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On the incentives of an integrated ISP to favor its own content*

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1 Introduction

One of the questions that has emerged in the *network neutrality* debate is the claim that integration between ISPs and CPs may conduct the market to a context where the integrated CPs have preferential treatment in the network to which they belong to, and therefore the non-integrated CPs tend to be squeezed to a small position in the market. For this purpose, one develops a model where two ISPs offer access to the internet to final consumers and offer network services to two rival CPs. The CPs sell content services to final consumers and advertising services.

The model has four novel aspects: **(i)** there are two rival ISPs, **(ii)** ISPs have full discretion on how to allocate quality of network services, i.e., on how to allocate bandwidth, **(iii)** CPs compete to sell content services directly to consumers, and in addition have advertising revenues, and **(iv)** ISPs may be integrated with CPs.

An important assumption of the model is that a consumer's choice of CP is independent of the choice of ISP. This assumption is motivated by tractability. However, it could be justified as follows. In general, the quality of service of CPs may differ across networks, depending on the quality of network service agreements they have with ISPs. However, consumers accede to the internet to consume many different services. The service provided by the CPs under analysis is only one among these many services. Hence, differences in quality of the service of the CPs under analysis across networks weight little in the consumers' choice of network. In section 3 one elaborates more on this issue.

The base model has three stages. In the first stage, ISPs and CPs bargain about the quality and price of network services. Each ISP makes a take-it-or-leave offer of network services to one CP, consisting of a price and quality level, and the CPs chooses to accept it or not. In the second stage, ISPs set the access price to the internet for consumers and consumers choose to which ISP they subscribe. In the third stage, CPs set the prices for their content services and consumers choose to which CP they subscribe.

The results show that, under no integration, both ISPs offer high quality network services to the CP that collects the highest advertising revenue. In fact, the increase in a CP's profit due to receiving high quality network services from one ISP is higher when it already benefits from high quality network services from the other ISP. Moreover, ISPs maximize the difference of quality of network services offered to the CPs, i.e., they offer the maximum quality level of network services to the CP that collects the highest advertising revenue, and offer the minimum quality level of network services to the other CP.

One then consider the case where one of the ISPs is integrated with one of the CPs, if

integration involves the CP that collects the highest advertising revenue, this CP gets the high quality network services from both ISPs. If integration involves the CP that collects the lowest advertising revenue, both ISPs still prefer to sell high quality network services to the CP that collects the highest advertising revenue, when the difference between the CPs' advertising revenues is large, in order to collect part of the higher advertising revenue. When the difference between advertising revenues is small, both ISPs sell the high quality network services to the vertically integrated CP, contrary to what happen under no-integration.

The remainder of the article is organized as follows. In Section 2 one reviews the literature. In section 3 the base model is presented and in section 5 one characterizes its equilibria. In section 5 the integration case is presented and in section 6 a welfare analysis is performed. Finally, in section 7 one concludes.

2 Literature Review

The model starts from a model similar to Choi and Kim (2010) and Cheng et al. (2011) regarding the incentives of ISP to discriminate a CP and the welfare impact of imposing network neutrality restrictions. Bandyopadhyay et al. (2010) is the closest article since it analyses the impact of vertical integration between ISPs and CPs but in a context where there is only ISP. They show that if network neutrality is not enforced, social welfare might increase or decrease with vertical integration. Moreover, they find that it is not always true that the ISP will degrade the delivery of the competing content, and in fact it will sometimes have the incentive to prioritize the latter over its own.

3 Model description

3.1 Environment

Consider an industry in which online *Content Providers* (CPs) deliver their content to consumers through broadband networks provided by *Internet Service Providers* (ISPs). There are two ISPs, firms $j = 1, 2$, and two CPs, firms $i = A, B$. The ISPs offer access to the internet to consumers, and network services to CPs. An ISP may offer the two CPs network services of different qualities and may charge a positive price for high quality network services. The CPs offer content services to consumers.

Denote by f_j on $[0, +\infty)$, the price ISP j charges consumers for internet access services, by q_{ji} on $[0, +\infty)$, the price ISP j charges CP i for network services, and by p_i on $[0, +\infty)$,

the price CP i charges consumers for content services.

The game has three stages that unfold as follows. In stage 1, each ISP makes a take-it-or-leave offer to a CP and the CPs accept or reject the offers they receive. The take-it-or-leave-it offer by each ISP consists of a price for higher quality. In stage 2, ISPs set the access price to the internet for consumers and consumers choose to which ISP they subscribe to. In stage 3, the CPs set prices for their content services and consumers choose to which CP they subscribe to.

3.2 Consumers

There is a large number of consumers, formally a continuum, whose measure is normalized to 1.

Regarding the preference for access to the internet, consumers are uniformly distributed along a Hotelling line segment of length 1 (Hotelling, 1929). A consumer whose most-preferred service is x on $[0, 1]$ has a quadratic disutility cost of $t_n x^2$ if he buys instead the service from the firm located at 0, with t_n on $(0, +\infty)$. Denote by u_j , the utility from getting access to the internet from ISP j . The utility of a consumer subscribing to the services of ISP j is then given by:

$$U_n = \begin{cases} (u_1 - f_1) - t_n x^2 & \text{if buys access from } ISP1 \\ (u_2 - f_2) - t_n (1 - x)^2 & \text{if buys access from } ISP2. \end{cases}$$

One assumes that ISPs may differ in the quality of access they offer to consumers. Hence, define the quality advantage of ISP 1 by:

$$\Delta := u_1 - u_2.$$

Parameter Δ can be positive, if ISP 1 offers a higher quality of access to the internet than ISP 2, or negative, otherwise. Moreover, one assumes $|\Delta| < 3t_n$ which ensures that both ISP's have a positive market share. ISP 1's market share is given by

$$\sigma_1 = \frac{1}{2} + \frac{1}{2t_n} (f_2 - f_1 + \Delta)$$

After getting access to the internet, consumers can buy content services from one of the two CPs. Regarding preferences for content services, consumers are also uniformly distributed along another Hotelling line segment of length 1. A consumer whose most-preferred service is x on $[0, 1]$ has a quadratic disutility cost of $t_c x^2$ if he buys instead the service from the firm located at 0, with t_c on $(0, +\infty)$.

Let u_c denote the utility derived from content, which one assumes to be equal for the two CPs, and $V_j\chi_{ji}$ denotes the quality of the network service that the ISP j offers to CP i , which is related with the time that each consumer has to wait in order to consume a CP's content. The utility of a consumer of ISP j subscribing the services of CP i is then given by:

$$U_c^j = \begin{cases} (u_c + V_j\chi_{jA} - p_A) - t_c x^2 & \text{if buys content from CP } A \\ (u_c + V_j\chi_{jB} - p_B) - t_c (1 - x)^2 & \text{if buys content from CP } B. \end{cases}$$

CP A 's market share within ISP j 's subscribers is

$$\sigma_{jA} = \frac{1}{2} + \frac{1}{2t_c} (p_B - p_A + V_j\chi_{Aj} - V_j\chi_{Bj})$$

Note that, in the decision about to which ISP to subscribe, consumers do not take into account the utility they will later obtain from CPs. This assumption is motivated by tractability. Even though, in general, the decision by a consumer about the best ISP is based on other factors such as the brand awareness and service quality (e.g. network speed). On the other hand, consumers accede to the internet to consume several services provided by different CPs, being the service provided by the CPs under analysis only one among these many services.

3.3 Firms

3.3.1 ISPs

ISP 1 is located at point 0 and ISP 2 at point 1 of the Hotelling line segment where consumers are distributed regarding their preferences for access to the internet services. All of the ISPs' marginal costs are constant and equal to zero.

Let D_j be the total demand of ISP j . The profit of ISP j is then given by:

$$\Pi_j = f_j D_j + \sum_{i=A}^B q_{ji} \chi_{ji},$$

where χ_{ji} takes value 1 if ISP j sells a higher quality of network services to CP i and 0 if not.

The difference in the quality of network services provided by ISP j to CP A and to CP B , v_j , is either V_j or $-V_j$. It is positive if ISP j provides a higher quality service to CP A than to CP B , and negative otherwise. V_j is assumed to belong to $(0, 3t_c)$ in order to ensure positive demand.

The network services bargaining game unfolds as follows. First, ISPs make, in sequence, a public take-it-or-leave offer to one CP, composed of a network services quality level and a

price. The ISPs can make an offer to the same CP, or to different CPs. Afterwards the CPs decide which offers they accept if any. If a CP rejects its offers, or is made no offer, it gets network services with the low quality free of charge.

3.3.2 CPs

CP A is located at point 0 and CP B at point 1 of the Hotelling line segment where consumers are distributed regarding their preferences for content services. Both CPs' marginal costs are constant and equal to c .

Each CP i has two revenue sources: **(i)** it charges each consumer a price, p_i , for access to its content, and **(ii)** obtains revenue stream, r_i , from advertisers for each consumer's content request it serves. Values r_i are exogenous, determined, possibly, in a competitive advertising market. Assume that CP A earns at least the same advertising revenue per consumer as CP B , i.e.,

$$R := r_A - r_B \text{ is on } [-r_{\max}, r_{\max}],$$

with $r_{\max} := 3t_c - V_j > 0$.¹ This means that CP A may be better or worse than its rival in getting the "right" consumers for its advertisers and therefore can charge higher advertising fees.²

Let D_i be the total demand of CP i . The profit of CP i is given by:

$$\Pi_i = (p_i + r_i - c)D_i - \sum_{j=1}^2 q_{ji}\chi_{ji}.$$

4 Equilibria with no integration between ISP and CP

Next, the equilibrium when there is no integration between ISPs and CPs is presented.

Proposition 1: *When there is no integration between ISPs and CPs,*

(i) CPs charge consumers for access to content:

$$\begin{aligned} p_A &= c + t_c - \frac{2r_A + r_B}{3} + \frac{\sigma_1 v_1 + \sigma_2 v_2}{3} \\ p_B &= c + t_c - \frac{r_A + 2r_B}{3} - \frac{\sigma_1 v_1 + \sigma_2 v_2}{3}; \end{aligned}$$

and have profit:

$$\begin{aligned} \pi_A(v_1, v_2) &= \frac{1}{18} t_c^{-1} (3t_c + R + v_1 \sigma_1 + v_2 \sigma_2)^2 \\ \pi_B(v_1, v_2) &= \frac{1}{18} t_c^{-1} (3t_c - R + v_1 \sigma_1 + v_2 \sigma_2)^2 \end{aligned}$$

¹These assumptions ensure that both CPs always get a positive market share.

²Cheng et al. (2011) refer that they find empirical evidence supporting this assumption.

(ii) ISPs charge consumers for access to the internet:

$$f_1 = t_n + \frac{\Delta}{3} \text{ and } f_2 = t_n - \frac{\Delta}{3};$$

(iii) a) If $R > 0$ both offers are made to CP A. ISP 1 offers $q_{1A} = \pi_A(V_1, V_2) - \pi_A(0, V_2)$ and ISP 2 offers $q_{2A} = \pi_A(0, V_2) - \pi_A(0, 0)$. CP A will get $\pi_A(0, 0)$ and CP B will get $\pi_B(-V_1, -V_2)$

b) If $R < 0$ both offers are made to CP B. ISP 1 offers $q_{1B} = \pi_B(V_1, V_2) - \pi_B(0, V_2)$ and ISP 2 offers $q_{2B} = \pi_B(0, V_2) - \pi_B(0, 0)$. CP B will get $\pi_B(0, 0)$ and CP A will get $\pi_A(-V_1, -V_2)$. ■

In equilibrium, both ISPs make an offer to the same CP since the increase in CP's profit due to the high quality network service in one ISP is higher when it already benefits from high quality network service in the other ISP. This implies that given an offer from an ISP to a given CP, the best response by the other CP is also to make an offer to the same CP. Moreover, the offer is made to the most profitable CP, since this is the one from which ISPs can extract a higher rent.

As concerns the internet access services market, one finds that the higher is the advantage of a given ISP, the higher are its price, market share and profits from selling access to the internet.

Finally, in what regards the content services market, one shows that for given advertising revenues, if a CP obtains network services from an ISP that are of a higher quality than those obtained by its rival CP, it can also offer higher quality content services to its consumers, and thereby charge a higher price for its content services. In turn, the CP that gets network services of lower quality has to reduce the price of its content services to compensate consumers for the lower quality of service. In addition, the CP that gets the network services of higher quality also gets a higher market share, in spite of charging a higher price for content services.

5 Equilibria with integration between ISP and CP

In the previous section, it was determined that, when there is no relation of ownership between ISPs and CPs, in equilibrium both ISPs offer high quality network services to CP A, the CP with the highest advertising revenue. In the present section it will be checked if this result is still true if an ISP 1 is integrated with CP A, while the other firms are independent.

One again assumes that each ISP makes one take-it-or-leave-it offer, however, and contrary to the previous section, the integrated ISP has always the option of offering the high quality network service to its CP when the outside CP rejects the offer. Moreover, one assumes that the integrated ISP will be the first to make the high quality network service offer.

The next Proposition presents the equilibrium. Let $R_1 = \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)} (2(V_1\sigma_1 - 3t_c) + V_2\sigma_2) < 0$.

Proposition 2: When ISP 1 is integrated with CP A :

- (i) If $R > \max\{-V_1\sigma_1, R_1\}$, ISP 1 does not make any offer and ISP 2 offers $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ to CP A.
- (ii) If $-V_1\sigma_1 < R < R_1$, ISP 1 makes offer $q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0)$ to CP B and ISP 2 makes offer $q_{2B} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ to CP B.
- (iii) If $R < -V_1\sigma_1$, ISP 1 offers $q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$ to CP B and ISP 2 offers $q_{2B} = \pi_B(-V_1, V_2) - \pi_B(-V_1, 0)$ to CP B. ■

As expected, and similarly to the non-integration case, when one of the ISPs is integrated with the CP with the highest advertising revenue, i.e. $R > 0$, in equilibrium CP A gets the high quality network service from both ISPs.

In case of integration of an ISP with the CP with the lowest advertising revenue, i.e., $R < 0$, both ISPs sell the highest quality network service to CP B when the difference between the advertising revenue it is able to collect as compared to the one collected by CP A takes high values. Note that in this case, although one of the ISPs is integrated with CP A, it still prefers to sell the highest quality network service to its rival in order to take advantage of the higher advertising revenues.

When R is negative but takes intermediate (less extreme) values, ISP 1 is forced to lower its offer to CP B. In fact, a higher offer to CP B would lead ISP 2 to react with an offer to CP A. In this case, ISP 1's offer to CP B would not be accepted. By lowering the offer to CP B, ISP 1 induces ISP 2 also to make an offer to CP B, both of which will be accepted.

6 Welfare analysis

The next Remark presents the comparison of each parties payoffs in the under integration and no integration. Let $R_2 = \frac{(\sigma_1 V_1 (V_1 \sigma_1 + 2V_2 \sigma_2 - 6t_c) + \sigma_2 V_2 (2V_1 \sigma_1 + V_2 \sigma_2 - 6t_c))}{2(3V_1 \sigma_1 + V_2 \sigma_2)}$

Proposition 3:

- (i) If $R > 0$, joint profit of the integrated firm is smaller than the sum of the profits under no integration. CP B payoff is the same under integration and no-integration. ISP 2 is better with integration. Consumer surplus is the same under integration and no-integration. Social welfare is the same under integration and no-integration.
- (ii) If $-V_1\sigma_1 < R < 0$ and $R > R_1$, joint profit of the integrated firm is larger than the sum of the profits under no integration if $R > R_2$ and smaller otherwise. CP B is worst with integration. ISP 2 is better with integration if $R > -\frac{\sigma_1 V_1}{2}$ and worst otherwise. Consumer surplus decreases with integration. Social welfare is lower with integration.
- (iii) If $-V_1\sigma_1 < R < 0$ and $R < R_1$, joint profit of the integrated firm is larger than the sum of the profits under no integration. CP B is worst with integration. ISP 2 is better with integration if $R > -\frac{\sigma_1 V_1}{2}$ and worst otherwise. Consumer surplus is the same under integration and no-integration. Social welfare is the same under integration and no-integration.
- (iii) If $R < -V_1\sigma_1$, joint profit of the integrated firm is larger than the sum of the profits under no integration. CP B is worst with integration. ISP 2 is worst with integration. Consumer surplus is the same under integration and no-integration. Social welfare is the same under integration and no-integration. ■

When one ISP is integrated with the CP with the highest advertising revenue, the integrated firm is worse-off than the two firms under no integration. In fact, after integration, aggregate profit is $\pi_A(V_1, 0)$ whereas under no integration, ISP 1 and CP A's profit are $\pi_A(V_1, V_2) - \pi_A(0, V_2) + \pi_A(0, 0) > \pi_A(V_1, 0)$. ISP 2, however, benefits from integration, getting $\pi_A(V_1, V_2) - \pi_A(V_1, 0) > \pi_A(0, V_2) - \pi_A(0, 0)$. The reason is that the integrated firm, is willing to allow ISP 2 to capture all the benefit of selling higher quality. The payoff of remaining players of the market does not change as well as social welfare since in both the scenario of integration and no-integration it is the same CP that gets the high quality network service.

When one of the ISPs is integrated with the CP with the highest advertising revenue, and the difference in advertising revenues is high, the total rent that ISPs are able to collect from CP B, $\pi_B(V_1, V_2) - \pi_B(-V_1, 0)$ is higher than in the non-integration scenario, $\pi_B(V_1, V_2) - \pi_B(0, 0)$, since in the case of integration there is one additional threat to CP B, which is the fact that if CP B refuses the offer by the vertically integrated ISP, the highest quality is given to its rival, contrary to the previous scenario where, in case of refusal to accept the offer,

CP B knew that nobody would benefit from the highest quality network service. Thus, the profits of the vertically integrated firm, $\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$ are higher than the sum of profits of the firms when separated, $\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(0, V_2)$. The opposite is true for the non-integrated ISP 2 ($\pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(0, V_2) - \pi_B(0, 0)$) if the difference in advertising revenues is sufficiently high, although if this is not very high the non-integrated ISP also benefits from collecting higher rents from CP 2 and gets higher profits in the scenario of integration. Consumer surplus and social welfare are the same under both scenarios since in both cases it is the same CP that benefits from the high quality network services.

When one of the ISPs is integrated with the CP with the highest advertising revenue, and the difference in advertising revenues assumes intermediate values. the profit of the integrated firm may be higher or lower than the two firms under no integration. In fact, on the one hand, it charges higher rents if non-integrated since it sells higher quality network services to the highest advertising revenue CP, however the CP suffers from a quality disadvantage. On the contrary under integration it does not extract the highest rent but assures higher network services for the CP. The lower the difference of advertising revenues the lower the increase in rents it can obtain from the rival CP under no integration. As concerns the non-integrated ISP a similar result happens. Under integration it has higher capacity to obtain the rents from the integrated ISP, but under no-integration it obtains rents from the highest advertising revenue CP. The lower the difference between the advertising revenue the lower is this latter effect and thus the more probable it obtains higher profits under integration.

Regarding social welfare it is higher under no-integration since the average quality of network services is higher in that scenario. In fact, under no-integration the CP that gets higher quality of network services is the one able to collect higher advertising revenues which induces it to charge a lower price. Thus this CP has a higher market share when it obtains the higher quality network services than its rival when it is the one obtaining the higher quality network services. Therefore, the average quality of network services consumed is higher under no-integration. Note that this also implies that transport costs are higher under no-integration but this effect is not sufficiently strong to compensate the previous one.

7 Conclusions

In this article, one starts to show that, when ISPs can give a discriminatory treatment to CPs, both ISPs sell high quality network services to the CP that collects the highest advertising revenue. As the fee they are able to charge a CP increases with the difference of treatment given to CPs, ISPs offer the maximum quality level of network services to the CP that collects the highest advertising revenue, and offer the minimum quality level of network services to the other CP.

One of the questions that has emerged in the *network neutrality* debate is the claim that integration between ISPs and CPs may conduct the market to a context where the integrated CPs have preferential treatment in the network to which they belong to, and therefore the non-integrated CPs tend to be squeezed to a small position in the market. In this paper, one shows that this may not always be true. In fact, both ISPs still prefer to sell the high quality network service to the highest advertising revenue CP, even if one of the ISPs is integrated with the other CP. Only when the difference between the advertising revenue of CPs takes low values, both ISPs opt for selling the high quality network service to the integrated CP.

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Appendix

Proof of Proposition 2:

Consider the case of no integration. Solving the game by backward induction, one needs to analyse two possible subgames:

I) ISP 1 has offered q_{1B} to CP B.

A) Consider first that ISP2 also makes an offer to CP B. An offer q_{2B} by ISP 2 to CP B is accepted if and only if:

$$\max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(0, V_2) - q_{2B} \} \geq \max \{ \pi_B(V_1, 0) - q_{1B}, \pi_B(0, 0) \} \quad (1)$$

Notice that $\max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(0, V_2) - q_{2B} \} = \pi_B(V_1, V_2) - q_{1B} - q_{2B}$ if and only if

$$\pi_B(V_1, V_2) - q_{1B} - q_{2B} > \pi_B(0, V_2) - q_{2B} \Leftrightarrow q_{1B} < \pi_B(V_1, V_2) - \pi_B(0, V_2)$$

and that $\max \{ \pi_B(V_1, 0) - q_{1B}, \pi_B(0, 0) \} = \pi_B(V_1, 0) - q_{1B}$ if and only if

$$\pi_B(V_1, 0) - q_{1B} > \pi_B(0, 0) \Leftrightarrow q_{1B} < \pi_B(V_1, 0) - \pi_B(0, 0)$$

with $\pi_B(V_1, V_2) - \pi_B(0, V_2) > \pi_B(V_1, 0) - \pi_B(0, 0)$ because

$$\pi_B(V_1, V_2) - \pi_B(0, V_2) - \pi_B(V_1, 0) + \pi_B(0, 0) > 0 \Leftrightarrow \frac{1}{9}t_c^{-1}\sigma_2\sigma_1V_2V_1 > 0$$

Hence,

1) if $q_{1B} < \pi_B(V_1, 0) - \pi_B(0, 0) < \pi_B(V_1, V_2) - \pi_B(0, V_2)$, then 1 is equivalent to

$$q_{2B} \leq \pi_B(V_1, V_2) - \pi_B(V_1, 0).$$

Therefore, the highest offer that will be accepted is

$$q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

and, in this case, both offers will be accepted.

2) if $\pi_B(V_1, 0) - \pi_B(0, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(0, V_2)$, then 1 is equivalent to

$$q_{2B} \leq \pi_B(V_1, V_2) - \pi_B(0, 0) - q_{1B}.$$

Therefore, the highest offer that will be accepted is

$$q_{2B} = \pi_B(V_1, V_2) - \pi_B(0, 0) - q_{1B}$$

and, in this case, both offers will be accepted.

3) if $q_{1B} > \pi_B(V_1, V_2) - \pi_B(0, V_2)$, then 1 is equivalent to

$$q_{2B} \leq \pi_B(0, V_2) - \pi_B(0, 0)$$

Therefore, the highest offer that will be accepted is

$$q_{2B} = \pi_B(0, V_2) - \pi_B(0, 0)$$

and, in this case, only ISP 2's offer will be accepted.

Summing up, ISP2's highest acceptable offer, when offering to CP B, given q_{1B} is then:

$$q_{2B} = \begin{pmatrix} \pi_B(V_1, V_2) - \pi_B(V_1, 0) \\ \pi_B(V_1, V_2) - \pi_B(0, 0) - q_{1B} \\ \pi_B(0, V_2) - \pi_B(0, 0) \end{pmatrix} \text{ if } \begin{pmatrix} q_{1B} < \pi_B(V_1, 0) - \pi_B(0, 0) \\ \pi_B(V_1, 0) - \pi_B(0, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(0, V_2) \\ q_{1B} > \pi_B(V_1, V_2) - \pi_B(0, V_2) \end{pmatrix}$$

If $q_{1B} > \pi_B(V_1, V_2) - \pi_B(0, V_2)$, ISP 2, if making an offer to CP B, will make an offer such that ISP 1's offer will not be accepted. In this case, ISP 1's payoff is 0. Otherwise it will get q_{1B} .

B) Consider now that ISP 2 makes an offer to CP A when ISP 1 had made an offer to CP B. The acceptance/rejection game played by the two CP's is as follows:

		CP B	
		Accept	Reject
CP A	Accept	$\pi_A(-V_1, V_2) - q_{2A}, \pi_B(V_1, -V_2) - q_{1B}$	$\pi_A(0, V_2) - q_{2A}, \pi_B(0, -V_2)$
	Reject	$\pi_A(-V_1, 0), \pi_B(V_1, 0) - q_{1B}$	$\pi_A(0, 0), \pi_B(0, 0)$

Acceptance of the two offers is an equilibrium if and only if these two inequalities hold:

$$\begin{aligned} \pi_A(-V_1, V_2) - q_{2A} &\geq \pi_A(-V_1, 0) \\ \pi_B(V_1, -V_2) - q_{1B} &\geq \pi_B(0, -V_2) \end{aligned}$$

Acceptance of q_{2A} and rejection of q_{1B} is an equilibrium if and only if these two inequalities hold:

$$\begin{aligned} \pi_A(0, V_2) - q_{2A} &\geq \pi_A(0, 0) \\ \pi_B(V_1, -V_2) - q_{1B} &< \pi_B(0, -V_2) \end{aligned}$$

ISP 2 will make the highest offer that is accepted (if positive), given q_{1B} . Hence,

$$q_{2A} = \begin{cases} \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) & \text{if } q_{1B} \leq \pi_B(V_1, -V_2) - \pi_B(0, -V_2) \\ \pi_A(0, V_2) - \pi_A(0, 0) & \text{if } q_{1B} > \pi_B(V_1, -V_2) - \pi_B(0, -V_2) \end{cases}$$

Note that

$$\pi_B(V_1, 0) - \pi_B(0, 0) > \pi_B(V_1, -V_2) - \pi_B(0, -V_2) \Leftrightarrow \frac{1}{9}t_c^{-1}\sigma_2\sigma_1V_2V_1 > 0$$

Hence:

$$\pi_B(V_1, -V_2) - \pi_B(0, -V_2) < \pi_B(V_1, 0) - \pi_B(0, 0) < \pi_B(V_1, V_2) - \pi_B(0, V_2)$$

and there are four possibilities to consider when comparing ISP 2's payoffs under the alternatives of making offers to CP A or to CP B, given that ISP 1 has made an offer q_{1B} to CP B:

i) When $q_{1B} < \pi_B(V_1, -V_2) - \pi_B(0, -V_2)$ it is preferable for ISP 2 to offer to CP A if and only if

$$\pi_A(-V_1, V_2) - \pi_A(-V_1, 0) > \pi_B(V_1, V_2) - \pi_B(V_1, 0) \Leftrightarrow R > V_1\sigma_1$$

ii) When $\pi_B(V_1, -V_2) - \pi_B(0, -V_2) < q_{1B} < \pi_B(V_1, 0) - \pi_B(0, 0)$ it is preferable for ISP 2 to offer to CP A if and only if:

$$\pi_A(0, V_2) - \pi_A(0, 0) > \pi_B(V_1, V_2) - \pi_B(V_1, 0) \Leftrightarrow R > \frac{V_1\sigma_1}{2}$$

iii) When $\pi_B(V_1, 0) - \pi_B(0, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(0, V_2)$ it is preferable for ISP 2 to offer to CP A if and only if:

$$\begin{aligned} \pi_A(0, V_2) - \pi_A(0, 0) &> \pi_B(V_1, V_2) - \pi_B(0, 0) - q_{1B} \Leftrightarrow \\ q_{1B} &> \frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2R(V_1\sigma_1 + 2V_2\sigma_2)) \quad (2) \end{aligned}$$

Checking if this is possible in the relevant interval for q_{1B} .

$$\pi_B(V_1, 0) - \pi_B(0, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(0, V_2) \\ \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 0 + \sigma_2 0)^2}{18t_c} < q_{1B} < \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 0 + \sigma_2 V_2)^2}{18t_c}$$

It can be showed that condition 2 is always true if $R > \frac{V_1\sigma_1}{2}$ and that it is impossible if $R < 0$.

iv) When $q_{1B} > \pi_B(V_1, V_2) - \pi_B(0, V_2)$ it is preferable for ISP 2 to offer to CP A if and only if:

$$\pi_A(0, V_2) - \pi_A(0, 0) > \pi_B(0, V_2) - \pi_B(0, 0) \Leftrightarrow R > 0.$$

Hence, given that ISP 1 has made an offer to CP B, ISP 2 will:

If $R < 0$ ISP 2 offers CP B

If $0 < R < \frac{V_1\sigma_1}{2}$ ISP 2 offers CP B if $q_{1B} < \frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2R(V_1\sigma_1 + 2V_2\sigma_2))$

If $0 < R < \frac{V_1\sigma_1}{2}$ ISP 2 offers CP A if $q_{1B} > \frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2R(V_1\sigma_1 + 2V_2\sigma_2))$

If $R > \frac{V_1\sigma_1}{2}$ ISP 2 offers CP A

Anticipating this, ISP 1, if offering to CP B, will offer the highest offer that will be accepted:

When ISP 2 offers CP B, ISP1 is rejected if $q_{1B} > \pi_B(V_1, V_2) - \pi_B(0, V_2)$.

When ISP 2 offers CP A, ISP1 is rejected if $q_{1B} > \pi_B(V_1, -V_2) - \pi_B(0, -V_2)$.

Hence, if offering to CP B, ISP 1 will offer:

If $R < 0$ ISP 1 offers $q_{1B} = \pi_B(V_1, V_2) - \pi_B(0, V_2)$

If $0 < R < \frac{V_1\sigma_1}{2}$ ISP 1 will offer $q_{1B} = \frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2R(V_1\sigma_1 + 2V_2\sigma_2))$

If $R > \frac{V_1\sigma_1}{2}$ ISP 1 will offer $q_{1B} = \pi_B(V_1, -V_2) - \pi_B(0, -V_2)$

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2 * 0 * (V_1\sigma_1 + 2V_2\sigma_2)) = \pi_B(V_1, V_2) - \pi_B(0, V_2)$$

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2 * 0 * (V_1\sigma_1 + 2V_2\sigma_2)) = \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c}$$

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2 * 0 * (V_1\sigma_1 + 2V_2\sigma_2)) - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c}$$

$$: \frac{1}{9}t_c^{-1}\sigma_1 V_1 R$$

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2 * \frac{V_1\sigma_1}{2} * (V_1\sigma_1 + 2V_2\sigma_2)) = \pi_B(V_1, -V_2) - \pi_B(0, -V_2)$$

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2 * \frac{V_1\sigma_1}{2} * (V_1\sigma_1 + 2V_2\sigma_2)) = \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 0 - \sigma_2 V_2)^2}{18t_c}$$

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2 * \frac{V_1\sigma_1}{2} * (V_1\sigma_1 + 2V_2\sigma_2)) - \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 0 - \sigma_2 V_2)^2}{18t_c} =$$

$$\frac{1}{18}t_c^{-1}(2 * \frac{V_1\sigma_1}{2} - V_1\sigma_1 + 2V_2\sigma_2) V_1\sigma_1 : \frac{1}{9}t_c^{-1}\sigma_2\sigma_1 V_2 V_1$$

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c}$$

II) Assume now that ISP 1 has made an offer to CP A. Then, everything is as above

but with A and B swapped and with $-R$ instead of R .

Thus, if offering to CP A, ISP 1 will offer

If $R > 0$ ISP 1 offers $q_{1A} = \pi_A(V_1, V_2) - \pi_A(0, V_2)$

If $-\frac{V_1\sigma_1}{2} < R < 0$ ISP 1 will offer $q_{1A} = \frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) + 2R(V_1\sigma_1 + 2V_2\sigma_2))$

If $R < -\frac{V_1\sigma_1}{2}$ ISP 1 will offer $q_{1A} = \pi_A(V_1, -V_2) - \pi_A(0, -V_2)$.

Establishing if it is better for ISP 1 to offer CP A or CP B.

If $R < -\frac{V_1\sigma_1}{2}$ it is better to offer CP A if and only if

$$\pi_A(V_1, -V_2) - \pi_A(0, -V_2) > \pi_B(V_1, V_2) - \pi_B(0, V_2)$$

$$\pi_A(V_1, -V_2) - \pi_A(0, -V_2) - \pi_B(V_1, V_2) + \pi_B(0, V_2) > 0$$

$$\frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 0 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c} = \left(-\frac{2}{9}\right)t_c^{-1}(V_2\sigma_2 - R)V_1\sigma_1$$

0

$R > V_2\sigma_2$ (impossible)

If $-\frac{V_1\sigma_1}{2} < R < 0$ it is better to offer CP A if and only if

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) + 2R(V_1\sigma_1 + 2V_2\sigma_2)) > \pi_B(V_1, V_2) - \pi_B(0, V_2)$$

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) + 2R(V_1\sigma_1 + 2V_2\sigma_2)) - \pi_B(V_1, V_2) + \pi_B(0, V_2) > 0$$

$$\frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) + 2R(V_1\sigma_1 + 2V_2\sigma_2)) - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c} =$$

$$\frac{2}{9}t_c^{-1}(V_1\sigma_1 + V_2\sigma_2)R > 0 \text{ (impossible)}$$

If $0 < R < \frac{V_1\sigma_1}{2}$ it is better to offer CP A if and only if

$$\pi_A(V_1, V_2) - \pi_A(0, V_2) > \frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2R(V_1\sigma_1 + 2V_2\sigma_2))$$

$$\frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c} - \frac{1}{18}t_c^{-1}(\sigma_1 V_1(6t_c + V_1\sigma_1 + 2V_2\sigma_2) - 2R(V_1\sigma_1 + 2V_2\sigma_2)) >$$

0

$$\frac{2}{9}t_c^{-1}(V_1\sigma_1 + V_2\sigma_2)R > 0 \text{ (always true)}$$

If $R > \frac{V_1\sigma_1}{2}$ it is better to offer CP A if and only if

$$\pi_A(V_1, V_2) - \pi_A(0, V_2) > \pi_B(V_1, -V_2) - \pi_B(0, -V_2)$$

$$\frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 0 - \sigma_2 V_2)^2}{18t_c} > 0$$

$$\frac{2}{9}t_c^{-1}(R + V_2\sigma_2)V_1\sigma_1 > 0 \text{ (always true).} \quad \blacksquare$$

8 Sequential

Timing is as follows: ISP 1 offers a public contract to CP B or makes an internal offer. ISP 2 observes this offer and makes an offer either to CP A or to CP B. Both CP's observe the two offers and simultaneously decide whether to accept the offer each one was made.

CP A and ISP 1 are integrated, meaning that CP A and ISP 1 both make their decisions to maximize aggregate profits.

8.1 Subgame type 1: ISP 1 does not make any offer (i.e. it offers internally to CP A)

a) An offer by ISP 2 to CP A is accepted if and only if:

$$\pi_A(V_1, V_2) - q_{2A} \geq \pi_A(V_1, 0)$$

$$q_{2A} \leq \pi_A(V_1, V_2) - \pi_A(V_1, 0) = \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} = \frac{1}{18}t_c^{-1}(2R + 6t_c + 2V_1\sigma_1 + V_2\sigma_2)$$

$\sigma_2 > 0$

b) An offer by ISP 2 to CP B is accepted if and only if:

$$\pi_B(-V_1, V_2) - q_{2B} \geq \pi_B(-V_1, 0)$$

$$q_{2B} \leq \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) = \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} = \left(-\frac{1}{18}\right)t_c^{-1}(2R - 6t_c + 2V_1\sigma_1$$

0.

ISP 2 will make the most profitable offer that is accepted. It is more profitable for ISP 2 to make the offer to CP A if and only if:

$$q_{2A} > q_{2B}$$

$$\pi_A(V_1, V_2) - \pi_A(V_1, 0) > \pi_B(-V_1, V_2) - \pi_B(-V_1, 0)$$

$$\pi_A(V_1, V_2) - \pi_A(V_1, 0) - \pi_B(-V_1, V_2) + \pi_B(-V_1, 0) > 0$$

$$\frac{(3t_c+R+\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c+R+\sigma_1V_1+\sigma_20)^2}{18t_c} - \frac{(3t_c-R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} + \frac{(3t_c-R-\sigma_1V_1+\sigma_20)^2}{18t_c} > 0 \frac{2}{9}t_c^{-1} (R + V_1\sigma_1) V_2\sigma_2 > 0 \Leftrightarrow R > -V_1\sigma_1.$$

Even if CP B has an advantage over the integrated CP A in terms of advertising revenue, ISP 2 may still make the offer to CP A. This result from the "complementarities" involved and from the fact that ISP 1 is not making any offer to CP B.

ISP 1's payoff is:

$$\left\{ \begin{array}{l} \pi_A(V_1, V_2) - q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, V_2) + \pi_A(V_1, 0) = \pi_A(V_1, 0) = \frac{(3t_c+R+\sigma_1V_1+\sigma_20)^2}{18t_c} \\ \pi_A(V_1, -V_2) = \frac{(3t_c+R+\sigma_1V_1-\sigma_2V_2)^2}{18t_c} \end{array} \right. \text{ if } \left\{ \begin{array}{l} R > -V_1\sigma_1 \\ R < -V_1\sigma_1 \end{array} \right.$$

ISP 2's payoff is:

$$\left\{ \begin{array}{l} \pi_A(V_1, V_2) - \pi_A(V_1, 0) = \frac{1}{18}t_c^{-1} (2R + 6t_c + 2V_1\sigma_1 + V_2\sigma_2) V_2\sigma_2 \\ \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) = \left(\frac{1}{18}\right) t_c^{-1} (-2R + 6t_c - 2V_1\sigma_1 + V_2\sigma_2) V_2\sigma_2 \end{array} \right. \text{ if } \left\{ \begin{array}{l} R > -V_1\sigma_1 \\ R < -V_1\sigma_1 \end{array} \right.$$

8.2 Subgame type 2: ISP 1 offers q_{1B} to CP B.

a) An offer q_{2B} by ISP 2 to CP B is accepted if and only if CP B's payoff of accepting both offers or only ISP 2's offer exceeds its payoff when rejecting both offers or only accepting ISP 1's offer. This happens when:

$$\max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(-V_1, V_2) - q_{2B} \} \geq \max \{ \pi_B(V_1, 0) - q_{1B}, \pi_B(-V_1, 0) \}$$

$$\text{Notice that } \max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(-V_1, V_2) - q_{2B} \} = \pi_B(V_1, V_2) - q_{1B} - q_{2B}$$

$$\text{iff } \pi_B(V_1, V_2) - q_{1B} - q_{2B} > \pi_B(-V_1, V_2) - q_{2B}$$

$$\pi_B(V_1, V_2) - \pi_B(-V_1, V_2) > q_{1B}$$

and that

$$\max \{ \pi_B(V_1, 0) - q_{1B}, \pi_B(-V_1, 0) \} = \pi_B(V_1, 0) - q_{1B}$$

$$\text{iff } \pi_B(V_1, 0) - q_{1B} > \pi_B(-V_1, 0) \Leftrightarrow$$

$$\pi_B(V_1, 0) - \pi_B(-V_1, 0) > q_{1B}$$

$$\text{with } \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) > \pi_B(V_1, 0) - \pi_B(-V_1, 0)$$

$$\text{because } \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) - \pi_B(V_1, 0) + \pi_B(-V_1, 0) > 0$$

$$\frac{(3t_c-R+\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c-R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c-R+\sigma_1V_1+\sigma_20)^2}{18t_c} + \frac{(3t_c-R-\sigma_1V_1-\sigma_20)^2}{18t_c} = \frac{2}{9}t_c^{-1} \sigma_2\sigma_1 V_2 V_1 >$$

0

Hence,

$$1) \text{ if } q_{1B} < \pi_B(V_1, 0) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$$

$$\max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(-V_1, V_2) - q_{2B} \} \geq \max \{ \pi_B(V_1, 0) - q_{1B}, \pi_B(-V_1, 0) \} \Leftrightarrow$$

$$\pi_B(V_1, V_2) - q_{1B} - q_{2B} > \pi_B(V_1, 0) - q_{1B} \Leftrightarrow$$

$$\pi_B(V_1, V_2) - \pi_B(V_1, 0) \geq q_{2B}$$

$$q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0).$$

In this case, ISP 1's offer is so low that it does not affect the best offer by ISP 2. Both offers are accepted.

$$2) \text{ if } \pi_B(V_1, 0) - \pi_B(-V_1, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$$

$$\max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(-V_1, V_2) - q_{2B} \} \geq \max \{ \pi_B(V_1, 0) - q_{1B}, \pi_B(-V_1, 0) \} \Leftrightarrow$$

$$\pi_B(V_1, V_2) - q_{1B} - q_{2B} \geq \pi_B(-V_1, 0) \Leftrightarrow$$

$$\pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{1B} \geq q_{2B}$$

$$q_{2B} = \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{1B}.$$

In this case, ISP 1's offer will limit the maximum that ISP 2 may offer that leads both offers to be accepted.

$$3) \text{ if } q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$$

$$\max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(-V_1, V_2) - q_{2B} \} \geq \max \{ \pi_B(V_1, 0) - q_{1B}, \pi_B(-V_1, 0) \} \Leftrightarrow$$

$$\pi_B(-V_1, V_2) - q_{2B} \geq \pi_B(-V_1, 0) \Leftrightarrow$$

$$\pi_B(-V_1, V_2) - \pi_B(-V_1, 0) \geq q_{2B}$$

$q_{2B} = \pi_B(-V_1, V_2) - \pi_B(-V_1, 0)$. In this case, ISP 1's offer is so high that it will be rejected and only ISP 2's offer is accepted.

ISP2's highest acceptable offer, when offering to CP B, given q_{2B} is then:

$$q_{2B} = \begin{pmatrix} \pi_B(V_1, V_2) - \pi_B(V_1, 0) \\ \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{1B} \\ \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) \end{pmatrix} \text{ if } \begin{pmatrix} q_{1B} < \pi_B(V_1, 0) - \pi_B(-V_1, 0) \\ \pi_B(V_1, 0) - \pi_B(-V_1, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \\ q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \end{pmatrix}$$

$$q_{2B} = \begin{pmatrix} \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} \\ \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - q_{1B} \\ \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} \end{pmatrix} \text{ if } \begin{pmatrix} q_{1B} < \pi_B(V_1, 0) - \pi_B(-V_1, 0) \\ \pi_B(V_1, 0) - \pi_B(-V_1, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \\ q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \end{pmatrix}$$

$$q_{2B} = \begin{pmatrix} \frac{1}{18} t_c^{-1} (6t_c - 2R + 2V_1\sigma_1 + V_2\sigma_2) V_2\sigma_2 \\ \frac{1}{18} t_c^{-1} (6t_c - 2R + V_2\sigma_2) (2V_1\sigma_1 + V_2\sigma_2) - q_{1B} \\ (-\frac{1}{18}) t_c^{-1} (2R - 6t_c + 2V_1\sigma_1 - V_2\sigma_2) V_2\sigma_2 \end{pmatrix} \text{ if } \begin{pmatrix} q_{1B} < \pi_B(V_1, 0) - \pi_B(-V_1, 0) \\ \pi_B(V_1, 0) - \pi_B(-V_1, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \\ q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \end{pmatrix}$$

If $q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$, ISP 2, if making an offer to CP B, will make an offer such that ISP 1's offer will not be accepted. In this case, ISP 1's payoff is $\pi_A(V_1, -V_2)$. Otherwise it will get $\pi_A(-V_1, -V_2) + q_{1B}$.

b) Consider now that ISP 2 makes an offer to CP A when ISP 1 had made an offer to CP B. The acceptance/rejection game played by the two CP's is as follows:

CP B

Accept

Reject

CP A Accept $\pi_A(-V_1, V_2) - q_{2A} + q_{1B}, \pi_B(V_1, -V_2) - q_{1B}$ $\pi_A(V_1, V_2) - q_{2A}, \pi_B(-V_1, -V_2)$
 +ISP 1 Reject $\pi_A(-V_1, 0) + q_{1B}, \pi_B(V_1, 0) - q_{1B}$ $\pi_A(V_1, 0), \pi_B(-V_1, 0)$

Acceptance of the two offers is an equilibrium if and only if

$$\pi_A(-V_1, V_2) - q_{2A} + q_{1B} \geq \pi_A(-V_1, 0) + q_{1B}$$

and

$$\pi_B(V_1, -V_2) - q_{1B} \geq \pi_B(-V_1, -V_2)$$

Acceptance of q_{2A} and rejection of q_{1B} is an equilibrium if and only if

$$\pi_A(V_1, V_2) - q_{2A} \geq \pi_A(V_1, 0)$$

and

$$\pi_B(V_1, -V_2) - q_{1B} < \pi_B(-V_1, -V_2)$$

ISP 2 will make the highest offer that is accepted (if positive), given q_{1B} .

Hence,

$$q_{2A} = \begin{cases} \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) & \text{if } q_{1B} \leq \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \\ \pi_A(V_1, V_2) - \pi_A(V_1, 0) & \text{if } q_{1B} > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \end{cases}$$

or

$$q_{2A} = \begin{cases} \frac{1}{18}t_c^{-1} (6t_c + 2R + V_2\sigma_2 - 2V_1\sigma_1) V_2\sigma_2 & \text{if } q_{1B} \leq \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \\ \frac{1}{18}t_c^{-1} (2R + 6t_c + 2V_1\sigma_1 + V_2\sigma_2) V_2\sigma_2 > 0 & \text{if } q_{1B} > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \end{cases}$$

Note that $\pi_B(V_1, 0) - \pi_B(-V_1, 0) > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2)$

$$\pi_B(V_1, 0) - \pi_B(-V_1, 0) - \pi_B(V_1, -V_2) + \pi_B(-V_1, -V_2) > 0$$

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} > 0$$

$$\frac{2}{9}t_c^{-1} \sigma_2 \sigma_1 V_2 V_1 > 0$$

Hence,

$$\pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) < \pi_B(V_1, 0) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$$

and there are four possibilities to consider when comparing ISP 2's payoffs under the alternatives of making offers to CP A or to CP B:

i) When $q_{1B} < \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2)$ it is preferable for ISP 2 to offer to CP A if and only if:

$$\pi_A(-V_1, V_2) - \pi_A(-V_1, 0) > \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

$$\frac{(3t_c + R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} > 0$$

$$\left(-\frac{2}{9}\right) t_c^{-1} (V_1 \sigma_1 - R) V_2 \sigma_2 > 0$$

$$R > V_1\sigma_1$$

ii) When $\pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) < q_{1B} < \pi_B(V_1, 0) - \pi_B(-V_1, 0)$ it is preferable for ISP 2 to offer to CP A if and only if:

$$\pi_A(V_1, V_2) - \pi_A(V_1, 0) > \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

$$\pi_A(V_1, V_2) - \pi_A(V_1, 0) - \pi_B(V_1, V_2) + \pi_B(V_1, 0) > 0$$

$$\frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} > 0$$

$$\frac{2}{9}t_c^{-1}\sigma_2 V_2 R > 0$$

$$R > 0$$

iii) When $\pi_B(V_1, 0) - \pi_B(-V_1, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$ it is preferable for ISP 2 to offer to CP A if and only if:

$$\pi_A(V_1, V_2) - \pi_A(V_1, 0) > \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{1B}$$

$$q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0)$$

$$q_{1B} > \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c}$$

$$q_{1B} > -\frac{2}{9}t_c^{-1}(RV_1\sigma_1 + RV_2\sigma_2 - 3V_1t_c\sigma_1)$$

Checking if this is possible in the relevant interval:

$$\pi_B(V_1, 0) - \pi_B(-V_1, 0) < q_{1B} < \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$$

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} < q_{1B} < \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c}$$

The condition is always true if

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} > -\frac{2}{9}t_c^{-1}(RV_1\sigma_1 + RV_2\sigma_2 - 3V_1t_c\sigma_1)$$

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} + \frac{2}{9}t_c^{-1}(RV_1\sigma_1 + RV_2\sigma_2 - 3V_1t_c\sigma_1) > 0$$

$$\frac{2}{9}t_c^{-1}\sigma_2 V_2 R > 0$$

$$R > 0$$

The condition is impossible if

$$-\frac{2}{9}t_c^{-1}(RV_1\sigma_1 + RV_2\sigma_2 - 3V_1t_c\sigma_1) > \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c}$$

$$-\frac{2}{9}t_c^{-1}(RV_1\sigma_1 + RV_2\sigma_2 - 3V_1t_c\sigma_1) - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} > 0$$

$$\left(-\frac{2}{9}\right)t_c^{-1}(R + V_1\sigma_1)V_2\sigma_2 > 0$$

$$R < -V_1\sigma_1$$

If $R > 0$, $q_{1B} > -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1)$ is always true in the relevant interval

If $-V_1\sigma_1 < R < 0$ the condition $q_{1B} > -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1)$ may or may not hold in the relevant interval in case iii) because $-\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) \in [\pi_B(V_1, 0) - \pi_B(-V_1, 0), \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)]$

If $R < -V_1\sigma_1$, $q_{1B} > -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1)$ is never true in the relevant interval

iv) When $q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$ it is preferable for ISP 2 to offer to CP A if and only if:

$$\begin{aligned} \pi_A(V_1, V_2) - \pi_A(V_1, 0) &> \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) \\ \pi_A(V_1, V_2) - \pi_A(V_1, 0) - \pi_B(-V_1, V_2) + \pi_B(-V_1, 0) &> 0 \\ \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} &> 0 \\ \frac{2}{9} t_c^{-1} (R + V_1 \sigma_1) V_2 \sigma_2 &> 0 \\ R &> -V_1 \sigma_1. \end{aligned}$$

Summing up:

$$\begin{array}{llll} & q_{1B} < & \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) & \\ & & < q_{1B} < & \\ & \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) & & \\ R < -V_1 \sigma_1 & \text{ISP 2 offers } q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0) & \text{ISP 2 offers } q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0) & \\ -V_1 \sigma_1 < R < 0 & \text{ISP 2 offers } q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0) & \text{ISP 2 offers } q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0) & \\ 0 < R < V_1 \sigma_1 & \text{ISP 2 offers } q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0) & \text{ISP 2 offers } q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0) & \\ R > V_1 \sigma_1 & \text{ISP 2 offers } q_{2A} = \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) & \text{ISP 2 offers } q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0) & \end{array}$$

Turning to ISP 1's decision. The decision to offer internally yields the same payoff as making an extremely high offer to CP B. Hence, one restricts the analysis to ISP 1's optimal offer to CP B. Assuming that ISP 1 makes an offer to CP B:

1) If $R > V_1 \sigma_1$, ISP 2 will always make an offer to CP A given by

$$q_{2A} = \begin{cases} \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) & \text{if } q_{1B} \leq \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \\ \pi_A(V_1, V_2) - \pi_A(V_1, 0) & \text{if } q_{1B} > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \end{cases}$$

ISP 1's offer to CP B is then accepted if and only if $q_{1B} \leq \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2)$.

ISP 1's payoffs, incorporating ISP 2's offer to CP A are:

$$\begin{cases} \pi_A(-V_1, V_2) - q_{2A} + q_{1B} = \pi_A(-V_1, 0) + q_{1B} & \text{if } q_{1B} \leq \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \\ \pi_A(V_1, V_2) - q_{2A} = \pi_A(V_1, 0) & \text{if } q_{1B} > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \end{cases}$$

It can be showed that

$$\begin{aligned} \pi_A(-V_1, 0) + \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) &< \pi_A(V_1, 0) \\ \pi_A(-V_1, 0) + \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) - \pi_A(V_1, 0) &< 0 \\ \frac{(3t_c + R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} &= \left(-\frac{2}{9}\right) t_c^{-1} (2R + V_2 \sigma_2) V_1 \sigma_1 \end{aligned}$$

0 because R is positive.

Hence, ISP 1 prefers to set $q_{1B} > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2)$ and obtain $\pi_A(V_1, 0)$. This is equal to making the offer internally. ISP 2 then sets $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$.

2) If $0 < R < V_1 \sigma_1$, ISP 2 offers CP A $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ iff $q_{1B} > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2)$. In this case, ISP 1's offer to CP B is not accepted. If $q_{1B} < \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2)$.

$\pi_B(-V_1, -V_2)$, ISP 2 offers CP B $q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0)$ and ISP 1's offer to CP B will be accepted. IPS 1's payoffs are then:

$$\begin{cases} \pi_A(-V_1, -V_2) + q_{1B} & \text{if } q_{1B} \leq \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \\ \pi_A(V_1, V_2) - q_{2A} = \pi_A(V_1, 0) & q_{1B} > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \end{cases}$$

It can be showed that

$$\pi_A(-V_1, -V_2) + \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) < \pi_A(V_1, 0)$$

$$\pi_A(-V_1, -V_2) + \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) - \pi_A(V_1, 0) < 0$$

$$\frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} = \frac{1}{18}t_c^{-1}(-\sigma_2 V_2(2R + 6t_c +$$

0 if $V_i < 3t_c$, which is true.

Hence, ISP 1 prefers to offer $q_{1B} > \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2)$ i.e., offer internally.

3) If $-V_1\sigma_1 < R < 0$, ISP 2 offers CP A $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ iff $q_{1B} > -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1)$. In this case, ISP 1's offer is not accepted and ISP 1 will get $\pi_A(V_1, V_2) - \pi_A(V_1, V_2) + \pi_A(V_1, 0) = \pi_A(V_1, 0)$.

If $q_{1B} < -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1)$ ISP 2 offers CP B $q_{2B} = \begin{pmatrix} \pi_B(V_1, V_2) - \pi_B(V_1, 0) \\ \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{1B} \end{pmatrix}$ if $\begin{pmatrix} q_{1B} < \pi_B(V_1, 0) - \pi_B(-V_1, 0) \\ \pi_B(V_1, 0) - \pi_B(-V_1, 0) < q_{1B} < -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) \end{pmatrix}$ and ISP 1's offer will be accepted.

ISP 1's payoffs are

$$\begin{cases} \pi_A(-V_1, -V_2) + q_{1B} = \pi_A(-V_1, -V_2) + \pi_B(V_1, 0) - \pi_B(-V_1, 0) & q_{1B} > -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) \\ \pi_A(-V_1, -V_2) + q_{1B} = \pi_A(-V_1, -V_2) - \frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) & \text{if } \pi_B(V_1, 0) - \pi_B(-V_1, 0) < q_{1B} < -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) \\ \pi_A(V_1, V_2) - q_{2A} = \pi_A(V_1, 0) & q_{1B} > -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) \end{cases}$$

Note that

$$\pi_A(-V_1, -V_2) - \frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) > \pi_A(V_1, 0)$$

$$\frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) - \frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} > 0$$

$$\frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1) - \frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} = \frac{1}{18}t_c^{-1}(-\sigma_2 V_2(-2V_1\sigma_1 + 6t_c$$

> 0

$$\frac{1}{18}t_c^{-1}(-\sigma_2 V_2(3t_c - V_1\sigma_1 - V_1\sigma_1 + 3t_c - V_2\sigma_2 + 6R) - 8RV_1\sigma_1) > 0$$

$$R < -\frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)}(6t_c - 2V_1\sigma_1 - V_2\sigma_2)$$

Obs: This value can be showed to be negative. But cannot be ranked it with $-V_1\sigma_1$:

$$\frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)}(2(V_1\sigma_1 - 3t_c) + V_2\sigma_2) - (-V_1\sigma_1) =$$

$$(-\frac{1}{2})(4V_1\sigma_1 + 3V_2\sigma_2)^{-1}(\sigma_2 V_2(6t_c - 8V_1\sigma_1 - V_2\sigma_2) - 8V_1^2\sigma_1^2) =$$

Hence, if $R < -\frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)}(6t_c - 2V_1\sigma_1 - V_2\sigma_2)$ both will offer CP B, with $q_{1B} = -\frac{2}{9}t_c^{-1}(R(V_1\sigma_1 + V_2\sigma_2) - 3V_1t_c\sigma_1)$ and $q_{2B} = \frac{1}{18}t_c^{-1}(6t_c - 2R + V_2\sigma_2)(2V_1\sigma_1 + V_2\sigma_2) - q_{1B} = \frac{1}{18}t_c^{-1}(2R + 6t_c + 2V_1\sigma_1 + V_2\sigma_2)V_2\sigma_2$, or, in terms of the original expressions:

$$q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0)$$

$$q_{2B} = \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{1B} = -\pi_B(-V_1, 0) + \pi_B(-V_1, 0) + \pi_A(V_1, V_2) - \pi_A(V_1, 0).$$

CP B's profit is

$$\pi_B(-V_1, 0)$$

Otherwise, ISP1 will not make an offer and ISP 2 will offer CPA.

4) If $R < -V_1\sigma_1$ ISP 2 never offers CP A. In this case, ISP 1's to CP B offer is not accepted if and only if $q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$. ISP 1's payoffs are

$$\begin{cases} \pi_A(-V_1, -V_2) + q_{1B} & \text{if } q_{1B} \leq \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \\ \pi_A(V_1, -V_2) & q_{1B} > \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \end{cases}$$

It can be showed that

$$\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) > \pi_A(V_1, -V_2)$$

$$\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) - \pi_A(V_1, -V_2) > 0$$

$$\frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} = \frac{4}{9}t_c^{-1} (V_2\sigma_2 - R) V_1\sigma_1 >$$

0 because R is negative. In this case it is better to offer $q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$ than offering internally, which would yield $\pi_A(V_1, -V_2)$.

Lemma 1:

If $R > 0$, ISP 1 does not make any offer, ISP 2 makes offer $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ to CP A.

If $-V_1\sigma_1 < R < 0$ and $R > \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)} (2(V_1\sigma_1 - 3t_c) + V_2\sigma_2)$, ISP 1 does not make any offer and ISP 2 offers $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ to CP A.

If $-V_1\sigma_1 < R < 0$ and $R < \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)} (2(V_1\sigma_1 - 3t_c) + V_2\sigma_2)$, ISP 1 makes offer $q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0)$ to CP B and ISP 2 makes offer $q_{2B} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ to CP B. This is impossible if $\frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)} (2(V_1\sigma_1 - 3t_c) + V_2\sigma_2) < -V_1\sigma_1$.

If $R < -V_1\sigma_1$, ISP 1 offers $q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$ to CP B, ISP 2 offers $q_{2B} = \pi_B(-V_1, V_2) - \pi_B(-V_1, 0)$ to CP B.

In all cases, the offers are accepted.

9 ISP2 makes the offers first:

9.1 ISP 2 offers q_{2A} to CPA

a) If ISP 1 does not make any offer:

CP A will accept ISP 2's offer

if and only if $\pi_A(V_1, V_2) - q_{2A} \geq \pi_A(V_1, 0) \Leftrightarrow q_{2A} \leq \pi_A(V_1, V_2) - \pi_A(V_1, 0)$

ISP 1 + CP A will have profit $\pi_A(V_1, V_2) - q_{2A}$

ISP 2 will have profit q_{2A} .

or

CP A will reject ISP 2's offer if and only if $q_{2A} > \pi_A(V_1, V_2) - \pi_A(V_1, 0)$.

ISP 1 + CP A will have profit $\pi_A(V_1, 0)$

ISP 2 will have no profit.

b) If ISP 1 makes an offer q_{1B} to CP B the acceptance/rejection game will be

CP B

Accept

Reject

CP A	Accept	$\pi_A(-V_1, V_2) + q_{1B} - q_{2A}, \pi_B(V_1, -V_2) - q_{1B}$	$\pi_A(V_1, V_2) - q_{2A}, \pi_B(-V_1, -V_2)$
+ISP 1	Reject	$\pi_A(-V_1, 0) + q_{1B}, \pi_B(V_1, 0) - q_{1B}$	$\pi_A(V_1, 0), \pi_B(-V_1, 0)$

ISP 1's offer will be accepted if and only if in equilibrium both are accepted:

$$\pi_B(V_1, -V_2) - q_{1B} \geq \pi_B(-V_1, -V_2) \text{ and}$$

$$\pi_A(-V_1, V_2) + q_{1B} - q_{2A} \geq \pi_A(-V_1, 0) + q_{1B}$$

or in equilibrium only ISP 1's offer is accepted

$$\pi_B(V_1, 0) - q_{1B} \geq \pi_B(-V_1, 0) \text{ and}$$

$$\pi_A(-V_1, V_2) + q_{1B} - q_{2A} < \pi_A(-V_1, 0) + q_{1B}$$

The highest acceptable offer by CP B is

$$q_{1B} = \begin{pmatrix} \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) \\ \pi_B(V_1, 0) - \pi_B(-V_1, 0) \end{pmatrix} \text{ if } \begin{pmatrix} q_{2A} \leq \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) \\ q_{2A} > \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) \end{pmatrix}$$

ISP 1's payoff is

$$\begin{pmatrix} \pi_A(-V_1, V_2) + q_{1B} - q_{2A} \\ \pi_A(-V_1, 0) + q_{1B} \end{pmatrix} \text{ if } \begin{pmatrix} q_{2A} \leq \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) \\ q_{2A} > \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) \end{pmatrix}$$

As ISP 1 will set the highest possible offer, payoffs will be:

$$\begin{pmatrix} \pi_A(-V_1, V_2) + \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) - q_{2A} \\ \pi_A(-V_1, 0) + \pi_B(V_1, 0) - \pi_B(-V_1, 0) \end{pmatrix} \text{ if } \begin{pmatrix} q_{2A} < \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) \\ q_{2A} > \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) \end{pmatrix}$$

Note that

$$\pi_A(-V_1, V_2) - \pi_A(-V_1, 0) < \pi_A(V_1, V_2) - \pi_A(V_1, 0)$$

$$\pi_A(-V_1, V_2) - \pi_A(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0) < 0$$

$$\frac{(3t_c + R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} = \left(-\frac{2}{9}\right) t_c^{-1} \sigma_2 \sigma_1 V_2 V_1 < 0$$

0

Hence, there are three possibilities:

$$\text{i) } q_{2A} < \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) < \pi_A(V_1, V_2) - \pi_A(V_1, 0)$$

ISP 1 prefers not to make offer if and only if

$$\pi_A(V_1, V_2) - q_{2A} > \pi_A(-V_1, V_2) + \pi_B(V_1, -V_2) - \pi_B(-V_1, -V_2) - q_{2A}$$

$$\pi_A(V_1, V_2) - \pi_A(-V_1, V_2) - \pi_B(V_1, -V_2) + \pi_B(-V_1, -V_2) > 0$$

$$\frac{(3t_c+R+\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c+R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c-R+\sigma_1V_1-\sigma_2V_2)^2}{18t_c} + \frac{(3t_c-R-\sigma_1V_1-\sigma_2V_2)^2}{18t_c} > 0$$

$$\frac{4}{9}t_c^{-1}(R+V_2\sigma_2)V_1\sigma_1 > 0$$

$$R > -V_2\sigma_2$$

$$\text{ii) } \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) < q_{2A} < \pi_A(V_1, V_2) - \pi_A(V_1, 0)$$

ISP 1 prefers not to make offer if and only if

$$\pi_A(V_1, V_2) - q_{2A} > \pi_A(-V_1, 0) + \pi_B(V_1, 0) - \pi_B(-V_1, 0)$$

$$\pi_A(V_1, V_2) - \pi_A(-V_1, 0) - \pi_B(V_1, 0) + \pi_B(-V_1, 0) > q_{2A}$$

$$q_{2A} < \pi_A(V_1, V_2) - \pi_A(-V_1, 0) - \pi_B(V_1, 0) + \pi_B(-V_1, 0)$$

$$q_{2A} < \frac{(3t_c+R+\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c+R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c-R+\sigma_1V_1+\sigma_2V_2)^2}{18t_c} + \frac{(3t_c-R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} = \frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) > 0$$

This is always true if

$$\frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) > \pi_A(V_1, V_2) - \pi_A(V_1, 0)$$

$$\frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) - \pi_A(V_1, V_2) + \pi_A(V_1, 0) > 0$$

$$\frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) - \frac{(3t_c+R+\sigma_1V_1+\sigma_2V_2)^2}{18t_c} + \frac{(3t_c+R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} > 0$$

0

$$\frac{4}{9}t_c^{-1}\sigma_1V_1R > 0$$

$$R > 0$$

This is impossible if

$$\frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) < \pi_A(-V_1, V_2) - \pi_A(-V_1, 0)$$

$$\frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) - \pi_A(-V_1, V_2) + \pi_A(-V_1, 0) < 0$$

$$\frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) - \frac{(3t_c+R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} + \frac{(3t_c+R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} < 0$$

0

$$\frac{2}{9}t_c^{-1}(2R + V_2\sigma_2)V_1\sigma_1 < 0$$

$$R < \frac{-V_2\sigma_2}{2}$$

If $\frac{-V_2\sigma_2}{2} < R < 0$ ISP 1 prefers not to make offer if and only if

$$q_{2A} < \frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2)$$

$$\text{iii) } \pi_A(-V_1, V_2) - \pi_A(-V_1, 0) < \pi_A(V_1, V_2) - \pi_A(V_1, 0) < q_{2A}$$

ISP 1 prefers not to make offer if and only if

$$\pi_A(V_1, 0) > \pi_A(-V_1, 0) + \pi_B(V_1, 0) - \pi_B(-V_1, 0)$$

$$\pi_A(V_1, 0) - \pi_A(-V_1, 0) - \pi_B(V_1, 0) + \pi_B(-V_1, 0) > 0$$

$$\frac{(3t_c+R+\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c+R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} - \frac{(3t_c-R+\sigma_1V_1+\sigma_2V_2)^2}{18t_c} + \frac{(3t_c-R-\sigma_1V_1+\sigma_2V_2)^2}{18t_c} > 0$$

$$\frac{4}{9}t_c^{-1}\sigma_1V_1R > 0$$

$$R > 0$$

Given the above, ISP 2 will make the highest offer that, given ISP 1's optimal move, is accepted.

This offer, which is equal to ISP 2's payoff is:

$$R < -V_2\sigma_2/2 : q_{2A} = \pi_A(-V_1, V_2) - \pi_A(-V_1, 0)$$

$$-V_2\sigma_2/2 < R < 0 : q_{2A} = \pi_A(V_1, V_2) - \pi_A(-V_1, 0) - \pi_B(V_1, 0) + \pi_B(-V_1, 0) = \frac{1}{18}t_c^{-1} (8RV_1\sigma_1 + 2RV_2\sigma_2)$$

$$R > 0 : q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$$

9.2 ISP 2 offers q_{2B} to CPB

a) ISP 1 does not make any offer.

Then, CP B will accept the offer if and only if $\pi_B(-V_1, V_2) - q_{2B} > \pi_B(-V_1, 0)$.

ISP 1's payoff is:

$$\begin{aligned} \pi_A(V_1, -V_2) \text{ if } q_{2B} < \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) \\ \pi_A(V_1, 0) \text{ if } q_{2B} > \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) \end{aligned}$$

b) ISP 1 also makes a q_{1B} offer to CP B.

ISP 1's offer is accepted if and only if

$$\max\{\pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(V_1, 0) - q_{1B}\} > \max\{\pi_B(-V_1, V_2) - q_{2B}, \pi_B(-V_1, 0)\}$$

$$\max\{\pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(V_1, 0) - q_{1B}\} = \pi_B(V_1, V_2) - q_{1B} - q_{2B} \text{ if and only if}$$

$$\pi_B(V_1, V_2) - q_{1B} - q_{2B} > \pi_B(V_1, 0) - q_{1B} \Leftrightarrow q_{2B} < \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

$$\max\{\pi_B(-V_1, V_2) - q_{2B}, \pi_B(-V_1, 0)\} = \pi_B(-V_1, V_2) - q_{2B} \text{ if and only if } \pi_B(-V_1, V_2) - q_{2B} > \pi_B(-V_1, 0) \Leftrightarrow q_{2B} < \pi_B(-V_1, V_2) - \pi_B(-V_1, 0).$$

Note that

$$\pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

$$\pi_B(-V_1, V_2) - \pi_B(-V_1, 0) - \pi_B(V_1, V_2) + \pi_B(V_1, 0) < 0$$

$$\frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} = \left(-\frac{2}{9}\right) t_c^{-1} \sigma_2 \sigma_1 V_2 V_1 < 0$$

0

Hence, there are three possibilities:

$$\text{i) } q_{2B} < \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

Then ISP 1's offer is accepted if and only if

$$\max\{\pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(V_1, 0) - q_{1B}\} \geq \max\{\pi_B(-V_1, V_2) - q_{2B}, \pi_B(-V_1, 0)\}$$

$$\pi_B(V_1, V_2) - q_{1B} - q_{2B} \geq \pi_B(-V_1, V_2) - q_{2B}$$

$$q_{1B} \leq \pi_B(V_1, V_2) - \pi_B(-V_1, V_2). \text{ (in this case both are accepted)}$$

$$\text{ii) } \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < q_{2B} < \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

$$\max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(V_1, 0) - q_{1B} \} > \max \{ \pi_B(-V_1, V_2) - q_{2B}, \pi_B(-V_1, 0) \}$$

$$\pi_B(V_1, V_2) - q_{1B} - q_{2B} > \pi_B(-V_1, 0)$$

$$q_{1B} < \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{2B}. \text{ (in this case both are accepted)}$$

$$\text{iii) } \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0) < q_{2B}$$

$$\max \{ \pi_B(V_1, V_2) - q_{1B} - q_{2B}, \pi_B(V_1, 0) - q_{1B} \} > \max \{ \pi_B(-V_1, V_2) - q_{2B}, \pi_B(-V_1, 0) \}$$

$$\pi_B(V_1, 0) - q_{1B} > \pi_B(-V_1, 0)$$

$$q_{1B} < \pi_B(V_1, 0) - \pi_B(-V_1, 0) \text{ (in this only ISP 1's offer is accepted)}$$

ISP 1's highest acceptable offer, when offering to CP B is then:

$$q_{1B} = \begin{pmatrix} \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \\ \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{2B} \\ \pi_B(V_1, 0) - \pi_B(-V_1, 0) \end{pmatrix} \text{ if } \begin{pmatrix} q_{2B} < \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0) \\ \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < q_{2B} < \pi_B(V_1, V_2) - \pi_B(V_1, 0) \\ \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0) \end{pmatrix}$$

ISP 2's offer is accepted if and only if $q_{2B} < \pi_B(V_1, V_2) - \pi_B(V_1, 0)$.

ISP 1's payoff is:

$$\begin{pmatrix} \pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \\ \pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{2B} \\ \pi_A(-V_1, 0) + \pi_B(V_1, 0) - \pi_B(-V_1, 0) \end{pmatrix} \text{ if } \begin{pmatrix} q_{2B} < \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0) \\ \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < q_{2B} < \pi_B(V_1, V_2) - \pi_B(V_1, 0) \\ \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0) \end{pmatrix}$$

One now considers each of these cases:

$$\text{If } q_{2B} < \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

ISP 1 prefers not to make an offer if and only if:

$$\pi_A(V_1, -V_2) > \pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$$

$$\pi_A(V_1, -V_2) - \pi_A(-V_1, -V_2) - \pi_B(V_1, V_2) + \pi_B(-V_1, V_2) > 0$$

$$\frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} = \left(-\frac{4}{9}\right) t_c^{-1} (V_2 \sigma_2 - R) V_1 \sigma_1$$

0

$$R > V_2 \sigma_2$$

$$\text{If } \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < q_{2B} < \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

ISP 1 prefers not to make an offer if and only if:

$$\pi_A(V_1, 0) > \pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - q_{2B}$$

$$q_{2B} > \pi_A(-V_1, -V_2) - \pi_A(V_1, 0) + \pi_B(V_1, V_2) - \pi_B(-V_1, 0)$$

$$q_{2B} > \frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} =$$

$$\frac{1}{9} t_c^{-1} (2V_1 \sigma_1 + V_2 \sigma_2) (V_2 \sigma_2 - 2R)$$

$$q_{2B} > \frac{1}{9} t_c^{-1} (2V_1 \sigma_1 + V_2 \sigma_2) (V_2 \sigma_2 - 2R)$$

(which is always true of $R > \frac{V_2 \sigma_2}{2}$.)

This is impossible if and only if

$$\frac{1}{9} t_c^{-1} (2V_1 \sigma_1 + V_2 \sigma_2) (V_2 \sigma_2 - 2R) > \pi_B(V_1, V_2) - \pi_B(V_1, 0) > 0$$

$$\frac{1}{9} t_c^{-1} (2V_1 \sigma_1 + V_2 \sigma_2) (V_2 \sigma_2 - 2R) - \pi_B(V_1, V_2) + \pi_B(V_1, 0) > 0$$

$$\frac{1}{9}t_c^{-1}(2V_1\sigma_1 + V_2\sigma_2)(V_2\sigma_2 - 2R) - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} = \frac{1}{18}t_c^{-1}(\sigma_2 V_2(2V_1\sigma_1 + V_2\sigma_2 - 2R)) > 0$$

$$R < \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(2V_1\sigma_1 + V_2\sigma_2 - 6t_c) < 0$$

This is always true if and only if

$$\frac{1}{9}t_c^{-1}(2V_1\sigma_1 + V_2\sigma_2)(V_2\sigma_2 - 2R) < \pi_B(-V_1, V_2) - \pi_B(-V_1, 0)$$

$$\frac{1}{9}t_c^{-1}(2V_1\sigma_1 + V_2\sigma_2)(V_2\sigma_2 - 2R) - \pi_B(-V_1, V_2) + \pi_B(-V_1, 0) < 0$$

$$\frac{1}{9}t_c^{-1}(2V_1\sigma_1 + V_2\sigma_2)(V_2\sigma_2 - 2R) - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} = \frac{1}{18}t_c^{-1}(\sigma_2 V_2(6V_1\sigma_1 + V_2\sigma_2 - 2R)) < 0$$

$$R > \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(6V_1\sigma_1 - 6t_c + V_2\sigma_2)$$

If $\frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(2V_1\sigma_1 - 6t_c + V_2\sigma_2) < R < \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(6V_1\sigma_1 - 6t_c + V_2\sigma_2)$ ISP 1 prefers not to make an offer if and only if: $q_{2B} > \frac{1}{9}t_c^{-1}(2V_1\sigma_1 + V_2\sigma_2)(V_2\sigma_2 - 2R)$

$$\text{If } \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) < \pi_B(V_1, V_2) - \pi_B(V_1, 0) < q_{2B}$$

ISP 1 prefers not to make an offer if and only if:

$$\pi_A(V_1, 0) > \pi_A(-V_1, 0) + \pi_B(V_1, 0) - \pi_B(-V_1, 0)$$

$$\pi_A(V_1, 0) - \pi_A(-V_1, 0) - \pi_B(V_1, 0) + \pi_B(-V_1, 0) > 0$$

$$\frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} + \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} > 0$$

$$\frac{4}{9}t_c^{-1}\sigma_1 V_1 R > 0$$

$$R > 0$$

Given the above, ISP 2 will make the highest offer that, given ISP 1's optimal move, is accepted.

This offer, which is equal to ISP 2's, payoff is:

$$R < \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(2V_1\sigma_1 - 6t_c + V_2\sigma_2) : q_{2B} = \pi_B(V_1, V_2) - \pi_B(V_1, 0)$$

$$\frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(2V_1\sigma_1 - 6t_c + V_2\sigma_2) < R < \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(6V_1\sigma_1 - 6t_c + V_2\sigma_2) : q_{2B} = \frac{1}{9}t_c^{-1}(2V_1\sigma_1 + V_2\sigma_2)(V_2\sigma_2 - 2R)$$

$$R > \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(6V_1\sigma_1 - 6t_c + V_2\sigma_2) : q_{2B} = \pi_B(-V_1, V_2) - \pi_B(-V_1, 0).$$

$$1) \pi_B(V_1, V_2) - \pi_B(V_1, 0) - (\pi_A(-V_1, V_2) - \pi_A(-V_1, 0))$$

$$\pi_B(V_1, V_2) - \pi_B(V_1, 0) - \pi_A(-V_1, V_2) + \pi_A(-V_1, 0) > 0$$

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} = \frac{2}{9}t_c^{-1}(V_1\sigma_1 - R)V_2\sigma_2 > 0$$

$$R < V_1\sigma_1$$

$$\text{As } \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)}(2V_1\sigma_1 - 6t_c + V_2\sigma_2) < V_1\sigma_1$$

always true

$$2) \pi_B(V_1, V_2) - \pi_B(V_1, 0) - \frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) > 0$$

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} - \frac{1}{18}t_c^{-1}(8RV_1\sigma_1 + 2RV_2\sigma_2 + 6V_2t_c\sigma_2 + 2V_1V_2\sigma_1\sigma_2 + V_2^2\sigma_2^2) > 0$$

$$\left(-\frac{2}{9}\right) t_c^{-1} (2V_1\sigma_1 + V_2\sigma_2) R > 0$$

$$R < 0$$

$$3) \pi_B(V_1, V_2) - \pi_B(V_1, 0) - (\pi_A(V_1, V_2) - \pi_A(V_1, 0)) > 0$$

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} > 0$$

$$\left(-\frac{2}{9}\right) t_c^{-1} \sigma_2 V_2 R > 0$$

$$R < 0$$

$$4) \frac{1}{9} t_c^{-1} (2V_1\sigma_1 + V_2\sigma_2) (V_2\sigma_2 - 2R) - (\pi_A(-V_1, V_2) - \pi_A(-V_1, 0)) > 0$$

$$\frac{1}{9} t_c^{-1} (2V_1\sigma_1 + V_2\sigma_2) (V_2\sigma_2 - 2R) - \pi_A(-V_1, V_2) + \pi_A(-V_1, 0) > 0$$

$$\frac{1}{9} t_c^{-1} (2V_1\sigma_1 + V_2\sigma_2) (V_2\sigma_2 - 2R) - \frac{(3t_c + R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} > 0$$

$$\frac{1}{18} t_c^{-1} (6V_1 V_2 \sigma_1 \sigma_2 - 6R V_2 \sigma_2 - 6V_2 t_c \sigma_2 - 8R V_1 \sigma_1 + V_2^2 \sigma_2^2) > 0$$

$$R < \frac{\sigma_2 V_2}{2(4V_1 \sigma_1 + 3V_2 \sigma_2)} (6V_1 \sigma_1 + V_2 \sigma_2 - 6t_c)$$

$$5) \frac{1}{9} t_c^{-1} (2V_1\sigma_1 + V_2\sigma_2) (V_2\sigma_2 - 2R) - \frac{1}{18} t_c^{-1} (8R V_1 \sigma_1 + 2R V_2 \sigma_2 + 6V_2 t_c \sigma_2 + 2V_1 V_2 \sigma_1 \sigma_2 + V_2^2 \sigma_2^2) >$$

0

$$R < \frac{\sigma_2 V_2}{2(8V_1 \sigma_1 + 3V_2 \sigma_2)} (2V_1 \sigma_1 - 6t_c + V_2 \sigma_2)$$

$$6) \frac{1}{9} t_c^{-1} (2V_1\sigma_1 + V_2\sigma_2) (V_2\sigma_2 - 2R) - \pi_A(V_1, V_2) - \pi_A(V_1, 0) > 0$$

$$\frac{1}{9} t_c^{-1} (2V_1\sigma_1 + V_2\sigma_2) (V_2\sigma_2 - 2R) - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} > 0 R < \frac{\sigma_2 V_2}{2(4V_1 \sigma_1 + 3V_2 \sigma_2)} (2V_1 \sigma_1 -$$

$$7) \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(-V_1, V_2) + \pi_A(-V_1, 0) > 0$$

$$\frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} > 0$$

$$\left(-\frac{2}{9}\right) t_c^{-1} \sigma_2 V_2 R > 0$$

$$R < 0$$

$$8) \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) - \frac{1}{18} t_c^{-1} (8R V_1 \sigma_1 + 2R V_2 \sigma_2 + 6V_2 t_c \sigma_2 + 2V_1 V_2 \sigma_1 \sigma_2 + V_2^2 \sigma_2^2)$$

$$\frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} - \frac{1}{18} t_c^{-1} (8R V_1 \sigma_1 + 2R V_2 \sigma_2 + 6V_2 t_c \sigma_2 + 2V_1 V_2 \sigma_1 \sigma_2 + V_2^2 \sigma_2^2) =$$

$$\left(-\frac{2}{9}\right) t_c^{-1} (R (2V_1 \sigma_1 + V_2 \sigma_2) + V_1 V_2 \sigma_1 \sigma_2) > 0$$

$$R < \frac{-V_1 V_2 \sigma_1 \sigma_2}{(2V_1 \sigma_1 + V_2 \sigma_2)}$$

$$9) \pi_B(-V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0)$$

$$\frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R + \sigma_1 V_1 - \sigma_2 0)^2}{18t_c} > 0 \left(-\frac{2}{9}\right) t_c^{-1} (R + V_1 \sigma_1) V_2 \sigma_2$$

0

$$R < -V_1 \sigma_1 \text{ (IMPOSSIBLE)}$$

Summing up:

ISP 2 offers to CP B if

$$\begin{array}{lll} R < \frac{\sigma_2 V_2}{2(4V_1 \sigma_1 + V_2 \sigma_2)} (2V_1 \sigma_1 + V_2 \sigma_2 - 6t_c) & \frac{\sigma_2 V_2}{2(4V_1 \sigma_1 + V_2 \sigma_2)} (2V_1 \sigma_1 + V_2 \sigma_2 - 6t_c) < R < \frac{\sigma_2 V_2}{2(4V_1 \sigma_1 + 3V_2 \sigma_2)} (2V_1 \sigma_1 + V_2 \sigma_2 - 6t_c) & \\ R < -V_2 \sigma_2 / 2 & R < V_1 \sigma_1 \text{ (always true)} & R < \frac{\sigma_2 V_2}{2(4V_1 \sigma_1 + 3V_2 \sigma_2)} (6V_1 \sigma_1 + V_2 \sigma_2 - 6t_c) \\ -V_2 \sigma_2 / 2 < R < 0 & R < 0 \text{ (always true)} & R < \frac{\sigma_2 V_2}{2(8V_1 \sigma_1 + 3V_2 \sigma_2)} (2V_1 \sigma_1 + V_2 \sigma_2 - 6t_c) \\ R > 0 & *** & R < \frac{\sigma_2 V_2}{2(4V_1 \sigma_1 + 3V_2 \sigma_2)} (2V_1 \sigma_1 + V_2 \sigma_2 - 6t_c) \end{array}$$

Lemma:

If $R > 0$ ISP 2 offers CP A $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$. ISP 1 does not make any offer:

If $R < -V_2\sigma_2/2$ or $-V_2\sigma_2/2 < R < 0$ and $R < \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + V_2\sigma_2)} (2V_1\sigma_1 - 6t_c + V_2\sigma_2)$ ISP 2 offers CP B.

If $R > 0$ ISP 1 offers CP A: It offers $q_{1A} = \pi_A(V_1, V_2) - \pi_A(0, V_2)$ and ISP 2 responds with an offer to CP A $q_{2A} = \pi_A(0, V_2) - \pi_A(0, 0)$. CP A will get $\pi_A(V_1, V_2) - (\pi_A(V_1, V_2) - \pi_A(0, V_2)) - (\pi_A(0, V_2) - \pi_A(0, 0)) = \pi_A(0, 0)$ and CP B will get $\pi_B(-V_1, -V_2)$

If $R < 0$ ISP 1 offers CP B: It offers $q_{1B} = \pi_B(V_1, V_2) - \pi_B(0, V_2)$ and ISP 2 responds with an offer to CP B $q_{2B} = \pi_B(0, V_2) - \pi_B(0, 0)$. CP B will get $\pi_B(V_1, V_2) - (\pi_B(V_1, V_2) - \pi_B(0, V_2)) - (\pi_B(0, V_2) - \pi_B(0, 0)) = \pi_B(0, 0)$ and CP A will get $\pi_A(-V_1, -V_2)$.

10 Comparison

If $R > 0$, ISP 1 does not make any offer, ISP 2 makes offer $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ to CP A.

If $-V_1\sigma_1 < R < 0$ and $R > \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)} (2(V_1\sigma_1 - 3t_c) + V_2\sigma_2)$, ISP 1 does not make any offer and ISP 2 offers $q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ to CP A.

If $-V_1\sigma_1 < R < 0$ and $R < \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)} (2(V_1\sigma_1 - 3t_c) + V_2\sigma_2)$, ISP 1 makes offer $q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0)$ to CP B and ISP 2 makes offer $q_{2B} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$ to CP B. This is impossible if $\frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)} (2(V_1\sigma_1 - 3t_c) + V_2\sigma_2) < -V_1\sigma_1$.

If $R < -V_1\sigma_1$, ISP 1 offers $q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$ to CP B, ISP 2 offers $q_{2B} = \pi_B(-V_1, V_2) - \pi_B(-V_1, 0)$ to CP B.

Let $R_1 = \frac{\sigma_2 V_2}{2(4V_1\sigma_1 + 3V_2\sigma_2)} (2(V_1\sigma_1 - 3t_c) + V_2\sigma_2) < 0$

Prices:

No integration

ISP1

ISP2

$$R < 0 \quad q_{1B} = \pi_B(V_1, V_2) - \pi_B(0, V_2) \quad q_{2B} = \pi_B(0, V_2) - \pi_B(0, 0)$$

$$R > 0 \quad q_{1A} = \pi_A(V_1, V_2) - \pi_A(0, V_2) \quad q_{2A} = \pi_A(0, V_2) - \pi_A(0, 0)$$

Integration

ISP1

ISP2

$$R < -V_1\sigma_1 \quad q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, V_2) \quad q_{2B} = \pi_B(-V_1, V_2) - \pi_B(-V_1, 0)$$

$$-V_1\sigma_1 < R < 0 \text{ and } R < R_1 \quad q_{1B} = \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0) \quad q_{2B} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$$

$$-V_1\sigma_1 < R < 0 \text{ and } R > R_1 \quad q_{1A} \quad q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$$

$$R > 0 \quad q_{1A} \quad q_{2A} = \pi_A(V_1, V_2) - \pi_A(V_1, 0)$$

Payoffs:

No integration

	ISP1	ISP2	CPA	CPB
$R < 0$	$\pi_B(V_1, V_2) - \pi_B(0, V_2)$	$\pi_B(0, V_2) - \pi_B(0, 0)$	$\pi_A(-V_1, -V_2)$	$\pi_B(0, 0)$
$R > 0$	$\pi_A(V_1, V_2) - \pi_A(0, V_2)$	$\pi_A(0, V_2) - \pi_A(0, 0)$	$\pi_A(0, 0)$	$\pi_B(-V_1, -V_2)$

Integration

ISP1+CPA

$R < -V_1\sigma_1$	$\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$	
$-V_1\sigma_1 < R < 0$ and $R < R_1$	$\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0)$	
$-V_1\sigma_1 < R < 0$ and $R > R_1$	$\pi_A(V_1, 0)$	
$R > 0$	$\pi_A(V_1, 0)$	

In this setting, ISP 1 and CP A may jointly lose with integration:

A) Let $R < -V_1\sigma_1$

with no integration, joint profit is given by

$$\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(0, V_2)$$

with integration,

$$\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)$$

joint profit is larger with integration.

$$\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(0, V_2) - (\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, V_2)) < 0$$

$$\pi_B(-V_1, V_2) - \pi_B(0, V_2) < 0$$

$$\frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c} = \left(-\frac{1}{18}\right) t_c^{-1} (6t_c - 2R - V_1\sigma_1 + 2V_2\sigma_2) V_1\sigma_1 < 0$$

B) Let $-V_1\sigma_1 < R < 0$ and $R < R_1$

with no integration, joint profit is given by

$$\pi_B(V_1, V_2) - \pi_B(0, V_2) + \pi_A(-V_1, -V_2)$$

with integration

$$\pi_A(-V_1, -V_2) + \pi_B(V_1, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0)$$

Joint profit with integration is larger:

$$\pi_B(0, V_2) - \pi_B(-V_1, 0) - \pi_A(V_1, V_2) + \pi_A(V_1, 0) > 0$$

$$\frac{(3t_c - R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R - \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} > 0$$

$$\left(-\frac{1}{18}\right) t_c^{-1} (2RV_1\sigma_1 + 4RV_2\sigma_2 - 6V_1t_c\sigma_1 + 2V_1V_2\sigma_1\sigma_2 + V_1^2\sigma_1^2) > 0$$

$$R < \frac{-\sigma_1 V_1}{2(V_1\sigma_1 + 2V_2\sigma_2)} (V_1\sigma_1 + 2V_2\sigma_2 - 6t_c) \text{ always true with } R < 0$$

C) $-V_1\sigma_1 < R < 0$ and $R > R_1$

with no integration, joint profit is given by

$$\pi_B(V_1, V_2) - \pi_B(0, V_2) + \pi_A(-V_1, -V_2)$$

with integration

$$\pi_A(V_1, 0)$$

Joint profit with integration is larger iff

$$\pi_B(V_1, V_2) - \pi_B(0, V_2) + \pi_A(-V_1, -V_2) < \pi_A(V_1, 0)$$

$$\pi_B(V_1, V_2) - \pi_B(0, V_2) + \pi_A(-V_1, -V_2) - \pi_A(V_1, 0) < 0$$

$$\frac{(3t_c - R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c - R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R - \sigma_1 V_1 - \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} = \frac{(\sigma_1 V_1 (V_1 \sigma_1 + 2V_2 \sigma_2 - 6t_c) + \sigma_2 V_2 (2V_1 \sigma_1 + 2V_2 \sigma_2 - 6t_c))}{2(3V_1 \sigma_1 + V_2 \sigma_2)}$$

R

D) Let $R > 0$

with no integration, joint profit is given by

$$\pi_A(V_1, V_2) - \pi_A(0, V_2) + \pi_A(0, 0)$$

with integration

$$\pi_A(V_1, 0)$$

Joint profit with no integration is larger:

$$\pi_A(V_1, V_2) - \pi_A(0, V_2) + \pi_A(0, 0) - \pi_A(V_1, 0) > 0$$

$$\frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 V_2)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 0 + \sigma_2 V_2)^2}{18t_c} + \frac{(3t_c + R + \sigma_1 0 + \sigma_2 0)^2}{18t_c} - \frac{(3t_c + R + \sigma_1 V_1 + \sigma_2 0)^2}{18t_c} = \frac{1}{9} t_c^{-1} \sigma_2 \sigma_1 V_2 V_1 >$$

0.

Regarding CP B it is worst when $R < 0$ since $\pi_B(0, 0) > \pi_B(-V_1, 0) > \pi_B(-V_1, -V_2)$, and is indifferent when $R > 0$.

Regarding IPS 2:

A) It is worst with integration since $[\pi_B(-V_1, V_2) - \pi_B(-V_1, 0)] - [\pi_B(0, V_2) - \pi_B(0, 0)] = -\frac{1}{9} \sigma_1 \sigma_2 V_1 \frac{V_2}{t_c} < 0$

B) and C) It may be better or worst with integration: $[\pi_A(V_1, V_2) - \pi_A(V_1, 0)] - [\pi_B(0, V_2) - \pi_B(0, 0)] = \frac{1}{9} \sigma_2 V_2 \frac{2R + \sigma_1 V_1}{t_c} > 0$ if $R > -\frac{\sigma_1 V_1}{2}$.

D) It is better with integration since $[\pi_A(V_1, V_2) - \pi_A(V_1, 0)] - [\pi_A(0, V_2) - \pi_A(0, 0)] = \frac{1}{9} \sigma_1 \sigma_2 V_1 \frac{V_2}{t_c} > 0$

Regarding consumer surplus:

A) and B) Consumer surplus does not change.

C) Under no integration, consumer surplus from a consumer choosing ISP j is given by:

$$CS_{CP}^{NI} = \int_0^{\frac{1}{2} + \frac{-3V_j + (R + 2(\sigma_1 V_1 + \sigma_2 V_2))}{6t_c}} \left(- \left(c + t_c - \frac{2r_A + r_B + \sigma_2 V_2 + \sigma_1 V_1}{3} \right) - t_c(x)^2 \right) dx + \int_{\frac{1}{2} + \frac{-3V_j + (R + 2(\sigma_1 V_1 + \sigma_2 V_2))}{6t_c}}^1 \left(V_j - \left(c + t_c - \frac{r_A + 2r_B - \sigma_2 V_2 - \sigma_1 V_1}{3} \right) - t_c(1-x)^2 \right) dx$$

Under integration, consumer surplus from a consumer choosing ISP j is given by:

$$CS_{CP}^I = \int_0^{\frac{1}{2} + \frac{3V_j + (R - 2(\sigma_1 V_1 + \sigma_2 V_2))}{6t_c}} \left(V_j - \left(c + t_c - \frac{2r_A + r_B - \sigma_2 V_2 - \sigma_1 V_1}{3} \right) - t_c(x)^2 \right) dx + \int_{\frac{1}{2} + \frac{3V_j + (R - 2(\sigma_1 V_1 + \sigma_2 V_2))}{6t_c}}^1 \left(- \left(c + t_c - \frac{r_A + 2r_B + \sigma_2 V_2 + \sigma_1 V_1}{3} \right) - t_c(1-x)^2 \right) dx$$

Variation in CS from a consumer choosing ISP j is given by: $CS_{CP}^I - CS_{CP}^{NI} = -\frac{1}{9}R \frac{2(\sigma_1 V_1 + \sigma_2 V_2) - 3V_j}{t_c}$.

The total variation in CS is given by: $\frac{1}{9}R \frac{\sigma_1 V_1 + \sigma_2 V_2}{t_c} < 0$.

D) Consumer surplus does not change.

Regarding total welfare (discounting advertising revenues):

C) Under no integration, social welfare from buying content in ISP j is given by:

$$SW_{CP}^{NI} = \int_0^{\frac{1}{2} + \frac{-3V_j + (R + 2(\sigma_1 V_1 + \sigma_2 V_2))}{6t_c}} (-t_c(x)^2) dx + \int_{\frac{1}{2} + \frac{-3V_j + (R + 2(\sigma_1 V_1 + \sigma_2 V_2))}{6t_c}}^1 (V_j - t_c(1-x)^2) dx$$

Under integration, social welfare from buying content in ISP j is given by:

$$SW_{CP}^I = \int_0^{\frac{1}{2} + \frac{3V_j + (R - 2(\sigma_1 V_1 + \sigma_2 V_2))}{6t_c}} (V_j - t_c(x)^2) dx + \int_{\frac{1}{2} + \frac{3V_j + (R - 2(\sigma_1 V_1 + \sigma_2 V_2))}{6t_c}}^1 (-t_c(1-x)^2) dx$$

Variation in SW from a consumer choosing ISP j is given by: $SW_{CP}^I - SW_{CP}^{NI} = \frac{2}{9}R \frac{\sigma_1 V_1 + \sigma_2 V_2}{t_c}$. The total variation in SW is given by: $\frac{2}{9}R \frac{\sigma_1 V_1 + \sigma_2 V_2}{t_c} > 0$.