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On the Taxation of Durable Goods and Housing

Abstract

This paper proposes a theory of commodity taxation in the presence of durable goods. Optimal commodity taxes depend on preference nonseparabilities between durable and nondurable consumption. In particular, the seminal Atkinson–Stiglitz result fails and differential commodity taxes are optimal even when the utility function is separable between labor and consumption. An application to housing decisions implies that housing should face higher tax rates than nondurable consumption. Moreover, the theory justifies housing provisions in the income tax code.

JEL-Code: D820, H210.

Keywords: optimal taxation, commodity taxation, durable goods, pre-committed goods, Atkinson-Stiglitz result.

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1 Introduction

Durable goods account for roughly 40 percent of a typical household's consumption expenditure.¹ Most durable goods (real estate, cars, furniture, consumer electronics, etc.) have a lifespan of many years. Their purchase and resale are subject to significant search costs, transaction costs or nondivisibilities.² Thus, the purchase of a durable good has a long-lasting effect on the household's future consumption.

This paper demonstrates how commodity taxes can utilize the long-lasting effects of durable goods in order to provide incentives. In a dynamic environment with durable and nondurable goods, we show that uniform commodity taxes are suboptimal even when the utility function is separable between labor and all goods. In this sense, the seminal Atkinson–Stiglitz theorem (Atkinson and Stiglitz, 1976) on uniform (intra-period) commodity taxation does not hold when some of the goods are durable.

As is well known from the analysis of static taxation problems, optimal commodity taxes generally depend on nonseparabilities between labor and consumption. The main insight of the present paper is that optimal commodity taxes also depend on nonseparabilities between durable and nondurable consumption. Hence, nonseparabilities between different consumption goods already motivate the use of differential taxes. The key idea is that durable goods change the valuation of future (nondurable) consumption and thereby affect the incentive to supply labor effort in the future. If durable and nondurable consumption are substitutable with each other, the durable good should be taxed, because durable consumption lowers the need for future nondurable consumption and diminishes the need to work hard in the future. On the other hand, if durable and nondurable consumption are complementary, the argument is reversed and the durable good should be subsidized. We show that our findings remain

¹The average annual expenditure on durable goods (shelter, household furnishings and equipment, apparel, vehicles, entertainment equipment) in the Consumer Expenditure Survey (CEX) 2011 is \$25,390. The average annual total expenditure amounts to \$63,972.

²Common estimates of the transaction costs for housing, for example, range from 4 to 14 percent of the house value (OECD, 2011).

valid when the durable good can be retraded over time. Moreover, for preferences that are nonseparable between labor and consumption, the same forces are at work and are supplemented by standard taxation motives based on relative complementarities between labor supply and the different goods.

In an application to housing, we present a novel rationale for housing taxes based on a nonseparability between housing and non-housing consumption. This finding calls into question the widespread tax advantages for housing wealth present in many countries.³ Different from conventional critiques of such tax advantages, our rationale is based on pure efficiency reasoning and is independent of redistributive goals. We show that empirical studies of the preferences for housing suggest that housing and nondurable consumption are Edgeworth substitutes. Therefore, our theoretical results imply that housing should face higher tax rates than nondurable consumption. We provide a simple calibrated version of our model to evaluate the quantitative importance of housing taxes. We find optimal tax rates (wedges) on the price of housing between 5% and 28% and welfare gains of housing taxes up to 0.3% of life-time consumption. Although the calibrated model is, admittedly, too stylized for a definite analysis of housing taxation, these numbers emphasize the quantitative relevance of our theoretical findings. Finally, we construct a housing-dependent income tax that allows to decentralize optimal allocations. This result provides a justification for housing provisions in the income tax code.

The paper is structured as follows. The remainder of this section surveys the related literature. Section 2 sets up our basic model of social insurance with durable and nondurable goods. Section 3 derives the optimal commodity wedges across goods and across time. We extend the basic model to allow for the retrade of durable goods. We also examine the role of preference nonseparabilities between consumption and labor effort. Section 4 applies our theory to the case of housing—a prime example of

³For instance, mortgage interest is (partly or fully) tax-deductible in the United States, the Netherlands, Switzerland, Belgium, Ireland, Norway and Sweden. In the UK, there is a reduced value added tax on the construction of new houses and renovations.

a durable good. Section 5 provides concluding remarks. Appendix A contains the proofs of all theoretical results.

Related literature. Our analysis builds on the study of pre-committed goods by Cremer and Gahvari (1995a,b). We extend their approach by proposing an explicit model of durability in a dynamic framework. We show that the tax implications of durable goods differ from those of pre-committed goods. At the same time, we suggest a novel interpretation of their findings: subsidies to pre-committed goods can be seen as a rationale for intertemporal wedges on *nondurable* goods in dynamic frameworks.

More generally, our paper is related to the vast literature on commodity taxation that emerges from the analysis by Atkinson and Stiglitz (1976). The original Atkinson–Stiglitz theorem is derived in a static environment that, by construction, cannot distinguish between durable and nondurable goods. Golosov, Kocherlakota, and Tsyvinski (2003) extend the theorem to a dynamic framework under the assumption that all goods in any period provide utility in the given period only. Therefore, neither the original Atkinson–Stiglitz result nor the dynamic extension apply to the case of durable goods.

We present our results in a moral hazard model of social insurance in an ex-ante homogeneous population, similar to the models of Eaton and Rosen (1980) and Varian (1980). As demonstrated in earlier research (da Costa and Werning, 2002), the role of commodity taxation coincides for hidden action models and private information (adverse selection) models. Therefore, it is not difficult to transfer our findings to the Mirrleesian framework adopted by Atkinson and Stiglitz (1976).⁴

Another argument for differential commodity taxes is proposed by Naito (1999) and recently Gomes, Lozachmeur, and Pavan (2014). Their papers study static environments with multiple sectors. One key finding is that, when sector-specific income taxation is impossible, commodity taxes can indirectly redistribute toward low-skilled

⁴The moral hazard framework employed in the present paper has an advantage of tractability. All tax implications can be derived with elementary methods, without the need for optimal control techniques and assumptions on the applicability of envelope theorems in dynamic environments (see LaFrance and Barney, 1991).

workers by affecting the wage distribution.

Grochulski and Kocherlakota (2010) and Koehne and Kuhn (2014) analyze labor and savings taxation when the consumption preferences are time-nonseparable due to habit formation. In both papers, there is only a single good in each period and so there is no scope for differential commodity taxation. The present findings on durable and nondurable goods suggest that habit formation may motivate commodity taxes when the habit formation relation differs across goods.

2 Setup

We study a two-period model of social insurance.⁵ A risk-neutral social planner provides insurance to a continuum of ex-ante identical, risk-averse agents. In each period, the agents influence their incomes by exerting labor effort. Income is subject to uncertainty and effort is private information. This creates a moral hazard problem. The planner faces a trade-off between insuring the agents against idiosyncratic income uncertainty and providing appropriate incentives for exerting labor effort.

2.1 Preferences

The agents discount the future at the rate $\beta \in (0, 1)$. The agents have identical von Neumann–Morgenstern preferences and maximize the expected value of

$$\tilde{u}(\tilde{c}, d) - \tilde{v}(\tilde{e}) + \beta (u(c, d) - v(e)),$$

where \tilde{c} and \tilde{e} represent the agent's nondurable consumption and effort in the first period, d represents durable consumption, and c and e represent the agent's nondurable consumption and effort in the second period. The consumption utilities $\tilde{u} : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ and $u : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ are twice continuously differentiable, strictly increasing in both ar-

⁵Our results generalize to longer time horizons but the notation becomes more cumbersome.

guments and strictly concave. As usual, we use subscript notation to represent partial derivatives. The labor disutilities $\tilde{v} : \mathbb{R}_+ \rightarrow \mathbb{R}$ and $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ are continuously differentiable, strictly increasing and weakly convex.

2.2 Production

In each period, the agents exert labor effort to generate a stochastic output. Output realisations for period 1 are denoted by $x_i, i = 1, \dots, N$, with associated probabilities $\tilde{\pi}_i(\tilde{e}) > 0$. Output realisations for period 2 are denoted by $x_j, j = 1, \dots, N$, with associated probabilities $\pi_j(e) > 0$. The planner observes output realisations, but does not observe effort decisions.

2.3 Social insurance problem

An *allocation* is a specification $(\mathbf{c}, \mathbf{d}, \mathbf{e})$ of nondurable consumption $\mathbf{c} = (\tilde{c}_i, c_{ij})_{i,j=1,\dots,N}$, durable consumption $\mathbf{d} = (d_i)_{i=1,\dots,N}$, and effort $\mathbf{e} = (\tilde{e}_0, e_i)_{i=1,\dots,N}$. In this notation, the indices i and j indicate that the respective variables depend on the output realisations x_i in period 1 and x_j in period 2.

The planner has access to a linear savings technology and a linear technology for transforming output into durable or nondurable consumption. Without loss of generality, we choose units of measurement such that the technological rates of transformation between all goods equal one. The budget constraint is therefore

$$\sum_{i,j} \tilde{\pi}_i(\tilde{e}_0) \pi_j(e_i) [x_i - \tilde{c}_i - d_i + x_j - c_{ij}] \geq 0. \quad (1)$$

Since effort is private information, allocations need to satisfy the incentive compatibility constraint:

$$\mathbf{e} \in \operatorname{argmax}_{e'_0, e'_i} \sum_{i,j} \tilde{\pi}_i(e'_0) \pi_j(e'_i) \left(\tilde{u}(\tilde{c}_i, d_i) - \tilde{v}(e'_0) + \beta [u(c_{ij}, d_i) - v(e'_i)] \right). \quad (2)$$

We assume that the first-order approach is valid.⁶ Therefore, we replace the incentive compatibility constraint (Eq. (2)) by the associated first-order conditions:

$$\tilde{v}'(\tilde{e}_0) \leq \sum_{i,j} \tilde{\pi}'_i(\tilde{e}_0) \pi_j(e_i) \left(\tilde{u}(\tilde{c}_i, d_i) + \beta [u(c_{ij}, d_i) - v(e_i)] \right), \quad (3)$$

$$v'(e_i) \leq \sum_j \pi'_j(e_i) u(c_{ij}, d_i), \quad i = 1, \dots, N. \quad (4)$$

The planner is benevolent and maximizes ex-ante welfare. An allocation $(\mathbf{c}, \mathbf{d}, \mathbf{e})$ is therefore called *optimal* if it maximizes ex-ante welfare

$$\sum_{i,j} \tilde{\pi}_i(\tilde{e}_0) \pi_j(e_i) \left(\tilde{u}(\tilde{c}_i, d_i) - \tilde{v}(\tilde{e}_0) + \beta [u(c_{ij}, d_i) - v(e_i)] \right) \quad (5)$$

subject to the budget constraint (Eq. (1)) and the incentive compatibility constraints (Eq. (3) and (4)). We assume throughout the paper that optimal allocations are interior, i.e., consumption and effort levels are strictly positive.

3 Optimal commodity taxation

This section derives the *commodity wedges* (tax distortions) imposed by optimal allocations. Commodity wedges describe the marginal distortions to the commodity prices required by any tax system that decentralizes an optimal allocation. Note that the robust insights from the present analysis are indeed about wedges and not about explicit tax instruments, because the decentralisation of optimal allocations in dynamic environments is generally not unique.⁷ One specific decentralisation is presented in

⁶The incentive problem in the first period is part of the model only for the sake of symmetry across periods. All our theoretical results can already be obtained in a simpler model with a realisation of uncertainty in the second period only. In this simpler model, the validity of the first-order approach is guaranteed under the well-known conditions from the static moral hazard framework (Rogerson, 1985; Jewitt, 1988). For example, the first-order approach is valid if the distribution function is convex in effort and the likelihood ratio is monotonic. The convexity of the distribution function can be interpreted as an empirically supported restriction on the elasticity of labor supply (Abraham, Koehne, and Pavoni, 2011).

⁷Note that the wedges of optimal allocations often provide guidelines for tax rates in settings with simple, linear instruments (Farhi and Werning, 2013).

Section 4.3.

The tax differential between the durable good d and the nondurable good \tilde{c} is captured by the *durability wedge*

$$\tau_i^d := \frac{\tilde{u}'_d(\tilde{c}_i, d_i) + \beta \sum_j \pi_j(e_i) u'_d(c_{ij}, d_i)}{\tilde{u}'_c(\tilde{c}_i, d_i)} - 1.$$

The durability wedge is the implicit tax rate on the durable good that equates the agent's marginal rate of substitution between durable and nondurable consumption with the marginal rate of transformation between the two goods. A positive durability wedge indicates that the durable good faces a higher implicit tax rate than the nondurable good. As suggested by the notation, the durability wedge may depend on the realisation x_i of the agent's output in period 1.

For comparison, we also consider the tax differential associated with nondurable consumption at different points in time. We define the *intertemporal wedge* as

$$\tau_i^s := 1 - \frac{\tilde{u}'_c(\tilde{c}_i, d_i)}{\beta \sum_j \pi_j(e_i) u'_c(c_{ij}, d_i)}.$$

This wedge represents a savings tax. Formally, the intertemporal wedge is the implicit tax rate on nondurable consumption in period 2 (relative to period 1) that equates the agent's marginal rate of intertemporal substitution with the marginal rate of intertemporal transformation.

Proposition 1 (Commodity wedges). *Let $(\mathbf{c}, \mathbf{d}, \mathbf{e})$ be an optimal allocation.*

- (i) *The durability wedge is weakly positive if durable and nondurable consumption are substitutes in the second period ($u''_{cd} \leq 0$). The durability wedge is strictly positive if the incentive constraint in the second period is binding and $u''_{cd} < 0$. The sign of the durability wedge is reversed if durable and nondurable consumption are complements in the second period.*
- (ii) *The intertemporal wedge is strictly positive if the incentive constraint in the second*

period is binding, and zero otherwise.

The first part of Proposition 1 shows that durable goods should be taxed differently from nondurables, provided that they are nonseparable with consumption in the following period. Hence, even though the preferences are additively separable between consumption and labor effort, the Atkinson–Stiglitz result on uniform (intra-period) commodity taxation does not apply to our setup.

The key difference between durable and nondurable consumption is that durable consumption affects the incentive problem in the following period. This dynamic incentive effect is precisely the reason why the Atkinson–Stiglitz result fails. Intuitively, if durable consumption is complementary with nondurable consumption in the following period ($u''_{cd} \geq 0$), durable consumption is socially desirable because it relaxes the incentive problem by raising the marginal utility of consuming in the following period. In that case, durable consumption should be taxed less than nondurable consumption ($\tau_i^d \leq 0$), as shown by Proposition 1(i). On the other hand, if durable consumption is substitutable with nondurable consumption in the following period, it should be taxed more than nondurable consumption ($\tau_i^d \geq 0$).

To make the intuitive argument more formal, we use the identity $\sum_j \pi'_j(e) = 0$ to write the second-period incentive constraint (Eq. (4)) as

$$v'(e_i) \leq \sum_j \frac{\pi'_j(e_i)}{\pi_j(e_i)} [u(c_{ij}, d_i) - u(c_{i1}, d_i)] \pi_j(e_i).$$

The right-hand side of this constraint measures the covariance between the likelihood ratio $\pi'_j(e_i)/\pi_j(e_i)$ and the utility increments $u(c_{ij}, d_i) - u(c_{i1}, d_i)$.⁸ The utility increments are increasing in the likelihood ratio, as can be seen from the optimality conditions for consumption. Hence, both random variables are monotonic in the same direction and their covariance is positive.⁹ Using the equation $u(c_{ij}, d_i) - u(c_{i1}, d_i) =$

⁸The condition is particularly transparent in the case of two outputs, $N = 2$, with $\pi_2 = 1 - \pi_1 =: \pi$. Then, the incentive constraint becomes $v'(e_i) \leq \pi'(e_i)[u(c_{i2}, d_i) - u(c_{i1}, d_i)]$.

⁹If we assume the monotone likelihood ratio property, both functions are increasing in j .

$\int_{c_{i1}}^{c_{ij}} u'_c(\xi, d_i) d\xi$, we see that the absolute value of the utility increments increases with the level of durable consumption d_i if $u''_{cd} \geq 0$. In that case, durable consumption increases the above covariance. This effect relaxes the second-period incentive constraint and thus motivates a subsidy to durable consumption.

The second part of Proposition 1 shows that a positive intertemporal wedge on nondurable goods is optimal. This result is well known and stems from the adverse effect of savings on the incentive to supply labor effort (see Diamond and Mirrlees, 1978; Golosov et al., 2003). Note that the two parts of Proposition 1 explicitly distinguish between intra-period and inter-period commodity wedges. This distinction leads to a novel interpretation of the analysis of pre-committed goods by Cremer and Gahvari (1995a,b). Assuming separability between pre-committed and post-uncertainty goods, their main result is that pre-committed goods should be subsidized relative to post-uncertainty goods. Our model becomes formally equivalent to their framework if we remove the durable good d from our model: nondurable consumption in the first period is pre-committed (i.e., decided before the realisation of uncertainty), whereas nondurable consumption in the second period is a post-uncertainty good that is separable from nondurable consumption in the first period. Proposition 1(ii) thus implies that the motive to subsidize pre-committed goods provides a rationale for intertemporal wedges on *nondurable* goods in a dynamic model. At the same time, Proposition 1(i) implies that the tax implications of durable goods will generally differ from those of pre-committed goods.¹⁰

3.1 Retrade of durable goods

To streamline the argument, our basic setup abstracted from the possibility that durable goods can be retraded/adjusted over time. In reality, adjustment opportunities exist for most durable goods. However, transaction costs (fees, taxes, search costs, etc.) are

¹⁰Pre-committed goods need not be durable—think of booking a holiday trip or buying tickets for the opera in advance. Moreover, both durable and nondurable consumption may take place before and after the resolution of uncertainty.

often an important barrier to adjustment. For instance, common estimates of the transaction costs for housing range from 4 to 14 percent of the house value (OECD, 2011) and so the typical household buys and sells a home only seldom. Estimates of the transaction costs for vehicles and other categories of durables also point to substantial costs and again make frequent adjustments suboptimal.¹¹

This section shows that the tax differentials from the basic setup persist if the durable good can be retraded subject to adjustment costs. To this end, we suppose that the durable good can be adjusted in the second period (after the realisation of uncertainty) by paying a fixed cost $\alpha > 0$. We write $\mathbf{d} = (\tilde{d}_i, d_{ij})_{i,j=1,\dots,N}$ for durable consumption \tilde{d}_i in the first period and durable consumption d_{ij} in the second period. We set the technological rate of transformation (“retrade price”) between the durable good and output in the second period to $p \in (0, 1)$.

The social insurance problem with retrade takes the following form:¹²

$$\max_{\mathbf{c}, \mathbf{d}, \mathbf{e}} \sum_{i,j} \tilde{\pi}_i(\tilde{e}_0) \pi_j(e_i) \left(\tilde{u}(\tilde{c}_i, \tilde{d}_i) - \tilde{v}(\tilde{e}_0) + \beta [u(c_{ij}, d_{ij}) - v(e_i)] \right) \quad (6)$$

s.t.

$$\sum_{i,j} \tilde{\pi}_i(\tilde{e}_0) \pi_j(e_i) \left[x_i - \tilde{c}_i - \tilde{d}_i + x_j - c_{ij} - p(d_{ij} - \tilde{d}_i) - \alpha \mathbb{1}_{\{d_{ij} \neq \tilde{d}_i\}} \right] \geq 0 \quad (7)$$

$$\tilde{v}'(\tilde{e}_0) \leq \sum_{i,j} \tilde{\pi}'_i(\tilde{e}_0) \pi_j(e_i) \left(\tilde{u}(\tilde{c}_i, \tilde{d}_i) + \beta [u(c_{ij}, d_{ij}) - v(e_i)] \right) \quad (8)$$

$$v'(e_i) \leq \sum_j \pi'_j(e_i) u(c_{ij}, d_{ij}), \quad i = 1, \dots, N. \quad (9)$$

Once we allow for retrade, adjustment costs become essential for the distinction between durable and nondurable goods. This insight is formalized in the following

¹¹For example, in addition to a profit margin, car dealers in the United States often levy documentary and administrative fees in the range of several hundred dollars to pay for the costs associated with registering a vehicle. For broader definitions of durable goods, indirect evidence on the size of transactions costs can be found in a recent paper by Alvarez, Guiso, and Lippi (2012). They calibrate the transaction costs for durable goods in order to replicate empirical trade frequencies and report transaction costs in the range of one percent.

¹²Implicitly, we model the adjustment cost as a social deadweight loss. Search costs are a prime example for this case.

result.

Remark 1. *If the adjustment cost is zero (or sufficiently low), retrade occurs for all realisations of uncertainty. In that case, the durable good is effectively nondurable and the durability wedge is zero.*

When adjustment costs are non-negligible, we obtain the following extension of Proposition 1.

Proposition 2 (Commodity wedges with retrade). *Let $(\mathbf{c}, \mathbf{d}, \mathbf{e})$ be an optimal allocation in the model with retrade.*

- (i) *The durability wedge is strictly positive if durable and nondurable consumption are strict substitutes in the second period ($u''_{cd} < 0$), the incentive constraint in the second period is binding and the adjustment cost α is sufficiently large. The sign of the durability wedge is reversed if durable and nondurable consumption are strict complements in the second period.*
- (ii) *The intertemporal wedge is strictly positive if the incentive constraint in the second period is binding, and zero otherwise.*

Remark 2. *Proposition 2 remains valid when the adjustment cost depends on the size of the adjustment or the level of durable consumption (in addition to a fixed cost of adjustment).*

Remark 3. *A similar result can be obtained when the durable good is not perfectly divisible. The logic of Proposition 2 applies whenever the nondivisibility is sufficiently large to prevent retrade for at least two realisations of uncertainty.*

Proposition 2 shows that the tax differentials from the basic setup generalize to the model with retrade. First, the results on intertemporal wedges are not at all affected by the retrade option for the durable good. Second, the sign of the durability wedge coincides with that of the basic setup provided that the adjustment cost α is sufficiently large. This condition is satisfied if there are at least two realisations of uncertainty in the second period for which the durable good is not adjusted.

The proof of Proposition 2 points out a peculiarity for the case of low adjustment costs. If there is exactly one state in the second period in which the durable good is not adjusted, the sign of the durability wedge is, in fact, determined by the likelihood ratio $\pi'_j(e_i)/\pi_j(e_i)$ in the no-adjustment state. The durability wedge is then negative if the likelihood ratio is positive in the no-adjustment state, and positive otherwise. Intuitively, durable consumption in the first period makes the no-adjustment state in the second period more attractive (relative to other states in the second period), because utility in the second period increases with durable consumption ($u'_d > 0$). Whether this is good or bad for incentives in the second period depends on the likelihood ratio. If the likelihood ratio in the no-adjustment state is positive, then the lasting effect of durable consumption on the no-adjustment state relaxes the incentive constraint in the second period, and thus durable consumption in the first period should be subsidized. Note, however, that this simple reasoning applies only if there is exactly one no-adjustment state. If there are two or more such states, durable consumption will affect the incentive problem through its *differential* impact on second-period states with heterogeneous nondurable consumption and the durability wedge will be shaped by the cross-derivative of the second-period utility function ($u''_{cd} \geq 0$) as in the basic setup.

3.2 Nonseparability between effort and consumption

Next, we allow for preference nonseparability between labor effort and consumption. We study utility functions of the form $\tilde{u}(\tilde{c}, d, \tilde{e}) + \beta u(c, d, e)$, where \tilde{u} and u are twice continuously differentiable, strictly concave, strictly increasing in the first two arguments and strictly decreasing in the third argument. By proceeding as in the proof of Proposition 1, we find that optimal allocations for nonseparable preferences generate the following durability wedge:

$$\tau_i^d = -\frac{\mu_i}{\tilde{\pi}_i (\lambda - \mu \tilde{u}''_{ce})} \sum_j \frac{\pi'_j}{\pi_j} u'_d \pi_j - \frac{\mu (\tilde{u}''_{de} - \tilde{u}''_{ce})}{\lambda - \mu \tilde{u}''_{ce}} - \frac{\mu_i \sum_j \pi_j u''_{de}}{\tilde{\pi}_i (\lambda - \mu \tilde{u}''_{ce})},$$

where λ , μ and μ_i denote the Lagrange multipliers associated with the planner's budget constraint and the incentive compatibility constraints.

It can be shown that all denominators (and Lagrange multipliers) in the above expression for τ_i^d are positive. Hence, the three components of the durability wedge can be interpreted as follows. The first term, $\sum_j \frac{\pi_j'}{\pi_j} u_d' \pi_j$, is well known from our analysis of the separable case (see Eq. (14) in the proof of Proposition 1). It captures the complementarity between durable and nondurable consumption. This term implies that the durable good should face a lower tax rate if durable and nondurable consumption are complements. The two remaining terms are novel and stem from nonseparabilities between consumption and labor effort. The second term shows that the durable good should face a lower tax rate if, in the first period, durable consumption is more complementary with labor effort than nondurable consumption ($\tilde{u}_{de}'' \geq \tilde{u}_{ce}''$). Finally, the third term shows that the durable good should face a lower tax rate if it is complementary with labor effort in the second period ($u_{de}'' \geq 0$).

4 Application to housing

In this section, we apply our theoretical insights to the case of housing. We derive qualitative and quantitative predictions for the optimal tax treatment of housing relative to nondurable consumption. Moreover, we show that our model justifies housing provisions in the income tax code. Note that housing is a prime example of a durable good. In particular, housing is only seldom changed over time, since the purchase and sale of a home imposes substantial transaction costs. Therefore, housing decisions generate the lasting effects on future preferences that are central to our model.

4.1 Qualitative findings for a common preference specification

In the economic literature on housing, preferences are commonly specified using a utility function with a constant elasticity of substitution,

$$u(c, h) = \frac{\left[(1 - \omega)c^{1-\frac{1}{\epsilon}} + \omega h^{1-\frac{1}{\epsilon}} \right]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\epsilon}}}}{1 - \frac{1}{\sigma}}, \quad (10)$$

where c denotes nondurable consumption, h denotes housing services, the parameter $\epsilon > 0$ measures the intratemporal substitutability between housing and nondurable consumption, $\omega \in (0, 1)$ controls the expenditure share on housing, and $\sigma > 0$ governs the intertemporal substitutability of the consumption-housing composite.¹³ This specification implies that housing and nondurable consumption are strict substitutes in the Edgeworth sense ($u''_{ch} < 0$) if and only if the parameters satisfy $\epsilon > \sigma$, i.e., if and only if the intratemporal elasticity of substitution exceeds the intertemporal elasticity. Therefore, the application of Proposition 1 yields the following result.

Corollary to Proposition 1. *If $\epsilon > \sigma$, housing should face higher tax rates than nondurable consumption.*

Li, Liu, and Yao (2009) provide one of the first structural estimations of the above preference parameters. Using a method of simulated moments based on household decision profiles, they obtain parameter values of $\epsilon = 0.323 > 0.162 = \sigma$. Bajari, Chan, Krueger, and Miller (2013) follow a similar approach for logarithmic utility functions and estimate $\epsilon = 4.550 > 1 = \sigma$. Piazzesi, Schneider, and Tuzel (2007) calibrate a model with $(\epsilon, \sigma) = (1.05, 0.2)$ based on macro-level consumption data.¹⁴ In sum, the available empirical evidence suggests that housing and nondurable consumption are Edgeworth substitutes. According to our model, *housing should therefore face higher tax*

¹³For $\sigma = 1$, preferences take a logarithmic form.

¹⁴See the references in their paper for further calibrations in which the intratemporal elasticity of substitution exceeds the intertemporal one.

rates than nondurable consumption.¹⁵

4.2 Numerical results for a calibrated model

To assess the quantitative role of a housing tax, we explore a calibrated version of the model. A period in the calibrated model corresponds to 20 years. This implies that the two-period model roughly covers the working life of a typical employee.¹⁶ The utility and probability functions are constant over time.

We model the consumption preferences with a CES utility function as in Eq. (10). The disutility of labor effort is specified as $v(e) = \alpha(1 - e)^{1-1/\eta} / (1 - 1/\eta)$, with a Frisch elasticity parameter of $\eta = 0.5$. The time-discount factor is 0.96 per annum. For output (gross earnings), we choose a binary process with $x_1 < x_2$. We measure labor effort in units such that the probability functions are linear: $\pi_2(e) = e$, $\pi_1(e) = 1 - e$, with $0 \leq e < 1$. With this specification, expected earnings are linear in labor effort, which allows us to interpret labor effort as a standard labor supply decision along the intensive margin (Abraham et al., 2011). We determine the output levels such that, when labor effort is one-third, the variance of log-output equals 0.432 and expected output is normalized to 1. This setup matches the cross-sectional variance of log-earnings in the United States in the period 1967–2006 (Heathcote, Storesletten, and Violante, 2014, Table 3).

We consider a range of empirically plausible elasticities (ϵ, σ) based on the three external estimation/calibration results referred to in Section 4.1. The remaining parameters of the model are the preference weights (α, ω) . These parameters are calibrated within the model to match specific targets for labor supply and the expenditure share on housing. We target an average labor effort of one-third, which roughly corresponds to the average fraction of working time over total disposable time in the United States.

¹⁵This insight is further strengthened for nonseparable preferences if we hypothesize that housing is complementary with leisure (or substitutable with labor).

¹⁶Our two-period model with binary output requires 11 allocation variables. The dimensionality of the problem grows exponentially in the number of time periods.

	Piazzesi et al. (2007)	Li et al. (2009)	Bajari et al. (2013)
(ϵ, σ)	(1.05, 0.2)	(0.323, 0.162)	(4.550, 1)
housing wedge (%)	27.7	15.6	5.1
intertemp wedge (%)	45.5	50.3	18.0
welfare gain (%)	0.26	0.03	0.06

Table 1: Expected housing wedges, expected intertemporal wedges and welfare gains of housing taxation for different preference scenarios

Based on CEX 2011 data, we target an expenditure share on housing of 0.23.¹⁷ Finally, we made one further adjustment to the setup from Section 2. Since housing and non-durable consumption are now measured in the same monetary unit, it is no longer valid to normalize the marginal rates of transformation between all goods to unity. We assume that the planner can transfer resources over time at rate $q = \beta$. One unit of housing (fixed across time) can then be converted into $1 + q$ units of nondurable consumption in the first period or $1/q + 1$ units of nondurable consumption in the second period. We adapt the planner's budget constraint and the definition of the commodity wedges accordingly. We provide a formal description of the quantitative model in Appendix B.¹⁸

Optimal housing taxes. Table 1 presents the expected durability wedges ("housing wedges") implied by the optimal allocations of different preference scenarios. We note that the durability wedge (implicit tax rate on housing) is quantitatively significant and ranges from 5 to 28 per cent. We also compute the welfare gains of housing taxation. To this end, we solve an auxiliary model that constrains the planner to equalize the marginal rate of substitution between nondurable consumption and housing with the marginal rate of transformation. Then, we compare welfare in the constrained model to welfare in the baseline model and express the welfare change in consumption equivalent terms. As the last row of Table 1 shows, the welfare gains of housing

¹⁷The average annual expenditure on shelter in the CEX 2011 amounts to \$11,780, compared with \$38,582 on nondurable consumption.

¹⁸We abstract from retrade in the calibrated model. As our theoretical analysis shows, a meaningful analysis of retrade requires a large number of income states, because such models generate very different results when the durable good remains fixed in less than two states.

taxation range from 0.03 to 0.26 per cent of life-time consumption. We note that the implied housing taxes and the welfare gains of housing taxation are largest for the preference parameters calibrated by Piazzesi et al. (2007). For those parameters, the discrepancy between the intratemporal and the intertemporal elasticity of substitution is particularly pronounced, which favors a stronger degree of Edgeworth substitutability between housing and nondurable consumption.¹⁹

Our quantitative results differ markedly from Cremer and Gahvari (1995b), who analyze a similar calibrated model but formalize housing as a pre-committed good in a static framework. They find *subsidies* on housing of 25 percent to be optimal. The associated welfare gain is 5 percent of consumption. Based on our interpretation in Section 3, subsidies to pre-committed goods can be related to intertemporal wedges (savings distortions) on nondurable goods in a dynamic model. The expected intertemporal wedges in our calibrated model are positive, indeed, and their quantitative magnitude (fourth row of Table 1) is broadly comparable to the subsidies found by Cremer and Gahvari (1995b).

4.3 An argument for housing provisions in the income tax code

We return to the general setup from Section 2 and present a decentralisation of optimal allocations through a tax system. The tax system combines a savings tax with an income tax that depends on the holdings of the durable good.²⁰ The savings tax creates the appropriate intertemporal wedge, whereas the durability wedge results from the income tax system. Although our arguments are general, housing constitutes a particularly natural example for the durable good in this system, because existing income tax codes often contain provisions related to housing.²¹

¹⁹The difference between the two elasticities is also pronounced for the calibration by Bajari et al. (2013). However, their model has a lower coefficient of risk aversion, which makes improvements to social insurance less valuable.

²⁰Of course, an income tax that depends on the durable good is formally equivalent to an income-dependent commodity tax.

²¹One example is the tax deductibility of mortgage interest. Note, however, that our system imposes a tax on housing, whereas provisions for mortgage interest often take the form of subsidies.

We fix an optimal allocation $(\mathbf{c}, \mathbf{d}, \mathbf{e})$ and define income taxes (t^1, t^2) for periods 1 and 2 and savings taxes t^s as follows:

$$t^1(x_i) := x_i - \tilde{c}_i - d_i, \quad (11)$$

$$t^2(x_i, x_j, d) := x_j - c_{ij} + (d - d_i) \frac{\tilde{u}'_d(\tilde{c}_i, d_i) + \beta u'_d(c_{ij}, d_i) - \tilde{u}'_c(\tilde{c}_i, d_i)}{\beta u'_c(c_{ij}, d_i)}, \quad (12)$$

$$t^s(x_i, x_j, s) := s \left(1 - \frac{\tilde{u}'_c(\tilde{c}_i, d_i)}{\beta u'_c(c_{ij}, d_i)} \right), \quad (13)$$

where the variable s represents a savings decision. Note that there is no loss in generality in restricting attention to income states that are distinct. The tax system may therefore relate to the indices i and j directly, even though the left-hand side of the definition is phrased in terms of the income states (x_i, x_j) .

We say that a tax system *implements* a given allocation if the allocation is the equilibrium outcome when agents make consumption and saving decisions and exert labor effort given taking the tax system as given.

Proposition 3 (Decentralisation). *Let $(\mathbf{c}, \mathbf{d}, \mathbf{e})$ be an optimal allocation. The tax system (t^1, t^2, t^s) defined by Eq. (11), (12), (13) implements $(\mathbf{c}, \mathbf{d}, \mathbf{e})$.*

The income taxes in Eq. (11) and (12) are defined such that the social planning allocation becomes affordable in the decentralized economy. Note that the income taxes depend on the amount of the durable good d . This dependency distorts the effective price of durable consumption. Specifically, the tax rate on the durable good conditions on the realisation of output in period 2 and can be written as

$$\frac{\tilde{u}'_d(\tilde{c}_i, d_i) + \beta u'_d(c_{ij}, d_i) - \tilde{u}'_c(\tilde{c}_i, d_i)}{\beta u'_c(c_{ij}, d_i)} = \left(\frac{\tilde{u}'_d(\tilde{c}_i, d_i) + \beta u'_d(c_{ij}, d_i)}{\tilde{u}'_c(\tilde{c}_i, d_i)} - 1 \right) \frac{\tilde{u}'_c(\tilde{c}_i, d_i)}{\beta u'_c(c_{ij}, d_i)}.$$

That is, the tax rate equals the *ex post* durability wedge multiplied by the *ex post* marginal rate of intertemporal substitution.²² If, for example, durable and nondurable

²²We cannot set the tax rate to the *ex ante* wedge τ_i^d because the agent chooses labor effort and consumption jointly, whereas the *ex ante* wedge considers only the decision margin for consumption.

consumption are substitutes ($u''_{cd} \leq 0$), the ex post durability wedge is high in period-2 states with low nondurable consumption. Thus, defining taxes in terms of the ex post wedge ensures that the durable good provides little insurance against the realization of uncertainty. As the proof of Proposition 3 shows, the effect of the durable good on the incentive to supply labor is, in fact, completely neutralized by this construction. We multiply the ex post durability wedge by the marginal rate of intertemporal substitution because the tax is collected in period 2, whereas the durable good is purchased in period 1. Finally, we complement the income tax system by a linear tax on savings defined in Eq. (13). The tax rate on savings corresponds to the ex post intertemporal wedge (Kocherlakota, 2005).

5 Concluding remarks

This paper shows that commodity taxation is optimal in the presence of durable goods. Even when the preferences are additively separable between labor effort and consumption, nonseparabilities between durable and nondurable consumption imply that differential commodity taxes improve welfare.

In our baseline model, the consumption flow from the durable good is formalized as a function of the stock in the first period. We show that, if adjustment costs are non-negligible, our findings generalize when the consumption flow can be adjusted over time in response to new information. Similarly, our results hold without modification when the consumption flow from the durable good changes over time in a deterministic way (due to depreciation, for example). Our implicit assumption is that the consumption flow in each period is a monotonic function of the stock of the durable good in the given period. The underlying mappings from stocks to flows, as well as changes (depreciation) of stocks over time, are implicitly embedded in the utility functions.

In an application to housing, we provide evidence for the quantitative relevance of our theoretical insights. We acknowledge that our approach abstracts from several

alternative motives for housing policy. For instance, capital market imperfections such as borrowing constraints may rationalize subsidies to housing. Moreover, political economy considerations may lead to outcomes that differ from the solution of a social planning problem. The interaction of these and further motives of housing policy is a challenging task for future research.

Appendix

A Proofs

Proof of Proposition 1. We introduce the multipliers $\lambda \geq 0$, $\mu \geq 0$, $\mu_i \geq 0$, $i = 1, \dots, N$, for the constraints given by Eq. (1), (3), (4), respectively. Based on the necessary first-order conditions of the social planner problem, we characterize optimal consumption as follows:

$$\begin{aligned} \frac{\lambda}{\tilde{u}'_c(\tilde{c}_i, d_i)} &= 1 + \mu \frac{\tilde{\pi}'_i(\tilde{e}_0)}{\tilde{\pi}_i(\tilde{e}_0)}, \\ \frac{\lambda - \frac{\mu_i}{\tilde{\pi}_i(\tilde{e}_0)} \sum_j \pi'_j(e_i) u'_d(c_{ij}, d_i)}{\tilde{u}'_d(\tilde{c}_i, d_i) + \beta \sum_j \pi_j(e_i) u'_d(c_{ij}, d_i)} &= 1 + \mu \frac{\tilde{\pi}'_i(\tilde{e}_0)}{\tilde{\pi}_i(\tilde{e}_0)}, \\ \frac{\lambda}{u'_c(c_{ij}, d_i)} &= \beta + \mu \beta \frac{\tilde{\pi}'_i(\tilde{e}_0)}{\tilde{\pi}_i(\tilde{e}_0)} + \frac{\mu_i}{\tilde{\pi}_i(\tilde{e}_0)} \frac{\pi'_j(e_i)}{\pi_j(e_i)}. \end{aligned}$$

Using the optimality conditions for \tilde{c}_i and d_i , we write the durability wedge as

$$\tau_i^d = -\frac{\mu_i}{\lambda \tilde{\pi}_i(\tilde{e}_0)} \sum_j \frac{\pi'_j(e_i)}{\pi_j(e_i)} u'_d(c_{ij}, d_i) \pi_j(e_i). \quad (14)$$

Differentiating the identity $\sum_j \pi_j(e) = 1$ for all e , we obtain $\sum_j \pi'_j(e) = 0$ for all e . Therefore, the likelihood ratio $\pi'_j(e_i) / \pi_j(e_i)$ is a centered random variable for the probability distribution defined by weights $\pi_j(e_i)$. This implies that the sum in the above expression for τ_i^d measures the covariance between the likelihood ratio $\pi'_j(e_i) / \pi_j(e_i)$

and the marginal utilities $u'_d(c_{ij}, d_i)$. The optimality condition for second-period consumption shows that c_{ij} increases with the likelihood ratio $\pi'_j(e_i)/\pi_j(e_i)$. Assuming $u''_{cd}(c_{ij}, d_i) \geq 0$ and using the fact that the covariance of two increasing functions of a random variable is nonnegative, we obtain $\sum_j \frac{\pi'_j(e_i)}{\pi_j(e_i)} u'_d(c_{ij}, d_i) \pi_j(e_i) \geq 0$. In this case, we have $\tau_i^d \leq 0$. Using the same argument, we obtain $\tau_i^d \geq 0$ if we assume $u''_{cd}(c_{ij}, d_i) \leq 0$. All the above inequalities become strict if $\mu_i > 0$ and $u''_{cd}(c_{ij}, d_i) \gtrless 0$.

(ii) We exploit the necessary first-order conditions for nondurable consumption to write the durability wedge as

$$\tau_i^s = 1 - \frac{1}{\beta \left[1 + \mu \frac{\tilde{\pi}'_i(\tilde{e}_0)}{\tilde{\pi}_i(\tilde{e}_0)} \right] \sum_j \frac{\pi_j(e_i)}{\beta + \mu \beta \frac{\tilde{\pi}'_i(\tilde{e}_0)}{\tilde{\pi}_i(\tilde{e}_0)} + \frac{\mu_i}{\tilde{\pi}_i(\tilde{e}_0)} \frac{\pi'_j(e_i)}{\pi_j(e_i)}}}.$$

The optimality condition for c_i implies that $1 + \mu \frac{\tilde{\pi}'_i(\tilde{e}_0)}{\tilde{\pi}_i(\tilde{e}_0)}$ is strictly positive. From Jensen's inequality (applied to the convex function $x \mapsto 1/x$), we obtain

$$\tau_i^s \geq 1 - \frac{\beta + \mu \beta \frac{\tilde{\pi}'_i(\tilde{e}_0)}{\tilde{\pi}_i(\tilde{e}_0)} + \frac{\mu_i}{\tilde{\pi}_i(\tilde{e}_0)} \sum_j \pi'_j(e_i)}{\beta \left[1 + \mu \frac{\tilde{\pi}'_i(\tilde{e}_0)}{\tilde{\pi}_i(\tilde{e}_0)} \right]} = 0,$$

where the last equality sign follows from the result $\sum_j \pi'_j(e_i) = 0$, shown in the proof of part (i). Finally, we note that Jensen's inequality becomes strict if and only if $\mu_i > 0$. \square

Proof of Proposition 2. As in the proof of Proposition 1(ii), the sign of the intertemporal wedge follows directly from the optimality conditions for nondurable consumption.

To analyze the durability wedge, we separate the social insurance problem with retrade (Eq. (6)) into two parts. The outer problem chooses for each realisation i a set D_i of second period states j in which the durable good is not adjusted: $D_i = \{j : d_{ij} = \tilde{d}_i\}$. The inner problem solves the social insurance problem given the restrictions imposed by the collection of sets D_i . We define the conditional probability distribution $\hat{\pi}_j(e_i|D_i) := \frac{\pi_j(e_i)}{\sum_{k \in D_i} \pi_k(e_i)}$, $j \in D_i$. We proceed as in the proof of Proposition 1(i) to derive

the durability wedge in the inner problem as

$$\tau_i^d = -\frac{\mu_i \sum_{j \in D_i} \pi_j(e_i)}{\lambda \tilde{\pi}_i(\tilde{e}_0) \left[1 - p + p \sum_{j \in D_i} \pi_j(e_i)\right]} \sum_{j \in D_i} \frac{\pi_j'(e_i)}{\pi_j(e_i)} u_d'(c_{ij}, \tilde{d}_i) \hat{\pi}_j(e_i | D_i).$$

If the set D_i contains at least two elements, then the logic of the proof of Proposition 1(i) applies and we obtain $\tau_i^d \geq 0$ if $\mu_i > 0$ and $u_{cd}''(c_{ij}, d_i) \leq 0$. Finally, if the adjustment cost α is sufficiently large, the optimal collection of sets in the outer problem will satisfy $|D_i| \geq 2$ for all i . \square

Proof of Proposition 3. We first show that, for any effort plan in the decentralized economy, the equilibrium allocation of consumption coincides with the social planning allocation (\mathbf{c}, \mathbf{d}) .

Given effort $(\hat{e}_0, \hat{e}_i)_{i=1, \dots, N}$, the decision problem in the decentralized economy is to maximize expected utility

$$\max_{\hat{c}, \hat{d}, s} \sum_i \tilde{\pi}_i(\hat{e}_0) \left[\tilde{u}(\hat{c}_i, \hat{d}_i) + \beta \sum_j \pi_j(\hat{e}_i) u(\hat{c}_{ij}, \hat{d}_i) \right]$$

subject to the following period-by-period budget constraints:

$$\begin{aligned} \hat{c}_i + \hat{d}_i + s_i &\leq x_i - t^1(x_i), \\ \hat{c}_{ij} &\leq x_j - t^2(x_i, x_j, \hat{d}_i) + s_i - t^s(x_i, x_j, s_i). \end{aligned}$$

We note that the taxes t^2 and t^s are linear in the quantities \hat{d}_i and s_i . Thus, the decision problem has a strictly concave objective and a linear constraint set. By monotonicity, we can use the budget constraints with equality and reduce the problem to the choice variables (\hat{d}_i, s_i) , $i = 1, \dots, N$. The necessary and sufficient conditions for optimality

are given by the following pair of first-order conditions for $i = 1, \dots, N$:

$$\begin{aligned} 0 &= -\tilde{u}'_c(\hat{c}_i, \hat{d}_i) + \tilde{u}'_d(\hat{c}_i, \hat{d}_i) + \beta \sum_j \pi_j(\hat{e}_i) \left[-\frac{dt^2(x_i, x_j, \hat{d}_i)}{d\hat{d}_i} u'_c(\hat{c}_{ij}, \hat{d}_i) + u'_d(\hat{c}_{ij}, \hat{d}_i) \right], \\ 0 &= -\tilde{u}'_c(\hat{c}_i, \hat{d}_i) + \beta \sum_j \pi_j(\hat{e}_i) \left(1 - \frac{dt^s(x_i, x_j, s_i)}{ds_i} \right) u'_c(\hat{c}_{ij}, \hat{d}_i). \end{aligned}$$

Setting $(\hat{c}_i, \hat{d}_i, \hat{c}_{ij}) = (\tilde{c}_i, d_i, c_{ij})$ for all i, j and using the definition of the tax system, the conditions become

$$\begin{aligned} 0 &= -\tilde{u}'_c(\tilde{c}_i, d_i) + \tilde{u}'_d(\tilde{c}_i, d_i) - [\tilde{u}'_d(\tilde{c}_i, d_i) - \tilde{u}'_c(\tilde{c}_i, d_i)] \sum_j \pi_j(\hat{e}_i), \\ 0 &= -\tilde{u}'_c(\tilde{c}_i, d_i) + \tilde{u}'_c(\tilde{c}_i, d_i) \sum_j \pi_j(\hat{e}_i). \end{aligned}$$

By the definition of a probability, $\sum_j \pi_j(\hat{e}_i) = 1$ for any \hat{e}_i . Hence, the social planning allocation satisfies the necessary and sufficient conditions for optimality. Moreover, it is easy to see that the social planning allocation satisfies the period-by-period budget constraints. Therefore, the social planning allocation solves the agent's decision problem for consumption in the decentralized economy for any given effort plan.

In the second step, we establish that the social planning solution for effort is optimal in the decentralized economy. Based on the first step, an effort plan is optimal in the decentralized economy if and only if it maximizes the expected utility given the social planning allocation of consumption. Since the last condition is equivalent to the incentive compatibility constraint, the proof is complete. \square

B Formal statement of the calibrated model

The formal statement of the calibrated model is as follows. We maximize social welfare

$$\sum_{i,j=1}^2 \pi_i(e_0) \pi_j(e_i) \left(u(c_i, d_i) - v(e_0) + \beta [u(c_{ij}, d_i) - v(e_i)] \right)$$

subject to the following budget and incentive constraints:

$$\begin{aligned} \sum_{i,j=1}^2 \pi_i(e_0) \pi_j(e_i) \left(x_i - c_i - d_i + q [x_j - d_i - c_{ij}] \right) &\geq 0 \\ \sum_{i,j=1}^2 \pi'_i(e_0) \pi_j(e_i) \left(u(c_i, d_i) + \beta [u(c_{ij}, d_i) - v(e_i)] \right) - v'(e_0) &\geq 0 \\ \sum_{j=1}^2 \pi'_j(e_i) u(c_{ij}, d_i) - v'(e_i) &\geq 0, \quad i = 1, 2. \end{aligned}$$

The associated commodity wedges are given by

$$\begin{aligned} \tau_i^d &:= \frac{u'_d(c_i, d_i) + \beta \sum_j \pi_j(e_i) u'_d(c_{ij}, d_i)}{(1+q) u'_c(\tilde{c}_i, d_i)} - 1, \\ \tau_i^s &:= 1 - \frac{q u'_c(c_i, d_i)}{\beta \sum_j \pi_j(e_i) u'_c(c_{ij}, d_i)}. \end{aligned}$$

We solve for all allocation variables simultaneously using standard numerical optimisation routines. Our calibration strategy generates income states of $x_1 = 0.3975$ and $x_2 = 1.6025$. The calibrated preference weights (α, ω) for the three preference scenarios are: $(6, 0.23)$ for the scenario in Piazzesi et al. (2007), $(6, 0.022)$ for the scenario in Li et al. (2009) and $(0.42, 0.43)$ for the scenario in Bajari et al. (2013).

For the auxiliary model with uniform commodity taxation, we impose $\tau_i^d = 0$, for $i = 1, 2$, as a further constraint to the optimisation problem. Then, to obtain the consumption equivalent variation of two welfare measures, we scale up all consumption levels (durable and nondurable) in both periods by a common rate Δ . For our CES

utility function, the consumption equivalent variation equals

$$\Delta = \begin{cases} \left(\frac{\tilde{V} + V_e}{V_c} \right)^{1/(1-1/\sigma)} - 1, & \sigma \neq 1, \\ \exp \left(\frac{\tilde{V} - V}{1+\beta} \right) - 1, & \sigma = 1, \end{cases}$$

where \tilde{V} is the welfare level of interest, $V = V_c - V_e$ is the welfare level in the baseline economy, and V_c and V_e are the parts of welfare created by consumption and effort, respectively.

References

- ABRAHAM, A., S. KOEHNE, AND N. PAVONI (2011): “On the first-order approach in principal–agent models with hidden borrowing and lending,” *Journal of Economic Theory*, 146, 1331–1361.
- ALVAREZ, F., L. GUIISO, AND F. LIPPI (2012): “Durable Consumption and Asset Management with Transaction and Observation Costs,” *American Economic Review*, 102, 2272–2300.
- ATKINSON, A. AND J. STIGLITZ (1976): “The design of tax structure: Direct versus indirect taxation,” *Journal of Public Economics*, 6, 55 – 75.
- BAJARI, P., P. CHAN, D. KRUEGER, AND D. MILLER (2013): “A Dynamic Model of Housing Demand: Estimation and Policy Implications,” *International Economic Review*, 54, 409–442.
- CREMER, H. AND F. GAHVARI (1995a): “Uncertainty and optimal taxation: In defense of commodity taxes,” *Journal of Public Economics*, 56, 291–310.
- (1995b): “Uncertainty, optimal taxation and the direct versus indirect tax controversy,” *Economic Journal*, 1165–1179.

- DA COSTA, C. AND I. WERNING (2002): "Commodity taxation and social insurance," MIT, Mimeo.
- DIAMOND, P. A. AND J. A. MIRPLEES (1978): "A model of social insurance with variable retirement," *Journal of Public Economics*, 10, 295 – 336.
- EATON, J. AND H. S. ROSEN (1980): "Optimal Redistributive Taxation and Uncertainty," *Quarterly Journal of Economics*, 95, 357–64.
- FARHI, E. AND I. WERNING (2013): "Insurance and taxation over the life cycle," *Review of Economic Studies*, 80, 596–635.
- GOLOSOV, M., N. KOCHERLAKOTA, AND A. TSYVINSKI (2003): "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, 70, 569–587.
- GOMES, R. D., J.-M. LOZACHMEUR, AND A. PAVAN (2014): "Differential Taxation and Occupational Choice," Working Paper 5054, CESifo Working Paper Series.
- GROCHULSKI, B. AND N. KOCHERLAKOTA (2010): "Nonseparable preferences and optimal social security systems," *Journal of Economic Theory*, 145, 2055 – 2077.
- HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2014): "Consumption and Labor Supply with Partial Insurance: An Analytical Framework," *American Economic Review*, 104, 1–52.
- JEWITT, I. (1988): "Justifying the First-Order Approach to Principal-Agent Problems," *Econometrica*, 56, 1177–1190.
- KOCHERLAKOTA, N. R. (2005): "Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation," *Econometrica*, 73, 1587–1621.
- KOEHNE, S. AND M. KUHN (2014): "Optimal taxation in a habit formation economy," *Journal of Public Economics*, <http://dx.doi.org/10.1016/j.jpubeco.2014.12.007>.

- LAFRANCE, J. T. AND L. D. BARNEY (1991): "The envelope theorem in dynamic optimization," *Journal of Economic Dynamics and Control*, 15, 355–385.
- LI, W., H. LIU, AND R. YAO (2009): "Housing over time and over the life cycle: a structural estimation," Working Paper 09-7, Federal Reserve Bank of Philadelphia.
- NAITO, H. (1999): "Re-examination of uniform commodity taxes under a non-linear income tax system and its implication for production efficiency," *Journal of Public Economics*, 71, 165 – 188.
- PIAZZESI, M., M. SCHNEIDER, AND S. TUZEL (2007): "Housing, consumption and asset pricing," *Journal of Financial Economics*, 83, 531–569.
- ROGERSON, W. P. (1985): "The First-Order Approach to Principal-Agent Problems," *Econometrica*, 53, 1357–1367.
- VARIAN, H. R. (1980): "Redistributive taxation as social insurance," *Journal of Public Economics*, 14, 49–68.