

Lang, Matthias

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## Legal uncertainty: A selective deterrent

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**Legal Uncertainty  
– a Selective Deterrent**

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November 2014, initial version February 2010

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## Abstract

I show that legal uncertainty, i.e., uncertainty about the legality of a specific action, has positive welfare effects. Legal uncertainty works as a screening device provided that the threshold of legality is uncertain. The uncertainty discourages controversial actions, while it encourages socially beneficial actions. Legal uncertainty is a selective deterrent, because the uncertainty changes the probability of being convicted in opposite directions. Hence, in designing optimal rules there is no reason to avoid legal uncertainty at all costs. For example, the positive effect of legal uncertainty influences the balance between per-se rules and rules of reason in competition law.

**JEL classifications:** D8, K2, K4, L5

**Keywords:** Enforcement, Deterrence, Legal Uncertainty, Rules of Reason, Regulation, Asymmetric Information

## 1 Introduction

Given the complexity of many legal procedures, e.g., in competition law, legal uncertainty is a major issue. With legal uncertainty, I refer here to circumstances where it is not clear ex ante whether a specific action is legal.<sup>1</sup> Previous literature has shown that legal uncertainty might deter the wrong actions – over-detering socially beneficial actions, while under-detering socially detrimental ones.<sup>2</sup> This paper shows that legal uncertainty inherent in a legal rule can advance the policymaker’s objectives. Legal uncertainty allows mitigating the restrictions of the enforcement authority, in particular, its ignorance of individuals’ private information. The enforcement authority uses legal uncertainty as a screening device. Therefore, some legal uncertainty increases welfare. Consequently,

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\*Humboldt University Berlin, Institute of Economic Theory I, Germany, lang@uni-bonn.de.

<sup>1</sup>This is similar to the notion of D’Amato (1983).

<sup>2</sup>See for instance Calfee and Craswell (1984), Craswell and Calfee (1986), Polinsky and Shavell (1989), Schinkel and Tuinstra (2006) or Katsoulacos and Ulph (2013).

policymakers should not dedicate themselves to eliminating legal uncertainty from legal rules. When considering a new or a modified rule, there are more important considerations than the inherent amount of legal uncertainty. Legal uncertainty could even make a rule more selective and increase social welfare.

Suppose that a specific action increases or decreases welfare depending on the circumstances and the enforcement authority cannot perfectly distinguish between these circumstances. The enforcement authority chooses the optimal policy by setting a threshold on the aspects observable to the authority. Then individuals decide whether or not to pursue the controversial action. Finally, the enforcement authority imposes fines on individuals who are above the threshold and pursue the action. With legal uncertainty, some individuals cannot anticipate with certainty whether they are above or below the threshold according to the estimates of the enforcement authority. In particular, legal uncertainty increases the probability of a conviction for individuals below the threshold and decreases the probability of a conviction for individuals above the threshold. This uncertainty about the threshold of legality deters individuals with few gains from the action. Individuals with large gains still pursue the action, especially if they are near the legal threshold. Hence, this uncertainty allows screening individuals according to unobservable characteristics. Therefore legal uncertainty makes the rules more selective and increases welfare. If individuals' private benefits do not enter welfare, e.g., in a consumer-welfare standard, welfare still increases, as legal uncertainty raises probabilities of conviction and reduces enforcement costs.

There are different reasons for this kind of legal uncertainty. For example 'it is difficult to predict ... how an antitrust court will distinguish between 'predatory' and 'competitive' price cuts' according to Calfee and Craswell (1984, p. 968). Alternative reasons are the existence of different procedures, measurement errors by the enforcement authority, different assessments of, e.g., efficiency defenses or uncertainty about what kind of evidence will be allowed. Consider two examples. First, vertical restraints, like resale price maintenance or exclusive dealings, are prohibited in the European Union under Article 101 (TFEU), formerly Article 81 (EC).<sup>3</sup> There is a Block Exemption Regulation, however, so that this rule does not apply if the market shares of the involved parties are below 30%. Although the European Commission gives guidelines how the relevant market shares are to be determined, it is extremely difficult to predict correctly the market share determined by the competition authorities. The causes are discrepancies in the definition of the relevant market, information asymmetries or imprecision in the measurement of sales, and other factors. This creates the kind of uncertainty analyzed in the model.

The second example is the case of Microsoft tying its operating system with additional software, in particular, a web browser and a media player.<sup>4</sup> In both instances the European

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<sup>3</sup>See European Commission (2010a) and Regulation No. 330/2010 for details.

<sup>4</sup>These are the cases COMP/39.530 and T-201/04 Microsoft vs. Commission. The commission summarizes its findings in the former case in European Commission (2010b).

Commission found an abuse of a dominant market position under Article 102 (TFEU), formerly Article 82 (EC). Think of a scale beginning with products where the bundling with the operating system is socially beneficial, as the integration allows for new features or higher performance and competing products are non-existent. On the other end of the scale are products where the bundling yields few or no efficiency gains, but competition is harmed considerably. While there is legal certainty on both ends of the scale, in the middle it is very difficult to exclude legal uncertainty completely. According to the model in this paper, this legal uncertainty could be socially beneficial.<sup>5</sup>

A caveat applies here. Although this model points out positive effects of legal uncertainty, welfare effects need not be monotone in the amount of uncertainty. Furthermore, there may be negative effects of legal uncertainty that are not captured in our analysis.<sup>6</sup> Policymakers, however, might positively influence the effects of legal uncertainty and steer deterrence towards harmful behavior by complementing a general rule with specific exceptions, like safe harbors, or detailed information with respect to some procedural aspects.<sup>7</sup>

The effects of legal uncertainty discussed in this paper directly influence the trade-off between per-se rules and rules of reason in competition law. With per-se rules, some clearly specified actions, like, e.g., certain rebates or resale price maintenance, are prohibited. A rule of reason, on the other hand, judges an action as illegal whenever the action is used in an anticompetitive way. Thus, the test of legality is whether competition was promoted or hindered.<sup>8</sup> Therefore an action may be legal in some cases, but not in others, depending on its consequences. Hence, rules of reason typically imply a certain amount of legal uncertainty. Recently, there has been a major shift away from per-se rules — exemplified by the case *Leegin vs. PSKS*, as the court's decision allowed resale price maintenance if it does not impede competition.<sup>9</sup> Also competition authorities in the European Union aim to pursue a 'more economic approach'. This approach focuses more on the market effects of the action under consideration. An example is the discussion of the European Commission about the enforcement of Article 102 (TFEU), formerly Article 82 (EC).<sup>10</sup> Previous literature has argued that rules of reason allow differentiating competition law in a more selective way at the price of some inherent legal uncertainty, because firms sometimes do not know whether their conduct is legal. Katsoulacos and Ulph (2010, p. 3) summarize this issue as follows: the 'legal uncertainty induced by effects-based procedures [i.e., rules of reason] is harmful and should lead [the competition] authority to favor per-se

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<sup>5</sup>Although both examples are from Europe, the results of this paper are also valid for the United States. In the United States, however, courts have frequently interpreted legal uncertainty in favor of the investigated party, thereby reducing overall deterrence.

<sup>6</sup>For instance, legal uncertainty may reduce the possibility to control the enforcement authority, as it becomes more difficult to detect incompetent or corrupt behavior. In addition, legal uncertainty might result in socially wasteful expenses in safeguards and evidence production.

<sup>7</sup>Ahlborn et al. (2004) and Christiansen and Kerber (2006) propose such modified or structured rules of reason.

<sup>8</sup>Kaplow and Shapiro (2007, p. 54ff) provide a good discussion of rules of reason in antitrust.

<sup>9</sup>Supreme Court of U.S. '*Leegin Creative Leather Products, Inc. vs. PSKS, Inc.*' Decision No. 06-480, June 28th, 2007.

<sup>10</sup>Cf. European Commission (2009), European Commission (2005) or Gual et al. (2005).

procedures.’ This paper shows that the conclusion depends on the kind of legal uncertainty. Legal uncertainty could even improve the balance in favor of rules of reason.

The remainder of the paper is organized as follows. Section 2 discusses the relevant literature. Section 3 sets up the model. Section 4 analyzes the welfare effects of legal uncertainty. Section 5 shows that the main result is robust to the introduction of risk aversion, correlation, and endogenous fines. Finally, Section 6 contains the concluding remarks. All proofs are gathered in the appendix.

## 2 Related Literature

Polinsky and Shavell (2000) summarize the literature on optimal deterrence. This literature, like, for example, Polinsky and Shavell (1979), Shavell (1987) and Polinsky and Shavell (1991), focuses mainly on optimal sanctions. With respect to legal uncertainty, the conventional wisdom is that legal uncertainty reduces deterrence and makes it more difficult or impossible to achieve optimal deterrence.

Calfee and Craswell (1984) discuss the kind of legal uncertainty I consider here and Craswell and Calfee (1986) formalize it. In their model, however, there is no information asymmetry about the individual’s type and her action. Therefore, legal uncertainty only hinders implementation of the optimal threshold of legality and either causes too much or too little deterrence. I show that legal uncertainty is beneficial and has positive effects on welfare. Polinsky and Shavell (1989) confirm that legal uncertainty lowers deterrence, because expected sanctions are reduced and less suits are brought to court. Schinkel and Tuinstra (2006) apply this reasoning to competition law. They consider type I and type II errors of the enforcement authority demonstrating that both lower deterrence.

In a different approach, Kaplow (1995) assumes mutual ignorance about the nature of the considered action, because individuals do not know the exact rules and the enforcement authority does not know the specific circumstances of the individual. Therefore both parties have to invest if they want to get the missing information. Thus, Kaplow (1995) models the trade-off between compliance costs and selectivity of rules. He shows that compliance costs are often low, even for quite complex rules. There is no legal uncertainty, however, if an individual decides to invest in learning the rules.

The beneficial effects of legal uncertainty have appeared in different contexts. Choné and Linnemer (2008) study the effect of uncertain efficiency gains on merger control. They characterize the market structure and demand elasticities that make such uncertainty beneficial. Strausz (2011) points out that regulatory risk might be advantageous and studies the necessary market structures. Lang and Wambach (2013) show for insurance fraud that uncertainty about enforcement might have a beneficial deterrence effect. Furthermore, the deterrence effect of uncertainty is already used in tax enforcement. According to Reinganum and Wilde (1988, p. 794), the Internal Revenue Service (IRS) in the U.S. confirms that ‘one of the tools in the arsenal of the IRS which promotes voluntary compliance is the

uncertainty in the minds of the taxpayers.’ Cronshaw and Alm (1995), Snow and Warren (2005), Osofsky (2011), and Gergen (2011) discuss different aspects of legal uncertainty in tax enforcement. In a different realm, Harel and Segal (1999) discuss the effects of ambiguity on individuals in criminal law.

Legal uncertainty inherent in legal rules is closely related to the trade-off between rules of reason and per-se rules. Ehrlich and Posner (1974) discuss the advantages and disadvantages of having per-se rules replaced by rules of reason. Rules of reason could better distinguish beneficial from harmful actions, but provide less guidance for the concerned parties. Yet, they do not analyze the overall effects on welfare. Katsoulacos and Ulph (2009) model this trade-off in competition law. They characterize conditions, such that rules of reason are welfare-enhancing compared to per-se rules. A sequence of extensions of Katsoulacos and Ulph (2010, 2013) scrutinizes, in particular, the arising legal uncertainty by introducing a second dimension of uncertainty about the nature of the considered action similarly to my model. They find that the selectivity of a rule of reason often outweighs the losses due to the arising legal uncertainty. I concentrate on legal uncertainty and do not consider the comparison between per-se rules and rules of reason. My model allows varying continuously the legal uncertainty inherent in legal rules. I show that the uncertainty itself might increase welfare if the amount of uncertainty is sufficiently small.

Finally, there is a related literature on costly state verification. Besanko and Spulber (1989) use such a model to analyze optimal enforcement of antitrust laws, but do not touch on the issue of legal uncertainty.

### 3 Enforcement Model

An enforcement authority faces a continuum of risk-neutral individuals with mass one. The assumption of risk-neutral individuals seems most appropriate when firms are concerned. I will consider risk-averse individuals in Section 5.1. The authority’s objective is to maximize welfare, i.e., the sum of externalities and individuals’ private benefits, weighting the individuals’ private benefits by  $\alpha \in [0, 1]$ .<sup>11</sup> The enforcement authority sets its policy by choosing an enforcement parameter  $\hat{x} \in \mathbb{R}$ .  $\hat{x}$  captures the threshold of legality. In the first example of vertical restraints this is a specific market share of 30% for the block exemption. Every individual has the binary choice whether to take an action, like, e.g., speeding, a parking violation, or a red-light running violation or to abstain from it. Depending on her choice, I refer to an individual as active or deterred.

The pay-offs of the action depend on the individual’s type  $(x, b)$  that is two-dimensional. The individual knows her type  $(x, b)$  in both dimensions. The first dimension  $x$  captures the aspects that the enforcement authority can observe with its auditing technology. Returning to my examples from the introduction, this refers to the market structure, like,

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<sup>11</sup>Besanko and Spulber (1993) and Neven and Roller (2005) study the relative merits of different welfare standards.



for example, market shares in the case of vertical restraints. In the case of Microsoft,  $x$  denotes the kind of software added to the operation system and whether the integration is socially beneficial or harmful.  $x$  is drawn from a distribution  $F$  on  $\mathbb{R}$  with a twice differentiable density  $f(x) > 0$  for all  $x \in \mathbb{R}$ . The value  $b$  is unobservable by the enforcement authority. As  $b$  influences the decision of the individual, I call  $b$  the individual's private benefits, which are independently distributed with a distribution  $G$  on  $[0, \bar{b}]$  with  $\bar{b} > 0$ , a differentiable density  $g(b) > \epsilon > 0$  for all  $b \in [0, \bar{b}]$ , and a non-decreasing hazard rate  $g(b)/(1 - G(b))$ .<sup>12</sup> A non-decreasing hazard rate is a common assumption in screening settings. Many familiar distributions, like the uniform, the normal or the exponential distribution, satisfy this property.

Welfare remains unchanged if the individual takes no action. If an individual of type  $(x, b)$  takes the action, she generates private benefits of  $b$  and externalities of  $-e(x)$  given by the twice differentiable function  $e(\cdot)$ . Thus, weighted welfare changes by  $\alpha b - e(x)$ . The first dimension of the individual's type  $x$  is ordered in such a way that a higher  $x$  signifies higher social harm, i.e.,  $e'(x) > 0$ . For some types taking the action is socially beneficial and for some it is socially harmful. Hence, there is a  $\tilde{x} \in \mathbb{R}$ , such that  $e(x) < 0$  for all  $x < \tilde{x}$  and  $e(x) > 0$  for all  $x > \tilde{x}$ . There are many examples for actions with positive or negative externalities. Price reductions, e.g., might reflect lower costs or an attempt at predatory pricing. The same holds for bidding patterns in procurement contests or standardization efforts, which might have beneficial effects or be part of some collusive agreement in order to harm other market participants. In the case of vertical restraints, a simplification would be to consider only the market shares. If these are very low, the restraints do not harm other market participants,  $e(x) < 0$ . Vertical restraints, however, could be very harmful,  $e(x) > 0$ , if the firms involved dominate the respective markets. In the case of Microsoft,  $e(x) > 0$ , on the one hand, corresponds to implementing a web browser in order to acquire a dominant position in the browser market by abusing its dominance in the market for operating systems.  $e(x) < 0$ , on the other hand, corresponds to integrating new and socially beneficial features, like a basic firewall, touchscreen support or improved USB drivers.

As a benchmark consider the first-best policy, where the individual's type is observable and verifiable. In this case an individual of type  $(x, b)$  should be active, whenever  $\alpha b - e(x) \geq 0$  or, equivalently,  $b \geq e(x)/\alpha$ . Then only the individuals depicted in Figure 1 are active and take the action. In the model the enforcement authority cannot perfectly observe and verify the individuals' type. In particular, the enforcement authority can only use the observable aspects, i.e., the first dimension, of their type. It does so by setting a threshold of legality  $\hat{x}$ . If the enforcement authority finds individuals to have  $x$  below  $\hat{x}$ , their actions may well be socially efficient and therefore the enforcement authority allows

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<sup>12</sup>As Section 5.2 discusses, the model is robust to the introduction of correlation between  $b$  and  $x$ . As long as correlation is not perfect, the mechanism in this model works. With perfect correlation the enforcement authority could infer  $b$  from  $x$  and therefore does not need legal uncertainty as a screening device.

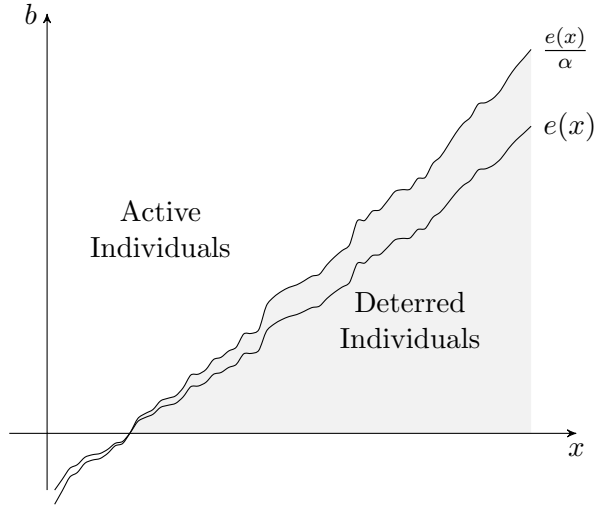


Figure 1: Active Individuals in the First-best Policy

the individuals to continue. Above  $\hat{x}$ , however, actions are judged as illegal and are prohibited. If the enforcement authority detects an individual violating this policy, it can make the individual pay a fine  $p$  with  $p > \bar{b}$ .<sup>13</sup> Yet for this purpose, the enforcement authority has to invest resources to produce evidence. The enforcement authority can make the observed  $x$  verifiable at costs  $\kappa$  with  $\kappa \leq (1 - \alpha)p$  and  $\kappa < p$ . The costs  $\kappa$  capture experts' testimonies, reports and other expenses to prove the enforcement authority's case. Alternatively, Polinsky and Shavell (2000) call  $\kappa$  the costs of imposing the fine. Figure 2 summarizes the timing of the model.

The auditing technology of the enforcement authority scrutinizes an exogenously given fraction  $a$  of all individuals with  $0 < a \leq \bar{b}/p$ . Yet, the auditing technology is imperfect. The enforcement authority does not learn the second dimension  $b$  of the type. In addition, it does not learn the first dimension  $x$  of the type exactly, but receives only a noisy signal  $x^M = x + \Delta\delta$  with a small  $\Delta \in [0, \infty)$  and  $\delta$  drawn from a distribution  $H$  on  $[-1, 1]$  with a differentiable density  $h(\cdot) > \epsilon > 0$ . In the case of vertical restraints this captures the difficulty in determining, whether the market share is 29% or 31%. With Microsoft the uncertainty might arise for products, like anti-virus software, where tying might offer great benefits, but also has the potential to harm other market participants considerably. This uncertainty about the threshold of legality or this measurement error is implied by the structure of the legal rules and is exogenous to the enforcement authority. Therefore the case without legal uncertainty, i.e.,  $\Delta = 0$ , serves only as a benchmark.

To sum up, the change in weighted welfare is

$$w(x, b) = \begin{cases} 0 & \text{if the individual } (x, b) \text{ is deterred} \\ \alpha b - e(x) & \text{if the individual } (x, b) \text{ is active and not fined} \\ \alpha b - e(x) + (1 - \alpha)p - \kappa & \text{if the individual } (x, b) \text{ is active and fined} \end{cases}$$

<sup>13</sup>The fine  $p$  is exogenous. Yet, making the fine endogenous does not change the model. The results in this paper just require a jump in the fine at  $\hat{x}$  which is optimal, even if the fine is completely endogenous. Section 5.3 discusses this case. For a discussion of the setting of fines in European competition law see European Commission (2006) and Wils (2007).

- At  $t = 0$ , the enforcement authority announces the policy  $\hat{x}$ .
- At  $t = 1$ , the individual's type  $(x, b)$  is realized and revealed to the individual.
- At  $t = 2$ , the individual chooses whether to take the action.
- At  $t = 3$ , the enforcement authority learns the signal  $x^M$  if an individual is audited.
- At  $t = 4$ , the individual has to pay the fine  $p$  if illegal behavior was detected and the enforcement authority pursues the claim at costs  $\kappa$ .

Figure 2: The Timing of the Model

with the weighted private benefits  $\alpha b$ , the externalities  $e(x)$ , the welfare effect  $(1 - \alpha)p$  of imposing a fine, and the enforcement costs  $\kappa$ . The individual's pay-offs are

$$\pi(x, b) = \begin{cases} 0 & \text{if the individual } (x, b) \text{ is deterred} \\ b & \text{if the individual } (x, b) \text{ is active and not fined} \\ b - p & \text{if the individual } (x, b) \text{ is active and fined.} \end{cases}$$

To guarantee an interior solution of the enforcement authority's optimization, another, more technical assumption is required:

$$e'(x) > \max \left\{ \frac{f'(x)}{f(x)} \left( \alpha \mathbb{E}(b|b \leq ap) - e(x) - \frac{1 - G(ap)}{G(ap)} a((1 - \alpha)p - \kappa) \right), \right. \quad (1) \\ \left. \frac{f'(x)}{f(x)} \left( \alpha ap - e(x) - \left( \frac{1 - G(ap)}{g(ap)} - ap \right) \frac{(1 - \alpha)p - \kappa}{p} \right), \right. \\ \left. \frac{f'(x)}{f(x)} \left( -e(x) - \frac{1}{g(0)} \frac{(1 - \alpha)p - \kappa}{p} \right) \right\}$$

for all  $x \in \mathbb{R}$ , which is equivalent to the slope of the externality function being sufficiently steep, such that the policy of the enforcement authority matters. It is easy to check that the second term is at least as big as the third term if and only if  $f'(x) \geq 0$ . Given the policy constraints of the enforcement authority, it usually cannot enforce the first-best policy. The next section shows that legal uncertainty allows mitigating the limitations of the enforcement authority.

## 4 Effects of Legal Uncertainty

First, I consider the individual's decision. The individual faces the fine  $p$  if she is caught by the enforcement authority taking the controversial action and the authority's signal  $x^M$  is above the policy threshold  $\hat{x}$ . Therefore the individual only takes the action if her private benefits  $b$  are high enough. Thus, there is a cut-off  $\hat{b}(x)$ , such that only individuals above  $\hat{b}(x)$  are active. The cut-off for the private benefit  $\hat{b}(x)$  varies with  $x$ . If the individual's type is low,  $x < \hat{x} - \Delta$ , the individual implements the action, as long as the private benefits are positive. Beginning at  $x = \hat{x} - \Delta$ , the cut-off  $\hat{b}(x)$  increases in  $x$  and equals

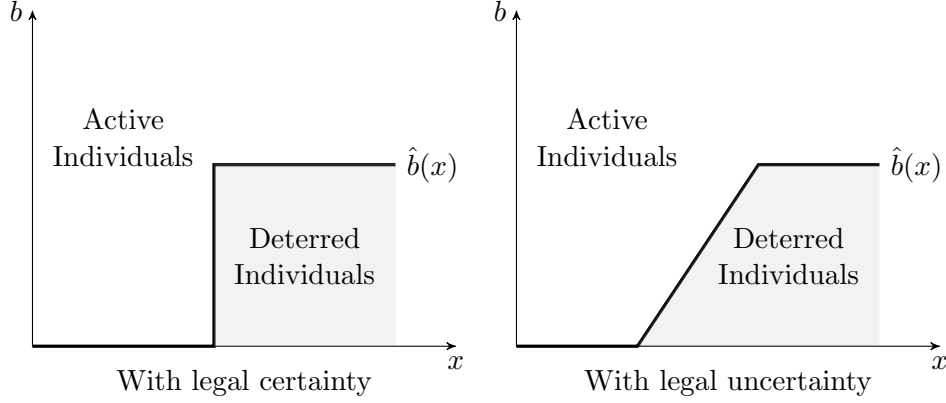


Figure 3: Changes in Active Individuals due to Legal Uncertainty

the probability of conviction multiplied by the penalty. Finally, for high types,  $x > \hat{x} + \Delta$ , the cut-off is constant and equals the expected penalty  $ap$ . Consequently, the cut-off is

$$\hat{b}(x) = \begin{cases} ap & \text{if } x > \hat{x} + \Delta \\ apH\left(\frac{x-\hat{x}}{\Delta}\right) & \text{if } \hat{x} - \Delta \leq x \leq \hat{x} + \Delta \\ 0 & \text{if } x < \hat{x} - \Delta. \end{cases} \quad (2)$$

Therefore legal uncertainty created by the imprecise measurement allows some screening of individuals. If they are close to the policy  $\hat{x}$ , individuals with low private benefits abstain from taking action for lower values of  $x$  than individuals with a high value of private benefits. If the legal rules would provide complete legal certainty, the measure of the enforcement authority would be perfect, i.e.,  $\Delta = 0$ , and the cut-off would be sharp. Then below the policy  $\hat{x}$ , all individuals take the action. Above  $\hat{x}$ , only those individuals with private benefits above the expected penalty  $ap$  will implement the action. Figure 3 depicts this pattern. The following lemma describes the effects of legal uncertainty on deterrence.

**Lemma 1.** *Below the policy  $\hat{x}$ , legal uncertainty (weakly) increases deterrence; above  $\hat{x}$ , legal uncertainty (weakly) decreases deterrence.*

The enforcement authority chooses the policy  $\hat{x}$  to maximize welfare  $W(\hat{x})$  which equals

$$\begin{aligned} & \int_{-\infty}^{\hat{x}-\Delta} \alpha \mathbb{E}(b) - e(x) dF(x) + \\ & + \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \left(1 - G(\hat{b}(x))\right) \left( \alpha \mathbb{E}(b|b \geq \hat{b}(x)) - e(x) + H\left(\frac{x-\hat{x}}{\Delta}\right)(p(1-\alpha) - \kappa)a \right) dF(x) + \\ & + \left(1 - G(ap)\right) \int_{\hat{x}+\Delta}^{\infty} \alpha \mathbb{E}(b|b \geq ap) - e(x) + (p(1-\alpha) - \kappa)a dF(x). \end{aligned} \quad (3)$$

The first term captures the region where the enforcement authority judges all actions as legal. In the intermediate region, there is legal uncertainty which decreases the probability of an individual taking the action while increasing its expected benefits. Finally, in the illegal region, activity is limited to the individuals with the highest private benefits.

Raising the policy threshold makes more individuals become active and raises the negative externalities. The optimal policy balances these additional externalities exactly with the expected private benefits and the reduction in enforcement costs.

**Lemma 2.** *With legal certainty,  $\Delta = 0$ , the optimal policy  $\hat{x}^N$  is determined by*

$$e(\hat{x}^N) = \alpha \mathbb{E}(b|b \leq ap) - \frac{1 - G(ap)}{G(ap)} a((1 - \alpha)p - \kappa). \quad (4)$$

With legal uncertainty, the same considerations apply in expectations.

**Proposition 1.** *With legal uncertainty,  $\Delta > 0$ , the optimal policy  $\hat{x}$  is determined by*

$$\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) h\left(\frac{x-\hat{x}}{\Delta}\right) \left( e(x) - \alpha \hat{b}(x) + \left( \frac{1 - G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1 - \alpha)p - \kappa}{p} \right) dF(x) = 0$$

Notice that only the intermediate region  $(x - \Delta, x + \Delta)$  matters for determining the optimal policy. Deterrence in this region is determined by the distribution  $G$  of the private benefits and the probability of being convicted by the enforcement authority that depends on the distribution  $H$  of the error term. While the equations in Lemma 2 and Proposition 1 differ, the optimal policies for small legal uncertainty and for legal certainty are closely related.

**Proposition 2.** *If legal uncertainty vanishes, the optimal policy  $\hat{x}$  converges to the optimal policy  $\hat{x}^N$  with legal certainty,  $\lim_{\Delta \searrow 0} \hat{x} = \hat{x}^N$ .*

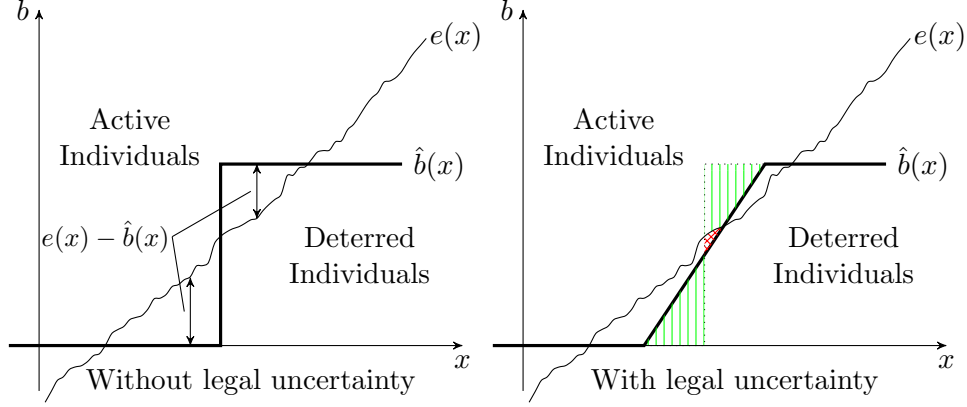
Now turn to the main result of this paper, namely the welfare effects of legal uncertainty. Legal uncertainty changes how many and which individuals are active as depicted in Figure 4. These changes increase welfare if the uncertainty is small.

**Theorem 1.** *Legal uncertainty increases welfare if the uncertainty is not too large.*

Consider the case  $\alpha = 1$  and  $\kappa = 0$  here, while postponing the general case to the appendix. The derivative of welfare with respect to legal uncertainty equals

$$\frac{\partial W(\hat{x})}{\partial \Delta} = \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} \left( e(x) - \hat{b}(x) \right) dF(x). \quad (5)$$

$\partial \hat{b}(x)/\partial \Delta$  captures the marginal influence of legal uncertainty on deterrence.  $e(x) - \hat{b}(x)$  captures the change in welfare if an individual of type  $(x, \hat{b}(x))$  becomes deterred. The densities  $g$  and  $f$  are less important here. The change in welfare  $e(x) - \hat{b}(x)$  decreases in  $x$  for sufficiently small legal uncertainty  $\Delta$  and  $x \in (\hat{x} - \Delta, \hat{x} + \Delta)$ . For small legal uncertainty, individuals close to the threshold are over- and under-deterred compared to the first-best. Legal uncertainty mitigates these deterrence problems. In particular, legal uncertainty makes individuals close to, but below the threshold of legality abstain from taking the action if their private benefits are low. In addition, legal uncertainty makes



On the left-hand side, actions in the absence of legal uncertainty are depicted and compared to the first-best. Introducing legal uncertainty on the right-hand side changes the implemented actions. The green striped areas show increases in welfare. The red checked areas show decreases in welfare. The change in welfare depends on  $e(x) - \hat{b}(x)$  as indicated by the arrows. The total change in welfare is positive.

Figure 4: Idea of the proof

individuals close to, but above the threshold of legality take the action if their private benefits are high. Both changes increase welfare as indicated by the green striped areas in Figure 4. Any potential losses in welfare as indicated by the red checked areas are small compared to the welfare gains. Formally, this comparison follows from the first-order condition with respect to the policy  $\hat{x}$ :

$$\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\Delta}{x-\hat{x}} \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) = 0 \quad (6)$$

according to Proposition 1. For sufficiently small legal uncertainty  $\Delta$ , this first-order condition (6) implies that there is a type  $x_0$  such that

$$\frac{\Delta}{x-\hat{x}} \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) \begin{cases} < 0 & \text{for all } x < x_0 \\ > 0 & \text{for all } x > x_0. \end{cases}$$

Hence, the enforcement agency chooses the policy such that the gains of deterring additional individuals with private benefits below the externalities are balanced by the losses of deterring additional individuals with private benefits above the externalities. Suppose for the moment  $x_0 > \hat{x}$ . Then the integrand  $g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x))$  in (5) is positive for  $x \in (\hat{x}-\Delta, \hat{x}) \cup (x_0, \hat{x}+\Delta)$  and for sufficiently small legal uncertainty  $\Delta$ . Intuitively, an increase in legal uncertainty makes some individuals in  $x \in (\hat{x}, \hat{x}+\Delta)$  take the action and some individuals in  $x \in (\hat{x}-\Delta, \hat{x})$  abstain from the action. This change increases welfare for each type  $x \in (\hat{x}-\Delta, \hat{x}) \cup (x_0, \hat{x}+\Delta)$ , but decreases welfare for each type  $x \in (\hat{x}, x_0)$ . Yet, this decrease in welfare is balanced by the gains for the types  $x \in (x_0, \hat{x}+\Delta)$ . Formally, the first-order condition (6) ensures that the derivative (5) of welfare with respect to legal uncertainty is positive for sufficiently small legal uncertainty  $\Delta$ . The case of  $x_0 < \hat{x}$  is analogous. Consequently, some legal uncertainty increases welfare.

Finally, consider the effects of legal uncertainty on the optimal policy  $\hat{x}$ . Whether legal

uncertainty increases or decreases the optimal policy depends on the curvature of the externality function and the mean of the image measure  $\hat{G}(x) := G(\hat{b}(x))/G(ap)$ . This image measure determines the optimal policy in Proposition 1.

**Proposition 3.** *Suppose  $f'(x) = 0$  for  $x \in (\min\{\hat{x}^N, \hat{x}\} - 2\Delta, \max\{\hat{x}^N, \hat{x}\} + 2\Delta)$ . Then if the externality function,  $e(x)$ , is convex and  $\mathbb{E}_{\hat{G}(x)}(x|x \in \{\hat{x} - \Delta, \hat{x} + \Delta\}) \geq \hat{x}$ , legal uncertainty decreases the optimal policy  $\hat{x}$ . If the externality function is concave and  $\mathbb{E}_{\hat{G}(x)}(x|x \in \{\hat{x} - \Delta, \hat{x} + \Delta\}) \leq \hat{x}$ , legal uncertainty raises the optimal policy  $\hat{x}$ .*

If the externality function is convex and the mean is undistorted, the optimal policy decreases and prohibits more actions than in the case with legal certainty. Yet, if the externality function is concave and the mean is undistorted, the optimal policy increases and permits more actions. Thus, the enforcement authority adapts its policy to the uncertainty. This completes the analysis of the model. The next section shows that this analysis is robust to the introduction of risk aversion, correlation, and endogenous fines.

## 5 Robustness

### 5.1 Risk Aversion

When legal uncertainty concerns individuals, the assumption of risk neutrality might be problematic. Therefore suppose individuals are risk-averse. The risk aversion is represented by a utility function  $u(\cdot)$ . The utility function is increasing, strictly concave and twice differentiable. Without loss of generality, I normalize  $u(0) = 0$ . If individuals are concerned, the most plausible welfare standard is a total welfare standard corresponding to  $\alpha = 1$ . In addition, I neglect the enforcement costs  $\kappa$  here, setting  $\kappa = 0$ .

The individuals' risk aversion changes the cut-off level  $\hat{b}(x)$  of the active individuals. Risk aversion will usually increase deterrence compared to risk neutrality. In particular, an individual of type  $(x, b)$  will take the action if

$$\left(1 - aH\left(\frac{x - \hat{x}}{\Delta}\right)\right) u(b) + aH\left(\frac{x - \hat{x}}{\Delta}\right) u(b - p) \geq 0.$$

If the inequality holds with equality, this equation implicitly defines the cut-off level  $\hat{b}(x)$  of the active individuals. As before, there is no deterrence in the legal region and  $\hat{b}(x) = 0$  for  $x \leq \hat{x} - \Delta$ . Additionally, the deterrence is constant in the illegal region and  $\partial \hat{b}(x)/\partial x = 0$  for  $x > \hat{x} - \Delta$ . Hence, deterrence effects of legal uncertainty are similar to the ones with risk-neutral individuals. Therefore, the screening effect in Theorem 1 also works for risk aversion.

**Proposition 4.** *Also for risk-averse individuals, legal uncertainty increases welfare if the uncertainty is not too large.*

Formally, risk aversion is equivalent to an adjusted distribution  $H$  of the error term  $\delta$ . In the proof, this equivalence allows me to proceed analogously to the proof of Theorem 1.

## 5.2 Correlation

Return to risk-neutral individuals. Suppose now that the private benefits are correlated with the externalities generated by the individual's action. In particular, there is some stochastic dependence between the two dimensions  $x$  and  $b$  of individuals' types. Formally,  $b$  is drawn from a distribution  $\tilde{G}(b|x)$  that depends on the value of  $x$ . For any  $x \in \mathbb{R}$ , the distribution  $\tilde{G}(b|x)$  is defined on  $[0, \bar{b}]$  and admits a differentiable density  $\tilde{g}(b|x) > \epsilon > 0$  for all  $b \in [0, \bar{b}]$ . For ease of exposition, I assume a total welfare standard with  $\alpha = 1$  and  $\kappa = 0$ . Notice that perfect correlation is impossible in this setting. Otherwise, any correlation is feasible.

**Lemma 3.** *For any  $\rho \in [0, 1)$ , there is a distribution  $\tilde{G}(b|x)$  and an increasing function  $z(\cdot)$ , such that the correlation coefficient between  $b$  and  $z(x)$  exceeds  $\rho$ .*

The function  $z$  allows to capture non-linear stochastic dependence. Now we can consider the welfare effects of legal uncertainty in this setting with correlation.

**Proposition 5.** *Also with correlation, legal uncertainty increases welfare if the uncertainty is not too large.*

The correlation makes the distribution of the private benefits  $b$  depend on the level of externalities  $x$ . This change in the distribution does not matter for the result, as Proposition 5 shows. Remember that the relevant value for the screening effect is the difference  $e(x) - \hat{b}(x)$ , i.e., between the externalities and the cut-off value for the private benefits. Both remain unchanged by the correlation. Therefore the result is robust to the introduction of correlation.

## 5.3 Endogenous Fines

Return to the setting of the main model. Another question is whether the enforcement authority could replicate the beneficial effects of legal uncertainty by setting the fine  $p$  appropriately. So far the fine was determined exogenously by the law. In reality, there might be different legal and organizational reasons why it is impossible to differentiate the fine very finely. Neglect these restrictions for a moment and assume that the enforcement authority endogenously sets the fine  $p$  at  $t = 0$ . In particular, the fine is a function of the externalities  $x$ , the observed part of the individual's type. Then the policy of the enforcement authority is entirely determined by the fine, as the threshold of legality is implicitly defined by the fine. The enforcement agency's decision whether to impose a fine at  $t = 4$  remains unchanged. I show that the optimal fine is discontinuous with a step.



**Proposition 6.** *If  $\alpha < 1$ ,  $\kappa > 0$  and there is legal certainty,  $\Delta = 0$ , the optimal fine equals*

$$p(x) = \begin{cases} 0 & \text{for } x < \hat{x} \\ \frac{\kappa}{1-\alpha} & \text{for } x \in [\hat{x}, x_1] \\ p^*(x) & \text{for } x > x_1 \end{cases}$$

with  $\hat{x} < x_1$  and  $p^*(x)$  determined by

$$e(x) + a\kappa = ap^*(x) - \frac{1 - G(ap^*(x))}{g(ap^*(x))}(1 - \alpha).$$

The values of  $\hat{x}$  and  $x_1$  are determined in the proof.

Thus, for socially beneficial types, i.e., with a low  $x$ , the enforcement authority tolerates active individuals by setting the fine to zero. As the threshold of legality  $\hat{x}$  is passed, a fine of  $\kappa/(1 - \alpha)$  is imposed, because lower fines do not justify spending the costs  $\kappa$  to enforce a fine. Then the fine  $p$  strictly increases in  $x$ . Notice that there is a step in the fine at  $\hat{x}$ . This discontinuity is sufficient for legal uncertainty to increase welfare. The reasoning is similar to Theorem 1 as the screening effect still applies around  $\hat{x}$ .

## 6 Conclusion

Legal studies frequently consider legal certainty as a value in itself. This paper takes a welfare perspective and studies the welfare effects of legal uncertainty. I show that some legal uncertainty raises social welfare. Suppose that the legal rules do not specify the threshold of legality exactly. In addition, there is asymmetric information between individuals and the enforcement authority. The enforcement authority cannot observe the individual's private benefits of an action. With legal certainty, hence, the individual's decision whether to take the action is independent of the action's private benefits.<sup>14</sup> With legal uncertainty, the probability of a conviction depends on the distance to the threshold of legality. If an individual is close to, but below the threshold, there is some probability of being convicted. Therefore individuals with low private benefits do not take the action. The legal uncertainty deters them. If an individual is close to, but above the threshold, there is some probability of not being convicted. Hence individuals with high private benefits take the action. The legal uncertainty encourages them to take the action. Both effects on average increase social welfare independently of underlying distributions and externalities. The effects are also robust to the introduction of risk aversion, correlation, endogenous fines and different welfare standards. Consequently, some legal uncertainty about the threshold of legality increases welfare.

Uncertainty about the threshold of legality might be due to imprecision in the mea-

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<sup>14</sup>I assume here that the expected penalty equals the maximum private benefits, but the proof does not rely on this assumption.

surement of the enforcement authority, missing precedents or unclear rules. In particular, the legal uncertainty is not by design, but inherent in the legal rules. I do not recommend designing especially ambiguous rules. The paper, however, points out that other criteria than legal certainty should decide about optimal rules. Even if rules contain some legal uncertainty, this uncertainty need not be a drawback, but might even increase welfare. This insight is very relevant for policy and allows policymakers to design better laws.

Obviously, there are limitations to the benefits of legal uncertainty. Welfare effects need not be monotone in the amount of uncertainty. Legal uncertainty reduces the accountability of the enforcement authority making it more challenging to deter corruption or regulatory capture. An interesting avenue for future research are the dynamic effects of legal uncertainty. Legal uncertainty might give individuals incentives to experiment and therefore implement more controversial actions with negative externalities than under legal certainty. Yet, the costs of such behavior, e.g., possible fines, are incurred by single individuals, while the benefits spill over to all players, as they learn, e.g., court decisions reducing the legal uncertainty.

## A Appendix

**Proof of Lemma 1:** The derivative of  $\hat{b}(x)$  with respect to the legal uncertainty  $\Delta$  equals

$$\frac{\partial \hat{b}(x)}{\partial \Delta} = \begin{cases} -aph\left(\frac{x-\hat{x}}{\Delta}\right) \frac{x-\hat{x}}{\Delta^2} & \text{for } x \in (\hat{x} - \Delta, \hat{x} + \Delta) \\ 0 & \text{otherwise.} \end{cases}$$

Therefore deterrence increases for  $x < \hat{x}$  and decreases for  $x > \hat{x}$ .  $\square$

**Proof of Lemma 2:** Suppose there is no legal uncertainty and  $\Delta = 0$ . Then the first derivative of welfare with respect to the policy  $\hat{x}$  yields the following first-order condition.

$$\begin{aligned} e(\hat{x}) &= \frac{1 - G(ap)}{G(ap)} \left( \alpha \frac{\mathbb{E}(b)}{1 - G(ap)} - \alpha \mathbb{E}(b|b \geq ap) - a((1 - \alpha)p - \kappa) \right) = \\ &= \alpha \mathbb{E}(b|b \leq ap) - \frac{1 - G(ap)}{G(ap)} a((1 - \alpha)p - \kappa). \end{aligned}$$

The second derivative with respect to the policy  $\hat{x}$  equals

$$-f(\hat{x})G(ap)e'(\hat{x}) + f'(\hat{x})G(ap) \left( \alpha \mathbb{E}(b|b \leq ap) - e(\hat{x}) - \frac{1 - G(ap)}{G(ap)} a((1 - \alpha)p - \kappa) \right).$$

Assumption (1) ensures that

$$e'(x) > \frac{f'(x)}{f(x)} \left( \alpha \mathbb{E}(b|b \leq ap) - e(x) - \frac{1 - G(ap)}{G(ap)} a((1 - \alpha)p - \kappa) \right)$$

for all  $x \in \mathbb{R}$ . Therefore the second derivative is negative and the optimization of the enforcement authority is globally concave. The concavity guarantees that the first-order condition (4) determines the solution.  $\square$

Lemma 4 determines the derivative of welfare with respect to the policy  $\hat{x}$  and legal uncertainty  $\Delta$ .

**Lemma 4.** *The derivative  $\partial W(\hat{x})/\partial y$  of welfare with respect to  $y \in \{\hat{x}, \Delta\}$  equals*

$$\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial y} \left( e(x) - \hat{b}(x)\alpha + \left( \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1-\alpha)p - \kappa}{p} \right) dF(x)$$

**Proof:** Taking the derivative of welfare with respect to  $y \in \{\hat{x}, \Delta\}$  results in

$$\begin{aligned} \frac{\partial W(\hat{x})}{\partial y} = & (\alpha \mathbb{E}(b) - e(\hat{x} - \Delta)) f(\hat{x} - \Delta) (1 - 1) + \\ & + (1 - G(ap)) (\alpha \mathbb{E}(b|b \geq ap) - e(\hat{x} + \Delta) + ((1 - \alpha)p - \kappa)a) f(\hat{x} + \Delta) (1 - 1) + \\ & + \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} -g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial y} \left( \alpha \mathbb{E}(b|b \geq \hat{b}(x)) - e(x) + \hat{b}(x) \frac{(1-\alpha)p - \kappa}{p} \right) + \\ & + \left( 1 - G(\hat{b}(x)) \right) \left( \alpha \frac{\partial \mathbb{E}(b|b \geq \hat{b}(x))}{\partial y} + \frac{\partial \hat{b}(x)}{\partial y} \frac{(1-\alpha)p - \kappa}{p} \right) dF(x). \end{aligned}$$

For  $y = \Delta$ , I use the envelope theorem here. Notice that the conditional expectation equals

$$\mathbb{E}(b|b \geq \tilde{b}) = \frac{1}{1 - G(\tilde{b})} \int_{\tilde{b}}^{\bar{b}} b dG(b).$$

Hence,

$$\frac{\partial \mathbb{E}(b|b \geq \tilde{b})}{\partial \tilde{b}} = \frac{1}{(1 - G(\tilde{b}))^2} \left( -(1 - G(\tilde{b})) \tilde{b} g(\tilde{b}) + g(\tilde{b}) \int_{\tilde{b}}^{\bar{b}} b dG(b) \right) = g(\tilde{b}) \frac{\mathbb{E}(b|b \geq \tilde{b}) - \tilde{b}}{1 - G(\tilde{b})}.$$

Together with the chain rule, this results in  $\partial \mathbb{E}(b|b \geq \hat{b}(x))/\partial y = g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial y} \frac{\mathbb{E}(b|b \geq \hat{b}(x)) - \hat{b}(x)}{1 - G(\hat{b}(x))}$  for  $y \in \{\hat{x}, \Delta\}$ . Therefore the derivative with respect to  $y \in \{\hat{x}, \Delta\}$  equals

$$\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial y} \left( e(x) - \hat{b}(x)\alpha + \left( \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1-\alpha)p - \kappa}{p} \right) dF(x). \quad \square$$

**Proof of Proposition 1:** By Lemma 4 (p. 16), the first derivative of welfare (3) with respect to the policy  $\hat{x}$  equals

$$\frac{\partial W(\hat{x})}{\partial \hat{x}} = \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) \quad (7)$$

with  $\mathcal{T}(x) = e(x) - \hat{b}(x)\alpha + \left( \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1-\alpha)p - \kappa}{p}$ . Hence, the first-order condition reads

$$\begin{aligned} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} e(x) dF(x) = \\ \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \left( \hat{b}(x)\alpha - \left( \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1-\alpha)p - \kappa}{p} \right) dF(x) \end{aligned} \quad (8)$$

The exact threshold of legality depends on the distributions  $F$ ,  $G$ , and  $H$  and the externality function  $e(x)$ . It remains to analyze the second-order condition. Differentiating (7) with respect to  $\hat{x}$  yields

$$\begin{aligned}\frac{\partial^2 W(\hat{x})}{\partial \hat{x}^2} &= -g(ap)\frac{ap}{\Delta}h(1)\mathcal{T}(\hat{x}+\Delta)f(\hat{x}+\Delta) + g(0)\frac{ap}{\Delta}h(-1)\mathcal{T}(\hat{x}-\Delta)f(\hat{x}-\Delta) - \\ &\quad - \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \frac{\partial g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})\mathcal{T}(x)}{\partial \hat{x}} dF(x) = \\ &= -g(ap)\frac{ap}{\Delta}h(1)\mathcal{T}(\hat{x}+\Delta)f(\hat{x}+\Delta) + g(0)\frac{ap}{\Delta}h(-1)\mathcal{T}(\hat{x}-\Delta)f(\hat{x}-\Delta) + \\ &\quad + \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} -\frac{\partial g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})\mathcal{T}(x)}{\partial \hat{x}} + \\ &\quad + g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})e'(x) - g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})e'(x) dF(x)\end{aligned}$$

For the second equality, add and subtract  $g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})e'(x)$  within the integral. Notice that

$$-\frac{\partial g(\hat{b}(x))}{\partial \hat{x}} = \frac{\partial g(\hat{b}(x))}{\partial x}, \quad -\frac{\partial \frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})}{\partial \hat{x}} = \frac{\partial \frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})}{\partial x}, \quad \text{and} \quad -\frac{\partial \mathcal{T}(x)}{\partial \hat{x}} + e'(x) = \frac{\partial \mathcal{T}(x)}{\partial x}.$$

Therefore

$$-\frac{\partial g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})\mathcal{T}(x)}{\partial \hat{x}} + g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})e'(x) = \frac{\partial g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})\mathcal{T}(x)}{\partial x}$$

and

$$\begin{aligned}\frac{\partial^2 W(\hat{x})}{\partial \hat{x}^2} &= -g(ap)\frac{ap}{\Delta}h(1)\mathcal{T}(\hat{x}+\Delta)f(\hat{x}+\Delta) + g(0)\frac{ap}{\Delta}h(-1)\mathcal{T}(\hat{x}-\Delta)f(\hat{x}-\Delta) + \\ &\quad + \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \frac{\partial g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})\mathcal{T}(x)}{\partial x} - g(\hat{b}(x))\frac{ap}{\Delta}h(\frac{x-\hat{x}}{\Delta})e'(x) dF(x).\end{aligned}$$

Partial integration yields

$$\begin{aligned}\frac{\partial^2 W(\hat{x})}{\partial \hat{x}^2} &= -\frac{ap}{\Delta} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x))h(\frac{x-\hat{x}}{\Delta})\mathcal{T}(x)f'(x) + g(\hat{b}(x))h(\frac{x-\hat{x}}{\Delta})e'(x)f(x) dx = \\ &= -\frac{ap}{\Delta} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x))h(\frac{x-\hat{x}}{\Delta}) \left( \mathcal{T}(x)\frac{f'(x)}{f(x)} + e'(x) \right) dF(x)\end{aligned}$$

Assumption (1) ensures that

$$e'(x) > \max \left\{ -\frac{f'(x)}{f(x)} \left( e(x) - \alpha ap + \left( \frac{1-G(ap)}{g(ap)} - ap \right) \frac{(1-\alpha)p - \kappa}{p} \right), \right. \\ \left. -\frac{f'(x)}{f(x)} \left( e(x) + \frac{1}{g(0)} \frac{(1-\alpha)p - \kappa}{p} \right) \right\}$$

for all  $x \in \mathbb{R}$ . As  $\mathcal{T}(x)$  decreases in  $\hat{b}(x)$ , this assumption ensures that  $-\mathcal{T}(x)\frac{f'(x)}{f(x)} < e'(x)$ .

Therefore

$$\begin{aligned} \mathcal{T}(x) \frac{f'(x)}{f(x)} + e'(x) &= \\ &= \frac{f'(x)}{f(x)} \left( e(x) - \hat{b}(x)\alpha + \left( \frac{1 - G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1 - \alpha)p - \kappa}{p} \right) + e'(x) > 0 \end{aligned}$$

for all  $x \in (\hat{x} - \Delta, \hat{x} + \Delta)$ . Consequently,  $\partial^2 W(\hat{x})/\partial \hat{x}^2 < 0$  and the optimization of the enforcement authority is globally concave.  $\square$

**Proof of Proposition 2:** According to Proposition 1, the optimal policy is determined by

$$\begin{aligned} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} e(x) dF(x) &= \\ \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \left( \hat{b}(x)\alpha - \left( \frac{1 - G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1 - \alpha)p - \kappa}{p} \right) dF(x) \end{aligned} \quad (9)$$

Notice that  $\partial \hat{b}(x)/\partial x = -\partial \hat{b}(x)/\partial \hat{x}$  for all  $x$  and

$$\begin{aligned} - \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} dx &= \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \frac{\partial G(\hat{b}(x))}{\partial x} dx = G(\hat{b}(\hat{x} + \Delta)) - G(\hat{b}(\hat{x} - \Delta)) = \\ &= G(ap) - G(0) = G(ap). \end{aligned}$$

Therefore

$$G(ap) \min_{x \in [\hat{x}-\Delta, \hat{x}+\Delta]} e(x)f(x) \leq \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} -g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} e(x) dF(x) \leq G(ap) \max_{x \in [\hat{x}-\Delta, \hat{x}+\Delta]} e(x)f(x).$$

In particular, the left-hand side of (9) converges to

$$\lim_{\Delta \searrow 0} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} e(x) dF(x) = -f(\hat{x})G(ap)e(\hat{x}). \quad (10)$$

Analogously, integration by substitution of  $x$  by  $b$  yields

$$\lim_{\Delta \searrow 0} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \hat{b}(x) dF(x) = -f(\hat{x}) \int_0^{ap} bg(b) db = -f(\hat{x})G(ap)\mathbb{E}(b|b \leq ap).$$

Finally,

$$\begin{aligned} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} -g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \left( \frac{1 - G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) dx &= \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \frac{\partial(1 - G(\hat{b}(x)))\hat{b}(x)}{\partial x} dx = \\ &= (1 - G(\hat{b}(\hat{x} + \Delta)))\hat{b}(\hat{x} + \Delta) - (1 - G(\hat{b}(\hat{x} - \Delta)))\hat{b}(\hat{x} - \Delta) = (1 - G(ap))ap. \end{aligned}$$

Hence,

$$\lim_{\Delta \searrow 0} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} -g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \left( \frac{1 - G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) dF(x) = f(\hat{x})(1 - G(ap))ap.$$

Consequently, the right-hand side of (9) converges to

$$-f(\hat{x})G(ap)\left(\alpha\mathbb{E}(b|b \leq ap) - \frac{1-G(ap)}{G(ap)}a((1-\alpha)p - \kappa)\right). \quad (11)$$

Rewrite condition (4) from Lemma 2 with legal certainty as

$$0 = \alpha\mathbb{E}(b|b \leq ap) - \frac{1-G(ap)}{G(ap)}a((1-\alpha)p - \kappa) - e(\hat{x}) \quad (12)$$

The right-hand side of (12) is decreasing and continuously differentiable in  $\hat{x}$ . Therefore, (12) yields a unique  $\hat{x}^N$ . Both sides of condition (9) are also continuously differentiable in  $\hat{x}$ . In addition, the solution  $\hat{x}$  to condition (9) is unique according to the proof of Proposition 1. The previous steps have shown that

$$\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \left( \hat{b}(x)\alpha - \left( \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1-\alpha)p - \kappa}{p} - e(x) \right) dF(x)$$

converges pointwise to the right-hand side of (12) multiplied by  $-f(\hat{x})G(ap) < 0$ . The convergence is even uniform, because

$$\begin{aligned} \sup_{\hat{x} \in \mathbb{R}} & \left| \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \left( \hat{b}(x)\alpha - \left( \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1-\alpha)p - \kappa}{p} - e(x) \right) dF(x) + \right. \\ & \left. + f(\hat{x})G(ap) \left( \alpha\mathbb{E}(b|b \leq ap) - \frac{1-G(ap)}{G(ap)}a((1-\alpha)p - \kappa) - e(\hat{x}) \right) \right| < \\ & < G(ap) \sup_{\hat{x} \in \mathbb{R}} \left( \max_{x \in [\hat{x}-\Delta, \hat{x}+\Delta]} |e(x)f(x) - e(\hat{x})f(\hat{x})| + \right. \\ & \left. + \left| \alpha\mathbb{E}(b|b \leq ap) - \frac{1-G(ap)}{G(ap)}a((1-\alpha)p - \kappa) \right| \max_{x \in [\hat{x}-\Delta, \hat{x}+\Delta]} |f(x) - f(\hat{x})| \right) \end{aligned}$$

converges to 0 for  $\Delta \searrow 0$ . Therefore, the optimal policy converges,  $\lim_{\Delta \searrow 0} \hat{x} = \hat{x}^N$ , as the uncertainty vanishes.  $\square$

**Proof of Theorem 1:** By Lemma 4 (p. 16), the derivative of welfare (3) with respect to  $\Delta$  equals

$$\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} \left( e(x) - \hat{b}(x)\alpha + \left( \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1-\alpha)p - \kappa}{p} \right) dF(x).$$

Denote the term in brackets by  $\mathcal{T}(x) = e(x) - \hat{b}(x)\alpha + \left( \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))} - \hat{b}(x) \right) \frac{(1-\alpha)p - \kappa}{p}$ .  $\mathcal{T}(x)$  decreases in  $x \in (\hat{x} - \Delta, \hat{x} + \Delta)$  for sufficiently small  $\Delta > 0$ , because the derivative of  $\mathcal{T}(x)$  with respect to  $x$  equals

$$\frac{\partial \mathcal{T}(x)}{\partial x} = e'(x) - \frac{\partial \hat{b}(x)}{\partial x} \alpha + \left( \frac{\partial \frac{1-G(\hat{b}(x))}{g(\hat{b}(x))}}{\partial \hat{b}} - 1 \right) \frac{\partial \hat{b}(x)}{\partial x} \frac{(1-\alpha)p - \kappa}{p} =$$

$$= e'(x) - \frac{ap}{\Delta} h\left(\frac{x - \hat{x}}{\Delta}\right) \left( \frac{(1 - \alpha)p - \kappa}{p} \left(1 - \frac{\partial \frac{1 - G(\hat{b}(x))}{g(\hat{b}(x))}}{\partial b}\right) + \alpha \right) < 0 \quad (13)$$

for sufficiently small  $\Delta > 0$ . The (weakly) increasing hazard rate of the distribution  $G$  of  $b$  implies a (weakly) decreasing inverse hazard rate. Therefore the term in brackets in Eq. (13) is positive as  $\kappa < (1 - \alpha)p$  by assumption. As  $\frac{ap}{\Delta}$  goes to infinity for  $\Delta$  to 0, the derivative  $\partial \mathcal{T}(x)/\partial x$  is negative for sufficiently small  $\Delta > 0$ .

Proposition 1 derives the first-order condition for the optimal policy  $\hat{x}$ :

$$\frac{\partial W(\hat{x})}{\partial \hat{x}} = \int_{\hat{x} - \Delta}^{\hat{x} + \Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) = 0 \quad (14)$$

Remember that  $g(b) > 0$  and  $\partial \hat{b}(x)/\partial \hat{x} < 0$  in the relevant range. As  $\mathcal{T}(x)$  decreases in  $x$ , this first-order condition (14) implies  $\mathcal{T}(\hat{x} - \Delta) > 0 > \mathcal{T}(\hat{x} + \Delta)$  for sufficiently small  $\Delta > 0$ . In particular, for sufficiently small  $\Delta > 0$  there is a  $x_0 \in (\hat{x} - \Delta, \hat{x} + \Delta)$  such that

$$\frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) \begin{cases} < 0 & \text{for all } x < x_0 \\ > 0 & \text{for all } x > x_0, \end{cases}$$

as  $\partial \hat{b}(x)/\partial \hat{x} = -\frac{ap}{\Delta} h(\frac{x - \hat{x}}{\Delta})$  is negative for  $x \in (\hat{x} - \Delta, \hat{x} + \Delta)$ .  $\frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x)$  changing sign exactly once in combination with the first-order condition (14) yields

$$- \int_{\hat{x} - \Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) = \int_{\hat{x}}^{\hat{x} + \Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) > 0. \quad (15)$$

Notice that according to Lemma 1 the derivative  $\partial \hat{b}(x)/\partial \Delta$  is positive for  $x \in (\hat{x} - \Delta, \hat{x})$  and negative for  $x \in (\hat{x}, \hat{x} + \Delta)$ . Moreover,  $\partial \hat{b}(x)/\partial \Delta = \frac{x - \hat{x}}{\Delta} \partial \hat{b}(x)/\partial \hat{x}$ .

Assume for the moment  $x_0 \leq \hat{x}$ . Then  $\frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) > 0$  for all  $x \in [\hat{x}, \hat{x} + \Delta)$  and

$$\begin{aligned} \int_{\hat{x}}^{\hat{x} + \Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} \mathcal{T}(x) dF(x) &= \int_{\hat{x}}^{\hat{x} + \Delta} g(\hat{b}(x)) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) > \\ &> 0 \int_{\hat{x}}^{\hat{x} + \Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) = 0 \end{aligned}$$

for sufficiently small  $\Delta > 0$ . In addition,

$$\begin{aligned} \int_{\hat{x} - \Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} \mathcal{T}(x) dF(x) &= \int_{\hat{x} - \Delta}^{\hat{x}} g(\hat{b}(x)) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) = \\ &= \int_{\hat{x} - \Delta}^{x_0} g(\hat{b}(x)) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) + \int_{x_0}^{\hat{x}} g(\hat{b}(x)) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) > \\ &> \frac{x_0 - \hat{x}}{\Delta} \int_{\hat{x} - \Delta}^{x_0} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) + \frac{x_0 - \hat{x}}{\Delta} \int_{x_0}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) = \end{aligned}$$

$$= \frac{x_0 - \hat{x}}{\Delta} \int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) \geq 0$$

for sufficiently small  $\Delta > 0$ , because  $x_0 \leq \hat{x}$  and  $\int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} \mathcal{T}(x) dF(x) < 0$  by (15). The case  $x_0 > \hat{x}$  is analogous and therefore omitted. Consequently,

$$\frac{\partial W(\hat{x})}{\partial \Delta} = \int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} \mathcal{T}(x) dF(x) + \int_{\hat{x}}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} \mathcal{T}(x) dF(x) > 0$$

for sufficiently small  $\Delta > 0$ . In addition, welfare (3) is continuous in  $\Delta \in [0, \infty)$ . Hence, some legal uncertainty increases welfare.  $\square$

**Proof of Proposition 3:** Lemma 2 states the first-order condition (4) for the optimal policy  $x$  if there is legal certainty,  $\Delta = 0$ . Rewrite this condition as

$$G(ap)e(\hat{x}^N) = \int_0^{ap} \alpha b dG(b) - (1 - G(ap))a((1 - \alpha)p - \kappa). \quad (16)$$

Notice that  $\partial \hat{b}(x)/\partial x = -\partial \hat{b}(x)/\partial \hat{x}$  for all  $x$  and

$$\begin{aligned} & (1 - G(ap))apf(x + \Delta') = \\ & = (1 - G(apH(1)))apf(x + \Delta') - \underbrace{(1 - G(0))apH(-1)f(x - \Delta')}_{=0} = \\ & = \int_{\hat{x}-\Delta'}^{\hat{x}+\Delta'} \frac{\partial(1 - G(apH(\frac{x-\hat{x}}{\Delta'})))apH(\frac{x-\hat{x}}{\Delta'})f(x)}{\partial x} dx = \\ & = \int_{\hat{x}-\Delta'}^{\hat{x}+\Delta'} ap \frac{\partial H(\frac{x-\hat{x}}{\Delta'})}{\partial x} \left(1 - G(apH(\frac{x-\hat{x}}{\Delta'})) - g(apH(\frac{x-\hat{x}}{\Delta'}))apH(\frac{x-\hat{x}}{\Delta'})\right) f(x) + \\ & \quad + (1 - G(apH(\frac{x-\hat{x}}{\Delta'})))apH(\frac{x-\hat{x}}{\Delta'})f'(x) dx = \\ & = - \int_{\hat{x}-\Delta'}^{\hat{x}+\Delta'} g(apH(\frac{x-\hat{x}}{\Delta'}))ap \frac{\partial H(\frac{x-\hat{x}}{\Delta'})}{\partial \hat{x}} \left(\frac{1 - G(apH(\frac{x-\hat{x}}{\Delta'}))}{g(apH(\frac{x-\hat{x}}{\Delta'}))} - apH(\frac{x-\hat{x}}{\Delta'})\right) dF(x) + \\ & \quad + \int_{\hat{x}-\Delta'}^{\hat{x}+\Delta'} (1 - G(apH(\frac{x-\hat{x}}{\Delta'})))apH(\frac{x-\hat{x}}{\Delta'})f'(x) dx \end{aligned}$$

for any  $\Delta' > 0$ . In addition,

$$\begin{aligned} & f(x + \Delta') \int_0^{ap} b dG(b) = f(x + \Delta') \int_0^{apH(1)} b dG(b) - f(x - \Delta') \int_0^{apH(-1)} b dG(b) = \\ & = \int_{\hat{x}-\Delta'}^{\hat{x}+\Delta'} \frac{\partial f(x) \int_0^{apH(\frac{x-\hat{x}}{\Delta'})} b dG(b)}{\partial x} dx = \\ & = \int_{\hat{x}-\Delta'}^{\hat{x}+\Delta'} f(x) g(apH(\frac{x-\hat{x}}{\Delta'}))apH(\frac{x-\hat{x}}{\Delta'})ap \frac{\partial H(\frac{x-\hat{x}}{\Delta'})}{\partial x} + f'(x) \int_0^{apH(\frac{x-\hat{x}}{\Delta'})} b dG(b) dx = \\ & = - \int_{\hat{x}-\Delta'}^{\hat{x}+\Delta'} g(apH(\frac{x-\hat{x}}{\Delta'}))ap \frac{\partial H(\frac{x-\hat{x}}{\Delta'})}{\partial \hat{x}} apH(\frac{x-\hat{x}}{\Delta'}) + \frac{f'(x)}{f(x)} \int_0^{apH(\frac{x-\hat{x}}{\Delta'})} b dG(b) dF(x) \end{aligned}$$

for any  $\Delta' > 0$ .



Inserting into the first-order condition (16) yields

$$\begin{aligned}
& -e(\hat{x}^N)G(ap) = \\
& = \int_{\hat{x}^N - \Delta'}^{\hat{x}^N + \Delta'} g(apH(\frac{x - \hat{x}^N}{\Delta'})) ap \frac{\partial H(\frac{x - \hat{x}^N}{\Delta'})}{\partial \hat{x}^N} \left( \alpha apH(\frac{x - \hat{x}^N}{\Delta'}) - \left( \frac{1 - G(apH(\frac{x - \hat{x}^N}{\Delta'}))}{g(apH(\frac{x - \hat{x}^N}{\Delta'}))} - apH(\frac{x - \hat{x}^N}{\Delta'}) \right) \frac{(1 - \alpha)p - \kappa}{p} \right) dx.
\end{aligned} \tag{17}$$

for any  $\Delta' > 0$ . Comparing the first-order conditions (8) with legal uncertainty and (17) without legal uncertainty, we see that the right-hand side of both conditions is the same if  $\Delta' = \Delta$  and  $f$  is constant in the relevant range. Additionally, the right-hand side of (17) is constant in  $\hat{x}^N$ . Suppose that, in addition,  $e(\cdot)$  is concave and

$$\hat{x}^N \geq \mathbb{E}_{\hat{G}(x)}(x | \hat{x}^N - \Delta \leq x \leq \hat{x}^N + \Delta) = \frac{1}{G(ap)} \int_{\hat{x}^N - \Delta}^{\hat{x}^N + \Delta} x g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial x} dx \tag{18}$$

with the image measure  $\hat{G}(x) = G(apH(\frac{x - \hat{x}^N}{\Delta}))/G(ap)$ . Therefore

$$\begin{aligned}
\int_{\hat{x}^N - \Delta}^{\hat{x}^N + \Delta} e(x) g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial x} dx &= -G(ap) \int_{\hat{x}^N - \Delta}^{\hat{x}^N + \Delta} e(x) \frac{g(\hat{b}(x))}{G(ap)} \frac{\partial \hat{b}(x)}{\partial x} dx \geq \\
&\geq -G(ap) e \left( \int_{\hat{x}^N - \Delta}^{\hat{x}^N + \Delta} \frac{x}{G(ap)} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial x} dx \right) \geq \\
&\geq -G(ap) e(\hat{x}^N).
\end{aligned}$$

Jensen's inequality guarantees the first inequality. Eq. (18) ensures the second inequality, because the externality function  $e(\cdot)$  is increasing. Consequently, legal uncertainty increases the threshold of legality if  $f$  is constant in the relevant range,  $e(\cdot)$  is concave and condition (18) is satisfied.

Analogously, legal uncertainty decreases the threshold of legality if  $f$  is constant in the relevant range,  $e(\cdot)$  is convex and  $\mathbb{E}_{\hat{G}(x)}(x | \hat{x} - \Delta \leq x \leq \hat{x} + \Delta) \geq \hat{x}$ .  $\square$

**Proof of Proposition 4:** The proof proceeds similarly to Theorem 1. By Lemma 4 (p. 16), the derivative of welfare (3) with respect to  $\Delta$  is

$$\int_{\hat{x} - \Delta}^{\hat{x} + \Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} \left( e(x) - \hat{b}(x) \right) dF(x).$$

$e(x) - \hat{b}(x)$  decreases in  $x \in (\hat{x} - \Delta, \hat{x} + \Delta)$  for sufficiently small  $\Delta > 0$ , because the cut-off type  $\hat{b}(x)$  increases in  $x$  by the implicit function theorem

$$\frac{\partial \hat{b}(x)}{\partial x} = \begin{cases} ah(\frac{x - \hat{x}}{\Delta}) \frac{1}{\Delta} \frac{u(\hat{b}(x)) - u(\hat{b}(x) - p)}{(1 - aH(\frac{x - \hat{x}}{\Delta}))u'(\hat{b}(x)) + aH(\frac{x - \hat{x}}{\Delta})u'(\hat{b}(x) - p)} > 0 & \text{for } x \in (\hat{x} - \Delta, \hat{x} + \Delta) \\ 0 & \text{otherwise.} \end{cases}$$

and  $\partial \hat{b}(x)/\partial x$  exceeds  $e'(x)$  for sufficiently small  $\Delta > 0$ . Proposition 1 derives the first-

order condition for the optimal policy  $\hat{x}$ :

$$\frac{\partial W(\hat{x})}{\partial \hat{x}} = \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = 0$$

Therefore

$$-\int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = \int_{\hat{x}}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x).$$

The implicit function theorem ensures that

$$\frac{\partial \hat{b}(x)}{\partial \Delta} = \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} = -\frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}}.$$

as in the case of risk neutrality. Hence, this derivative of  $\hat{b}(x)$  is positive for  $x \in (\hat{x} - \Delta, \hat{x})$  and negative for  $x \in (\hat{x}, \hat{x} + \Delta)$ .

As  $e(x) - \hat{b}(x)$  decreases in  $x$ , the first-order condition implies  $e(\hat{x} - \Delta) - \hat{b}(\hat{x} - \Delta) > 0 > e(\hat{x} + \Delta) - \hat{b}(\hat{x} + \Delta)$  for sufficiently small  $\Delta > 0$ . In particular, for sufficiently small  $\Delta > 0$  there is a  $x_0 \in (\hat{x} - \Delta, \hat{x} + \Delta)$  such that

$$\frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) \begin{cases} < 0 & \text{for all } x < x_0 \\ > 0 & \text{for all } x > x_0, \end{cases}$$

as  $\partial \hat{b}(x)/\partial \hat{x}$  is negative.

Assume for the moment  $x_0 \leq \hat{x}$ . Then  $\frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) > 0$  for all  $x \in [\hat{x}, \hat{x} + \Delta)$  and

$$\begin{aligned} \int_{\hat{x}}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) &= \int_{\hat{x}}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) > \\ &> 0 \int_{\hat{x}}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = 0 \end{aligned}$$

for sufficiently small  $\Delta > 0$ . In addition,

$$\begin{aligned} \int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) &= \int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = \\ &= \int_{\hat{x}-\Delta}^{x_0} g(\hat{b}(x)) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) + \int_{x_0}^{\hat{x}} g(\hat{b}(x)) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) > \\ &> \frac{x_0 - \hat{x}}{\Delta} \int_{\hat{x}-\Delta}^{x_0} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) + \frac{x_0 - \hat{x}}{\Delta} \int_{x_0}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = \\ &= \frac{x_0 - \hat{x}}{\Delta} \int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) > 0 \end{aligned}$$

for sufficiently small  $\Delta > 0$ , because  $x_0 - \hat{x} < 0$  and  $\int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) < 0$ . The case  $x_0 > \hat{x}$  is analogous and therefore omitted. Consequently,

$$\frac{\partial W(\hat{x})}{\partial \Delta} =$$

$$= \int_{\hat{x}-\Delta}^{\hat{x}} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) + \int_{\hat{x}}^{\hat{x}+\Delta} g(\hat{b}(x)) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) > 0$$

for sufficiently small  $\Delta > 0$ . Finally, welfare is continuous in  $\Delta \in [0, \infty)$ . Hence, some legal uncertainty increases welfare.  $\square$

**Proof of Lemma 3:** Consider any  $\rho \in [0, 1)$  and a small  $\gamma > 0$ . Set  $z(x) = \frac{\bar{b}}{1+\exp(-x)}$ ,  $\beta_\gamma(x) = \int_0^{\bar{b}} \exp(-\frac{(b-z(x))^2}{\gamma}) db$  and

$$\tilde{g}_\gamma(b|x) = \frac{1}{\beta_\gamma(x)} \exp\left(-\frac{(b-z(x))^2}{\gamma}\right)$$

for  $b \in [0, \bar{b}]$ . For any  $\gamma > 0$ ,  $\tilde{g}_\gamma(b|x)$  is a differentiable density on  $[0, \bar{b}]$ . In addition,  $\tilde{g}_\gamma(b|x)$  is positive and bounded away from zero for a given  $\gamma > 0$ . Finally, the correlation coefficient between  $b$  and  $z(x)$  decreases in  $\gamma$  and in the limit for  $\gamma \rightarrow 0$  equals 1. Therefore, choosing a sufficiently small  $\gamma > 0$  ensures that the correlation coefficient between  $b$  and  $z(x)$  exceeds  $\rho$ .  $\square$

**Proof of Proposition 5:** With correlation, welfare equals

$$\begin{aligned} & \int_{-\infty}^{\hat{x}-\Delta} \mathbb{E}_{\tilde{G}(b|x)}(b) - e(x) dF(x) + \\ & + \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \left(1 - \tilde{G}(\hat{b}(x)|x)\right) \left(\mathbb{E}_{\tilde{G}(b|x)}(b|b \geq \hat{b}(x)) - e(x)\right) dF(x) + \\ & + \int_{\hat{x}+\Delta}^{\infty} (1 - \tilde{G}(ap|x)) (\mathbb{E}_{\tilde{G}(b|x)}(b|b \geq ap) - e(x)) dF(x). \end{aligned}$$

By Lemma 4 (p. 16), the derivative of welfare with respect to  $\Delta$  is

$$\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x).$$

$e(x) - \hat{b}(x)$  decreases in  $x$  for sufficiently small  $\Delta > 0$ . The first-order condition for the optimal policy  $\hat{x}$  equals

$$\frac{\partial W(\hat{x})}{\partial \hat{x}} = \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = 0. \quad (19)$$

Remember that  $\tilde{g}(b|x) > 0$  and  $\partial \hat{b}(x)/\partial \hat{x} < 0$  in the relevant range. As  $e(x) - \hat{b}(x)$  decreases in  $x$ , this first-order condition (19) implies  $e(x - \Delta) - \hat{b}(x - \Delta) > 0 > e(x + \Delta) - \hat{b}(x + \Delta)$  for sufficiently small  $\Delta > 0$ . In particular, for sufficiently small  $\Delta > 0$  there is a  $x_0 \in (\hat{x} - \Delta, \hat{x} + \Delta)$  such that

$$\frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) \begin{cases} < 0 & \text{for all } x < x_0 \\ > 0 & \text{for all } x > x_0, \end{cases}$$

as  $\partial \hat{b}(x)/\partial \hat{x} = -\frac{ap}{\Delta} h(\frac{x-\hat{x}}{\Delta})$  is negative for  $x \in (\hat{x} - \Delta, \hat{x} + \Delta)$ .  $\frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) > 0$

changing sign exactly once in combination with the first-order condition (19) yields

$$\begin{aligned} & - \int_{\hat{x}-\Delta}^{\hat{x}} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = \\ & = \int_{\hat{x}}^{\hat{x}+\Delta} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) > 0. \end{aligned} \quad (20)$$

Notice that according to Lemma 1 the derivative  $\partial \hat{b}(x)/\partial \Delta$  is positive for  $x \in (\hat{x} - \Delta, \hat{x})$  and negative for  $x \in (\hat{x}, \hat{x} + \Delta)$ . Moreover,  $\partial \hat{b}(x)/\partial \Delta = \frac{x-\hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}}$ .

Assume for the moment  $x_0 \leq \hat{x}$ . Then  $\frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) > 0$  for all  $x > \hat{x}$  and

$$\begin{aligned} & \int_{\hat{x}}^{\hat{x}+\Delta} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) = \\ & = \int_{\hat{x}}^{\hat{x}+\Delta} \tilde{g}(\hat{b}(x)|x) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) > \\ & > 0 \int_{\hat{x}}^{\hat{x}+\Delta} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = 0 \end{aligned}$$

for sufficiently small  $\Delta > 0$ . In addition,

$$\begin{aligned} & \int_{\hat{x}-\Delta}^{\hat{x}} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) = \\ & = \int_{\hat{x}-\Delta}^{\hat{x}} \tilde{g}(\hat{b}(x)|x) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = \\ & = \int_{\hat{x}-\Delta}^{x_0} \tilde{g}(\hat{b}(x)|x) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) + \\ & \quad + \int_{x_0}^{\hat{x}} \tilde{g}(\hat{b}(x)|x) \frac{x - \hat{x}}{\Delta} \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) > \\ & > \frac{x_0 - \hat{x}}{\Delta} \int_{\hat{x}-\Delta}^{x_0} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) + \\ & \quad + \frac{x_0 - \hat{x}}{\Delta} \int_{x_0}^{\hat{x}} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) = \\ & = \frac{x_0 - \hat{x}}{\Delta} \int_{\hat{x}-\Delta}^{\hat{x}} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) \geq 0 \end{aligned}$$

for sufficiently small  $\Delta > 0$ , because  $\int_{\hat{x}-\Delta}^{\hat{x}} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \hat{x}} (e(x) - \hat{b}(x)) dF(x) < 0$  by (20) and  $x_0 - \hat{x} \leq 0$ . The case  $x_0 > \hat{x}$  is analogous and therefore omitted. Consequently,

$$\begin{aligned} \frac{\partial W(\hat{x})}{\partial \Delta} & = \int_{\hat{x}-\Delta}^{\hat{x}} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) + \\ & \quad + \int_{\hat{x}}^{\hat{x}+\Delta} \tilde{g}(\hat{b}(x)|x) \frac{\partial \hat{b}(x)}{\partial \Delta} (e(x) - \hat{b}(x)) dF(x) > 0 \end{aligned}$$

for sufficiently small  $\Delta > 0$ . In addition, welfare is continuous in  $\Delta \in [0, \infty)$ . Hence, some legal uncertainty increases welfare.  $\square$

**Proof of Proposition 6:** Welfare equals

$$\int (1 - G(ap(x))) \left( \alpha \mathbb{E}(b|b \geq ap(x)) - e(x) + (p(x)(1 - \alpha) - \kappa)a \right) dF(x).$$

Pointwise optimization of welfare with respect to the fine  $p(x)$  yields

$$\begin{aligned} & -g(ap(x))a \left( \alpha \mathbb{E}(b|b \geq ap(x)) - e(x) + (p(x)(1 - \alpha) - \kappa)a \right) + \\ & + (1 - G(ap(x))) \left( \alpha \frac{\partial \mathbb{E}(b|b \geq ap(x))}{\partial p(x)} + (1 - \alpha)a \right) = 0. \end{aligned}$$

Lemma 4 (p.16) together with the chain rule results in  $\partial \mathbb{E}(b|b \geq ap(x))/\partial p(x) = g(ap(x))a \frac{\mathbb{E}(b|b \geq ap(x)) - ap(x)}{1 - G(ap(x))}$ . Inserting this derivative into the last equation yields

$$g(ap(x))a \left( e(x) - \alpha ap(x) - (p(x)(1 - \alpha) - \kappa)a + \frac{1 - G(ap(x))}{g(ap(x))}(1 - \alpha) \right) = 0$$

or

$$e(x) + a\kappa = ap^*(x) - \frac{1 - G(ap^*(x))}{g(ap^*(x))}(1 - \alpha). \quad (21)$$

By assumption, the hazard rate is increasing. Hence, the inverse hazard rate decreases and the right-hand side of the last equation increases in  $p^*(x)$ . Therefore the solution  $p^*(x)$  to (21) is unique and Eq. (21) implicitly defines a candidate  $p^*(x)$  for the optimal fine. The left-hand side of (21) increases in  $x$ . Hence,  $p^*(x)$  increases in  $x$ . Yet, the fine has to satisfy additional constraints.

If the enforcement authority sets a fine  $p(x) \in (0, \frac{\kappa}{1-\alpha})$  for some  $x$ , individuals anticipate that ex post the authority does not pursue its claims. Then individuals behave as if the penalty were 0. Making the claim verifiable is too costly for the enforcement authority. Therefore the authority does not enforce any penalties  $p(x) \in (0, \frac{\kappa}{1-\alpha})$ . Hence, the optimal penalty is either 0 or at least  $\frac{\kappa}{1-\alpha}$ . Consequently, the optimal fine is

$$p(x) = \begin{cases} 0 & \text{for } x < \hat{x} \\ \frac{\kappa}{1-\alpha} & \text{for } x \in [\hat{x}, x_1] \\ p^*(x) & \text{for } x > x_1. \end{cases}$$

$\hat{x}$  is determined – as in Lemma 2 – by  $e(\hat{x}) = \alpha \mathbb{E}(b \leq a \frac{\kappa}{1-\alpha})$ . Notice that  $p(1 - \alpha) - \kappa = 0$  for  $p = \frac{\kappa}{1-\alpha}$ .  $x_1$  is determined by  $p^*(x_1) = \frac{\kappa}{1-\alpha}$ . It remains to show  $\hat{x} < x_1$ . Plugging  $p(x) = \frac{\kappa}{1-\alpha}$  into Eq. (21) yields  $e(x) = \alpha a \frac{\kappa}{1-\alpha} - \frac{1 - G(ap(x))}{g(ap(x))}(1 - \alpha)$ . As  $\frac{1 - G(ap(x))}{g(ap(x))}(1 - \alpha) > 0$  and  $a \frac{\kappa}{1-\alpha} > \mathbb{E}(b \leq a \frac{\kappa}{1-\alpha})$ , the inequality  $\alpha \mathbb{E}(b \leq a \frac{\kappa}{1-\alpha}) < \alpha a \frac{\kappa}{1-\alpha} - \frac{1 - G(ap(x))}{g(ap(x))}(1 - \alpha)$  is valid. Therefore  $\hat{x} < x_1$ .  $\square$

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