

von Hoyningen-Huene, Wiebke; Kiesmüller, Gudrun P.

Working Paper

Maintenance and Production Scheduling on a Single Machine with Stochastic Failures

Arbeitspapiere des Instituts für Betriebswirtschaftslehre der Universität Kiel

Suggested Citation: von Hoyningen-Huene, Wiebke; Kiesmüller, Gudrun P. (2015) : Maintenance and Production Scheduling on a Single Machine with Stochastic Failures, Arbeitspapiere des Instituts für Betriebswirtschaftslehre der Universität Kiel, ZBW - Leibniz-Informationszentrum Wirtschaft, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/106608>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Maintenance and Production Scheduling on a Single Machine with Stochastic Failures

W. von Hoyningen-Huene, G.P. Kiesmüller

Abstract In this paper we study the problem of determining a production schedule for an order of n jobs on a single machine which is subject to stochastic machine failures. To avoid long downtime of the machine caused by unexpected failures, preventive maintenance should be planned as well. If a failure cannot be averted, a corrective maintenance activity is performed. Both maintenance activities are assumed to restore the machine to become ‘as good as new’. Furthermore it is assumed that jobs, interrupted by a machine failure, have to get restarted after the corrective maintenance is finished (non-resumable case). The aim is to minimise the average cost, composed of cost through order tardiness, when exceeding a due date, and cost for preventive as well as corrective maintenance activities. In order to regard practical relevance we compare simple production and maintenance scheduling rules for a huge number of jobs. In the course of a simulation study we first illustrate that in general an adjusted First Fit Decreasing algorithm generates the best results compared to other common used scheduling rules. We secondly show that the optimal length of the preventive maintenance interval can be estimated by using an extended decomposition approach which regards the non-resumable case, even if the processing times of the jobs are very long.

Keywords: Maintenance, Scheduling, Stochastic Failures, Non-Resumable, Simulation Study

1 Introduction

The problem discussed in this paper is to schedule an order of n non-resumable jobs on a single machine, while this machine is affected by stochastic breakdowns. If jobs are non-resumable, these jobs have to be started anew when they are interrupted by a machine failure. To avoid the production stops, preventive maintenance (PM) activities are embedded in the production schedule. If PM activities are conducted, cost for staff and material accrue as well as cost for the time the activity delays the order. But, if a breakdown occurs, a corrective maintenance (CM) activity is performed. CM activities cause repair and delay cost, including also the order delay through the repeating times of those jobs, which have to get restarted. The aim is to find simple decision rules to gain a sequence of jobs and PM activities, such that the average cost for maintenance activities and for delaying the finishing time of the order are minimised.

In practice, PM activities are often scheduled periodically at a fixed point in time, for example once a week, and therewith independently of production scheduling decisions. If jobs are non-preemptive, this results in high cost through machine idle times and production delays. Additionally, the possibility of unexpected machine failures is often ignored when the schedule for the production process is determined. But, machine failures cause cost for CM activities as well as for production delays and therewith are affecting the completion time of the jobs. In literature these deterministic problems of integrating production scheduling and periodically conducted maintenance activities are modelled by using an availability constraint. Overviews of these problems are given in Lee (1996), Sanlaville and Schmidt (1998), Schmidt (2000) and Ma et al. (2010). Similar to that, Chen (2008) solves an integrated scheduling and maintenance problem where maintenance activities are inserted periodically, but in a flexible time interval. He applies the First Fit Decreasing (FFD) algorithm for this deterministic case. Since the occurrence of failures is not included and the number of PM activities is predetermined, the algorithm is used to arrange the batches of jobs and the positions of the PM activities in its flexible interval to minimise makespan. In a note on this paper Xu et al. (2009) show this algorithm to be the

best possible approximation in polynomial time for the considered problem.

If the maintenance and production scheduling is considered in an integrated manner while failures can occur stochastically, this is mostly done under the assumption that jobs are resumable. If jobs are resumable, it is possible to continue an interrupted job after machine repair. For example Cassady and Kutanoglu (2003) investigate a model for the integrative problem minimising job tardiness if scheduling and maintenance decisions are made simultaneously. They solve the problem by complete enumeration and compare the optimal solution with a heuristic. This heuristic first schedules the jobs optimal, ignoring the occurrence of failures, and secondly inserts the PM activities. The length of the time interval between two consecutive PM activities is determined by maximising the availability of the machine. The integrated approach leads to improvements of around 30%, but the enumeration can only be used for orders with a maximum of eight jobs. A similar approach is used in Cassady and Kutanoglu (2005) to find schedules minimising the total weighted completion time. Sortrakul et al. (2005) also minimise the total weighted completion time, but solve the problem using a genetic algorithm and for up to 30 jobs. Pan et al. (2010) present a model to minimise the maximum weighted tardiness if the time to perform a PM activity is dependent on the age of the machine. For a small number of jobs schedules are determined by complete enumeration. The presented papers operate with a PM interval calculated by maximising the availability of a machine. If jobs are non-resumable, the availability is not a reliable quantity, since the availability of a machine increases when jobs are repeated. Another observation is, that these papers just address a small number of jobs since they make use of complex solution approaches. We are looking for practical relevance and therewith for simple decomposition approaches to regard a huge number of jobs.

Besides, few work has been done regarding the non-resumable case. If jobs are non-resumable, an interruption of a job results in an even higher impact of machine failures on the cost of a production process and job order delay. Adiri et al. (1989) assume the jobs to be non-resumable and consider the occurrence of a single breakdown but do not schedule PM activities. They found that the Longest Processing Time (LPT) is the optimal sequence for minimising the makespan if the distribution function for the time to failure is convex. Otherwise, if the distribution function

is concave, the Shortest Processing Time (SPT) is shown to be optimal. Later we expose that this is not true if several failures are possible during a schedule and PM activities are included in the decision process.

In order to gain simple approaches for practical usage, we are considering those priority rules suggested in the literature (SPT, LPT and FFD) for the scheduling part and examine suitable formula for PM intervals. To determine the PM interval we make use of a decomposition approach also found in literature. Since this common used formula is not suitable for an order of non-resumable jobs, adaptations are made to regard the non-resumable case. The estimations of the PM interval are compared to the optimal interval, which is found by simulation. We also show how the PM intervals are reacting on the different scheduling rules and parameter values as for example on the time to failure.

The paper is organised as follows: In the next section the problem is described in more detail. In section 3 the tested scheduling and maintenance insertion rules are presented. Section 4 contains the simulation study. It includes the settings and the results of the simulation study comparing different job scheduling rules in section 4.1 and 4.2. In section 4.3 and 4.4 the explanation of our decomposition approaches follows. The decomposition approaches are proposed to estimate the optimal preventive maintenance interval and connect them with the considered scheduling rules. In the simulation study these approaches are compared and the results are discussed in section 4.5 and 4.6. The last section contains a summary and a conclusion.

2 Problem Definition

We consider an order of n jobs to be scheduled on a single machine. A job j ($j = 1, 2, \dots, n$) has a processing time p_j . These given processing times are assumed to be constant and can be different for each job. The whole job order is available at time zero. Further, a common due date dd for the whole order is given. Exceeding this due date results in cost c_1 for every time unit the delivery is delayed. We further assume that the machine wears out with ongoing production process. The time to failure T is therefore modelled in an Increasing Failure Rate (IFR) mode.

If a machine fails during processing a job, this job has to get restarted (non-resumable case). To avoid machine failures and therewith unexpected expensive production stops, PM tasks are inserted in the production schedule. It is assumed that each PM causes cost c_p for material or payment of maintenance providers and requires t_p time units. A PM is not allowed to interrupt a job, thus the non-preemptive case is assumed. If a failure could not be averted, a CM is performed to repair the machine and renew damaged items. This results in cost c_c for repair (with $c_c > c_p$) and in t_c (with $t_c > t_p$) time units of conduction. We address time as well as cost aspects to regard the schedule time preference and the maintenance cost preference respectively. The expected cost can be written as:

$$K = cm \cdot c_c + pm \cdot c_p + (C_{\max} - dd) \cdot c_l \quad (1)$$

with cm and pm being the expected number of CM activities and PM activities, respectively. C_{\max} is the expected makespan of the whole schedule. The makespan is defined as the completion time of the last produced job. Here, the expected makespan consists of the sum of processing times, the expected time which is spent for PM activities and unexpected repairs, and the time units t_{repeat} resulting from repeated production due to job interruption:

$$C_{\max} = \sum_{j=1}^n p_j + cm \cdot t_c + pm \cdot t_p + t_{\text{repeat}}. \quad (2)$$

We consider the expected number of PM activities, because we are assuming an age replacement policy (see also for example Barlow and Hunter (1960), Bosch and Jensen (1983) and Beichelt (1993)). An illustration of this policy is given in Figure 1 for the maintenance part. The age of the machine increases continuously while producing the jobs. We suppose that a PM activity is inserted by no later than a predetermined time interval denoted as τ , thus a specific age of the machine. If a PM activity is inserted, the machine will be restored to become ‘as good as new’. This implies that the age of the machine returns to zero and increases again with the further production process. Whenever the machine fails, a CM is conducted to restore the

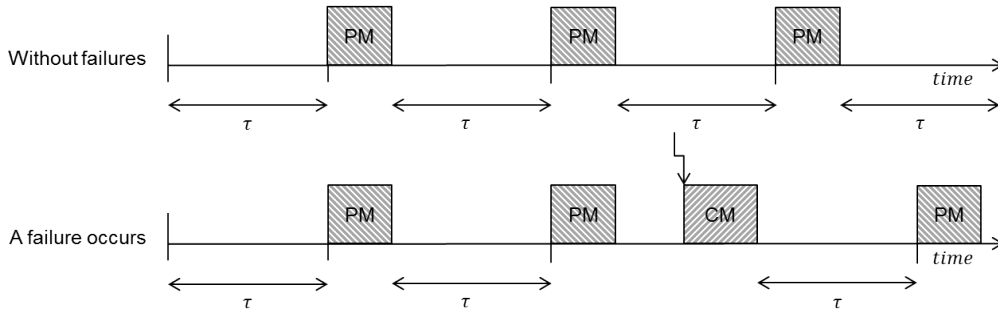


Figure 1: The age replacement policy.

machine to be ‘as good as new’. If the resumable case is assumed, it makes sense to conduct a minimal repair after a CM activity. A minimal repair causes that the probability of a failure rises further after the CM. This assumption simplifies the calculation of PM intervals, because a breakdown does not change this prior determined PM interval. Since we are interested in the non-resumable case, a minimal repair is no realistic assumption, because a restart of the interrupted job might result in an endless loop of failures and restarted jobs. Thus, we assume that if a failure occurs, a CM is conducted, the machine age becomes zero and the next time interval starts right after the repair. It means that the following originally planned PM activities are reordered. Therefore, at the beginning of production the actual number of PM tasks is not known.

However, some PM activities have to be included into the schedule independent from the job schedule, at least (pm_{least}) to avoid machine failure. Since the distribution of the time to failure is assumed to be known, the number of expected failures, if no PM is inserted, is also known. It is calculated by dividing the sum of the processing times of the jobs by the Mean Time To Failure ($MTTF$):

$$pm_{\text{least}} = \frac{\sum_{j=1}^n p_j}{MTTF}. \quad (3)$$

Thus, it is assumed that for these expected failures a number of PM activities is always needed. Since these activities are not relevant for our decision process, they can be excluded from the average cost. In the cost function we just include cost due to additional PM activities ($pm =$

$pm_{\text{all}} - pm_{\text{least}}$).

In the next section the used solution approaches are explained regarding the scheduling rules as well as the decision rule, when to insert a PM activity.

3 Solution Approaches

The aim of this paper is to compare the performance of simple scheduling decisions and PM insertion rules. As mentioned before, we are using a PM time interval (τ) which is the maximal time between two consecutive PM activities. In special, a PM task is inserted in the production schedule right in front of a job, if the age of the machine after finishing this job would exceed the time interval. Thus, a τ has to be found which minimises the expected cost, but the actual intervals are created by the production schedule itself. To find a formula for PM intervals regarding the non-resumable case in order to minimise the expected cost, we underlie several job sequences. We consider three different scheduling rules mentioned in literature as well as a random order of jobs. The Shortest Processing Time (SPT) and the Longest Processing Time (LPT) are chosen. An example of scheduling jobs in LPT order is given in Figure 2 with processing times given in Table 1 and $\tau = 10$.

Table 1: Job processing times.

Job	1	2	3	4	5	6	7
p_j	1	2	3	4	4	7	8

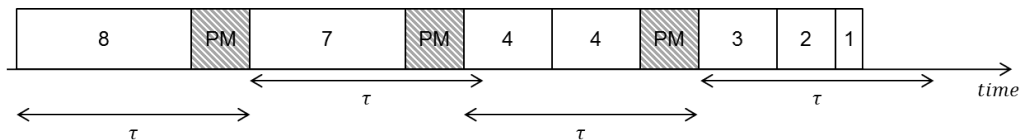


Figure 2: Jobs scheduled in LPT rule with $\tau = 10$.

A hypothesis was, that scheduling jobs in LPT must be more advantageous than scheduling them by SPT rule with regard to the cost for restarted jobs as shown in Adiri et al. (1989).

Despite that hypothesis, both rules (SPT and LPT) have in common that they mostly schedule either just long jobs between two consecutive PM activities or just short jobs. Since failures are more likely at the end of a renewal cycle, it should be beneficial to schedule long jobs right after a PM when the age of the machine is low, and small jobs at the end of an interval. Thus, if a failure occurs, the repeating times of a job will be rather small. To consider this, another scheduling rule was created which is based on the First Fit Decreasing (FFD) algorithm in bin packaging problems and is therefore called the FFD rule, here. The FFD algorithm is a heuristic which fills bins of equal length in the way, that first the items which have to be packed are ordered in non-increasing order of their size. Then one item after another is inserted into that bin where it fits first (see Coffman et al. (1997)). The aim is to minimise the number of used bins. Assigned to our problem, the bins describe the time intervals between the PM activities and the items are the jobs. In detail, the job is chosen next, to get inserted into the production schedule, which fits best in the remaining space between the finishing time of the latest inserted job and the next assumed PM activity. Figure 3 shows the example of Table 1 in FFD scheduling sequence. Since this rule has a close upper bound to the optimal solution in the bin packaging

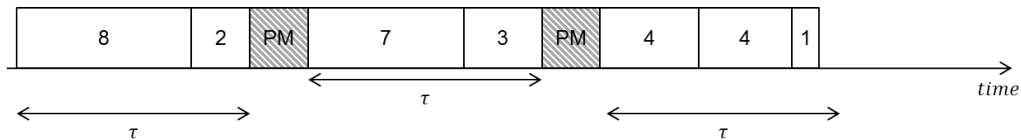


Figure 3: Jobs scheduled in FFD rule with $\tau = 10$.

problem (see Li and Yue (1997)), it is assumed that it is also able to generate few additional PM activities. Additionally, the rule shall insert the short jobs when the probability of a failure is high, and uses the advantage of the age replacement policy in the case of also reordering jobs after a failure. Thus, if a CM activity has to be conducted, the interrupted job does not have to get repeated right after the repair, but is scheduled in the following intervals where it fits best. Scheduling in SPT or LPT has no effect on this, because the next job chosen after repair is again the shortest respectively the longest remaining job which is the same as the interrupted one.

4 Simulation Study

In this simulation study we first compare the scheduling rule performances using their optimal PM interval. We further create and analyse different formulae to estimate the optimal PM intervals with the help of the explained scheduling rules.

4.1 Parameter

In the simulation study $n=100$ jobs are scheduled with processing times which are uniformly distributed in different intervals, where the mean processing time is fixed to 50 (Table 2): five times in the interval of 1 to 100, two times with 50 jobs having processing times uniformly distributed in the interval of {15-35} and 50 out of {65-85}, one production schedule with 50 jobs having processing times of 90 and 50 of 10. These different jobs are created to evaluate if the length and the variability of processing times do have an impact on the results.

Table 2: 8 different job orders.

Order No.	number of jobs x processing times
1-5	100 x uniform {1-100}
6-7	50 x uniform {15-35}; 50 x uniform {65-85}
8	50 x 90; 50 x 10

It is assumed that the time to failure (T) can be modelled as a Weibull random variable with shape parameter β , scale parameter α and the cumulative distributions function:

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^\beta}. \quad (4)$$

In order to have an IFR (and therewith a convex distribution function), β has to be above one. To consider different quantities of possible failures during a schedule, 6 scenarios were created which differ in the $MTTF$ and in the coefficient of variation ($c_v = \frac{\sigma}{MTTF}$) of the time to failure (Table 3). σ is the standard deviation of the time to failure and the $MTTF$ of the Weibull

distribution is

$$MTTF = \alpha \cdot \Gamma \left(1 + \frac{1}{\beta} \right) \quad (5)$$

with Γ being the complete Gamma function. The two blocks differ in their $MTTF$ so that in the first block on average 10 failures occur during the production schedule if no PM is inserted and in the second block round about 5 failures. It was abandoned to evaluate scenarios with less failures, because this resulted in weak estimators. Within each block the coefficient of variation and therewith the variation of the time to failure increases.

Table 3: 6 scenarios of time to failure for 100 jobs and mean processing time 50.

k	MTTF	c_v	number of failures
1	500	0.1	≈ 10 failures
2	500	0.3	
3	500	0.5	
4	1000	0.1	≈ 5 failures
5	1000	0.3	
6	1000	0.5	

The parameter value for t_p is set to 7 and for c_p to 20. For the remaining parameters different values are chosen: $t_c \in \{10, 20, 30\}$, $c_c \in \{40, 69, 100\}$, $c_l \in \{2, 10, 20\}$. In total 1296 instances are tested for every scheduling rule with each running 50,000 times to ensure accuracy of our estimates.

4.2 Comparison of the Scheduling Rules

In the first part of the simulation study a comparison of the four scheduling rules ($v \in V = \{\text{SPT, LPT, Random, FFD}\}$) is conducted for all instances underlying their optimal PM interval. In order to be able to find the optimal PM interval (τ_{opt}) by simulation, a complete enumeration is used for each of the 1296 instances. Since all jobs have integer processing times, just integer interval lengths were tested. To compare the performances of the scheduling rules the relative percentage deviation is calculated which is determined by the comparison of

the average cost \bar{K}_ν underlying scheduling rule ν , and the minimum average cost obtained by the four scheduling rules:

$$RPD_\nu = \frac{\bar{K}_\nu - \min_{s \in V}(\bar{K}_s)}{\min_{s \in V}(\bar{K}_s)} \cdot 100\%. \quad (6)$$

The best scheduling rule has a $RPD_\nu = 0$. The results are shown in Table 4 for the different $MTTF$, the values of c_ν , the 8 different orders with unequal processing times and the cost respectively time parameter values. For every scheduling rule the average RPD_ν ‘Av’ value is listed as well as the maximum and the minimum value found if the parameter in the left row is fixed on the corresponding value and all other parameters are enumerated.

Table 4: RPD_ν of the four tested scheduling rules.

	SPT			LPT			Random			FFD		
	Av	Max	Min	Av	Max	Min	Av	Max	Min	Av	Max	Min
$MTTF=500$	5.1	15.9	0.9	4.7	14.7	0.0	3.6	22.3	0.0	0.0	2.4	0.0
$MTTF=1000$	4.3	14.6	0.5	2.5	13.8	0.0	2.3	13.5	0.0	0.2	2.4	0.0
$c_\nu = 0.1$	8.2	15.9	1.5	6.2	14.7	0.0	5.4	22.3	0.0	0.1	2.4	0.0
$c_\nu = 0.3$	3.8	13.8	0.7	3.1	13.8	0.0	2.5	11.1	0.0	0.1	2.4	0.0
$c_\nu = 0.5$	2.1	6.3	0.5	1.5	5.6	0.0	1.0	5.3	0.0	0.1	1.4	0.0
Order1	3.9	10.1	0.7	3.4	9.6	0.1	2.4	9.5	0.0	0.0	0.3	0.0
Order2	4.2	11.9	0.7	2.7	11.0	0.0	1.4	3.7	0.0	0.1	1.4	0.0
Order3	5.2	14.6	0.7	4.1	14.5	0.4	2.1	9.5	0.0	0.2	1.8	0.0
Order4	4.4	12.1	0.5	3.2	14.1	0.0	2.8	10.1	0.0	0.1	1.5	0.0
Order5	4.3	12.6	0.9	2.9	12.6	0.1	2.1	10.2	0.0	0.1	0.9	0.0
Order6	4.7	15.0	0.9	4.0	11.8	0.2	3.6	15.4	0.0	0.0	0.3	0.0
Order7	5.0	14.8	0.8	4.3	14.6	0.2	3.8	11.0	0.0	0.1	2.4	0.0
Order8	5.9	15.9	1.4	4.2	14.7	0.0	5.3	22.3	0.0	0.2	2.4	0.0
$t_c = 10$	5.3	15.9	0.8	4.0	14.7	0.0	3.2	22.3	0.0	0.1	2.4	0.0
$t_c = 20$	4.7	13.4	0.6	3.6	12.0	0.0	2.9	19.7	0.0	0.1	2.0	0.0
$t_c = 30$	4.1	12.4	0.5	3.3	12.0	0.0	2.7	14.4	0.0	0.1	2.4	0.0
$c_c = 40$	5.0	15.9	0.9	3.7	14.7	0.0	3.0	22.3	0.0	0.1	2.4	0.0
$c_c = 69$	4.7	14.7	0.7	3.6	14.1	0.0	2.9	18.0	0.0	0.1	2.4	0.0
$c_c = 100$	4.5	14.4	0.5	3.5	13.3	0.0	2.9	13.5	0.0	0.1	2.4	0.0
$c_l = 2$	3.6	15.9	0.5	2.9	14.7	0.0	2.2	22.3	0.0	0.1	2.4	0.0
$c_l = 10$	5.1	13.6	0.9	3.9	14.5	0.0	3.2	18.3	0.0	0.1	1.6	0.0
$c_l = 20$	5.4	14.8	1.2	4.1	14.6	0.0	3.4	15.7	0.0	0.1	2.4	0.0

It is obvious that the FFD rule outperforms the other scheduling rules in every instance. Since the RPD_{FFD} is on average no more than 0.2%. Even the maximum values of the FFD

rule are always better than the average RPD_v values of SPT and LPT. We found out that this is also true for the makespan. Thus, if more than one breakdown is possible to occur and PM activities are inserted in the scheduling process, LPT is not optimal anymore as for the problem discussed in Adiri et al. (1989).

For the FFD rule the variation of the parameter values does not have any impact. For a closer look on the other scheduling rules Figure 4 illustrates the effect of ascending the $MTTF$ on the left side and increasing c_v on the right side, with respect to the RPD_v . If the $MTTF$ increases, the RPD_{LPT} decreases significantly, because the longer the time to failure, the shorter are the jobs in front of a PM if the jobs are scheduled in LPT sequence. Thus, the repeating times are getting smaller if the $MTTF$ moves from 500 to 1000. Another significant effect evolves if c_v , and therewith the variance of the time to failure, increases. The RPD_v of SPT, LPT and Random decreases with increasing variance, making the performance of all scheduling rules more similar. Thus, if the variance is high it is nearly unimportant which scheduling rule is implemented, since it is quite impossible to anticipate the failures.

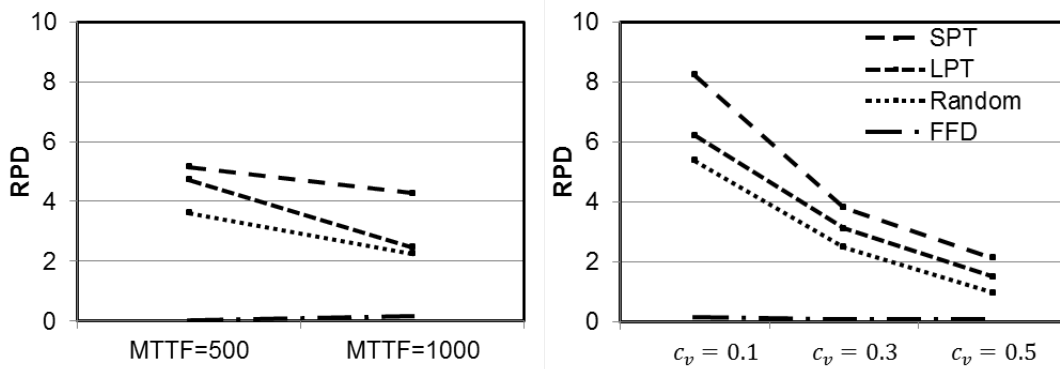


Figure 4: RPD_v of the scheduling rules for different $MTTF$ and c_v values.

Since the FFD rule outperforms the other scheduling rules, it is most interesting if the optimal PM interval of the FFD rule can be estimated. Nevertheless, SPT and LPT are easier to implement. Thus, it might also be easier to find their optimal intervals. Therefore, in the next section decomposition approaches are determined to find out if the optimal length of the PM interval can be estimated isolated from the scheduling rule.

4.3 Decomposition Approaches

To find the optimal PM intervals, we investigate two different approaches based on different cost expressions. These expressions are used to derive estimates for the PM intervals.

The first one is often used in maintenance optimisation literature without considering production scheduling decisions (see for example Barlow and Hunter (1960), Bosch and Jensen (1983), Beichelt (1993), Brandolese et al. (1996)), where the optimal time interval is derived by minimising average maintenance cost per time unit given as:

$$K_1(\tau) = \frac{E(K_{cy}(\tau))}{E(L)} = \frac{c_c F(\tau) + c_p \cdot \bar{F}(\tau)}{\int_0^\tau \bar{F}(t) dt}. \quad (7)$$

$E(K_{cy}(\tau))$ denotes the expected cost in a maintenance cycle and $E(L)$ is the expected length of this cycle. $E(K_{cy}(\tau))$ consists of the expected cost composed of the repair cost weighted by the probability that the machine fails before reaching the time interval τ and the PM cost weighted by the converse probability ($\bar{F}(\tau) = 1 - F(\tau)$). This cost function is derived based on the assumption of an infinite time horizon and does not take into account any production scheduling challenges. However, the derived PM interval depends on the cost for maintenance performance as well as on the distribution of the time to failure. This influence can also be observed for our cost function, as shown in Figure 5. Here, the cost averaged over all simulation runs is illustrated as a function of the PM interval length for different failure scenarios. Scenario 1 reflects about 10 possible failures and a low variance in the time to failure, scenario 3 a high variance of the time to failure and scenario 4 about 5 possible failures and a low variance in the time to failure. All other scenarios were spared for the reason of clarity. But, the functions of the other scenarios behave consistently to those illustrated. The average cost of the shown three scenarios were calculated with high values for repair and lateness cost parameters ($t_c = 30$, $c_c = 100$, $c_l = 20$). The jobs are scheduled in SPT rule. All scenarios have in common, that for low interval length lots of unnecessary PM activities are inserted, resulting in high average cost. With increasing interval length less preventive, but more CM activities occur. The average cost decrease until the additional summarised cost for CM activities exceed the reduction of the

cost for PM activities. Further, the average cost increase until they terminate at a level where only CM activities are conducted. If the curves are considered separately, it is obvious that

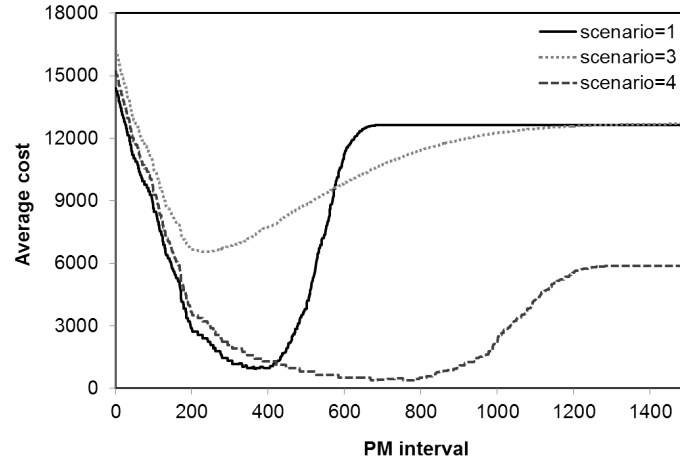


Figure 5: The average cost for different PM interval length; scenario 1, 3 and 4 in SPT, $t_c = 30$, $c_c = 100$, $c_1 = 20$.

their properties depend on the value of the $MTTF$ and the coefficient of variation of the time to failure. With increasing τ the average cost for scenario 1 decrease to a much lower minimum than for scenario 3. This implies that with a high variance in the time to failure it is not possible to abandon many PM activities, making the curve much flatter and the differences in cost for different τ values lower. Since the $MTTF$ is higher for scenario 4 than for 1, τ can be increased much more without additional failures.

Of course, cost calculated by equation (7) are much smaller and the curves are flatter for small and high values of τ . Nevertheless, at first sight the behaviour of our cost function seems quite similar to those generated by (7). But contrary to (7), the curves are not smooth as can be seen in Figure 6 for scenario 1 and 4. The steps emerge as a result of production scheduling. In the course of a step, the occurrence of failures rises until τ increases to a level where lots of PM activities can be saved compared to the increasing number of breakdowns. Increasing a small τ excludes the PM activities between small jobs. Additionally, the higher τ , the higher the increase of τ has to be to save the next set of PM activities. Therefore, the steps of the cost function for scenario 1 are much smaller than those for scenario 4. The general slope of these steps is highly driven by the non-resumable jobs, making it even harder to find the optimal

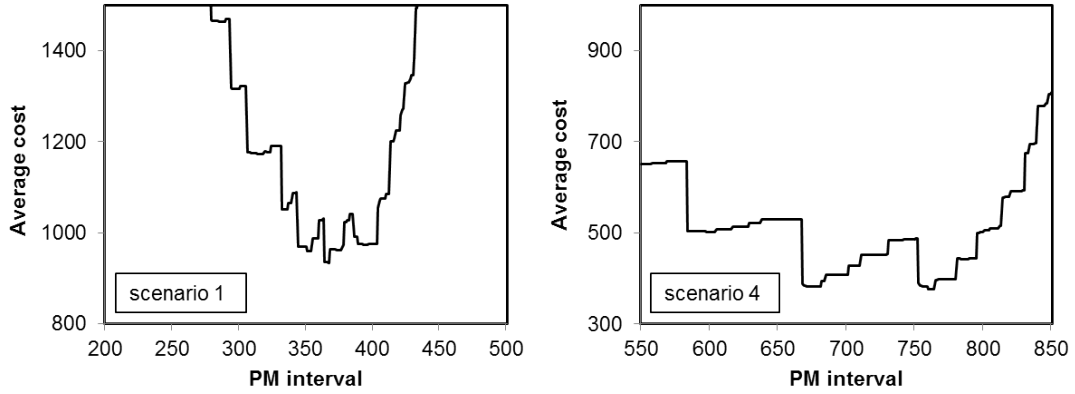


Figure 6: The average cost for different PM interval length around the minimum; scenario 1 and 4 in SPT, $t_c = 30$, $c_c = 100$, $c_l = 20$.

PM interval since a small deviation in τ leads to high cost deviations. Thus, cost function (7) does not fit exactly to our model assumptions, since it does not regard jobs repeating times, the finite time horizon and cost for order lateness. Therefore, we adapt the approach to the studied situation and obtain a second cost function. In the second cost function, the average cost for the whole schedule are estimated, taking into account the whole working time of the machine ($\sum_{j=1}^n p_j + t_{\text{repeat}}$). The working time of the machine consists of the processing times of the jobs and the time units induced by the repetition of jobs interrupted by breakdowns. Divided through the cycle length, the expected number of maintenance cycles for the whole schedule can be estimated as

$$A_c(\tau) = \frac{\sum_{j=1}^n p_j + t_{\text{repeat}}}{\int_0^\tau \bar{F}(t) dt}. \quad (8)$$

Since t_{repeat} is dependent on the number of breakdowns, it can be written as:

$$t_{\text{repeat}} = \frac{\sum_{j=1}^n p_j}{\int_0^\tau \bar{F}(t) dt} \cdot F(\tau) \cdot t_r \quad (9)$$

with t_r defined as the averaged time units per failure spent for the repetition of jobs due to interruption. How to determine t_r is explained in section 4.4. It is multiplied with the number of estimated expected maintenance cycles $\left(\frac{\sum_{j=1}^n p_j}{\int_0^\tau \bar{F}(t) dt} \right)$, if the repeating times are excluded, and the

probability that the machine fails during a time interval. The number of expected maintenance cycles becomes:

$$A_c(\tau) = \frac{\sum_{j=1}^n p_j \left(1 + \frac{F(\tau) \cdot t_r}{\int_0^\tau \bar{F}(t) dt} \right)}{\int_0^\tau \bar{F}(t) dt}. \quad (10)$$

The assessed cost function based on equation (1) and equation (7) can thus be written as:

$$K_2(\tau) = F(\tau) \cdot A_c(\tau) \cdot c_c + \bar{F}(\tau) \cdot (A_c(\tau) - pm_{\text{least}}) \cdot c_p + (C_{\text{max}} - dd) \cdot c_1. \quad (11)$$

The expected cost in a maintenance cycle are calculated similar as in equation (7) but are multiplied with the expected number of cycles in the schedule. Another adaption is made to calculate just the additional PM activities: pm_{least} is the expected number of PM activities, which have to be conducted at least (see equation (3)). Therewith the number of expected maintenance cycles weighted by the probability that no failure occurs during τ , is decreased by the number of predictable PM activities. Since pm_{least} is constant, it has no effect on the calculation of τ .

Finally, the makespan is estimated as follows with regard to equation (2):

$$C_{\text{max}} = \sum_{j=1}^n p_j + F(\tau) \cdot A_c(\tau) \cdot (t_c + t_r) + \bar{F}(\tau) \cdot A_c(\tau) \cdot t_p. \quad (12)$$

To find the time intervals by minimising $K_1(\tau)$ or $K_2(\tau)$, a numerical optimisation is conducted, which is explained in the next section.

4.4 The Due Date and the Time t_r

In order to be able to compare the different decomposition approaches, a due date and an estimation of t_r is needed. Since the expected number of failures without PM activities is known, it is assumed that the planner considers this in lead time negotiations. Thus, the due date is assumed to consist of the sum of processing times of the jobs plus the work time which is

needed to fulfil the minimum number of expected PM activities:

$$dd = \sum_{j=1}^n p_j + pm_{\text{least}} \cdot t_p. \quad (13)$$

The assessed cost function (11) for the decomposition approach, with the makespan and the due date inserted, turns to:

$$K_2(\tau) = F(\tau) \cdot A_c(\tau) \cdot c_c + \bar{F}(\tau) \cdot (A_c(\tau) - pm_{\text{least}}) \cdot c_p + \left(\sum_{j=1}^n p_j + F(\tau) \cdot A_c(\tau) \cdot (t_c + t_r) + \bar{F}(\tau) \cdot A_c(\tau) \cdot t_p - \left(\sum_{j=1}^n p_j + pm_{\text{least}} \cdot t_p \right) \right) \cdot c_1 \quad (14)$$

which can be shortened to:

$$K_2(\tau) = F(\tau) \cdot A_c(\tau) \cdot (c_c + (t_c + t_r) \cdot c_1) + (\bar{F}(\tau) \cdot A_c(\tau) - pm_{\text{least}}) \cdot (c_p + t_p \cdot c_1). \quad (15)$$

To determine the PM intervals for $K_2(\tau)$, the parameter t_r still needs to be estimated. t_r contains the repeated time units of a job per breakdown. To get an idea, if this parameter is decisional relevant for the estimation of τ_{optv} , in a first trial t_r is set to zero. With another calculation we tested the dependence of the repeating times on the mean \bar{p} of the processing times per order. This is motivated by the fact that longer jobs have a higher probability of long repeating times if a failure occurs. Here just the results are shown if t_r is set to $0.5 \cdot \bar{p}$. Table 5 summarises the three resulting different approaches with τ_d defining the calculated PM interval of decomposition method d . All other used parameters are set to the values as described in section 4.

Table 5: Decomposition approaches with different values of t_r .

d	cost function	t_r	derived τ_d
1	$K_1(\tau)$	-	τ_1
2	$K_2(\tau)$	0	τ_2
3	$K_2(\tau)$	$0.5 \cdot \bar{p}$	τ_3

4.5 Comparison of the Decomposition Approaches

In this section the performance of the estimated PM intervals τ_d is shown through the use of simulation. The average cost $\bar{K}_v(\tau_d)$, using scheduling rule v and PM interval τ_d , created by the decomposition method d , are compared with the average cost $\bar{K}_v(\tau_{optv})$, if integrating the optimal interval of a scheduling rule v into the considered scheduling rule. The relative percentage deviation of these cost is calculated by the following equation in order to assess the quality of the decomposition approach:

$$Div_{vd} = \frac{\bar{K}_v(\tau_d) - \bar{K}_v(\tau_{optv})}{\bar{K}_v(\tau_{optv})} \cdot 100\% \quad (16)$$

This implies, that for example the performance of the FFD rule using an estimated time interval is compared to the performance of the FFD rule with its optimal PM interval.

The relative percentage deviation of the PM intervals is calculated by:

$$Div\tau_{vd} = \frac{\tau_d - \tau_{optv}}{\tau_{optv}} \cdot 100\%. \quad (17)$$

In Table 6 the percentage cost deviations of scheduling in SPT, LPT and FFD are given for all instances and intervals generated by $K_1(\tau_1)$. The results for the random scheduled sequence are not included, because of clarity and because the sequence has no added value for the comparison. Table 6 shows Div_{v1} for the three scheduling rules on average 'Av' as well as the maximum value found, when the parameter on the left side is fixed. Additionally, the percentage average relative deviations $Div\tau_{v1}$ from the estimated τ_1 to τ_{optv} is listed in the last column of each scheduling rule block. The estimation of τ_1 through K_1 results in relative deviations from the minimum cost of about 17% for SPT and LPT sequence and about 22% if scheduled in FFD rule. Since K_1 does not regard the finite time horizon and cost for lateness, this estimation creates time intervals which are too high. On average the estimated time intervals are at least 20% higher than the optimal. The deviations of the PM interval are even higher for the FFD rule, because τ_{optFFD} is smaller than those of the other scheduling rules. Figure 7 shows this as an

Table 6: Div_{v1} between $\bar{K}_v(\tau_1)$ and $\bar{K}_v(\tau_{optv})$ and $Div\tau_{v1}$ for each scheduling rule.

	SPT			LPT			FFD		
	Av	Max	Av $Div\tau_{v1}$	Av	Max	Av $Div\tau_{v1}$	Av	Max	Av $Div\tau_{v1}$
$MTTF=500$	11.5	61.1	21.6	11.7	63.4	20.9	19.8	93.7	34.5
$MTTF=1000$	22.9	112.0	28.0	22.3	116.8	27.4	24.5	121.8	34.1
$c_v = 0.1$	21.9	112.0	3.4	22.5	116.8	3.2	28.9	121.8	9.3
$c_v = 0.3$	16.1	85.1	18.0	15.5	81.4	17.4	22.1	100.7	25.9
$c_v = 0.5$	13.6	58.2	53.0	12.9	54.5	51.9	15.4	60.5	67.7
Order1	18.7	104.9	24.1	16.4	75.1	24.6	22.9	116.0	35.5
Order2	15.6	99.1	24.5	15.0	86.2	23.7	22.8	115.2	33.8
Order3	17.8	92.9	25.0	18.3	110.0	25.0	23.8	113.5	36.2
Order4	16.8	112.0	25.2	17.8	116.8	25.0	23.6	121.8	35.2
Order5	17.7	84.2	24.9	18.6	91.8	24.1	22.0	107.3	34.6
Order6	19.1	106.7	23.4	17.8	87.8	23.2	22.1	112.7	32.7
Order7	16.9	82.9	23.1	16.8	88.5	23.6	21.3	101.9	32.8
Order8	14.9	94.6	28.3	15.3	97.7	23.9	18.6	100.7	33.5
$t_c=10$	13.6	90.1	18.5	13.2	89.5	17.7	16.0	78.4	26.6
$t_c=20$	17.1	101.3	24.7	16.9	104.4	23.8	22.2	102.0	34.3
$t_c=30$	20.9	112.0	31.2	20.9	116.8	30.9	28.3	121.8	42.0
$c_c=40$	39.5	112.0	54.5	37.7	116.8	53.5	48.0	121.8	66.2
$c_c=69$	6.8	26.2	17.2	7.4	33.7	16.4	14.5	51.8	25.7
$c_c=100$	5.3	45.1	2.8	5.8	50.7	2.5	3.9	14.8	10.9
$c_l=2$	11.3	76.7	16.3	11.3	80.1	15.2	13.1	59.4	24.2
$c_l=10$	19.1	105.6	27.8	18.8	110.5	27.1	25.1	110.2	37.7
$c_l=20$	21.2	112.0	30.3	20.8	116.8	30.1	28.1	121.8	41.0

example for scenario 1 and high cost and lateness parameters. The FFD curve seems to be a left and down shifted version of the other scheduling rule curves. The FFD rule can generate smaller cost with a lower time interval, because of its manner to fill the intervals between to PM activities more equal than the other scheduling rules.

Since t_c and c_l are not included in the cost function K_1 , c_c is the only driver of τ_1 . τ_1 does not change if t_c or c_l increase. In contrast to this τ_{optv} decreases with increasing cost parameters. Therefore, the deviation from the optimum increases when t_c and c_l are raised. If c_c increases, τ_1 decreases as well as the deviation in average cost. Noticeable is also that the relative deviation of cost decreases if the variance in the time to failure increases, although the relative deviation of the PM interval is much higher. This is caused by the fact that the cost curve is flatter for higher variance in the time to failure, especially when τ is overestimated (see again Figure 5).

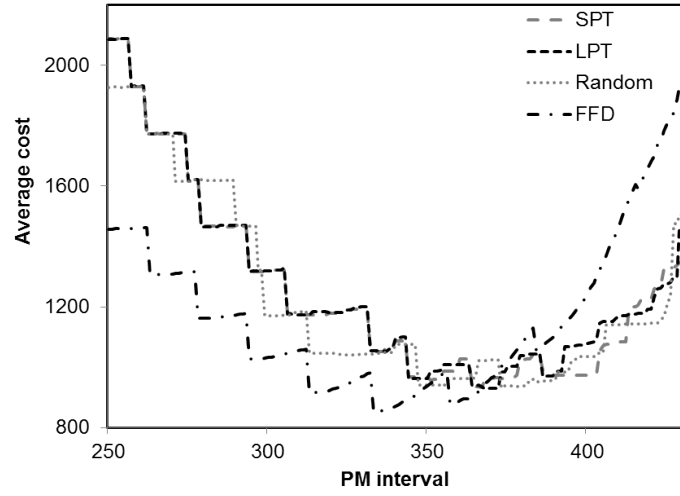


Figure 7: Comparison of the average cost for the scheduling rules at different PM interval length; scenario 1 in SPT, $t_c = 30$, $c_c = 100$, $c_1 = 20$.

Table 7: Div_{v2} between $\bar{K}_v(\tau_2)$ and $\bar{K}_v(\tau_{optv})$ and $Div\tau_{v2}$ for each scheduling rule.

	SPT			LPT			FFD		
	Av	Max	Av $Div\tau_{v2}$	Av	Max	Av $Div\tau_{v2}$	Av	Max	Av $Div\tau_{v2}$
$MTTF=500$	15.2	189.1	29.4	14.8	205.1	28.5	25.8	237.8	43.1
$MTTF=1000$	24.4	192.9	36.3	23.4	228.4	35.6	32.1	227.8	42.6
$c_v = 0.1$	21.5	192.9	5.3	22.1	228.4	5.2	40.4	237.8	11.3
$c_v = 0.3$	21.2	127.5	23.6	19.9	115.1	22.9	28.2	145.2	31.8
$c_v = 0.5$	16.6	67.6	69.5	15.3	60.5	68.2	18.3	68.2	85.5
Order1	20.5	138.0	32.2	19.4	108.0	32.6	29.9	168.1	44.3
Order2	20.0	105.1	32.5	18.8	127.8	31.6	29.9	175.8	42.4
Order3	20.0	108.0	33.1	20.2	140.0	33.1	32.0	185.2	45.3
Order4	19.7	113.3	33.4	19.4	96.4	33.0	30.7	184.9	43.8
Order5	19.4	126.1	32.9	19.2	106.7	32.2	29.4	163.7	43.2
Order6	20.0	125.8	31.1	18.6	103.3	31.0	27.4	154.3	40.8
Order7	18.1	103.0	30.8	17.5	135.9	31.5	27.7	168.4	40.9
Order8	20.5	192.9	36.5	19.6	228.4	31.6	24.7	237.8	42.3
$t_c = 10$	42.0	192.9	58.6	39.2	228.4	57.7	53.4	237.8	70.0
$t_c = 20$	11.7	54.8	24.8	12.3	60.4	24.0	21.6	72.3	34.5
$t_c = 30$	5.5	38.7	15.0	5.7	46.0	14.6	11.9	45.7	24.1
$c_c = 40$	27.7	192.9	43.1	26.5	228.4	42.1	38.8	237.8	53.5
$c_c = 69$	18.0	104.3	31.4	17.4	99.2	30.5	27.1	139.3	41.1
$c_c = 100$	13.7	95.9	24.0	13.4	89.8	23.6	21.0	97.4	34.0
$c_1 = 2$	8.4	67.9	17.4	8.4	62.4	16.1	13.9	69.1	25.2
$c_1 = 10$	21.8	113.3	36.6	20.8	104.5	35.9	31.9	150.8	47.2
$c_1 = 20$	29.1	192.9	44.4	28.0	228.4	44.3	41.0	237.8	56.3

Table 7 illustrates the results if τ_2 is estimated with cost function K_2 when the repeating time t_r is not considered. These results are on average even worse than the results created with cost function K_1 , although cost for lateness are involved. But, since the time horizon changed from infinity to a specific length, failures are less punished and τ is even more overestimated. Since the repair time is now considered, the relative deviation of τ_2 from the optimal interval is getting less if t_c increases. But still the relative deviation from the optimal time interval gets worse if the cost for lateness increase. Thus, it is preferable to accept more failures instead of paying the delay of the order caused through additional PM activities. This implies that the penalty for failures is still too low for the estimated cost function.

In summary, the results of estimating τ_2 are rather insufficient. It is possible to find the minimum average cost for each scheduling rule, but since the maximum values are very high, the variance of the results is high, too.

Table 8: Div_{v3} between $\bar{K}_v(\tau_3)$ and $\bar{K}_v(\tau_{optv})$ and $Div\tau_{v3}$ for each scheduling rule.

	SPT			LPT			FFD		
	Av	Max	Av $Div\tau_{v3}$	Av	Max	Av $Div\tau_{v3}$	Av	Max	Av $Div\tau_{v3}$
$MTTF=500$	3.7	35.6	-5.5	3.9	37.3	-6.0	2.8	21.5	4.0
$MTTF=1000$	7.4	47.9	-0.3	8.5	52.7	-0.8	5.0	44.1	4.4
$c_v = 0.1$	14.0	47.9	-4.0	15.6	52.7	-4.1	8.7	44.1	1.5
$c_v = 0.3$	2.1	13.1	-2.8	2.3	13.5	-3.4	2.3	15.5	3.6
$c_v = 0.5$	0.4	2.6	-1.8	0.6	2.9	-2.6	0.7	6.1	7.5
Order1	6.9	47.9	-3.6	7.0	49.0	-3.1	3.7	41.9	5.1
Order2	5.8	46.6	-2.8	6.9	50.1	-3.4	3.3	26.5	4.3
Order3	5.8	45.2	-2.7	6.8	47.6	-2.6	4.3	43.4	5.8
Order4	4.9	41.7	-2.5	5.3	45.3	-2.6	4.1	43.7	5.0
Order5	6.5	42.6	-3.0	7.7	52.7	-3.4	3.5	41.6	4.4
Order6	6.5	47.7	-4.0	6.8	48.4	-4.2	3.7	40.1	2.9
Order7	5.0	45.1	-4.3	5.2	38.7	-3.7	4.3	44.1	2.9
Order8	2.7	35.6	-0.2	3.7	37.3	-3.9	4.3	26.6	3.2
$t_c = 10$	4.5	45.1	-0.9	4.7	50.9	-1.5	3.6	26.5	5.8
$t_c = 20$	6.7	46.6	-3.4	7.5	52.7	-3.9	2.6	42.9	3.8
$t_c = 30$	5.4	47.9	-4.4	6.3	49.0	-4.8	5.6	44.1	3.0
$c_c=40$	4.8	47.9	-2.1	5.4	50.5	-2.6	4.4	43.7	4.9
$c_c=69$	5.4	46.0	-2.7	6.0	52.7	-3.4	3.7	42.2	4.1
$c_c=100$	6.3	46.6	-3.8	7.1	50.7	-4.1	3.7	44.1	3.6
$c_l = 2$	5.3	47.9	-3.1	5.8	52.7	-4.0	3.0	44.1	3.4
$c_l = 10$	5.7	45.0	-2.8	6.5	50.9	-3.2	4.2	43.2	4.4
$c_l = 20$	5.5	42.4	-2.8	6.2	50.5	-2.9	4.5	43.7	4.8

In Table 8 the results are illustrated estimating τ_3 through assuming t_r to be 50% of the mean processing time referred to the considered order. The relative deviations in cost are for the FFD rule around 4% and for SPT and LPT around 6%. Negative values in the columns of 'Av τ ' imply, that the estimated interval is less than the optimal. Thus, if t_r is considered, the estimation of τ_{optv} is much better than before. Precisely, τ_{optv} is slightly overestimated for the FFD rule, but for SPT and LPT rule the time interval is now underestimated. It is superior to estimate the time interval a bit above τ_{optv} instead of a bit beneath, like Figure 6 shows. This is also in evidence if the PM intervals, which lead to the minimum relative deviation in average cost, are compared to those leading to a maximum relative deviation. In Figure 8 the relative deviations of τ_3 from τ_{optv} per order are plotted, which lead to the maximum deviation in cost (left side) and to the minimum deviation (right side). The intervals at the minimum are small positive numbers and at the maximum mostly small negative values. Because of this fact on

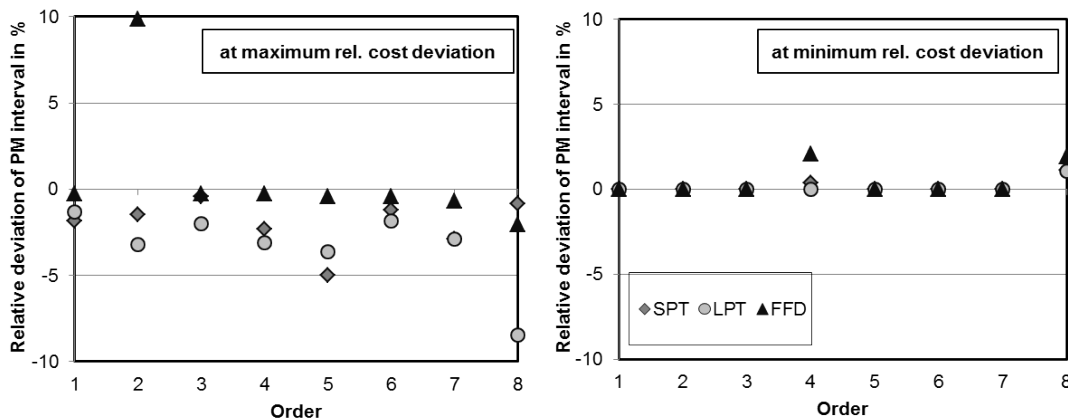


Figure 8: Relative deviation of τ_3 from τ_{opt} at maximum and minimum relative deviation of cost.

average the FFD rule creates the best results using τ_3 , although the distinctions in the deviations between the different scheduling rules are small. But, since the FFD rule creates smaller average cost (see Table 4), the estimation of τ_3 , integrated to the FFD rule, leads to less average cost than integrated into the other scheduling rules.

4.6 Impact of the Processing Times

After comparing the quality of the different estimations of τ , it is obvious, that the approach including t_r in the decision process works well. But, all evaluated orders have the same mean in processing times of about 50. Therefore, no assertion can be made if the intervals depend on the mean of processing times. To evaluate this, other orders are tested differing in the mean of processing times. The three orders are: Order50100: 50 jobs with mean processing time 100, Order12540: 125 jobs with mean processing time 40 and Order25020: 250 jobs with mean processing time 20. For a better comparison, the sum of processing times is still 5000. For the three new orders the results are presented with estimations of τ by decomposition method 2 and 3. In Table 9 the RPD_v of the different scheduling rules are given for the new orders if all other parameter values are enumerated. It can be seen that the relative differences in cost are getting smaller with decreasing mean processing time of the jobs. The FFD is still significantly the best scheduling rule if the mean of the order is 100 and 40. But, with a mean of 20, just the SPT rule is slightly worse than the other rules and never generates the best results. Thus, with smaller mean job length the minimal average cost generated through different scheduling rules become more similar and therewith the advantage of scheduling in FFD sequence decreases. Figure 9

Table 9: RPD_v of 50 jobs with $\bar{p} = 100$, 125 jobs with $\bar{p} = 40$ and 250 jobs with $\bar{p} = 20$.

	SPT			LPT			Random			FFD		
	Av	Max	Min	Av	Max	Min	Av	Max	Min	Av	Max	Min
Order50100	11.8	22.6	2.6	10.7	22.5	1.2	9.0	26.0	0.0	0.0	1.5	0.0
Order12540	3.2	8.0	0.7	2.3	9.4	0.0	3.0	10.0	0.0	0.3	1.5	0.0
Order25020	1.4	5.4	0.2	0.5	5.6	0.0	0.3	1.8	0.0	0.3	2.5	0.0

illustrates the relative deviations from the minimum average cost for the scheduling rules SPT, LPT and FFD, when t_r is set to zero and when $t_r = 0.5 \cdot \bar{p}$. If the mean of the jobs is 100, the average cost using FFD rule are on average 56% higher than the minimum cost integrating the optimal τ into the FFD rule and it never reaches the optimum. Since the jobs are longer, it becomes more important to create small and equal intervals between two PM activities to keep the repeating times low. The simulation indicated that the repeating times are higher if the

processing times of the jobs are longer (round about 30% higher if the mean of the processing times doubles). Thus, if t_r is not considered, the relative deviation of cost is larger for longer jobs. Integrating \bar{p} in the scheduling decision results in relative deviations from the minimum

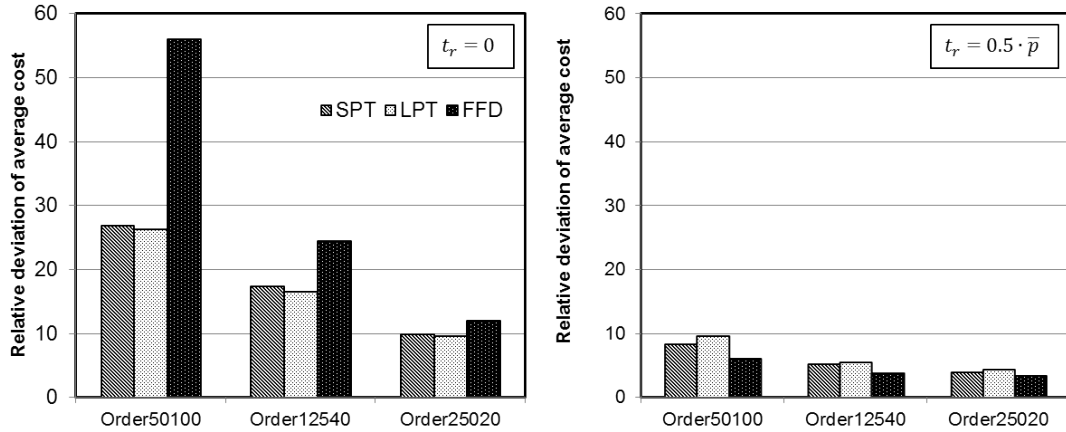


Figure 9: Average relative deviation from minimum average cost for the scheduling rules if $t_r = 0$ and if $t_r = 0.5 \cdot \bar{p}$ in %.

average cost of less than 10%. And if the mean of processing times decreases, the deviations are even smaller. If the repeating times are smaller, the average cost curves get flatter. This is the reason why it is easier to estimate the optimal interval and becomes nearly unimportant, which scheduling rule to take. Nevertheless, if the non-resumable case is considered as a whole, then it is recommended to schedule in FFD rule and using a problem adapted PM interval like τ_3 .

5 Conclusion

In this paper we studied the problem of scheduling jobs and PM activities if failures can occur stochastically and jobs are non-resumable. The aim was to find simple decision rules to minimise cost for the maintenance activities and for order lateness. Through simulation studies we showed that the optimal PM interval is dependent on the production schedule. It was also exposed, that the FFD rule, which is more responsive to the problem assumptions, creates better results than the common used SPT and LPT sequences. During the estimation of the PM interval it was shown that for the non-resumable case it is advantageous to adapt the

problem assumptions on the calculation of τ instead of using other existing methods, since the optimal cost are very sensitive to the estimation of τ if jobs are non-resumable. But even if the processing times of the jobs are very long, we were able to find good estimates of the optimal τ setting the amount of repeating times on the mean processing time of a job order.

References

- Adiri, I., Bruno, J., Frostig, E., and Rinnooy Kan, A. H. G. (1989). Single Machine Flow-Time Scheduling with a Single Breakdown. *ACTA Informatica*, (26):679–696.
- Barlow, R. and Hunter, L. (1960). Optimum Preventive Maintenance Policies. *Operations Research*, (8(1)):90–100.
- Beichelt, F. (1993). *Zuverlässigkeits- und Instandhaltungstheorie*. Teubner, Stuttgart.
- Bosch, K. and Jensen, U. (1983). Instandhaltungsmodelle -Eine Übersicht: Teil 1. *OR Spektrum*, (5):105–118.
- Brandolese, M., Franci, M., and Pozzetti, A. (1996). Production and Maintenance Integrated Planning. *International Journal of Production Research*, (34(7)):2059–2075.
- Cassady, C. R. and Kutanoglu, E. (2003). Minimizing Job Tardiness Using Integrated Preventive Maintenance Planning and Production Scheduling. *IIE Transactions*, (35):503–513.
- Cassady, C. R. and Kutanoglu, E. (2005). Integrating Preventive Maintenance Planning and Production Scheduling for a Single Machine. *IEEE Transactions on Reliability*, (54(2)):304–309.
- Chen, J. (2008). Scheduling of Nonresumable Jobs and Flexible Maintenance Activities on a Single Machine to Minimize Makespan. *European Journal of Operational Research*, (190):90–102.

- Coffman, E. G., Garey, M. R., and Johnson, D. S. (1997). Approximation Algorithms for Bin Packing: A Survey. *D.S. Hochbaum (Ed.), Approximation Algorithms for NP-hard Problems, PWS Publishing Company, Boston*, pages 46–93.
- Lee, C. Y. (1996). Machine Scheduling with an Availability Constraint. *Journal of Global Optimization*, (9):395–416.
- Li, R. H. and Yue, M. Y. (1997). The Proof of $FFD(L)=11/9 OPT(L)+7/9$. *Chinese Science Bulletin*, (42, No. 15):1262–1265.
- Ma, Y., Chu, C., and Zuo, C. (2010). A Survey of Scheduling with Deterministic Machine Availability Constraints. *Computers & Industrial Engineering*, (58):199–211.
- Pan, E., Liao, W., and Xi, L. (2010). Single-Machine-Based Production Scheduling Model Integrated Preventive Maintenance Planning. *International Journal of Advanced Manufacturing Technology*, (50):365–375.
- Sanlaville, E. and Schmidt, G. (1998). Machine Scheduling with Availability Constraints. *ACTA Informatica*, (35):795–811.
- Schmidt, G. (2000). Scheduling with Limited Machine Availability. *European Journal of Operational Research*, (121):1–15.
- Sortrakul, N., Nachtmann, H. L., and Cassady, C. R. (2005). Genetic Algorithms for Integrated Preventive Maintenance Planning and Production Scheduling for a Single Machine. *Computers in Industry*, (56):161–168.
- Xu, D., Yin, Y., and Li, H. (2009). A Note on "Scheduling of Nonresumable Jobs and Flexible Maintenance Activities on a Single Machine to Minimize Makespan". *European Journal of Operational Research*, (197):825–827.