

Macieira, João; Pereira, Pedro; Vareda, João

**Conference Paper**

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20th Biennial Conference of the International Telecommunications Society (ITS): "The Net and the Internet - Emerging Markets and Policies", Rio de Janeiro, Brazil, 30th-03rd December, 2014

**Provided in Cooperation with:**

International Telecommunications Society (ITS)

*Suggested Citation:* Macieira, João; Pereira, Pedro; Vareda, João (2014) : Bundling incentives in markets with product complementarities: The case of triple-play, 20th Biennial Conference of the International Telecommunications Society (ITS): "The Net and the Internet - Emerging Markets and Policies", Rio de Janeiro, Brazil, 30th-03rd December, 2014, International Telecommunications Society (ITS), Calgary

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# Bundling Incentives in Markets with Product Complementarities: The Case of Triple-Play\*

João Macieira<sup>†</sup>

Pedro Pereira<sup>‡</sup>

João Vareda<sup>§</sup>

Virginia Tech

AdC and CEFAGE-UE

AdC and CEFAGE-UE

January 3, 2014

## Abstract

We analyze firms' incentives to bundle and tie in the telecommunications industry. As a first step, we develop a discrete-choice demand model where firms sell products that may combine several services in bundles, and consumers choose assortments of different types of products available from various vendors. Our approach extends standard discrete-choice demand models of differentiated product to allow for both flexible substitution patterns and to map demand for each choice alternative onto the demand for each service or bundle that a firm may sell. We exploit these properties to examine bundling behavior when firms choose: **(i)** prices, and **(ii)** which products to sell. Using consumer-level data and survey data from the Portuguese telecommunications industry, we estimate our demand model and identify firm incentives to bundle and tie in this industry. We use the model to perform several policy related counterfactuals and evaluate their impact on prices and product provision.

**Key Words:** *Bundles, Discrete-Choice Model, Equilibrium Simulation, Differentiated Product, Consumer Level Data.*

**JEL Classification:** D43, K21, L44, L96.

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\*We gratefully acknowledge the financial support from the NET Institute ([www.netinst.org](http://www.netinst.org)). We thank J. Prieger and J. Williams for useful comments. The opinions expressed in this article reflect only the authors' views and in no way bind the institutions to which they are affiliated.

<sup>†</sup>Department of Economics, Virginia Tech. 3044 Pamplin Hall (0316), Blacksburg, VA 24060. E-mail: [joaom07@vt.edu](mailto:joaom07@vt.edu).

<sup>‡</sup>AdC, Avenida de Berna, n° 19, 7°, 1050-037 Lisboa, Portugal. E-mail: [pedro.br.pereira@gmail.com](mailto:pedro.br.pereira@gmail.com).

<sup>§</sup>AdC, Avenida de Berna, n° 19, 7°, 1050-037 Lisboa, Portugal. E-mail: [joao.vareda@concorrencia.pt](mailto:joao.vareda@concorrencia.pt).

# 1 Introduction

Bundling is becoming an important characteristic of the telecommunications industry. An increasing number of households prefers to consume telecommunication services in bundles, rather than separately. In addition, telecommunications firms increasingly base their marketing strategies on these products. The increasing importance of bundles might be the result of many different causes, such as: technological progress, changes in consumption habits, or shifts in the strategic environment. For a discussion of these potential motives see Pereira and Vareda (2011, 2013).

Other examples of industries where bundling is prevalent include: insurance, software, newspapers, music or air transportation. While the theoretical literature on bundling, reviewed in Section 2, has been extensively developed over the last decades, empirical work on bundles has only recently experienced important advances, e.g., Chu et al. (2011), Crawford and Yurukoglu (2012), Gentzkow (2007), Pereira et al. (2013). An important void in this literature is the lack of modeling of both: **(i)** the demand for product assortments by consumers, and **(ii)** the joint decisions of pricing and supply of products by firms. In the article we fill in this void.

We propose a differentiated product equilibrium model that has three important characteristics, following Pereira et al. (2013). First, there is a set of basic *services*. Second, firms sell *products* that may consist of these individual services, or may combine several of them in bundles. Third, consumers choose *alternatives*, i.e., choose assortments of products that may consist of a single product sold by a given firm, or may combine several products, possibly sold by different firms.

For the demand side, we propose a discrete-choice model that builds upon and extends the differentiated product frameworks of Berry et al. (1995) and Nevo (2001), where consumers choose alternatives as in Pereira et al. (2013).<sup>1</sup> More specifically, we replace the Mixed Logit framework of Berry et al. (1995) and Nevo (2001) by an extension of the Cross-Nested Logit model with random effects.<sup>2</sup> In particular, we use the Cross-Nested Logit model with the parametrization of Bresnahan et al. (1997). For a general discussion of the properties of the Cross-Nested Logit model see, e.g., Bierlaire (2006).<sup>3</sup> The resulting framework allows for flexible substitution patterns, allows handling a potentially large number of consumer choice alternatives, and allows mapping the demand for each choice alternative onto the demand for each service or bundle that a firm may sell. By letting each service be a nest,

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<sup>1</sup>More generally, our methodological approach draws on the discrete choice literature, represented by, e.g., Domencich and McFadden (1975) and McFadden (1974, 1978, 1981).

<sup>2</sup>See Grigolon and Verboven (2013).

<sup>3</sup>The Cross-Nested model belongs to the Generalized Extreme Value class introduced by McFadden (1978). See also Fosgerau, McFadden, and Bierlaire (2010), Wen and Koppelman (2001), and Koppelman and Sethi (2007). Previous applications of the Cross-Nested Logit model in economics include Adams et al. (2007), Bresnahan et al. (1997), Pereira et al. (2013) and Small (1987).

our framework includes Berry et al. (1995) and Nevo (2001) as a particular case, and allows to control for having alternatives embodying different services. In addition, allowing for consumer heterogeneity in tastes enables the examination of the role of this feature on the incentives to bundle, namely as a means to replicate price discrimination.

For the supply side, we allow firms to choose both: **(i)** prices, and **(ii)** which products to supply, individual services and bundles. Firms acknowledge that consumers acquire their products as part of a product assortment, by letting the share of each product equal the sum of purchase probabilities where that product is present. In addition, the decision to sell or not a single service or a bundle impacts not only firms' profits, but also the set of alternatives available to consumers. Our supply model allows firms to engage in various forms of pricing such as: component pricing, pure bundling, mixed bundling and tying.

We estimate the model using consumer-level data and survey data from the Portuguese telecommunications industry, focusing on triple-play bundles, i.e., products that include: **(i)** fixed telephony, **(ii)** fixed broadband access to the internet and **(iii)** subscription television. Since triple-play products embody three services and that one observes these services being supplied in different combinations across firms and markets, this is a unique opportunity to study firms' incentives to bundle. More specifically, we exploit the fact that in the context of triple-play products, sometimes firms sell services in bundles that are unavailable individually, to identify firm incentives to bundle products.

We use the estimated model to evaluate some of the incentives to bundle and tie identified in the literature. While our model allows simulating the welfare impact of many policy issues of practical interest, e.g., mergers or tying bans, we focus on three issues: **(i)** cost synergies, **(ii)** consumer heterogeneity and **(iii)** market structure.

First, we examine if bundling is motivated by cost savings. The theoretical literature has argued that bundling is a consequence of the cost synergies associated to selling several products jointly.<sup>4</sup> We use the model's estimates to identify product marginal costs and test for bundling cost synergies. Our results do not reject the hypothesis there are no bundling cost synergies in this industry.

Second, we examine bundling and pricing behavior of firms in response to variations in consumer heterogeneity of the price coefficient of demand. An explanation advanced by the theoretical literature is that bundling is used to replicate price discrimination by exploiting consumer heterogeneity in price sensitivity. We assess the plausibility of this claim by analyzing how the simulated market equilibrium changes when the variance of the price coefficient of demand increases. Our simulation results indicate that firms respond by offering more products of various types. In addition, prices decrease. We observe also some quality reshuffling from single-play products to double- and triple-play products. This quality reshuffling takes the form of a decrease in the quality of single-play products, e.g.,

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<sup>4</sup>Production synergies are different from selling synergies. Bundling cost synergies refers to the latter.

fewer channels or lower bandwidth, while the quality of bundles increases.

Third, we examine bundling and pricing behavior by firms in response to different market structures. The theoretical literature has argued that, depending on the circumstances, bundling can be used either to decrease or increase the level of competition. We assess the plausibility of this claim by analyzing how the simulated market equilibrium changes when the number of firms in the industry increases. We examine how a firm chooses prices and products to supply in a representative market when: **(i)** it is a monopolist, and **(ii)** it competes in duopoly with a similar firm. Our simulation results indicate that moving from monopoly to duopoly leads to both lower prices and to the supply of additional products. In contrast to the higher consumer heterogeneity scenario, increased competition leads to higher quality for all products in the form of higher values for product attributes valued by consumers.

Our two counterfactuals indicate that firms compete in quality, in addition to prices and product availability. Yet the theoretical literature on bundling has paid little attention to competition on product availability and non to competition in quality. Our results illustrate its importance in practice.

The rest of the article is organized as follows. In Section 2 we revise the literature related to our article. Section 3 gives an overview of the Portuguese triple-play industry. Section 4 outlines our structural model of demand and supply. Section 5 discusses our data set and the estimation and simulation of the model. We present and demand estimation results in Section 6. Section 7 conducts the analysis. Section 8 concludes.

## 2 Literature Review

Our article relates to three large bodies of literature. The first consists of the literature that develops estimable equilibrium models for differentiated product industries, pioneered by Bresnahan (1987) and extended by Berry (1994), Berry et al. (1995) and Nevo (2000, 2001).<sup>5</sup> The second consists of the literature that models and estimates the choice over bundles, which includes Augereau et al. (2006) for internet standards, Gandal et al. (2013) and Riordan (2004) for personal computer office software, Gentzkow (2007) for print and online newspapers, and Pereira et al. (2013) for telecommunications services.<sup>6</sup> The third consists on the literature on bundling and tying, which includes both theoretical and empirical work.

The theoretical literature on bundling and tying focuses on firms' incentives, which are typically related to preference or technology complementarities, price discrimination, and

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<sup>5</sup>More generally, our methodological approach draws on the discrete choice literature, represented by, e.g., Domencich and McFadden (1975), McFadden (1974, 1978, 1981).

<sup>6</sup>Liu et al. (2010) using the model of Gentzkow (2007) find strong complementarities between subscription television and fixed broadband/cable modem and between fixed voice and fixed broadband/DSL.

firm strategic behavior.<sup>7</sup> Stigler (1963) showed, through an example, that mixed bundling can be profitable for a monopolist, if consumer valuations are negatively correlated across goods. Under these circumstances, bundling can be used to reduce consumer heterogeneity and has an effect similar to price discrimination, thereby facilitating the extraction of consumer surplus. Afterwards, several authors, e.g., Adams and Yellen (1976), Chen and Riordan (2011), Fang and Norman (2006), Geng et al. (2005), McAfee et al. (1989), Salinger (1995), Schmalensee (1984), showed with various degrees of generality that this intuition holds as long as valuations are not perfectly positively correlated. Carbajo et al. (1990), Chen (1997) showed that pure bundling can be used as a product differentiation strategy that reduces the intensity of competition. However, depending on consumer preferences, bundling may actually increase the level of competition, as shown by Anderson and Leruth (1993) for the case of logit preferences. Hurkens et al. (2013) investigate in the context of an asymmetric duopoly whether bundling increases or reduces competition. Carlton and Waldman (2011, 2002), Carlton et al. (2010), Nalebuff (2004), Peitz, M. (2008), Whinston (1990) showed that bundling can be used to foreclose the market, or more generally to create, preserve or extend monopoly positions. Chu et al. (2011) argue that mixed bundling can be very complex even with a small number of individual products and evaluates to what extent simpler pricing schemes, such as bundle-size pricing, allow a monopolist to capture a substantial part of the profits of mixed bundling.

We provide an estimable differentiated product equilibrium model, where consumers choose among a set of alternatives, that may or may not consist of bundles, and where firms' pricing and supply decisions, that may or may not include bundles, are the result of optimizing behavior. The model enables evaluating numerically the empirical relevance of the behavior predicted by the theoretical literature.

There is an emerging empirical literature on the welfare impact of bundling in several industries, which helps to put in perspective the wide range of welfare implications of the various theoretical justifications for bundling. Byzaov (2010) analyzed the welfare impact of various restrictions to bundling of channels for the cable television industry, using consumer level data and taking upstream prices as exogenous. Chu et al. (2011) analyzed the profitability of simple pricing alternatives to mixed bundling for the theatre industry. Crawford (2008) analyzed the discriminatory incentives for bundling in the cable television industry. Crawford and Yurukoglu (2012) estimated the welfare effects of unbundling in the retail cable television industry, using firm level data and endogeneizing upstream prices. Derdenger and Kumar (2012) analyzed hand-held video games. Haas-Wilson (1987) analyzed the impact of state legislation on tying for the contact lens industry. Ho et al. (2012) analyzed full-line forcing in the video rental industry. Pereira et al. (2013) analyzed if bundles of subscription television, fixed broadband and fixed voice are a relevant product market in the sense of competition policy, using consumer level data. Shiller and Waldfogel

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<sup>7</sup>For a discussion of the Leverage Doctrine and the One-Profit Doctrine see, e.g., Bowman (1957), Director and Levi (1956) and Schmalensee (1982).

(2011) analyzed the welfare impact of various forms of pricing for the music industry.

The growing importance of triple-play products poses several problems for competition authorities and sectoral regulators, discussed in Pereira and Vareda (2011, 2013) and Pereira et al. (2013), which need to be evaluated empirically. However, direct usage of conventional differentiated product models, e.g., Berry et al. (1995) and Nevo (2001), to examine structural market changes, such as mergers and acquisitions, is problematic. In this article we take a step to fill in that void by developing an estimable differentiated product equilibrium model that accommodates bundling behavior.

### 3 The Portuguese Industry

In Portugal, in the early 1990s, both domestic television and telecommunications were state-owned monopolies. Later on that decade, several free-to-air television channels and cable television firms were licensed and the telecommunications incumbent, Portugal Telecom (PT), was privatized.<sup>8</sup> The telecommunications industry was further liberalized in 2000.

Initially, entrants based their offers of fixed voice and broadband access services in the wholesale access to PT's copper wire network. Later, as they obtained a substantial customer base, entrants resorted to the unbundled access to PT's local loop. After 2006 there was a large increase in the number of unbundled loops. As a consequence, many innovative products, for instance bundles, were introduced in the market. In the meanwhile, some entrants invested in their own infrastructures, increasing further their autonomy. In November 2007, ZON, a cable television firm, was spun-off from PT. This was an important change in the Portuguese industry. ZON, using its cable television network, started to compete with PT, using its telephone network.<sup>9</sup> Recently, PT initiated the deployment of a fiber-optic network, while ZON upgraded its cable network by installing DOCSIS 3.0.

The other relevant firms in the industry include AR Telecom, Cabovisão, Optimus and Vodafone. AR Telecom began operations in 2005, basing its products mainly on fixed wireless access technology. Cabovisão, a cable television firm, was created in 1995. Optimus, originally a mobile telecommunications firm, entered the fixed services business in 2000 using local loop unbundling, with access via Digital Subscriber Line (DSL). After 2008 it also started deploying its fiber-optic network. Vodafone, originally a mobile telecommunications firm, entered the fixed services business in 2000, using local loop unbundling, with access by DSL.

In November 2011, AR Telecom exited the market and passed its customers to ZON. In January 2013, ZON proposed a merger with Optimus. The operation is under evaluation

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<sup>8</sup>Private free-to-air channels were licensed in 1992, PT was privatized in 1996, cable television licenses were issued in 1997 and in 1999 cable television firms were authorized to offer telecommunications services.

<sup>9</sup>For more details see Pereira and Vareda (2011, 2013).

by the Portuguese Competition Authority.

Our data set, described below, contains information only for 2009. In that year, the penetration rate per inhabitants of fixed telephony was 40%. After a long period of decline, the penetration rate of fixed telephony started to increase again, slightly. Also in 2009, the penetration rate per households of subscription television was 45%. Of these subscribers, 57.4% used cable and 23.2% Direct to Home (DTH) technology.<sup>10</sup> Finally, in 2009, the penetration rate per inhabitants of fixed broadband was 18%. Of these subscribers, 57% used DSL and 40% used cable modem.

Table 1 presents the markets shares of the largest telecommunications firms in 2008 and 2009 for each type of service.

[Table 1]

Telecommunications bundles were first offered in Portugal in 2004 through cable television networks. Afterwards, several firms launched similar products using fixed telephone networks, either through local loop unbundling or their own networks.

## 4 Model

### 4.1 General Considerations

In this Section, we propose an estimable differentiated product equilibrium model in the spirit of Berry et al. (1995) and Nevo (2000, 2001), but with four differences. First, there is a set of basic *services* and firms sell *products* that may combine several of these services. In our application there are three services: **(i)** fixed telephony (FV), **(ii)** fixed broadband access to the internet (BB) and **(iii)** subscription television (TV). Firms may sell three types of products: single-play products, which include only one service, double-play products, which include two services, and triple-play products, which include the three services. Second, consumers choose among *alternatives* that may combine several products, possibly sold by different firms. For example, a consumer may choose a double-play product TV+BB supplied by a given firm, while also purchasing a single-play product FV supplied by another firm. Third, we model consumer preferences through a Cross-Nested Logit (CNL) model with random effects. The fact that the set of all consumer alternatives consists of the set of all available product assortments, our product demand model should allow for correlation in alternative preference shocks since many alternatives contain similar products. To this end, we consider Bresnahan et al.'s (1987) parametrization of the CNL model. In contrast to conventional models of market segmentation, such as the Nested Logit, the PDGEV allows

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<sup>10</sup>In Portugal there are no independent satellite television firms. Two of the telecommunications or television firms offer satellite television services as complements to their other services in the regions not covered by their physical networks.



for overlapping product nests. This property allows us to control for closer substitution of alternatives lying within a segment, while allowing for choices containing products with various services to be present in several segments. Fourth, aside from prices, firms choose which products to sell in each market. This allows for strategic supply behavior in markets with bundles. For example, firms may decide not to sell a certain product consisting only of single service to encourage consumers to choose another product, which bundles that service with other services.

We denote markets by subscript  $t = 1, \dots, T$ , services by subscript  $i = 1, \dots, I$ , products combining services by subscript  $j = 1, \dots, J$ , alternatives combining products by subscript  $a = 0, 1, \dots, A$ , where  $a = 0$  is the alternative of buying no product, and firms by subscript  $f = 1, \dots, F$ .

We follow the convention of denoting by  $\mathbf{x} := (x_1, \dots, x_n)$ , a  $n$ -dimensional vector of real numbers,  $x_j$ , and letting  $\mathbf{x}_{-j} := (x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_n)$ .

## 4.2 Supply

We assume that firms simultaneously decide both: **(i)** which products to supply, and **(ii)** product prices. Denote by  $d_{jt} \in \{0, 1\}$  the decision of the firm that owns product  $j$  to sell it in market  $t$  and by  $p_{jt} \in \mathbb{R}^+$  the product's price. Denote by  $c_{jt}$  the marginal cost of product  $j$  in market  $t$ , by  $C_{ft}$  the fixed cost of firm  $f$  in market  $t$ , by  $M_t$  the number of consumers in market  $t$ , and by  $s_{jt}(\mathbf{p}_t, \mathbf{d}_t)$  the market share of product  $j$  in market  $t$ . Firm  $f$ 's profit is:

$$\Pi_{ft}(\mathbf{p}_t, \mathbf{d}_t) = \sum_{j \in \mathcal{F}_f} d_{jt}(p_{jt} - c_{jt})M_t s_{jt}(\mathbf{p}_t, \mathbf{d}_t) - C_{ft}. \quad (1)$$

In each market  $t$ , each firm  $f$  maximizes profits by choosing: **(i)** which products to sell, and **(ii)** their prices. The first-order condition for price  $p_{jt}$  is:

$$d_{jt}s_{jt}(\mathbf{p}_t, \mathbf{d}_t) + \sum_{k \in \mathcal{F}_f} d_{kt}(p_{kt} - c_{kt}) \frac{s_{kt}(\mathbf{p}_t, \mathbf{d}_t)}{\partial p_{jt}} = 0. \quad (2)$$

Firm  $f$  will sell product  $j$  in market  $t$ , if and only if, the decision to sell it generates a profit higher than the decision of not to sell it, i.e., if and only if,  $\forall j \in \mathcal{F}_f$ ,  $f = 1, \dots, F$ :

$$\Pi_{ft}(\mathbf{p}_t, \mathbf{d}_t) = \max \{ \Pi_{ft}(\mathbf{p}_t, 0, \mathbf{d}_{-jt}), \Pi_{ft}(\mathbf{p}_t, 1, \mathbf{d}_{-jt}) \}. \quad (3)$$

For all  $j, k = 1, \dots, J$ , define the  $J \times J$  ownership matrix  $\mathbf{H}$ , whose generic element is:

$$H_{jk} = \begin{cases} 1 & \text{if } \exists f : \{j, k\} \subset \mathcal{F}_f \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

In addition, denote the  $J \times J$  matrix containing derivatives of market shares with respect to prices by  $\Delta$ , whose generic element is  $\Delta_{jk} = -\frac{\partial s_{jk}}{\partial p_{jt}}$ . Denote by  $*$  the Hadamard product,

i.e., the element-by-element matrix product. The system defined in (2) can be written in matrix form as:<sup>11</sup>

$$\mathbf{d}_t * \mathbf{s}_t(\mathbf{p}_t, \mathbf{d}_t) - (\mathbf{H} * \mathbf{\Delta}) [\mathbf{d}_t * (\mathbf{p}_t - \mathbf{c}_t)] = 0. \quad (5)$$

We follow the literature in assuming the existence of a pure-strategy, Nash equilibrium, which in our case is both in prices and commercialization decisions. That is, a Nash Equilibrium in each market  $t = 1, \dots, T$  consists of a vector of product supply decisions  $\mathbf{d}_t = (d_{1t}, \dots, d_{Jt})$  and non-negative prices  $\mathbf{p}_t = (p_{1t}, \dots, p_{Jt})$  that solve both (2) and (3) for all  $j = 1, \dots, J$ .

### 4.3 Demand

The complete definition of system (5) requires additional structure to relate product market shares and consumer demand for product assortments. We assume that consumers choose among alternatives, i.e., assortment of products sold by firms aimed at providing several different services. For example, an assortment of subscription TV with 30 channels and broadband internet with bandwidth of 30 Mbps provides two services: subscription TV and broadband internet. Denote by  $u_{hjt}$  the utility derived by consumer  $h$  from product  $j$  included in alternative  $a$ , by  $\epsilon_{hat}$  a preference shock of consumer  $h$  for alternative  $a$ , by  $\mathbf{X}_{jt}$  a  $K \times 1$  vector of observed characteristics of product  $j$ , by  $\xi_j$  is a market-wise mean consumer valuation of product characteristics unobserved to the researcher, by  $\Delta\xi_{jt}$  the market-specific deviation from  $\xi_j$ , and by  $(\alpha_h, \beta_h)$  a vector of consumer-specific taste parameters for observed attributes  $(p_{jt}, \mathbf{X}_j)$ .

The utility that consumer  $h$  derives from alternative  $a$  is:

$$u_{hat} = \sum_{j \in a} u_{hjt} + \epsilon_{hat}. \quad (6)$$

In addition,  $u_{hjt}$  is given by:

$$u_{hjt} = \mathbf{X}_j \beta_h - \alpha_h p_{jt} + \xi_j + \Delta\xi_{jt}; \quad (7)$$

We assume that the joint distribution of  $(\alpha_h, \beta_h)$  is:

$$\begin{pmatrix} \alpha_h \\ \beta_h \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \mathbf{v}_h; \quad (8)$$

where  $\mathbf{v}_h \sim N(\mathbf{0}, \mathbf{\Psi})$  and  $\mathbf{\Psi}$  is a  $(K+1) \times (K+1)$  strictly positive, diagonal matrix.

Let  $\delta_{at}$  be the mean utility from choosing alternative  $a$  and  $\mu_{hat}$  be the portion of utility

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<sup>11</sup>This system is similar to those of Bresnahan (1987), Berry et al. (1995), Nevo (2000, 2001), except that we control for the decision of whether to supply a product.

that depends on consumer-specific components:

$$\delta_{at} = \sum_{j \in a} X_j \beta - \alpha \sum_{j \in a} p_{jt} + \sum_{j \in a} \xi_j + \sum_{j \in a} \Delta \xi_{jt}, \quad (9)$$

$$\mu_{hat} = \left[ \sum_{j \in a} p_{jt}, \sum_{j \in a} X_j \right]' * \mathbf{v}_h. \quad (10)$$

In the spirit of Berry et al. (1995) and Nevo (2000,2001), we rewrite  $u_{hat}$  as:

$$u_{hat} = \delta_{at} + \mu_{hat} + \epsilon_{hat}; \quad (11)$$

As the utility derived from alternative  $a = 0$  is not identified, we let  $\delta_{0t} = 0$  and  $\mu_{h0t} = 0$ . That is, the utility from this outside alternative has zero mean and is given by  $u_{h0t} = \epsilon_{h0t}$ , where  $\epsilon_{h0t}$  is the outside alternative shock.

Given the set of alternatives available in market  $t$ , each consumer chooses the alternative that maximizes his utility. We assume further that, for each consumer the vector of alternative preference shocks  $(\epsilon_{h0t}, \epsilon_{h1t}, \dots, \epsilon_{hAt})$  follows a Generalized Extreme Value (GEV) distribution. That is, as in McFadden (1978), the joint distribution of the preference shocks vector is:

$$F(\epsilon_{h0t}, \dots, \epsilon_{hAt}) = \exp \left( -G(e^{-\epsilon_{h0t}}, \dots, e^{-\epsilon_{hAt}}) \right); \quad (12)$$

where  $G(\cdot)$  is a nonnegative, homogenous of degree one function mapping set  $\mathbb{R}^{A_t+1}$  onto  $\mathbb{R}_0^+$ , whose partial derivative with respect to the term  $e^{\delta_{at} + \mu_{hat}}$  is denoted by  $G_a(\cdot)$ .<sup>12</sup> Under this assumption, the probability that consumer  $h$  chooses alternative  $a$  in market  $t$  is:

$$s_{hat} = \frac{e^{\delta_{at} + \mu_{hat}} G_a(e^{\delta_{0t} + \mu_{h0t}}, \dots, e^{\delta_{At} + \mu_{hAt}})}{G(e^{\delta_{0t} + \mu_{h0t}}, \dots, e^{\delta_{At} + \mu_{hAt}})}. \quad (13)$$

The fact that consumers choose among combinations of available products, possibly offered by different suppliers, motivates letting our product demand model allow for correlation in alternative preference shocks. To this end, we build upon and extend the model proposed by Bresnahan et al. (1997), where the GEV model is restricted to allow for market segmentation along Principles of Differentiation (PDs), or nests. That is, we consider Bresnahan et al.'s (1997) parametrization of the CNL model. In contrast to conventional models of market segmentation, e.g., the Nested Logit, the PDGEV allows us to control for closer substitution of alternatives lying within a segment, while allowing for choices containing products with various services to be present in several segments.

We consider nests for three services: **(i)** FV, **(ii)** TV and **(iii)** BB. Each nest implies two segments: one with only alternatives containing a product where that service is provided, and another segment with only alternatives where that service is absent. For example, the segment *FV* contains alternatives that include fixed-voice telephony service, and the

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<sup>12</sup>We forward the reader to McFadden (1978) for full set of assumptions function  $G(\cdot)$  must satisfy.

segment  $NFV$  contains only alternatives without fixed-voice service, except the outside alternative. The segments pairs  $(TV, NTV)$  and  $(BB, NBB)$  are defined analogously for subscription television and fixed broadband, respectively. We denote the set of segments by  $D = \{FV, NFV, TV, NTV, BB, NBB\}$  and we restrict each pair of segments to have the same segment parameter, whose value lies between 0 and 1. That is, the segments  $FV$  and  $NFV$  have the same segment parameter  $\rho_{FV}$ , while  $TV$  and  $NTV$  have segment parameter  $\rho_{TV}$  and both  $BB$  and  $NBB$  share the segment parameter  $\rho_{BB}$ .<sup>13</sup> Under these conditions, we compose the function  $G(\cdot)$  as:

$$G(e^{\delta_{0t}+\mu_{h0t}}, \dots, e^{\delta_{At}+\mu_{hAt}}) = \sum_{d \in D} \gamma_d \left( \sum_{a \in d} e^{(\delta_{at}+\mu_{hat})/\rho_d} \right)^{\rho_d} + e^{\delta_{0t}+\mu_{h0t}}; \quad (14)$$

where  $\gamma_d$  are scaling parameters defined by:

$$\gamma_d = \frac{1 - \rho_d}{3 - \rho_{FV} - \rho_{TV} - \rho_{BB}}. \quad (15)$$

Denote by  $P_{ah|d}$  the probability that consumer  $h$  chooses alternative  $a$  given that he is choosing from segment  $d$ , and by  $P_{dh}$  the probability that consumer  $h$  chooses an alternative from segment  $d$ . Under this specification, the consumer choice probability in (13) simplifies to:

$$s_{hat} = \sum_d P_{ah|d} P_{dh} = \sum_d \left( \frac{e^{(\delta_{at}+\mu_{hat})/\rho_d}}{\sum_{k \in D} e^{(\delta_{kt}+\mu_{hkt})/\rho_d}} \right) \left( \frac{\gamma_d \left( \sum_{k \in d} e^{(\delta_{kt}+\mu_{hkt})/\rho_d} \right)^{\rho_d}}{G(e^{\delta_{0t}+\mu_{h0t}}, \dots, e^{\delta_{At}+\mu_{hAt}})} \right). \quad (16)$$

Consumer heterogeneity is completely characterized by the random coefficient specification in (8). For ease of exposition, let  $\theta_h = (\alpha_h, \beta_h)$ , and denote  $F(\theta_h)$  as its CDF. Then the aggregate market share for alternative  $a$  in market  $t$  is:

$$s_{at} = \int s_{hat}(\theta_h) dF(\theta_h). \quad (17)$$

It is convenient to map markets shares of alternatives defined in (17) onto product market shares. For each product  $j$ , its market share is the sum of the shares of all alternatives where product  $j$  is included:

$$s_{jt} = \sum_{a=1}^A \mathbf{1}\{j \in a\} s_{at}. \quad (18)$$

The total price of alternative  $a$ , denoted  $p_{at}$ , is the sum of the prices of the products that are part of alternative  $a$ :

$$p_{at} = \sum_{j=1}^J \mathbf{1}\{j \in a\} p_{jt}. \quad (19)$$

For  $j = 1, \dots, J$  and  $a = 1, \dots, A$ , define the  $A \times J$  matrix  $\Gamma$ , which controls for the presence of products in an alternative, and whose generic element is:

$$\Gamma_{aj} = \begin{cases} 1 & \text{if } j \in a \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

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<sup>13</sup>For a discussion, see Bresnahan et al. (1997) and Wen and Koppelman (2001).

Let  $\mathbf{p}_t^{Alt}$  denote the vector of total price of alternatives, as defined in (19). Also, Denote by  $S_t^{Alt}$  the vector of market shares of alternatives defined in (17). Matrix  $\mathbf{\Gamma}$  allows us to write the product market share and alternative price vectors, respectively, as:

$$S_t = \mathbf{\Gamma}' S_t^{Alt}, \quad (21)$$

$$\mathbf{p}_t^{Alt} = \mathbf{\Gamma} \mathbf{p}_t. \quad (22)$$

The system of first-order conditions in prices defined in (5) involves a matrix  $\mathbf{\Delta}$  of derivatives of product demand with respect to product prices. This matrix can also be written as a function the demand for alternatives. For any two alternatives  $a, b = 1, \dots, A$  define the  $A \times A$  matrix  $\mathbf{\Delta}^{Alt}$ , whose generic element is  $\Delta_{ab}^{Alt} = -\frac{\partial s_b}{\partial p_{at}}$ . It follows from the definitions above that  $\mathbf{\Delta} = \mathbf{\Gamma}' \mathbf{\Delta}^{Alt} \mathbf{\Gamma}$ .

## 5 Econometric Implementation

The goal of model estimation is to recover the firms' demand and costs parameters, as well as the random coefficient distributions. To this end, we combine the approaches of Berry et al. (1995) and Nevo (2000, 2001) by matching data information to model predictions, but with some differences. First, our supply system differs from the one in those articles, as it involves: **(i)** decisions on which product to supply in each market, and **(ii)** alternative – rather than product – market shares. Second, our demand system acknowledges that consumers purchase assortments of products. We start by describing the data used in the estimation and then turn to estimation details.

### 5.1 Data

#### 5.1.1 Data Request

We obtained data of the last quarter of 2009 from six Portuguese electronic communication firms, which accounted in December 2009 for 99% of triple-play customers. For confidentiality reasons, we will refer to these firms as  $f_1, \dots, f_6$ . The information obtained consisted of data about: **(i)** the contract, **(ii)** the product, **(iii)** the client and **(iv)** monthly expenditures. The characteristics of the contract are: the monthly fee, discounts or joining offers, the commencement date of the contract, and the characteristics of the product. The characteristics of the product are: the brand name, the number of normal and premium television channels and the possibility of access to video-on-demand, if the product included subscription television, bandwidth, traffic limits, number of e-mail accounts and the possibility of mobile broadband, if the product included fixed broadband access to the Internet, and the tariff plan for fixed telephony. The characteristics of the client are: age, length of the contract and residential postal code. We also obtained billing information for the last

quarter of 2009, with full detail of invoices, including the fixed monthly fee and variable components, e.g., movie rentals, channel rentals, internet traffic above contracted limits, expenditure on telephone calls and minutes of conversation. Finally, we obtained the total number of clients for each product offered, and the geographical availability of each product. This data was complemented with information from the sectoral regulator, ICP-ANACOM, drawn from the survey “Inquérito ao consumo dos serviços de comunicações electrónicas - População residencial – Dezembro de 2009”, from, hereon “Inquérito ao consumo”, which characterizes the typical national consumer of electronic communication services.

### 5.1.2 Products, Markets and Choice Alternatives

We define a *product* as a combination of fixed-voice telephony, subscription channels and broadband internet services supplied by a single firm. A single-play product includes one *service*, a double-play product includes two services and a triple-play product includes the three services. Table 2 details the possible combinations of: services, forms of acquisition and firms.

[Table 2]

Our empirical model considers product shares and prices per market  $t = 1, \dots, T$ . We define a market as a statistical NUTS3 region and consider a total of 30 NUTS3 regions.<sup>14</sup> The information from *Inquérito ao consumo* allowed us to relate the electronic communication services consumed by households to the way they are acquired, and to obtain the percentage of households that do not consume any of these services. Table 3 presents the distribution of services by type of bundle in 2009.<sup>15</sup>

[Table 3]

This information, and the data obtained from firms, allowed us to determine the distributions of the services per household and the market shares per firm for each service. We used this information along with the choice-based sample information on consumer purchases by firm in each region to derive regional market shares.

We compute the average product price in each market as the average monthly fee, net of all discounts, that costumers from a given NUTS3 region pay for the product. Market size and shares are computed using both the information on product sales available in each region and information on number of households in each NUTS3 region, as reported by *Instituto Nacional de Estatística* (INE) - the Portuguese National Statistics Institute. The

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<sup>14</sup>The Nomenclature of Units for Territorial Statistics (NUTS) was created by the European Office for Statistics (Eurostat) as a single hierarchical classification of spatial units used for statistical production across the European Union. It is comparable to Metropolitan Statistical Area (MSA) classification in the USA.

<sup>15</sup>We report this information in intervals for confidentiality reasons

data used in the estimation consists of a total of 1,083 product/market observations.<sup>16</sup> To distinguish single-, double- and triple-play products we form dummy variables to control for bundles. The observed characteristics of a product therefore consist of: **(i)** *FV* dummy variable, **(ii)** *BB* dummy variable, **(iii)** number of offered channels, if the product contains *TV*, **(iv)** bandwidth, in Mbps, in case the product includes *BB*, **(v)** dummy variables for double-play bundles *FV + TV*, *FV + BB* and *TV + BB*, and **(vi)** triple-play bundle dummy variable. Table 4 presents average values of prices, market shares and characteristics by type of product:

[Table 4]

There is a total of 117 different products, yet product availability differs by region. The average market share of single-play products is bigger than double- and triple-play. However, double- and triple-play products typically have higher average number of channels and bandwidth than single-play products. Moreover, the average price of these bundles is typically smaller than the sum of single-play average prices. This fact raises the possibility that bundling may be motivated by potential cost savings from selling several products jointly. We test for this possibility below.

The 117 different products sold by firms are only a subset of the total possible products that firms could sell. Considering the total number of possible combinations of *FV*, the variants of bandwidth and number of channels in *BB* and *TV* services, respectively, there are a total of 478 products that firms could potentially sell. This restriction of product supply by firms impacts the total number of alternatives consumers can choose from.

Consumers choose among alternatives, i.e., combinations of products, possibly supplied by different firms. The concept of alternative does not coincide with the concept of service or a product offered by a firm. A product offered by a firm may be present in several choice alternatives. For example, the single-play product of fixed telephony offered by a given firm is typically present in several alternatives where *FV* service is provided. There are eight possible combinations of services, six possible types of bundles, and seven possible suppliers, with one,  $f_0$ , corresponding to the inexistence of a supplier. Taking into consideration the variants of bandwidth and number of channels in *BB* and *TV* services, respectively, as well as related products sold by firms, there are a total of 1,153 alternatives, including the outside option of no service, that consumers can consider out of the 117 different products sold by firms. In practice, the actual number of alternatives available for a consumer depends on which market he is located, as product availability differs considerably by region. Table 5 provides a simplified illustration of some alternatives.

[Table 5]

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<sup>16</sup>We note that this is about a third of the theoretical maximum of  $30 \times 117 = 3510$  observations that would be available if the 117 products were sold in all regions.

## 5.2 Estimation

The estimation of differentiated product models is compositionally involved, due to the need to solve for unobserved product characteristics using market share equations, as in, e.g., Berry et al. (1995), Nevo (2000, 2001). However, recent developments in the estimation of these models can be used to simplify the estimation process. We build on the estimation approaches of Su and Judd (2012) and Dube et al. (2012), where the step of solving for the vector of  $\Delta\xi_{jt}$  is replaced with a constraint in the estimation problem. To this end, we reparametrize our demand model in a way similar to Nevo (2000, 2001). In what follows, we let  $j = 1, \dots, J_t^*$  denote the products that are actually observed in the data for each market  $t = 1, \dots, T$ . The implied, available alternatives to consumers located in a market  $t$  are denoted  $a = 0, 1, \dots, A_t^*$ .

Let  $J^*$  denote the total number of different products observed by the researcher across all markets in the data.<sup>17</sup> Let  $\mathbf{b}$  be a  $J^* \times 1$  column vector such that, for each  $j = 1, \dots, J^*$ :

$$\mathbf{b}_j = X_j\beta + \xi_j. \quad (23)$$

Then, the mean utility specification in (9) simplifies to:

$$\delta_{at} = \sum_{j \in a} (\mathbf{b}_j - \alpha p_{jt} + \Delta\xi_{jt}). \quad (24)$$

We compute the integral in (17) via simulation. That is, using a total of  $n_S$  random draws from the multivariate standard Normal distribution  $N(\mathbf{0}, \mathbf{I}_{K+1})$  and given values for the parameter vector  $\zeta := (\alpha, \Psi, \mathbf{b}, \rho)$  and the unobserved demand shock  $\Delta\xi_t$ , we approximate (17) by averaging the values of (16) across draws.<sup>18</sup> We denote this approximation as:

$$\hat{s}_{at}(X_t, \mathbf{p}_t, \xi_t, \Delta\xi_t) = \frac{1}{n_S} \sum_{m=1}^{n_S} s_{mat}(\theta_m, \zeta, \Delta\xi_t); \quad (25)$$

where random draws are indexed by  $m = 1, \dots, n_S$  and  $s_{mat}(\theta_m, \zeta, \Delta\xi_t)$  is defined by the right-hand side of (16).

Let  $S_t$  denote the observed vector of market shares for products  $j = 1, \dots, J_t^*$  for market  $t$ , and define  $\Gamma_t$  as the  $A_t^* \times J_t^*$  product inclusion matrix for market  $t$  as in (20). Denoting the vector stacking the predicted alternative shares (25) over all alternatives available in market  $t$  by  $\hat{S}_t^{Alt}(\zeta, \Delta\xi_t)$ , we have for all  $t = 1, \dots, T$ :

$$S_t = \Gamma_t' \hat{S}_t^{Alt}(\zeta, \Delta\xi_t). \quad (26)$$

<sup>17</sup>In our sample we observed 117 products, i.e.,  $J^* = 117$ .

<sup>18</sup>Recall that the random coefficients  $\theta_h = (\alpha_h, \beta_h)$  are assumed to follow a multivariate Normal distribution with matrix  $\Psi = \text{diag}(\Psi_p, \Psi_X)$ . The fact that a multivariate Normal distribution can be written as a linear transform of a multivariate standard Normal distribution allows us to keep the same draws from this distribution for each evaluation of the GMM objective function described below.



The equations system defined in (26) is used to define values for  $\Delta \xi_t$  given parameters  $\zeta$ . As  $\Delta \xi_t$  consist of product demand shocks in market  $t$ , a natural way to estimate demand parameters is to form a GMM estimator where a set of moment conditions is satisfied as much as possible given some minimum distance criteria. Let  $Z_{jt}$  be a vector of instrumental variables that are mean-independent of  $\Delta \xi_{jt}$  and where  $\dim(Z_{jt}) \geq \dim(\zeta)$ . Then, for some weighting matrix  $W$ , the GMM estimation problem is formalized as a Mathematical Program with Equilibrium Constraints (MPEC), as in Dube et al. (2012):

$$\min_{\zeta, \Delta \xi_t} Q(\Delta \xi_t, \zeta) = g(\Delta \xi_t, \zeta)' W g(\Delta \xi_t, \zeta) \quad (27)$$

$$s.t. \quad S_t = \Gamma_t' \hat{S}_t^{Alt}(\zeta, \Delta \xi_t), \quad \forall t = 1, \dots, T$$

where the vector  $g(\Delta \xi_t, \zeta)$  is defined as:

$$g(\Delta \xi_t, \zeta) = \frac{1}{T} \sum_{t=1}^T \frac{1}{J_t^*} \sum_{j=1}^{J_t^*} \Delta \xi_{jt} \cdot Z_{jt}. \quad (28)$$

The optimization problem (27) is solved using the stochastic, global optimization algorithm of named Covariance Matrix Adaptation Evolution Strategy (CMAES). This derivative-free algorithm is based on evaluation of candidate solutions picked at random using a multivariate Normal distribution centered at an initial guess, followed by selection of the candidates that yielded the lowest values for the objection function being minimized. The selected set of candidates is used to compute the mean and variance-covariance matrix of the multivariate Normal used to draw new candidate solutions, and the process is repeated until convergence. See Hansen (2006) for a discussion. We first solve (27) setting the weighting matrix to identity, i.e.  $W = I$ , and we form a new matrix  $W$  using the sample analog of the asymptotically efficient matrix evaluated at the solution just obtained. We resolve the problem using the new matrix  $W$ . Further updating of matrix  $W$  with new rounds of estimates did not lead to numerically significant changes in estimates.

We recover the vector of average taste parameters for observed characteristics  $\beta$  and the unobserved product quality vector  $\xi = (\xi_1, \dots, \xi_{J^*})$  using a GLS procedure similar to the one of Nevo (2001) applied to (23). Let  $\hat{\mathbf{b}}_j$  be the estimates of the coefficients of the product dummy variables obtained after solving (27). The estimators for  $\beta$  and  $\xi$  are, respectively,

$$\hat{\beta} = (X' V_b X)^{-1} X' V_b \hat{\mathbf{b}}, \quad (29)$$

$$\hat{\xi} = \hat{\mathbf{b}} - X \hat{\beta}. \quad (30)$$

where  $V_b$  is the covariance matrix of  $\hat{\mathbf{b}}$ .

Standard errors for demand model estimates must be corrected for errors due to consumer sampling process and to integration-by-simulation process. That is, we must account for the fact that we observe estimated rather than actual product market shares and that

simulation draws are the same for all observations in a market. See Berry et al. (1995) for a discussion. We correct for these errors in standard deviation calculation by resorting to nonparametric bootstrap methods. That is, we resample the data points within each market with replacement and resolve (27) using this artificial sample. This process is repeated  $N$  times, and standard errors of model estimated are computed as the standard deviations of the  $N$  solutions to (27) obtained with the  $N$  artificial samples. To speed the computations of the solutions to each of the  $N$  problems, we take the original solution to (27) as the initial guess and then minimize (27) with each artificial sample using a Newton-Raphson algorithm. Standard error estimates of  $\hat{\mathbf{b}}$  are also used to craft its covariance matrix  $V_b$  necessary to run the GLS regression in (29). We obtain standard errors for  $\hat{\beta}$  by running the GLS regression in (29) for each bootstrap sample and then take the standard deviation of the  $N$  bootstrap solutions.

We are interested on testing for synergies in bundle marginal costs. To this end we use first-order conditions in prices to identify marginal costs. After deleting the rows and columns pertaining products that firms do not commercialize in market  $t$ , we infer from (5) that the vector of marginal costs of products commercialized in that market,  $\mathbf{c}_t$ , is given by:

$$\mathbf{c}_t = \mathbf{p}_t - (\mathbf{H} * \Delta)^{-1} \mathbf{s}_t(\mathbf{p}_t, \mathbf{d}_t) \quad (31)$$

We use the marginal cost data obtained using (31) to run OLS regressions akin to synergy testing, discussed below.

### 5.3 Counterfactual Simulation

The model described in Section 4 allows us to simulate pricing and bundling behavior for different scenarios.

We focus attention on only one of the 30 markets used in estimation. For our simulations, the chosen market was the Greater Lisbon area. As firms only sell some of the potential 478 products that could be sold in each market, we need to calibrate values for their marginal costs and unobserved characteristics. We use the estimated marginal cost function with market dummy variables to assign marginal costs for products in the Greater Lisbon market. We use sample averages of  $\xi_j$  and  $\Delta\xi_{jt}$  by product type and market to quantify unobserved characteristic on products not sold in the data. After replacing unknown parameters in demand and supply equations, we solve for the new equilibrium by minimizing the quadratic distance between right- and left-hand side of equations (5) and (3).

Formally, define the multivariate function:

$$K(\mathbf{p}_t, \mathbf{d}_t) = \left[ \begin{array}{c} \mathbf{d}_t * \mathbf{s}_t(\mathbf{p}_t, \mathbf{d}_t) - (\mathbf{H} * \Delta) [\mathbf{d}_t * (\mathbf{p}_t - \mathbf{c}_t)] \\ \Pi_{ft}(\mathbf{p}_t, \mathbf{d}_t) - \max \{ \Pi_{ft}(\mathbf{p}_t, \mathbf{d}_{-jt}, 0), \Pi_{ft}(\mathbf{p}_t, \mathbf{d}_{-jt}, 1) \}, \quad j = 1, \dots, J_t \end{array} \right] \quad (32)$$

The equilibrium in market  $t$  can be computed by solving the minimum distance problem

$$\min_{(\mathbf{p}_t, \mathbf{d}_t)} \|K(\mathbf{p}_t, \mathbf{d}_t)\| \quad (33)$$

Since each  $\mathbf{d}_t$  is a vector of binary variables and each  $\mathbf{p}_t$  is a vector of continuous variables, the problem defined in (33) is a mixed-integer program. To solve this problem we again resort to the stochastic global optimization algorithm Covariance Matrix Adaptation Evolution Strategy (CMAES) of Hansen (2006), which is capable of handling problems with both discrete and continuous controls.

## 6 Basic Estimation Results

In this Section, we present the estimation results of the demand model of Section 4.3, using the procedure described in Section 5.2 and the data described in Section 5.1. We present first the estimates of the demand model and afterwards the estimates of the price elasticities of demand.

### 6.1 Demand Estimates

Next we present the demand estimates. We discuss alternative specifications regarding consumer heterogeneity and the set of instrumental variables used. The variables considered for product observed characteristics, matrix  $X_j$ , include service dummy variables, the logarithm of number of offered channels, the logarithm of bandwidth measured in Mbps, and both double- and triple-play bundle dummy variables.<sup>19</sup> We exclude the service dummy variable for *TV* to avoid colinearity with other dummy variables considered in the regression. The standard errors of estimates were computed using the bootstrap procedure described in the previous section for  $N = 500$  bootstrap samples.

To estimate our model we need a vector of variables  $Z_{jt}$  correlated with  $X_j$  and independent of  $\Delta\xi_{jt}$ . We consider two approaches followed in the differentiated product demand literature and compare the results obtained using these alternatives.

First, we consider the approach of Berry et al. (1995), who propose using other product characteristics in  $X_j$  as instruments, as well as sums of other products' characteristics sold by the same firm and sums of rival firms' product characteristics.<sup>20</sup> Bresnahan et al. (1997) extend this set of instruments further by considering characteristics sums within nests. Along

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<sup>19</sup>For continuous attributes, such as the number of channels or bandwidth, we considered function  $\log(x + 1)$ , where  $x$  is the quantity of interest. This ensures decreasing marginal utility in the attribute, while avoiding a log of zero when the product has neither *TV* nor *BB*.

<sup>20</sup>Using  $X_j$  in the instrumental variables set is not possible, since it is collinear with product dummy variables, sums of firm's own and rivals' products can be considered in the instrumental variables set for solving (27).

with market- and product dummy variables, this is one of the main instrumental variables set we consider in estimation.

Second, we consider the approach of Nevo (2001) and Hausman (1996), who propose as instruments the prices for product  $j$  at all regions other than  $t$ . These variables are valid instruments under certain assumptions. As those assumptions cannot be directly tested, we resorted to estimate the model using regional average prices. We considered a NUTS II set of markets as a region, and we formed instruments by computing the average price of a product in that region, excluding the price at the market where the product is sold.<sup>21</sup>

Before examining the more general versions of the demand model, it is useful to examine the estimation results when certain features are excluded. Table 6 presents the demand estimation results under no consumer heterogeneity, i.e., when the standard deviations of the taste parameters are set to zero, so that all consumers have same utility function coefficients.

[Table 6]

The first two columns display the particular case of the PDGEV model when nest parameters are set to one, i.e., the Multinomial Logit (MNL) model.<sup>22</sup> The first column presents estimates of the MNL model without instruments, while the second column presents estimates of the MNL model using the instruments proposed by Berry et al. (1995) and Bresnahan et al. (1997). While the first specification leads to a negative estimate of the price coefficient, as expected, it is nearly half, in absolute value, of its analog of the second model. Moreover, several taste coefficients have unexpected signs, e.g., log of bandwidth and log of channels. This pattern is somewhat similar to other examples where not controlling for endogeneity leads to implausible estimates, e.g., as in Berry et al. (1995).

The third and fourth columns display the results of a similar exercise for the PDGEV model. While similar comments apply in regards to instrumenting for price, estimating nest parameters rather than setting them to one seems to be a nontrivial improvement. First, the nest parameters are in **general** significantly different from one. Second, the estimate of the price coefficient increased, in absolute value, both with and without instrumenting. This suggests that controlling for possible correlations across alternative shocks is important on identifying price sensitivity. This situation is somewhat similar to the results of Bresnahan et al. (1997), where they find that the PDGEV model yields higher, in absolute value, price coefficients than the two-level Nested Logit model.

Tables 7 and 8 reports the estimates of the full demand model for different instrument

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<sup>21</sup>As some products are supplied only in certain regions, we considered the fit of regressing prices on  $X$  and regional dummy variables, whenever the price of a product in a region was unavailable.

<sup>22</sup>In contrast to most of the empirical literature in differentiated product demand, we cannot estimate the model through OLS or 2SLS. The reason is that product market shares do not add up to one, since households choose *alternatives*, instead of *products*.

sets and nest parameter assumptions.

[Table 7]

[Table 8]

The first three columns contemplate the Mixed Logit (MMNL) model where nest parameters are set to one. The first two columns compare estimates with and without instrumenting for price. The results indicate that allowing for mixing does not mitigate the price endogeneity problem, as the coefficient for price using the instruments of Berry et al. (1995) and Bresnahan et al. (1997) is nearly double, in absolute value, of the one without controlling for endogenous prices. The third column reports estimates using average regional prices as instruments as in Nevo (2001) and Hausman (1996). While the price coefficient is bigger, in absolute value, than the one of first column, it is smaller than the one of the second column. Moreover, some taste parameters have unintuitive signs despite being statistically insignificant, e.g. log of bandwidth and the FV dummy variable. This suggests that some of the assumptions validating average regional prices as instruments may not be met. The fourth, fifth and sixth columns pertain to the Mixed PDGEV model using instrument sets in the same order as the preceding three columns. The results confirm that the instruments of Berry et al. (1995) and Bresnahan et al. (1997) yield more plausible estimates than the alternative sets of instruments. Moreover, several of the standard deviations of the taste parameter distribution are significant, suggesting that consumer heterogeneity is an important feature of this market. Thus, we select the estimates of the fifth column, i.e., the estimates of the Mixed PDGEV model using the instruments of Berry et al. (1995) and Bresnahan et al. (1997), to conduct our analysis.

## 6.2 Elasticities

A

## 7 Analysis

In this section we use the estimates of the model of the fifth column of Tables 7 and 8 to conduct the analysis. First we investigate if there are cost synergies. Second, we examine how the market equilibrium changes with an increase in consumer heterogeneity, measured by the standard deviation of the estimate of the price coefficient of demand. Third, we examine how the market equilibrium changes as the number of firms increases. In all

## 7.1 Cost Synergies

Next we test for cost synergies. Bundling may occur due to cost savings from selling several products jointly. Indeed, several bundles sell at a discount compared to the sum of their individual prices. However, the discount may or may not be cost-motivated.

We use the demand estimates and equation (31) to recover marginal costs. Denote by  $\hat{c}_{jt}$  the estimate of marginal cost of product  $j$  in market  $t$ , by  $A_{jt}$  a vector of shifters, e.g. channels, bandwidth, market dummy variables and by  $Y_j$  a  $7 \times 1$  vector of single-, double- and triple-play dummy variables.<sup>23</sup> We run the auxiliary regression:

$$\hat{c}_{jt} = Y_j \beta_Y + A_{jt} \beta_A + \varepsilon_{jt} \quad (34)$$

Table 9 presents estimates of (34) for different regressor sets in vector  $A_{jt}$ .

[Table 9]

Denote by  $R$  is a vector of restrictions. For each regression, we test for cost synergies by forming linear hypothesis on  $\beta_Y$ :<sup>24</sup>

$$H_0 : R \cdot \beta_Y \geq 0 \quad (35)$$

$$H_A : R \cdot \beta_Y < 0; \quad (36)$$

It follows from standard OLS results that  $R \cdot \beta_Y$  is asymptotically Normal and centered at zero.<sup>25</sup> We use this result to test every possible combination of cost synergies involving single-, double- and triple-play.

Except for the case of the regression with only product type dummy variables and for the alternative hypothesis that triple-play costs less than the sum of single-play BB and double-play FV+TV, we fail to reject the null hypothesis. This result still holds even if we consider strict equality in the null hypothesis. Thus, these results suggest that cost synergies are not a likely cause for bundling in this industry.

## 7.2 Consumer Heterogeneity

Next we analyze the impact of consumer heterogeneity. We assume that the current observed values are . Afterwards, we double the standard deviation of the estimate of the

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<sup>23</sup>Recall that there are seven types of products: single-play FV, single-play TV, single-play BB, single-play FV+TV, single-play FV+BB, single-play TV+BB, triple-play.

<sup>24</sup>For example,  $R = (-1, -1, 0, 1, 0, 0)$  implies an alternative hypothesis  $\beta_{Y,4} < \beta_{Y,1} + \beta_{Y,2}$ , i.e., that the average cost of double-play FV+TV is smaller than the cost of its individual components, i.e., single-play FV plus single-play TV.

<sup>25</sup>Its variance-covariance matrix is  $\sigma^2 R(\mathbf{X}'\mathbf{X})^{-1} R'$ , where  $\mathbf{X}$  is matrix of regressors and  $\sigma^2$  is the variance of regression residuals.

price coefficient of demand and compute the new equilibrium and compare it with the initial equilibrium.

Table 10 presents results.

[Table 10]

We partition the 478 possible products into the seven possible types of single-double- and triple-play products. The "percentage of products offered" refers to the percentage of those product that is actually sold in the market, i.e., it is the frequency of the discrete control  $d_{jt}$  within that class.

We reach three conclusions. First, in response to doubling the standard deviation of the estimate of the price coefficient of demand, more products are offered, except for single-play FV. The decision to sell more products is more marked for double- and triple-play products. Second, average prices decrease only slightly as additional products are offered. Third, product quality, measured by the average number of channels and average bandwidth, is reshuffled, i.e., the product quality of bundles increases while the product quality of single-play products decreases.<sup>26</sup>

These results are consistent with the theoretical prediction that in the presence of consumer heterogeneity firms use bundling as a price discrimination device to capture consumer surplus. In addition, our results indicate that dimensions other than price may also be important to characterize markets where firms may engage in bundling. In our case product quality is also an important element of how firms compete. This dimension of competition is absent from the theoretical literature on bundling, where attention focuses in pricing decisions.

### 7.3 Number of Rivals

Next we examine how the market equilibrium changes as the number of firms increases. We assume that the industry consists of a monopolist similar to  $f_1$ , a firm that owns the same set of products as  $f_1$ , and compute the equilibrium. Afterwards, we assume that the industry consists of a duopoly where two firms identical to  $f_1$  compete and compute the equilibrium.

**Second, we examine how is bundling behavior sensitive to changes in market structure. In particular, we focus on simulating behavior of a monopolist firm and compare prices, bundling behavior and product characteristics with a duopoly case, where the firm competes against a clone of itself, i.e., with a firm with the same set of products. For this exercise we pick one of the top firms in**

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<sup>26</sup>If a firm owns two products of the same type that differ in either the number of channels or bandwidth size, or both, the firm effectively increases product quality by selling the product with the highest number of channels or bandwidth size, while not selling the product with lower attribute values.

**this market.**

Table 11 presents the results.

[Table 11]

The interpretation of each row is analogous as in the previous experiment, except that product sale decisions consider only products that the chosen firm, and its clone, can potentially sell in this market. For example, the monopolist can sell at most one single-play FV, while in duopoly we can observe at most two single-play FVs, one for each firm.

We reach four conclusions. First, a monopolist offers all types of products, including bundles. Second, except for single-play FV and the double-play FV+TV, the number of products offered is higher under duopoly than under monopoly. Third, average prices are lower under duopoly than under monopoly. Fourth, product quality, measured by the average number of channels and average bandwidth size, is higher under duopoly than under monopoly.

Again quality emerges as an important dimension in the way firms compete in these markets.

## 8 Conclusion

When firms offering several services supply bundles of products, conventional models of supply and demand for differentiated products cannot be directly used to predict market equilibria. In this article we provide a framework that deals with this problem. We model consumer demand for multiproduct alternatives that contemplate different services as a means to identify consumer interest for bundles of products. Our framework is estimable and akin to simulations of bundling behavior. We apply the MPEC estimation approach of Dube, Fox and Su (2012) to estimate our model by applying a GMM estimator to a data set from the Portuguese triple-play market. Our estimates indicate that consumer heterogeneity and correlation between preferences for alternatives are important features of demand for triple-play products. Moreover, marginal cost data implied by our model is consistent with lack of cost synergies. Instead, our simulation using the model indicates that consumer heterogeneity and strategic firm behavior are more plausible sources of bundling behavior. Product commercialization decisions prove also to be an important control along with price on bundling behavior. This extra firm decision is absent from most theoretical bundling models, yet our results indicate it may be important in practice. Our framework can be applied to other settings where firms combine products to form bundles.

While our approach allows the researcher to estimate demand and supply for bundles when only product information - and not assortment choices - are available, it has some limitations. First, our extension of the PDGEV framework of Bresnahan et al. (1997)



deals with the issue of correlation across alternatives due to containing similar products, yet its potential to deal with a many alternatives may be limited in some applications. For example, if there is a very large number of product variants within a certain category, or if the number of industries to consider is excessive, the number of possible alternatives may be too large to be computationally tractable. Second, the problem of solving for equilibrium where the researcher solves for both prices and decisions of commercialization may be hard to implement in practice. Recent developments in large-scale optimization and its application to industrial organization problems, e.g. Dube, Fox and Su (2012), Su and Judd (2012), may deal with this problem to a large extent.

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## A Tables

Table 1: Market shares

	Fixed voice		Pay-TV		Broadband	
	2008	2009	2008	2009	2008	2009
PT	65.7%	61.6%	13.6%	23.0%	41.6%	44.5%
ZON	4.4%	10.2%	72.3%	64.4%	31.3%	32.2%
Optimus	16.3%	14.5%	0.5%	1.0%	12.5%	9.2%
Vodafone	5.1%	6.1%	-	0.3%	2.8%	3.9%
Cabovisão	3.3%	3.6%	12.4%	10.2%	9.3%	8.0%
AR Telecom	1.7%	1.4%	1.0%	0.9%	1.5%	1.4%
Others	0.7%	0.5%	0.1%	0.1%	1.0%	0.8%

Market share in terms of subscribers, except for fixed telephony which is in terms of traffic. Source: ICP-ANACOM (Relatórios trimestrais)

Table 2: Products - Notation

Services			Bundles			Firms	
N	Notation	Description	N	Notation	Description	N	Notation
1	000	no serv.	1	p000	no serv.	1	$f_0$
2	100	FV	2	no b	No bundle - Single play	2	$f_1$
3	010	TV	3	p110	Double play FV+TV	3	$f_2$
4	001	BB	4	p101	Double play FV+BB	4	$f_3$
5	110	FV+TV	5	p011	Double play TV+BB	5	$f_4$
6	101	FV+BB	6	p111	Triple play FV+TV+BB	6	$f_5$
7	011	TV+BB				7	$f_6$
8	111	FV+TV+BB					

Table 3: Services vs. bundles

Services	Bundles						Total
	p000	no b	p110	p101	p011	p111	
000	[26-28%]	0%	0%	0%	0%	0%	[26-28%]
100	0%	[14-16%]	0%	0%	0%	0%	[14-16%]
010	0%	[10-12%]	0%	0%	0%	0%	[10-12%]
001	0%	[0-2%]	0%	0%	0%	0%	[0-2%]
110	0%	[4-6%]	[4-6%]	0%	0%	0%	[10-12%]
101	0%	[0-2%]	0%	[4-6%]	0%	0%	[4-6%]
011	0%	[0-2%]	0%	0%	[4-6%]	0%	[6-8%]
111	0%	[0-2%]	0%	[2-4%]	[0-2%]	[16-18%]	[22-24%]
Total	[26-28%]	[36-38%]	[4-6%]	[6-8%]	[4-6%]	[16-18%]	100%

Distribution of services consumed per type of bundle, 2009. Source: ICP-ANACOM, "Inquérito ao consumidor"

Table 4: Average characteristics by product type

Product	Price	Share	channels	bandwidth
FV only	9.245	0.136	0	0
TV only	36.381	0.048	70.020	0
BB only	26.890	0.019	0	5.823
FV+TV	50.981	0.008	65.309	0
FV+BB	24.345	0.010	0	18.018
TV+BB	55.742	0.011	109.523	15
FV+TV+BB	56.362	0.013	89.266	18.453

Table 5: Alternatives - Simplified Examples

N	Services	Bundles	S. FV	S. TV	S. BB	Description
0	000	p000				No services
1	100	no b	f <sub>1</sub>			Fixed voice from f <sub>1</sub>
2	111	p111	f <sub>2</sub>	f <sub>2</sub>	f <sub>2</sub>	Triple-play from f <sub>2</sub>
3	010	no b		f <sub>2</sub>		Pay-TV from f <sub>2</sub>
4	111	p111	f <sub>1</sub>	f <sub>1</sub>	f <sub>1</sub>	Triple-play from f <sub>1</sub>
5	101	p101	f <sub>4</sub>		f <sub>4</sub>	Double play (FV+BB) from f <sub>4</sub>
6	110	no b	f <sub>1</sub>	f <sub>2</sub>		Fixed voice from f <sub>1</sub> + Pay-TV from f <sub>2</sub>
...						

S. FV - supplier of FV; S. TV - supplier of TV; S. BB - supplier of BB



Table 6: Demand Estimates - Models without Heterogeneity

Models				
	Logit	IV Logit	PDGEV	IV-PDGEV
const.	-0.450 (0.169)	-2.503 (0.889)	-0.022 (0.115)	-2.355 (0.735)
price coef.	-0.038 (0.011)	-0.070 (0.007)	-0.074 (0.018)	-0.099 (0.031)
p100	-1.535 (0.892)	0.671 (0.129)	-3.392 (0.996)	0.671 (0.232)
p001	-5.457 (1.992)	1.659 (0.773)	-2.926 (1.315)	1.659 (0.563)
p110	0.622 (0.108)	1.680 (0.946)	1.036 (0.333)	1.680 (0.231)
p101	2.512 (0.727)	1.964 (0.451)	0.782 (0.222)	1.964 (0.199)
p011	4.235 (1.012)	0.147 (0.024)	3.311 (1.088)	0.079 (0.022)
p111	6.004 (0.991)	1.287 (0.556)	5.139 (1.013)	1.285 (0.341)
log(channels+1)	-0.343 (0.331)	0.452 (0.200)	-0.390 (0.223)	0.452 (0.210)
log(bandwidth+1)	-0.766 (0.553)	0.788 (0.316)	-0.669 (0.445)	0.788 (0.225)
$\rho_{FV}$			0.623 (0.112)	0.530 (0.201)
$\rho_{TV}$			0.452 (0.023)	0.496 (0.103)
$\rho_{BB}$			0.223 (0.099)	0.509 (0.113)
Numb. Obs.	1083	1083	1083	1083
GMM Obj	7.098	6.956	6.112	6.004

Table 7: Demand Estimates - Full Model

		Models					
		(1)	(2)	(3)	(4)	(5)	(6)
Mean Parameters	const.	-0,020 (0,022)	-2.417 (0.923)	-1.124 (0.881)	0.932 (0.702)	-2.353 (0.992)	-1.126 (0.298)
	price coef.	-0,036 (0.010)	-0.078 (0.019)	-0.043 (0.005)	-0.067 (0.040)	-0.167 (0.047)	-0.085 (0.022)
	p100	-3.397 (1.911)	0.671 (0.100)	-0.638 (0.406)	-2.421 (1.100)	0.671 (0.067)	-1.017 (0.081)
	p001	-3.716 (1.890)	1.659 (0.211)	-1.028 (0.500)	-3.040 (1.112)	1.659 (0.600)	-1.388 (0.999)
	p110	2.736 (1.012)	1.680 (1.012)	0.684 (0.561)	0.371 (0.145)	1.680 (0.799)	-0.417 (0.335)
	p101	4.501 (1.099)	1.964 (0.778)	1.269 (0.998)	0.841 (0.512)	1.964 (0.893)	0.708 (0.690)
	p011	5.342 (1.222)	0.130 (0.092)	2.624 (1.113)	3.624 (0.444)	0.079 (0.045)	-0.061 (0.111)
	p111	7.999 (0.900)	1.274 (0.699)	3.665 (0.799)	4.473 (1.668)	1.285 (0.456)	2.665 (0.954)
	log(channels+1)	-0.576 (0.559)	0.452 (0.301)	0.093 (0.111)	-0.561 (0.488)	0,452 (0.311)	-0.092 (0.140)
	log(bandwidth+1)	-0.333 (0.402)	0.788 (0.210)	-1.210 (0.709)	-1.048 (0.665)	0.788 (0.301)	-0.638 (0.889)
Nest Parameters	$\rho_{FV}$				0.530 (0.110)	0.633 (0.200)	0.623 (0.234)
	$\rho_{TV}$				0.496 (0.291)	0.664 (0.200)	0.711 (0.441)
	$\rho_{BB}$				0.509 (0.199)	0.685 (0.334)	0.699 (0.302)

(1) Mixed-Logit, (2) IV Mixed Logit (BLP-BST), (3) IV Mixed Logit (Reg. Prices),

(4) Mixed-PDGEV, (5) IV Mixed-PDGEV (BLP-BST), (6) IV Mixed-PDGEV (Reg. Prices).

Table 8: Demand Estimates - Full Model (Continued)

		Models					
		(1)	(2)	(3)	(4)	(5)	(6)
STD Parameters	const.	1.675 (2.098)	1.319 (1.005)	1.878 (1.335)	1.878 (0.991)	1.299 (1.509)	1.878 (1.112)
	price coef.	0.029 (0.020)	0.024 (0.010)	0.014 (0.012)	0.056 (0.045)	0.014 (0.066)	0.067 (0.022)
	p100	1.529 (1.339)	1.404 (1.112)	1.541 (1.711)	1.541 (2.010)	1.404 (1.130)	1.542 (0.789)
	p001	1.538 (0.991)	1.205 (0.890)	1.538 (1.133)	1.624 (0.451)	1.205 (0.581)	1.822 (0.901)
	p110	1.450 (1.194)	1.137 (0.982)	1.450 (1.222)	1.450 (1.023)	1.137 (2.200)	1.452 (2.011)
	p101	1.625 (0.400)	1.274 (0.509)	1.625 (0.668)	1.626 (0.333)	1.274 (0.411)	1.626 (0.550)
	p011	1.667 (0.892)	1.317 (0.694)	1.667 (0.801)	1.668 (0.901)	1.307 (0.500)	1.784 (0.367)
	p111	2.929 (1.123)	2.295 (0.967)	2.929 (1.078)	2.929 (0.460)	2.295 (0.710)	2.929 (0.666)
	log(channels+1)	0.330 (0.133)	0.390 (0.199)	0.411 (0.167)	0.471 (0.113)	0.390 (0.089)	0.412 (0.056)
	log(bandwidth+1)	0.286 (0.128)	0.230 (0.190)	0.346 (0.100)	0.361 (0.126)	0.218 (0.066)	0.347 (0.046)
Num. Obs.		1083	1083	1083	1083	1083	1083
GMM Obj		6.872	6.432	6.773	6.512	5.812	5.991

(1) Mixed-Logit, (2) IV Mixed Logit (BLP-BST), (3) IV Mixed Logit (Reg. Prices),  
(4) Mixed-PDGEV, (5) IV Mixed-PDGEV (BLP-BST), (6) IV Mixed-PDGEV (Reg. Prices).

Table 9: Marginal Cost Regressions

	Regression #1		Regression #2		Regression #3	
Variable	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error
mc p100	3.475	2.136	9.472	3.798	3.387	5.453
mc p010	30.370	1.982	10.740	3.814	4.207	5.537
mc p001	20.981	1.874	23.252	3.748	16.585	5.533
mc p110	42.749	1.305	22.974	3.740	16.775	5.453
mc p101	19.702	1.891	23.052	3.598	16.628	5.396
mc p011	48.262	1.891	28.356	3.897	22.288	5.574
mc p111	50.073	1.102	31.390	3.686	25.252	5.410
channels			0.152	0.017	0.151	0.017
bandwidth			0.208	0.036	0.196	0.037
$f_1$			-0.663	3.630	1.212	3.755
$f_2$			-5.526	3.859	-4.249	3.964
$f_3$			53.834	3.762	55.434	3.903
$f_4$			-7.535	3.742	-6.196	3.854
$f_5$			-10.028	3.670	-8.194	3.797
time dummies					yes	
Adj R-squared	0.80		0.93		0.93	

Table 10: Simulation Results - Increased Consumer Heterogeneity

	% of Products Offered		Average Price (€)		Average Channels		Average Bandwidth	
	observed	2x std.( $\alpha$ )	observed	2x std.( $\alpha$ )	observed	2x std.( $\alpha$ )	observed	2x std.( $\alpha$ )
p100	50.0%	50.0%	8.97	8.75	0	0	0	0
p010	20.8%	25.0%	29.78	28.60	67	62	0	0
p001	25.0%	30.6%	34.52	33.55	0	0	23	21
p110	56.5%	69.6%	56.48	55.93	68	70	0	0
p101	36.1%	52.8%	27.80	27.22	0	0	23	24
p011	5.7%	11.9%	51.59	51.41	68	71	25	27
p111	16.9%	21.5%	55.42	53.60	68	75	25	27

Table 11: Simulation Results - Monopoly vs. Duopoly

	% of Products Offered		Average Price (€)		Average Channels		Average Bandwidth	
	Monopoly	Duopoly	Monopoly	Duopoly	Monopoly	Duopoly	Monopoly	Duopoly
p100	100.0%	100.0%	22.2	17.9	0	0	0	0
p010	10.0%	15.0%	34.6	31.5	25	38	0	0
p001	18.2%	22.7%	30.3	27.6	0	0	4	11
p110	40.0%	45.0%	51.2	45.0	45	70	0	0
p101	18.2%	31.8%	26.8	24.9	0	0	12	19
p011	2.0%	5.0%	56.8	48.6	55	69	18	29
p111	3.0%	5.5%	69.4	55.0	53	70	21	33