natural to wonder if the reason for those price oscillations might be connected to the output market rather than the permit market in the sense that permits could be used somehow to obtain windfall profits either in the output market or the market of some important input such as electricity.

In this third line, some articles have shown that perfect competition in the permit market might not be enough to render a cost-effective outcome if the product market is not perfectly competitive. In the framework of a Cournot duopoly, Sartzetakis (1997) compares the efficiency of a competitive emissions market to a command-and-control

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<sup>1</sup> As an example regarding Annex 1 countries in the Kyoto Protocol, Russia initially received roughly a fifth of the permits and a third went to USA. Countries with market power can easily manipulate prices up (down) through tariffs on permit exports (domestic subsidies to cleaner technologies) and also implement policies regarding the linkage between domestic and foreigner markets. See Barrett (1998) for a related discussion.

regulation in which the emissions of each firm are legally fixed. Emission trading modifies the allocation of emissions among firms and consequently their production choices. Sartzetakis (2004) shows that welfare can decrease when emission trading is allowed between asymmetric firms endowed with different abatement and production technologies. The permit price that clears the market is a weighted average of the value of emissions of firms under command and control and, therefore, the cost of the more inefficient firm is reduced while the cost of the more efficient one is increased when permits trade is introduced.

Meunier (2011) analyzes the efficiency of emission permit trading between two imperfectly competitive product markets and concludes that even if the firms are price takers in the permit market, the integration of permit markets can decrease welfare because of imperfect competition in product markets. Theoretically, if markets are perfectly competitive, a unique global permit market that covers all polluting activities would be efficient to allocate an aggregate emissions level. If markets are not perfect but some firm enjoys market power instead, several permit markets may be more efficient than an integrated one.

The closest to our paper is the one by Ehrhart *et al.* (2008), which shows that collusion in the product market may occur even if the firms are price takers in the permit market. Under some conditions, a permit price increase leads to higher profits due to a decrease in product quantities which increases the output price. In the industrial organization literature this strategy is generally known as “raising rivals’ costs”. In the particular case of an emission permit market, Erhart *et al.* (2008) conclude that firms might have incentives to collude in order to push the price of permits upwards. Although this movement has the direct effect of increasing one’s cost, since it also raises the rival’s, both firms could benefit by restricting the quantity and increasing the price. They argue that in the EU ETS, even if there is no explicit market power in the permit market itself, there are loopholes in the trading law that allow collusive behavior among firms to manipulate the price of permits.

This paper addresses the question if firms’ interests could be aligned to push the price of permits up (and, therefore, if there are incentives to collude) under Stackelberg competition. So, we check if the colluding incentives reported by Erhart *et al.* (2008) might still arise in a setting that is asymmetric in nature in the sense that there is a leader and a follower, whereas Erhart *et al.* restrict to purely symmetric settings.

We first set a general model in which we show that the effect of a higher permit

price on the leader's and the follower's profit is ambiguous. So, the possibility that firms benefit from a price increase still exists, but the asymmetric role of the firms entail that such a possibility happens under different conditions for the leader and the follower, which introduces the possibility that one firm is interested in rising and the other in decreasing the permit price.

Then, we explore a particular case with a separable cost function to come up with more accurate insights. As a first central finding, under the reasonable assumption that the solution is interior (both firms produce, pollute and abate to some extent), we conclude that both firms face a profit function that is convex in the permit price. Moreover, within the relevant range, when the price is low enough, both firms will benefit from a further price reduction, and for high enough prices, the follower would benefit from a price increase while the leader would still prefer the price to decrease. Therefore, in the latter range there is no room for collusion. This is in contrast to Ehrhart *et al.* (2008), who set a symmetric model and therefore, both firms' interests are always aligned. The implication of this finding is that the existence of leadership in output markets reduces the room for collusive agreements in the permit market. Actually, in our specific example with a separable function, we conclude that the collusive region shrinks to the extent that it disappears.

As a first extension, we consider the possibility that some permits are distributed for free (by means of grandfathering) and we conclude that this possibility opens up the way for collusive agreements. In fact, apart from the two regions identified in the simple case, there is a third region in which both firms are interested in pushing the price up and this region is wider the more permits are distributed for free. This result points out an important argument against grandfathering in the sense that it could introduce incentives to foster collusive behavior.

As a second extension, we explore the effect of asymmetries and we conclude that the chances to face an environment that is propitious for a collusive agreement are very sensitive to the configuration of the cost parameters of both firms and the allocation of free permits received by the leader, but not by the follower. In short, those parameter changes that tend to undermine the leader's advantage in output production (i.e., an increase in the leader's cost or a decrease in the follower's cost) have the effect of making the firms more symmetric in a certain sense and, therefore, it increases the chances to observe a collusive behaviour. The opposite happens with the abatement costs: the chances for a collusive behavior tend to decrease with the leader's and to

increase with the follower's abatement cost. The reason is that, since the leader produces more output than the follower, its cost is more sensitive to the permit price and thus it is more difficult for him to get benefited by such a price increase, and this is truer the higher his abatement cost. On the other hand, an increase in the follower's abatement cost reduces the possibility that it is optimal for him to pollute zero, which widens the interior solution range and, consequently, also the scope for agreement.

The remainder of the paper is organized as follows. Section 2 states the basic model. In Section 3 a particular abatement cost function is considered, including the basic case and the two extensions. Concluding remarks are given in Section 4 and all the mathematical proofs are gathered in an appendix.

## 2. The general model

We set up a simple duopoly Stackelberg model of a polluting industry sector with tradable permits. Firm 1 is a leader and firm 2 is a follower in the output market. Following Erhart *et al.* (2008) we assume no explicit market power in the permit market, although we implicitly consider the possibility that they might find some extra-market ways to agree in lobbying activities to push the price up or down.<sup>2</sup> For the sake of comparability with Ehrhart *et al.* (2008), in this section we assume that the firms do not enjoy any initial allocation of permits, and so they have to buy all the permits they need in a market at a given price  $p$  (the possibility of an initial endowment of permits is considered as an extension in Section 3). The game has two stages: in the first stage, firms sequentially decide on their output levels,  $x_1$  and  $x_2$ , a la Stackelberg, facing the inverse demand function  $P(X)$ , where  $X := x_1 + x_2$ . In the second stage, they simultaneously choose on their cost-minimizing emission levels,  $e_1$  and  $e_2$ .

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<sup>2</sup> Erhart *et al.* (2008) argue that, in the EU ETS, there are several loopholes in the trading law that foster collusion. These loopholes consist in the existence of some mechanisms that create the possibility for a price manipulation even in the absence of market power. The most important of these mechanisms are first, the possibility to influence the initial allocation of permits (to make it more stringent), second, the 'opt-in' rule that enables industries not committed to participate in the permits trading system to do so voluntarily, third, the possibility to implement project-based mechanisms and pay more for these credits than they would at the market and, fourth, by paying additional emissions duties. Hinterman (2011) claims that this type of price manipulation could explain the behavior of the price for allowances during the first phase of the EU ETS, although his model is not fully comparable to ours since he assumes explicit market power in both output and permit markets.

The cost function of firm  $i \in (1, 2)$ ,  $C_i(x_i, e_i)$ , depends on output ( $x_i$ ) and emissions ( $e_i$ ) and is continuous and twice differentiable in both arguments with the following properties:

$$\frac{\partial C_i}{\partial x_i} > 0, \quad \frac{\partial C_i}{\partial e_i} < 0, \quad \frac{\partial^2 C_i}{\partial e_i^2} > 0, \quad \frac{\partial^2 C_i}{\partial x_i \partial e_i} < 0. \quad (1)$$

This function integrates production and abatement costs and reflects the fact that producing clean (with low emissions) is more costly than producing dirty. Every unit of emissions must be covered by a permit that can be obtained in the market at a given price  $p$ . Considering the cost of permit purchasing, total cost of firm  $i$  is given by

$$TC_i(x_i, e_i) := C_i(x_i, e_i) + pe_i. \quad (2)$$

The model is solved by backward induction. In the second stage of the game both firms decide their emissions levels to minimize their total cost,  $TC_i(x_i, e_i)$ , while taking their output levels and the price of permits as given. If the solution is interior, we get the standard first-order condition (FOC),<sup>3</sup>

$$\frac{\partial C_i}{\partial e_i} + p = 0, \quad (3)$$

from which we obtain each firm's (inverse) demand for permits,  $e_i^*(x_i, p)$ . Total differentiation of the FOC shows that optimal emissions are increasing in output and decreasing in the permit price:

$$\frac{\partial^2 C_i}{\partial e_i^2} de_i + \frac{\partial^2 C_i}{\partial e_i \partial x_i} dx_i = 0 \Rightarrow \frac{\partial e_i}{\partial x_i} = \frac{-\frac{\partial^2 C_i}{\partial e_i x_i}}{\frac{\partial^2 C_i}{\partial e_i^2}} > 0, \quad (4)$$

$$\frac{\partial^2 C_i}{\partial e_i^2} de_i + dp = 0 \Rightarrow \frac{\partial e_i}{\partial p} = \frac{-1}{\frac{\partial^2 C_i}{\partial e_i^2}} < 0. \quad (5)$$

Using the envelope theorem, we conclude that the minimized total cost function defined as

$$TC_i^*(x_i, p) := TC(x_i, e_i^*(x_i, p)) = C(x_i, e_i^*(x_i, p)) + pe_i^*(x_i, p) \quad (6)$$

has the following properties:

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<sup>3</sup> The second order condition is always fulfilled due to the convexity of  $C_i$  in emissions.

$$\frac{\partial TC_i^*}{\partial p} = e_i^* \geq 0, \quad (7)$$

$$\frac{\partial TC_i^*}{\partial x_i} = \frac{\partial C_i}{\partial x_i} + \left( \frac{\partial C_i}{\partial e_i} + p \right) \frac{\partial e_i}{\partial x_i} = \frac{\partial C_i}{\partial x_i}, \quad (8)$$

$$\frac{\partial^2 TC_i^*}{\partial x_i^2} = \frac{\partial^2 C_i}{\partial x_i^2} + \frac{\partial^2 C_i}{\partial x_i \partial e_i} \frac{\partial e_i}{\partial x_i} = \frac{\partial^2 C_i}{\partial x_i^2} - \frac{\left( \frac{\partial^2 C_i}{\partial x_i \partial e_i} \right)^2}{\frac{\partial^2 C_i}{\partial e_i^2}}, \quad (9)$$

$$\frac{\partial^2 TC_i^*}{\partial x_i \partial p} = \frac{\partial^2 C_i}{\partial x_i \partial e_i} \frac{\partial e_i}{\partial p} = - \frac{\frac{\partial^2 C_i}{\partial x_i \partial e_i}}{\frac{\partial^2 C_i}{\partial e_i^2}} > 0. \quad (10)$$

Now we move on to the first stage, where the firms choose their output levels. We start analyzing the follower's behavior, which faces the following maximization problem:

$$\text{Max}_{x_2} \Pi_2(x_1, x_2, e_2^*(x_2, p), p) = P(x_1 + x_2)x_2 - TC_2^*(x_2, p). \quad (11)$$

The FOC of this problem is

$$P(x_1 + x_2) + \frac{\partial P}{\partial X} x_2 - \frac{\partial TC_2^*}{\partial x_2} = 0, \quad (12)$$

which, solving for  $x_2$ , gives the reaction function of the follower,  $x_2^*(x_1, p)$ . Differentiating the FOC and operating we conclude that the optimal follower's output is decreasing in the leader's output and the price of permits:

$$\frac{dx_2^*}{dx_1} = \frac{-\frac{dP}{dX}}{2\frac{dP}{dX} - \frac{\partial^2 TC_2^*}{\partial x_2^2}} < 0, \quad \frac{dx_2^*}{dp} = \frac{\frac{\partial^2 TC_2^*}{\partial x_2 \partial p}}{2\frac{dP}{dX} - \frac{\partial^2 TC_2^*}{\partial x_2^2}} < 0. \quad (13)$$

The leader takes the follower's reaction function into account when maximizing its own profit. The FOC of the corresponding problem is

$$P(x_1 + x_2) + \frac{\partial P}{\partial x_1} x_1 \left( 1 + \frac{\partial x_2^*}{\partial x_1} \right) - \frac{\partial TC_1}{\partial x_1} = 0 \quad (14)$$

and, by differentiating the FOC, we conclude that the leader's output supply is also decreasing in the price of permits:

$$\frac{dx_1^*}{dp} = \frac{\frac{\partial^2 TC_1^*}{\partial x_1 \partial p}}{\frac{\partial P}{\partial x_1} \left( 2 + \frac{\partial x_2^*}{\partial x_1} \right) - \frac{\partial^2 TC_1^*}{\partial x_1^2}} < 0. \quad (15)$$

Now we are ready to address the main question of this paper, namely the effect of an increase in the price of permits on the firms' profit. The question is: could both firms benefit simultaneously from a price increase as predicted by Erhart et al (2008) in a symmetric setting? The motivation behind this question is that, if the answer happens to be positive, both firms might have incentives to collude or, by any means, to lobby in order to manipulate the price of permits up. For the sake of realism it is relevant to ask this question in a setting in which the firms play different roles regarding their market power as we can observe this situation in reality.

By direct differentiation of the profit functions we conclude that the marginal effect of the price of permits on the profits of both firms is given by the following expressions:

$$\frac{\partial \Pi_1}{\partial p} = \frac{\partial P}{\partial X} \frac{\partial x_2^*}{\partial p} x_1^* - e_1^*, \quad (16)$$

$$\frac{\partial \Pi_2}{\partial p} = \frac{\partial P}{\partial X} \left( 1 + \frac{\partial x_2^*}{\partial x_1^*} \right) \frac{\partial x_1^*}{\partial p} x_2^* - e_2^*. \quad (17)$$

There are two important points to be stressed here. First, the sign of both expressions is ambiguous. The reason is that an increase in the price of permits has two different effects. On the one hand, it drives cost up, which tends to reduce firms' profit but, on the other hand, it also causes output to decrease and, therefore, the product price to increase, which can be beneficial for both firms. If the second effect happens to dominate the first, then profit will increase with the price of permits. If this happens simultaneously for both firms, there exist incentives to collude in order to manipulate the price up, as noted by Ehrhart *et al.* (2008).

Second, the conditions under which a higher price is profit-enhancing are different for the leader and the follower. This opens up the possibility of a disagreement between the firms in the sense that one of them is interested in a price increase and the other one in a price decrease. This is in contrast to Ehrhart *et al.* (2008), where both firms are symmetric and, therefore, either both firms are better-off or both are worse-off after a price increase. This asymmetry seems to reduce the scope for a price

manipulation agreement. At this level of generality it is not possible to get more specific insights. For that reason, we explore a specific case in the next section.

### 3. A Separable Function

To get some additional insight, in this section we assume a particular abatement cost function which is separable from production cost. We initially consider that there is no grandfathering and the cost functions of both firms are exactly the same, so that the only difference between them is due to their roles as leader and follower. After studying this basic case, we explore, first, the effect of distributing free permits to the firms and, second, the consequences of considering cost asymmetries.

#### 3.1. Basic case

Assume that the production and abatement costs are separable in the following way. The production cost of firm  $i$  is given by  $cx_i$ , so there is a constant marginal production cost equal to  $c$ . The (inverse) demand function for output has the linear form  $P(X) = a - bX$ . Every unit of output generates  $r$  units of pollution, where  $r > 0$  is a constant coefficient of pollution intensity, (thus, gross emissions of firm  $i$  are given by  $rx_i$ ). By performing abatement activities, firms can reduce their flow of pollution. Denote as  $q_i \geq 0$  the amount of emissions abated by firm  $i$ . Then, net emissions are given by  $e_i = rx_i - q_i$ . Following Sarzetakis (1997) we assume the following quadratic abatement cost function, which is common for both firms:

$$AC(q_i) = q_i(d + tq_i), \quad (18)$$

where  $d$  and  $t$  are positive parameters. Adding up the costs of production, abatement and permit purchasing, and using the definition of  $q_i$ , we can write total cost as a function of output and emissions as in the general model:

$$TC_i(x_i, e_i) = cx_i + (rx_i - e_i)(d + t(rx_i - e_i)) + pe_i. \quad (19)$$

To ensure interior solution we bound the relevant parameters by including the following technical assumption:

$$\mathbf{Assumption 1:} \quad d < p < \bar{p}, \quad \text{where} \quad \bar{p} := p / e_2^*(x_2^*, p) = 0. \quad (20)$$

This assumption rules out uninteresting solutions in which any of the firms either produces zero, pollutes zero or abates zero. The lower bound for  $p$  prevents abatement from being negative (see equation (22) below). To understand this result,

note that  $d$  is the marginal cost of abatement at  $q = 0$ . If the price of permits is even lower than the cost of the first unit of abatement, this means that it will never be profitable for the firms to abate, since buying permits is a cheaper option. The upper bound for  $p$  is defined as that value of the permit price such that, in equilibrium, it is optimal for the follower to pollute zero.<sup>4</sup> The reason to include this assumption is that, in our setting, the follower's emissions is the first variable to reach a zero value as  $p$  increases and, therefore, this is a sufficient condition to ensure a nonnegative solution.<sup>5</sup>

Proceeding as in the general model, we solve first the second stage, where both firms choose their emission levels. Endowed with our specific analytical expressions, we can compute the optimal amount of emissions of firm  $i$  as a function of output:

$$e_i^*(x_i, p) = \frac{d-p}{2t} + rx_i, \quad (21)$$

from which it is straightforward to conclude that firm  $i$ 's optimal abatement is

$$q_i^*(p) = \frac{p-d}{2t} > 0, \quad (22)$$

which, due to separability, is independent of output. Using (21) in (19) we get the expression for the minimized cost function, which reveals that marginal product cost is constant in output and increasing in the permit price:

$$TC_i^*(x_i, p) = x_i(c + pr) - \frac{(d-p)^2}{4t}. \quad (23)$$

Now, we move on to solve the first stage, in which both firms decide on their output levels. By standard methods,<sup>6</sup> we get

$$x_1^* = \frac{a-c-rp}{2b}, \quad (24)$$

$$x_2^* = \frac{a-c-rp}{4b}. \quad (25)$$

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<sup>4</sup> The specific expression for  $\bar{p}$  can be found in the appendix. Specifically, it is given by (A3) in the basic model, but it takes a different form in the extensions developed below.

<sup>5</sup> If both the follower's abatement and the follower's net emissions are nonnegative, it is straightforward to conclude that the follower's gross emissions,  $rx_2$ , are nonnegative, which implies that the follower's output is nonnegative. As we show below, in equilibrium the leader always produces more and pollutes more than the follower and, thus, Assumptions 1 ensures that all the relevant variables of the model are nonnegative in equilibrium.

<sup>6</sup> The follower chooses  $x_2$  to maximize its profit while taking  $x_1$  as given. The leader chooses  $x_1$  to maximize its own profit taking into account the follower's reaction function.

From (24) and (25) we conclude that the leader's output is twice as much as the follower's as in the classical Stackelberg model with linear demand and constant marginal cost. As always, both firms' output depend positively on the demand intercept,  $a$ , and negatively on the demand slope,  $b$ , and all the cost parameters  $c$ ,  $r$  and  $p$ . We can now compute the equilibrium value of both firms' profit as a function of the price of permits, defined as:

$$\Pi_i^*(p) := \left[ a - b(x_1^* + x_2^*) \right] x_i^* - TC_i^*(x_i^*, p), \quad (26)$$

and for notational convenience, we define

$$\hat{p}_i := \arg \min_p \Pi_i^*(p) \quad i = 1, 2. \quad (27)$$

Lemma 1 and Proposition 1 show the main results of this part of the paper. Lemma 1 determines the shape of the equilibrium profit functions and, as a straightforward implication, Proposition 1 splits the relevant range for the permit price in two regions with different consequences on the interests of the firms regarding the evolution of  $p$ .

#### LEMMA 1

$\Pi_1^*(p)$  and  $\Pi_2^*(p)$  are strictly convex functions of  $p$  with  $d < \hat{p}_2 < \hat{p}_1 = \bar{p}$ . ■

#### PROPOSITION 1

*If  $d < p < \hat{p}_2$ , a price decrease would make the profit of both firms to increase. If  $\hat{p}_2 < p < \bar{p}$ , a price increase will decrease the leader's profit and increase the follower's profit.* ■

The results in Lemma 1 and Proposition 1 are illustrated in Figure 1. There are two important facts to be underlined in this figure. First, the profit of firm  $i$  is strictly convex in  $p$  with a minimum at  $\hat{p}_i$  (for  $i=1, 2$ ). We can get some insight about the reasons that underline the shape of the profit functions by dissecting the effects of an increase in the price of permits.

When  $p$  increases, both firms respond by producing less, abating more and, thus, emitting less. These responses have effects on both revenues and costs. Revenue is a strictly concave, inverted-U shaped function of  $p$ . When  $p$  is low, the resulting reduction in output induces a revenue-enhancing increase in the price of output,

whereas, for large values of  $p$ , the negative effect on output prevails. Regarding costs, there are three effects: production cost decreases linearly, abatement cost increases quadratically due to the shape of the abatement function and the cost of purchasing permits decreases quadratically due to the substitution of permits for abatement and, in marginal terms, the latter effect turns out to dominate all the others for large values of  $p$ .

The second insight from Figure 1 is that the minima of the profit functions are unambiguously ordered such that  $\hat{p}_2 < \hat{p}_1$ , i.e., the follower reaches a minimum for a lower price than the leader. As a matter of fact, it can be proved that  $\frac{\partial \Pi_1}{\partial p} < \frac{\partial \Pi_2}{\partial p}$ , i.e., for any value of  $p$ , the leader's profit decreases more or increases less than the follower's. As a consequence, we have that, if  $p < \hat{p}_2$ , both firms are in the decreasing part of their profit functions, which implies that their profit would increase if the permit price decreased. If, instead,  $\hat{p}_2 < p < \hat{p}_1$ , the follower is in the increasing part (and so it would benefit from a price increase) whereas the leader is still in the decreasing part (and, therefore, it would still prefer the price to decrease). Intuitively, the reason why the profit of firm 2 reaches a minimum before firm 1 is that, being a Stackelberg follower, it is optimal for firm 2 to produce less than firm 1 and, therefore, to pollute less. This implies that the direct effect of a price increase is milder as compared to the indirect effect (the rival's cost rise with the resulting increase in the output price).

Apparently, if  $p > \hat{p}_1$  the leader enters the increasing part of its profit function and, therefore, both firms would benefit from a higher price, but under our specification, it is the case that  $\hat{p}_1 = \bar{p}$ , i.e., the minimum of the leader's profit function is reached precisely at the highest value of the price that is compatible with an interior solution. (specifically,  $e_2 < 0$  for any  $p > \hat{p}_1$ ) and so there is no feasible range under which both firms would benefit from a price increase.<sup>7</sup>

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<sup>7</sup> For analytical simplicity (and also for the sake of realism) we do not discuss in detail the corner solution range, since the case in which any of the firms does not pollute at all seems uninteresting. Actually, if we rule out negative emissions, what happens to the right of  $\bar{p}$  is that, for firm 2, it is optimal not to pollute at all ( $e_2 = 0$ ) and, therefore, in this range its cost is not directly affected by the price of permits, while the leader's is. The expressions for output and profits that we have derived are valid only under the interior solution scenario. If we allow for corner solutions with  $e_2 = 0$ , then we come up with different expressions and, using these expressions we conclude that, in the vicinity (to the right) of  $\bar{p}$ , the leader's output and profit are still decreasing (in the price) while the follower's output and profit are increasing.

The main consequence of Lemma 1 and Proposition 1 is that, in our example, there is a range where both firms are interested in decreasing the price but, unlike the symmetric case developed by Ehrhart *et al.* (2008), it is never the case that both firms simultaneously profit from a price increase, and therefore they never have incentives to lobby in order to press the price up. Moreover, there is a range of disagreement in which the interests of both firms diverge, which can never happen in the symmetric case.

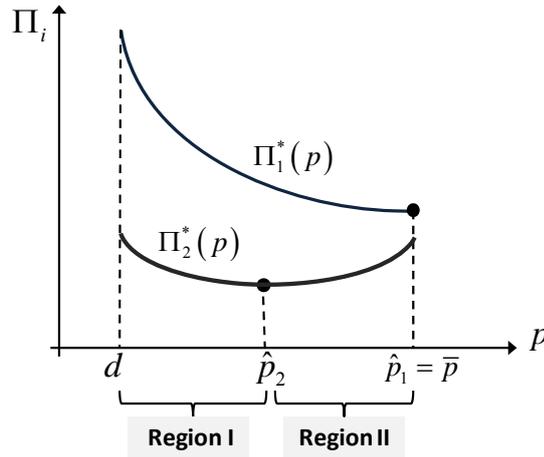


FIGURE 1: Equilibrium profits as a function of  $p$  (basic case)

In this example we have illustrated how the asymmetry between the firms (in the sense of a leader-follower relationship) reduces the chances for collusive behavior so much that they disappear. In the next subsections we show two generalizations of this example where the result is not so extreme in the sense that the chances for collusive agreements decrease with respect to a purely symmetric setting but they do not fully disappear.

### 3.2. Grandfathering

Up to now, for comparability with Ehrhart *et al.* (2008), we have assumed that the firms do not have any initial allocation of permits and, therefore, they have to buy all the permits they need in the market. In reality, it is common that the participants in CAP systems receive a number of permits for free by means of a grandfathering scheme. In fact, as it is discussed for example in Alvarez and André (2014), grandfathering has traditionally been the most widespread method to distribute permits.

We now extend our setting to consider the possibility that some permits are initially distributed with no cost for the firms by a grandfathering scheme.<sup>8</sup> So, the firms only need to buy those permits that exceed their initial allocation and, moreover, they have the option to sell permits if they pollute less than their initial allocation.

Consider that both firms receive an equal allocation of free permits,  $S$ , and denote as  $y_i$  the amount of permits that firm  $i$  buys (if  $y_i > 0$ ) or sells (if  $y_i < 0$ ) in the market, which can be calculated as the difference between net emissions and the allocation of permits:

$$y_i = e_i - S = rx_i - q_i - S, \quad (28)$$

from which we get  $e_i = y_i + S$ , i.e., the net emissions of a firm must be covered by permits that either come from her free allocation or are bought in the market. Therefore firm  $i$ 's total cost function is now given by the expression:

$$TC_i(x_i, y_i) = cx_i + (rx_i - y_i - S)(d + t(rx_i - y_i - S)) + py_i, \quad (29)$$

which can be written in terms of output and net emissions as

$$TC_i(x_i, e_i) = cx_i + (rx_i - e_i)(d + t(rx_i - e_i)) + p(e_i - S). \quad (30)$$

Solving the second stage of the game we conclude that the optimal levels of emissions and abatement for each firm are still given by (21) and (21') respectively, and it is straightforward to obtain the optimal traded permits and the corresponding minimized cost function:

$$y_i^*(x_i, p) = \frac{d - p}{2t} + rx_i - S, \quad (31)$$

$$TC_i^*(x_i, p) = x_i(c + pr) - \frac{(d - p)^2}{4t} - pS, \quad (32)$$

where separability entails that the minimized cost function has the same structure as in the basic case, except for the fact that the value of the endowment of free permits appears as a reduction in the cost. Lemma 2 and Proposition 2 are the main results of this part. We still use the notation introduced in (26) to refer to the value of the permit price that minimizes each profit function. For notational convenience we also define

$$\tilde{S} := \frac{r(a - c - dr)}{8b}.$$

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<sup>8</sup> Actually, the fact that the permits are distributed for free is not crucial for our results. The only important assumption is that the firms enjoy an exogenously given amount of permits.

## LEMMA 2

When both firms are initially endowed with the same free allocation of permits,  $S$ , the equilibrium profit functions for both firms are strictly convex with a unique minimum each at  $\hat{p}_i$  for  $i=1,2$ , with  $\frac{\partial \hat{p}_i}{\partial S} < 0$ . Moreover, the relevant thresholds are ordered in the following way:

- If  $S < \tilde{S}$ , then  $d < \hat{p}_2 < \hat{p}_1 < \bar{p}$ .
- If  $\tilde{S} < S < 2\tilde{S}$ , then  $\hat{p}_2 < d < \hat{p}_1 < \bar{p}$ .
- If  $S > 2\tilde{S}$ , then  $\hat{p}_2 < \hat{p}_1 < d < \bar{p}$ .

## PROPOSITION 2

When both firms are initially endowed with a free allocation of permits, the following results hold:

- If  $S < \tilde{S}$  the relevant range of values for  $p$  has three regions: In region I, defined by  $d < p < \hat{p}_2$ , both firms get better off when  $p$  decreases. In region II, defined by  $\hat{p}_2 < p < \hat{p}_1$ , the leader gets better off when  $p$  decreases and the follower gets better off when  $p$  increases. In region III, defined by  $\hat{p}_1 < p < \bar{p}$ , both firms get better off when  $p$  increases.
- If  $\tilde{S} < S < 2\tilde{S}$ , region I disappears and region II is delimited by  $d < p < \hat{p}_1$ .
- If  $S > 2\tilde{S}$ , regions I and II disappear and region III is defined by the whole feasible range,  $[d, \bar{p}]$ . ■

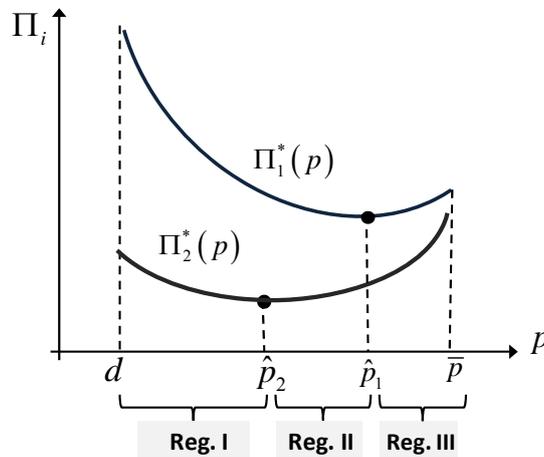


FIGURE 2: Equilibrium profits as a function of  $p$  (basic case)

The consequences of Lemma 2 and Proposition 2 are the following. The profit of both firms is still strictly convex in the price of permits, with a minimum at price  $\hat{p}_i$ ,  $i=1, 2$ . When grandfathering is introduced, the values of the permit price at which the minima are reached,  $\hat{p}_1$  and  $\hat{p}_2$ , shift to the left and they shift more the higher the value of  $S$ . This shift implies, for each firm, that there is wider range of the permit price such that it gets better-off when the price increases. The reason is that the existence of free permits makes permit purchasing less costly for firms and, moreover, it opens the way from getting positive revenues by selling some permits.

More importantly, if we focus on case *a*) (with  $S < \tilde{S}$ ), when grandfathering is included, we have three regions instead of two, as illustrated in Figure 2. In region III, to the right of  $\hat{p}_1$ , both firms profit from a price increase, while the solution is still interior ( $e_1, e_2 > 0$ ). Technically, the reason why this new region arises is that, now, the direct effect of a price increase on cost is softer since the firms have to buy fewer permits. It can also be the case, if the price is high enough, that it is profitable for the firms to sell part of their free endowment instead of buying additional permits, which provides a new opportunity to increase profits. Anyway, at  $\hat{p}_1$  we have that  $y_1 > 0$  i.e., at the point where the leader starts finding it profitable to increase the price, it is still a net buyer of permits and so the profit-enhancing effect is not due to selling permits yet.

Moreover, if the initial allocation of permits is large enough, it could be the case that region I disappears, which implies that the follower is always interested in manipulating the price of permits up (case *b*) in Lemma 2 and Proposition 2), or even that both regions I and II disappear, which implies that both the leader and the follower are always interested in manipulating the price up. This is, of course, the most favorable case for collusion.

The focus of this paper is on region III since this is the only one in which firms can find it profitable to collude in order to push the price up. One natural question is how large this region is or, in other words, how likely it is that we fall in this region. To answer this question, we focus on case *a*) ( $S < \tilde{S}$ ), which is the richer one and perhaps the most realistic (the discussion of the other two cases is straightforward). Then, region III is delimited by two threshold values for  $p$ : First,  $\hat{p}_1$ , which is the price above which it is profitable, not only for the follower, but also for the leader to push the price up. The

second threshold is the upper bound,  $\bar{p}$ , which is the highest value of the price compatible with an interior solution. By computing the difference between these two thresholds, we get the size of region III,

$$\bar{p} - \hat{p}_1 = \frac{4btS}{2b + tr^2},$$

which depends positively on the number of free permits, as well as the slope of the demand curve,  $b$ , and the abatement cost parameter  $t$ , whereas it depends negatively on the emissions intensity parameter  $r$ .

### 3.3. Asymmetric cost

In the previous developments we have considered that both firms were fully symmetric in terms of cost functions and also, in the case of grandfathering, in terms of free permit endowment. There are two reasons to hold this assumption. The first is simplicity. The second is to focus on the leader-follower relationship as the (only) source of asymmetry between firms.

In this subsection we consider the possibility that firms are asymmetric in terms of cost and/or initial permit endowment and we explore the effect of these asymmetries on the chances to generate a propitious environment for a collusive behavior. In other words, we explore the effect of different parameters on the size of region III as it was defined in the previous subsection.

To account for cost asymmetry, we denote the production cost of firm  $i$  as  $c_i x_i$ , where  $c_i$  is a firm-specific unit cost parameter. Since we have postulated that firm 1 is a leader and firm 2 is a follower in the output market, it is natural to conjecture that  $c_1 < c_2$ , i.e., the position of the leader might well be due to the fact that it enjoys a cost advantage, but nothing prevents us from considering the opposite case. Analogously, firm  $i$ 's abatement cost function is given by:

$$AC_i(q_i) = q_i(d_i + t_i q_i), \quad i = 1, 2. \quad (33)$$

Finally, each firm might receive an initial free endowment of permits,  $S_i$ , which is not necessarily constant across firms. Proceeding as in the basic case, we conclude that, in the second stage, the optimal amounts of emissions, abatement and purchase of permits for each firm are given, respectively, by<sup>9</sup>

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<sup>9</sup> Unlike the rest of parameters, we assume that the emissions intensity parameters,  $r$ , is common for both firms, i.e.,  $r_1 = r_2 = r$ . There are two pragmatic reasons for this simplification. First, the sensitivity analysis

$$e_i^*(x_i, p) = \frac{d_i - p}{2t_i} + rx_i, \quad (34)$$

$$q_i^*(p) = \frac{p - d_i}{2t_i},$$

$$y_i^*(x_i, p) = rx_i - \frac{p - d_i}{2t} - S_i,$$

and moving on to the first stage we can compute the equilibrium levels of output:

$$x_1^* = \frac{a + c_2 - 2c_1 - rp}{2b}, \quad (35)$$

$$x_2^* = \frac{a + 2c_1 - 3c_2 - rp}{4b}. \quad (36)$$

To investigate the chances to observe a collusive behavior we proceed by analyzing the effect of different parameters on the size of region III. In the previous subsection we concluded that, simply by introducing a constant initial allocation of permits, we come up with three different cases. Now, due to the larger number of varying parameters, by choosing the right combination of them we could generate almost any imaginable case. Hence, we need to bound the range of possibilities in some way to avoid, on the one hand, meaningless results (such as negative output, negative abatement or negative emissions) and, on the other hand, a qualitative change in the nature of the solution. For this reason, in this subsection we introduce the following assumptions:

**Assumption 1'**:  $\max\{d_1, d_2\} < p < \bar{p}$ , where  $\bar{p}$  is defined in (20).

**Assumption 2**:  $e_1 > e_2$ .

**Assumption 3**:  $\hat{p}_2 < \hat{p}_1$ .

The two first assumptions ensure nonnegative values for all the relevant variables. The idea is that, under any plausible scenario, the leader will still be the one who produces a larger amount of output and a larger amount of emissions. As a consequence, the follower will still be the one who finds it profitable to pollute zero for a lower value of  $p$  and such a value determines the upper bound for the range that is

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results related to these parameters are unclear and so we do not come up with any valuable insight by exploring them. Second, the sign of some equilibrium values for some of the key variables are affected by the terms  $2r_1 - r_2$  and/or  $3r_1 - 2r_2$  and this fact, anyway, forces us to keep the asymmetry between these parameters bounded to avoid meaningless results.

compatible with an interior solution,  $\bar{p}$ . If this is the case, it is natural to accept that Assumption 3 also holds, i.e., it is easier for the follower than it is for the leader to benefit from a price increase.

Under these assumptions, region III is delimited by  $\hat{p}_1$  and  $\bar{p}$  and hence its size increases if  $\bar{p}$  increases and/or  $\hat{p}_1$  decreases. Proposition 3 summarizes how the size of this region depends on the parameters of the model. Table 1 presents a taxonomy of all the relevant effects.

### PROPOSITION 3

*The size of region III is increasing in the following cases:*

- a) *If the leader's marginal production cost,  $c_1$ , increases or the follower's marginal production cost,  $c_2$ , decreases.*
- b) *If the parameter of the linear term in the abatement cost function decreases for the leader ( $d_1$ ) or increases for the follower ( $d_2$ ).*
- c) *If the parameter of the quadratic term in the leader abatement cost function,  $t_1$ , decreases (provided the number of free permits is moderate) or the equivalent follower's parameter,  $t_2$ , increases.*
- d) *If the number of free permits received by the leader,  $S_1$ , is increasing regardless of the free permits received by the follower.*

Effects on thresholds	Changes in parameters of the model							
	$c_1$	$c_2$	$d_1$	$d_2$	$t_1$	$t_2$	$S_1$	$S_2$
$\Delta\bar{p}$	+	-	0	+	0	+	0	0
$\Delta\hat{p}_1$	-	+	+	0	+	0	-	0
$\Delta(\bar{p} - \hat{p}_1)$	+	-	-	+	-	+	+	0

Table 1. Summary of sensitivity analysis results.

(\*) For a moderate value of  $S_1$ .

Regarding point a) in Proposition 3, increasing the leader's production cost or reducing the follower's cost tends to erode the leader's advantage with respect to the follower, which has the effect of making the firms more symmetric in terms of their position in the market. The more symmetric the firms are, the more aligned their interests are and, hence, it is more likely that they find it profitable to collude. Table 1 reveals that increasing  $c_1$  has a double effect: on the one hand,  $\bar{p}$  grows because the output of the follower increases, which makes it less likely that firm 2 decides not to emit at all (in other words, the range of prices under which there is an interior solution

widens). On the other hand,  $\hat{p}_1$  decreases since, due to a higher cost, firm 1 tends to produce less, to emit less and, therefore, its total cost is less sensitive to an increase in the price of permits. Both of these effects tend to enlarge the agreement region. Just the opposite happens when  $c_2$  increases: firm 1 tends to produce more and pollute more and hence its cost becomes more sensitive to an increase in the price of permits (which increases the value of  $\hat{p}_1$ ) whereas the follower tends to produce less and to reach sooner the point where it finds it profitable to stop polluting ( $\bar{p}$  decreases), which reduces the size of the agreement region.

As it comes to the parameters of the abatement cost function ( $d_i$  and  $t_i$ ) notice that, due to separability, each firm's parameters are only relevant for the own firm, but not for its rival. Both the linear and the quadratic term of firm 2 are irrelevant to determine the value of  $\hat{p}_1$  but increasing any of them makes the follower's abatement cost to increase, which in turn makes it less likely to reach the point where it decides to pollute zero or, in other words, it enlarges the relevant feasible range. The corresponding parameters for firm 1 do not matter to determine the value of  $\bar{p}$  and their only relevant effect is on  $\hat{p}_1$ . Assuming a moderate value of the leader's initial endowment of permits, any increase in  $d_1$  and  $t_1$  makes the leader's abatement cost higher, which makes firm 1 becomes more sensitive to increases in the price of permits.

Finally, the initial allocation of permits is irrelevant for the upper bound of  $p$  since it represents just a fixed term in the cost (and the profit) function and so the optimal decisions are not affected. The value of a firm's profits is affected by its own endowment (not the rival's), and hence only  $S_1$  is relevant to determine the size of region III. When the leader's free endowment increases, its cost becomes less sensitive to an increase in the permits price and, therefore, it would be more receptive to the idea of pushing the price up, which increases the chances of a collusive behavior.

#### **4. Conclusions and policy implications**

We have explored the possibility that two firms that compete a la Stackelberg in the output market and are subject to a CAT system could have incentives to manipulate the price of permits up. We do so in a framework similar to the one by Erhart *et al.* (2008) with the difference that they restrict to symmetric situations whereas we explore a situation that is asymmetric in nature. The main research question is if the incentives

for this type of collusive behavior, which have been reported by Erhart *et al.* (2008) for symmetric models, still exist in a situation in which some firm has a dominant position and other or others act as followers.

In a general model, we have shown that the effect of a permit price increase on the firms' profit has an ambiguous sign. This opens the way for the firms to benefit from a price increase and the possibility to make collusive agreements in order to manipulate the price up. Nevertheless, the asymmetric role of each firm causes that the conditions under which a price is profit-enhancing are different for each of them.

Under a separable cost function we show, first, that the profit functions are strictly convex in the permit price and, second, that the minima of the profit functions are different for both firms, which creates a region of disagreement where the leader prefers that the price goes down whereas the follower prefers it to go up. This situation is ruled out in Erhart *et al.* (2008) by construction since the interests of fully symmetric firms are always aligned.

The main message is that a leader-follower relationship between firms reduces the scope for an agreement between the firms to manipulate the price up. Actually, in the basic case in which there are no free permits distributed among the firms and the cost functions are symmetric, the region where there incentives to collude shrinks to the extent to disappear. Then main policy implication of this finding is that a situation of market power in the product market can prevent the existence of incentives for collusion in the permit market.

Another policy implication of our research is that distributing some permits for free (e.g. by means of grandfathering) opens up the possibility for collusive behaviour and the more permits are distributed by a non-market scheme, particularly to the firms that enjoy market power, the more incentives for collusion. The European Union is reducing the use of grandfathering and increasing the use of auctioning to distribute emission permits. The 2008 revised European Emission Trading Directive established as a fundamental change for the third trading period, starting in 2013, the mandate that auctioning of allowances is to be the default method for allocating allowances. The arguments posed by the European Commission (EC) to support the introduction of auctions are that auctioning "best ensures the efficiency, transparency and simplicity of the system, creates the greatest incentives for investment in a low-carbon economy and

eliminates windfall profits".<sup>10</sup> Our results suggest an additional argument to reduce the use of grandfathering (and, arguably, to increase the use of auctioning) as it might introduce incentives for price manipulation.

Our final insight is that the chances for the firms to find collusion profitable are very sensitive to the cost asymmetries between them. In general terms, the more asymmetric the firms are, the more difficult collusion becomes.

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<sup>10</sup> See [http://ec.europa.eu/clima/policies/ets/cap/auctioning/faq\\_en.htm](http://ec.europa.eu/clima/policies/ets/cap/auctioning/faq_en.htm), section "Why are allowances being auctioned?". Alvarez and André (2014) study the efficiency argument.

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## APPENDIX

### Proof of Lemma 1

Using (23) and (24) in (21) we get the equilibrium values for emissions:

$$e_1^*(x_1^*, p) = \frac{db + rt(a - c) - p(b + tr^2)}{2bt}, \quad (\text{A1})$$

$$e_2^*(x_2^*, p) = \frac{2bd + tr(a - c) - p(2b + tr^2)}{4bt}, \quad (\text{A2})$$

and using the definition given in (20), we compute the value of  $\bar{p}$  by equating (A2) to zero:

$$e_2^*(x_2^*, p) = 0 \Rightarrow p = \bar{p} = \frac{2bd + tr(a - c)}{2b + tr^2}. \quad (\text{A3})$$

Using (23) and (24) in the inverse demand expression  $P(X) = a - bX$  we get the equilibrium price of output:  $P = \frac{a + 3(c + pr)}{4}$ . Using the expressions for  $x_1$ ,  $x_2$ ,  $e_1$ ,  $e_2$  and  $P$  together with (19), we get the expressions for the equilibrium profits of both firms:

$$\Pi_1^*(p) = \frac{t(a - c - pr)^2 + 2b(d - p)^2}{8bt},$$

$$\Pi_2^*(p) = \frac{t(a - c - pr)^2 + 4b(d - p)^2}{16bt}.$$

Differentiating twice with respect to  $p$  we conclude that the second derivative of both functions is positive and, thus, both of them are strictly convex in  $p$ . By inspection of the first derivative, and bearing (A3) in mind, we conclude that  $\Pi_1$  has a minimum at  $p = \hat{p}_1 = \bar{p}$ , which implies that  $\Pi_1$  is decreasing in  $p$  for all the feasible values of  $p$  below  $\bar{p}$ . In a similar way, we conclude that  $\Pi_2$  has a minimum at  $\hat{p}_2 := \frac{tr(a - c) + 4bd}{4b + tr^2}$ , which implies that  $\Pi_2$  is decreasing in  $p$  for  $p < \bar{p}_2$  and increasing for  $p > \bar{p}_2$ .

The last step is to check that the thresholds are ordered in the right way. By direct comparison we conclude that

$$d < \hat{p}_2 < \bar{p} \Leftrightarrow a - c > dr.$$

To prove that the last inequality is true, using (21') and the definition of abatement ( $q_i = rx_i - e_i$ ) we conclude that, in the relevant range,  $x_2 > \frac{e_2}{r} > 0$ . Using the expression for  $x_2^*$  given in (24) we conclude that  $x_2 > 0$  implies  $a - c > rp$ , and this inequality, together with the first part of Assumption 2 ( $d < p$ ), implies  $a - c > dr$ . QED.

### Proof of Proposition 1

The result straightforwardly follow from Lemma 1: the relevant range for  $p$  is delimited by  $d$  and  $\bar{p}$ . Since  $\Pi_2^*$  is strictly convex and it reaches at minimum at  $\hat{p}_2$ , we conclude that it is strictly decreasing between  $d$  and  $\hat{p}_2$  and strictly increasing between  $\hat{p}_2$  and  $\bar{p}$ . Since  $\Pi_1^*$  is strictly convex and it reaches at minimum at  $\hat{p}_1 = \bar{p}$ , it is strictly decreasing between  $d$  and  $\bar{p}$ . This completes the proof. QED.

### Proof of Lemma 2

Since the expressions for  $e_i$  ( $i=1,2$ ) are the same as in the basic case and the minimized cost function (31) is the same as (22) except for a constant term, it immediately follows that the expressions for  $x_i$  ( $i=1,2$ ) are also the same as in the basic case. Using these values we get the equilibrium profits of both firms

$$\begin{aligned}\Pi_1^*(p) &= P(x_1^* + x_2^*)x_1^* - TC_1^*(x_1^*, p) = \frac{t(a-c-pr)^2 + 2b(d-p)^2 + 8btpS}{8bt}, \\ \Pi_2^*(p) &= P(x_1^* + x_2^*)x_2^* - TC_2^*(x_2^*, p) = \frac{t(a-c-pr)^2 + 4b(d-p)^2 + 16btpS}{16bt}.\end{aligned}$$

The second derivative reveals that these functions are still strictly convex. Differentiating them with respect to  $p$  we conclude that they have respective minima at

$$\begin{aligned}\text{Arg min}_p \Pi_1^*(p) &\equiv \hat{p}_1 = \frac{rt(a-c) + 2bd - 4btS}{2b + tr^2}, \\ \text{Arg min}_p \Pi_2^*(p) &\equiv \hat{p}_2 = \frac{rt(a-c) + 4bd - 8btS}{4b + tr^2},\end{aligned}$$

and it is straightforward that both  $\hat{p}_1$  and  $\hat{p}_2$  depend negatively on  $S$ .

Regarding the order of the thresholds, by direct comparison we conclude that  $\hat{p}_1 > \hat{p}_2 \Leftrightarrow 2brt(a - c - dr + 2rSt) > 0$ , but in the proof of Proposition 1 we have proved  $a - c - dr \geq 0$ , which ensures that  $\hat{p}_1 > \hat{p}_2$ . Moreover, using (20') we also conclude that  $\bar{p} = \hat{p}_1 + \frac{4btS}{2b + tr^2} > \hat{p}_1$ . So, we have  $\hat{p}_2 < \hat{p}_1 < \bar{p}$ . To determine the relative position of  $d$ , first recall that, from Lemma 1, we know  $d < \bar{p}$  and so we only have to check if  $d$  is below  $\hat{p}_2$ , in the interval  $(\hat{p}_2, \hat{p}_1)$  or in the interval  $(\hat{p}_1, \bar{p})$ . By direct comparison we conclude the following:

$$\hat{p}_1 > d \Leftrightarrow S < \frac{r(a - c - dr)}{4b} = 2\tilde{S}, \quad (\text{A4})$$

$$\hat{p}_2 > d \Leftrightarrow S < \frac{r(a - c - rd)}{8b} = \tilde{S}. \quad (\text{A5})$$

This completes the proof.

QED.

### Proof of Proposition 2

Consider first statement a). The results in regions I and II follow from Lemma 2 according to a similar reasoning to that used in the proof of Proposition 1. In region III, between  $\hat{p}_1$  and  $\bar{p}$ , it is straightforward to conclude that both  $\Pi_1^*(p)$  and  $\Pi_2^*(p)$  are strictly increasing in  $p$ . Statements b) and c) straightforwardly follow from (A4), (A5) and Assumption 2.

QED.

### Proof of Proposition 3

Using (34) and (35) in (33) we get the equilibrium values for emissions:

$$e_1^*(x_1^*, p) = \frac{b(d_1 - p) + t_1 r [a + c_2 - 2c_1 - rp]}{2bt_1}$$

$$e_2^*(x_2^*, p) = \frac{2b(d_2 - p) + rt_2(a + 2c_1 - 3c_2 - rp)}{4bt_2}$$

By imposing the non-negativity conditions to the follower's emissions we get the upper bound value for the permit price,  $\bar{p}$ :

$$e_2^* \geq 0 \Leftrightarrow p \leq \bar{p} := \frac{2bd_2 + rt_2(a - 3c_2 + 2c_1)}{2b + r^2t_2} \quad (\text{A6})$$

By substitution of the relevant variables in the profit function we obtain the expression for the leader's profit function in terms of the parameters of the model:

$$\Pi_1(x_2^*, x_1^*, y_2^*, p) = \frac{[a + c_2 - 2c_1 - rp]^2}{8b} + \frac{(d_1 - p)^2}{4t_1} + pS_1$$

Differentiating with respect to  $p$  we obtain

$$\frac{\partial \Pi_1}{\partial p} = \frac{2b(p - d_1) + 4bt_1S_1 - rt_1[a + c_2 - 2c_1 - rp]}{4bt_1}$$

And by equating this derivative to zero we get the minimum value of  $p$  such that the leader finds it profitable to push the price up,  $\hat{p}_1$ :

$$\frac{\partial \Pi_1}{\partial p} > 0 \Leftrightarrow p > \hat{p}_1 := \frac{rt_1(a + c_2 - 2c_1) + 2bd_1 - 4bt_1S_1}{r^2t_1 + 2b} \quad (A7)$$

By direct differentiation of the values of  $\bar{p}$  and  $\hat{p}_1$  we obtain the results in the proposition:

$$\begin{aligned} \frac{\partial \bar{p}}{\partial c_1} &= \frac{2rt_2}{2b + r^2t_2} > 0; & \frac{\partial \hat{p}_1}{\partial c_1} &= \frac{-2rt_1}{2b + rt_1} < 0; \\ \frac{\partial \bar{p}}{\partial c_2} &= \frac{-3rt_2}{2b + r^2t_2} < 0; & \frac{\partial \hat{p}_1}{\partial c_2} &= \frac{rt_1}{2b + rt_1} > 0; \\ \frac{\partial \bar{p}}{\partial d_1} &= 0; & \frac{\partial \hat{p}_1}{\partial d_1} &= \frac{2b}{2b + rt_1} > 0; \\ \frac{\partial \bar{p}}{\partial d_2} &= \frac{2b}{2b + r^2t_2} > 0; & \frac{\partial \hat{p}_1}{\partial d_2} &= 0; \\ \frac{\partial \bar{p}}{\partial S_1} &= \frac{\partial \bar{p}}{\partial S_2} = 0; & & \\ \frac{\partial \hat{p}_1}{\partial S_1} &= \frac{-4bt_1}{2b + rt_1} < 0; & \frac{\partial \hat{p}_1}{\partial S_2} &= 0; \\ \frac{\partial \hat{p}_1}{\partial t_1} &= \frac{2br(a + c_2 - 2c_1 - d_1r) - 8b^2S_1}{[r^2t_1 + 2b]} \leq 0 \Leftrightarrow S_1 \leq \frac{r(a + c_2 - 2c_1 - d_1r)}{4b} \\ \frac{\partial \bar{p}}{\partial t_2} &= \frac{2br[a - 3c_2 + 2c_1 - rd_2]}{(2b + r^2t_2)^2} > 0 \end{aligned}$$

where, in an interior solution, the numerator of the last expression must be positive for the follower's output to be positive. QED.

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