Hegemann, Annika; Kunoth, Angela; Rupp, Kristina; Sureth, Caren

Working Paper
Impact of capital gains taxation on the holding period of investments under different tax systems

arqus Discussion Paper, No. 183

Provided in Cooperation with:
arqus - Working Group in Quantitative Tax Research

Suggested Citation: Hegemann, Annika; Kunoth, Angela; Rupp, Kristina; Sureth, Caren (2015): Impact of capital gains taxation on the holding period of investments under different tax systems, arqus Discussion Paper, No. 183, Arbeitskreis Quantitative Steuerlehre (arqus), Berlin

This Version is available at:
http://hdl.handle.net/10419/107592

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Impact of Capital Gains Taxation
on the Holding Period of Investments
under Different Tax Systems

arqus Discussion Paper No. 183
February 2015
Impact of Capital Gains Taxation on the Holding Period of Investments under Different Tax Systems

Annika Hegemann*, Angela Kunoth#, Kristina Rupp† and Caren Sureth‡

Abstract: An investment that is characterized by exit flexibility requires both decisions on investment and holding period. As selling an investment often leads to tax-liable capital gains and capital gains crucially depend on the duration of an investment we investigate the impact of capital gains taxation on the holding period under three different tax systems. In our analytical investigation we examine whether there is an optimal exit time and if there is no optimal exit time, what would be an appropriate time of sale. Moreover, we determine the worst exit time which should be avoided by investors. We find that while often an immediate sale is optimal longer holding periods may be beneficial under certain conditions. Beyond the well-known impact of the retention policy we clarify that the minimal holding period particularly depends on the degree of income and corporate tax integration. We find, e.g., a high retention rate to extend the minimum holding period under a shareholder relief tax system but is likely to accelerate sales under a classic corporate tax system. These results help to anticipate the economic implications of capital gains taxes on investment. Obviously, depending on the underlying tax system the after-tax profitability of long-term and sustainable investments is particularly affected by capital gains taxes. These results are interesting for both investors and tax politicians.

Keywords: Capital Gains Taxation, Holding Period, Exit Flexibility, Investment Decisions, Timing Decisions

JEL-Classification: H20, H21, H25

Acknowledgements: We thank Stephan Alberternst and Pia Kortebusch for valuable suggestions on an earlier draft. We are particularly grateful to Boqiang Huang for technical support and helpful advices on using MatLab. The usual disclaimer applies. Financial support by the German Research Fund (DFG), project no. SU 501/4-2 is gratefully acknowledged.

* University of Paderborn, Faculty of Business Administration and Economics, Warburger Str.100, 33098 Paderborn, Germany. Tel.: ++49-5251-60-1875, email: annika.hegemann@wiwi.upb.de, www.upb.de/taxation.
# University of Cologne, Mathematical Institute, Weyertal 86-90, 50931 Köln, Germany. Tel.: ++49-221-470-6080, email: kunoth@math.uni-koeln.de, www.mi.uni-koeln.de/AG-Kunoth/.
† University of Paderborn, Faculty of Business Administration and Economics, Warburger Str.100, Paderborn, Germany. Email: ruppk2@mail.uni-paderborn.de.
‡ University of Paderborn, Faculty of Business Administration and Economics, Warburger Str.100, 33098 Paderborn, Germany. Tel.: ++49-5251-60-1781, email: caren.sureth@wiwi.upb.de, www.upb.de/taxation and Vienna University of Economics and Business, Vienna, Austria.
1 INTRODUCTION

Entrepreneurial investments not only are characterized by decisions on carrying out an investment but also on the holding period. Taxes are well known to be important parameters of the institutional environment for investments. They may significantly affect investment decisions and particularly, lead to a lock-in effect. Furthermore, the duration of an investment may affect an investment’s after-tax profitability. The relevance of the holding period in investment decisions is highlighted by, e.g., Cheng et al (2010) and Alles and Murray (2009). Thus, when should an investor divest? To what extent does capital gains tax affect exit decisions? To what extent does the underlying tax system, i.e., the integration of corporate tax into the income tax impact the effect of capital gains taxation on the holding period?

Several analytical and empirical studies investigate the impact of taxation on investment in corporate stocks and asset prices, particularly of the effects that arise from capital gains taxation (Feldstein et al 1980; Bradford 1981; Stiglitz 1983; Auerbach 1989; Auerbach 1991; Lang and Shackelford 2000; Blouin et al 2002; Ayers et al 2003; Guenther and Sansing 2006; Dai et al 2008; Becker et al 2013; Campbell et al 2013; for an overview see also, Shackelford and Shevlin 2010; Graham 2008; Hundsdoerfer et al 2008; Hanlon and Heitzman 2010). The literature provides evidence that stock prices react on capital gains tax rate changes. Furthermore, there are empirical studies across different (capital gains) tax systems. E.g., Poterba and Weisbenner 2001 examine U.S. (turn-of-the-year) stock returns across three capital gains tax regimes (total of 34 years). In their empirical analysis they find significant differences across these three systems. They explain this observation by individual investors adjusting their trading behavior to the respective tax regime. Another empirical study is conducted by Jacob 2014. Based on a sample of Swedish individuals he provides evidence that the tax regime, either a progressive or proportional regime, impacts the realization of capital gains. Both studies Poterba and Weisbenner 2001 and Jacob 2014 account for restrictions in the deductibility of capital losses.

Distortive tax effects under different tax regimes with capital gains taxation have also been identified analytically by König and Wosnitza (2000) and Schreiber and Rogall (2000). The latter analyze the German Corporate Tax Reform 2001, which abolished the full imputation system and introduced shareholder relief in Germany. They find distortions after the tax reform but also a reduction of double taxation of capital gains. Moreover, Sureth (2006) and Sureth and Langeleh (2007) identify a distorting effect of capital gains tax on investments under a full imputation system, the shareholder relief and a classical tax system.
Knoll and Wenger (2007) find that the introduction of a flat tax on dividends particularly discriminates private domestic equity investors in high tax brackets.

Selling an investment often leads to capital gains disclosures and consequently to capital gains taxation. As capital gains crucially depend on the duration of an investment we investigate investment decisions with exit flexibility. Despite the body of empirical literature on capital gains taxes and trading behavior (Bogart and Gentry 1995; Ivković et al 2005; Ayers et al 2008; Haesner and Schanz 2013) and theoretical studies that account for loss-offset opportunities in this context (Constantinides 1983; Stiglitz 1983; Nippel and Podlech 2011; Ehling et al 2013) all of these studies mainly focus on listed corporations. Yet only little attention has been paid to the impact of capital gains taxation on both investment in corporate shares in general and the holding period under different tax systems. In our analytical investigation we examine whether and if so when there is an optimal exit time.

We take the investor's point of view and focus on investment into corporate shares. After an initial investment the investor receives dividends and/or capital gains in the subsequent periods. In addition or alternatively to the investment in a corporation, the investor can invest in the capital market and earn interest. Interest, dividends, and capital gains are all considered as capital income, but typically subject to different tax rules, e.g., different tax scales. We abstract from loss-offset opportunities. Against this background, we study the impact of taxes on the investment decisions under the three most common tax systems that differ in the degree of income-corporate tax integration. We focus on the variable “time of sale” to analyze whether the investment becomes more or less profitable under a classic corporate tax system, a full imputation tax system or a shareholder relief tax system for different exit times. As exit time drives the magnitude of capital gains taxation it is particularly important to investigate the impact of the capital gains tax on investment decisions. This is true, particularly as many countries tax capital gains, e.g., Canada in 1972, Ireland in 1975, Australia in 1985, imposed a capital gains taxation not long ago but other countries abolished capital gains taxation between 2000 and 2012, e.g., Cyprus, Korea, Slovenia and China.1 By contrast, Portugal in 2010 and Austria in 2012 have implemented a capital gains taxation for long term gains in the last years.2 Furthermore in Germany, since 2009 capital gains are no longer tax-exempted but are generally tax liable.3

---

3 For an overview of the top marginal tax rates on capital gains of the OECD countries see Pomerleau (2014), p. 7.
As the impact of capital gains taxation on timing, i.e. the choice of exit time in a capital gains tax setting with exit flexibility is yet still underresearched we include both ordinary and capital gains taxes in the decision calculus to answer the following question about the investment decisions in this paper. To what extent does capital gains taxation affect the holding period under different tax regimes?

There is literature on tax effects on investment decisions and holding periods. E.g., Protopapadakis (1983) estimates the effective marginal tax rate of capital gains and emphasizes the dependency of the expected holding period and these expected effective tax rates and using U.S. IRS data estimates on average rather long holding periods of 24 and 31 years. Cook and O’Hare (1987) study the relationship between holding periods and after-tax rates of return in a capital gains tax setting. They distinguish between finite and infinite (realized at death) durations. A tax rate increase reduces the after-tax rate of return of short-run realizations for finite durations, while the after-tax rate of return of long-run realizations either decreases or increases depending on the relation of investment rate, and internal growth. In their empirical investigation they find evidence that the marginal capital gains tax rate from the year before death does not have a significant impact on capital gains realized at death. In line with these findings Gau and Wang (1994), who empirically investigate the determinants of holding period decisions of real investment, find that current market interest rate, consumption, and investment preferences are more important than tax issues. By contrast, Klein (1999) finds capital gains taxes to be an important holding decision driver. He elaborates a general equilibrium asset pricing model with capital gains taxation that occurs upon realization. He illustrates that the optimal holding in a given stock depends on the individual deferral terms, accrued capital gains and the expected holding periods of all investors. Also, Niemann and Sureth (2013) examine the impact of capital gains taxation on investment timing under simultaneous investment and abandonment flexibility. They show numerically that capital gains taxation accelerates risky investment under specific conditions, e.g., high liquidation proceeds, more conservative tax accounting, low interest rates, and low volatilities (Cremer et al 2010). Jacob (2013) studies empirically the impact of capital gains taxes on holding behavior after the German Corporate Tax Reform 2001. He provides evidence that capital gains realizations are deferred if marginal income tax rates are high.

To summarize, prior literature offers mixed results on the impact of taxes on holding and exit decisions. Already Niemann and Sureth (2013) illustrate that the capital gains taxation under certain conditions can accelerate investments in simultaneous entry and exit
decisions. In contrast to our study they neither distinguish between different tax systems 
and nor focus on the impact of profit retention rates, which we identify as important drivers 
for holding decision.

No study yet analytically addresses the impact of capital gains taxation in different tax 
environments on holding periods. We want to fill this void. We focus on both different tax 
systems including capital gains taxes and tax rate differentials and the impact of retention 
policy. We determine minimum holding periods and worst exit scenarios that should be 
avoided by investors for different tax systems. Our analysis is based on theoretical investiga-
tions as well as numerical schemes elaborated in Rupp (2012). Obviously, if investors are 
flexible with respect to holding time, it is important to anticipate tax differences that may 
arise from different holding behavior.

To answer our research question in section 2 we determine the present value of the fu-
ture cash flows from the investment as a decision criterion. We use Gordon’s growth model 
(Gordon and Shapiro 1956; Gordon 1962) as the basic asset pricing model in order to theo-
retically derive a solution for the underlying investment decision. This model enables you 
to investigate the impact of dividends and capital gains taxation on holding decisions. First 
we describe the pre-tax model and afterwards we integrate taxes into the model. To show 
the differences that arise from the three underlying tax systems we focus on the change in 
the present value of the investment in each scenario and conduct a sensitivity analysis that 
provides a first impression on the influence of different parameters, i.e., retention rate. In 
section 3 we determine analytically the exit time with the lowest present value for different 
tax systems. We find that this worst exit time varies depending on the underlying tax sys-
tems significantly. Often an immediate sale is optimal. Employing Newton’s method as a 
umerical scheme to find zeroes of the relevant equation, we identify the minimal holding 
period, which must be exceeded to obtain present values that are higher than in case of an 
immediate sale. In Section 4 we conclude.

2 Model

In the Gordon growth model the present value of an investment \( V_0 \) is determined by the 
future dividends \( D(t) \), which are generated by the investment project. An investor realizes 
all investment projects that, in present value terms, earn more than the initial investment 
outlay. The Gordon growth model is a discrete time model, thus we look at time \( t \in \mathbb{N}_0 \).
König and Wosnitza (2000) use the Gordon’s growth model to compare the imputation tax system with and without capital gains taxation. They show that the capital gains taxation distorts the price formation on the stock market by a temporary double taxation. The result is a discrimination of equity investment against debt capital, which makes business foundations more difficult. Sureth and Langeleh (2007) theoretically investigate the effect of different degrees of integrating corporate and capital gains tax into income tax and its impact on investment decisions. Using the growth model they cannot determine theoretically a general solution for the investment problem, i.e., the decision to invest in shares or at the capital market under different tax regimes. Only under restrictive assumptions they find that the shareholder relief tax system induces more severe distortions than the full imputation tax system. In a sensitivity analysis they investigate the influence of different retention rates as well as different holding periods on the investment value under an imputation system. They find that the distorting tax effects increase in the retention rate. Moreover, they highlight the relevance of the time of sale under a capital gains tax.

2.1 TAX-FREE MODEL

In line with these previous studies we assume a private investor A spends $I_0$ for an investment in shares of a corporation. To determine the present value of the investment $V_0$, the investor discounts all future dividends $D$ with the exogenously given interest rate $i \in (0, 1)$. The observed time horizon is determined by $t \in [0, \infty)$. Profits arise after one period starting at $t = 1$. The rate of return of the alternative investment at the capital market is denoted by $i$.\(^4\) We assume the capital market is perfect and certain. Therefore, the present value can be described as a function of $\gamma$ and $i$,\(^5\)

$$V_0[\gamma, i] = \sum_{t=1}^{\infty} \frac{D[t, \gamma]}{(1+i)^t}, \tag{1}$$

where $\gamma$ denotes the retention rate. Here, dividends $D[t, \gamma]$ are the distributed part of the investment profit $P[t]$. On firm level a constant fraction $\gamma \in [0, 1)$ of the profit is retained\(^6\)

$$R[t, \gamma] = \gamma \cdot P[t], \tag{2}$$

\(^6\) See Gordon and Shapiro (1956), p. 105. Note, that in the following we consider $\gamma$ to be exogenously given. In line with prior literature, we do not account for information asymmetry or other signalling causes. We further abstract from the retention rate being less sensitive to cash flows if dividend taxes are sufficiently high as suggested by empirical evidence. See, e.g., Jacob and Jacob (2013).
where $R[t, \gamma]$ describes the retained profit in $t$, while distributing the rest as dividends

$$D[t, \gamma] = (1 - \gamma) \cdot P[t]. \quad (3)$$

The profit from the previous period $P[t-1]$ and the return on retained profits serve as a basis for the profit at $t$. The profit of the current period is $P(1) = iI_0$. Hence we obtain

$$P[t] = (1 + \gamma i) \cdot P[t-1]. \quad (4)$$

Here, $\gamma i$ is the constant growth rate that characterizes the Gordon growth model which, in the literature, is typically denoted by $g$.

By assumption both the internal and external rate of return are equal to $i$. Under this assumption the underlying investment can only be more attractive than the alternative capital market investment in case of a (non-neutral) tax system. Consequently, in the absence of taxes, the investor will always be indifferent between the investment in the corporation and the alternative investment.

The investor $A$ holds the investment object for a fixed period of time. At time $t = z$ she or he sells her or his investment to another private investor $B$ at a price $S[z, \gamma, i]$ with $z \geq 0$. If the price is greater than the investment outlay $I_0$, the investor earns a capital gain $G[z, \gamma, i]$.

$$G[z, \gamma, i] = S[z, \gamma, i] - I_0. \quad (5)$$

On the other hand, the investor realizes a capital loss if the price is smaller than $I_0$.7 Investor $B$ holds the investment object until $T = \infty$. The present value $V_0[z, \gamma, i]$ captures all future cash flows, i.e., it contains the discounted cash flow and the discounted price.8 We obtain

$$V_0[z, \gamma, i] = \sum_{t=1}^{z} \frac{D[t, \gamma]}{(1 + i)^t} + \frac{S[z, \gamma, i]}{(1 + i)^z}. \quad (6)$$

Finally, we determine the price $S[z, \gamma, i]$, which, for the investor $B$, is determined by the future dividends earned after the sale at $t = z$. At the same time the price is the minimal

---

7 See eq. (5).
price the investor A is willing to accept to sell the investment. In the Gordon growth model this price for A and B is given by

\[ S[z, y, i] = \sum_{t=z+1}^{\infty} \frac{D[t, y]}{(1 + i)^{t-z}} = I_0 (1 + yi)^z. \] (7)

The present value \( V_0[z, y, i] \) is a function of \( i \) and thus includes a comparison of the two investment alternatives, namely the real investment in shares and the alternative investment at the capital market. If \( V_0 \) is greater than \( I_0 \), the real investment is more attractive than the capital market investment. If \( V_0[z, y, i] \) is smaller than \( I_0 \), the investor prefers the alternative capital market investment. If \( V_0[z, y, i] \) equals \( I_0 \), the investor is indifferent between the two alternatives.

### 2.2 General Tax Model

In the following we integrate taxes into the growth model. We model a general tax system which can be easily transformed to a classic corporate tax system, a full imputation tax system and a shareholder relief tax system.\(^{10}\)

The profit \( P[t, \tau^c] \) is taxed at the corporate tax rate \( \tau^c \in [0, 1) \),

\[ P[t, \tau^c] = P[t] \cdot (1 - \tau^c). \] (8)

After corporate taxation the profit can be distributed. The retained fraction is \( R[t, y, \tau^c] = yP[t] \cdot (1 - \tau^c) \). The dividend fraction is given by \( D[t, y, \tau^c] = (1 - y)P[t] \cdot (1 - \alpha \tau^c) \). Dividends are subject to shareholder level taxation. Income taxes at rate \( \tau \) have to be paid on dividends. Depending on the tax system a fraction of corporate tax is imputable to the income tax. We denote the fraction of corporate tax that cannot be imputed for income tax purposes by \( \alpha \in [0, 1] \) and obtain

\[ P[t, \tau^c] = yP[t] \cdot (1 - \tau^c) + (1 - y)P[t] \cdot (1 - \alpha \tau^c). \] (9)

We restrict \( y \) to

\[ y < 1 \quad \text{and} \quad y < \frac{i \tau}{i - \tau}. \] (10)

---

\(^9\) Determined by applying the geometric series.

for all tax systems. With this restriction we avoid obtaining results that are absurd from an economic point of view.\footnote{If we allowed for $\gamma \geq \frac{\iota}{\tau}$ we would model infinite growth. Infinite growth is unrealistic and for this reason excluded from our model for plausibility reasons. See Sureth (2006), pp. 58, 74; Sureth and Langeleh (2007), p. 317.}

At shareholder level, both interest and dividends are subject to income tax at a tax rate $\tau \in [0, 1)$. Investor A and B have the same income tax rate. The income tax rate is equal to the personal tax rate of the investor. We assume that capital gains are taxed with a special tax rate $\tau^g = \frac{\tau}{2}$. The parameter $\beta \in [0, 1]$ denotes the fraction of dividends and capital gains that is subject to income taxation.

In our model we do not account for any tax privilege on interest income. Therefore, the after-tax market rate of return is given by

$$i^\tau = i \cdot (1 - \tau).$$

(11)

Dividends rate of income tax is calculated as follows

$$D[t, \gamma, \tau, \tau^c] = (1 - \gamma)P[t] \cdot (1 - \alpha \cdot \tau^c) \cdot (1 - \beta \cdot \tau).$$

(12)

The after-tax price amounts to:

$$S[z, \gamma, i, \tau, \tau^c, \tau^g] = \sum_{t=z+1}^{\infty} \frac{D[t, \gamma, \tau, \tau^c]}{(1 + i^\tau)^{t-z}} - \left( \sum_{t=z+1}^{\infty} \frac{D[t, \gamma, \tau, \tau^c]}{(1 + i^\tau)^{t-z}} - I_0 \right) \cdot \beta \cdot \tau^g.$$  

(13)

First the after-tax dividends of investor B are discounted to time $z$. The difference between the discounted dividends and the initial outlay $I_0$ is subject to capital gains taxation at a rate $\tau^g \in [0, 1)$.

The present value after taxation for real investment is given by

$$V_0^{GT}[z, \gamma, i, \tau, \tau^c, \tau^g] = \sum_{t=1}^{z} \frac{(1 - \gamma)P[t] \cdot (1 - \alpha \cdot \tau^c) \cdot (1 - \beta \cdot \tau)}{(1 + i^\tau)^t} + \frac{S[z, \gamma, i, \tau, \tau^c, \tau^g]}{(1 + i^\tau)^z}. $$

(14)
Here, we use $GT$ as a suffix for the present value under the general tax system. Then, based on equation (14) we model the three specific tax systems.\textsuperscript{12} The classic corporate tax system, indicated by suffix $CC$, is given by $\alpha = 1$ and $\beta = 1$ and thus\textsuperscript{13}

$$V_0^{CC}[z, y, i, \tau, \tau^c, \tau^g] = \sum_{t=1}^{\infty} \frac{(1 - y)P[t] \cdot (1 - \tau^c) \cdot (1 - \tau)}{(1 + i^t)^t} + \frac{S[z, y, i, \tau, \tau^c, \tau^g]}{(1 + i^t)^z}. \quad (15)$$

Here, the corporate tax burden is unrefundable.

The shareholder relief tax system, indicated by $SR$, is exemplarily modeled with $\alpha = 1$ and $\beta = 0.5$, which implies that 50\% of dividends and capital gains are assumed to be subject to income taxation,\textsuperscript{14}

$$V_0^{SR}[z, y, i, \tau, \tau^c, \tau^g] = \sum_{t=1}^{\infty} \frac{(1 - y)P[t] \cdot (1 - \tau^c) \cdot (1 - 0.5\tau)}{(1 + i^t)^t} + \frac{S[z, y, i, \tau, \tau^c, \tau^g]}{(1 + i^t)^z}. \quad (16)$$

The full imputation tax system, indicated by $FI$, is characterized by $\alpha = 0$ and $\beta = 1$, and thus given by\textsuperscript{15}

$$V_0^{FI}[z, y, i, \tau, \tau^c, \tau^g] = \sum_{t=1}^{\infty} \frac{(1 - y)P[t] \cdot (1 - \tau)}{(1 + i^t)^t} + \frac{S[z, y, i, \tau, \tau^c, \tau^g]}{(1 + i^t)^z}. \quad (17)$$

3 HOLDING DECISION

3.1 PARAMETER SENSITIVITY

To investigate the holding decision we use the general form of the present value function\textsuperscript{16} and simplify in the following $D[t, y, \tau, \tau^c]$ to $D_t$.


\textsuperscript{13} Under a classic corporate tax system income and corporate taxes are levied independently on shareholder and company level. The withholding tax system, which has been introduced in Germany in 2009 can be regarded as a classic corporate tax system.

\textsuperscript{14} E.g., the shareholder relief tax system was introduced in Germany in 2001 and later reformed and accompanied by a flat tax on most types of capital income. Today, the shareholder relief is only rarely applicable. If dividends and capital gains qualify for shareholder relief, then a fraction of 40\% is tax-exempted. The shareholder relief tax system actually still is in force in several countries, e.g., Luxemburg, Norway and France. See Bundesministerium der Finanzen (2013), pp. 12-13.

\textsuperscript{15} In Germany the full imputation tax system was implemented from 1977 to 2000. A full imputation tax system is actually present in Malta. See Bundesministerium der Finanzen (2013), p. 14.

\textsuperscript{16} See eqs. (13) and (14), and Sureth and Langeleh (2007), p. 317.
$$V_0^\tau = \sum_{t=1}^{z} \frac{D_t \cdot (1 - \alpha \tau^c) \cdot (1 - \beta \tau)}{(1 + i\tau)^t} + \sum_{t=z+1}^{\infty} \frac{D_t \cdot (1 - \alpha \tau^c) \cdot (1 - \beta \tau)}{(1 + i\tau)^t}$$

\begin{align*}
&- \left( \sum_{t=z+1}^{\infty} \frac{D_t \cdot (1 - \alpha \tau^c) \cdot (1 - \beta \tau)}{(1 + i\tau)^{t-z}} - I_0 \right) \cdot \beta \tau^g \cdot (1 + i\tau)^{-z} \\
&= I_0 \cdot \frac{(1 - \beta \tau)(1 - \gamma)(1 - \alpha \tau^c)}{(1 - \tau) - \gamma(1 - \tau^c)} \left( 1 - \beta \tau^g \cdot \frac{(1 + \gamma i\tau^c)^z - \frac{(1 - \gamma)(1 - \alpha \tau^c)}{(1 - \tau) - \gamma(1 - \tau^c)}}{(1 + i\tau)^z} \right) \\
&= I_0 \cdot \phi \cdot \left( 1 - \beta \tau^g \cdot \frac{(1 + \gamma i\tau^c)^z - \frac{1}{\phi}}{(1 + i\tau)^z} \right)
\end{align*}

with \( \phi = \frac{(1 - \beta \tau)(1 - \gamma)(1 - \alpha \tau^c)}{(1 - \tau) - \gamma(1 - \tau^c)}. \)

To address our research question on the influence taxes on the exit time we conduct sensitivity analyses. This numerical investigations provide a first impression about the extent to which the exit time drives the after-tax present value. First, we illustrate the present value for the three tax systems under predefined conditions. In a second step we vary the retention rates. The retention rate determines the capital gains tax base, which is also determined by the exit time. For our further investigation we set the interest rate to \( i = 0.1. \)

Figure 1 provides an overview of the present values under the three tax systems depending on time \( t = z. \) This figure depicts the present value with capital gains taxation \((V_0^\tau)\) and without capital gains taxation \((I_0 \cdot \phi).\) For the after tax present value \(V_0[z, \gamma, i, \tau, \tau^c, \tau^g]\) we write for simplicity \(V_0^\tau.\)

While the present value without capital gains taxation \(I_0 \phi\) does not depend on time \( t = z\) under all three tax system, considering capital gains taxation \(V_0^\tau\) generates a non-linear function of \( z.\) Hence, there is an obvious time effect caused by capital gains taxation. Capital gains taxation affects the exit time and hence the after-tax present value of the project. We observe two reciprocal effects. On the one hand there is a strong growth effect: The later the exit the greater the capital gain (taxable base) because retained earnings are increasing in time. On the other hand for late exits the capital gains tax is relatively low because the tax is paid upon realization and thus the discount effect reduces the tax payment in present value terms, especially for late exits. If one effect perfectly outweighs the other one there will be no minimum and the present value will be monotonically increasing or...
decreasing. If the two effects do not balance out, a minimum can be found due to the fact that the present value $V_0^T$ increases at a certain rate and the difference of present value and investment outlay ($V_0^T - I_0 = 1$), which is subject to the capital gains tax at time $z$, has to be discounted at another rate, i.e., the after-tax capital market rate. Therefore, the present value after taxes decreases until a minimum value (growth effect). For later exit times $z$, the capital gains tax is paid later and the tax payments have less impact on the present value (discount effect). After a minimum value is reached the discount effect exceeds the growth effect and the curve increases until the capital gains tax converges against zero.

If there is no capital gains tax then $V_0^T$ collapses to $I_0 \phi = V_0^T$. Figure 1 shows that the present value in the long-run converges asymptotically to the present value without capital gains taxation. This pattern holds for all three tax systems.\(^{17}\)

Under both the shareholder relief tax system and the classic corporate tax system the present value with capital gains taxation is greater than the present value without capital gains taxation for early exits. This relation is due to a capital loss tax refund. A capital loss results if $S_T^z - I_0 < 0$ even if $z = 0$. If $z = 0$ we invest and divest in the same period. While we

\(^{17}\) See Sureth and Langeleh (2007), p. 320, for full imputation tax system. Extending their study we show the development for all three different tax systems over time.
invest \( I_0 \) under all these tax systems our return differs. This distortion is due to the inter-
play of dividends, interest effects, corporate tax rates, and retention rate. We find a trade
off under full imputation, whereas a discrimination of investments into corporate shares in
comparison to alternative investments arises under the other two systems. Even for \( z = 0 \)
there is a tax refund for capital losses under shareholder relief and classic corporate tax
system caused by these distortional effects from current taxation (see figure 1).

Figure 2 illustrates the present values \( V^\tau_0 \) for different retention rates \( \gamma \) under a full
imputation tax system.\(^{18}\)

![Graph](image.png)

*Figure 2: \( V^\tau_0 \) for various \( \gamma \) under the full imputation system with \( I_0 = 1 \), \( \tau^c = 0.4 \) and \( \tau = 0.5 \)*

The higher the retention rate \( \gamma \) the more pronounced the curvature of the present value
function and the greater the difference between \( \phi \) and \( V^\tau_0 \). This relation is essential for the
pronounced impact of capital gains taxation for high retention ratios. Especially for early
exit times \( z \) the curvature of the graph is strong.\(^{19}\) The effect of capital gains taxation for
high retention ratios is visible for late exit times, too. On the other hand, for low retention
ratios it only affects early exit times. In figure 2 for a retention rate of \( \gamma = 0.7 \) the influence

---


\(^{19}\) If the capital gains taxation is paid early, the influence on the present value after taxes \( V^\tau_0 \) is strong. The
discount effect is very weak for early exit times \( z \).
of capital gains taxation is visible until \( z = 500 \), while for a retention rate of \( \gamma = 0.1 \) this effect occurs only until approximately \( z = 80 \). Hence, this finding indicate that the investor should consider capital gains taxation particularly for short holding periods in decision-making.

Beyond the characteristics of the tax systems \((FI, SR, CC)\) and the retention rate \( \gamma \) the tax rates influence the present value after taxes for various exit times \( z \), too. Therefore, we vary the tax rates in our sensitive analysis, because the tax rates are exposed to multiple changes by tax reforms and severely affect our results.\(^{20}\)

![Figure 3](image1.png)  \hspace{1cm}  ![Figure 4](image2.png)

**Figure 3:** \( V_{\tau}^T \) for various \( \tau \) under the full imputation system with \( I_0 = 1 \), \( \tau^c = 0.4 \) and \( \gamma = 0.2 \)

**Figure 4:** \( V_{\tau}^T \) for various \( \tau^c \) under the full imputation system with \( I_0 = 1 \), \( \tau = 0.4 \) and \( \gamma = 0.2 \)

Figure 3 (left hand side) illustrates the after-tax present value as a function of \( z \) for various income tax rates \( \tau \) and figure 4 (right hand side) displays the after-tax present value as a function of \( z \) for various corporate tax rates \( \tau^c \). We plot the result for the full imputation tax system as an example for our analysis.\(^{21}\)

Again the same curve characteristics that are already known from figures 1 and 2 can be observed.\(^{22}\) The tax rates have an opposite influence on the after-tax present values. While an increasing income tax rate \( \tau \) in figure 3 increases the value of \( V_{\tau}^T \), an increasing

\(^{20}\) For example the business tax reform 2008 in Germany and the tax reform 2000 in Germany.


\(^{22}\) The curvature is driven by the growth and the discount effect.
corporate tax rate reduces \( V_0^T \). If the income tax rate \( \tau \) increases, the alternative investment on the capital market is hit hardest. Interest payments are fully taxed, so are the current earnings from the investment in shares, i.e., dividends\(^{23}\). By contrast, capital gains, by assumption, are only subject to halved income tax rate, i.e., \( \tau^g = \frac{\tau}{2} \). The slope of the curve is more emphasized for higher income tax rates because the capital gains taxation increases proportionally with the income tax rate. This result is in line with Jacob (2013), who finds empirical evidence for deferred capital gains realizations under German shareholder relief if marginal income tax rates are high.\(^{24}\)

An increasing corporate tax rate \( \tau^c \) in figure 4 leads to a decrease of \( V_0^T \) because retained earnings are reduced and weaken the internal growth. As a result the after-tax present value decreases. The slope of the curve is more stressed for lower corporate tax rates because capital gains are higher and therefore have more influence on the present value.

In summary an investor especially has to consider high income tax rates \( \tau \) and low corporate tax rates \( \tau^c \) when deciding on the exit time \( z \). This is because the present value of the investment is particularly sensitive to these parameters.

### 3.2 Analytical Approach

Until now our conclusion, just as in prior literature, is restricted to the underlying numerical examples. To improve the explanatory power of our results in the next step we aim to determine the optimal exit time in a theoretical fashion.

In the following we study \( V_0^T \) under the three tax regimes with capital gains taxation. We use the first derivative to calculate the extreme values of the present value equation with respect to \( z \). If we find a maximum, we have identified the optimal exit time, at which the value of \( V_0^T \) is highest.

For our analytical calculation we set all variables, except \( z \), and assume the function for the present value of the investment can be continuously differentiated. We set

\[
V_0^T[z] := V_0^T[z, \gamma, \tau, \tau^c, \tau^g, i]. \tag{19}
\]

\(^{23}\) Dividends are also fully subject to the income tax rate.

\(^{24}\) Note, he uses data from 2001 and 2004 where short-term capital gains where tax-exempted.
To determine the optimal exit time we use eq. (19) and the first and the second derivative of $V_0^\tau$ with respect to $z$\textsuperscript{25}

\[
\frac{d}{dz} V_0^\tau [z] = -I_0 \cdot \phi \cdot \beta \tau^g \cdot \frac{(\ln(1 + y \tau_c) - \ln(1 + i^\tau)) \cdot (1 + y \tau_c)^z + (\ln(1 + i^\tau)) \cdot \frac{1}{\phi}}{(1 + i^\tau)^z}.
\] (20)

\[
\frac{d^2}{dz^2} V_0^\tau [z] = -I_0 \cdot \phi \cdot \beta \tau^g \cdot \frac{(\ln(1 + y \tau_c) - \ln(1 + i^\tau))^2 \cdot (1 + y \tau_c)^z - (\ln(1 + i^\tau))^2 \cdot \frac{1}{\phi}}{(1 + i^\tau)^z}.
\] (21)

The necessary condition for the optimum is given by:

\[
\frac{d}{dz} V_0^\tau [z] = -I_0 \cdot \phi \cdot \beta \tau^g \cdot \frac{(\ln(1 + y \tau_c) - \ln(1 + i^\tau)) \cdot (1 + y \tau_c)^z + (\ln(1 + i^\tau)) \cdot \frac{1}{\phi}}{(1 + i^\tau)^z} = 0.
\] (22)

Since $I_0 > 0$, $\beta > 0$, $\tau^g > 0$, and $(1 + i^\tau)^z > 0$ per definition\textsuperscript{26}, we have to examine whether $\phi$ with

\[
\phi = \frac{(1 - \beta \tau)(1 - y)(1 - \alpha \tau^c)}{(1 - \tau) - y(1 - \tau^c)}
\] (23)

is positive or negative. The numerator in equation (23) is always greater than zero. This is true as $(1 - \beta \tau) > 0$ because $\beta \in [0, 1]$, and $\tau \in [0, 1)$; $(1 - y) > 0$ because $y \in [0, 1)$; $(1 - \alpha \tau^c) > 0$ because $\alpha \in [0, 1]$ and $\tau^c \in [0, 1)$. The denominator of the fraction is greater than zero if

\[
(1 - \tau) - y(1 - \tau^c) > 0 \Rightarrow y < \frac{1 - \tau}{1 - \tau^c}.
\] (24)

While in equation (10) we have assumed that $y < \frac{i^\tau}{\tau^c}$, the denominator is positive and thus $\phi > 0$. We simplify equation (22) to\textsuperscript{27}

\[
0 = (\ln(1 + y \tau_c) - \ln(1 + i^\tau)) \cdot (1 + y \tau_c)^z + (\ln(1 + i^\tau)) \cdot \frac{1}{\phi}
\] (25)

\textsuperscript{25} See Rupp (2012), pp. 29-30.

\textsuperscript{26} We have defined $\beta$ with $\beta \in [0, 1]$. For the underlying tax systems $\beta$ is set either equal to 0.5 or 1 and always greater than zero.

\textsuperscript{27} See Rupp (2012), pp. 31-32.
and solve the equation with respect to $z$

$$z = \frac{\ln\left(\frac{-\ln(1+i) \cdot \frac{1}{\Phi}}{\ln(1+y^{\tau_c})}\right)}{\ln(1+y^{\tau_c})}. \tag{26}$$

Equation (26) describes the exit time $z$ that fulfills the necessary condition in equation (22). To figure out whether it is a maximum or a minimum, we have to study the second derivative

$$\frac{d^2}{dz^2} V_0^T[z] = -I_0 \cdot \phi \cdot \beta \tau^g \cdot \frac{(\ln(1+y^{\tau_c}) - \ln(1+i^T))^2 \cdot (1+y^{\tau_c})^z - (\ln(1+i^T))^2 \cdot \frac{1}{\Phi}}{(1+i^T)^z}. \tag{27}$$

Given that $I_0 > 0$, $\phi > 0$, $\beta \tau^g > 0$ and $(1+i^T)^z > 0$, in the final step we investigate whether the numerator in equation (21) is negative, thus whether

$$(\ln(1+y^{\tau_c}) - \ln(1+i^T))^2 \cdot (1+y^{\tau_c})^z - (\ln(1+i^T))^2 \cdot \frac{1}{\Phi} < 0. \tag{28}$$

As all parts of inequality (27) are negative we can simplify to:

$$1 < \frac{(\ln(1+i^T))^2 \cdot \frac{1}{\Phi}}{(1+y^{\tau_c})^z \cdot (\ln(1+y^{\tau_c}) - \ln(1+i^T))^2}. \tag{28}$$

After transposing the inequality it is obvious that the right hand side of the inequality (28) is greater than zero. Overall the second derivative is positive. Hence, as the function is strictly convex the extreme value is a global minimum. A global maximum does not exist.

Despite we cannot determine the optimal exit time, our results (see eq. (28)) imply that there is a worst time to exit which generates the minimum present value.\(^{28}\)

### 3.3 Worst Exit Time

The worst exit time is interesting for the investor, because she or he should avoid to sell the investment at this time or close to this time. In other words, she or he has to minimize the possibility of a liquidity squeeze especially for this time of sale because then the lowest present value would be realized. Figure 5 displays the exit times with the lowest present values for different retention ratios for the underlying three tax systems.\(^{29}\)

For small retention ratios the worst exit time depends on the respective tax system, whereas for high retention ratios the lowest present values are similar for all three tax

---

\(^{28}\) See also figure 2.

\(^{29}\) $\tau^c = 0.4; \tau = 0.4$. 
systems. Figure 5 illustrates that the tax treatment of dividends deviates stronger among the three tax systems than the tax treatment of capital gains. This result holds for all tax rate relations, i.e., $\tau^c = \tau$, $\tau^c < \tau$ and $\tau^c > \tau$. For a low retention rate of $\gamma = 0.1$ the worst exit time is reached after 18 periods under the full imputation tax system while it takes 104 (55) periods in the classic corporate (shareholder relief) tax system. For a high retention rate ($\gamma = 0.9$) the worst exit time is reached after 44 periods (FI), 54 periods (CC), and 49 periods (SR).

![Figure 5: Worst exit time $\hat{z}$ for various $\gamma$ with $\tau^c = 0.4$ and $\tau = 0.4$](image)

We conclude that the type of tax system is particularly important for the holding decision if the dividend ratio is huge. Against this background, especially in countries where the tax system is volatile, the investor should be aware that the worst exit time possibly changes tremendously after a substantial tax reform. Prior studies clarify that tax systems in many countries often change:\footnote{E.g., Becker et al. (2013) investigate the effect of payout taxes on the allocation of investment and report on the tax regimes across 25 countries between 1990 and 2008. 13 of these countries changed their tax system within the observed period. Spain and Mexico even have two variations of their tax systems. See Becker et al (2013), p. 6.} If a corporation that is situated in a country with a volatile tax system distributes most of the profits (huge dividend rate), the change in the tax system possibly changes their exit timing as the reform is likely to change the worst and avoidable exit time. Furthermore, dividend ratios between 27 % and 61 % on average (i.e, relatively high retention rates) can be observed in several countries.\footnote{See ap Gwilym et al (2006), p. 39, who provide a descriptive overview over the payout ratios between 1965/1973/1990-2004 of eleven countries, that are considered representative for the industrialized world.} Some empirical studies provide
evidence that the dividend ratios in tendency increase every year.\textsuperscript{32} By contrast, other studies indicate on average a decline in dividend payout. Nevertheless, their findings also indicate that “… larger firms, firms with higher profitability, and firms with lower growth opportunities have a greater propensity to pay dividends.”\textsuperscript{33} Taking German stock exchange indices\textsuperscript{34} as an example shows that especially some of DAX and MDAX listed firms have rather high dividend ratios (approximately 70 % of DAX listed firms have a dividend ratio above 35 %, and 30 % of DAX listed firms above 65 %). In other countries, e.g., Australia, a dividend ratio of more than $2/3$ of their stock performance on average can be observed.\textsuperscript{35} In those countries with such high payout ratios it is even more important to account for the holding implications of possible tax reforms.

3.4 Extrema at the Boundaries

Until now we have determined the extreme values of the present value function. It turned out that there is only a global minimum, that is the worst time to sell the shares. This point of time depends on the underlying tax systems and is especially sensitive to high dividend ratios. To find out which point of time is the best exit time we examine the boundaries of the representation (19). The present value functions under all three tax systems are convex and thus the highest present values emerge at the boundaries. Given that at $z = 0$ we observe a local maximum we investigate the development for $z > z^{\text{min}}$.\textsuperscript{36} We want to figure out whether a present value for $z > z^{\text{min}}$ can be realized by sales that exceed the local maximum at $z = 0$.

For example in figure 1 we illustrate that the local maximum of the present value is at $z = 0$ for all tax systems. Note, in this figure we only depict a finite time horizon of $T = 100$. Here, the present value decreases with later exit times until a minimum present value (worst exit time) is reached. For later exits the present value increases again but does not exceed the initial present value at $z = 0$ until $z = T = 100$. Additionally figure 1 illustrates that the present value with capital gains taxation seems to converge asymptotically towards the present value without capital gains.

\textsuperscript{34} See Frère et al (2012), p. 4.
\textsuperscript{36} $z^{\text{min}}$ denotes the exit time $z$ where the global minimum is reached.
To study these effects in more detail we determine in general under what conditions a local extremum that exceeds the present value at $z = 0$ can be reached. Here we examine which of the local extrema (of the boundaries) is the global maximum. For this purpose we compare analytically the present value at $z = 0$ with the present value at $z = T$. At $z = 0$ the present value equation is

$$V_0^T[z = 0] = I_0\phi \left( 1 - \beta T^g \cdot \left( 1 - \frac{1}{\Phi} \right) \right). \quad (29)$$

For $z = T$ the present value equation becomes

$$V_0^T[z = T] = I_0\phi \cdot \left( 1 - \beta T^g \cdot \frac{(1 + \gamma T^c)^T - \frac{1}{\Phi}}{(1 + i^c)^T} \right). \quad (30)$$

Depending on $T$ we obtain:

$$V_0^T[z = 0] \preceq V_0^T(z = T). \quad (31)$$

For late exits the present value converges to

$$\lim_{z \to \infty} V_0^T = I_0\phi,$$ \quad (32)

i.e., the present value without capital gains taxation. We obtain for the last term in brackets in equation (30), the capital gains tax multiplier,

$$\lim_{z \to \infty} \frac{(1 + \gamma T^c)^z - \frac{1}{\Phi}}{(1 + i^c)^z} = 0.$$ \quad (33)

The multiplier converges to zero. This finding implies that the maximum present value at the right boundary is reached whenever the capital gains taxation has no influence on the investment value. This result is robust for all investigated tax systems with capital gains taxation, i.e., also for different tax rates and retention rates. The maximally reachable present value of the right boundary is $I_0\phi$, the present value without capital gains taxation. We denote the earliest exit time which reaches a present value of $I_0\phi$ with $z^{max}$. Then, equation (30) simplifies to

$$V_0^T[T = z^{max}] = I_0\phi. \quad (34)$$

---

38 See figure 6.
39 Since it is a matter of a limit value of $V_0^T$, there is an error of $1e-8$. See eq. (33).
Figure 6 depicts the exit time $z^{\text{max}}$.

In the following we compare $V_0^\tau[z = 0]$ with $V_0^\tau[z = z^{\text{max}}] = I_0 \phi[z = z^{\text{max}}]$ to find out, which of the local extrema is the global maximum and thus the exit time with the highest present value. Hence, we concentrate on the difference between the local extrema. If the right extreme value is the global maximum we have

$$V_0^\tau[z = 0] = I_0 \cdot \phi \left(1 - \beta \tau^g \cdot (1 - \frac{1}{\phi})\right) < I_0 \cdot \phi[z = z^{\text{max}}].$$  \hspace{1cm} (35)$$

Inequality (36) describes the difference of the local extrema which reflects the capital gains taxation respectively the capital loss tax refund for an exit at $z = 0$.

$$0 < \beta \tau^g \left(1 - \frac{1}{\phi}\right).$$  \hspace{1cm} (36)

If $\beta \tau^g > 0$, the following condition holds:

$$\phi > 1.$$  \hspace{1cm} (37)

If $\phi > 1$, which implies that the income tax rate is greater than the corporate tax rate, inequality (35) is satisfied. The capital gains taxation for $V_0^\tau[z = 0]$ leads to the highest present value with a sell-off at $z = z^{\text{max}}$ for the investor.\textsuperscript{40}

\textsuperscript{40} If $\phi < 1$, than the inequality $0 > \beta \tau^g \left(1 - \frac{1}{\phi}\right)$ is satisfied. The capital loss tax refund for $V_0^\tau[z = 0]$ leads to the highest present value with a sell-off in $z = 0$ for the investor.
We find, that if $\phi > 1$, the global maximum value is reached at $z = z_{\text{max}}$. So the relation

$$V_0^T [z = z_{\text{max}}] = V_0^T [z] |_{\tau g = 0}$$

(38)

holds. It is interesting whether there is an exit time $z = z_{\text{max}}$ that leads to identical present values for investments with capital gains taxation ($V_0^T$) and without capital gains taxation ($I_0 \cdot \phi$). At this exit time the capital gains taxation is irrelevant. For any other holding period the investor has to consider capital gains taxation in his or her investment decision. To determine $z_{\text{max}}$ we set the two present value functions equal and obtain

$$I_0 \cdot \phi = I_0 \cdot \phi \cdot \left(1 - \beta \tau g \cdot \frac{(1 + i \tau_c) z_{\text{max}} - \frac{1}{\phi}}{(1 + i) z_{\text{max}}} \right)$$

(39)

$$\iff 0 = \frac{(1 + y i \tau_c) z_{\text{max}} - \frac{1}{\phi}}{(1 + i) z_{\text{max}}}.$$

Unfortunately, we cannot determine $z_{\text{max}}$ from this theoretically. In fact, the denominator of equation (40) is greater than zero. Thus, multiplying with the non-vanishing denominator and writing $\phi$ on one side, the equation (40) holds if and only if

$$\phi = \frac{1}{(1 + y i \tau_c) z_{\text{max}}}$$

(40)

Since the second summand on the right hand side is always nonnegative, only $\phi \leq 1$ is possible. In this case, the highest present value of the investment is in $z = 0$. The investor realizes a capital loss and receives a tax refund at $z = 0$. In general the investment with $\phi < 1$ would be rejected by the investor because she or he prefers the alternative investment at the capital market. Only investments with $\phi > 1$ are beneficial but this case cannot be derived from (40). The reason for this result is that we deal with a limit value of $V_0^T$ which never reaches the value $I_0 \phi$. If we accept a certain deviation of the two present values from each other when determining $z_{\text{max}}$ for $\phi > 1$ analytically, the investment must be held for 300 to 7,000 periods to come as close as an error term of $1e - 8$ to $z_{\text{max}}$. Obviously, holding periods of this dimension are unrealistic or at least not decision relevant. Against this background we conclude that it is not worthwhile to conduct a numerical investigation to obtain a more precise value for $z_{\text{max}}$.

In the next step we examine another interesting point in time. It is interesting for the investor to know when a late exit is as attractive as an immediate exit, i.e., when the present

---

$^41$ $i \in (0, 1), \tau \in [0, 1), z \geq 0.$

$^42$ See eq. (33).
value for exits later than \( z = 0 \) reaches the present value for an exit at \( z = 0 \) again. Only if late exits are at least as attractive as immediate sales an investor will hold the investment. We define the exit time \( z = z^g \) with \( z^g \in [0, z^{max}] \), as the point in time at which the present value in case of an exit at \( z^g \) \( (V^T_0 [z = z^g]) \) is equal to the present value for an immediate exit at \( z = 0 \) \( (V^T_0 [z = 0]) \). Only for exit times later than \( z^g \) an investment earns more than in case of an immediate sale and thus should be held minimal for \( z^g \) periods. In figure 7 we illustrate \( z^g \) as the intersection of the present value in case of an immediate sale and a sale at \( z^g \).

![Figure 7: \( z^{max} \) and \( z^g \) for various \( z \)](image)

If \( \phi < 1 \), the investor realizes a capital loss at \( z = 0 \) and hence receives a tax refund. This tax refund causes the global maximum value at \( z = 0 \). For later exit times \( z > 0 \) this value \( V^T_0 [z = 0] \) cannot be reached again.

We concentrate on \( \phi > 1 \) as this is the only case in which the present value is greater than 1 and the investment is beneficial in comparison to the financial investment. We have \( \phi > 1 \) if the income tax rate is greater than the corporate tax rate. This relation holds for all tax systems.

We compare \( V^T_0 [z = 0] \) with \( V^T_0 [z = z^g] \) to answer the question for which exit time \( z \) the initial value at \( z = 0 \) can be reached again.

\[
V^T_0 [z = 0] = V^T_0 [z = z^g],
\]

(41)

---

43 See figure 1. The investor realizes a capital loss at \( z = 0 \) under the classic corporate tax system and shareholder relief tax system.

44 See Appendix.
For holdings with \( z > z^g \) the investment will be better off than in case of an immediate sale. Unfortunately, we could not find a way to analytically determine the tangential point \( z^g \) from this equation. In contrast to the \( z^{\text{max}} \)-function, \( z^g \) in equation (41) exists at least for \( \phi > 1 \). Hence, we can identify (beneficial) investments that generate after-tax cashflows, which exceed the initial present value at \( z = 0 \) for later sellings. Nevertheless, to determine this point in time we resort to a numerical approach.

### 3.5 Numerical Analysis

As we cannot calculate \( z^g \) analytically, we use Newton’s method to determine the break-even point \( z^g \). Recall that Newton’s method is an iterative scheme to compute numerically a zero of an equation of the form \( f(x) = 0 \) with a given differentiable function \( f \).

In the following we denote \( z^g \) as a break even point. From this point of time it is beneficial for the investor to hold the investment.

We assume the following parameters:

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_0 )</td>
<td>1</td>
</tr>
<tr>
<td>( i )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \tau^c )</td>
<td>0.4 (FI) and 0.25 (SR and CC)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \tau^g )</td>
<td>( \frac{\tau}{2} ) (FI and CC) and ( \tau ) (SR)</td>
</tr>
</tbody>
</table>

Table 1: Data for the numerical example

---

\[ I_0 \cdot \phi \left( 1 - \beta \tau^g \cdot (1 - \frac{1}{\phi}) \right) = I_0 \cdot \phi \cdot \left( 1 - \beta \tau^g \cdot \frac{(1 + yi \tau^c)z^g - \frac{1}{\phi}}{(1 + i \tau)z^g} \right), \]

\[ \iff 1 - \frac{1}{\phi} = \frac{(1 + yi \tau^c)z^g - \frac{1}{\phi}}{(1 + i \tau)z^g}, \]

\[ \iff 0 = \frac{(1 + yi \tau^c)z^g - \frac{1}{\phi}}{(1 + i \tau)z^g} + \frac{1}{\phi} - 1. \]
The assumed tax rates are based on real-world tax rates.\textsuperscript{46} We assume an income tax rate of $\tau = 0.5$, according to the average marginal tax rate in Germany (1990 - 2012).\textsuperscript{47} The capital gains tax rate is linked to the income tax rate. If the underlying tax system requires a uniform income tax rate on dividends and interest ($\beta = 1$), we set $\tau^g = \frac{\tau}{2}$. If the tax system is characterized by a preferential lower income tax rate on dividends we set $\beta = 0.5$ and choose $\tau^g = \tau$. Under all tax systems capital gains are assumed tax-exempted.

Figure 8 displays $z^g$ as a result of the Newton iteration for the three tax systems and different retention rates $\gamma$.

This figure illustrates that, e.g., under the full imputation (shareholder relief) tax system with a retention ratio of $\gamma = 0.2$ the investment yields the present value given for $z = 0$ after 66 (17) periods, while it takes 87 (47) periods for a high retention ratio of $\gamma = 0.6$.\textsuperscript{48}

If we assume that the investor strives to reach the break even point ($z^g$) as fast as possible and the preferred tax system depends on the profit distribution, the development of the profit distribution is essential for the investor. Under full imputation and shareholder relief

\textsuperscript{46} In the German full imputation tax system the corporate tax rate from 1999 until 2000 was 40 %. Within the shareholder relief tax system the corporate tax rate was 25 %. We choose the same tax rate for classic corporate tax system.

\textsuperscript{47} The marginal tax rate in Germany for 1990-1999: 53 %, 2005: 42 % and 2012: 45 %.

\textsuperscript{48} See also Protopapadakis (1983), who estimates average holdings periods of 21 to 34 years.
$V_0^g$ can be realized for earlier exits for low retention ratios. By contrast, under the classic corporate tax system we observe high retention rate to stimulate an earlier break even.\footnote{Under the classic corporate tax system for retention rates smaller than 0.4 a disadvantageous present value emerges, which we exclude. See section 3.4.}

To understand the mechanisms at work properly we have to separate the driving forces. Interest, captured in the discount factor, is taxed equally under all three systems. The capital gains tax rate is the same under all three systems, too. The difference is due to the corporate taxation, the dividend taxation and thus also taxes that have reduced accumulated capital gains, which will be tax-liable at sale.

Under the full imputation tax system the capital gains tax burden exceeds taxes on dividends.\footnote{The corporate taxation can be imputed only against the income tax on dividends.} With an increasing retention rate the present value decreases and the growth effect can only later be compensated by the discount effect. These effects delay $z^g$.

Under the shareholder relief tax system dividends and capital gains are subject to the same tax rate. Nevertheless, capital gains taxation occurs later and multiple taxation of profits is likely to occur: Both types of capital income are taxed at a corporate tax rate of 25\%. Dividends are subject to a capital income tax of 25\% in the same period. Capital gains are taxed at 25\% at the time of sale $z$. Note, the capital gains tax basis has been affected by the anticipated dividend tax of investor B. Capital gains are therefore at a tax disadvantage, particularly for high retention ratios. Then, the break even point $z^g$ is reached even later.

Under the classic corporate tax system the tax burden for capital gains is smaller than for dividends.\footnote{Dividends are taxed without tax shield with a tax rate of 50\% and capital gains are taxed at a rate 25\%.} This tax advantage leads to an increase in the present value for increasing retention rates. Consequently the break even point, $z^g$, is reached earlier.

The temporary difference in $z^g$ is higher for small retention rate ratios. This result is consistent with our results for the worst exit time.\footnote{To highlight the relevance of the assumed range of retention rates we give an example. If we look at the annual reports McDonald’s Corporation (2011) and McDonald’s Corporation (2013) we find a retention ratio of, e.g., 24\% in 2007, 57\% in 2008 and of 44\% in 2013. Relating to the example above, which is characterized by similar dividend rates, the holding period, which is necessary for a beneficial investment differs between 17 (shareholder relief tax system, $y = 0.2$) and 113 (classic corporate tax system, $y = 0.4$) periods.}

We find that from the investor’s perspective a change of the tax system as well as of the retention policy affects tax-optimal holding strategies. Under shareholder relief an early break even point results from a low retention rate whereas low retention rates are causal for a late break even point under a classic corporate tax system of countries which have changed their tax system from classic corporate tax system to shareholder relief tax system.
as, e.g., The Netherlands in 2001, United States in 2003 and Switzerland in 2007. These reforms highlight the relevance of our findings. In countries with such tax reforms the minimal holding period change dramatically in the aftermath of a reform.

To illustrate the influence of the capital gains taxation in a different way, figure 9 depicts the relative difference between the present value without and with capital gains taxation \( \frac{\phi - V_0}{\phi} \) for an immediate exit at \( z = 0 \). Although this is a stylized setting, this illustration of the present value with an exit time in \( z = 0 \) enables us to abstract from time effects of capital gains taxation, e.g., the growth and the discount effect. Here, we only capture the effect from capital gains taxation independent of the exit time.

![Figure 9](attachment:image.png)

\( \phi \): present value without capital gains taxation
\( V_0 \): present value with capital gains taxation
\( z \): exit time

\( \gamma \): retention ratio

**Figure 9:** \( \frac{\phi - V_0}{\phi} \) for different tax systems and various \( \gamma \) with \( \tau = 0.5 \) and \( z = 0 \)

Obviously, the greatest difference occurs under the shareholder relief tax system, i.e., the distortion of capital gains taxation is highest in this tax system. The differences of all tax systems increase with increasing retention ratios. This development is due to the higher influence of capital gains taxation. The reason for this development is the present value without capital gains taxation (\( \phi \)). The higher \( \phi \) the bigger the distortion of capital gains taxation. A closer consideration on the value of \( \phi \) is appropriate: From a tax perspective \( \phi \) is affected by income tax and corporate tax. Retained earnings are most intensively taxed under a full imputation tax system because there a corporate tax rate of 40% is applicable while the other systems are characterized by a corporate tax rate of 25%. Dividends are most taxed under the classic corporate tax system at a corporate tax 25% and income tax.

---

50% without tax exemptions.\footnote{By contrast, under a full imputation system dividends are only subject to the income tax at 50\% and under shareholder relief to the corporate tax at a rate of 25\% and only the half of dividends are subject to the income tax.} The moderate taxation under shareholder relief leads to the highest present value without capital gains and is crucial for the maximal deviation between the present value without and with capital gains taxation \((\phi - V_{\tau 0})\) in a shareholder relief tax system.\footnote{See figure 9.}

If we look at \(V_{\tau 0}\), capital gains are differently taxed in the three tax systems. The strong slope of the classic corporate curve is remarkable. The retention ratio for classic corporate tax system is most influential. In a classic corporate tax system capital gains taxation is most powerful. Here, the capital gains are not tax exempted and the tax base is relatively high with respect to a corporate tax rate of 25\%. The slope of the full imputation curve is flat. Indeed the capital gains tax rate is not tax exempted but the capital gains itself is the smallest with respect to the high corporate tax rate of 40\%. The slope of shareholder relief tax system is similar to full imputation tax system. In this system the capital gain is indeed higher than under full imputation\footnote{See corporate tax rate in shareholder relief tax system is 25\%.} but only the half of capital gain is taxed.

\section*{4 Conclusion}

As investments are often characterized by entry and exit flexibility investors need to make both decisions on investment and holding period. Both decision are typically affected by taxation. Particularly selling an investment often leads to capital gains inducing capital gains taxes. Capital gains crucially depend on the duration of an investment. Focusing on investments in corporate shares we investigate the impact of capital gains taxation on the holding period under three different tax systems. In our analytical investigation we examine whether and if so when there is an optimal exit time.

Unfortunately, we find that there is no optimal holding period for a present value maximizing investor. We show that an immediate sell-off often is more attractive than short holdings. Nevertheless, if an investor wants to hold the stocks for a longer period of time she or he is well advised to wait not only until the worst exit time but rather until the minimum exit time, i.e., until the present value for an immediate sell-off can be recovered. Obviously, capital gains taxation delays exit decisions. We show that this minimal holding period further crucially depends on the degree of income and corporate tax integration.
Our results indicate that shareholder relief allows those shareholders that aim for mid- or long-term investment time horizons for the earliest profitable sales. Hence shareholder relief provides the highest degree of exit time flexibility among the underlying three tax systems. The minimum exit time is a function of the retention ratio. While a high retention rate extends the minimal holding period under a shareholder relief tax system it is likely to accelerate sales under a classic corporate tax system. Under a classic corporate tax system the postponement is particularly pronounced due to the relatively high capital gains tax base.

Furthermore, besides being able to identify minimal holding periods, we extend the literature by determining the worst exit time, which should be avoided by investors. The worst exit time differs particularly among the underlying three tax systems if the retention rate is low. If a large share of the profits is distributed we show that under a classic corporate tax system investors are very inflexible in their exit decision and have to stick to their investment for long periods of time while the other two systems allow for exits with present values above the worst outcome earlier in time. Under full imputation given a large share of profits is distributed the worst exit time is reached earliest. These results are helpful in divestment decisions.

Despite the limitations that arise from our set of assumptions our investigation highlights the complexity of the mechanisms at work that affect the after-tax present value and thus investment and divestment behavior. Our results indicate that the capital gains tax lock-in effect is only a temporary effect, which leads to delayed exits. Investments under the classic tax system and full imputation are more affected by this distortion than investments under shareholder relief. Our findings provide interesting implications for tax reform discussions, in particular on the effects of capital gains taxes on mid- and long-term investments.
REFERENCES


APPENDIX

For the full imputation tax system $\alpha = 0$, $\beta = 1$. Hence we obtain\(^57\)

\[
\phi > 1 \quad (42)
\]

\[
\Leftrightarrow \quad \frac{(1 - \beta \tau)(1 - y)(1 - \alpha \tau^c)}{(1 - \tau - y(1 - \tau^c))} > 1
\]

\[
\Leftrightarrow \quad (1 - \tau)(1 - y) > (1 - \tau - y(1 - \tau^c))
\]

\[
\Leftrightarrow \quad (1 - \tau)((1 - y) - 1) > -y(1 - \tau^c).
\]

As $y \neq 0$

\[
1 - \tau < 1 - \tau^c
\]

\[
\Leftrightarrow \quad \tau > \tau^c.
\]

Under the shareholder relief tax system $\alpha = 1$, $\beta = 0.5$ and thus\(^58\)

\[
\phi > 1 \quad (43)
\]

\[
\Leftrightarrow \quad \frac{(1 - 0.5\tau)(1 - y)(1 - \tau^c)}{(1 - \tau - y(1 - \tau^c))} > 1
\]

\[
\Leftrightarrow \quad -\frac{0.5\tau(1 + \tau^c) - \tau^c}{0.5\tau(1 - \tau^c)} < y.
\]

Since $y < \frac{\tau^c}{i}$ we obtain

\[
-\frac{0.5\tau(1 + \tau^c) - \tau^c}{0.5\tau(1 - \tau^c)} \leq \frac{(1 - \tau)\overline{i}}{(1 - \tau^c)\overline{i}}
\]

\[
\Leftrightarrow \quad -0.5\tau - 0.5\tau \tau^c + \tau^c < 0.5\tau - 0.5\tau^2
\]

\[
\Leftrightarrow \quad (-0.5\tau + 1)\tau^c < (-0.5\tau + 1)\tau^2
\]

\[
\Leftrightarrow \quad \tau > \sqrt{\tau^c} \geq \tau^c.
\]

From $0 \leq \tau^c < 1$ follows

\[
\tau > \tau^c.
\]
For the classic corporate tax system we have to set \( \alpha = 1, \beta = 1 \) and get\(^{59} \)

\[
\phi > 1 \quad (44)
\]

\[
\Leftrightarrow \quad \frac{(1 - \beta \tau)(1 - \gamma)(1 - \alpha \tau^c)}{(1 - \tau) - \gamma(1 - \tau^c)} > 1
\]

\[
\Leftrightarrow \quad \gamma > \frac{\tau^c(1 - \tau)}{\tau(1 - \tau^c)}.
\]

Given that \( \gamma < 1 \) we can write

\[
1 > \frac{\tau^c(1 - \tau)}{\tau(1 - \tau^c)} \quad (45)
\]

\[
\Leftrightarrow \quad \tau(1 - \tau^c) > \tau^c(1 - \tau)
\]

\[
\Leftrightarrow \quad \tau > \tau^c.
\]

---

Impressum:
Arbeitskreis Quantitative Steuerlehre, arqus, e.V.
Vorstand: Prof. Dr. Ralf Maiterth (Vorsitzender),
Prof. Dr. Kay Blaufus, Prof. Dr. Dr. Andreas Löffler
Sitz des Vereins: Berlin

Herausgeber: Kay Blaufus, Jochen Hundsdörfer,
Martin Jacob, Dirk Kiesewetter, Rolf J. König,
Lutz Kruschwitz, Andreas Löffler, Ralf Maiterth,
Heiko Müller, Jens Müller, Rainer Niemann,
Deborah Schanz, Sebastian Schanz, Caren Sureth,
Corinna Treisch

Kontaktadresse:
Prof. Dr. Caren Sureth, Universität Paderborn,
Fakultät für Wirtschaftswissenschaften,
Warburger Str. 100, 33098 Paderborn,
www.arqus.info, Email: info@arqus.info
ISSN 1861-8944