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Aging and pension reform: Extending the retirement age and human capital formation

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Aging and Pension Reform: Extending the Retirement Age and Human Capital Formation

SAFE Working Paper No. 82

SAFE | Sustainable Architecture for Finance in Europe

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Non-Technical Summary

The world will experience major changes in its demographic structure in the next decades. In all countries, this process is driven by increasing life expectancy and falling birth rates. The fraction of the population in working-age will decrease and the fraction of people in old-age will increase. This process is already well under way in industrialized countries with many developing countries following suit in a few decades.

Standard economic analyses predict that these demographic processes will lead to a scarcity of the factor labor. This implies that there will be an upward pressure on wages. At the same time, because capital will be a relatively abundant factor, the rate of return to capital is projected to decrease. By decreasing the return on pension savings, this will lead to losses in welfare for cohorts who will be retired when the rate of return is low.

This paper takes the perspective of the three demographically oldest European nations – France, Germany and Italy – and asks how strongly three channels of adjustment to these ongoing developments and their interactions dampen such adverse welfare effects:

1. investing abroad
2. endogenous human capital formation, i.e., a more intense education
3. increasing the retirement age

Human capital formation in our context captures educational efforts in its broadest interpretation. It means both tertiary University education as well as on-the-job training, i.e., continuous learning over a worker's life-cycle.

Our quantitative finding is that such behavioral responses to the ongoing demographic change in combination with an increase in the retirement age have strong implications for economic aggregates and welfare, in particular in a globalized world. These adjustments substantially reduce the maximum welfare losses of demographic change for households alive in 2010. Our analysis shows that appropriate human capital policies in combination with pension policies are an important topic for future research and the policy agenda.

Aging and Pension Reform: Extending the Retirement Age and Human Capital Formation*

Edgar Vogel^{†‡} Alexander Ludwig[§] Axel Börsch-Supan[¶]

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Abstract

Projected demographic changes in industrialized and developing countries vary in extent and timing but will reduce the share of the population in working age everywhere. Conventional wisdom suggests that this will increase capital intensity with falling rates of return to capital and increasing wages. This decreases welfare for middle aged asset rich households. This paper takes the perspective of the three demographically oldest European nations—France, Germany and Italy—to address three important adjustment channels to dampen these detrimental effects of ageing in these countries: investing abroad, endogenous human capital formation and increasing the retirement age. Our quantitative finding is that endogenous human capital formation in combination with an increase in the retirement age has strong implications for economic aggregates and welfare, in particular in the open economy. These adjustments reduce the maximum welfare losses of demographic change for households alive in 2010 by about 2.2 percentage points in terms of a consumption equivalent variation.

JEL classification: C68, E17, E25, J11, J24

Keywords: population aging; human capital; welfare; pension reform; retirement age; open economy

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1 Introduction

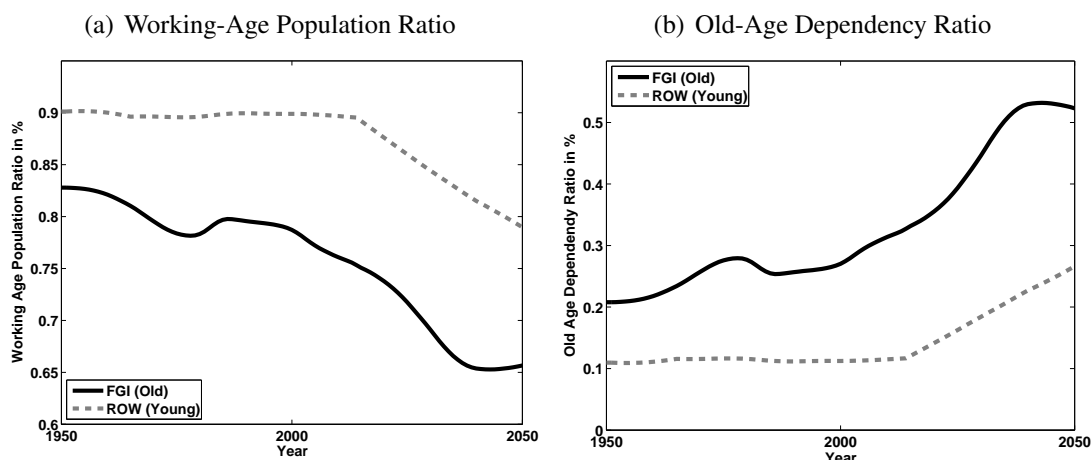
The world will experience major changes in its demographic structure in the next decades. In all countries, this process is driven by increasing life expectancy and falling birth rates. The fraction of the population in working-age will decrease and the fraction of people in old-age will increase. This process is already well under way in industrialized countries with many developing countries following suit in a few decades. Standard economic analyses predict that these demographic processes will increase the capital-labor ratio. Hence, rates of return to capital will decrease and wages increase, which has adverse welfare consequences for current cohorts who will be retired when the rate of return on assets is low.

The purpose of this paper is to ask how strongly three channels of adjustment to these ongoing developments and their interactions dampen such adverse welfare effects. We focus on France, Germany and Italy (FGI) as the three European countries that are most affected by the ongoing demographic change. First, compared to these countries, the rest of the world is relatively young. In autarky, rates of return to capital in FGI are therefore higher. From the perspective of industrialized countries such as FGI, globalization and investing capital abroad may therefore stabilize the return to capital. Second, as raw labor will become a relatively scarce factor and as life expectancy increases, strong incentives to invest in human capital emanate. This improves productivity. Such endogenous human capital adjustments may thereby substantially mitigate the effects of demographic change on macroeconomic aggregates and individual welfare. Third, while human capital adjustment increases the *quality* of the factor labor, a parametric pension reform through increasing the retirement age will increase the *quantity* of labor. By increasing the fraction of the population in the labor force, this will further increase per capita output. In addition to this direct effect, increasing the retirement age will also extend the worklife planning horizon of households, thus amplifying the incentives to accumulate human capital.

Point of departure of our analysis is the demographic evolution in two world regions, FGI and the rest of the world (ROW). The left panel of Figure 1 illustrates the impact of demographic change on the working-age population ratio—the ratio of the working-age population (of age 16 – 64) to the total adult population (of age 16 – 90)—and the right panel the old-age dependency ratio—the ratio of the old population of age 65 – 90 to the working-age population—in these regions. As the figure shows, the demographic structure is subject to significant changes over time in both regions. Currently, there are large level differences but overall demographic trends are very similar.

We feed these demographic data into an Auerbach and Kotlikoff (1987) style overlapping generations (OLG) model with two integrated world regions, endogenous labor supply decisions and endogenous human capital formation. Our model builds on Ludwig, Schelkle, and Vogel (2012) who focus at the US as a closed economy and ignore any adjustments of the retirement age. Our extensions of this earlier work allow us to compare different adjustments—which have been identified as important in the previous literature—within one coherent framework and to highlight their interactions. Despite these conceptual differences to earlier work, we also direct the view towards an analysis of FGI in an integrated world and not of the US in isolation. Importantly, FGI is small relative to the rest of the world. In contrast to standard small open economy models where constant interest

Figure 1: Old-Age Dependency Ratio and Working-Age Population Ratio



Notes: Data taken from United Nations (2007) and own projections. FGI: France, Germany and Italy, ROW: Rest of the world.

rates are assumed, we however discipline the calibration by considering FGI as embedded in the world economy. Hence, relative prices are determined by worldwide demographic processes.

As the central part of our analysis we work out the quantitative differences between a benchmark model—with open economies and endogenous human capital formation—and counterfactual models where countries operate as closed economies and where human capital may be exogenous. Along this line we emphasize the role of pension policy. We combine our pension reform of increasing the retirement age with two pension scenarios of a stylized pay-as-you-go (PAYG) pension system. In these scenarios either the contribution rate or the relative benefit level is held constant and—given a balanced budget and the demographic trends as displayed in Figure 1—benefits or contributions adjust.

Our main findings for our baseline scenario with constant contribution rates are as follows. In absence of endogenous human capital formation and without a fundamental pension reform, demographic change would lead to a substantial reduction of economic growth. Expressed relative to a constant trend growth scenario, this leads to an accumulated output reduction by about 5% until 2040 and a reduction of the real rate of return to capital by more than one percentage point. This implies that currently (as of year 2010) 50-year old households experience welfare losses. They would be willing to give up roughly 7% of their consumption each period of their remaining expected life to rather live in a world without the ongoing demographic change. However, this picture is too gloomy in light of potential adjustments through human capital formation and a pension reform. Taking these two effects into consideration, the overall output response is particularly large in the open economy scenario. There, GDP per capita in year 2040 is about 15 percent higher without and 20 percent higher with the pension reform, relative to a path with constant growth.¹

¹It is important to emphasize that we here refer to GDP per capita, not per efficient worker. The latter concept would take into account the endogenous human capital adjustment also in the denominator.

The decrease of the rate of return—which is the relevant aggregate statistic to evaluate the welfare consequences for middle aged asset rich households—is only 0.6 percentage points. Hence, maximum welfare losses reduce to 4%.

The reason for the reduction of output and rates of return (and the accompanying welfare losses) in the closed economy, exogenous human capital model without a pension reform is that demographic change leads to scarcity of raw labor. This increases the capital intensity, decreasing the rate of return and—given our maintained assumption of constant contribution rates—leads to increasing net wages. An important feedback is labor supply of households, which increases, but the effect is not strong enough to compensate for the reduction of raw labor. Hence, output and rates of return decrease in general equilibrium.

However, the adjustment of the quantity of labor through the endogenous increase of hours worked is only one feedback channel. Once human capital endogenously adjusts, then the simultaneous decrease of asset returns and increase of wages leads households to reduce their labor supply when young to invest in human capital. This increases the quality of the work force thereby also leading to a lower increase of capital per effective worker. The higher amount of human capital leads to output gains (relative to a constant growth scenario) along the transition and the rate of return decreases by less. This feedback is additionally amplified by the pension reform: it increases the raw amount of labor and triggers additional incentives to invest in human capital. The pension reform is particularly effective in the open economy. Because FGI is a relatively small region compared to the rest of the world, general equilibrium feedback is absent in the open economy which would otherwise dampen the effects of the reform.

In closed economies İmrohoroğlu et al. (1995), Fuster, İmrohoroğlu, and İmrohoroğlu (2007) Huang et al. (1997), and De Nardi et al. (1999) quantify the effects of social security adjustments on factor prices and welfare. In open economies, Domeij and Flodén (2006), Bösch-Supan et al. (2006), Fehr et al. (2005), Attanasio et al. (2007) and Krüger and Ludwig (2007), among others, investigate the role of international capital flows during the demographic transition. Storesletten (2000) examines the effect of migration to industrialized countries as a means to take pressure from social security systems. The effects of increased human capital accumulation is examined by Fougère and Mérette (1999), Sadahiro and Shimasawa (2002), Buysse et al. (2012), Ludwig, Schelkle, and Vogel (2012) and Heijdra and Reijnders (2012). This work uses some version of the seminal paper by Ben-Porath (1967)² and concludes that human capital adjustments may significantly mitigate the adverse consequences of demographic change.

While evidence of the effect of changes in the mandatory retirement age in the quantitative literature is scarce, there is a growing number of empirical papers estimating the effect of pension reforms on old-age labor supply and the actual retirement age. For instance, Mastrobuoni (2009), Hurd and Rohwedder (2011) and French and Jones (2012) document that the response of older workers to changes in retirement age legislation is large (extensive margin) whereby younger workers do not react much (intensive margin), just as we

²The model developed by Ben-Porath (1967) is the workhorse model to understand questions linked to any sort of human capital accumulation and wage growth over the life cycle (see Browning, Hansen, and Heckman (1999) for a review). Further, Heckman, Lochner, and Taber (1998), Guvenen and Kuruscu (2009), and Huggett, Ventura, and Yaron (2012) used the model to explain changes in income inequality.

find.³ While in this paper we ignore the link between human capital accumulation and endogenous growth in the long-run, there is a considerable number of contributions shedding light on this topic.⁴

The remainder of our analysis is organized as follows. In Section 2 we present the formal structure of our quantitative model. Section 3 describes the calibration strategy and our computational solution method. Our results are presented in Section 4. Finally, Section 5 concludes the paper. Detailed descriptions of computational methods and additional results are relegated to separate appendices.

2 The Model

We use a large scale multi-country OLG model in the spirit of Auerbach and Kotlikoff (1987) with endogenous labor supply, human capital formation and a standard consumption-saving decision. Our model extends Ludwig, Schelkle, and Vogel (2012) to an open economy setup and a flexible treatment of the retirement age. The population structure is exogenously determined by time and region specific demographic processes for fertility, mortality, and migration, the exogenous driving force of the model.⁵ The world population is divided into 2 regions, FGI and ROW.

2.1 Timing, Demographics and Notation

The model is cast in discrete time with time t being measured in calendar years. Each year, a new cohort enters the economy. Since agents are inactive before they enter the labor market, entering the economy refers to the first time agents make own decisions and is set to real life age of 16 (model age $j = 0$). In the benchmark scenario agents retire at an exogenously given age of 65 (model age $jr = 49$) and live at most until age 90 (model age $j = J = 74$). Both numbers are identical across regions. At a given point in time t , individuals of age j in country i survive to age $j + 1$ with probability $\varphi_{t,j,i}$, where $\varphi_{t,J,i} = 0$. The number of agents of age j at time t in country i is denoted by $N_{t,j,i}$ and $N_{t,i} = \sum_{j=0}^J N_{t,j,i}$ is total population in t, i . In the demographic projections migration happens at the age of 16. Thus, we implicitly assume that new migrants are born with the initial human capital endowment and human capital production function of natives. This assumption is consistent with Hanushek and Kimko (2000) who show that individual productivity (and thus human capital) of workers is mainly related to a country's level of schooling and not to cultural factors. This assumption on the age of migration also implies that we can treat newborns and immigrants alike, which is technically convenient.

³Similarly, Imrohoroglu and Kitao (2009) find in a calibrated life cycle model that privatizing the social security system has large effects on the reallocation over the life cycle but small effects on aggregate labor supply.

⁴See, e.g., de la Croix and Licandro (1999), Echevarría and Iza (2006), Ludwig, Schelkle, and Vogel (2007), Heijdra and Romp (2009) and Lee and Mason (2010).

⁵Although changes in prices may have—via numerous mechanisms—feedback effects on life expectancy, fertility, and migration we abstract from examining these channels. See Liao (2011) for a decomposition of economic growth into effects caused by demographics (endogenous fertility) and technological progress.

2.2 Households

Households are populated by one representative agent deciding about consumption, saving, labor supply, and time investment into human capital formation. The remaining time is consumed as leisure. A household in region i maximizes lifetime utility at the beginning of economic life ($j = 0$) in period t ,

$$\max \sum_{j=0}^J \beta^j \pi_{t,j,i} \frac{1}{1-\sigma} \{c_{t+j,j,i}^\phi (1 - \ell_{t+j,j,i} - e_{t+j,j,i})^{1-\phi}\}^{1-\sigma}, \quad \sigma > 0, \quad (1)$$

where the per period utility function takes consumption c , working hours ℓ and time spent on increasing the stock of human capital e , as inputs. Standardizing the time endowment to unity leaves $1 - \ell - e$ as leisure time. ϕ is the consumption elasticity in utility, β is the raw time discount factor, and σ is the inverse of the inter-temporal elasticity of substitution with respect to the consumption-leisure aggregate. $\pi_{t,j,i}$ denotes the unconditional probability to survive until age j , $\pi_{t,j,i} = \prod_{k=0}^{j-1} \varphi_{t+k,k,i}$, for $j > 0$ and $\pi_{t,0,i} = 1$.

Agents earn labor income (pensions if retired), interest payments on their physical assets, and receive accidental bequests. Social security contributions are a share $\tau_{t,i}$ of their gross wages. Net wage income in period t of an agent of age j living in region i is given by $w_{t,j,i}^n = \ell_{t,j,i} h_{t,j,i} w_{t,i} (1 - \tau_{t,i})$, where $w_{t,i}$ is the (gross) wage per unit of supplied human capital at time t in region i . Annuity markets are missing and accidental bequests are distributed by the government as lump-sum payments to households. The household's dynamic budget constraint is given by

$$a_{t+1,j+1,i} = \begin{cases} (a_{t,j,i} + tr_{t,i})(1 + r_{t,i}) + w_{t,j,i}^n - c_{t,j,i} & \text{if } j < jr \\ (a_{t,j,i} + tr_{t,i})(1 + r_{t,i}) + p_{t,j,i} - c_{t,j,i} & \text{if } j \geq jr, \end{cases} \quad (2)$$

where $a_{t,j,i}$ denotes assets, $p_{t,j,i}$ is pension income, $tr_{t,i}$ are transfers from accidental bequests, and $r_{t,i}$ is the real interest rate, the rate of return to physical capital. Households start their economic life with zero assets ($a_{t,0,i} = 0$) and do not intend to leave bequests to the next generation ($a_{t,J+1,i} = 0$).

2.3 Formation of Human Capital

The initial level of human capital $h_{t,0,i} = h_0$ is exogenously given, identical across households of a birth cohort and cohort invariant. Then, at any point in time agents can spend a fraction of their time to build human capital. We employ a frequently used twist of the Ben-Porath (1967) human capital technology given by

$$h_{t+1,j+1,i} = h_{t,j,i} (1 - \delta_i^h) + \xi_i (h_{t,j,i} e_{t,j,i})^{\psi_i} \quad \psi_i \in (0, 1), \quad \xi_i > 0, \quad \delta_i^h \geq 0, \quad (3)$$

where ξ_i is a scaling factor, ψ_i determines the curvature of the human capital technology and δ_i^h is the depreciation rate of human capital. Parameters of the production function vary across regions to allow for region-specific human capital profiles during our calibra-

tion period.⁶ Since we do not model any other labor market frictions⁷ or costs of human capital acquisition this is the only way to replicate observed differences in age-wage profiles. However, we adjust parameters such that they are eventually identical in both regions and thus agents will have—everything else equal—the same life cycle human capital profile in the final steady state (see Section 3.3).

Investment into human capital requires only the input of time. Opportunity costs of human capital accumulation are not only forgone wages but also the utility loss due to less leisure. As we do not model formal education and on-the-job-experience (learning-by-doing) separately, the accumulation of human capital is a mixture of formal and informal training programs. Human capital can be accumulated at all stages of the life-cycle but optimal behavior implies that agents will spend more time on building human capital early in life and factually stop investing some years before retirement.

2.4 Firms

There is a large number of firms in a perfectly competitive environment producing a homogenous good (which can be consumed or invested) using the Cobb-Douglas technology

$$Y_{t,i} = K_{t,i}^\alpha (A_{t,i} L_{t,i})^{1-\alpha}. \quad (4)$$

Here, α denotes the share of capital used in production. $K_{t,i}$, $L_{t,i}$ and $A_{t,i}$ are region-specific stocks of physical capital, effective labor and the level of technology, respectively. Labor inputs and human capital of different agents (of different age) are perfect substitutes.⁸ Aggregate effective labor input $L_{t,i}$ is given by $L_{t,i} = \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i}$. Factors of production are paid their marginal products, i.e.,

$$w_{t,i} = (1 - \alpha) A_{t,i} k_{t,i}^\alpha \quad (5a)$$

$$r_{t,i} = \alpha k_{t,i}^{\alpha-1} - \delta, \quad (5b)$$

where $k_{t,i} \equiv \frac{K_{t,i}}{A_{t,i} L_{t,i}}$ is the capital intensity, i.e., the capital stock per efficient unit of labor, $w_{t,i}$ is the gross wage per unit of efficient labor, $r_{t,i}$ is the interest rate and δ denotes the (constant) depreciation rate of physical capital. Total factor productivity, $A_{t,i}$, is growing at the region-specific exogenous rate $g_{t,i}^A$: $A_{t+1,i} = A_{t,i}(1 + g_{t,i}^A)$.

2.5 Capital Markets

We assume that both regions are initially closed economies where we solve for the equilibrium transition path of both economies with agents using only prices and transfers from the closed economy scenario. Then, we repeat the same exercise but assume that both economies are open. We follow Buiter and Kletzer (1995) and assume that physical capital

⁶See Browning, Hansen, and Heckman (1999) for a summary of the literature and an overview over empirical estimates of the parameters.

⁷de la Croix, Pierrard, and Sneessens (2013) emphasize the role of labor market frictions in the context of demographic change.

⁸See Prskawetz and Fent (2007) for a model with imperfect substitutability of different worker types

is perfectly mobile whereas human capital (labor) is immobile. This implies that, in the world capital market equilibrium, the interest rate is the same across regions. Given the choice of our country aggregates, the weight of FGI decreases over time. Hence, the open economy market clearing interest rate will be largely dominated by demographic developments in the other part of the world.

2.6 The Pension System

The pension system is a simple pay-as-you-go system that replicates key mechanics of many real-world public pension schemes. The system is balanced in every period by adjusting the contribution rate or the replacement rate. Workers contribute a fraction $\tau_{t,i}$ of their gross wages, and pensioners receive a fraction $\rho_{t,i}$ of their average indexed yearly earnings over their employment history.⁹ The level of pensions for each period t in region i of an agent of age j is given by $p_{t,j,i} = \rho_{t,i} w_{t+jr-j,i} \bar{h}_{t+jr-j,i} \frac{s_{t,j,i}}{jr-1}$, where $w_{t+jr-j,i} \bar{h}_{t+jr-j,i} \frac{s_{t,j,i}}{jr-1}$ are average indexed yearly earnings over the working life (AIYE)¹⁰, $w_{t+jr-j,i} \bar{h}_{t+jr-j,i}$ are average earnings of all workers in period t when a retiree of age j reaches retirement age jr . Further, $\bar{h}_{t,i}$, the region specific average effective human capital of a worker, is defined as

$$\bar{h}_{t,i} = \frac{\sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i}}{\sum_{j=0}^{jr-1} N_{t,j,i}}. \quad (6)$$

Past individual earnings of an agent relative to average economy-wide earnings in the respective year is given by $s_{t,j,i} = \sum_{i=0}^j \frac{\ell_{t-j+i,i} h_{t-j+i,i}}{\bar{h}_{t-j+i}}$ which links contributions and pensions. Lastly, the budget constraint of the system is given by

$$\tau_{t,i} w_{t,i} \sum_{j=1}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i} = \rho_{t,i} \sum_{j=jr}^J N_{t,j,i} w_{t+jr-j,i} \bar{h}_{t+jr-j,i} \frac{s_{t,j,i}}{jr-1} \quad \forall t, i. \quad (7)$$

where we have substituted $p_{t,j,i}$ into the equation.

We consider two policy scenarios. In our first scenario we keep the retirement age at the baseline level (65 years) and hold the contribution rate constant $\tau_{t,i} = \bar{\tau}_i$ (labeled “const. τ ”). We endogenously adjust the replacement rate to balance the budget of the pension system. The alternative adjustment with constant replacement rates is briefly discussed in Subsection 4.4.

As the second dimension of pension reforms we increase the normal retirement age in region FGI. This reform scenario captures two effects on incentives to acquire human capital: a lengthening of the working life combined with—everything else equal—lowering

⁹Most pension systems apply some non-linear transformation of “earning points” into pensions to foster intra-generational redistribution (Whitehouse 2003). For instance, the U.S. system applies an additional bend-point formula to pensions (Diamond and Gruber 1999). However, as in our model there is only one representative agent, we do not model this feature.

¹⁰Some pension systems do not take the full employment history but a limited number of the “best” years into account. We ignore this issue for computational reasons.

the tax burden on currently working individuals. It mimics elements of the actual reform debate in FGI.¹¹

2.7 Equilibrium

Denoting current period/age variables by x and next period/age variables by x' , a household of age j solves in region i , at the beginning of period t , the maximization problem

$$V(a, h, s, t, j, i) = \max_{c, \ell, e, a', h', s'} \{u(c, 1 - \ell - e) + \varphi_i \beta V(a', h', s', t + 1, j + 1, i)\} \quad (8)$$

subject to $w_{t,j,i}^n = \ell_{t,j,i} h_{t,j,i} w_{t,i} (1 - \tau_{t,i})$, (2), (3) and the constraint $e \in [0, 1 - \ell]$ and $\ell \in [0, 1]$.

Definition 1. *Given the exogenous population distribution and survival rates in all periods $\{\{N_{t,j,i}, \varphi_{t,j,i}\}_{j=0}^J\}_{t=0}^T\}_{i=1}^I$, an initial physical capital stock and an initial level of average human capital $\{K_{0,i}, \bar{h}_0\}_{i=1}^I$, and an initial distribution of assets and human capital $\{a_{t,0,i}, h_{t,0,i}\}_{j=0}^J\}_{i=1}^I$, a competitive equilibrium is sequences of individual variables $\{\{c_{t,j,i}, e_{t,j,i}, a_{t+1,j+1,i}, h_{t+1,j+1,i}, s_{t+1,j+1,i}\}_{j=0}^J\}_{t=0}^T\}_{i=1}^I$, sequences of aggregate variables $\{\{L_{t,i}, K_{t+1,i}, Y_{t,i}\}_{t=0}^T\}_{i=1}^I$, government policies $\{\{\rho_{t,i}, \tau_{t,i}\}_{t=0}^T\}_{i=1}^I$, prices $\{\{w_{t,i}, r_t\}_{t=0}^T\}_{i=1}^I$, and transfers $\{\{tr_{t,i}\}_{t=0}^T\}_{i=1}^I$ such that*

1. *given prices, bequests and initial conditions, households solve their maximization problem as described above,*
2. *interest rates and wages are paid their marginal products, i.e. $w_{t,i} = (1 - \alpha) \frac{Y_{t,i}}{L_{t,i}}$ and $r_{t,i} = \alpha \frac{Y_{t,i}}{K_{t,i}} - \delta$,*
3. *per capita transfers are determined by*

$$tr_{t,i} = \frac{\sum_{j=0}^J a_{t,j,i} (1 - \varphi_{t-1,j-1,i}) N_{t-1,j-1,i}}{\sum_{j=0}^J N_{t,j,i}}, \quad (9)$$

4. *government policies are such that the budget of the social security system is balanced in every period and region, i.e. equation (7) holds $\forall t, i$, and household pension income is given by $p_{t,j,i} = \rho_{t,i} w_{t+jr-j,i} \bar{h}_{t+jr-j,i} \frac{s_{t,j,i}}{jr-1}$,*

¹¹Germany raised the retirement age to 67 years for cohorts born after 1964. In Italy, the retirement age will be raised to 68 years by 2050. In France, the minimum retirement age is unchanged but the number of contribution years for a full pension will increase by about 2 years until 2020.

5. all regional labor markets clear and allocations are feasible in all periods:

$$L_{t,i} = \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i} \quad (10a)$$

$$Y_{t,i} = \sum_{j=0}^J c_{t,j,i} N_{t,j,i} + K_{t+1,i} - (1 - \delta) K_{t,i} + F_{t+1,i} - (1 + r_t) F_{t,i}, \quad (10b)$$

$$Y_t = \sum_{i=1}^I Y_{t,i} \quad (10c)$$

$$K_{t+1} = \sum_{i=1}^I \sum_{j=0}^J a_{t+1,j+1,i} N_{t,j,i} \quad (10d)$$

6. and the world capital market clears at the world interest rate $r_t = r_{t,i}, \forall i$, hence the sum of foreign assets $F_{t,i}$ across all regions is zero

$$\sum_{i=1}^I F_{t,i} = 0 \quad \Leftrightarrow \quad r_{t,i} = r_t \forall i \quad \Leftrightarrow \quad k_{t,i} = k_t, \frac{K_{t,i}}{Y_{t,i}} = \frac{K_t}{Y_t} \forall i. \quad (11)$$

While our assumption of frictionless international capital markets implies that capital intensities, $k_{t,i}$, adjust such that the rate of return is equalized across regions, human capital is immobile by assumption. Hence, wages differ across regions and are a function of the country specific productivity $A_{t,i}$.

Definition 2. A stationary equilibrium is a competitive equilibrium in which per capita variables grow at constant rate $1 + \bar{g}^A$ and aggregate variables grow at constant rate $(1 + \bar{g}^A)(1 + n)$.

2.8 Thought Experiments

The exogenous driving force of our model is the time-varying and region specific demographic structure. The solution of our model is done in two steps. We first assume that both regions are closed and solve for the region specific artificial initial steady state. We then compute the closed economy equilibrium transition paths to the new steady state. While computing the transition paths, we include sufficiently many “phase-in” and “phase-out” periods¹² to ensure convergence. We recompute the equilibrium transition path assuming open capital markets. To display the effects of our pension reform on macroeconomic variables we report simulation results for the main projection period of interest, from 2000 to 2050. To capture the welfare effects of the pension reform, changes in welfare are reported for agents alive in 2010. We use data from 1960 – 2005 in order to calibrate the vector of structural model parameters (cf. Section 3).

¹²In total we use 750 periods in the simulation with 250 phase-in periods (before changes in the population) and 300 phase-out periods (after the population structure has settled to its steady-state level). However, changes in variables which are constant in steady state are numerically irrelevant already around 100 periods before the we impose the steady state restriction.

Our baseline model variant (which is also used in calibration) is one with endogenous human capital for the baseline retirement age. We use the results from this model variant to compute average time investment and the associated human capital profile. We use this as input in the alternative model with exogenous human capital. Specifically, we obtain the life-cycle profile of time investment into education $\bar{e}_{j,i}$ for each age $j = 1, 2, \dots, J$ by averaging over all life-cycle profiles of agents living during the calibration period.¹³ The corresponding human capital profile is computed by using the time series $\bar{e}_{j,i}$ in (3).

3 Calibration and Computation

The calibration of the model is standard. We choose parameters such that simulated moments match their counterparts in the data. For the wage profile, we choose parameters such that the endogenous wage profiles match the empirically observed wage profile during the calibration period 1960 – 2005 (cf. Section 3.3).¹⁴ We provide a condensed overview over all parameters in Table 1.

3.1 Demographics

Population data from 1950 – 2005 are taken from the United Nations (2007). For the period until 2050 we use the same data source and choose the UN’s “medium” variant for the fertility projections. However, we have to forecast population dynamics beyond 2050 to solve our model. The key assumptions of our projection are as follows: First, for both regions the total fertility rate is constant at 2050 levels until 2100. Then we adjust fertility such that the number of newborns is constant for the rest of the simulation period. Second, we use the life expectancy forecasted by the United Nations (2007) and extrapolate it until 2100 at the same (region and gender-specific) linear rate.¹⁵ Then we assume that life expectancy in FGI stays constant. Life expectancy in ROW keeps rising until it reaches the level in FGI by the year 2300. These assumptions imply that a stationary population structure is reached in about 2200 in the old nations and in 2300 in the rest of the world. Our assumptions ensure that a steady state is reached eventually also in the economic model which is necessary to close the dynamics of the system. At the same time, we choose these somewhat artificial and technically convenient adjustments to take place in the distant future so that their impact on our window of interest—years 2000 to 2050—is negligible.

¹³Formally, we compute $\bar{e}_{j,i} = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} e_{t,j,i}$.

¹⁴We do the moment matching exercise in the model variant with endogenous human capital and constant contribution rate scenario with the benchmark retirement age. We do not re-calibrate model parameters across social security scenarios or for the alternative human capital model, mainly because any parametric change would make comparisons (especially welfare analysis) across models impossible.

¹⁵Life expectancy estimated by the UN for cohort born in 2050 is in the industrialized nations 81.5 year for men and 86.8 year for women. In the rest of the world, life expectancy is 71.7 for men and 75.7 for women. The estimates of the trend are as follows: in the industrialized countries life expectancy at birth increases for each cohort at a linear rate of 0.12 years for men and 0.117 years for women. For the rest of the world the slope coefficient for is 0.204 for men and 0.217 for women. See also Oeppen and Vaupel (2002) for the evolution of life expectancy.

3.2 Households

We set σ to 2. This corresponds to a standard estimate of the IES of 0.5 (Hall 1988). The pure time discount factor β is chosen to match a capital-output ratio of 2.9 in FGI which requires $\beta = 0.99$. To calibrate the weight of consumption in the utility function, we set $\phi = 0.37$ by targeting an average labor supply of 1/3 of the total available time. We constrain the parameters of the utility function to be identical across regions.

3.3 Individual Productivity and Labor Supply

We follow Ludwig, Schelkle, and Vogel (2012) and choose the parameters of the human capital production function such that average wage profiles resulting from endogenous human capital model replicate empirically observed wage profiles. As in Börsch-Supan, Härtl, and Ludwig (2014) we assume that an estimated age productivity profile for the U.S. is a good proxy for the age productivity in region FGI. Our estimates are based on PSID data, adopting the procedure of Huggett et al. (2012). Accordingly, life-cycle efficiency peaks at around age 50 when they are about 60% higher than at labor market entry. This is in line with estimates by Fitzenberger et al. (2001) for Germany. After normalizing the initial value of human capital to $h_0 = 1$ we determine the value of the structural parameters $\{\xi_i, \psi_i, \delta_i^h\}_{i=1}^I$ using indirect inference methods (Smith 1993; Gourieroux et al. 1993). To do this, we run regressions on the wage profiles obtained from the simulation and the observed data on a 3rd-order polynomial in age defined as

$$\log w_{j,i} = \lambda_{0,i} + \lambda_{1,i}j + \lambda_{2,i}j^2 + \lambda_{3,i}j^3 + \varepsilon_{j,i}. \quad (12)$$

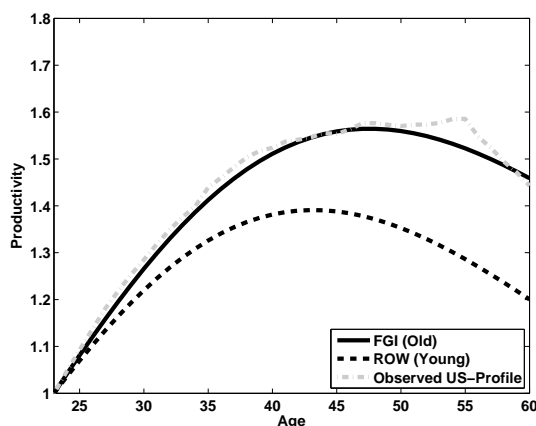
where $w_{j,i}$ denotes age specific productivity. We write the coefficient vector from the regression on the observed wage data as $\lambda_i^d = [\lambda_{1,i}, \lambda_{2,i}, \lambda_{3,i}]'$ and the one from the simulated human capital profile of cohorts born in 1960 – 2005 by $\hat{\lambda}^s = [\hat{\lambda}_{1,i}, \hat{\lambda}_{2,i}, \hat{\lambda}_{3,i}]'$. The vector $\hat{\lambda}^s$ is then a function of the deep structural parameters $\{\xi_i, \psi_i, \delta_i^h\}_{i=1}^I$. We choose the values for the structural parameters by minimizing the distance between the values of the polynomial obtained from the regression on the actual data and the simulated data, i.e., by minimizing $\|\lambda_i^d - \hat{\lambda}_i^s\| \forall i$, see Subsection 3.6 for computational details.

As the demographically younger region ROW is a mix of developing and developed countries, we cannot use the profile from FGI. Instead, we take the polynomial estimated on the U.S.-profile and scale coefficient λ_1 by a factor of 0.95. The resulting age-wage profile corresponds to a profile estimated on Mexican data by Attanasio, Kitao, and Violante (2007). This choice is motivated by the fact that GDP per capita of Mexico is very close to the global (weighted) average of region ROW. The main difference between the two profiles is that wages in the U.S. drop by 10% and Mexican wages by 20% from their peak to retirement age and that the maximal wage in the U.S. is about 100% higher than the wage at entry into the labor market. The same number in Mexico is about 90%. Attanasio et al. (2007) attribute these differences—US profiles are steeper and drop less towards the end of working life—to differences in the physical requirements in the two economies. The flatter profile probably reflects less human capital intensive and more physically demanding tasks of the “representative” worker. Further supportive evidence on flatter profiles is provided by

Lagakos, Moll, Porzio, and Qian (2012). Using a panel with 48 developing and developed countries, they find that age-experience profiles are much steeper in developed countries.

Figure 2 presents the empirically observed productivity profile and the estimated polynomials for the different regions. The coefficients¹⁶ and the shape of the wage profile are in line with the literature, see, e.g., Altig et al. (2001) and Hansen (1993). The value of $\psi \approx 0.60$ is also in the middle of the range reported in Browning, Hansen, and Heckman (1999). The depreciation rate of human capital is $\delta^h = 1.4\%$ for ROW and $\delta^h = 0.9\%$ for FGI. Although there is a considerable disagreement about δ^h in the literature, our numbers are in a reasonable range, see, e.g., Arrazola and de Hevia (2004), and Browning, Hansen, and Heckman (1999).

Figure 2: Wage Profiles



Notes: Data standardized by the wage at the age 23. Source: PSID, own calculations.

We adjust the parameters of the human capital production function such that they are eventually identical in both regions. To this end we parameterize the adjustment path and calibrate it such that parameters start to change for the cohort born in year 2100 and are identical for the cohort born in year 2300. We denote the vector of parameters $\{\xi_i, \psi_i, \delta_i^h\} = \vec{\chi}_i$ and assume that

$$\vec{\chi}_{i,k} = \vec{\chi}_{i \neq j,k} + \Delta(\chi_{j,k}) \cdot t \quad k = 1, 2, 3, \quad (13)$$

for the adjustment process where $\Delta(\chi_{j,k})$ denotes the per period linear adjustment of the parameter, t is the length of the adjustment period, and k is an element from χ_i .

3.4 Production

The production elasticity of capital is set to $\alpha = 0.33$ such that we match the share of capital income in national accounts. The average growth rate of total factor productivity, \bar{g}_i^A , is calibrated such that we match the region-specific growth rate of GDP per capita, taken

¹⁶The coefficient estimates from the regression on the US profiles are λ_0 : -1.6262, λ_1 : 0.1054, λ_2 : -0.0017 and λ_3 : 7.83e-06. The coefficients for ROW are identical except for λ_1 which is scaled by 0.95.

from Maddison (2003). Growth of output per capita in FGI during our calibration period is 2.8%. Accordingly, we set the growth rate of TFP to 1.85% to meet our calibration target. To match the observed growth of GDP per capita of 2.2% in ROW, we let TFP grow at a rate of 1.5%. From 2100 onwards we let the growth rate of TFP in ROW adjust smoothly to the growth rate in FGI. This adjustment process is assumed to be completed in 2300. Further, we compute relative GDP per capita from Maddison (2003) for both regions in 1950 and use this ratio to calibrate the relative productivity levels at the beginning of the calibration period. Initially, per capita GDP in ROW is only 20% of income per capita in the old nations. Finally, we calibrate δ such that our simulated data match an average investment output ratio of 20% in FGI which requires $\delta = 0.035$.

3.5 The Pension System

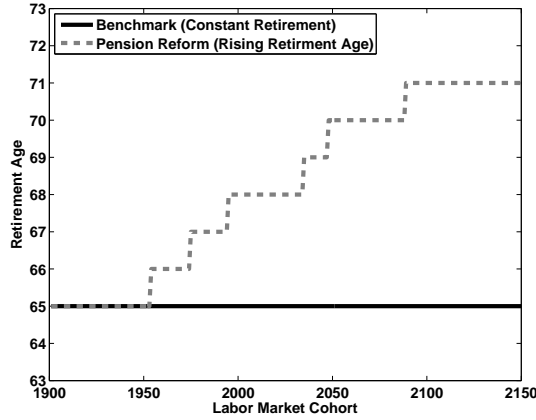
In our first social security scenario (“const. τ ”) we fix contribution rates and adjust replacement rates of the pension system. Since there are no yearly data on contribution rates for sufficiently many countries, we use data from Palacios and Pallarés-Miralles (2000) for the mid 1990s and assume that the contribution rate was constant through the entire calibration period. On the individual country level, we use the pension tax as a share of total labor costs weighted by the share of contributing workers to compute a national average. Then we weight these numbers by total GDP to compute a representative number for the two world regions. The contribution rate in the young (old) region is then 4.1% (10.9%). Given the initial demographic structure, the replacement rate is 13.8% (16.4%) in the young (old) region. In our baseline social security scenario we freeze the country specific contribution rate at the level used for the calibration period for all following years. We also assume that the retirement age is fixed at 65 years and agents do not expect any change. We label this scenario as “Benchmark” (“BM”) in the following analysis.

For the second type of policy reform we increase the retirement age by linking it to remaining life expectancy at age 65 (the current retirement age). We assume that for an increase in conditional life expectancy by 1.5 years, retirement increases by one year. We model this change—labeled “Pension Reform” (“PR”)—by assuming that this reform affects already workers in the labor market in 1955 (birth cohort 1939) by raising their retirement age immediately by one year and thereby effectively increasing the number of workers already in 2001. We then apply this rule for all following cohorts. This pattern mimics recent pension reforms in many old countries, e.g., recent pension reforms in Germany. The reform has direct effects via lengthening expected lifetime labor supply of workers and changing prices for retirees. Given our projections of life expectancy, the retirement age will eventually settle down at 71 years, a value also discussed in the public debate about pension reforms. We show the stepwise increase in the retirement age in Figure 3 as a function of the respective labor market cohort.

3.6 Computational Method

For a given set of structural model parameters, we solve the model by iterating on household related variables (inner loop) and aggregate variables (outer loop). In the outer loop,

Figure 3: Retirement Age



Notes: The jumps in the broken line indicate the labor market cohort which is affected by the change in the retirement age and not the actual time when the number of workers is increasing.

we solve for the equilibrium by making an initial guess about the time path of the following variables: the capital intensity, the ratio of bequests to wages, the replacement rate (or contribution rate) of the pension system and the average human capital stock for all periods from $t = 0, 1, \dots, T$. For the open economy we impose the restriction of identical capital intensity for both regions but require all other variables from above to converge for each country separately. On the household level (inner loop), we start by guessing $\{c_J, h_J\}$, i.e. the terminal values for consumption and human capital. Then we iterate on them until convergence of the inner loop as defined by some metric. In each outer loop, household variables are aggregated in each iteration for all periods. Values for the aggregate time series are then updated using the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007) until convergence.

To calibrate the model (we do this in the “const. τ ” scenario, benchmark retirement), we run additional “outer outer” loops on the vector of structural model parameters in order to minimize the distance between moments computed from the simulated data and their corresponding calibration targets for the calibration period 1960 – 2005. In a nutshell, the common parameter values determined in this procedure are β , ϕ , δ , and the country specific parameters of the human capital production function are $\{\xi_i, \psi_i, \delta_i^h\}$.

4 Results

Our prime focus is to work out the effects of a parametric pension reform—the increase of the retirement age as described above—on macroeconomic aggregates and on welfare of households living through the demographic transition. We divide the presentation of results into three parts. Subsection 4.1 looks at the evolution of macroeconomic aggregates such as the replacement rate to the pension system, the rate of return to capital, aggregate labor supply and detrended GDP per capita in closed and open economies with exogenous or endogenous human capital formation. As this analysis shows, a key factor is the endogenous reaction of labor supply and human capital formation. To shed more light on

Table 1: Model Parameters

			Young	Old
Preferences	σ	Inverse of Inter-Temporal Elasticity of Substitution	2.00	
	β	Pure Time Discount Factor	0.990	
	ϕ	Weight of Consumption	0.369	
Human Capital	ξ	Scaling Factor	0.169	0.154
	ψ	Curvature Parameter	0.604	0.623
	δ^h	Depreciation Rate of Human Capital	1.4%	0.7%
	h_0	Initial Human Capital Endowment	1.00	1.00
Production	α	Share of Physical Capital in Production	0.33	
	δ	Depreciation Rate of Physical Capital	4.7%	
	g^A	Exogenous Growth Rate		
		Calibration Period	1.5%	1.9%
		Final Steady State	1.9%	1.9%

Notes: “Young” and “Old” refer to the region. Only one value in a column indicates that the parameter is identical for both regions.

the interplay between these margins, we ask in Subsection 4.2 how much of the exogenous increase in the retirement age (extensive margin) is potentially offset by adverse endogenous labor supply reactions at the intensive margin. Finally, Subsection 4.3, evaluates the welfare consequences for households who live through the demographic transition.

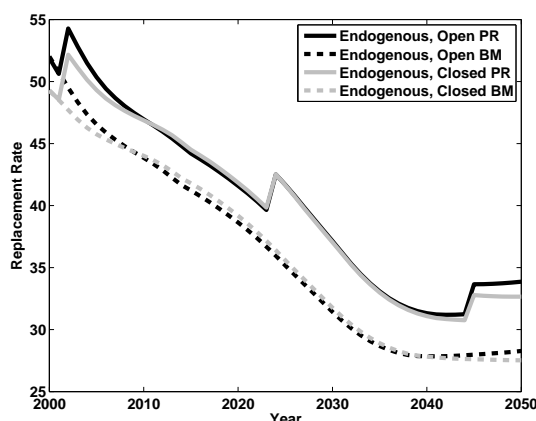
Throughout, we focus the analysis on a pension scenario with constant contribution rates. Once these effects are understood, the consequences under the polar case of constant replacement rates immediately follow. We briefly discuss those in Subsection 4.4, relegating all details to the Supplementary Appendix.

4.1 Macroeconomic Aggregates

Figure 4 shows the evolution of the replacement rate for the benchmark pension system (“BM”) and the pension reform (“PR”). Keeping the contribution rate unchanged during the entire period at 11.3% requires a drop in the replacement rate to about 28% until 2050. This decline in the replacement rate is substantially dampened by the pension reform (“PR”). Because of the constant contribution rate, the distortions for labor supply and human capital investment decisions are similar across human capital model variants. Hence, differences between endogenous and exogenous human capital formation are small.

Figure 5 shows the evolution of the rate of return as an index. The index is constructed such that we normalize the closed economy rate of return to zero in 2000 in both the exogenous (left panel) and the endogenous human capital (right panel) model. Thereby the change of the rate of return caused by the ongoing demographic change in the two respective model variants can be immediately read off. At the same time we normalize the open economy rate of return such that the return differential between the open and the closed

Figure 4: Adjustment of the Replacement Rate

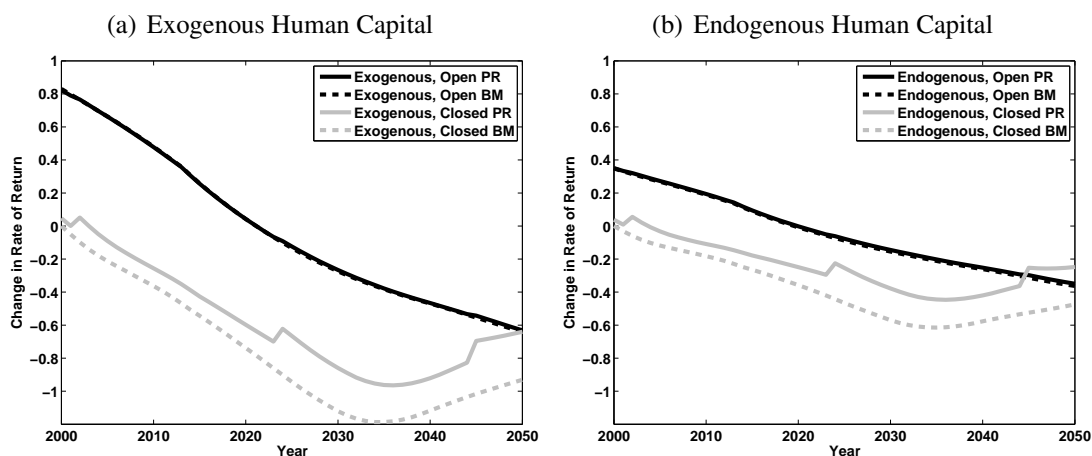


Notes: “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “Exogenous” (left panel) and “endogenous” (right panel) refer to the respective human capital model. “PR” and “BM” denote the pension reform and benchmark retirement scenario. All results obtained with constant contribution rate, τ .

economy in year 2000 is made explicit. In both panels we observe the well established finding of falling returns due to demographic change. While the level of the rate of return in the year 2000 is higher in the open economy scenario, it decreases more strongly. The reason is that countries outside FGI are currently younger in terms of levels of the key demographic indicators, but age more rapidly after about year 2015, cf. Figure 1. Furthermore, the overall decrease of the rate of return is quite substantially dampened if human capital adjusts endogenously. Relative scarcity of raw labor and abundance of physical capital and the associated decrease of the rate of return and increase of wages, trigger human capital investments in the endogenous human capital model, also see Figure 6 below. As a consequence of this feedback, the capital intensity increases by less and the rate of return correspondingly falls by less. Finally, the pension reform increases the supply of raw labor thereby leading to a lower reduction of the rate of return in the closed economy. In the open economy, however, this effect dissipates because region FGI, in which the reform takes place, is small relative to the rest of the world. Therefore the return is virtually unaltered between the BM and the PR scenarios.

Figure 6 shows effective labor supply—i.e., hours worked times human capital weighted with the respective population shares and summed across all ages—relative to the total number of workers—i.e., the population aged 16 to 64—again for the exogenous human capital model in the left and the endogenous human capital model in the right panel. We normalize the simulated time series of relative effective labor supply to one in year 2000. In case of exogenous human capital, labor supply per worker increases by slightly more than 10% until 2050 in the most optimistic scenario, i.e., the scenario PR in the open economy. In case of endogenous human capital adjustments, however, this increase is far more pronounced with total effective labor supply per worker increasing by 40 percent until year 2050. As explained above, this is a consequence of the strong incentives to invest into human capital, triggered by relative price movements in aging economies. In the PR model

Figure 5: Rate of Return [Index]



Notes: “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “Exogenous” (left panel) and “endogenous” (right panel) refer to the respective human capital model. “PR” and “BM” denote the pension reform and benchmark retirement scenario. All results obtained with constant contribution rate, τ .

this is further amplified by the extension of the working life and the associated incentives to increase human capital investments, also see Subsection 4.2 below.

Finally, we investigate the evolution of GDP per capita in Figure 7, again expressed as an index. We detrend by the constant growth factor. In consequence the normalization is such that detrended GDP per capita would stay at the constant value of 1 in a stationary economy. In case of exogenous human capital (left panel) we confirm findings from the earlier literature simulating a decrease of detrended GDP per capita in light of demographic change caused by the relative scarcity of labor, cf., e.g., Krüger and Ludwig (2007) and Ludwig, Schelkle, and Vogel (2012). The decrease is less strong in the open economy and basically absent in scenario PR for the open economy. In case of endogenous human capital adjustments, the increase of human capital already documented above leads to a substantial increase of GDP per capita by up to about 25% until 2040 in scenario PR for the open economy.

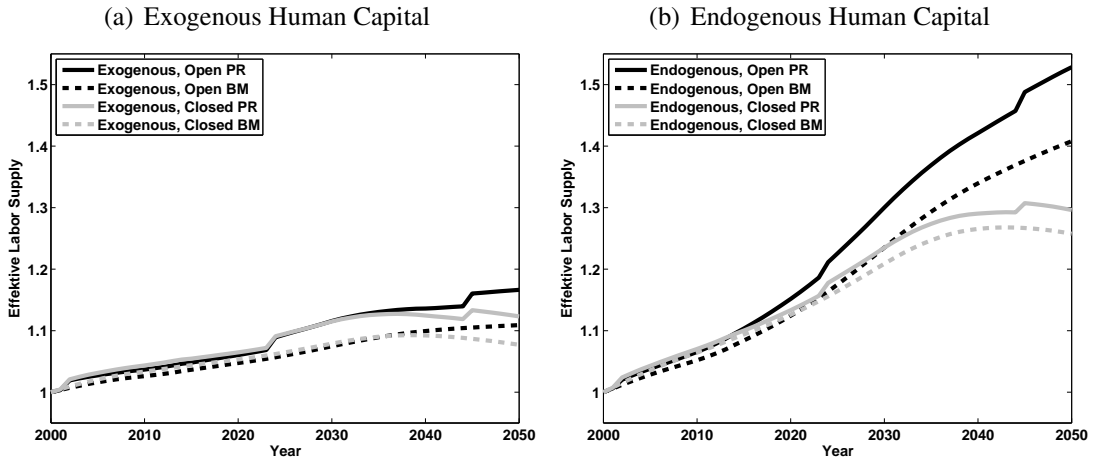
4.2 Labor Supply over the Life Cycle

To understand the effects of the pension reform on aggregate hours and human capital, it is key to understand the response over the life cycle. Theoretical insights into these life cycle effects are developed in Appendix B.2 by using a simple two-period model. To shed light on these adjustments quantitatively, we isolate the effects induced by a change in the retirement age from changes induced by general equilibrium feedback.

Results of this decomposition analysis for the closed economy are summarized in Figure 8 as percent deviations from the benchmark life cycle profile.¹⁷ Ignoring general equi-

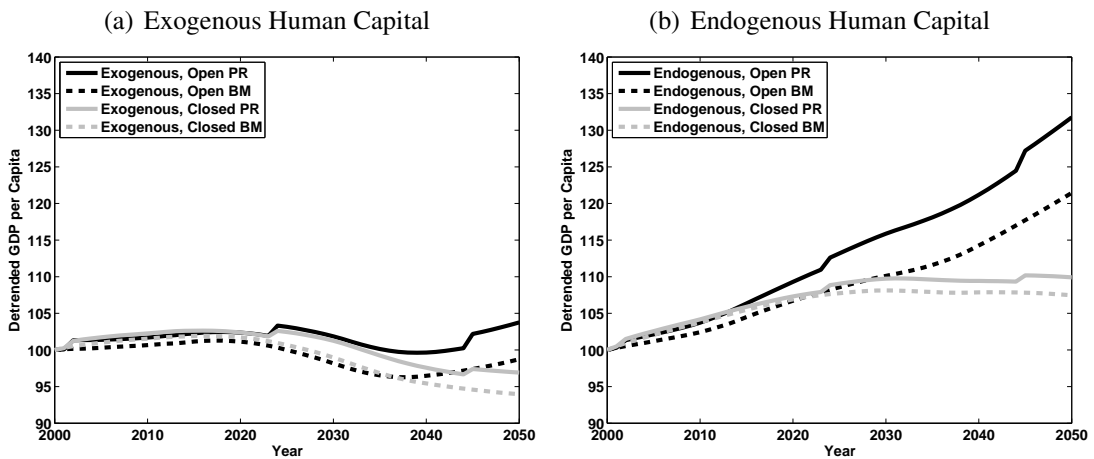
¹⁷First, we define our benchmark to be the cohort entering the labor market in 1955. This is the first cohort affected by

Figure 6: Effective Labor Supply per Worker [Index]



Notes: “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “Exogenous” (left panel) and “endogenous” (right panel) refer to the respective human capital model. “PR” and “BM” denote the pension reform and benchmark retirement scenario. All results obtained with constant contribution rate, τ .

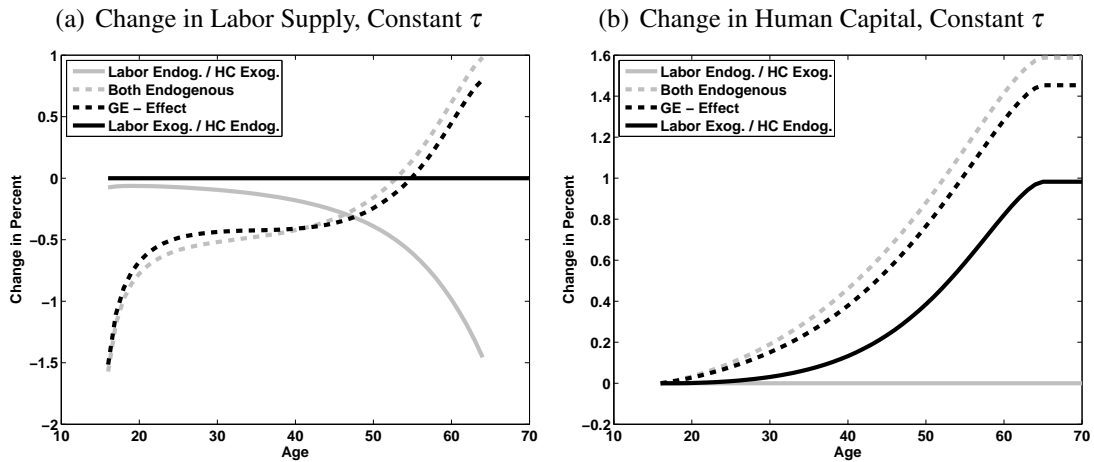
Figure 7: Detrended GDP per Capita [Index]



Notes: “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “Endogenous” (left panels) and “exogenous” (right panels) refer to the human capital profile. “PR” and “BM” denote the pension reform and benchmark retirement scenario. All results obtained with constant replacement rate τ .

librium adjustments and human capital accumulation, an increase in the retirement age has a negligible impact on labor supply at the intensive margin early in life but labor supply decreases considerable at higher ages. With both margins of adjustment at work (“Both Endogenous”) the young invest more time in education and hence labor supply decreases relative to the benchmark. Older agents work more to reap the benefits of higher levels of human capital. In a nutshell, when both are endogenous, agents substitute labor supply against human capital early in life. These predictions are consistent with the theoretical model in the appendix (cf. equation (31)). Finally, in general equilibrium, the endogenous labor supply and human capital response leads to higher pension payments, lower increases in wages and relatively higher interest rates. This mitigates the incentives to invest in human capital and increases (decreases) labor supply when young (old) relative to the case with constant prices. As a consequence, once all general equilibrium adjustments are considered, incentives to work and invest into human capital are weakened. The total effect on the intensive margin of labor supply is negative (see Table 2). Turning to the human capital investment, we observe that with human capital and labor supply being endogenous, there is a sizable positive effect of a longer working life on human capital. Similarly to labor supply, the general equilibrium effects dampen the increase via relatively lower wages and relatively higher interest rates.

Figure 8: Reallocation over the Life Cycle



Notes: “Endogenous” and “Exogenous” refer to the human capital production profile. Results with closed capital markets.

an increase of the normal retirement age. Then, we take general equilibrium prices and policy instruments with retirement age at 65 and increase the retirement age. This enables us to compare household decisions in partial equilibrium. In order to isolate the different adjustment channels—labor supply and human capital—in this partial equilibrium setting we perform three experiments. We start by holding constant the human capital profile and endogenously compute labor supply (“Labor Endog. / HC Exog.” and vice versa). We next hold the labor supply profile fixed and allow human capital to adjust (“Labor Exogenous / HC Endogenous”). Finally, we allow labor supply and human capital to be endogenous and thereby capture the total (partial equilibrium) effect (“Both Endogenous”). As a last step, we compare our results to the life cycle profiles of households observed in general equilibrium after implementation of the pension reform (“GE Effect”). In order to understand the role of general equilibrium feedback effects from relative prices, we show the results of the experiment only for the closed economy.

Table 2 documents the total effect of these behavioral adjustments as the sum of changes of life cycle allocations shown in Figure 8. Moving from left to right in the table, we observe lower labor supply at the intensive margin in partial equilibrium. The drop in hours worked is smallest when education and hours worked are simultaneously endogenous. However, this is compensated by the additional working year which leads to a sizable increase in total labor supply. We can therefore conclude that the exogenous increase of the retirement age has a significant impact on total aggregate effective labor supply mainly by inducing agents to work more in the marginal year and investing more into human capital. On the contrary, reactions at the intensive margin only shift allocations over the life cycle with a small total negative effect. Analyzing the results of the same reform in an open economy gives qualitatively the same findings. There, the difference between the scenarios “Both Endogenous” and “GE effect” is negligible.

Table 2: Change in Life Cycle Allocations

	Endogenous Variable			
	ℓ	e	ℓ, e	ℓ, e (GE)
	Open			
Labor (Intensive)	-0.3%	-0.3%	-0.3%	0.1%
Labor (Total)	0.5%	0.5%	0.5%	1.0%
	Closed			
Labor (Intensive)	-0.4%	-0.4%	-0.2%	-0.3%
Labor (Total)	1.2%	1.2%	1.4%	1.4%

Notes: “Intensive” refers to the total amount of effective labor supply up to the “old” retirement age (as defined in the “BM” scenario), “total” adds the additional working year. The variables reported (title of column) is always the endogenous variable, the other is assumed to be exogenous (see description of methodology above). The upper (lower) panel refers to the open (closed) economy scenario.

Such quantitative responses are in line with empirical estimates. As Mastrobuoni (2009) shows, more than half of the increase in retirement age in the US is taken up by agents. In Table 2 (rightmost column) we show that the increase in hours in the *additional* year corresponds to an implicit pass-through of roughly $\frac{1}{2} - \frac{2}{3}$ which we believe to be a reasonable approximation of the empirical results. If workers would work full time, an increase in the retirement age by one year constitutes an increase of total time by about $\frac{1}{65-16} \approx 0.02$. However, total hours worked over the working life increase by only about 1.0 – 1.3% which is a pass-through of about between $\frac{1}{2}$ and $\frac{2}{3}$.

4.3 Welfare

In our model, households are affected by three distinct consequences of demographic change and policy reform. First, for given prices utility increases because survival probabilities increase. Second, households are affected by changes in prices and transfers due to

general equilibrium effects of aging. For cohorts currently alive, these profound changes can have—depending on the position in the life cycle—positive or negative welfare effects. Third, when the retirement age is increased, a constraint is relaxed which will lead to welfare gains.

Furthermore, as shown above, increasing the retirement age in the closed economy leads to higher levels of rates of return (lower wages) and lower decreases of the rate of return as societies are aging—cf. Figures 5(b) and 5(a)—with associated feedback into welfare consequences.

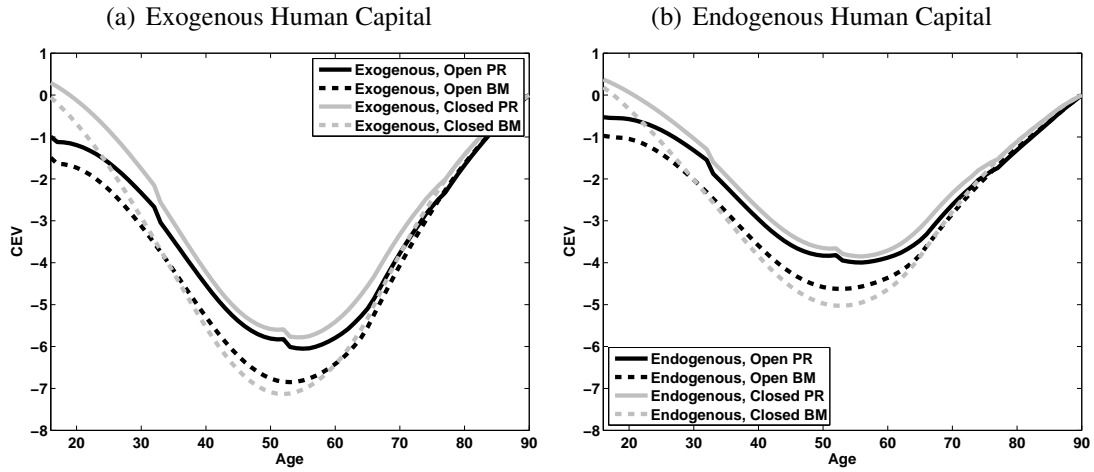
We want to isolate and quantify the effect of changing prices, taxes and transfers as well as increasing the retirement age on households' lifetime utility. To this end, we first compute the (remaining) lifetime utility of an agent of age j born in year t using the full set of (time varying) general equilibrium prices, taxes and transfers. Then, we hold all prices and transfers constant at their respective year 2010 value and recompute agent's remaining lifetime utility. For both scenarios we keep constant survival probabilities at their year 2010 values. We compute the consumption equivalent variation, $g_{t,j,i}$, i.e., the percentage of consumption that needs to be given to the agent at each date for her remaining lifetime at prices from 2010 in order to make her indifferent between the two scenarios. Positive values of $g_{t,j,i}$ thus indicate welfare gains from the general equilibrium effects of aging.¹⁸ In order to isolate the effects of changing prices, taxes and transfers, we do not account for the gain in households' lifetime utility during the additional life years generated by the increase in life expectancy.

Welfare of Generations Alive in 2010

Results on welfare for generations alive in 2010 are displayed in Figure 9 and can be summarized as follows: in the closed economy versions, newborns in 2010 marginally benefit from demographic change, confirming earlier findings in the literature by, e.g., Krüger and Ludwig (2007) and Ludwig, Schelkle, and Vogel (2012). Hence we find that, when the contribution rate is held constant, increasing wages dominate for newborn households who experience welfare gains. The converse applies to old and asset rich households. In welfare terms, the increase of wages dominates the reduction of the rate of return for this group. Recall that we hold the contribution rate constant so that any increase in gross wages feeds one to one into an increase in net wages. These gains vanish in the open economy variants because here the relative price movements are less favorable for this cohort. Table 3 provides a summary. All other generations are loosing with losses peaking around age 52. The reason is relative asset richness and hence lower asset income from decreasing interest rates. These losses are substantially lower in the endogenous human capital model variants and decrease to about 4% in the PR scenarios.

¹⁸Using the functional form from equation (1) the consumption equivalent variation is given by $g_{t,j,i} = \left(\frac{V_{t,j,i}}{V_j^{2010}} \right)^{\frac{1}{\phi(1-\sigma)}} - 1$ where $V_{t,j,i}$ denotes lifetime utility using general equilibrium prices and V_j^{2010} is lifetime utility using constant prices from 2010.

Figure 9: Consumption Equivalent Variation of Agents alive in 2010, Constant τ



Notes: “Endogenous” and “Exogenous” refer to the human capital model.

Table 3: Welfare Gains / Losses - Newborns 2010

Pension System	Open		Closed	
	Exog.	Endog.	Exog.	Endog.
BM	-1.5%	-1.0%	-0.1%	0.2%
PR	-1.0%	-0.5%	0.3%	0.4%

Notes: “Endog.” and “Exog.” refer to the endogenous and exogenous human capital production profile. “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “BM” denotes the benchmark pension system (constant retirement age), and “PR” denotes the pension reform (increase in the retirement age). Results from the constant τ scenario.

Table 4: Maximum Welfare Losses - Agents alive 2010

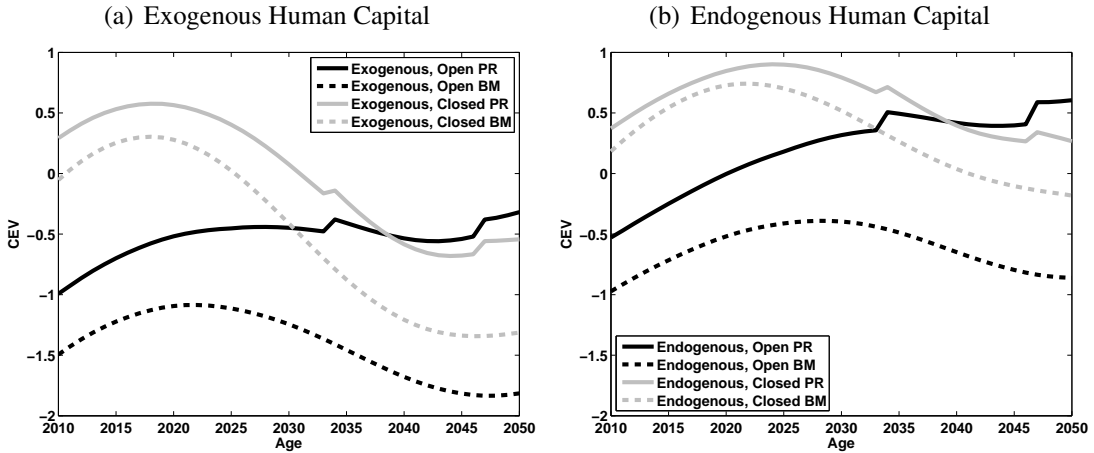
Pension System	Open		Closed	
	Exog.	Endog.	Exog.	Endog.
BM	-6.8%	-4.6%	-7.1%	-5.0%
PR	-6.1%	-4.0%	-5.8%	-3.9%

Notes: “Endog.” and “Exog.” refer to the endogenous and exogenous human capital production profile. “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “BM” denotes the benchmark pension system (constant retirement age), and “PR” denotes the pension reform (increase in the retirement age). Results from the constant τ scenario.

Welfare of Future Generations

We now report welfare changes for newborns between 2010 and 2050 in Figures 10. In the closed economy model with endogenous human capital adjustments and the pension reform, all newborn generations are benefiting from the reform. While all future newborn generations generally lose from demographic change in the open economy, they are substantially better off if the pension reform takes place.

Figure 10: Consumption Equivalent Variation of Future Generations, Constant τ



Notes: “Endog.” and “Exog.” refer to the endogenous and exogenous human capital production profile. “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “BM” denotes the benchmark pension system (constant retirement age), and “PR” denotes the pension reform (increase in the retirement age). Results from the constant τ scenario.

4.4 Results with Fixed Replacement Rate

Our analysis concentrates on the case with a constant contribution rate, τ . We here briefly comment on how these results alter when we consider the polar pension scenario with a constant replacement rate, ρ . All details are relegated to the Supplementary Appendix. The upshot of that analysis is that the endogenous increase of the contribution rate caused by demographic change in a world with constant replacement rates leads to a mitigation of the effects on the effective labor supply. Effective labor supply relative to all workers increases only by roughly 10% in the open economy PR scenario, compared to 40% observed for the pension scenario with constant contribution rates. As the pension system is now rather generous, household do not build up that much retirement savings. Therefore, the increase of the capital intensity is not as strong as in the constant- τ scenario with the consequence that the reduction of the rate of return is smaller. On top of this effect, keeping ρ constant causes a substantial increase in the labor market distortions due to a higher τ . These trends of capital formation and effective labor supply in comparison to the constant- τ scenario also imply that GDP per capita increases less strongly. Overall, the effects of the pension reform are slightly less pronounced. In terms of welfare, all generations currently alive as

well as all future generations experience losses from demographic change under constant replacement rates. These losses are somewhat mitigated under the pension reform PR but the effect is far less positive than we observed for the constant- τ scenario.

5 Conclusion

This paper revisits the literature on the consequences of demographic change—aging—for welfare of generations who live through the demographic transition in industrialized countries focusing on France, Germany and Italy (FGI) in a globalized world. We ask how the potentially detrimental consequences of aging for FGI may be mitigated by two margins of adjustment, namely by investing abroad and by human capital formation. We address this question in combination with pension policy. That is, we ask how the design of pension policy may contribute to dampening via these endogenous channels.

We conclude that endogenous human capital adjustments in combination with a pension policy reform by increasing the retirement age and a constant contribution rate has strong implications, both for economic aggregates such as the future trends of per capita output and for welfare. Welfare losses of currently 50-year old households decrease by roughly 3 percentage points when such adjustments take place. Furthermore, we emphasize that world prices are influenced by global demographic developments. Hence a fundamental pension reform in FGI (in isolation) has very strong effects on per capita output because mitigating general equilibrium feedback is largely absent. Therefore, labor market policies focusing at the extensive margin (by increasing the retirement age) and adjustments at the education margin are powerful policy options to mitigate the adverse welfare effects of demographic change.

However, we ignore the endogenous adjustment of retirement to policy, as done by, e.g., Heijdra (2009) and Buyse et al. (2012). While we document that our estimates of endogenous labor supply adjustments to changes in the exogenous reform of the retirement age are reasonable, one may still argue that it is important to explicitly model the endogenous choice of retirement at the extensive margin. Yet, our approach is a valid first order approximation. In order to replicate actual retirement patterns observed in the data, a model with endogenous retirement requires the specification of pension pay adjustment factors for late/early retirement. These adjustment factors would also change with a reform that increases the statutory retirement age. Such a reform was recently implemented, e.g., in Germany. Moreover, as such adjustment factors do not suffice to replicate actual retirement patterns, specification and calibration of some fixed costs of work participation is needed as, e.g., in French (2005) and Fehr, Kindermann, and Kallweit (2013). It is reasonable to assume that such fixed costs decrease as health and therefore life-expectancy is improving. For these reasons, the average and the statutory retirement age would co-move in such an extended model.

Furthermore, in our work, all endogenous human capital adjustments are driven by relative price changes, increases in life expectancy and increases of the statutory retirement age. If, instead, human capital formation is affected by market imperfections, such as borrowing constraints, then these automatic adjustments will be inhibited. In this case,

appropriate human capital policies in combination with pension policies are an important topic for future research and the policy agenda.

References

- Altig, D., A. J. Auerbach, L. J. Kotlikoff, and K. A. S. J. Walliser (2001). Simulating fundamental tax reform in the United States. *American Economic Review* 91(3), 574–594.
- Arrazola, M. and J. de Hevia (2004). More on the estimation of the human capital depreciation rate. *Applied Economic Letters* 11, 145–148.
- Attanasio, O., S. Kitao, and G. L. Violante (2007). Global demographic trends and social security reform. *Journal of Monetary Economics* 54(1), 144–198.
- Auerbach, A. J. and L. J. Kotlikoff (1987). *Dynamic Fiscal Policy*. Cambridge: Cambridge University Press.
- Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *Journal of Political Economy* 75(4), 352–365.
- Börsch-Supan, A., K. Härtl, and A. Ludwig (2014). Reforms, international diversification, and behavioral reactions. *American Economic Review P&P* 104(5), 224–229.
- Börsch-Supan, A., A. Ludwig, and J. Winter (2006). Ageing, pension reform and capital flows: A multi-country simulation model. *Economica* 73, 625–658.
- Browning, M., L. P. Hansen, and J. J. Heckman (1999). Micro data and general equilibrium models. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Chapter 8, pp. 543–633. Amsterdam: Elsevier.
- Buiter, W. H. and K. M. Kletzer (1995). Permanent international productivity growth differentials in an integrated global economy. *Scandinavian Journal of Economics* 95(4), 467–493.
- Buyse, T., F. Heylen, and R. Van de Kerckhove (2012). Pension reform, employment by age, and long-run growth. *Journal of Population Economics*, forthcoming.
- de la Croix, D. and O. Licandro (1999). Life expectancy and endogenous growth. *Economics Letters* 65, 255–263.
- de la Croix, D., O. Pierrard, and H. Sneessens (2013). Aging and pension in general equilibrium: Labor market imperfections matter. *Journal of Economics Dynamics and Control* 37, 104–124.
- De Nardi, M., S. İmrohoroğlu, and T. Sargent (1999). Projected U.S. demographics and social security. *Review of Economic Dynamics* 2(1), 575–615.
- Diamond, P. and J. Gruber (1999). Social security and retirement in the united states. In J. Gruber and D. Wise (Eds.), *Social Security and Retirement Around the World*, Chapter 11, pp. 437–474. Chicago: University of Chicago Press.
- Domeij, D. and M. Flodén (2006). Population aging and international capital flows. *International Economic Review* 47(3), 1013–1032.
- Echevarría, C. Á. and A. Iza (2006). Life expectancy, human capital, social security and growth. *Journal of Public Economics* 90, 2324–2349.
- Fehr, H., S. Jokisch, and L. J. Kotlikoff (2005). Will China eat our lunch or take us out for dinner? Simulating the transition paths of the U.S., EU, Japan, and China. *NBER Working Paper No. 11668*.
- Fehr, H., F. Kindermann, and M. Kallweit (2013). Should pensions be progressive? *European Economic Review* 63, 94–116.

- Fitzenberger, B., R. Hujer, T. E. MaCurdy, and R. Schnabel (2001). Testing for uniform wage trends in west-germany: A cohort analysis using quantile regressions for censored data. *Empirical Economics* 23, 41–86.
- Fougère, M. and M. Mérette (1999). Population ageing and economic growth in seven OECD countries. *Economic Modelling* 16, 411–427.
- French, E. (2005). The effects of health, wealth, and wages on labor supply and retirement behaviour. *Review of Economic Studies* 72, 395–427.
- French, E. and J. Jones (2012). Public pensions and labor supply over the life cycle. *International Tax and Public Finance*, 1–20. forthcoming.
- Fuster, L., A. İmrohoroğlu, and S. İmrohoroğlu (2007). Elimination of social security in a dynastic framework. *Review of Economic Studies* 74(1), 113–145.
- Gourieroux, C., A. Monfort, and E. Renault (1993). Indirect inference. *Journal of Applied Econometrics* 8, S85–S118.
- Guvenen, F. and B. Kuruscu (2009). A quantitative analysis of the evolution of the u.s. wage distribution: 1970–2000. In *NBER Macroeconomics Annual*. NBER. forthcoming.
- Hall, R. (1988). Intertemporal substitution in consumption. *Journal of Political Economy* 96(2), 339–357.
- Hansen, G. D. (1993). The cyclical and secular behaviour of the labour input: Comparing efficiency units and hours worked. *Journal of Applied Econometrics* 8(1), 71–80.
- Hanushek, E. A. and D. D. Kimko (2000). Schooling, labor-force quality, and the growth of nations. *American Economic Review* 90(2), 1184–1208.
- Heckman, J., L. Lochner, and C. Taber (1998). Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents. *Review of Economic Dynamics* 1, 1–58.
- Heijdra, B. and L. S. M. Reijnders (2012). Human capital accumulation and the macroeconomy in an ageing society. *CESifo Working Paper Series* 4046.
- Heijdra, B. J. and W. E. Romp (2009). Human capital formation and macroeconomic performance in an ageing small open economy. *Journal of Economic Dynamics and Control* 33, 725–744.
- Heijdra, B. Romp, W. (2009). Retirement, pensions, and ageing. *Journal of Public Economics* 93, 586–604.
- Huang, H., S. İmrohoroğlu, and T. J. Sargent (1997). Two computations to fund social security. *Macroeconomic Dynamics* 1(1), 7–44.
- Huggett, M., G. Ventura, and A. Yaron (2012). Sources of lifetime inequality. *American Economic Review* 101(7), 2923–54.
- Hurd, M. and S. Rohwedder (2011). Trends in labor force participation: How much is due to changes in pensions? *Population Ageing* 4, 81–96.
- İmrohoroğlu, S. and S. Kitao (2009). Labor supply elasticity and social security reform. *Journal of Public Economics* 93(7–8), 867–878.
- İmrohoroğlu, A., S. İmrohoroğlu, and D. H. Joines (1995). A life cycle analysis of social security. *Economic Theory* 6, 83–114.

- Krüger, D. and A. Ludwig (2007). On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of Monetary Economics* 54(1), 49–87.
- Lagakos, D., B. Moll, T. Porzio, and N. Qian (2012). Experience matters: human capital and development accounting. *NBER Working Paper 18602*.
- Lee, R. D. and A. Mason (2010). Fertility, human capital, and economic growth over the demographic transition. *European Journal of Population* 26(2), 159–182.
- Liao, P. (2011). Does demographic change matter for growth? *European Economic Review* 55(5), 659–677.
- Ludwig, A. (2007). The Gauss-Seidel-Quasi-Newton method: A hybrid algorithm for solving dynamic economic models. *Journal of Economic Dynamics and Control* 31(5), 1610–1632.
- Ludwig, A., T. Schelkle, and E. Vogel (2007). Demographic change, human capital and endogenous growth. *MEA Discussion Paper 151-07*.
- Ludwig, A., T. Schelkle, and E. Vogel (2012). Demographic change, human capital and welfare. *Review of Economic Dynamics* 15(1), 94–107.
- Maddison, A. (2003). *The World Economy: Historical Statistics*. Paris: OECD.
- Mastrobuoni, G. (2009). Labor supply effects of the recent social security benefit cuts: Empirical estimates using cohort discontinuities. *Journal of Public Economics* 93(11-12), 1224–1233.
- Oeppen, J. and J. W. Vaupel (2002). Broken limits to life expectancy. *Science* 296(5570), 1029–1031.
- Palacios, R. and M. Pallarés-Miralles (2000). International patterns of pension provision. *World Bank Social Protection Discussion Paper Series No. 9*.
- Prskawetz, A. and T. Fent (2007). Workforce ageing and the substitution of labor: the role of supply and demand of labour in Austria. *Metroeconomica* 58(1), 95–126.
- Sadahiro, A. and M. Shimasawa (2002). The computable overlapping generations model with endogenous growth mechanism. *Economic Modelling* 20, 1–24.
- Smith, A. A. (1993). Estimating nonlinear time-series models using simulated vector autoregressions. *Journal of Applied Econometrics* 8, S63–S84.
- Storesletten, K. (2000). Sustaining fiscal policy through immigration. *Journal of Political Economy* 108(2), 300–323.
- United Nations (2007). *World Population Prospects: The 2002 Revision*. United Nations Population Division. New York.
- Whitehouse, E. R. (2003). The value of pension entitlements: A model of nine oecd countries. *OECD Social, Employment and Migration Working Papers No. 9*.

A Computational Appendix

A.1 Household Problem

To simplify the description of the solution of the household model for given prices (wage and interest rate), transfers and social security payments, we focus on steady states and therefore drop the time index t and the country index i . Furthermore, we focus on a de-trended version of the household problem in which all variables x are transformed to $\tilde{x} = \frac{x}{A}$ where A is the technology level growing at the exogenous rate g . To simplify notation, we do not denote variables by the symbol $\tilde{\cdot}$ but assume that the transformation is understood. The de-trended version of the household problem is then given by

$$\begin{aligned}
 V(a, h, s, j) &= \max_{c, \ell, e, a', h'} \left\{ u(c, 1 - \ell - e) + \beta s(1 + g)^{\phi(1 - \sigma)} V(a', h', s', j + 1) \right\} \\
 &\text{s.t.} \\
 a' &= \frac{1}{1 + g} ((a + tr)(1 + r) + y - c) \\
 y &= \begin{cases} \ell h w (1 - \tau) & \text{if } j < jr \\ \rho w_{jr} (1 + g)^{jr - j} \bar{h}_{jr} \frac{s_{jr}}{jr - 1} & \text{if } j \geq jr \end{cases} \\
 h' &= g(h, e) \tag{14}
 \end{aligned}$$

$$s' = s + \ell \frac{h}{\bar{h}} \tag{15}$$

$$\ell \in [0, 1], \quad e \in [0, 1].$$

Here, $g(h, e)$ is the human capital technology. Let $\tilde{\beta} = \beta s(1 + g)^{\phi(1 - \sigma)}$ be the transformation of the discount factor. Using the budget constraints, now rewrite the above as

$$\begin{aligned}
 V(a, h, s, j) &= \max_{c, \ell, e, a', s', h'} \left\{ u(c, 1 - \ell - e) + \tilde{\beta} V \left(\frac{1}{1 + g} ((a + tr)(1 + r) + y - c), g(h, e), s + \ell \frac{h}{\bar{h}}, j + 1 \right) \right\} \\
 &\text{s.t.} \\
 \ell &\geq 0.
 \end{aligned}$$

where we have also replaced the bounded support of time investment and leisure with a one-side constraint on ℓ because the upper constraints, $\ell = 1$, respectively $e = 1$, and the lower constraint, $e = 0$, are never binding due to Inada conditions on the utility function and the functional form of the human capital technology (see below). Denoting by μ_ℓ the Lagrange multiplier on the inequality constraint for ℓ , we can write the first-order conditions as

$$c : \quad u_c - \tilde{\beta} \frac{1}{1 + g} V_{a'}(a', h', s'; j + 1) = 0 \tag{16a}$$

$$\ell : \quad -u_{1 - \ell - e} + \tilde{\beta} \left[h w (1 - \tau) \frac{1}{1 + g} V_{a'}(\cdot) + V_{s'}(\cdot) \frac{h}{\bar{h}} \right] + \mu_\ell = 0 \tag{16b}$$

$$e : \quad -u_{1 - \ell - e} + \tilde{\beta} g_e V_{h'}(a', h', s', j + 1) = 0 \tag{16c}$$

and the envelope conditions read as

$$a: \quad V_a(a, h, s, j) = \tilde{\beta} \frac{1+r}{1+g} V_{a'}(a', h', s', j+1) \quad (17a)$$

$$h: \quad V_h(\cdot) = \begin{cases} \tilde{\beta} \left(\ell w (1-\tau) \frac{1}{1+g} V_{a'}(\cdot) + g_h V_{h'}(\cdot) + V_{s'}(\cdot) \ell \frac{1}{h} \right) & \text{if } j < jr \\ \tilde{\beta} V_{h'}(\cdot) g_h & \text{if } j \geq jr \end{cases} \quad (17b)$$

$$s: \quad V_s(\cdot) = \begin{cases} \tilde{\beta} V_{s'}(\cdot) & \text{if } j < jr \\ \tilde{\beta} \left(V_{s'}(\cdot) + \rho w_{jr} (1+g)^{jr-j} \bar{h}_{jr} \frac{1}{j_{r-1}} \frac{1}{1+g} V_{a'} \right) & \text{if } j \geq jr \end{cases} \quad (17c)$$

Note that for the retirement period, i.e. for $j \geq jr$, equations (16b) and (16c) are irrelevant and equation (17b) has to be replaced by

$$V_h(a, h, s, j) = \tilde{\beta} g_h V_{h'}(a', h', s', j+1).$$

From (16a) and (17a) we get

$$V_a = (1+r)u_c \quad (18)$$

and, using the above in (16a), the familiar inter-temporal Euler equation for consumption follows as

$$u_c = \tilde{\beta} \frac{1+r}{1+g} u_{c'}. \quad (19)$$

From (16a) and (16b) we get the familiar intra-temporal Euler equation for leisure,

$$u_{1-\ell-e} = u_c h \left(w(1-\tau) + (1+g) \frac{V_{s'}(\cdot)}{V_{a'}(\cdot)} \frac{1}{h} \right) + \mu_\ell. \quad (20)$$

From the human capital technology (3) we further have

$$g_e = \xi \psi(eh)^{\psi-1} h \quad (21a)$$

$$g_h = (1-\delta^h) + \xi \psi(eh)^{\psi-1} e. \quad (21b)$$

We loop backwards in j from $j = J-1, \dots, 0$ by taking an initial guess of $[c_J, h_J]$ as given and by initializing $V_{a'}(\cdot, J) = V_{h'}(\cdot, J) = 0$. During retirement, that is for all ages $j \geq jr$, our solution procedure is by standard backward shooting using the first-order conditions. However, during the period of human capital formation, that is for all ages $j < jr$, the first order conditions would not be sufficient if the problem is not a convex-programming problem. And thus, our backward shooting algorithm will not necessarily find the true solution. In fact this may be the case in human capital models such as ours because the effective wage rate is endogenous (it depends on the human capital investment decision). For a given initial guess $[c_J, h_J]$ we therefore first compute a solution via first-order conditions and then, for each age $j < jr$, we check whether this is the unique solution. As an additional check, we consider variations of initial guesses of $[c_J, h_J]$ on a large grid. In all of our scenarios we never found any multiplicities.

The details of our steps are as follows:

1. In each j , $h_{j+1}, V_{a'}(\cdot, j+1), V_{h'}(\cdot, j+1)$ are known.
2. Compute u_c from (16a).
3. For $j \geq jr$, compute h_j from (3) by setting $e_j = \ell_j = 0$ and by taking h_{j+1} as given and compute c_j directly from equation (25) below.
4. For $j < jr$:
 - (a) Assume $\ell \in [0, 1)$ so that $\mu_\ell = 0$.
 - (b) Combine (3), (16b), (16c) and (21a) to compute h_j as

$$h_j = \frac{1}{1 - \delta^h} \left(h_{j+1} - \xi \left(\frac{\xi \psi (1+g) V_{h'}(\cdot)}{w(1-\tau) V_{a'}(\cdot) + (1+g) V_{s'} \frac{1}{h}} \right)^{\frac{\psi}{1-\psi}} \right). \quad (22)$$

- (c) Compute e from (3) as

$$e_j = \frac{1}{h_j} \left(\frac{h_{j+1} - h_j(1 - \delta^h)}{\xi} \right)^{\frac{1}{\psi}}. \quad (23)$$

- (d) Calculate $lcr_j = \frac{1 - e_j - \ell_j}{c_j}$, the leisure to consumption ratio from (20) as follows:
From our functional form assumption on utility marginal utilities are given by

$$\begin{aligned} u_c &= \left(c^\phi (1 - \ell - e)^{1-\phi} \right)^{-\sigma} \phi c^{\phi-1} (1 - \ell - e)^{1-\phi} \\ u_{1-\ell-e} &= \left(c^\phi (1 - \ell - e)^{1-\phi} \right)^{-\sigma} (1 - \phi) c^\phi (1 - \ell - e)^{-\phi} \end{aligned}$$

hence we get from (20) the familiar equation:

$$\frac{u_{1-\ell-e}}{u_c} = hw(1 - \tau) = \frac{1 - \phi}{\phi} \frac{c}{1 - \ell - e},$$

and therefore:

$$lcr_j = \frac{1 - e_j - \ell_j}{c_j} = \frac{1 - \phi}{\phi} \frac{1}{hw(1 - \tau)}. \quad (24)$$

- (e) Next compute c_j as follows. Notice first that one may also write marginal utility from consumption as

$$u_c = \phi c^{\phi(1-\sigma)-1} (1 - \ell - e)^{(1-\sigma)(1-\phi)}. \quad (25)$$

Using (24) in (25) we then get

$$\begin{aligned} u_c &= \phi c^{\phi(1-\sigma)-1} (lcr \cdot c)^{(1-\sigma)(1-\phi)} \\ &= \phi c^{-\sigma} \cdot lcr^{(1-\sigma)(1-\phi)}. \end{aligned} \quad (26)$$

Since u_c is given from (16a), we can now compute c as

$$c_j = \left(\frac{u_{c_j}}{\phi \cdot lcr_j^{(1-\sigma)(1-\phi)}} \right)^{-\frac{1}{\sigma}}. \quad (27)$$

(f) Given c_j, e_j compute labor, ℓ_j , as

$$\ell_j = 1 - lcr_j \cdot c_j - e_j.$$

(g) If $\ell_j < 0$, set $\ell_j = 0$ and iterate on h_j as follows:

- i. Guess h_j
- ii. Compute e as in step 4c.
- iii. Noticing that $\ell_j = 0$, update c_j from (25) as

$$c = \left(\frac{u_c}{\phi(1-e)^{(1-\sigma)(1-\phi)}} \right)^{\frac{1}{\phi(1-\sigma)-1}}.$$

iv. Compute μ_ℓ from (16b) as

$$\mu_\ell = u_{1-\ell-e} - \tilde{\beta} \left[hw(1-\tau) \frac{1}{1+g} V_{a'}(\cdot) + V_{s'}(\cdot) \frac{h}{\bar{h}} \right]$$

v. Finally, combining equations (16b), (16c) and (21a) gives the following distance function f

$$f = e - \left(\frac{\tilde{\beta}[\cdot] + \mu_\ell}{\tilde{\beta} V_{h'}(\cdot) \xi \psi h^\psi} \right)^{\frac{1}{\psi-1}}, \quad (28)$$

where e is given from step 4(g)ii. We solve for the root of f to get h_j by a non-linear solver iterating on steps 4(g)ii through 4(g)v until convergence.

5. Update as follows:

- (a) Update V_a using either (17a) or (18).
- (b) Update V_h using (17b).

Next, loop forward on the human capital technology (3) for given h_0 and $\{e_j\}_{j=0}^J$ to compute an update of h_J denoted by h_J^n . Compute the present discounted value of consumption, PVC , and, using the already computed values $\{h_j^n\}_{j=0}^J$, compute the present discounted value of income, PVI . Use the relationship

$$c_0^n = c_0 \cdot \frac{PVI}{PVC} \quad (29)$$

to form an update of initial consumption, c_0^n , and next use the Euler equations for consumption to form an update of c_J , denoted as c_J^n . Define the distance functions

$$g_1(c_J, h_J) = c_J - c_J^n \quad (30a)$$

$$g_2(c_J, h_J) = h_J - h_J^n. \quad (30b)$$

In our search for general equilibrium prices, constraints of the household model are occasionally binding. Therefore, solution of the system of equations in (30) using Newton based methods, e.g., Broyden's method, is instable. We solve this problem by a nested

Brent algorithm, that is, we solve two nested univariate problems, an outer one for c_J and an inner one for h_J .

Check for uniqueness: Observe that our nested Brent algorithm assumes that the functions in (30) exhibit a unique root. As we adjust starting values $[c_J, h_J]$ with each outer loop iteration we thereby consider different points in a variable box of $[c_J, h_J]$ as starting values. For all of these combinations our procedure always converged. To systematically check whether we also always converge to the unique optimum, we fix, after convergence of the household problem, a large box around the previously computed $[c_J, h_J]$. Precisely, we choose as boundaries for this box $\pm 50\%$ of the solutions in the respective dimensions. For these alternative starting values we then check whether there is an additional solution to the system of equations (30). We never detected any such multiplicities.

A.2 The Aggregate Model

To solve the open economy general equilibrium transition path we proceed as follows: for a given $r \times 1$ vector $\vec{\Psi}$ of structural model parameters, we first solve for an “artificial” initial steady state in period $t = 0$ which gives initial distributions of assets and human capital. We thereby presume that households assume prices to remain constant for all periods $t \in \{0, \dots, T\}$ and are then surprised by the actual price changes induced by the transitional dynamics. Next, we solve for the final steady state of our model which is reached in period T and supported by our demographic projections. In the sequel, the superscripts c and o refer to the closed or open economy and M denotes the number of regions.

For the closed economy steady state, for each region j we solve for the equilibrium of the aggregate model by iterating on the $m^c \times 1$ steady state vector $\vec{P}_{ss,j}^c = [p_{1,j}, \dots, p_{m^c,j}]'$. $p_{1,j}$ is the capital intensity, $p_{2,j}$ are transfers (as a fraction of wages), $p_{3,j}$ are social security contribution (or replacement) rates and $p_{4,j}$ is the average human capital stock for region j . We perform this procedure separately for both world regions.

To compute the open economy steady state we solve for the equilibrium of the aggregate model by iterating on the $m^o \times 1$ steady state vector $\vec{P}_{ss}^o = [p_1, \dots, p_{m^o,j}]'$ where the number of variables is given by $m^o = M(m^c - 1) + 1$. p_1 is the *common (world)* capital intensity, $p_{2,j}$ are transfers (as a fraction of wages), $p_{3,j}$ are social security contribution (or replacement) rates and $p_{4,j}$ is the average human capital stock for region j . Notice that all elements of \vec{P}_{ss}^c and \vec{P}_{ss}^o are constant in the steady state.

Solution for the steady states for each *closed* region j (where we drop the region index for brevity) of the model involves the following steps:

1. In iteration q for a guess of $\vec{P}_{ss}^{c,q}$ solve the household problem.
2. Update variables in \vec{P}_{ss}^c as follows:
 - (a) Aggregate across households to get aggregate assets and aggregate labor supply to form an update of the capital intensity, p_1^n .
 - (b) Calculate an update of bequests to get p_2^n .
 - (c) Using the update of labor supply, update social security contribution (or replacement) rates to get p_3^n .

- (d) Use labor supply and human capital decisions to form an update of the average human capital stock, p_4^n .
3. Collect the updated variables in $\vec{P}_{ss}^{c,n}$ and notice that $\vec{P}_{ss}^{c,n} = H(\vec{P}_{ss}^c)$ where H is a vector-valued non-linear function.
4. Define the root-finding problem $G(\vec{P}_{ss}^c) = \vec{P}_{ss}^c - H(\vec{P}_{ss}^c)$ and iterate on \vec{P}_{ss}^c until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss} .

Solution for the steady states of the *open* economy of the model involves the following steps:

1. In iteration q for a guess of $\vec{P}_{ss}^{o,q}$ solve the household problem.
2. Update variables in \vec{P}_{ss}^o as follows:
 - (a) Use the guess for the global capital intensity to compute the capital stock for region j compatible with the open economy, perfect competition setup. Use this aggregate capital stock with the aggregate labor supply to form an update of the global capital intensity, p_1^n .
 - (b) Calculate an update of bequests to get $p_{2,j}^n \forall j$.
 - (c) Using the update of labor supply, update social security contribution (or replacement) rates to get $p_{3,j}^n \forall j$.
 - (d) Use labor supply and human capital decisions to form an update of the average human capital stock, $p_4^n \forall j$.
3. Collect the updated variables in $\vec{P}_{ss}^{o,n}$ and notice that $\vec{P}_{ss}^{o,n} = H(\vec{P}_{ss}^o)$ where H is a vector-valued non-linear function.
4. Define the root-finding problem $G(\vec{P}_{ss}^o) = \vec{P}_{ss}^o - H(\vec{P}_{ss}^o)$ and iterate on \vec{P}_{ss}^o until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss} .

Next, we solve for the transitional dynamics for each of the closed economies (where we again drop the region index j) by the following steps:

1. Use the steady state solutions to form a linear interpolation to get the starting values for the $m^c(T-2) \times 1$ vector of equilibrium prices, $\vec{P}^c = [\vec{p}'_1, \dots, \vec{p}'_{m^c}]'$, where $p_i, i = 1, \dots, m^c$ are vectors of length $(T-2) \times 1$.
2. In iteration q for guess $\vec{P}^{c,q}$ solve the household problem. We do so by iterating backwards in time for $t = T-1, \dots, 2$ to get the decision rules and forward for $t = 2, \dots, T-1$ for aggregation.
3. Update variables as in the steady state solutions and denote by $\tilde{\vec{P}}^c = H(\vec{P}^c)$ the $m^c(T-2) \times 1$ vector of updated variables.
4. Define the root-finding problem as $G(\vec{P}^c) = \vec{P}^c - H(\vec{P}^c)$. Since T is large, this problem is substantially larger than the steady state root-finding problem and we use the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007) to form and update guesses of an approximate Jacobi matrix of the system of $m^c(T-2)$ non-linear equations. We initialize these loops by using a scaled up version of B_{ss} .

We then solve for the transitional dynamics for the open economy setup by the following steps:

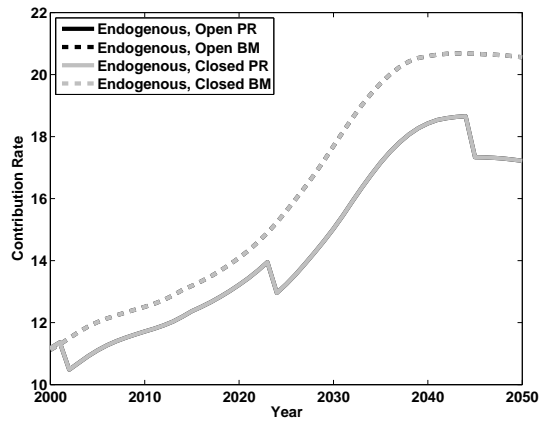
1. Use the equilibrium transition solutions from the closed economies to get the starting values for the $m^o(T - \tilde{t} - 2) \times 1$ vector of equilibrium prices, $\vec{P}^o = [\vec{p}'_1, \dots, \vec{p}'_{m^o}]'$, where $p_i, i = 1, \dots, m^o$ are vectors of length $(T - \tilde{t} - 2) \times 1$ where \tilde{t} is the year of opening up.
2. In iteration q for guess $\vec{P}^{o,q}$ solve the household problem. We do so by iterating backwards in time for $t = T - \tilde{t} - 1, \dots, 2$ to get the decision rules and forward for $t = 2, \dots, T - \tilde{t} - 1$ for aggregation. For agents already living in year \tilde{t} we use their holdings of physical assets and human capital from year \tilde{t} as state variables and solve their household problem only for their remaining lifetime.
3. We then proceed as in the case for the closed economies (updating) but define the root-finding problem now for the open economy as $G(\vec{P}^o) = \vec{P}^o - H(\vec{P}^o)$ which we solve by the same method as above.

B Supplementary Appendix

B.1 Additional Results: Constant Replacement Rate

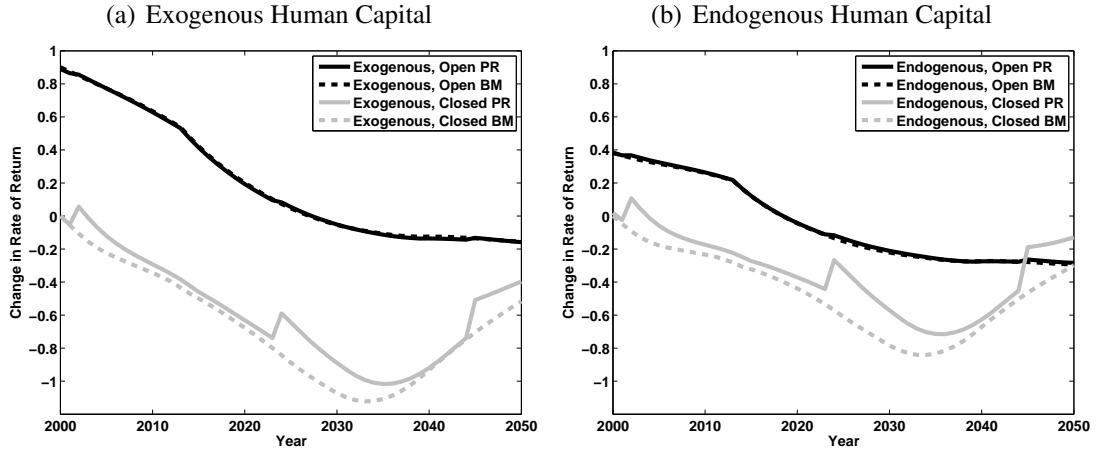
Aggregate Variables

Figure 11: Adjustment of Contribution Rates



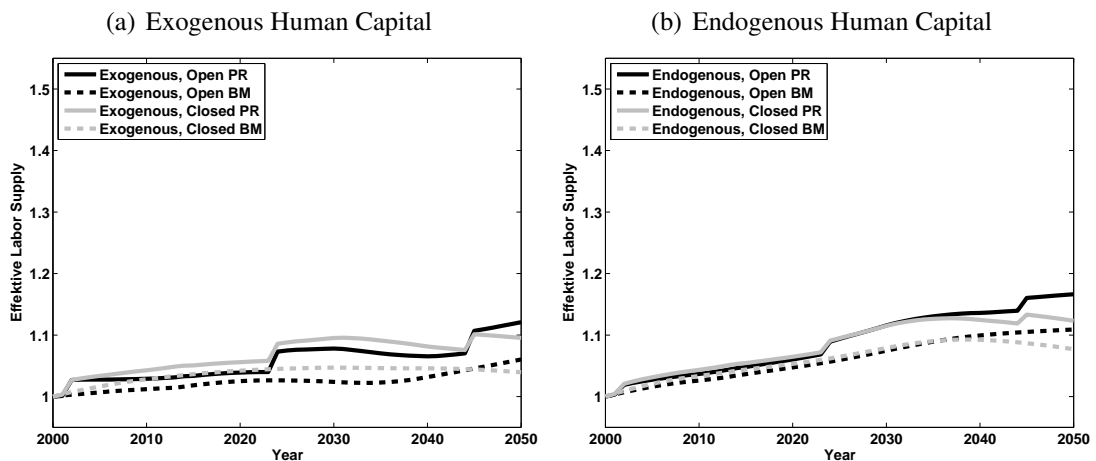
Notes: “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “Exogenous” (left panel) and “endogenous” (right panel) refer to the respective human capital model. “PR” and “BM” denote the pension reform and benchmark retirement scenario. All results obtained with constant contribution rate, τ .

Figure 12: Rate of Return [Index]: Constant Replacement Rates



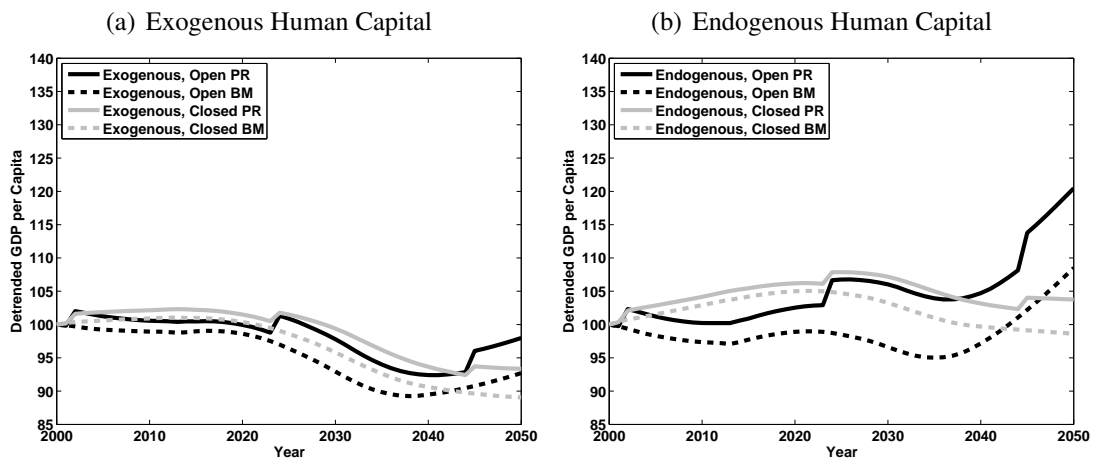
Notes: “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “Endogenous” (left panels) and “exogenous” (right panels) refer to the human capital profile. “PR” and “BM” denote the pension reform and benchmark retirement scenario. All results obtained with constant replacement rate ρ .

Figure 13: Effective Labor Supply [Index]: Constant Replacement Rates



Notes: “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “Endogenous” (left panels) and “exogenous” (right panels) refer to the human capital profile. “PR” and “BM” denote the pension reform and benchmark retirement scenario. All results obtained with constant replacement rate ρ .

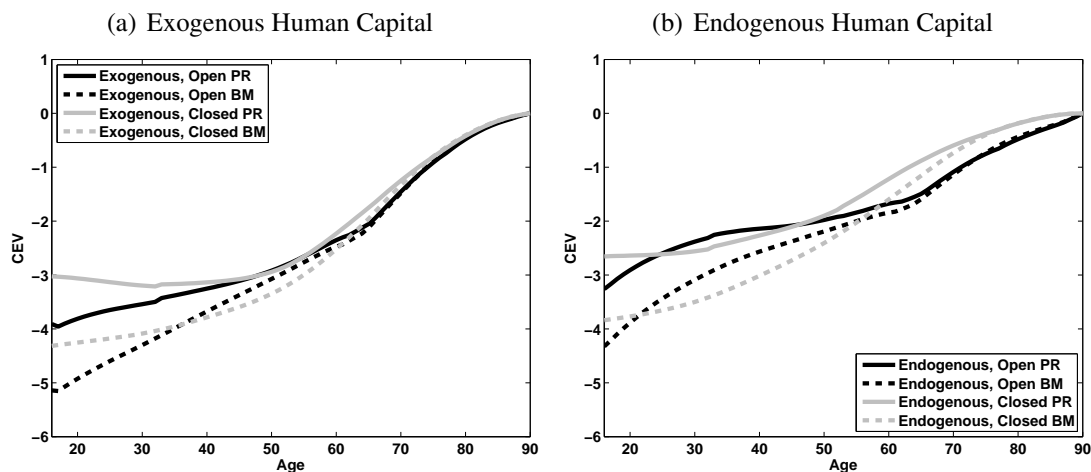
Figure 14: Detrended GDP per Capita [Index]: Constant Replacement Rates



Notes: “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “Endogenous” (left panels) and “exogenous” (right panels) refer to the human capital profile. “PR” and “BM” denote the pension reform and benchmark retirement scenario. All results obtained with constant replacement rate ρ .

Welfare of Generations Alive in 2010

Figure 15: Consumption Equivalent Variation of Agents Alive in 2010: Constant Replacement Rates



Notes: “Endogenous” and “Exogenous” refer to the human capital production profile.

Table 5: Welfare Gains / Losses - Newborns 2010: Constant Replacement Rates

Pension System	Open		Closed	
	Endog.	Exog.	Endog.	Exog.
BM	-4.3%	-5.1%	-3.8%	-4.3%
PR	-3.3%	-3.9%	-2.7%	-3.0%

Notes: “Endog.” and “Exog.” refer to the endogenous and exogenous human capital production profile. “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “BM” denotes the benchmark pension system (constant retirement age), and “PR” denotes the pension reform (increase in the retirement age). Results from the constant ρ scenario.

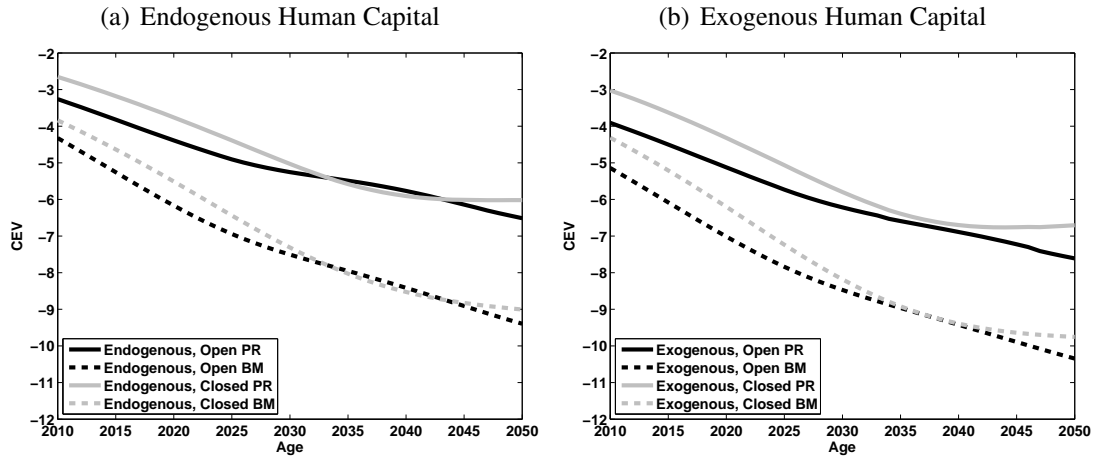
Table 6: Maximum Welfare Losses - Agents alive 2010: Constant Replacement Rates

Pension System	Open		Closed	
	Endog.	Exog.	Endog.	Exog.
BM	-4.3%	-5.2%	-3.8%	-4.3%
PR	-3.3%	-4.0%	-2.7%	-3.2%

Notes: “Endog.” and “Exog.” refer to the endogenous and exogenous human capital production profile. “Open” and “Closed” refer to the results obtained from the closed and open economy versions. “BM” denotes the benchmark pension system (constant retirement age), and “PR” denotes the pension reform (increase in the retirement age). Results from the constant ρ scenario.

Welfare of Future Generations (Benchmark Model & Pension Reform)

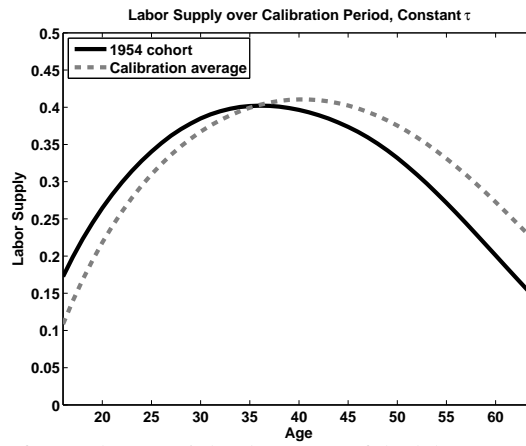
Figure 16: Consumption Equivalent Variation of Future Generations: Constant Replacement Rate



Notes: “Endogenous” and “Exogenous” refer to the human capital production profile.

Life Cycle Labor Supply for Calibration Period

Figure 17: Life Cycle Labor Supply for Calibration Period: Constant Contribution Rate



Notes: “Calibration average” refers to the unweighted average of the labor supply profiles during the calibration period and “1954 cohort” refers to the life-cycle labor supply of the cohort born in 1954.

B.2 Simple Model: Reallocation of Time over the Life Cycle

We want to understand theoretically the effects on labor supply and human capital at the intensive and extensive margin. We further decompose household's reaction when we keep prices and policy instruments fixed — i.e., wages, interest rates, the contribution rate and pension payments — and when we allow for general equilibrium feedback.

To understand the mechanisms theoretically, consider a simplified two-period version of the model used in the quantitative part. Households maximize utility

$$\begin{aligned}
 U &= \phi \ln(c_1) + (1 - \phi) \ln(1 - \ell_1) + \beta (\phi \ln(c_2) + (1 - \phi) \mathbb{1} \ln(1 - \ell_2)) \\
 \text{s.t.} \\
 c_1 + \frac{c_2}{1+r} &= w_1 \ell_1 (1 - e) + \frac{1}{1+r} (w_2 \ell_2 h(e) \mathbb{1} + (1 - \mathbb{1})p)
 \end{aligned}$$

with standard notation. $h(e) \geq 1$ is a strictly concave human capital production function where e is time investment which has to be made in the first period. $\mathbb{1}$ is an indicator function taking on the value of 1 if the agent is working in the last (second) period and 0 if he is retired and receives a pension p . Hence, changing the value of the indicator function from 0 to 1 mimics the pension reform of the quantitative model in a consistent way. Without loss of generality we normalize $w_1 = 1$. Denote first-period labor supply in the benchmark model — where $\mathbb{1} = 0$ — by ℓ_1^{BM} and labor supply with the higher retirement age — where $\mathbb{1} = 1$ — by ℓ_1^{PR} . Then, the difference in labor supply after increasing the retirement age is

$$\ell_1^{PR} - \ell_1^{BM} = \frac{\beta(1-\phi)^2}{(1+\beta)(1+\beta\phi)} - \frac{1-\phi}{R(1+\beta)} \left(w_2 \frac{h(e^*)}{1-e^*} - p \frac{1+\beta}{1+\beta\phi} \right) \quad (31)$$

with e^* being the equilibrium investment into human capital. This means that — keeping human capital constant — increasing the retirement age can in general either increase or decrease labor supply in the first period. Labor supply in the first period increases if

$$\frac{\beta(1-\phi)}{(1+\beta\phi)} > \frac{1}{R} \left(w_2 \frac{h(e^*)}{1-e^*} - p \frac{1+\beta}{1+\beta\phi} \right)$$

whereby the right-hand-side of this condition can be interpreted as reflecting the (adjusted) difference between human capital wealth — i.e., the discounted value of future income — between the model without retirement — in term $w_2 \frac{h(e^*)}{1-e^*}$ — and with retirement — in term $p \frac{1+\beta}{1+\beta\phi}$. If future wages are relatively small, i.e., if $w_2 \frac{h(e^*)}{1-e^*} < p \frac{1+\beta}{1+\beta\phi}$, then the discounted value of future income in case of the reform is small such that labor supply in the first period increases. Effects are however ambiguous if future labor income is sufficiently high. An unambiguous finding is that allowing for endogenous human capital increases $\frac{h(e^*)}{1-e^*}$ and therefore decreases labor supply when the retirement age increases.

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