

Stolzenburg, Ulrich

Working Paper

The agent-based Solow growth model with endogenous business cycles

Economics Working Paper, No. 2015-01

Provided in Cooperation with:

Christian-Albrechts-University of Kiel, Department of Economics

Suggested Citation: Stolzenburg, Ulrich (2015) : The agent-based Solow growth model with endogenous business cycles, Economics Working Paper, No. 2015-01, Kiel University, Department of Economics, Kiel

This Version is available at:

<https://hdl.handle.net/10419/106998>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

C | A | U

Christian-Albrechts-Universität zu Kiel

Department of Economics

Economics Working Paper
No 2015-01

the agent-based solow growth model with endogenous business cycles

by Ulrich Stolzenburg

issn 2193-2476



THE AGENT-BASED SOLOW GROWTH MODEL WITH ENDOGENOUS BUSINESS CYCLES

ULRICH STOLZENBURG

Department of Economics Olshausenstr. 40, 24118 Kiel
University of Kiel, Germany E-Mail: stolzenburg@economics.uni-kiel.de

January 2015

Abstract: This article describes a simulated monetary macro model with different types of interacting agents. As such, it is assigned to the field of agent-based computational economics (ACE), where agents become virtual objects in a computer simulation. The ACE model core with labor market and goods market interaction between households and firms is adopted from *Lengnick (2013)*, whereas production technology and technological progress of firms are adopted from the neoclassical *Solow (1956)* model. Nominal interest rates are set in accordance with the *Taylor (1993)* principle, characterized by strong responses of monetary policy to deviations from inflation target. Although inflation desirably follows lagged output in a pro-cyclical manner, the dynamic system allows for long-run stability of inflation rates. Firms on aggregate level endogenously generate waves of higher and lower investment. A recurrent cyclical movement of aggregate economic activity, in particular demand, employment and inflation, is transmitted from these waves of investment activity. Cyclical patterns of boom and bust emerge with a frequency of approximately seven years just like *Juglar*-type cycles. Moreover, the model generates a short-run *Phillips*-curve relationship, long-run neutrality of monetary policy and business cycle patterns similar to the *Goodwin (1967)* model. Fiscal stabilization policy is shown to dampen macroeconomic fluctuations, thus allowing for a higher level of average employment. Calibration of model parameters is conducted to generate realistic orders of magnitude of important macroeconomic proportions. The newly developed model is a combination of ideas from different economic perspectives and contributes to macroeconomic model-building under the paradigm of agent-based computational economics.

Keywords: Agent-Based Computational Economics, Demand-Led Economic Growth, Solow-Swan, Monetary Policy, Phillips curve, Stabilization policy

Contents

1. Introduction	1
2. Model	5
2.1. Structure and Timing	5
2.2. Banking System	6
2.3. Household	7
2.3.1. Job Market	7
2.3.2. Income and Taxes	8
2.3.3. Goods Market Trading Partners	9
2.3.4. Consumption	10
2.4. Firm	10
2.4.1. Technology	10
2.4.2. Capital Stock Development	11
2.4.3. Key Data for Firm Decisions	12
2.4.4. Decision: Price	13
2.4.5. Decision: Hire or Fire	14
2.4.6. Decision: Wage	15
2.4.7. Decision: Investment	16
2.4.8. Decision: Profit Payout	17
2.5. Government	18
2.6. Monetary Policy	18
3. Simulation and Model Behavior	19
3.1. Single Firm	20
3.2. Macro Behavior: The Business Cycle	25
3.3. (Non-)Neutrality of Money	28
3.4. Stabilization policy	30
3.5. Calibration	33
3.6. Limitations and Parameter Sensitivity	34
4. Conclusion	37
A. Appendix	43
A.1. List of Symbols and Model Parameters	43
A.2. National Accounting	45

1. Introduction

Agent-Based Computational economics (ACE) developed recently as a new branch of macroeconomic modeling, which falls in the paradigm of complex adaptive systems (*Tesfatsion, 2003*). Economic agents become artificial objects in a computer simulation, with endowments and behavioral rules. Agent-based models consist of computational objects which interact according to behavioral rules (*Page, 2008*). A newly developed ACE macro model is outlined in this article.

A key advantage of the ACE approach should be underscored here, which is that analytic tractability is not a key requirement for model equations. There is no need to solve them for an equilibrium equation or set of equations, thereby forcing assumptions to be overly simple, well behaved and tractable. Instead, model outcome is analyzed by observation of emergent time series. As a result, modeling of agent behavior becomes less restricted and allows to incorporate and combine research findings from neighboring fields such as experimental economics and behavioral economics. Another advantage of agent-based models in general is that macro results may occur that differ completely from disaggregate micro level agent behavior. A famous generally understandable example for such an emergent property on aggregate level is *Schellings (1969)* model of racial segregation in cities, emerging from rather open-minded individual attitudes with regard to mixed neighborhoods.

There are some disadvantages to acknowledge: First, it requires a good deal of effort to get familiar with programming and typical challenges of calibrating these models. *Oeffner (2009)* provides a detailed and comprehensible introduction to the virtues and challenges of ACE model building. Secondly, there is no analytic solution of equilibrium or model dynamics, so people used to it may feel a lack of mathematical certainty. According to *Page (2008)*, ACE models “occupy a middle ground between stark, dry rigorous mathematics and loose, possibly inconsistent, descriptive accounts”. Thirdly, freedom concerning modeling of agent behavior is accompanied by a growing level of complexity, which complicates understanding and interpretation of results. The term “wilderness of bounded rationality” points to the difficulty of transferring certain findings about non-rational agent behavior to functional forms. It is accompanied by the problem of (too) many parameters in large models (*Sims, 1980*). As *Lengnick (2013)* argues, ACE modelers are “tempted to over-increase the level of complexity”.

Until recently, macroeconomic model building is rather dominated by Dynamic Stochastic General Equilibrium (DSGE) models, which became the standard models of monetary policy analysis (cf. *Woodford, 2003; Clarida et al., 1999*). However, DSGE modeling is subject to ongoing criticism (*Mankiw, 2006; Solow, 2010; Colander et al., 2009*). Some standard paradigms appear questionable, especially in the light of possibilities offered by the ACE approach.

To begin with, most General Equilibrium models assume existence of a representative agent, who optimizes utility with infinite horizons. *Kirman (1993)* argues why the assumption of a representative agent is questionable. *Solow (2010)* challenges the idea “that the whole economy can be thought of if it were a single [...] person carrying out a rationally designed, long-term plan, occasionally disturbed by unexpected shocks, but adapting to them in a rational, consistent way.” He also argues that DSGE models by construction provide no reasonable way to cope with

involuntary unemployment, since the representative agent only rationally chooses to substitute work with leisure a little more. Moreover, there may also occur the need to analyze consequences of certain policies with regard to the income distribution; or to analyze how the distribution of wealth influences growth (e.g. *Alesina and Rodrick*, 1994; *Dynan, Skinner and Zeldes*, 2004; *Galor*, 2009). The ACE approach allows agents to be modeled as heterogeneous individuals, who partly can become involuntarily unemployed. Topics related to wealth and income distribution can also be tackled.

Another assumption in general equilibrium models is that relative prices are simply set optimally, thus allowing for permanent market clearing as long as no frictions are imposed. The concept of permanent fulfillment of an equilibrium condition, brought about by a fictitious *Walrasian* auctioneer, who calculates an optimal vector of relative prices in meta time, is not convincing (*Ackerman*, 2002; *Gaffeo et al.*, 2008; *Kirman*, 2006). In ACE models, the process and coordination of relative price adjustments can be modeled explicitly, possibly resulting in a temporary equilibrium situation. Finally, the famous *Lucas* (1976) critique argues that relations between macroeconomic variables may change with policy, because agents incorporate new policies into economic decision making. This idea induced not only the spread of microfoundations as an acknowledged requirement for macro models, it also made the rational expectations hypothesis a key ingredient of model building. As long as the consequence derived from *Lucas*' critique concerns the requirement of microfounded agent behavior, ACE models allow for a much more complex and realistic set of assumptions applicable to simulated heterogeneous agents. However, an application of rational expectations to ACE models appears inappropriate, since it implies that (simulated) agents are able to fully understand a complex dynamic system of agent interactions and to calculate expected values for aggregate outcomes. Not even the designer of the dynamic ACE system is able to calculate an accurate probability distribution of possible outcomes. A more general critique of rational expectations in macro models is provided by *Syll* (2012).

It should be noted that former arguments do not disqualify DSGE models to analyze and estimate real aggregate economic behavior in a valuable way. *Friedman* (1951) argues that accuracy of predictions derived from a theory is more important than the underlying assumptions, as long as they are consistent. Nevertheless, the goal of economic theory is not restricted to prediction and data-fitting; it also consists of providing convincing explanations of real-world phenomena and processes, so contrary to *Friedman's* claim assumptions actually *do* matter. The ACE approach offers a new way for macroeconomic model building, facilitating more realistic designs of agent behavior.

Point of departure for the newly developed ACE macro model is the baseline model of *Lengnick* (2013), later referred to as *L13* model.¹ It is a simple model of a closed economy, which partly draws on former models of *Dosi et al.* (2008) and *Gaffeo et al.* (2008). The simulated economy consists of households and firms interacting on a goods market and on a labor market, where

¹ Java Source code of the *Lengnick* model was adopted, which was very helpful and facilitated the start. Program output such as macroeconomic time series were analyzed with MATLAB. The source code of the extended model described here is available upon request.

each type of agent follows simple adaptive rules. The model generates endogenous business cycles and some desired characteristics on aggregate level.² The basic structure of the model has been adopted, such as the sequence of activities, connections between agents and the organization of the goods market and of the labor market.

Certain restrictions of the *Lengnick* model prompted several changes and extensions: (1) Labor is sole production factor input in *L13*, while in the extended model each firm is endowed with a capital stock, which is subject to depreciation. Technological progress in combination with *Cobb-Douglas* production technology is incorporated, so that firms become customers of other firms by purchasing investment goods. This extension simply forces production technology and architecture of the *Solow-Swan* growth model (*Solow*, 1956; *Swan*, 1956) on single firms of an agent-based framework.³ (2) In *L13*, firm ownership remains unspecified. Here, households are shareholders of firms, so that profit is paid out as a return rate of firm shares. (3) In *L13*, firms are frequently unable to meet customer demand, followed by a loss of customers. To prevent a constant flow of restricted, disappointed customers between firms, firms are modeled with excess production capacity in the new model. (4) A fixed quantity of money is circling between agents in *L13*. Instead, endogenous money is introduced with credit, savings, interest rates and a monetary policy rule. Availability of credit also avoids frequent emergency wage cuts of firms, if they run out of money in *L13*. (5) Finally, consumption paths of unemployed households in *L13* are unrealistic, since consumption almost immediately drops to low one-digit percentages of former levels. The new model introduces a government, providing for unemployment benefits and collecting taxes.

The presented *monetary* model incorporates elements from different economic schools: It is *Keynesian*, since economic activity of firms is strictly demand-driven and features involuntary unemployment, while there is no hypothetical lower wage that allows for market-clearing. *Say's law* does not apply, when firms are designed to have excess production capacities, thus allowing for unsold quantities. Fluctuating consumption and investment, combined with unknown household savings and availability of credit weaken the link between production and aggregate demand even more. In the context of this model, the notion of “equilibrium” may be understood as a situation when aggregate demand fluctuates around a target percentage of production capacity. Business cycles are created endogenously, generated by higher or lower firm investment as the leading determinant of economic dynamics. Both *Keynes* (1936) and *Kalecki* (1937) also uncovered investment as a major source of periodical macroeconomic fluctuations. As it turns out, firm profits depend on aggregate investment, but not on investment of the one single firm. On the other hand, firm investment in turn depends on past firm profits. A reinforcing spiral

² Crises in the *L13* emerge as follows: After a period of abundant aggregate demand, firm inventories decline to critical lower values. Firms are permanently unable to acquire more workers for production enlargement, thus inducing rising wages and correspondingly decreasing profits. As soon as profits reach a lower bound, price increases are triggered, by which the money supply is devalued in a system with a fixed quantity of money. Finally, the reduced value of real money in the system causes a drop in firm sales, which induces firms to fire workers.

³ To name a few major differences of the *Solow* model: It only captures the *aggregate* level and assumes all factors of production to be constantly employed; there is no role for money; savings are generated as a fixed proportion of output and are directly invested.

of investment, demand and profits is an emergent property on aggregate level, both upwards and downwards. As it turns out, the emergent cyclical pattern resembles dynamics of the *Goodwin* (1961) model. Elements from *neoclassical* theory are incorporated, namely the *Solow-Swan* growth model from which firm technology and technological progress are adopted on firm level. Monetary policy is conducted in accordance with the *New Macroeconomic Consensus* where the *Taylor* principle is applied to automatically adjust nominal interest rates as a strong response to deviations from inflation target. A short-run *Phillips curve* emerges from aggregate model dynamics, similar to *New Keynesian* models.

Naturally, there have been other approaches to ACE macro modeling before. *Lengnick* (2013) distinguishes two categories of such models: The first category models the economy in considerable detail and complexity such as the EURACE project (*Dawid et al.*, 2011) with even a spatial structure. The second category abstracts from reality to a larger degree, which is where *L13* and the presented model belong to. *Dosi et al.* (2008) develop a model of investment with R&D, where aggregate demand and output are driven by lumpy investment. They obtain macro behavior in line with a number of stylized facts. However, contrary to the approach followed here, neither a labor market nor a goods market are modeled, instead firms are simply assigned a proportion of aggregate demand. Moreover, households fully consume their income, which also differs from the more general consumption decision applied here. *Gaffeo et al.* (2008) explicitly model the goods market, but do not capture the capital side of the economy, since their production function employs labor as the sole input. *Oeffner* (2009) develops an ACE macro model characterized by demand-driven economic activity, business cycles amplified by firm investment, Cobb-Douglas technology, growth and inflation. It resembles in many ways the model presented here. However, it simulates three firm sectors instead of one: Two consumption goods sectors (with a capital stock) and a capital goods sector (modeled without a capital stock); each of them populated by a fixed number of firms. Another major difference is that firm employment is fixed in *Oeffners* model, so there is no analysis of unemployment.

The ACE model presented here is the first to combine an explicit modeling of goods market and labor market, firm capital stock, investment, growth, inflation and endogenously created business cycles. Contrary to numerous other models, it encompasses the demand-led character of firm decisions on employment and price setting. This article provides a comprehensive description of the model structure and analyzes simulation behavior. As will be shown, the model reproduces a number of stylized facts and is calibrated to generate realistic proportions on aggregate level. To further demonstrate the usefulness of ACE models in general and of this extended model in particular, the response to a monetary shock is analyzed. Additionally, three different fiscal policy regimes are analyzed: As it turns out, a policy aiming at demand stabilization performs best with respect to average employment. Yet this result, admittedly, is a rather expectable property of a demand-led macro model.

The remainder of this article is organized as follows: Section 2 explains the building blocks of the agent-based economy step-by-step, such as the basic structure, banking system, agent classes of households, firms and the government as well as the conduct of monetary policy. Section 3

describes the model behavior in the running simulation. Time series for individual firms (3.1) are followed by macroeconomic variables and analysis of business cycle dynamics (3.2). In addition, Phillips curves, monetary policy shocks (3.3), fiscal policy (3.4), calibration issues (3.5) and limitations (3.6) are also treated. Section 4 concludes.

2. Model

2.1. Structure and Timing

The sequence of activities consists of in two different time intervals (cf. *Lengnick, 2013*). All relevant decisions take place on a monthly basis, as well as payment of wages, profits, interest, taxes and unemployment benefits. On daily basis, goods are only produced and sold to customers. Figure 1 provides an overview of the sequence of important activities.

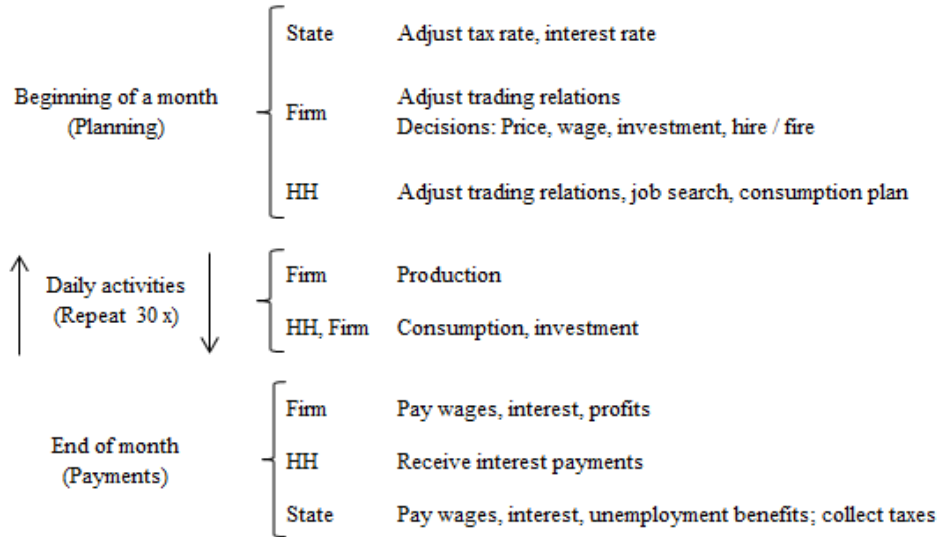


Figure 1: Sequence of monthly events.

The developed *monetary* model distinguishes carefully between real variables, which are counted in natural units (goods), and nominal measures counted in currency units. Regarding notation, all nominal measures are written consequently with a preceding letter n in order to avoid confusion, while real measures are written without this preceding letter.⁴ The model internally calculates on a monthly basis, so that inflation rates, interest rates, the depreciation rate and return rates are actually very small values. In this article, however, corresponding values are presented as annualized percentages in order to simplify understanding. In the following sections, variables are in monthly notation with a subscript t with months $t = 1, \dots, T$.

There are three types of agents: households, firms, and a government sector (state). The number of households ($N^{hh} = 2000$) and firms ($N^{fi} = 100$) is fixed in order to exclude demographic

⁴ The only exception is the *nominal* inflation rate i_t , which is measured as a percentage, so it is actually not a nominal variable with respect to its unit of measurement.

aspects as well as firm entry and exit. A central bank is also present, but has no other function but to set the nominal interest rate. Figure 2 depicts the model structure.

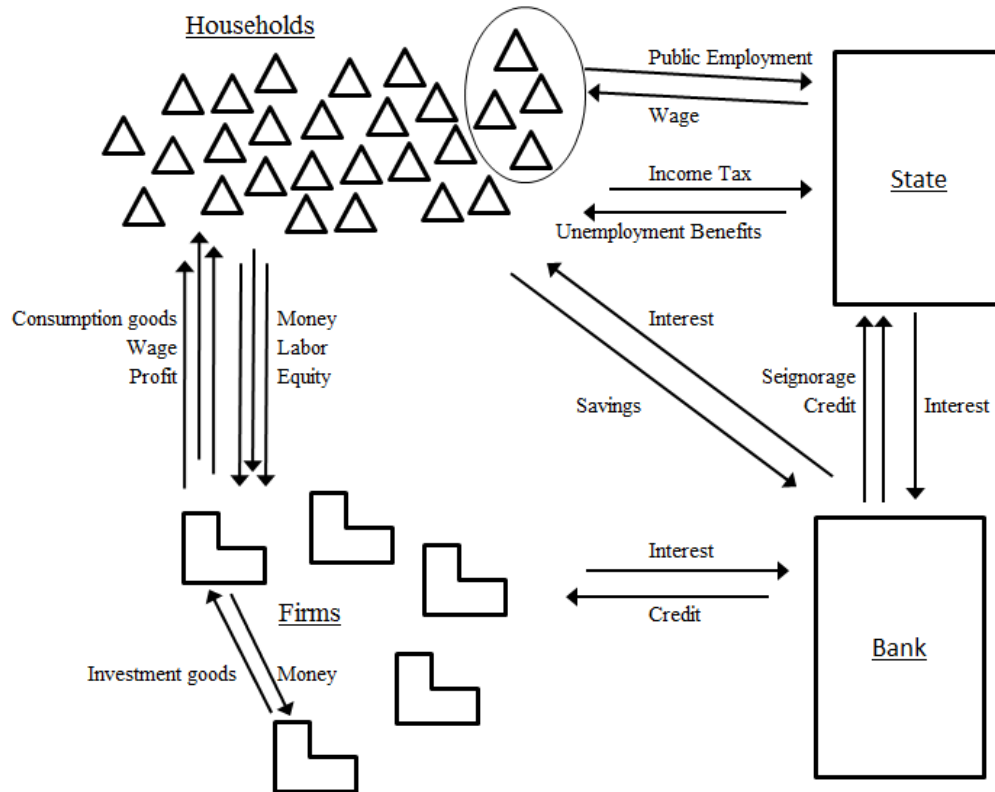


Figure 2: Model structure with financial and real flows between agents.

2.2. Banking System

All households, firms and the government are endowed with a bank account. There is only one bank representing the banking system. However, it is abstracted from administration costs, employees and the objective to generate profits. Each bank account contains a money account, which is used for payments⁵, and a savings account, which can be used as an interest-bearing financial asset; and which can also become a credit account, if the balance is negative. The money account is restricted to positive values and it is interest-free. The savings account can be positive and negative, so it is either a financial asset or a credit account.

All agents are free to transfer arbitrary amounts from the money account to the savings account and back, so there is no credit restriction and no credit risk evaluation. Money is created endogenously once an agent demands for additional liquidity, thereby increasing aggregate money. The created monetary amount is used for payments, thus circling between agents. Once it is

⁵ The model abstracts from cash payments, so deposits of the money account are transferred between agents in order to execute payments. The two-staged structure of real-world banking systems with a role for central bank money is also not captured.

transferred to a savings account, the aggregate quantity of money is reduced respectively.⁶ At the start of each month, agents decide about their liquidity need, so they decide to hold transaction money based on their past monthly cost. Spare money is moved to the savings/credit account, either to gain interest income (households) or to avoid unnecessary cost (firms).

All Money and savings accounts sum up to zero at all times.⁷

$$0 = \underbrace{nM_t^{hh} + nM_t^{fi}}_{\text{transaction money}} + \underbrace{nSC_t^{hh}}_{+} + \underbrace{nSC_t^{fi}}_{-} + \underbrace{nSC_t^{st}}_{-}, \quad (1)$$

Aggregate money and savings/credit accounts of the household sector and firm sector are simple aggregates: $nM_t^{hh} = \sum_{h=1}^{N^{hh}} nM_{t,h}^{hh}$, $nM_t^{fi} = \sum_{i=1}^{N^{fi}} nM_{t,i}^{fi}$, $nSC_t^{hh} = \sum_{h=1}^{N^{hh}} nSC_{t,h}^{hh}$ and $nSC_t^{fi} = \sum_{i=1}^{N^{fi}} nSC_{t,i}^{fi}$. The government does not hold any transaction money at trading days. Therefore, $nM_t^{st} = 0$.

The interest rate is adjusted monthly by a central bank. It is imposed on all savings/credit account balances and is paid by debtors and received by creditors of the bank. For simplicity, the *current* nominal interest rate is valid for the whole stock of savings and credit.⁸

2.3. Household

2.3.1. Job Market

Job market decisions take place at the beginning of a month. Each household is connected to one employer unless he is unemployed, while a firm is able to employ an arbitrary number of employees. Households offer inelastically one unit of labor per month, so that wage payment is monthly labor income. Each firm pays the same wage to all of its employees, though there may be differences between firms.

If a household is fired, it will remain employed during the current month and becomes unemployed at the beginning of the following month. A reservation wage, which is the minimum wage for acceptance of a new job, is set at its latest wage payment. Each month of unsuccessful job search reduces the reservation wage by 5%, which is effectively a small obstacle to employment. The household consults up to five firms per month to ask for a job. As soon as a firm offers a job, for which the wage exceeds the current reservation wage, the household is employed instantly. With probability 10%, an employed household will also contact one firm to ask for a better-paid

⁶ Concerning double bookkeeping, money is created in a credit contract, once an indebted firm asks for more credit, which is a balance sheet extension. If a household transfers an amount of money from its savings account to the money account, it is a mere asset swap. Both ways, aggregate money is increased. On the other hand, the quantity of money is reduced if a firm transfers it to the savings/credit account in order to pay back debt, which is a balance sheet contraction. A household transferring money to its stock of savings is experiencing an asset swap again. The aggregate quantity of money is decreased in both cases.

⁷ Firms and the government are usually indebted, so they are debtors of the banking system. Households accumulate savings, so they are creditors of the banking system. Money holders (households and firms) are also creditors.

⁸ In a more complex setting, savings and credit are modeled as financial contracts featured by a contract duration of a fixed number of months. This is actually more plausible, since firms consider investments dependent on the current nominal interest rate and expected inflation. An investment decision financed by a fixed credit contract at least preserves decisive credit conditions.

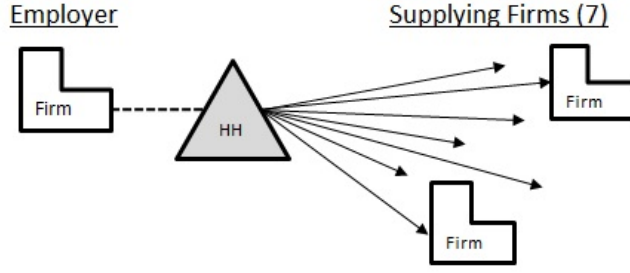


Figure 3: Household Connections: Seven firms as trading partners for consumption goods (arrows) and one employer (dashed line).

job. These job offers are refused directly, if the new wage is lower than the current one. If the offered wage is higher, the job is accepted with a probability dependent on the wage difference according to $Prob(Accept) = 1 - e^{-\gamma_w \cdot \ln(2) \cdot \Delta nW}$ (see Figure 4), which ensures a restricted level of wage competition.



Figure 4: Household decision: Probability to change employer (supplier) dependent on wage (price) difference.

2.3.2. Income and Taxes

All monthly payments to households will be carried out at the end of a month, thus determining household income for the following month. Primary income consists of up to three sources: (1) wage from employer i , (2) paid-out profit of firms the household is shareholder of and (3) nominal interest payment on nominal savings/credit account $nSC_{t,h}$. If a household is indebted, $nSC_{t,h}$ and interest payments are negative.

$$nInc_{t,h}^{prim} = nW_{t,i} + n\Pi_{t,h}^{paid} + i_t \cdot nSC_{t,h} \quad (2)$$

A tax rate τ_t is imposed equally on all sorts of primary income: $nTax_t = \tau_t \cdot nInc_{t,h}^{prim}$. Unemployment benefits (nUB_t) are paid if the household was unemployed during the running month. Instead of wage, it receives unemployment benefits, which is 50% of net average wage:

$$nUB_t = 0.5 \cdot (1 - \tau_t) \cdot nWI_{t-1}, \quad (3)$$

where nWI_{t-1} is the average wage (or the wage index), calculated as a mean value of previous month' firm wages, weighted by the employment share of each firm.⁹ Household h is left with net income

$$nInc_{t,h}^{net} = \begin{cases} \left(nW_{t,i} + n\Pi_{t,h}^{paid} + i_t \cdot nSC_{t,h} \right) \cdot (1 - \tau_t) & \text{if employed} \\ \left(n\Pi_{t,h}^{paid} + i_t \cdot nSC_{t,h} \right) \cdot (1 - \tau_t) + nUB_t & \text{if unemployed} \end{cases} \quad (4)$$

2.3.3. Goods Market Trading Partners

Each household maintains a fixed number of seven connections to firms (cf. *Lengnick, 2013*). Firms, on the other hand, are not limited in the number of connections to customers. Connections to supplying firms are adjusted slowly and infrequently, thus expressing loyalty of customers and stability of trading relations. Nevertheless, each household adapts its list of firm connections monthly due to price consideration, customer restrictions and randomly.

1. With a fixed probability of $p_p = 25\%$, households search for cheaper trading partners. One existing (old) connection is chosen randomly, and one other (new) firm is also chosen randomly. If the new price is higher than the old price, the existing connection is kept. Otherwise, the price difference is translated into a probability to replace the existing connection: $Prob(Switch) = 1 - e^{-\gamma_p \cdot \ln(2) \cdot \Delta P}$ (see Figure 4). This way, imperfect price competition is established, but customers do not react strongly when price differences are negligible.
2. If a firm is sold out, it is unable to satisfy further customer requests. In the respective trading day, restricted customers simply buy from the next firm. However, being restricted more often by the same firm induces households to replace the respective trading connection. Again, with a probability $p_r = 25\%$ one (restricted) connection is reconsidered and eventually replaced by a random new firm. The probability for a switch is dependent on the severeness of the restriction compared to consumption plans, while negligible restrictions will have no consequences.¹⁰
3. Finally, trading connections are exchanged randomly with a low percentage of $p_s = 2\%$. With a sum of $2,000 \cdot 7 = 14,000$ firm connections, random rearrangement concerns 40 connections per months. This random customer redistribution ensures that small firms do not run out of customers but are stabilized after losing customers.

⁹ $nWI_{t-1} = \sum_{i=1}^{N_f^i} nW_{t-1,i} \cdot \frac{L_{t-1,i}}{\sum_{j=1}^{N_f^i} L_{t-1,j}}$

¹⁰ Restrictions of demand are measured in daily consumption packages: RD_t . Probability to change a connection depends on last month' restriction: $Prob(Switch) = 1 - e^{-\gamma_r \cdot \ln(2) \cdot RD_{t-1}}$. Restrictions by more than one firm will induce replacement of one connection dependent on the relation of the restriction between the two (or more) restricting firms.

2.3.4. Consumption

Simulated households plan consumption based on current income, not taking into consideration the stock of savings, which may be a multitude of income. However, it is assumed that households offset expected real devaluation of savings stocks ($nSC_{t,h} \cdot \pi_t^e$) by directly reinvesting the respective amount, where π_t^e is expected inflation (as defined below in section 2.4.3). Therefore, consumption-relevant net income is reduced, so that interest income is only relevant for consumption, as long as it is generated by the *real* interest rate r_t .¹¹ A personal price index $nPI_{t,h}^c$, which is the average price of current trading partners, is calculated to determine the purchasing power of individual income. Consumption-relevant real net income is therefore

$$RNI_{t,h} = \frac{nInc_{t-1,h}^{net} - nSC_{t,h} \cdot \pi_t^e}{nPI_{t,h}^c} \quad (5)$$

All households are assigned a common intercept parameter \bar{C}_t and a common marginal rate of consumption c .¹² Consumption also depends negatively on the expected real interest rate $r_t = i_t - \pi_t^e$, which is not only plausible, but also derived from optimizing behavior of representative agents in DSGE models like *Woodford* (2003).

Households plan real consumption at the beginning of each month.¹³ It is strongly affected by latest consumption, so it adjusts only gradually to a new income level, modeled with a parameter of consumption inertia $\lambda_c = 0.9$. Therefore, a household adjusts at a monthly rate of $(1 - \lambda_c) = 10\%$ to a new income level, for example if there is a change in employment status. Planned real consumption of household h is:

$$C_{t,h} = \lambda_c \cdot C_{t-1,h} + (1 - \lambda_c) \cdot (\bar{C}_t + c \cdot e^{-r_t} \cdot RNI_{t,h}) \quad (6)$$

2.4. Firm

2.4.1. Technology

Firms employ *Cobb-Douglas* production technology with factor inputs capital $K_{t,i}$, labor $L_{t,i}$ and technology parameter A_t . Daily production capacity of firm i with capital exponent $\alpha = 0.2$ is:

¹¹ *Oeffner* (2009) went further and made interest payments reinvested completely, not only the part of nominal interest that offsets inflationary devaluation. He argues at length why reinvestment of interest payments is a crucial stability condition for the simulated economy. Otherwise, rising interest rate would increase household income and induce higher consumption expenditure, so that economic activity is stimulated by “tight” monetary policy, which is counter-factual.

¹² A simple linear Keynesian consumption function is adopted here as explained in *Mankiw* (2000, p. 480). Parameter $c = 0.85$ reflects the marginal propensity to consume, whereas the intercept parameter is set at a small share of previous month’ net average wage: $\bar{C}_t = 0.18 \cdot (1 - \tau_t) \cdot nWI_{t-1}$, therefore the intercept grows at the same rate as the entire economy.

¹³ Usually, *planned* real consumption becomes *actual* real consumption. If one of the randomly chosen trading partners is sold out, the household simply chooses randomly the next firm out of 7 (see section 2.3.3) to satisfy its daily demand. Since all firms provide excess production capacities, it practically never happens that a household is restricted completely by all seven firms, even if the case is a theoretical possibility.

$Y_{t,i}^{c,day} = K_{t,i}^\alpha \cdot (A_t \cdot L_{t,i})^{1-\alpha}$. Thus, monthly production capacity multiplies the former equation by 30 days per month:

$$Y_{t,i}^c = 30 \cdot K_{t,i}^\alpha \cdot (A_t \cdot L_{t,i})^{1-\alpha} \quad (7)$$

Technology parameter A_t is identical for all firms and grows at a constant exogenous rate of labor-enhancing technological progress $g_A = 1.2\%$ annually (or 0.1% per month), so $A_t = A_{t-1} \cdot e^{g_A}$ (cf. *Barro and Sala-i-Martin*, 2003, chapter 3).

2.4.2. Capital Stock Development

In contrast to the original *Lengnick* (2013) model, each firm not only acts as a supplier but also as a customer of goods and services, when it invests in real capital. There is no distinction between consumption goods and investment goods.¹⁴ Investing firms and consuming households contribute to a unique demand flow. To purchase goods or services for investment, each firm maintains a limited number of seven connections to other supplying firms. These supplier connections are reconsidered monthly equivalent to household connections in section 2.3.3 with regard to prices, restrictions and randomly. At the beginning of each month, connections are reconsidered and replaced with some probability.

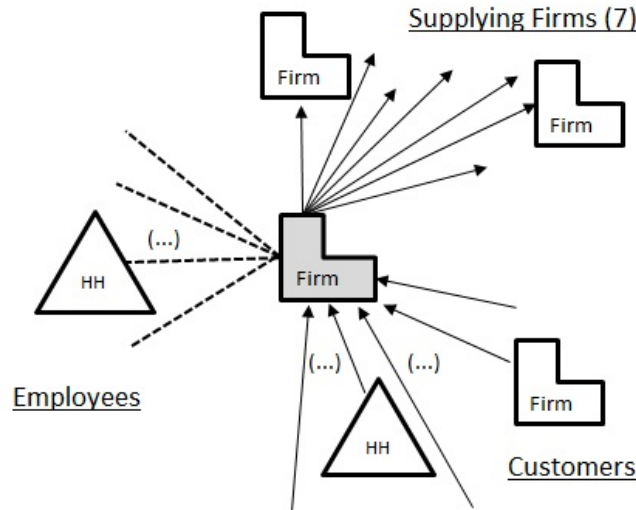


Figure 5: Firm Connections: Number of trading partners for investment goods supply is limited to 7, number of employees (households) and customers (households and firms) is not limited.

Each firm maintains a stock of real capital goods, which is devalued by parameter ρ at the end of each month at an annual rate of 9.6% (or monthly 0.8%). A firm invests according to its monthly gross investment plan, which is discussed later (see section 2.4.7). On a daily basis, each firm purchases goods from other firms to accomplish its investment plan. As soon as the month has passed, the sum of newly purchased investment goods is added to the capital stock.

¹⁴The same implicit assumption is part of the *Solow* (1956) model, where aggregate output is split into investment ($s \cdot Y$) and consumption.

It is ready for productive use in the following month. Firm capital evolves according to

$$K_{t,i} = K_{t-1,i} + \underbrace{I_{t-1,i} - \rho \cdot K_{t-1,i}}_{I_{t-1,i}^{net}} \quad (8)$$

2.4.3. Key Data for Firm Decisions

Firm decisions are built upon few key variables, some of which are explained here:

Utilization: Capacity utilization is defined as real sales divided by production capacity:

$$U_{t,i} = \frac{Y_{t,i}^{sales}}{Y_{t,i}^c} \quad (9)$$

In order to prevent supply shortages with a likely loss of trading relations to customers, firms provide for excess capacity. The target value U^* is 85%, permitting firms to accommodate demand fluctuations.¹⁵ Prices are set such that they offset cost of temporarily idle resources. For decision-making, firms determine a short-run weighted average level of utilization of last $T_u = 6$ months, where weights are highest for the most current month and decline linearly:

$$\bar{U}_{t,i} = \sum_{s=1}^{T_u} U_{t-s,i} \cdot \frac{(T_u + 1 - s)}{0.5 \cdot (T_u \cdot (T_u + 1))} \quad (10)$$

Expected Inflation: Inflation is measured by the price index PI_t which is a mean price of all $N^{fi} = 100$ firms in month t , weighted by their (real) market shares:

$$nPI_t = \sum_{i=1}^{N^{fi}} nPt_{t,i} \cdot \frac{Y_{t,i}^{sales}}{\sum_{i=1}^{N^{fi}} Y_{t,i}^{sales}} \quad (11)$$

Annual inflation is the logarithmic difference of the price index with 12 months lag: $\pi_t = (\ln(nPI_t) - \ln(nPI_{t-12}))$. Annualized *monthly* inflation is the logarithmic difference of values of the price index: $\pi_t^m = 12 \cdot (\ln(nPI_t) - \ln(nPI_{t-1}))$. For simplicity, inflation expectations are homogeneous among all agents. It is assumed that medium-run expected inflation is adaptive based on the last $T^\pi = 24$ monthly inflation rates as a weighted mean value with linearly declining weights. It is further assumed that central bank announcements of the inflation target π^* influence expectations directly to a some extent with $\lambda_\pi = 0.1$ reflecting central bank credibility. Expected inflation is

$$\pi_t^e = \lambda_\pi \cdot \pi^* + (1 - \lambda_\pi) \cdot \sum_{s=1}^{T^\pi} \pi_{t-s}^m \cdot \frac{(T^\pi + 1 - s)}{0.5 \cdot T^\pi \cdot (T^\pi + 1)} \quad (12)$$

Profit Rate: Let return on capital (RoC , elsewhere also termed h , e.g. in *Hein and Schoder*, 2011) relate profit with capital stock value. In order to assess profitability per unit of invested

¹⁵ Idle resources are quite normal: Think of restaurants, empty shops, hotel rooms, or food production. Apparently, the degree of utilization of available resources is not 100% in many branches, but on average well below, particularly in the service sector. Moreover, consumption goods are often short-lived (food), go out of fashion or out of date (clothes, electronics) and are costly to store. Respective firms will have to deal with idle resources and emergent cost of produced goods that can not be sold immediately.

capital, nominal interest payments on firm debt are left out of the calculation. Therefore, return on capital is calculated by subtracting costs of wages and depreciation from firm turnover, and to divide it by the current capital stock, valued by the mean price $nPI_{t,i}^K$ of currently connected suppliers of investment goods:

$$RoC_{t,i} = \frac{nP_{t,i} \cdot Y_{t,i}^{sales} - nW_{t,i} \cdot L_{t,i} - \rho \cdot K_{t,i} \cdot PI_{t,i}^K}{K_{t,i} \cdot nPI_{t,i}^K} \quad (13)$$

The profit rate of last 12 months $\overline{RoC}_{t,i}$ is calculated as a simple mean value of monthly profit rates. Firm decisions for investment also consider the development of the profit rate, in particular the difference between $\overline{RoC}_{t,i}$ and the average profit rate 12 months before, which is $\overline{RoC}_{t-12,i}$.

2.4.4. Decision: Price

Firms adjust their prices only infrequently as a result of menu costs. The current firm market price $nP_{t,i}$ is accompanied by a target price nP_i^* which reflects the exact price the firm would be willing to choose in the absence of menu costs. If the target price deviates by more than (an arbitrary threshold of) 1.5% from the current market price, the firm sets the current target price as a new market price.

$$nP_{t,i} = \begin{cases} nP_{t-1,i} & \text{if } \frac{nP_{t,i}^* - nP_{t-1,i}}{nP_{t-1,i}} \in (0.985; 1.015) \\ nP_{t,i}^* & \text{else} \end{cases} \quad (14)$$

Similar to the *Calvo* (1983) model, this price setting behavior ensures that only a small proportion of firms changes its price in a certain period. However, unlike the *Calvo* approach, where firms are forced to wait for a random event (the “*Calvo* fairy”) that finally allows them to adjust the price, firms decide freely about the timing of price adjustments here.

The target price evolves monthly with expected inflation and capacity utilization: With a high level of capacity utilization, a firm is more likely and willing to increase the price, considering itself in a strong market position. Deviations below target utilization trigger price drops, utilization above target leads to a rising target price. The probability of an additional target price movement is modeled with a random decision, whose probability is given by a reversed bell curve with standard deviation $\sigma = 0.14$ as shown in Figure 6. The price change decision is given by:

$$DP_{t,i} = \begin{cases} 1 & \text{if } (\overline{U}_{t,i} - U^* \geq 0) \text{ with } Prob = 1 - e^{-\left(\frac{\overline{U}_{t,i} - U^*}{\sigma}\right)^2} \\ -1 & \text{if } (\overline{U}_{t,i} - U^* < 0) \text{ with } Prob = 1 - e^{-\left(\frac{\overline{U}_{t,i} - U^*}{\sigma}\right)^2} \\ 0 & \text{with } Prob = e^{-\left(\frac{\overline{U}_{t,i} - U^*}{\sigma}\right)^2} \end{cases} \quad (15)$$

Then, the target price is actually moved up or down by ε_p , which again is a random variable that follows a uniform distribution between 0% and 1.5%.¹⁶ Therefore, utilization below target

¹⁶ Since latest utilization is correlated with neighboring values, a high degree of utilization will likely be followed by another one, so the target price may rise several months in a row.

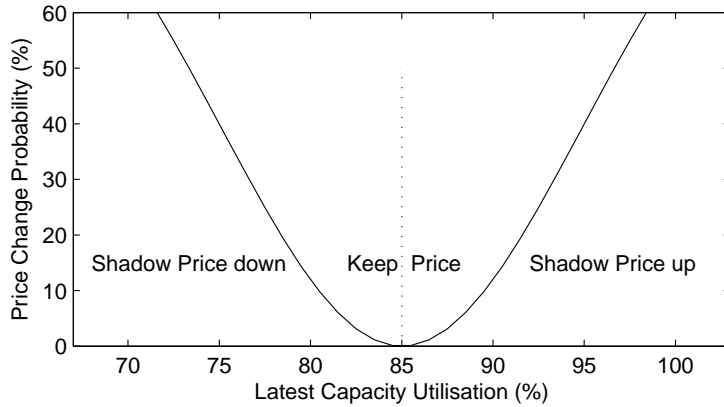


Figure 6: Probability to change the firm shadow price. When capacity utilization is low, price is likely decreased, for high utilization, price is likely increased.

implies a likely reduction (or slower increase) of the target price. Utilization above target level implies a higher probability of additional increases. In the absence of such acceleration of the slow inflationary price drift, an annual inflation of approximately 1.2% leaves the market price unchanged for more than 12 months on average, before price adjustments are triggered.

$$nP_{t,i}^* = nP_i^* \cdot (1 + \pi_t^e + DP_{t,i} \cdot \varepsilon_p), \quad \varepsilon_p \sim U(0, 0.015), \quad (16)$$

Please note that deviation of firm utilization from target, $\bar{U}_{t,i} - U^*$, is a similar concept as the output gap in *New Keynesian* (NK) models. Price setting of an individual firm depends on this “utilization gap” and is effectively doing the same thing as the *New Keynesian Phillips Curve* (cf. *Woodford*, 2003). Target utilization at 85% of production capacity equates to production at 100% of the so-called production potential of a representative firm. In NK models, the output gap directly affects inflation with an estimated parameter. Here, deviation from target utilization of a single firm generates price movements only with a certain probability, while on aggregate this stochastic element reliably generates inflation dynamics following economic activity. One key difference is that NK models assume rational expectations, while the presented ACE model applies adaptive inflation expectations. However, NK models often use hybrid *Phillips* curves which incorporate forward-looking and also backward-looking inflation (hybrid NKPC).

2.4.5. Decision: Hire or Fire

Short-run fluctuations of demand and utilization will be accommodated within a corridor (U^{low}, U^{up}) around the target value $U^* = 0.85$, without any adjustments to production. However, if latest utilization rises to levels above $U^{up} = 0.91$, the firm will create an open position by increasing the employment target $L_{t,i}^*$. A household asking for a job in that month is employed immediately, if the firms’ offered wage is convincingly high for the household to accept it. On the other hand, if latest utilization falls below $U^{low} = 0.78$, the employment target is decreased

so that a random worker is fired at the beginning of the following month:

$$\text{Firm Decision: } \begin{cases} L_{t,i}^* = L_{t-1,i}^* - 1 \text{ (Fire)} & \bar{U}_{t,i} < U^{low} \\ L_{t,i}^* = L_{t-1,i}^* + 1 \text{ (Hire)} & \bar{U}_{t,i} > U^{up} \\ L_{t,i}^* = L_{t-1,i}^* & U^{low} \leq \bar{U}_{t,i} \leq U^{up} \end{cases} \quad (17)$$

However, if employment of the firm has changed during the last $T_u = 6$ months, the firm has to recalculate its degree of latest utilization $\bar{U}_{t,i}$ with production capacity values based on current employment. Then, firm employment decisions are based on hypothetical values of past utilization. For example, if the firm just fired a worker, past sales are compared to now reduced production capacity.

2.4.6. Decision: Wage

A firm-specific wage contract for several months is fixed. Similar to price setting behavior, a target wage develops permanently, while the actual market wage is adjusted infrequently. When a new wage contract is due at time s , the current target wage is set as new firm wage.

$$nW_{t,i} = nW_{t,i}^* \quad \text{if } t = s \quad (18)$$

The new wage contract runs until month $s = t + 10 + \nu$, while ν follows a discrete uniform distribution between 0 and 4, so duration for the new contract is a random number between 10 and 14 months. Only a part of firms adjusts its wage in each month; average contract duration is 12 months.

Owners of the capital stock and workers struggle for their proper share of generated value added. The development of firm target wages depends on (1) expected inflation, (2) labor productivity growth, (3) latest utilization (4) the deviation of current markup (of price over unit wage cost) from its target value and (5) whether the firm was able to fulfill its employment target lately. The target wage is adjusted monthly:

1. It is increased by expected monthly inflation, so $nW_{t,i}^*$ is multiplied by $(1 + \pi_t^e)$.
2. The rate of labor productivity growth is a natural part of wage negotiations. The target wage is therefore multiplied by: $(1 + g_A)$
3. High utilization increases firm profit, so negotiation power of employees increases with high utilization. Monthly deviations from utilization target are translated with factor $a_u = 0.05$ to changes in the target wage. nW^* is multiplied by $(1 + a_u \cdot (\bar{U}_{t,i} - U^*))$.
4. The firm compares wage cost per unit of production with its current market price. Assuming a target value of $m^* = 60\%$ for a markup of price over unit wage cost, the current markup $m_{t,i}$ is closing the gap to its target value m^* at a rate of $a_m = 3\%$ per month. The target wage is multiplied monthly by $(1 + a_m \cdot \ln(\frac{m_{t,i}}{m^*}))$.

5. If the firm was unable to fulfill its employment target $L_{t,i}^*$ in the last month, for example when no household was willing to get hired, the shadow wage is increased by a random variable ε_w , which follows a uniform distribution between 0 and 0.01. If the firm was able to fulfill its employment target for the past $T_w = 6$ months, the shadow wage is decreased by that random variable. Therefore, the target wage is changed additionally, if $DW_{t,i}$ is different from 0:

$$DW_{t,i} = \begin{cases} 1 & \text{if } L_{t,i}^* > L_{t,i} \\ -1 & \text{if } L_{t-s,i}^* = L_{t-s,i}, \quad s = 0, 1, \dots, T_w - 1 \\ 0 & \text{else} \end{cases} \quad (19)$$

The target wage is multiplied by $(1 + DW_{t,i} \cdot \varepsilon_w)$ with $\varepsilon_w \sim U(0, 0.01)$. Note that this last part of the wage setting mechanism is adopted from *Lengnick* (2013).

In addition, the target wage evolves according to:

$$nW_{t,i}^* = nW_{t-1,i}^* \cdot \left(1 + \pi_t^e + g_A + a_u \cdot (\bar{U}_{t,i} - U^*) + a_m \cdot \ln \left(\frac{m_{t,i}}{m^*} \right) + DW_{t,i} \cdot \varepsilon_w \right) \quad (20)$$

2.4.7. Decision: Investment

In the context of investment decisions, hypothetical profit $n\Pi_{t,i}^{hyp}$ is maximized, which ignores *nominal* interest payments, but optimizes capital input with respect to the *real* interest rate:

$$n\Pi_{t,i}^{hyp} = nP_{t,i} \cdot Y_{t,i}^{Sales} - nW_{t,i} \cdot L_{t,i} - (r_t + \rho) \cdot K_{t,i} \cdot nPI_{t,i}^K. \quad (21)$$

$nPI_{t,i}^K$ is the mean price of current trading partners for capital investment (which is close to the general price index nPI_t), $r_t = i_t - \pi_t^e$ is the expected real interest rate. The firm calculates with output at target utilization U^* :

$$Y_{t,i}^{Sales} \approx U^* \cdot Y_{t,i}^c = U^* \cdot 30 \cdot K_{t,i}^\alpha \cdot (A_t \cdot L_{t,i})^{1-\alpha} \quad (22)$$

The capital stock is optimal when the marginal productivity of capital equals its marginal running cost, i.e. capital depreciation and real interest payments.

$$\frac{\partial n\Pi_{t,i}^{hyp}}{\partial K_{t,i}} = 0 = nP_{t,i} \cdot U^* \cdot \frac{\partial Y^c}{\partial K} - (i_t - \pi_t^e + \rho) \cdot nPI_{t,i}^K \quad (23)$$

Rearranging yields the target capital stock:

$$K_{t,i}^* = \left(\frac{nP_{t,i}}{nPI_{t,i}^K} \cdot \frac{U^* \cdot 30 \cdot \alpha}{(i_t - \pi_t^e + \rho)} \right)^{\frac{1}{1-\alpha}} \cdot A_t \cdot L_{t,i} \quad (24)$$

All terms in brackets of equation (24) are constant or stable in the long run. Therefore, we see that the target capital depends linearly on labor input $L_{t,i}$ and technology parameter A_t . Once a firm hires a worker, marginal productivity of capital rises, so that K^* rises proportionally. When employment is constant, A_t grows at a constant rate g_A , so K^* also grows at that rate. For the whole economy, aggregate capital also grows at g_A .

Investment of a firm depends on (1) capital depreciation, (2) distance to the target capital stock $K_{t,i}^*$, (3) last years average profit rate $\overline{RoC}_{t,i}$ and (4) the change in the average profit rate.

1. The base level of gross investment is set by real capital depreciation $\rho \cdot K_{t,i}$.
2. It is multiplied by target capital divided by current capital. The resulting convergence to target capital also brings about a dependence of current investment on employment $L_{t,i}$, current capital $K_{t,i}$ and real interest rates r_t). Constant growth of A_t also determines average net investment to be positive.
3. The return rate to the capital stock of the last 12 months influences investment, since capital is invested where it is most productive and profitable. Gross investment is multiplied by $(1 + a_h \cdot \overline{RoC}_{t,i})$.
4. The development of profit also influences investment, as it was claimed by *Kalecki* as described by *Dobb* (1973, p.222). Investment is multiplied by $(1 + a_{\Delta h} \cdot (\overline{RoC}_{t,i} - \overline{RoC}_{t-12,i}))$. As it turns out, this term strongly determines business cycle dynamics on aggregate level.

Finally, since investment decisions are often carried out with some lag, actual gross investment is adjusted slowly with an investment inertia parameter $\lambda_I = 0.9$. In sum, planned gross investment is:¹⁷

$$I_{t,i} = \lambda_I \cdot I_{t-1,i} + (1 - \lambda_I) \cdot \left(\rho \cdot K_{t,i} \cdot \frac{K_{t,i}^*}{K_{t,i}} \cdot (1 + a_h \cdot \overline{RoC}_{t,i} + a_{\Delta h} \cdot (\overline{RoC}_{t,i} - \overline{RoC}_{t-12,i})) \right),$$

which boils down to

$$I_{t,i} = \lambda_I \cdot I_{t-1,i} + (1 - \lambda_I) \cdot \rho \cdot K_{t,i}^* \cdot (1 + (a + b) \cdot \overline{RoC}_{t,i} - b \cdot \overline{RoC}_{t-12,i}). \quad (25)$$

2.4.8. Decision: Profit Payout

Payout decisions are based on calculated realized profit $n\Pi_{t,i}$. It is different from hypothetical profit (as explained above in section 2.4.7), since realized profit is determined by calculating nominal sales less realized cost for wages, *nominal* interest payments and capital depreciation:

$$n\Pi_{t,i} = nP_{t,i} \cdot Y_{t,i}^{sales} - w_{t,i} \cdot L_{t,i} + i_t \cdot SC_{t,i} - \rho \cdot K_{t,i} \cdot nPI_{t,i}^K \quad (26)$$

Each firm decides about paid-out profit, which is distributed equally among shareholders. Overall Profit is either paid out or kept in to increase equity, i.e. to reduce the debt ratio: $n\Pi_{t,i} = n\Pi_{t,i}^{paid} + n\Pi_{t,i}^{kept}$. If realized profit is negative, no profit is paid out. Otherwise, as long as the debt ratio is lower than 50%, all profits are paid out to shareholders. If the debt ratio increases to levels above 50%, only half of the profit is paid out.

$$n\Pi_{t,i}^{paid} = \begin{cases} 0 & n\Pi_{t,i} \leq 0 \\ 0.5 \cdot n\Pi_{t,i} & (n\Pi_{t,i} > 0) \& \left(\frac{nSC_{t,i}}{nPI_{t,i}^K \cdot K_{t,i}} < -0.5 \right) \\ n\Pi_{t,i} & (n\Pi_{t,i} > 0) \& \left(\frac{nSC_{t,i}}{nPI_{t,i}^K \cdot K_{t,i}} \geq -0.5 \right) \end{cases} \quad (27)$$

¹⁷Usually, *planned* real gross investment becomes *actual* real gross investment. If one of the randomly chosen trading partners is sold out, the firm simply randomly chooses the next supplier (out of 7) to satisfy its daily investment demand. Since all firms have excess production capacities, it practically never happens that a firm is restricted completely by all seven trading partners.

2.5. Government

Ten per cent of all households are employed by public authorities at private sectors average wage. Randomly chosen workers are publicly employed at the start of the simulation and never change their employer. Public employment does not play a vital role in the model: Employees simply receive monthly wages and do not produce any goods.¹⁸

The fiscal surplus (or deficit) nFS_t is calculated as public revenue minus public cost. Revenue is composed of taxes and seignorage gain (see below). Cost includes unemployment benefits, wages for public employment $L^{st} = 200$ and interest on public debt:

$$nFS_t = \underbrace{nTax_t + nSG_t}_{\text{revenue}} - \underbrace{UP_t \cdot nUB_t - L^{st} \cdot nWI_{t-1} + i_t \cdot nSC_t^{st}}_{\text{public cost}}. \quad (28)$$

Monthly tax revenue $nTax_t = \tau_t \cdot nInc_t$ depends on the tax base, which is current household income $nInc_t$. Public employees receive wage payments as high as previous month' wage index nWI_{t-1} . UP_t is the number of unemployed persons receiving unemployment benefits nUB_t (see section 2.3.2). There is also a rather small seignorage gain from the banking system, which is transferred to the treasury and is therefore a public revenue. It emerges because money as a liability of the banking system is interest-free, while the opposing credit on the banks balance sheet is met with interest payments. Interest payments *to the bank* by debtors outweigh interest payments *by the bank* to creditors. The difference is $nSG_t = i_t \cdot (nM_t^{hh} + nM_t^{fi})$.

The sum of average public cost subtracted by the seignorage gain represents the financing requirement of the general income tax. The respective tax base is composed of all sorts of primary income, on which the general income tax is applied. The tax rate is set to balance long-run public cost and long-revenue so that public debt remains stable. It is calculated as an average long-run financing requirement. However, the government approves a small budget deficit on average, since nominal output is constantly growing. The tax rate is set so that it generates revenue as high as 95% of financing requirements *on average*, adjusting very slowly.

$$\tau_t = \frac{1}{300} \cdot \sum_{s=t-300}^{t-1} 0.95 \cdot \frac{UP_t \cdot nUB_t + L^{st} \cdot nWI_{t-1} - i_t \cdot nSC_t^{st} - nSG_s}{nInc_s} \quad (29)$$

In this long-run perspective, the tax rate remains stable across business cycles and serves as an automatic stabilization mechanism. In a recession, for example, when public revenue stagnates and public cost rises, a stable tax rate ensures that the fiscal deficit (and public debt) rises, so that aggregate demand is dampened automatically (see section 3.4).

2.6. Monetary Policy

The central banks aims at price stability by employing the nominal interest rate i_t as its sole instrument. Price stability is accomplished if medium-run inflation remains close to its target value of $\pi^* = 1.2\%$ annually (0.1% per month). The central bank influences inflation rates

¹⁸Public employment was introduced to permanently generate a considerable amount of public cost, so that the income tax rate ranges around 10% rather than 0.5%.

directly via inflation expectations (cf. equation (12)). Moreover, the aggregate level of economic activity is also influenced by i_t . It is the sum of aggregate consumption and investment.

$$\sum_{i=1}^{N^{fi}} Y_{t,i}^{sales} = \sum_{h=1}^{N^{hh}} C_{t,h} + \sum_{i=1}^{N^{fi}} I_{t,i}$$

$$Y_t^{Sales} = C_t + I_t. \quad (30)$$

Firm investment as well as household consumption both depend negatively on the real interest rate. Therefore, monetary policy transmission runs along a consumption channel and an investment channel. For comparison, *New Keynesian* models apply an IS-curve without investment activity, which is derived from a representative consumer, optimizing the level of economic activity dependent on real interest rates. Both cases share negative dependence of economic activity on r_t . In the ACE model, consumption also depends strongly on current income.

The nominal interest rate is set by the central bank according to the *Taylor* (1993) rule, except that the output gap is not part of the equation.

$$i_t = r^* + \pi^* + 1.5 \cdot (\pi_{t,i} - \pi^*), \quad (31)$$

where r^* is the long-run real interest rate that is considered neutral with respect to monetary policy (*Blinder*, 1999), $\pi^* \approx 1.2\%$ is the annual inflation target (0.1% monthly) and $\pi_{t,i}$ is inflation during last 12 months.¹⁹ In the simulation, r^* is adjusted if the central bank does not meet its inflation target over a long period. Deviation of measured inflation above (below) target are responded by more than proportionate raises (declines) of the nominal interest rate, thus dampening (stimulating) economic activity by raising (shrinking) the real interest rate. The *Taylor* principle is expressed in the more than proportionate policy reaction with a factor of 1.5 to deviations from target.

3. Simulation and Model Behavior

The simulation is started with initialization. At the beginning, all virtual agents are created and randomly connected with respect to employment and trading relations. 100 shares per firm are also randomly distributed equally to 2,000 households. Then, reasonable starting values for capital stock and household consumption are calculated based on model parameters. Firms are initially indebted by 50% of the capital stock value, thus firms are partly financed externally.²⁰ To ensure that all monetary accounts sum up to zero, the sum of firm credit is accompanied by the transfer of an identical amount of deposits to bank accounts of households, both as transaction money and savings. Household savings are also distributed equally, rendering the initial wealth

¹⁹To be precise, the central bank uses a weighted average of inflation rates of different horizons. It consists of 3-month-inflation (weight 0.25), 6-month-inflation (weight 0.25) and annual inflation (weight 0.5) in order to respond directly and properly to short-run developments.

²⁰Given that firms pay out profits as explained in section 2.4.8, simulation shows that firm debt ratios are stable at around 50%, as long as the simulated economy does not experience strong movements of the price level like a deflation.

distribution completely equal with small random fluctuations. Finally, the simulation starts and requires some "burn-in" adjustment time to adjust initial firm values and random connections to stable proportions.

3.1. Single Firm

This section covers a closer investigation of individual firm behavior. Key firm variables across business cycles are analyzed. The left side of figure 7 shows how price setting behavior of firms

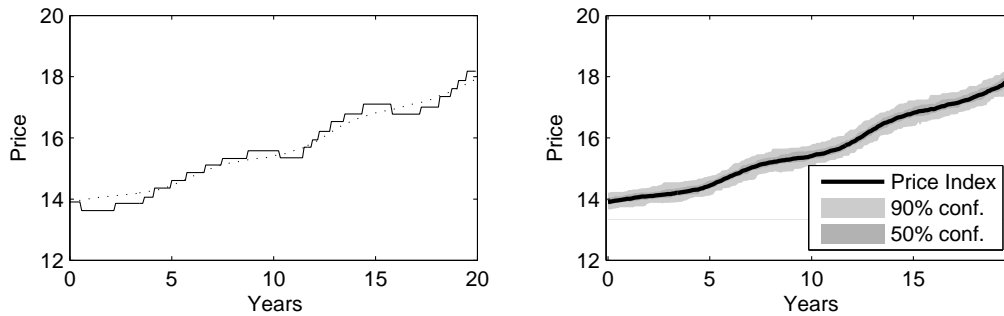


Figure 7: Firm price (left) with price index (dotted) and distribution of firm prices across business cycles (right).

works in the simulated economy. On average, the firm adjusts its price less than once in a year, while in times of recessions, price adjustment is delayed even more due to reduced pressure on prices. For comparison, the dotted line depicts the current aggregate price index. A considerable deviation of a firm price from competing prices affects firm sales strongly, so price competition prevents firm prices from diverging. The graph on the right side depicts the aggregate price index surrounded by shaded areas, which indicate the (narrow) distribution of firm prices. Competition for customer connections keeps prices on a comparable level.

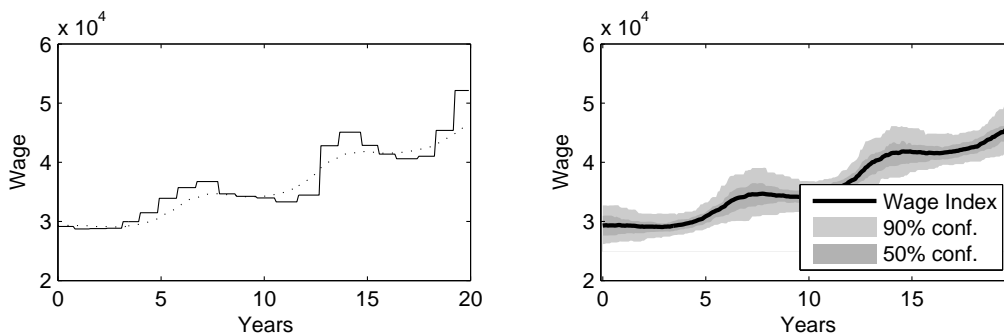


Figure 8: Firm wage (left) with wage index (dotted) and distribution of firm wages across business cycles (right).

Similarly, the diagram on the left side of figure 8 shows how wage contracts are actually fixed across business cycles. On average, firm wage is adjusted once every 12 months, while sometimes there are bigger "jumps", when employees are in a good position for negotiation. For comparison, the dotted line depicts the current aggregate wage index. A considerable deviation of a firm wage from competing wages has consequences for the firm's ability to fulfill its employment

target. Since wage competition is less intense than price competition in presented setting, wages fluctuate more than prices. The diagram on the right side depicts the wage index surrounded by shaded areas, indicating the distribution of firm wages. In times of recession, firm wages deviate stronger from average, while increasing competition for employees during boom induces wages to converge.

More firm-specific variables are shown in Figure 9, such as monthly real firm sales (solid line) together with production capacity (dotted line) in the upper left. Excess capacity accommodates short-run fluctuations of monthly demand. Below, an internal variable of the modeled firm is shown, namely the number of trading connections to customers (mostly households, but also firms). Actual sales fluctuate, but follow on average the number of connections, whereas there is an upward trend of sales due to rising labor productivity and therefore rising income. Please note that the number of connections tends to rise whenever the firm price is below average and vice versa (compare to Figure 7, left side). The bottom graph on the left is latest utilization as a weighted average of last six months, which is a key variable for firm decisions as explained above. Whenever average utilization leaves the corridor around its target, firm employment is adjusted, which is shown on the upper right. Employment of this firm ranges between 22 and 29 in the shown time period, whereas changes in employment are triggered by latest utilization.²¹

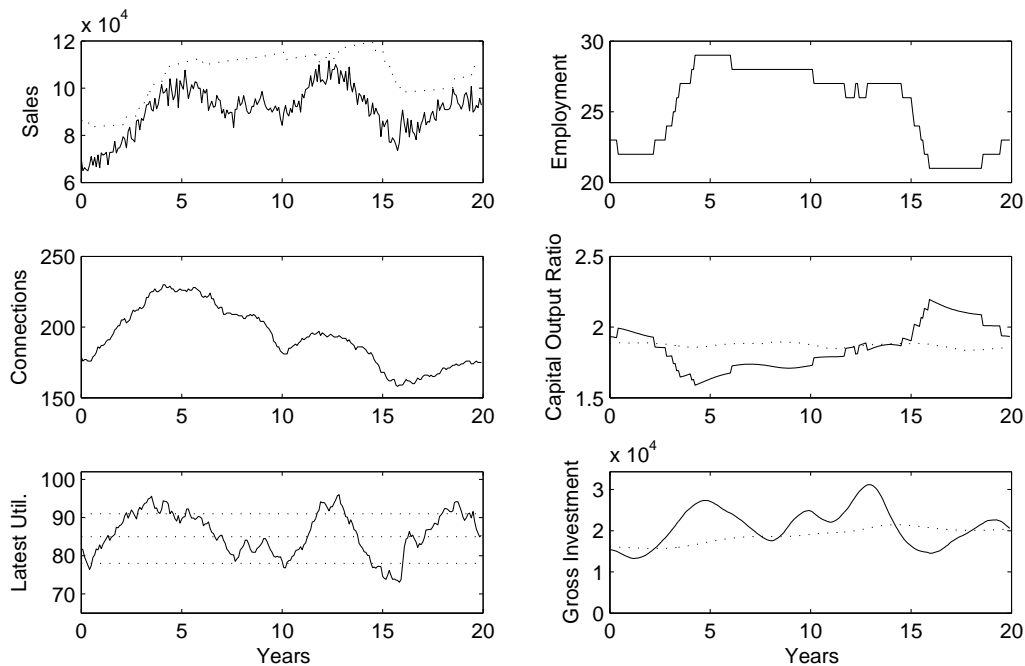


Figure 9: Firm-specific variables across business cycles: Demand, customer connections, average utilization (left), employment, capital-output ratio, gross investment (right).

Firms' capital-output ratio is depicted below employment. Whenever a worker is hired or fired, there is a jump in this ratio. Please note the dotted line, which is the average capital-

²¹Firm employment is 17 – 18 on average: 200 households are public employees, so that 100 firms employ up to 1,800 households.

output-ratio of all firms (≈ 1.85). If a firm's capital output ratio deviates from the average value, there is a tendency of convergence back to the dotted line. Since investment activity depends on the optimal capital stock, which itself depends on employment, hiring and firing decisions are followed by investment decisions that support the new level of employment. Gross investment is depicted as solid line in the bottom diagram on the right side, together with a dotted line, indicating the amount of capital depreciation. When the firm reduces employment in the first years shown, net investment becomes negative, while hiring decisions are followed by growing investment activity. Therefore, investment of a firm clearly follows employment²², although it also depends on past profits and the current interest rate.

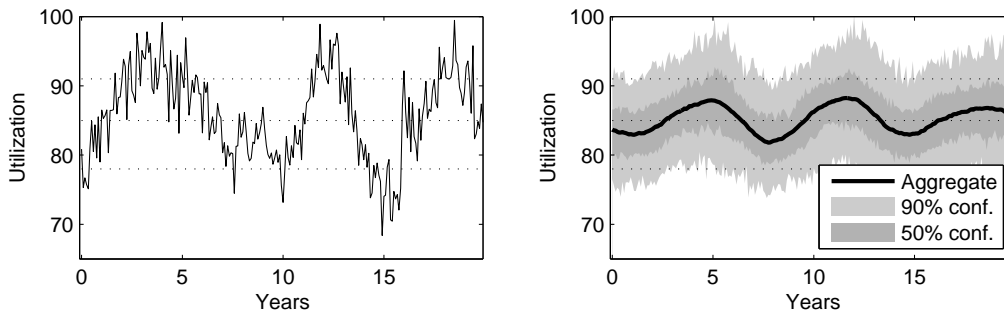


Figure 10: Distribution of firm utilization across business cycles.

Figure 10 shows monthly utilization of a firm (left) and the distribution of firm utilization (right). Firm sales and corresponding utilization degrees are fluctuating from month to month. Therefore, the distribution of utilization values is scattered with considerable variance. If monthly utilization leaves the corridor bounded by dotted lines only once in a while, the firm does not immediately hire or fire workers. Yet if the short-run average *latest* utilization (left side of figure 11) leaves the respective corridor, employment decisions are immediately triggered. The right side shows the distribution of latest utilization, whose variance is considerably lower compared to monthly utilization, since fluctuations of firm sales from one month to the next are smoothed away to a large extent. The business cycle is clearly visible, with a higher percentage of firms beyond the upper corridor line in booms and a higher percentage beyond the lower dotted line in times of recession.

A firm's return on capital (RoC) (as defined in equation (13)) is shown in the left diagram of figure 12, whereas the graph on the right side shows the distribution of return rates to capital across business cycles. Please compare to firm utilization in figure 10 to recognize that monthly returns are strongly correlated with sales and utilization. The rate of return to capital of most firms ranges between 0% and 15%, while average returns are approximately 5% during recessions and 8% during booms. However, return rates of individual firms comprise a continuum of values waving up and down with aggregate economic activity.

Finally, Figure 13 plots a histogram of return rates as they occur during 20 years. In the

²²The *Solow* (1956) growth model predicts this dependency on aggregate level, i.e. population growth induces additional capital accumulation.

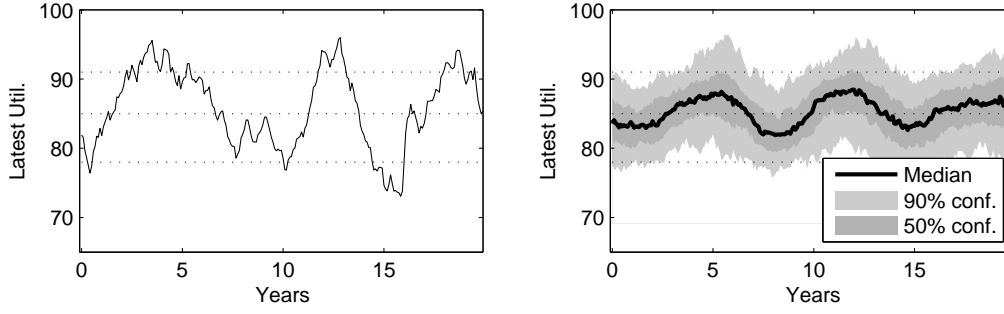


Figure 11: Distribution of short-run average firm utilization across business cycles.

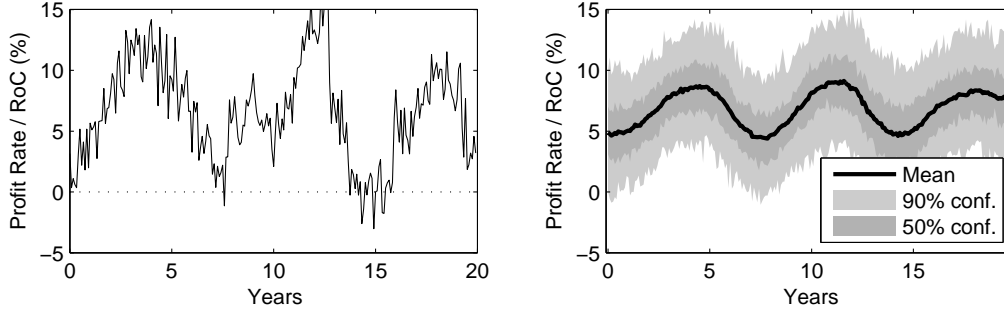


Figure 12: Monthly return on capital (RoC) distribution across business cycles.

upper graph, a histogram of return rates to capital of 100 firms with 240 monthly observations is shown. Below, return on equity is shown for comparison, which is defined as realized firm profit, which is already subtracted by nominal interest payments, divided by equity. Note that equity owners also experience capital gains when nominal firm debt is devalued by inflation, which is also considered. Firm equity is calculated by subtracting firm debt from the capital stock value. The return rate of shareholders is:

$$RoE_{t,i} = \frac{nP_{t,i} \cdot Y_{t,i}^{sales} - nW_{t,i} \cdot L_{t,i} - \rho \cdot K_{t,i} \cdot nPI_{t,i}^K + (i_t - \pi_t) \cdot nSC_{t,i}^{fi}}{K_{t,i} \cdot nPI_{t,i}^K + nSC_{t,i}^{fi}} \quad (32)$$

Equity investment is risky, so variance of the RoE distribution is considerably larger. Moreover, since the real interest rate is lower than corresponding return on capital in the current model setting, the mean of the RoE distribution (of approximately 10 – 12%) is considerably higher compared to the RoC distribution (6 – 7%).²³

A closer look at firms marginal cost of production provides an argument why firm production is actually limited by demand. In the short run, production is adjusted by hiring or firing employees, while capital depreciation and interest payments are fixed cost components. Therefore, marginal

²³ Return rates seem to be normally distributed. The reason is that households and firms split monthly spending to 30 daily consumption packages and choose one random supplier per day from their list of suppliers. As a result, this stochasticity of individual demand decisions generates fluctuations in firm sales, utilization and profit. The law of large numbers applies to both distributions, rendering the sum of random choices to follow a normal distribution.

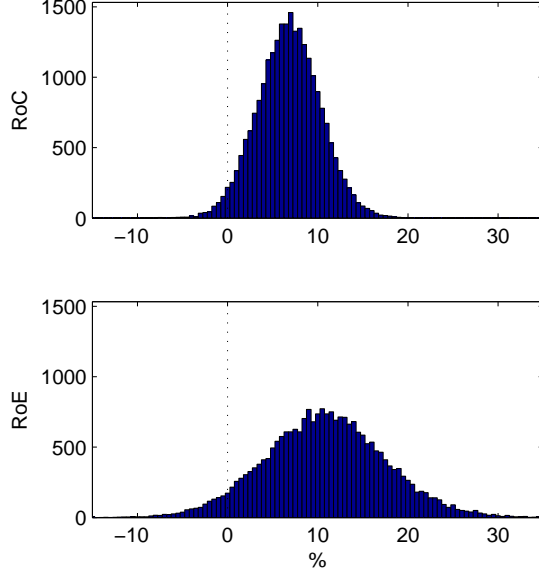


Figure 13: Return distributions of 100 firms across business cycles. Upper graph: Return on capital stock, lower graph: Return on equity ($\approx 50\%$ of capital stock).

cost of production $nMC_{t,i}$ is given by wage cost of one produced good:

$$nMC_{t,i} = \frac{\partial (nW_{t,i} \cdot L(Y^c, K))}{\partial Y^c} \Big|_{(K_{t,i}, Y_{t,i}^c)} = \frac{nW_{t,i}}{A_t \cdot (1 - \alpha) \cdot 30^{\frac{1}{1-\alpha}}} \cdot \left(\frac{K_{t,i}}{Y_{t,i}^c} \right)^{\frac{-\alpha}{1-\alpha}} \quad (33)$$

Similarly, marginal productivity of labor $nMPL_{t,i}$ is calculated by evaluating the partial derivative of monthly production capacity with regard to labor input at current values for capital and labor and multiplying it with the current price:

$$nMPL_{t,i} = P_{t,i} \cdot \frac{\partial Y^c(K, L)}{\partial L} \Big|_{(K_{t,i}, L_{t,i})} = P_{t,i} \cdot 30 \cdot K_{t,i}^\alpha \cdot A_t^{1-\alpha} \cdot (1 - \alpha) \cdot L_{t,i}^{-\alpha} \quad (34)$$

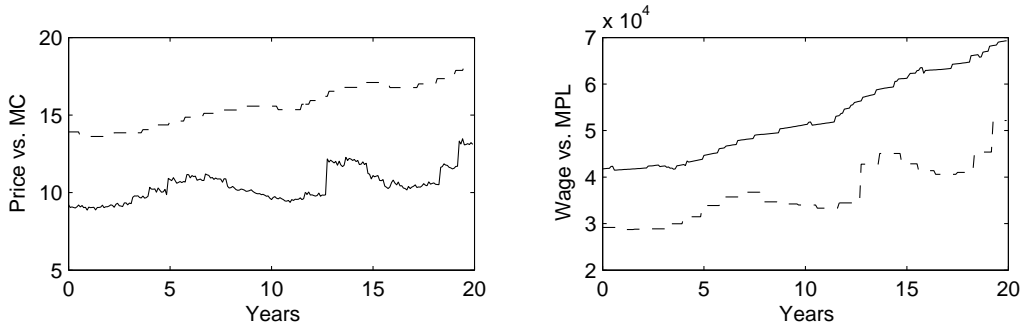


Figure 14: Left: Marginal cost (solid) compared to price (dashed), right: Marginal productivity of labor (solid) compared to wage (dashed).

The left side of Figure 14 shows that marginal cost of production is always below current firm price, with certain “jumps” occurring when the firm wage is adjusted. The right side indicates that

marginal productivity of labor is always considerably higher than the current wage. Again, there are small “jumps” in marginal productivity occurring when the firm’s market price is adjusted. Both graphs suggest that firm production is effectively limited by demand, since each additional unit sold increases profit. Therefore, firms in general would enjoy to increase output by large amounts, if there were the possibility to sell the additionally produced units.

3.2. Macro Behavior: The Business Cycle

Figure 15 shows aggregate model behavior and emergent business cycle dynamics by plotting six macroeconomic variables over 240 months. Endogenous business cycles are generated by recurrent waves of investment activity as shown in the upper left diagram, with a frequency of about 7 years.

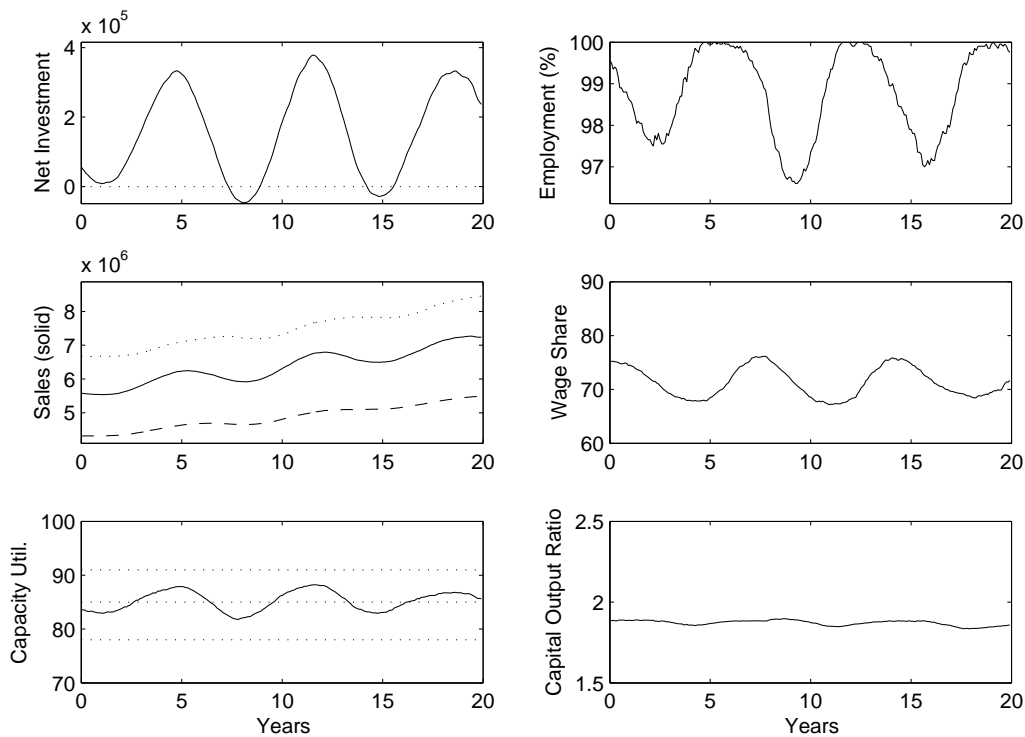


Figure 15: Model behavior on macro level across business cycles. Left side: Net investment, aggregate sales (GDP), utilization. Right side: employment, wage share of national income, capital output ratio. Second graph on the left (Sales, solid line) also shows production capacity (dotted) and consumption (dashed).

The second graph on the left depicts aggregate firm sales (solid) with investment as a component of it. With a small lag, aggregate consumption (dashed line) also shows a small cyclical movement, since increasing sales are accompanied by increasing household incomes. Production capacity (dotted line) rises with technology growth, but experiences cyclical periods of slightly higher and lower growth. Slope differences in the development of production capacity result from different levels of employment and net investment. Aggregate capacity utilization is depicted in the lower graph on the left with dotted lines of target utilization and the corridor around it determining employment decisions. The business cycle is clearly visible also in this measure of

aggregate economic activity, when aggregate utilization falls below target in recessions and rises above in times of boom.

The graph on top right side of figure 15 shows how employment develops during business cycles. The economy is near full employment, since no relevant labor market frictions are modeled and labor is a universal factor input, thus rendering each worker to match in all firms. During recession, unemployment rises by about three per cent, unless recovering investment activity increases utilization levels above target again. Then more firms decide to hire workers, instead of firing them. The second graph on the right shows the sum of wage income compared to GDP, which rises up to 75% during recession, accompanied by a drop in firm profits. During boom, it falls slightly below 70% when firm profit is at its peak. Finally, the lower graph on the right depicts the capital-output ratio, calculated as real capital stock divided by *annual* production capacity, which is constant in a steady-state equilibrium in the *Solow* (1956) model. Except from small variations over the business cycles, resulting from variations in aggregate employment, it is actually stable at about 185% of annual production capacity.

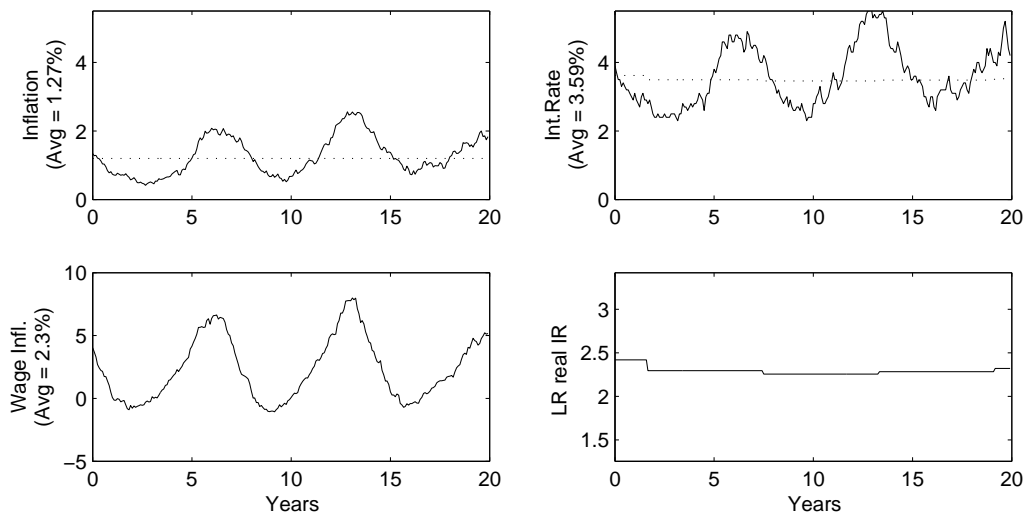


Figure 16: Monetary variables across business cycles. Left side: inflation, wage inflation. Right side: nominal interest rate, long-run real interest rate.

Concerning long-run growth of aggregate sales, it is known that technology parameter A_t grows at 1.2%. In section 2.4.7 it was argued that the firm capital stock grows at the same rate. With constant returns to scale, firm production capacity also grows at 1.2% in the long run, as long as employment is stable:

$$Y^c(\lambda \cdot K, \lambda \cdot A, L) = 30 \cdot (\lambda \cdot K)^\alpha \cdot (\lambda \cdot A \cdot L)^{1-\alpha} = \lambda \cdot Y^c(K, A, L). \quad (35)$$

with an arbitrary factor λ . If a single firm (with constant employment) grows at a certain rate, the aggregate of all firms with employment on a constant level will also grow at that rate. As it turns out, aggregate real firm turnover (GDP) actually grows at an average annual rate of approximately 1.2% when it is calculated for a long horizon of 1,000 months or more, thus capturing several cycles. Therefore, aggregate long-run growth is determined by the exogenous

rate of technological progress, similar to the *Solow* (1956) model.

Figure 16 shows the development of monetary variables. On the left, (annual) inflation and wage inflation both follow economic activity along the business cycle. On average, inflation is quite close to its target value (dotted line). Wage inflation fluctuates stronger than inflation and is higher on average, because it additionally incorporates labor productivity growth. Over long horizons, growth of the wage rate converges to 2.4%, which is the sum of labor productivity growth and the inflation target. On the top right of Figure 16, the nominal interest rate which is set by the central bank is depicted. Below, the almost fixed long run real interest rate is shown, which is relevant for monetary policy decisions. The average nominal interest rate is approximately the sum of average inflation and the long run real rate (dotted line). Note that the interest rate time series amplifies fluctuations of inflation, which reflects over-proportionate reactions of monetary policy to deviations from inflation target (*Taylor* principle).

In its presented setting, the model runs at a long-run real interest rate of about 2.3% on average.²⁴

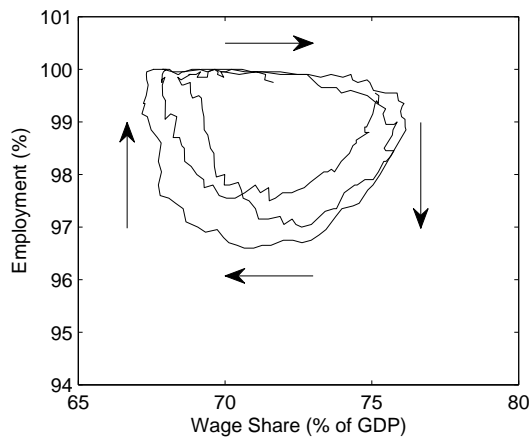


Figure 17: Cyclical behavior of employment and the wage share similar to *Goodwin* cycles.

As it turns out, dynamics of the business cycle in the ACE model resembles *Goodwin* (1967) cycles. Figure 17 depicts these Goodwin cycles in a scatterplot of employment level and the wage share. The mechanism behind the cyclical process visible in ACE time series is described in the following. As soon as the economy reaches a level near full employment, competition for workers and high utilization induces firms to raise wages faster than prices. Subsequently, profits fall and the share of wages is increased. Investment, which is strongly influenced by past profits, is reduced as a consequence. Aggregate demand is stagnating or even falling, firm utilization declines. Eventually, some of the firms fire workers, thereby reducing consumption and

²⁴ A drop in household's consumption propensity, however, would induce the model dynamics to converge to a lower real interest rate. This mechanism is known as "paradox of thrift" and was described by *Keynes* (1936, chapter 23). An increase in the savings rate (thrift) induces adverse effects on aggregate demand and other macroeconomic variables. In the adjustment process, the simulated ACE economy faces a drop in aggregate consumption, demand and employment. The central bank will eventually recognize that the current value r^* generates average inflation rates below target, so that r^* is reduced.

aggregate demand even more. The downward spiral ends when profits stop falling and availability of unemployed workers allows for moderate wage contracts. Now, a declining wage share with corresponding rise in profits induces a new wave of investment with increasing consumption and output. Increasing demand and utilization induces firms to hire more workers, thus triggering even more investment to endow additional workers with capital. Soon, the business cycle returns to the point where it started.

3.3. (Non-)Neutrality of Money

A major discussion of macroeconomic theory of the past concerns the question of neutrality of money. The classical dichotomy claims that real variables and monetary variables may be analyzed separately, because economic agent's decisions are claimed not to be affected by changes in nominal units of scale. Therefore, changes in the money supply are claimed not to affect real variables (*Patinkin*, 1987). In this context, the quantity theory of money is based on the equation of exchange. Translated to the notation of simulated ACE aggregates, it is:

$$nM_t \cdot v_t = nPI_t \cdot Y_t^{sales} \quad (36)$$

The left diagram of figure 18 shows the equation of exchange, solved for velocity v_t . Apparently, the velocity of money turnover is a constant relation between economic activity and money. A monetarist might interpret this graph by claiming that an increase of the quantity of money $M_t = M_t^{hh} + M_t^{fi}$ per unit of output Y_t^{sales} causes an increase in price level PI_t . However, since money is created endogenously in this ACE model, with agents that decide freely about the desired amount of transaction money, such an interpretation is invalid. Actually, agents hold transaction money as high as previous month's individual cost, plus a liquidity buffer of 20%. As a result, velocity of monthly money holding is slightly higher than 1, when compared to monthly turnover. Causality runs from price level to the desired and realized amount of transaction money, not vice versa.

Economists in the tradition of *Keynes* argue that money actually affects economic activity. Recent *New Keynesian* DSGE models incorporate frictions of price and wage developments that render short-run economic activity influenceable by monetary policy., However, in the long run, the classical dichotomy holds (see, for example, *Benchimol and Fourçans*, 2012). The *New Keynesian Phillips curve* marks the short run efficiency of monetary policy.

The presented ACE model also generates a *Phillips* curve relationship, which will be derived here: Inflation is measured as a growth rate of the price index PI_t , which itself depends on price decisions of each single firm (cf. equations (11), (12) and (16) above). Now, firm prices are driven by a common value for expected inflation π_t^e , therefore aggregate inflation is also directly influenced by it. Moreover, firms accelerate or slow down their individual target price development with capacity utilization. On aggregate, there is a continuum of firm capacity utilization levels. As the right diagram of figure 11 indicates, the positioning of this continuum

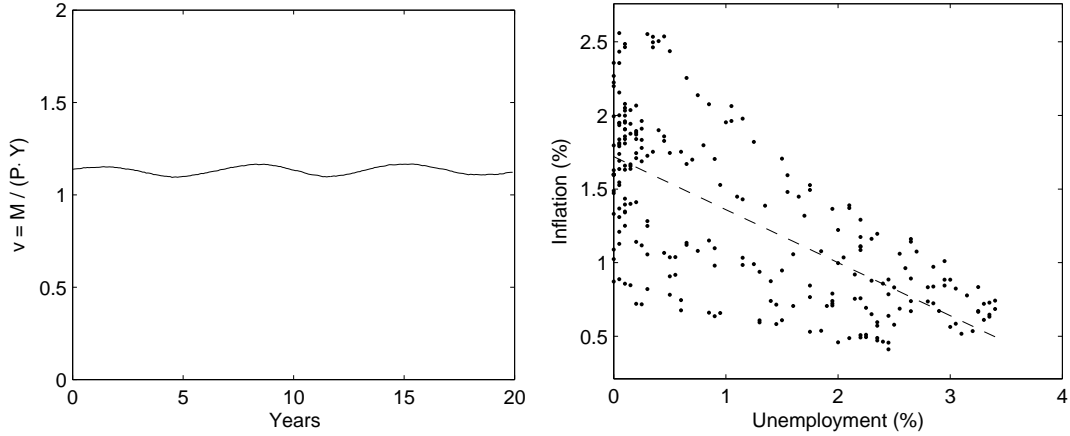


Figure 18: Left: Velocity of money turnover, Right: Phillips curve at $\pi^* = 1.2\%$ of 240 subsequent monthly observations. Dashed line depicts OLS regression.

shifts with aggregate capacity utilization U_t :

$$U_t = \frac{\sum_{i=1}^{N^{fi}} Y_{t,i}^{Sales}}{\sum_{i=1}^{N^{fi}} Y_{t,i}^c} \quad (37)$$

If U_t increases (decreases), a larger number of firms faces high (low) levels of capacity utilization, thus inflation accelerates (slows down). On aggregate, we can state that inflation is driven by aggregate utilization, or rather its deviation from target:

$$\pi_t \approx \pi_t^e + f(U_t - U^*) \quad (38)$$

Please note the similarity to a *New Keynesian Phillips curve* (Woodford, 2003), except that the output gap has been replaced by a “utilization gap” as another measure for aggregate economic activity. Another difference is that inflation expectation π_t^e is purely adaptive, so there is no rational expectations hypothesis employed. Moreover, no stochastic term is explicitly given, although there are stochastic elements incorporated in the ACE simulation, for example the number of firms that actually adjusts its price in a certain month.

In the ACE economy, a *Phillips curve* is clearly visible in the right diagram of figure 18, where annual inflation is plotted against current unemployment, indicating non-neutrality of money at least in the short run. Due to adaptive expectations, a central bank that tries to reduce unemployment by increasing its inflation target will increase demand in the short run. However, there is no trade-off in a way that higher inflation *causes* low unemployment, as was implicitly assumed in the “rules vs. discretion” debate.²⁵

²⁵See *Kydland and Prescott* (1982). According to this view, a representative agent balances leisure and consumption. Surprise inflation, in turn, induces the agent to work more, as a result of erroneously over-estimating the real purchasing power of nominal wages. This is a good example for the explanatory value of microfoundations, if assumptions on individual behavior are questionable. In the real world, *most* households determine their level of work time by a mixture of labor contracts with fixed hours (as a convention of society); and the need to satisfy their employer, who might ask for temporary overtime to accommodate demand variations.

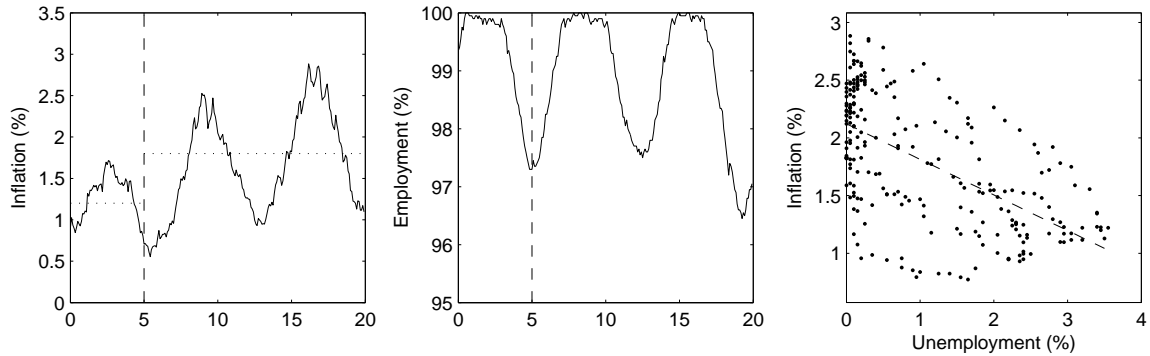


Figure 19: Adjustment after shock to inflation target from 1.2% to 1.8%. Left: Inflation, Middle: Employment, Right: Phillips curve 240 monthly observations right after shock month.

The direction of causality runs strictly from economic activity to rising prices in this model. Nevertheless, monetary policy is not necessarily inefficient: A sudden increase of the inflation target induces monetary policy to lower the nominal interest rate, which moves economic activity upward in the short run. Adaptive expectations subsequently adjust to the new level of inflation and afterwards, the central bank faces a shifted Phillips curve that emerges from a higher inflation target. See Figure 19 to see the adjustment process after the inflation target is increased to $\pi^* = 1.8\%$ annually at year 5. Inflation almost immediately switches to cyclical fluctuation on a higher average level of inflation. Employment also continues in its cyclical pattern. The graph on the right side depicts the *Phillips* curve relationship for 240 monthly observations after the shock month. Clearly, the new *Phillips* curve relation is simply shifted upward. Therefore, monetary policy is neutral with respect to real variables in the medium run.

3.4. Stabilization policy

The tax regime of “automatic stabilization” as described above (section 2.5, equation (29)) is applied in all diagrams throughout this article, except partly for this section. Its mechanism is shown in the left side of figure 20. The upper graph shows public cost and revenue; the corresponding tax rate is depicted in the middle and the bottom graph depicts aggregate employment. Please note that the tax rate remains stable slightly above 9%. In recessions, tax revenue is stagnating, while public cost for unemployment benefits is increasing strongly. Therefore, the fiscal deficit is increasing during economic downturn, so that we have an effect of automatic stabilization of aggregate demand: Public debt growth is increased in times of recession.²⁶ Fluctuations of demand are dampened automatically by the state, which is the reason for the term “Automatic Stabilization” assigned to this tax regime. A second tax regime is termed “Deficit Spending”, which is shown in the middle column of figure 20. Since this is a *Keynesian* model characterized by demand-led economic activity, it is not surprising that fiscal policy aiming at demand stabilization actually works. The government in this ACE model does not directly buy goods and

²⁶ Public debt varies between 15 and 20% of the capital stock value in the presented setting. When compared to annual turnover, the public debt burden is stable at 25-35% of GDP.

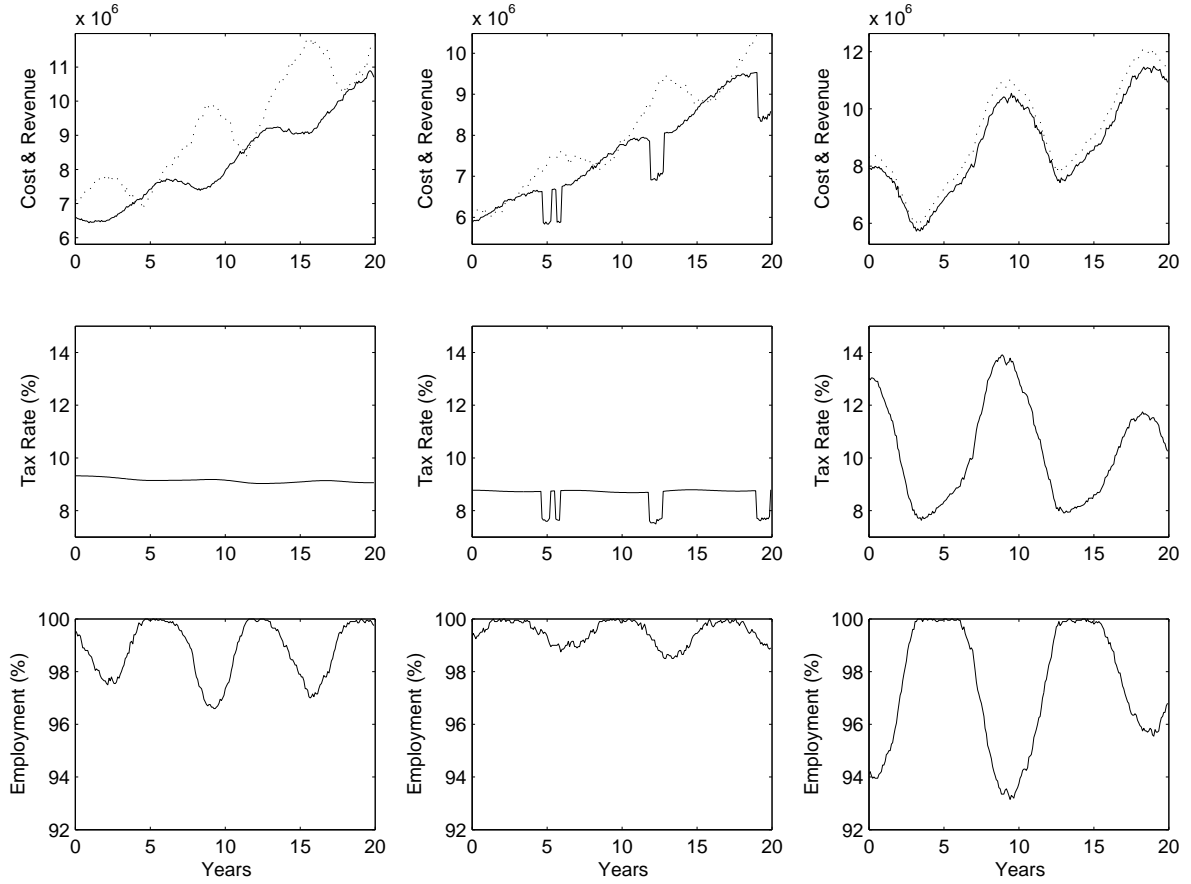


Figure 20: Fiscal Policy Regimes: Automatic Stabilization (left side), Deficit Spending (middle) and Pro-Cyclical (right). All columns show: Upper diagram: Public cost (dotted) and revenue (solid), middle: Tax rate, bottom: Employment.

services. Instead, aggregate demand is influenced by adjusting the general income tax. Whenever aggregate utilization falls considerably below target, the state reacts with a tax cut in order to stimulate consumption expenditure. Otherwise, the tax rate is adjusted according to the regime of “Automatic Stabilization”, defined as τ_t in equation (29). The deficit-spending tax rate is adjusted according to

$$\tau_t^{\text{Def}} = \begin{cases} \tau_t & \text{if } U_t - U^* \geq -0.01 \\ \tau_t + (U_t - U^*) & \text{if } U_t - U^* < -0.01 \end{cases} \quad (39)$$

The second column of diagrams in figure 20 depicts the fiscal regime of deficit spending. As the employment plot (bottom diagram) shows, business cycles are dampened considerably in this *Keynesian* regime of demand stabilization. As a result, there is less unemployment on average, so that financing requirements of the general income tax are even reduced. Therefore, the average tax rate is slightly lower at approximately 8.5 – 9%. Public debt is also lower in this regime, because high deficits in times of recession are reduced in frequency and magnitude.²⁷

²⁷Public debt in the deficit spending regime varies between 7 and 11% of the capital stock value, given the same

In principle, the ACE framework allows for stabilization of aggregate demand with a higher average level of output. In General Equilibrium frameworks such as *Lucas* (2003) as well as *Gali, Gertler and López-Salido* (2007), deviations from potential output are symmetric and can only be dampened in both directions. However, the modeled stabilization policy of deficit spending shows how ACE models it becomes possible to “fill in troughs without shaving off peaks” (*DeLong and Summers*, 1988, p. 434). The employment plot in the middle column of figure 20 shows the reduction of unemployment in recessions and on average. Therefore, stabilization policy in an ACE framework offers additional welfare gains compared to General Equilibrium models as applied by *Lucas* (2003). Risk-averse individuals will not only appreciate reduced variability of consumption and employment, they will capture additional welfare gains from a higher level of average consumption.²⁸

Finally, a third tax regime is termed “pro-cyclical”, which is shown in the right column of figure 20. Again, the government conducts fiscal policy in this ACE model by adjusting the general income tax. Here, the state aims at permanent balancing of the fiscal deficit throughout business cycles. To this end, a recession, which is accompanied by a stagnating tax base and rising cost, is answered with an increasing tax rate, thereby dampening aggregate consumption even more. The pro-cyclical tax regime aims at balancing public finance requirements and collected taxes permanently. The respective tax rate is given by:

$$\tau_t^{\text{Pro}} = 0.95 \cdot \frac{UP_{t-1} \cdot nUB_{t-1} + L^{st} \cdot nWI_{t-1} - i_t \cdot nSC_t^{st} - nSG_{t-1}}{nInc_{t-1}} \quad (40)$$

Observe the parallel development of cost and revenue in the upper diagram of the right column of figure 20, thus indicating a stable budget deficit as was intended by this policy regime. However, the tax rate is increased pro-cyclically whenever demand and utilization is low already, thus dampening consumption even more. In the other extreme, the tax rate is lowered when the economy is at full employment levels and aggregate demand is at its peak, thus stimulating economic activity even more. As a result, business cycles are reinforced and amplified, with considerably higher unemployment burdens and longer duration. Therefore, fiscal policy aiming at permanently balancing the state deficit is accompanied by high cost to society.²⁹

Keynes (1936) showed that anti-cyclical fiscal policy is effective when the goal is to dampen recessions. It should have become public knowledge by now that pro-cyclical fiscal policy will have strongly adverse effects on the real economy, as shown in former analysis. However, in the current Euro crisis, economic policy of austerity is forced on southern Euro-zone countries. When investment activity and aggregate demand in general are low already, accompanied by lower tax revenue and increasing cost, governments in these countries are “encouraged” to balance the

model parametrization. When compared to annual turnover, the debt burden is stable at 15-20% of GDP. That is, public debt is considerably lower than in the benchmark regime of automatic stabilization. Compared to real-world experience with *Keynesian* fiscal policy in the twentieth century, this result appears counter-factual.

²⁸ Moreover, we might determine additional gains from analyzing welfare on disaggregate level. Income and consumption paths of individual heterogeneous agents are considerably less volatile, because a smaller proportion of agents becomes unemployed at all. Therefore, welfare gains of stabilization are even higher.

²⁹ Public debt in the pro-cyclical regime varies between 8 and 9% of the capital stock value, given the same model parametrization. When compared to annual turnover, the debt burden is stable at 15-20% of GDP.

budget pro-cyclically. As a result, southern Euro-zone countries face high unemployment rates and deflationary tendencies.³⁰ In a system of double-entry bookkeeping, it can be shown that it is not possible to reduce public debt and firm sector debt at the same time, if there is no other group of agents which is willing to spend considerably more.³¹

3.5. Calibration

The model is calibrated to produce a number of desired properties and proportions on aggregate level. Business cycles are of 7 years frequency, which corresponds quite well to *Juglar* cycles with a length between 7 and 11 years as reported by *Korotayev and Tsirel* (2010). However, in the absence of exogenous shocks in the simulation, endogenous simulated cycles are more predictable and even compared to real-world business cycle occurrences.

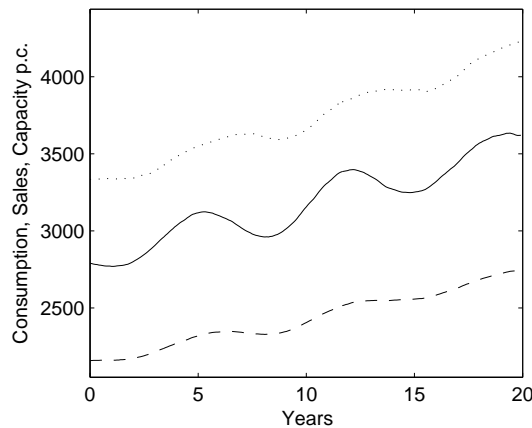


Figure 21: Per capita real production capacity (dotted line), real firm sales (solid) and real consumption (dashed).

Figure 21 (again) depicts real values for production capacity (dotted line), firm sales (solid) and consumption (dashed) per capita (as shown before). Please note the upward trend of shown measures. Even production capacity fluctuates around a trend, since firms reduce employment and capital accumulation during recessions. Note that consumption is considerably less volatile than investment, given as distance between sales and consumption, which is in line with stylized facts (cf. *Stock and Watson*, 1999). The annual growth rate of aggregate firm sales (real GDP) varies approximately between -1.5% in economic downturn and $+3.5\%$ in times of boom. All real variables grow annually at 1.2% in the long run. Over 20 years, exponential growth of 0.1% per month amounts to an increase of 27.1% .

The annual depreciation rate of productive firm capital is set at 9.6% . *Madsen et al.* (2012) distinguish average depreciation rates of firm capital into structures (buildings) with a depreciation rate of 1.75% and machinery and equipment with a depreciation rate of 17.6% . So the chosen depreciation rate is a compromise between both components of firms capital stocks. *Chiarella*,

³⁰ At this point, it is still unclear whether the strategy will at least accomplish the goal of an internal devaluation of countries in a crisis of competitiveness in the end. The future will show whether imbalances within the Euro zone can be reduced sufficiently, before pro-cyclical policy is perceived too costly to society.

³¹ See proof in appendix A.2.

Flaschel and Franke (2005, p. 85) refer to detailed data on the US economy and calculate with a similar capital depreciation rate (9.5%) as this model. The ACE model produces a capital-output ratio of approximately 185%. This amounts to a reciprocal value of output to firm capital Y/K of 0.54 and is a reasonable order of magnitude for developed economies. *Madsen et al.* (2012) estimate a long-run average for 16 OECD countries for Y/K and find it to converge to levels of about 0.5 since 1980. *Chiarella et al.* (2005, p.85) calculate with 0.7 for the Y/K ratio referring to US data.

A wage share of approximately 70 – 75% is slightly higher than respective proportions in real-world developed economies. According to OECD data (stats.oecd.org) for the total economy in Germany and the US, the wage share ranged between 65% and 70% recently. However, unlike the simulated economy, in real world economies there are considerable amounts of capital in the form of real estate; returns of invested capital in real estate also contribute to overall GDP. Closely related, the average profit rate of simulated firms ranges between 5% in recessions and 8% in booms, so we have an average return rate to invested capital of approximately 6 – 7%. The average return rate to (debt-free) equity is considerably higher at 10 – 12%.³²

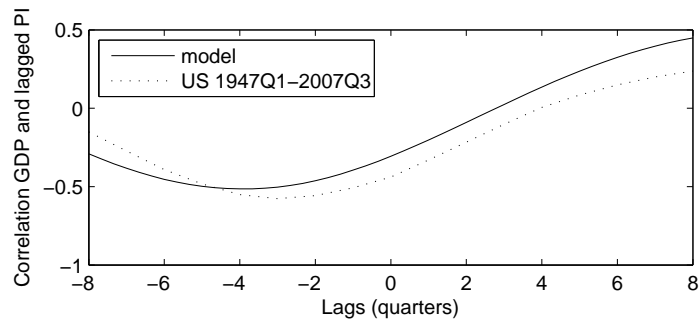


Figure 22: Correlation of detrended GDP and detrended Price Index from ACE model (solid line) and with data for the US economy between 1947 and 2007 (dotted line).

Finally, figure 22 depicts the correlation between detrended GDP and price level for 24 months backward and forward. For comparison, respective lags (on a quarterly basis) for US data between 1947 and 2007 are shown as reported by *Lengnick* (2013). Apparently, the pattern is rather similar, so the model qualitatively fits real data quite well.

3.6. Limitations and Parameter Sensitivity

Since the developed model does not perfectly mimic real-world behavior on individual level, consider this (not necessarily exhaustive) list of limitations: Concerning firms, (1) competition between them is reduced to price competition (for customers) and wage competition (for employees), which does not even closely capture the complexity of actual firm decisions. To name a few, innovation, marketing, product quality and corporate governance are left out. (2) The

³²The implied distribution of simulated GDP (100%): Wages (70-75%), Depreciation (17-18%), Profit (9-15%). Multiplying profit (as percentage of GDP) by simulated output-capital ratio ($Y/K = 0.54$) yields a profit rate ranging between 5 and 8 per cent again.

model abstracts from firm entry and exit. (3) The firm size distribution as depicted in Figure 23 is skewed to the right, which is a stylized fact (*Delli Gatti et al., 2010*). However, the range of firm sizes is very limited. Realism in this respect is beyond the scope of this article. (4) Firms are behaving according to fixed rules instead of adapting their strategies in a process of learning. (5) Technological progress is exogenous instead of being the product of innovative activities; a criticism that also applies to the *Solow* model, which induced more recent developments of modern growth theory.

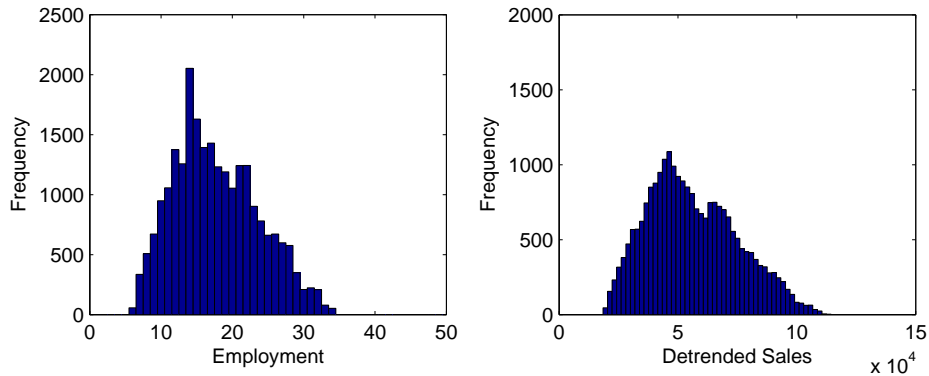


Figure 23: Distributions of monthly firm size (100 firms, 20 years): Employees (left) and detrended sales (right).

Workers, on the other hand, (1) are modeled with identical labor supply, so there are no individual differences in capabilities, human capital, specialization or productivity. (2) Household agents are infinitely living. (3) The distribution of firm shares is fixed from the beginning, so there is no market to trade shares and no market determining asset prices. (4) Savings and asset prices are not subject to agent behavior, for example asset prices inflation might increase household consumption. (5) ACE modeling permits investigation of topics related to wealth and income distribution. However, the current model specification is not calibrated with respect to actual distributions.

Lorenz (1905) curves are depicted in figure 24, which are useful to visualize the concentration of wealth (left) and income (right) in the population. It was measured at the end of the period presented throughout the diagrams above. Starting from an initially equal wealth distribution, even after more than 100 years wealth inequality did not increase to extreme levels. Moreover, the income distribution is almost egalitarian. The reason is that households are modeled homogeneous with respect to the value of individual labor supply, which renders wages between households almost identical. Since about 80% of national income consist of wages, the income distribution would be quite close to equality, even if the wealth distribution reflected a maximum level of inequality. Moreover, the modeled income tax applies equally to all levels of wage and capital income. It appears to be a fruitful area of future research to apply realistic patterns for a wealth and income distribution.

Regarding sensitivity of model parameters, it should be distinguished between two different goals: Stability and establishment of business cycles. First, in order to render the simulated economy somewhat stable, hyper inflation or a deflationary depression with increasing debt

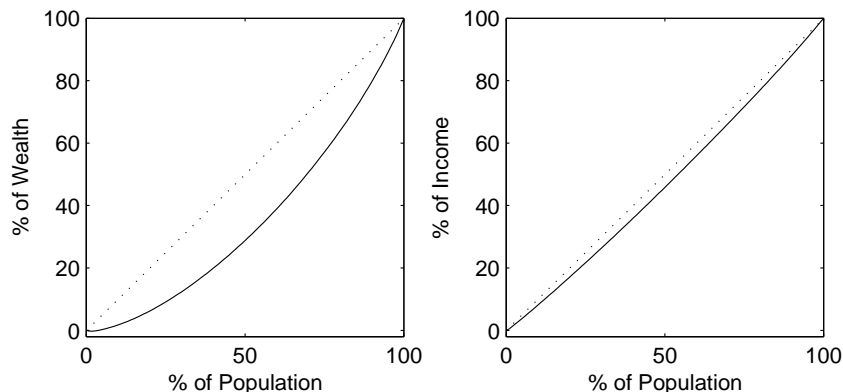


Figure 24: Lorenz curve of wealth (left) and income distribution (right).

burdens should be avoided. To this end, some parameters are crucial: (1) Model parameters of the consumption function have to be chosen, so that the sum of consumption and investment is sufficiently utilizing firm capacities. Aggregate demand needs to be in a range that allows the central bank to stabilize aggregate demand at a level, that corresponds to firms' target capacity utilization. (2) Another important parameter is λ_π , which allows the central bank to directly influence expected inflation. In the current setting it is 0.1, however, if it is 0, adaptive inflation expectations become much more likely to destabilize the economy in a deflationary or inflationary spiral. (3) Upper and lower boundaries for acceptable firm utilization levels should not be too narrow. Otherwise, employment becomes volatile with small variations in aggregate demand. This becomes critical, when fired workers reduce consumption, so that aggregate demand is lowered even more, thus triggering firing decisions of other firms. (4) Finally, limited variability of firm investment is a desired property, but it should also remain reliably stable. If, for example, a firm tried to invest as much to directly “jump” to the target capital stock in a single month, aggregate investment might become extremely volatile. In that situation, firms might suddenly be unable to satisfy demand on aggregate scale, rendering the central bank unable to influence aggregate output sufficiently.

Secondly, the establishment of a business cycle pattern as presented in this article is quite sensible to model parameters determining firm investment. Small variations may influence the shape and intensity of endogenously created business cycles strongly, with the possible result, that variability vanishes almost completely. Among the most important parameters for emergence of business cycles are: (1) Parameter $a_{\Delta h}$ which determines the dependence of investment on changes in profits is particularly critical. (2) Parameter a_h which determines the degree to which investment is influenced by past profits. (3) The depreciation rate, which determines the level of gross investment as a major component of aggregate demand. (4) The frequency of business cycles is strongly affected by investment inertia λ_I . However, seemingly unrelated parameters in their interplay may also influence the pattern to some extent. The presented set of parameter values was determined in a process of trial and error by using common-sensical reasoning. It is a characteristic feature of agent-based computational models with a considerable degree of complexity, that aggregate dynamic model behavior can not be traced back to a single

parameter. Therefore, it is a characteristic of ACE models that can hardly be displaced.

4. Conclusion

“Yet another model” might be a comprehensible first comment on a newly developed macro model. There have been numerous attempts to establish new models and new variants of existing models. These models differ with respect to the underlying paradigm, thus highlighting different mechanisms or key variables. Surely it constitutes a sound request to ask whether a new model actually contributes to this large reservoir of ideas and elaborate model constructions.

First, agent-based computational economics is a fairly new and yet densely populated field of research, compared to widespread paradigms like general equilibrium models. The mere number of economists employing this way of model-building is limited, so chances are that yet undiscovered ideas emerge when conducting research off the beaten track. Secondly, the presented model combines modeling of economic growth and business cycle dynamics in a comprehensive approach, where even unemployment is modeled explicitly, instead of simply relying on the link of output and employment. Usually, growth and business cycle dynamics are analyzed separately, although they are related closely. Thirdly, the newly presented model is capable of incorporating elements and ideas from different perspectives and economic schools. It encompasses *neoclassical* technology and investment decisions; *Keynesian* ideas enter in multiple ways such as demand-led output determination, involuntary unemployment, income-dependent consumption and business cycles, that are created and reinforced by investment activity as the main engine of market economies. Monetary policy is conducted in accordance with the *Taylor* rule. Moreover, it is designed as a *monetary* model with double-entry bookkeeping, allowing for close supervision of monetary flows between agents and groups of agents.

The model simulates households and firms as artificial objects in a computer simulation, who interact according to behavioral rules on a goods market and on a labor market. A state provides for unemployment benefits and collects taxes, a central bank adjusts the nominal interest rate in order to stabilize inflation close to a target level. Firm investment is modeled such that it is reinforced by rising profit, whereas firm profit in turn depends on aggregate investment activity. This way, a virtuous circle (or a vicious circle) in the interplay of investment and profit generates recurrent waves of increasing and declining economic activity on aggregate level. Business cycles are created endogenously from investment activity, whose fluctuations are transmitted to firm demand, capacity utilization, employment and price setting behavior. Model parameters are calibrated in order to mimic reasonable macroeconomic proportions like cycle length, the capital-output ratio and the wage share.

As it turns out, the model is capable to generate *Goodwin* cycles and a well-behaved *Phillips* curve relationship. After a monetary policy shock with a higher inflation target, the Phillips curve is simply shifted upward, so money neutrality holds in the long run. Demand stabilization policy is shown to dampen business cycles considerably, while on the contrary, a pro-cyclical fiscal regime is characterized by business cycles of larger amplitude and cycle length.

In addition, it appeals to common-sense, when modeled firm agents resemble important characteristics of real-world firms. Decisions are constructed to actually depend on the amount of goods or services, that a firm is able to sell. This explicit demand-led property of firm behavior is absent in many other models. As a consequence, production decisions are generally not limited by restricted availability of resources or rising marginal cost of production, which is also counter-factually assumed in many models (e.g. *Lindbeck and Snower, 1994*). Instead, modeled firms are restricted by demand and would be glad to sell as many units as possible. As *Sraffa (1926)* puts it: “*The chief obstacle against which [business men] have to contend when they want gradually to increase their production does not lie in the cost of production – which, indeed, generally favors them in that direction – but of the difficulty of selling the larger quantity of goods without reducing the price [...]*”

Avenues for future research are plenty in the yet densely populated world of ACE modeling. For example, starting from this presented model, imposing a more realistic distribution of income and wealth among households would be required to investigate suppositions about relationships of certain macro variables with measures of wealth concentration. Another example would be the design of herd behavior among firms concerning economic outlook, combined with firm investment depending on this level of sentiment. Waves of high and low average sentiment index values might thus be transmitted to real variables, so that business cycles might emerge from a process of social opinion formation (*Lux and Stolzenburg, 2011*).

References

- [1] Ackerman, F. (2002). Still dead after all these years: interpreting the failure of general equilibrium theory. *Journal of Economic Methodology* 9(2), 119–139.
- [2] Alesina, A., Rodrick, D. (1994). Distributive Politics and Economic Growth. *Quarterly Journal of Economics* 109(2), 465–90.
- [3] Barro, R.J., Sala-i-Martin, X. (2003). *Economic Growth*. MIT Press, ed.2, vol.1.
- [4] Blinder, A. (1999), *Central Banking in Theory and Practice*. MIT Press Cambridge.
- [5] Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics*, No. 12(3), pp. 383–398.
- [6] Chiarella, C., Flaschel, P., Franke, R. (2011). *Foundations for a Disequilibrium Theory of the Business Cycle*. Cambridge University Press.
- [7] Clarida, R., Gali, J., Gertler, M. (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature* 37(2), 1661–1707.
- [8] Colander, D., Föllmer, H., Haas, A., Goldberg, M., Juselius, K., Kirman, A., Lux, T., Sloth, B. (2009). The financial crisis and the systemic failure of the economics profession. *Critical Review* 21(2–3), 249–267.
- [9] De Long, J.B., Summers, L.H. (1988). *Brookings Papers on Economic Activity* 19(2), 433–494.
- [10] Delli Gatti, D., Gaffeo, E., Gallegati, M. (2010). Complex agent-based macroeconomics: a manifesto for a new paradigm. *Journal of Economic Interaction and Coordination* 5(2), 111–135.
- [11] Dobb, M. (1973). *Theories of value and distribution since Adam Smith*. Cambridge University Press.
- [12] Dosi, G., Fagiolo, G., Roventini, A. (2008). The microfoundations of business cycles: an evolutionary, multi-agent model. *Journal of Evolutionary Economics* 18, 413–432.
- [13] Dynan, K.E. Skinner, J. and Zeldes, S.P. (2004). Do the Rich Save More? *Journal of Political Economy* 112(2), 397–444.
- [14] Benchimol, J., Fourçans, A. (2012). Money and risk in a DSGE framework: A Bayesian application to the Eurozone. *Journal of Macroeconomics* 34, 95–111.
- [15] Gaffeo, E., Delli Gatti, D., Desiderio, S., Gallegati, M. (2008). Adaptive microfoundations for emergent macroeconomics. *Eastern Economic Journal* 34 (4), pp. 441–463.

- [16] Galí, J., Gertler, M., López-Salido, J.D. (2007). Markups, Gaps, and the Welfare Costs of Business Fluctuations. *Review of Economics and Statistics* 89(1), 44–59.
- [17] Galor, O. (2009). *Inequality and Economic Development*. Edward Elgar Publishing Limited, Cheltenham.
- [18] Goodwin, R.M. (1967). A Growth Cycle, in C.H. Feinstein (edt.): *Socialism, Capitalism and Economic Growth*. Cambridge.
- [19] Hein, E., Schoder, C. (2011). Interest rates, distribution and capital accumulation – A post-Kaleckian perspective on the US and Germany. *International Review of Applied Economics* 25(6), 693–723.
- [20] Kalecki, M. (1937). A Theory of the Business Cycle, *Review of Economic Studies* 4, 77–97. Reprinted in: Kalecki, M. (1971), *Selected Essays on the Dynamics of the Capitalist Economy. 1933-1970*. Cambridge University Press.
- [21] Keynes, J.M. (1936). *The General Theory of Employment, Interest and Money*, London (reprinted 2007).
- [22] Kirman, A.P. (1992). Whom or what does the representative individual represent? *Journal of Economic Perspectives* 6(2), pp. 117–136.
- [23] Kirman, A.P. (2006). Demand theory and general equilibrium: from explanation to introspection, a journey down the wrong road. *History of Political Economy* 38, 246–280.
- [24] Korotayev, Andrey V., Tsirel, Sergey V. (2010). A Spectral Analysis of World GDP Dynamics: Kondratieff Waves, Kuznets Swings, Juglar and Kitchin Cycles in Global Economic Development, and the 2008–2009 Economic Crisis. *Structure and Dynamics* 4(1), 3–57.
- [25] Kydland, F., and E. C. Prescott (1977). Rules Rather than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy*, 473–492.
- [26] Lengnick, M. (2013). Agent-based macroeconomics: A baseline model. *Journal of Economic Behavior and Organization* 86, pp. 102–120.
- [27] Lindbeck, A., Snower, D.J. (1993). How are Product Demand Changes Transmitted to the Labor Market? *Economic Journal* 104 (423), 386–398.
- [28] Lorenz, M.O. (1905). Methods of measuring the concentration of wealth. *Publications of the American Statistical Association* 9(70), 209–219.
- [29] Lucas, R. (1976). Econometric Policy Evaluation: A Critique. In: Brunner, K.; Meltzer, A. (edt.) *The Phillips Curve and Labor Markets*. Carnegie-Rochester Conference Series on Public Policy 1, 19–46.
- [30] Lucas, Jr., R.E. (2003). Macroeconomic priorities, *American Economic Review* 93(1), 1–14.

- [31] Lux, T., Stolzenburg, U. (2011). Identification of a Core-Periphery Structure Among Participants of a Business Climate Survey. *European Physical Journal B* 84, 521–533.
- [32] Madsen, J.B., Mishra, V. and Smyth, R. (2012). Is the Output-Capital Ratio Constant in the Very Long Run? *The Manchester School* 80(2), pp. 210–236.
- [33] Mankiw, N.G. (2000). *Makroökonomik*. 4th ed., Schäffer-Poeschel, Stuttgart.
- [34] Mankiw, N.G. (2006), The Macroeconomist as Scientist and Engineer, *Journal of Economic Perspectives* 20(4), 29–46.
- [35] Oeffner, M., (2008). Agent-Based Keynesian Macroeconomics - An Evolutionary Model Embedded in an Agent-Based Computer Simulation, MPRA Paper 18199, University Library of Munich.
- [36] Page, S.E. (2008). "Agent-based models," *The New Palgrave Dictionary of Economics*, 2nd Edition.
 Patinkin, D. (1987). Neutrality of money, *The New Palgrave: A Dictionary of Economics* v.3, 639–644.
- [37] Solow, R.M. (1956), A contribution to the Theory of Economic Growth, *Quarterly Journal of Economics* 70, 5–94.
- [38] Solow, R. (2010). Building a Science of Economics for the Real World. Prepared Statement of Robert Solow, Professor Emeritus, MIT, to the House Committee on Science and Technology, Subcommittee on Investigations and Oversight: July 20, 2010. Link: <http://www2.econ.iastate.edu/classes/econ502/tesfatsion/Solow.StateOfMacro.CongressionalTestimony.July2010.pdf>
- [39] Schelling, T.C. (1969). Models of segregation. *American Economic Review* 59(2), 488–493.
- [40] Sims, C.A., (1980). Macroeconomics and reality. *Econometrica* 48, 1–48.
- [41] Sraffa, P. (1926). The Law of Returns Under Competitive Conditions. *Economic Journal* 40, S.538–550.
- [42] Stock, J., Watson, M. (1999). Business cycle fluctuations in U.S. macroeconomic time series. In: Taylor, J., Woodford, M. (eds.). *Handbook of macroeconomics*. Elsevier Science, Amsterdam.
- [43] Swan, T.W. (1956). Economic Growth and Capital Accumulation. *Economic Record* 32 (2): 334–361.
- [44] Syll, L.P. (2012). Rational expectations - a fallacious foundation for macroeconomics in a non-ergodic world. *real-world economics review* 62, 34–50.

- [45] Taylor, J.B. (1993). Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.
- [46] Tesfatsion, L. (2003). Agent-based Computational Economics: Modeling Economies as Complex Adaptive Systems. *Information Sciences* 149(4), 262–268.
- [47] Woodford, M. (2003). *Interest and prices: Foundations of a theory of monetary policy*. Princeton, New Jersey: Princeton University Press.

A. Appendix

A.1. List of Symbols and Model Parameters

Listed symbols are split into four groups: Subscripts, model parameters, real variables (measured, for example, in natural units or per cent) and monetary variables (measured in currency units).

Symbol	Type	Unit/Value	Description
i	subscript	#	Firm number
h	subscript	#	Household number
t	subscript	#	Month
a_h	Parameter	7	Dependence of firm investment on profit rate
$a_{\Delta h}$	Parameter	5	Dependence of firm investment on change in profit rate
a_u	Parameter	0.03	Dependence of firm wage setting on utilization
a_m	Parameter	0.05	Dependence of firm wage setting on price markup
α	Parameter	0.2	Cobb-Douglas capital exponent
c	Parameter	0.85	Marginal rate of consumption
g_A	Parameter	(annually) 1.2%	Technology growth rate
γ_p	Parameter	5	Price competition intensity
γ_r	Parameter	0.5	Restriction aversion
γ_w	Parameter	2.5	Wage competition intensity
λ_C	Parameter	0.9	Consumption inertia
λ_I	Parameter	0.9	Investment inertia
λ_π	Parameter	0.1	central bank credibility (exp. inflation)
L^{st}	Parameter	200	Public Employment (fixed)
m^*	Parameter	60%	Target price markup over unit wage cost
N^{hh}	Parameter	2,000	Number of households
N^{fi}	Parameter	100	Number of firms
p_p	Parameter	25%	Probability to search for a cheaper supplying firm
p_r	Parameter	25%	Probability to reconsider 1 connection to restricting firm(s)
p_s	Parameter	2%	Probability to replace 1 connection randomly
π^*	Parameter	(annually) 1.2%	Inflation target
ρ	Parameter	(annually) 9.6%	Capital stock depreciation rate
σ_p	Parameter	0.14	Standard deviation of bell curve (price setting behavior)
T	Parameter	3,000	Number of simulated months
T^π	Parameter	24	Backward-looking months of adaptive inflation expectation
T^u	Parameter	6	Backward-looking months of short run utilization average
T^w	Parameter	6	Backward-looking months of employment target fulfillment
U^*	Parameter	85%	Firm target utilization
U^{low}	Parameter	78%	Firm lower bound utilization
U^{up}	Parameter	91%	Firm upper bound utilization
A_t	Variable	real number	Technology parameter (growing)
C_t	Variable	goods	Aggregate real consumption
$C_{t,h}$	Variable	goods	Real household consumption
\bar{C}_t	Variable	goods	Autonomous real household consumption
dur	Variable	#	Wage contract duration (month number)
$DP_{t,i}$	Variable	$\in \{-1, 0, 1\}$	Decision target price change (Price setting behavior)
$DW_{t,i}$	Variable	$\in \{-1, 0, 1\}$	Decision target wage change (Wage setting behavior)
ε_p	RV	$\in (0, 0.015)$	Stochastic price change

Continued on next page

Table – *Continued*

Symbol	Type	Unit/Value	Description
ε_w	RV	$\in (0, 0.01)$	Stochastic wage change
i_t	Variable	%	Nominal interest rate
I_t	Variable	goods	Aggregate real gross investment
$I_{t,i}$	Variable	goods	Firm real gross investment
$I_{t-1,i}^{net}$	Variable	goods	Firm real net investment
$K_{t,i}$	Variable	goods	Firm real capital stock
$K_{t,i}^*$	Variable	goods	Firm target capital stock
$L_{t,i}$	Variable	#	Firm employees
$L_{t,i}^*$	Variable	#	Firm employment target
$m_{t,i}$	Variable	%	Firm markup over unit wage cost
ν	RV	$\in \{0, 1, \dots, 4\}$	Stochastic wage contract duration (+10)
π_t	Variable	%	Inflation rate during last 12 months
π_t^e	Variable	%	Expected inflation rate
π_t^m	Variable	%	Monthly inflation rate (annualized)
r_t	Variable	%	Expected real interest rate
r^*	Variable	%	Long run real interest rate
RD_t	Variable	%	Restricted demand of household
$RNI_{t,h}$	Variable	goods	Real net disposable household income
$RoC_{t,i}$	Variable	%	Profit rate (Return on capital stock)
$\overline{RoC}_{t,i}$	Variable	%	Profit rate (RoC) during last 12 months
$RoE_{t,i}$	Variable	%	Return on equity
τ_t	Variable	%	Tax rate in default fiscal policy regime “Automatic Stabil.”
τ_t^{Def}	Variable	%	Tax rate in fiscal policy regime “Deficit spending”
τ_t^{Pro}	Variable	%	Tax rate in fiscal policy regime “Pro-cyclical”
$U_{t,i}$	Variable	%	Firm utilization
$\overline{U}_{t,i}$	Variable	%	Latest firm utilization (short-run average)
U_t	Variable	%	Aggregate utilization
UP_t	Variable	#	Unemployed persons
$Y_{t,i}^c$	Variable	goods	Firm real production capacity
$Y_{t,i}^{sales}$	Variable	goods	Firm real sales
Y_t^{sales}	Variable	goods	Firm turnover / Aggregate sales
$nC_{t,h}$	Variable	currency	Nominal household consumption expenditure
nFS_t	Variable	currency	Fiscal surplus / deficit
$nI_{t,i}$	Variable	currency	Firm nominal investment expenditure
$nInc_{t,h}^{prim}$	Variable	currency	Primary household income
$nInc_{t,h}^{net}$	Variable	currency	Net household income (secondary)
nM_t	Variable	currency	Aggregate transaction money
nM_t^{hh}	Variable	currency	Aggregate household sector money
$nM_{t,h}^{hh}$	Variable	currency	Household money
nM_t^{fi}	Variable	currency	Aggregate firm sector money.
$nM_{t,i}^{fi}$	Variable	currency	Firm money
nM_t^{st}	Variable	currency	State money
$nMC_{t,i}$	Variable	currency	Marginal cost of production (1 good)
$nMPL_{t,i}$	Variable	currency	Marginal productivity of labor
$nO_{t,i}$	Variable	currency	Firm nominal operative result
nO_t	Variable	currency	Aggregate firm nominal operative result
$nP_{t,i}$	Variable	currency	Firm price

Continued on next page

Table – *Continued*

Symbol	Type	Unit/Value	Description
$nP_{t,i}^*$	Variable	currency	Firm target price
nPI_t	Variable	currency	Price index
$nPI_{t,h}^C$	Variable	currency	Average price of supplying firms (HH consumption)
$nPI_{t,i}^K$	Variable	currency	Average price of supplying firms (firm investment)
$n\Pi_{t,i}$	Variable	currency	Realized firm profit
$n\Pi_{t,i}^{hyp}$	Variable	currency	Hypothetical firm profit (for optimal investment)
$n\Pi_{t,i}^{kept}$	Variable	currency	Accumulated kept-in part of firm profit
$n\Pi_{t,i}^{paid}$	Variable	currency	Firm profit paid-out to shareholders
$nS_{t,h}$	Variable	currency	Nominal household savings
nS_t	Variable	currency	Aggregate nominal household savings
nSC_t^{hh}	Variable	currency	Aggregate household sector savings
$nSC_{t,h}^{hh}$	Variable	currency	Household Savings/Credit account
nSC_t^{hh}	Variable	currency	Aggregate firm sector savings (debt)
$nSC_{t,h}^{hh}$	Variable	currency	Firm Savings/Credit account
nSG_t	Variable	currency	Seignorage gain (public revenue)
$nTax_t$	Variable	currency	Tax revenue
$nW_{t,i}$	Variable	currency	Firm wage
$nW_{t,i}^*$	Variable	currency	Firm target wage
nWI_t	Variable	currency	Wage index
nUB_t	Variable	currency	Unemployment benefit

A.2. National Accounting

The monetary ACE model features double-entry bookkeeping, that is, each movement of an amount of money has a counterpart on another agents' bank account. Agent-Based model simulation allows a closer look on aggregate and disaggregate movement of economic variables. By defining aggregate monetary flows of sectors, macroeconomic relations are derived that allow for economic policy consequences.

1. Let nominal household savings $nSC_{t,h}$ be calculated as monthly net income of a household $nInc_{t,h}^{net}$, subtracted by consumption expenditures $nC_{t,h}$, which equals the sum of changes in households money account $\Delta nM_{t,h}$ and savings/credit account $\Delta nSC_{t,h}$, both individually and on aggregate.³³

$$nS_{t,h} = nInc_{t,h} - nC_{t,h} = \Delta nM_{t,h} + \Delta nSC_{t,h} \quad (41)$$

$$\sum_{h=1}^{N^{hh}} nS_{t,h} = \sum_{h=1}^{N^{hh}} (nInc_{t,h} - nC_{t,h}) = \sum_{h=1}^{N^{hh}} (\Delta nM_{t,h} + \Delta nSC_{t,h})$$

$$nS_t = \Delta nM_t^{hh} + \Delta nSC_t^{hh} \quad (42)$$

³³ Due to random choice of trading partners, nominal consumption expenditures are only translated approximately to real consumption: $nC_{t,h} \approx PI_{t,h}^c \cdot C_{t,h}$.

2. Let nominal firm operating result $nO_{t,i}$ be calculated as firm nominal turnover, subtracted by all monetary outflows (nominal wages, interest payments, gross investment expenditure³⁴ and profit payout):

$$nO_{t,i} = nP_{t,i} \cdot Y_{t,i}^{sales} - nW_{t,i} \cdot L_{t,i} + i_t \cdot nSC_{t,i}^{fi} - n\Pi_{t,i}^{paid} - nI_{t,i}, \quad (43)$$

$nO_{t,i}$ equals changes in firms money and savings accounts, both individually and on aggregate.

$$nO_{t,i} = \Delta nM_{t,i} + \Delta nSC_{t,i} \quad (44)$$

$$\sum_{i=1}^{N^{fi}} (nO_{t,i}) = \sum_{i=1}^{N^{fi}} (\Delta nM_{t,i} + \Delta nSC_{t,i})$$

$$nO_t = \Delta nM_t^{fi} + \Delta nSC_t^{fi} \quad (45)$$

3. The fiscal surplus/deficit is public revenue minus cost for unemployment benefits, public employment $L_t^{st} = 200$ and interest on public debt:

$$nFS_t = \underbrace{nTax_t + nSG_t}_{\text{revenue}} - \underbrace{UP_t \cdot nUB_t - L_t^{st} \cdot nWI_{t-1} + i_t \cdot nSC_t^{st}}_{\text{public cost}} \quad (46)$$

with tax revenue $nTax_t$, seignorage gain nSG_t , public wage nWI_{t-1} and UP_t the number of unemployed households. The fiscal surplus also equals the change in the states' wealth position (public debt):

$$nFS_t = \Delta nSC_{t,i}^{st} + \underbrace{\Delta M_{t,i}^{st}}_{=0} \quad (47)$$

As introduced in section 2.2, positive and negative monetary balances of all agents sum up to zero (deposit equals debt) at all times:

$$0 = \underbrace{nM_t^{hh} + nM_t^{fi}}_{\text{transaction money}} + \underbrace{nSC_t^{hh}}_{\text{Savings}} + \underbrace{nSC_t^{fi} + nSC_t^{st}}_{\text{Credit}} \quad (48)$$

By taking differences and rearranging, we have:

$$0 = \underbrace{\Delta nM_t^{hh} + \Delta nSC_t^{hh}}_{\text{nominal HH savings}} + \underbrace{\Delta nM_t^{fi} + \Delta nSC_t^{fi}}_{\text{firm sector operating result}} + \underbrace{\Delta nSC_{t,i}^{st}}_{\text{fiscal deficit}} \quad (49)$$

Therefore, the sum of nominal household savings, nominal firm operating result, and state surplus/deficit is zero, as the running simulation confirms every month:

$$0 = nS_t + nO_t + nFS_t \quad (50)$$

That is, improvement of aggregate financial position of one sector³⁵ is possible if and only if someone else's financial position is worsened, given a closed economy as is modeled here. For

³⁴Translation of nominal investment expenditures to real investment holds only approximately due to random choice of trading partners: $nI_{t,i} \approx PI_{t,i}^K \cdot I_{t,h}$.

³⁵Sum of all money and savings/credit accounts.

example, if the household sector increases savings, either firms or the state will have to take a higher debt burden with mathematical certainty. Put differently, if firms on aggregate and the state both try to reduce their nominal debt burden, they will only succeed if households on aggregate are willing reduce the aggregate stock of savings.