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MINIMUM QUALITY STANDARDS AND NON-COMPLIANCE

Laura Birg, Jan S. Voßwinkel
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Abstract

This paper studies the effect of non-compliance with a minimum quality standard on prices, quality, and welfare in a vertical differentiation model. Non-compliance with a minimum quality standard by a low-quality firm reduces quality levels of both firms, increases the price for the high-quality product, decreases the price for the low-quality product, and shifts demand from the low-quality to the high-quality firm. Under non-compliance, an increase in the standard increases the quality difference, increases the price difference, and shifts demand from the high-quality to the low-quality firm. Stricter government enforcement decreases the quality level of the low-quality firm, increases the price of the high-quality product and shifts demand from the low-quality firm to the high-quality firm. Non-compliance of the low quality firm increases profits for both firms, reduces consumer surplus and increases or decreases welfare depending on the market size, the effect of quality levels of the externality, the detection probability, and the minimum quality level.

JEL Classification: K42, L13, L50

Keywords: minimum quality standard, non-compliance, enforcement

1 Introduction

This paper studies the effect of non-compliance with a minimum quality standard on prices, quality levels, and welfare in a vertical differentiation model. Also, it explores the effect of an increase in the minimum quality standard and the effect of a higher level of government enforcement effort under non-compliance.

In the European Union firms’ investments in product quality are not only driven by consumer preferences, but also by mandatory minimum quality standards that are applied in order to limit external effects such as harmful emissions or risks to consumers and third parties. However, non-compliance with these minimum quality standards seems to be not just an exception,
but it appears to be the rule in many cases. A significant number of household electrical products imported from outside of the EU does not comply with the respective minimum quality standards. The European Commission reports that only 5% of the household lights tested fully comply with the respective administrative or technical requirements. Only about 17% of all cord extension sets meet all requirements, while 58% of the cord extension sets tested are considered unsafe. Other examples are energy saving lamps (23% technical non-compliance), consumer entertainment electronic products (50% technical non-compliance), imported toys (55% technical non-compliance), and Christmas lightning (European Commission, 2013b, Annex 7).

Effective market surveillance aims at identifying unsafe or environmentally harmful products, which are then to be taken off the market. Market surveillance is carried out by the authorities of the member states (European Commission, 2013a). The internal market with products circulating freely within the European Union poses a particular challenge to market surveillance. Cross-border coordination of member states’ activities is vital for effective consumer protection in the internal market.

Market surveillance is incomplete today. The market surveillance rules are fragmented and different legal bases apply (Regulation 765/2008 and the General Product Safety Directive 2001/95/EC, or sector-specific European Union harmonization legislation). This may create confusion among national market authorities, consumers, and firms. Product safety requirements determining whether a product is safe and may be marketed are not always clear and consistent. Moreover, unsafe products not fulfilling product safety requirements may enter the EU market via third countries, if national activities are not coordinated sufficiently (European Commission, 2013a).

In 2013, the European Commission has proposed the so called “Product Safety and Market Surveillance Package” with the aim to improve consumer product safety and to strengthen market surveillance of products circulating on the internal market by better coordinating member states’ activities and streamlining the various legal bases (European Commission 2013a). The main idea of the “Product Safety and Market Surveillance Package” is to increase the probability that products that do not fulfill all requirements are detected by simplifying the safety rules, streamlining market surveillance procedures, and better coordinating the market surveillance activities in the EU (European Commission, 2013a).

Consumers often might not be aware of products not fulfilling all safety requirements. One reason could be that consumers trust in the competent market surveillance authorities. Since the question of fulfilling the standards is a complex issue, many consumers may simply have to rely on market surveillance authorities, because they are unable to observe the quality of a product in all its dimensions and/or lack sufficient knowledge of the respective standards. Alternatively fragmented surveillance rules could explain consumers’ unawareness. If national market authorities treat the same products differently within the single market, consumers lack a clear signal of confidence. Also, consumers might expect that a product not meeting all respective requirements completely will not automatically cause safety concerns. Another explanation for
the fact that consumers buy products that do not meet the relevant standards would be a lower preference for safety or environmental issues by consumers compared to the standard-setting authority.

This paper relates to the literature of minimum quality standards in several ways. Like the majority of papers on minimum quality standards (e.g. Ronnen, 1991 and Crampes & Hollander, 1995) we consider duopolistic markets, where single product firms face minimum quality standards as exogenous constraints. Unlike Ronnen (1991) or Boom (1995), however, we assume that quality improvements do not result in higher fixed cost for firms, but in higher variable costs as in Crampes & Hollander (1995), Ecchia & Lamberti (1997), and Petropoulou (2013). This assumption may be appropriate for many household appliances, toys etc., where quality improvements stem (partly) from using high-quality materials or more complex production processes.

The literature on minimum quality standards has stressed that quality choices of oligopolistic firms differ from socially optimal levels (Scarpa, 1998). The literature on minimum quality standards typically finds that quality levels of products increase with the level of the minimum quality standard (Ronen 1991, Motta & Thisse, 1993) as long as the quality standard does not reduce the number of firms in the market (Motta & Thisse, 1993).

Recently, several papers have analyzed the effects of non-compliance with a minimum quality standard. Faure, Schleich & Schlomann (2013) test non-compliance with the EU Energy Labeling Directive in a sample of 100,000 appliances from 1,400 retail stores in 27 EU member states. They show that perceived costs and benefits, normative motives, and social influence may explain retailer compliance with the EU energy labeling program. Other papers analyze mainly the effect of non-compliance with environmental standards. For instance, Hatcher (2007) compares emission level standards and standards expressed in terms of emissions per unit of output (ratio standards) under non-compliance. He shows that emission level standards and ratio standards lead to different results with respect to emissions and output. Arguedas, Camacho & Zofio (2010) analyze firm’s incentives to adopt abatement technologies, if non-compliance occurs. They find out that under certain assumptions imperfect compliance increases firms’ incentives to invest in abatement technologies, if emission standards are applied. Arguedas (2013) studies optimal fines for exceeding pollution standards. She shows that under non-compliance an optimal fine should decrease with investment effort and positive social costs of sanctioning, whereas under full-compliance the fine should be independent of investment efforts.

In this paper, we study the effect of non-compliance with a minimum quality standard on prices, quality levels, and welfare in a vertical differentiation model following Ecchia & Lambertini (1997). The paper assumes a duopolistic market structure with one firm selling a high-quality product and the other selling a low-quality product. We endogenize quality and assume variable cost of quality improvements. Consumers are heterogeneous with respect to their preference for quality. The introduction of an exogenous minimum standard may be motivated by external effects such as environmental harmful pollution or risks to consumers and third parties.

We assume a level of the minimum quality standard that is “tough”, i.e. a minimum quality
level that is set equal to the highest quality level available in the market or even exceeds this quality level. Exceeding the highest quality level available on the market does not imply that this quality level is technically infeasible. It only implies that this level is not profit-maximizing for any firm without regulation. Referring to the maximum quality level on the market in standard-setting corresponds to the common approach by the European Commissions of referring to “Best Available Techniques”\(^1\). Recently, the EU has decided on maximum CO\(_2\) emissions levels for passenger cars and light domestic vehicles. Especially for some heavy passenger cars and light domestic vehicles firms still lack technical solutions to reach the future maximum emission levels at reasonable cost and without decreasing other quality dimensions too much. This case corresponds to a minimum quality level that exceeds available quality levels at present on the market. In a dynamic perspective, governments increase minimum quality levels over time. So future minimum quality levels will exceed quality levels available on the market today. There are also methodological reasons for this assumption: If the level of the minimum quality standard would be lower than the quality level of the \(H\)-firm without regulation, non-compliance of the \(L\)-firm would be equivalent to the case of no regulation.

Non-compliance with a minimum quality standard may increase the low-quality firm’s profit. This behavior reduces both quality levels, it increases the price for the high-quality product, decreases the price for the low-quality product, and shifts demand from the low-quality to the high-quality firm. Under non-compliance, an increase in the standard increases the quality difference, increases the price difference, and shifts demand from the high-quality firm to the low-quality firm. A higher level of government enforcement decreases the quality level of the low-quality firm, but shifts demand from the low-quality firm to the high-quality firm. Non-compliance reduces consumer surplus, but increases producer surplus, and increases or decreases welfare, depending on the market size, the effect of quality levels of the externality, the detection probability, or the minimum quality level. Although overall consumer surplus declines, a subgroup of consumers might gain from non-compliance.

The rest of the paper is organized as follows. In the next section, the vertical differentiation model is presented and the case of no government intervention, the case of full compliance with the minimum quality standard, and the case of non-compliance by the low-quality firm with the standard are analyzed. Section 3 analyzes the effect of enhancing the minimum quality standard as well as stricter enforcement of a given standard and discusses the choice of policy instruments. Section 4 studies welfare. Section 5 concludes.

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\(^1\)The main idea of this paper also holds for industrial processes. Non-compliance with maximum pollution levels of industrial emissions is a major concern in some member states. The The Directive on industrial emissions defines “available techniques” as “those developed on a scale which allows implementation in the relevant industrial sector, under economically and technically viable conditions, taking into consideration the costs and advantages, whether or not the techniques are used or produced inside the Member State in question, as long as they are reasonably accessible to the operator”. It is not necessary that this technique is used in the market under consideration.
2 The Model

Following Ecchia & Lambertini (1997), consider a duopolistic market with vertical product differentiation. Assume that a product is supplied in two quality levels, $s_H$ and $s_L$, with $s_H > s_L$, and that each firm supplies only one quality level.

The production technology is characterized by variable cost, which are convex in quality and linear in quantity.

$$C_i = s_i^2 q_i. \tag{1}$$

Firms use higher quality materials or more complex production processes to enhance the quality level of their products. This may be an appropriate assumption for many household appliances like vacuum cleaners, for consumer entertainment products or for toys. For these products, non-compliance is a frequent phenomenon (European Commission, 2013b, Annex 7).

Consumers are heterogeneous with respect to their preference for quality, as in Mussa & Rosen (1978). They are characterized by a preference parameter $\theta$, which is uniformly distributed on the interval $[a, b]$ with $b = a + 1^2$. Each consumer buys at most one unit of the most preferred good. The utility derived from no purchase is zero, while a consumer who buys one unit of the good obtains a net utility of

$$U = \theta s_i - p_i, \quad i = H, L. \tag{2}$$

A consumer with a positive net utility of the good chooses the most preferred version of the good by trading off quality against the price. The higher $\theta$, the higher is the willingness to pay for quality. The consumer heterogeneity can be interpreted as differences in income or as differences in consumption patterns. Note that $\theta$ can also be interpreted as the marginal rate of substitution between income and quality (Tirole, 1988). Frequent usage may be accompanied by a higher willingness to pay for quality.

The marginal consumer indifferent between purchasing the high-quality good and the low-quality good is given by $\theta^* = \frac{p_H - p_L}{s_H - s_L}$. Hence, demand for the high-quality good and the low-quality good respectively is given as

$$q_H = b - \frac{p_H - p_L}{s_H - s_L}, \quad q_L = \frac{p_H - p_L}{s_H - s_L} - a. \tag{3}$$

Firms’ profits are given as

$$\pi_i = (p_i - s_i^2) q_i. \tag{4}$$

Competition follows a three-stage game: In the first stage, the government chooses a minimum quality level and a the intensity of market surveillance. In the second stage, firms choose quality levels. In the third stage, firms compete in prices.

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Assume $b > b_{\text{min}} = \frac{5}{4}$ to guarantee equilibrium existence (Ecchia & Lambertini, 1997).
2.1 No Regulation

Consider first a system with no government intervention. Prices and quality levels can be found in the Appendix. Firms are free to choose quality levels. Both quality levels increase in the maximum willingness to pay $b$. The difference between quality levels $\Delta s = s_H - s_L$ is independent of $b$. An increase in $b$ raises both quality levels equally.

Both equilibrium prices increase in $b$. Also, the price differential $\Delta p = p_H - p_L$ increases in $b$. An increase in the maximum willingness to pay increases the price of the high-quality product to a larger extent than the price of the low-quality product.

The duopoly is symmetric, quantities are $q_H = q_L = \frac{1}{2}$.

2.2 Minimum Quality Standard and Compliance

Now assume the introduction of a minimum quality standard, with which both firms comply. We assume a level of the minimum quality standard that is “tough”, i.e. a minimum quality level that is set equal to the highest quality level available in the market or even exceeds this quality level. Exceeding the highest quality level available on the market does not imply that this quality level is technically infeasible. It only implies that this level is not profit-maximizing for any firm without regulation.

Assume an exogeneously given “tough” standard that is binding for both firms, i.e. $S \geq s_H$. Also assume $S \leq S^{\text{max}}$ to guarantee that no firm exits the market. Prices, quality levels, and quantities can be found in the Appendix.

The low-quality firm sets the quality level to the required minimum quality level. The high-quality firm’s optimal response is to raise its quality level to sustain product differentiation. The introduction of the minimum quality standard increases both quality levels ($s_C^H > s_H$, $s_C^L > s_L$).

Both quality levels increase in the minimum quality standard, with the increase in the quality level of the low-quality firm exceeding the increase in the quality level of the high-quality firm ($0 < \frac{\partial s_C^H}{\partial S} < \frac{\partial s_C^L}{\partial S}$). Thus, an increase in the standard decreases the quality difference.

The introduction of the minimum quality standard increases both prices ($p_C^H > p_H$, $p_C^L > p_L$), as firms skim off consumers’ higher willingness to pay (consumers are willing to pay more for higher quality). In addition, firms’ cost increase in quality. Both prices increase in the minimum quality standard. An increase in the standard decreases the price differential ($\frac{\partial p_C^H}{\partial S} > \frac{\partial p_C^L}{\partial S} > 0$).

The introduction of the minimum quality standard shifts demand from the high-quality firm to the low-quality firm, due to the increase in quality levels ($q_C^H < q_H$, $q_C^L > q_L$). An increase in the standard enhances this demand-shifting effect ($\frac{\partial q_C^H}{\partial S} < 0$, $\frac{\partial q_C^L}{\partial S} > 0$). Proposition 1 summarizes the effect of a minimum quality standard for both firms meeting the standard.

Proposition 1 Suppose a “tough” minimum quality standard is introduced and both firms comply with the standard. Then the standard i) increases both quality levels, ii) increases both prices,
and iii) shifts demand from the high-quality to the low-quality firm. An increase in the standard i) decreases the quality difference, ii) decreases the price difference, and iii) enhances the demand-shifting effect.

Proof. See Appendix. ■

2.3 Minimum Quality Standard and Non-Compliance

Now assume that the regulatory authority cannot monitor compliance with the standard perfectly. Violations against the standard are detected with probability \( \phi \). If discovered by the government, products not complying with the standard are confiscated.\(^4\) Prices, quality levels, and quantities can be found in the Appendix. The low-quality firm does not comply with the standard. This is profitable \((\pi_{NC}^L > \pi_{C}^L)\), for many combinations of market size \( b \), detection probability \( \phi \) and levels of the minimum quality standard \( S \) (see Figures 6-9 in the Appendix). So in this paper non-compliance is not a mere assumption, but an endogenous result of profit-maximizing behavior.

Consumers may distinguish products of a higher quality level from products of a lower quality level, but cannot observe, whether the standard is fulfilled. Alternatively, they do not base their purchase decisions on standard fulfillment, because they either have a lower preference for safety or environmental issues than the standard-setting authority, or because they do not expect that a product not meeting all respective requirements completely will cause safety concerns.

Firms’ profits are given as

\[
\begin{align*}
\pi_{NC}^H &= \left( p_{NC}^H - s_{NC}^H \right) q_{HC}^NC, \\
\pi_{NC}^L &= \left( (1 - \phi) p_{NC}^L - s_{NC}^L \right) q_{LC}^NC.
\end{align*}
\]

In response to the low-quality firm not complying with the standard, the high-quality firm lowers its quality level to the required minimum quality level. The choice of a higher quality level is never optimal for the high-quality firm. So in this setting the minimum quality standard imposed by the government defines in fact a maximum quality level available on the market. Quality levels are strategic complements. So non-compliance with the standard of the low-quality firm reduces both quality levels \((s_{HC}^NC < s_{HC}^C, s_{LC}^NC < s_{LC}^C)\), the average quality level and the maximum quality available in the market.

An increase in the minimum quality requirement under non-compliance increases both quality levels, with the quality level of the low-quality firm increasing by less than the quality level of the high-quality firm \((0 < \frac{\partial s_{HC}^NC}{\partial S} < \frac{\partial s_{HC}^C}{\partial S})\). This is, in contrast to the case of full compliance, the quality difference increases in the standard \((\frac{\partial s_{HC}^NC}{\partial S} > 0)\).

Non-compliance of the low-quality firm increases the price for the high-quality product and

\(^4\)In the EU, this is one of the common options for action by the member states’ customs authorities, if a product presents a serious risk.
decreases the price for the low quality product \((p^{NC}_H > p^{NC}_L, p^{NC}_L < p^{NC}_H)\). Both prices increase in the standard. An increase in the standard raises the price difference \(\left(\frac{\partial p^{NC}_H}{\partial S} > \frac{\partial p^{NC}_L}{\partial S}\right)\).

Non-compliance of the low quality firm induces a lower price for low-quality products because of the lower willingness to pay for lower quality and lower cost. The price for the high quality product \(p_H\) depends positively on the quality difference \(\Delta s\), since a higher relative quality level leads to a competitive advantage of the high-quality firm, whereas \(p_L\) depends negatively on \(\Delta s\).

The increase of the price difference is lower than the increase of the quality difference due to non-compliance.

Non-compliance of the low-quality firm increases the quantity of the high-quality product and decreases the quantity of the low-quality good \((q^{NC}_H > q^{C}_H, q^{NC}_L < q^{C}_L)\). Thus, non-compliance has the opposite effect compared to the introduction of a minimum quality standard: it shifts demand from the low-quality firm to the high-quality firm. An increase in the standard, however, again shifts demand from the high-quality firm to the low-quality firm \((\frac{\partial q^{NC}_H}{\partial S} < 0, \frac{\partial q^{NC}_L}{\partial S} > 0)\).

Proposition 2 summarizes the effect of non-compliance with the minimum quality standard by the low-quality firm.

**Proposition 2** Suppose the low-quality firm does not comply with the minimum standard. Then non-compliance i) reduces both quality levels, ii) causes the high-quality firm to set its quality level according to the minimum quality level, iii) increases the price for the high-quality product and decreases the price for the low-quality product, and iv) shifts demand from the low-quality firm to the high-quality firm. An increase in the standard i) increases the quality difference, ii) increases the price difference, and iii) shifts demand from the high-quality to the low-quality firm.

**Proof.** See Appendix. ■

### 3 Government Policies

This section compares the consequences of two government policies: Raising the minimum quality standard and stricter enforcement of an existing minimum quality standard. Prices, quality levels, and quantities can be found in the Appendix.

#### 3.1 Raising the Minimum Quality Standard

Governments regularly raise minimum quality levels in order to strengthen consumer protection or environmental policy. Thus, it is important to analyze the effects of raising the minimum quality standard in both cases, compliance and non-compliance. Quality levels of both types of the product, \(H\) and \(L\), are strategic complements. An increase in the minimum quality standard \(S\) leads to an increase of both quality levels \(s_H\) and \(s_L\) under compliance and under non-compliance. It causes both prices to rise in the case of compliance as well as in the case of non-compliance. This can be explained by a higher willingness to pay for the increased quality
level and by higher variable cost for producing products of a higher quality level. An increase in the standard decreases the quantity of the high-quality product.

In the case of compliance, the low-quality firm sets its quality level to the minimum quality level $S$. So an increase in $S$ has a direct effect on $s_L$. The best response of the high-quality firms to an increase in $S$ and $s_C^L$ is an increase of $s_C^H$. An increase in $S$ decreases the difference in quality levels, as the low-quality firm increases quality by more than the high-quality firm. The high-quality firm already has a relatively high quality level. Since costs are convex in quality, costs increase faster than the marginal willingness to pay. The price difference between high-quality products and low-quality products decreases due to an increase in $S$. Since the high-quality firm increases quality by less then the low-quality firm, it also increases prices by less. The quantity of the high-quality product decreases, the quantity of the low-quality product increases in an increase of the minimum quality standard.

Under non-compliance, which seems to be the more realistic case, an increase in $S$ has a direct effect on $s_{NC}^H$, because the $H$-firm meets exactly the minimum quality standard. Via best response of the $L$-firm this causes an increase of $s_{NC}^L$. An increase in $S$ causes an increase of the difference in quality levels, because the high-quality firm increases quality more than the low-quality firm. Compared to the case of compliance, the high-quality firm realizes a lower quality level, so the marginal cost of an increase of the quality level of is lower. The low quality firm faces the risk that a share of its production gets confiscated. This leads to a lower incentive to invest in quality.

The price difference between high-quality products and low-quality products increases due to an increase in $S$, since the low quality firm invests in quality by less. In addition, the ability to increase prices following an increase in costs due to higher quality (but not fulfilling the minimum quality standard) is dampened by the risk of non-complying products being detected and confiscated by the market authorities. An increase in $S$ shifts demand from the high-quality firm to the low-quality firm.

Proposition 3 summarizes the main results

**Proposition 3** An increase in the minimum quality level leads to an increase of both quality levels and both prices under compliance as well as under non-compliance. The quantity of the high-quality product decreases, the quantity of the low-quality product increases in an increase of the minimum quality standard. Under compliance, an increase in the minimum quality level causes a decrease of the difference in quality levels and a decrease of the price difference between the high-quality product and the low-quality product. Under non-compliance, an increase of the minimum quality level causes an increase of the difference in quality levels and an increase of the price difference between the high-quality product and the low-quality product.

**Proof.** See Appendix. ■
3.2 Stricter Enforcement

The regulatory authority may also spent more resources to increase the detection probability \( \phi \) in order to reduce the share of non-compliant products to increase the average quality levels of products that are available on the market. This is one of the main ideas of the “Product Safety and Market Surveillance Package” proposed by the European Commission (European Commission, 2013a).

An increase in government enforcement decreases the quality level of the low-quality firm: 
\[
\frac{\partial q_{NC}}{\partial \phi} < 0.
\]

The reason for this is that an increase of the detection probability \( \phi \) makes it less profitable for the low quality firm to invest in quality, as a larger share of products is confiscated.

The quality level of the high-quality firm remains unchanged due to stricter enforcement.

A higher level of government enforcement leads to a price increase of the high-quality product and a price decrease of the low-quality product, if the maximum willingness to pay for quality is sufficiently high \(^5\). It shifts demand from the low-quality firm to the high quality firm.

Proposition 4 summarizes the effect of increased government enforcement.

**Proposition 4** Suppose the low-quality firm does not comply with the minimum standard. An increase in government enforcement i) decreases the quality level of the low-quality product, ii) increases the price for the high-quality product and decreases the price for the low-quality product, if the maximum willingness to pay for quality is high enough, and iii) shifts demand from the low-quality firm to the high-quality firm.

**Proof.** See Appendix.

3.3 Choice of Policy Instrument

An increase of \( S \) and an increase of \( \phi \) may be seen as policy substitutes under non-compliance, both with the aim to increase the quality of products on the market. But both instruments have different consequences on the regulatory authority, the quality of non-compliant products, prices, and quantities of compliant and non-compliant products.

An increase in \( S \) is free of cost from the government’s point of view. It increases the average quality and increases the lowest quality level. The last effect is important, if a low quality level is associated with a risk for consumers, because it makes low-quality products more safe. An increase in \( S \) increases prices. In addition, it shifts demand from high-quality products to low quality products.

An increase in \( \phi \), in contrast, causes an increase in government spending for market surveillance. It decreases the average quality level and the lowest quality level available on the market, while the quality of the high quality product remains unchanged. This effect again is important, if a low quality level is associated with a risk to consumers, because it makes low-quality prod-

\(^5\) \( \frac{\partial q_{NC}}{\partial \phi} < 0 \) if \( b \geq 1.7 \).

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ucts more unsafe. It increases prices of for the high-quality goods and decreases prices for the low-quality good. It shifts demand from the low-quality products to the high-quality products.

If the intention of the government is to guarantee a minimum quality level of all products sold on the market, an increase in $S$ seems to be favorable to an increase in $\phi$, because it increases both average and lowest quality level. If, however, the intention of the government is protection of high-quality firms, an increase of $\phi$ is the best instrument. If consumers buying the low-quality products are at risk or pollute the environment, while there is no such risk or pollution effect associated with high-quality products increasing $\phi$ might be preferable, because it shifts demand to the high quality firm, although it makes low-quality products even worse and more money is spent. Of course, this does not hold when it is the increased detection probability that induces a decrease of $s_L$ below a critical level and thereby causes a new risk.

In addition, higher level of government enforcement may misguide consumers to have more confidence in all products available on the market. Therefore a higher level of government enforcement should be accompanied by an additional program that raises the awareness for safety problems of consumer products.

4 Welfare Analysis

A minimum quality standard could be motivated by a negative externality like environmental harmful emissions or risks to third parties that depends on the quality level of both products. Assume the regulator’s payoff $R = - (\eta - \alpha q_H s_H - \alpha q_L s_L)$, where $\eta$ denotes the externality and $\alpha$ denotes the effect of the product quality on reducing the externality from the regulator’s perspective. Profits, consumer surplus, the regulator’s payoff, and welfare can be found in the Appendix.

If both firms comply with the minimum quality standard, the minimum quality standard lowers profits for both firms, increases consumer surplus, if $S < S^*$, and decreases consumer surplus, if $S > S^*$. The introduction of the minimum quality standard increases the regulator’s payoff. The minimum quality standard increases (decreases) welfare, if the effect of increased quality levels on the externality $\alpha$ is sufficiently high (low). An increase in the minimum quality standard decreases profits of both firms, decreases consumer surplus, but increases the regulator’s payoff. It increases total welfare, if $\alpha$ is sufficiently high.

Compared to the case of full compliance, non-compliance of the low-quality firm increases the $H$-firm’s profits. Depending on the detection probability $\phi$, non-compliance may also increase profits for the low-quality firm – otherwise, it would comply (see figures 6-9 in the Appendix). Non-compliance reduces consumer surplus. Non-compliance also decreases the and the regulator’s payoff, if the market size $b$ is sufficiently small. But for a sufficiently large market size, the regulator’s payoff under non-compliance may exceed the payoff under compliance, if the detection probability $\phi$ is sufficiently high.

\footnote{Similar results would hold, if $\alpha$ is the weight of the regulator’s payoff in the social welfare function.}
If the market size $b$ is sufficiently small and if $S$ is sufficiently high, welfare under non-compliance may exceed welfare under compliance, if $S$ is sufficiently high. If the effect of the product quality on the externality $\alpha$ is sufficiently high and $S$, and $\phi$ are sufficiently low, welfare is lower under non-compliance compared to the case of compliance. Welfare increases in market size $b$, because the regulator’s payoff increases in market size.

There is a dynamic effect of an increase in the standard: A higher standard increases the incentive for the low-quality firm for non-complying behavior for a given detection probability $\phi$. So it may be the increase in the minimum quality standard that may cause the switch of the low-quality firm from compliance to non-compliance. This switch also changes the behavior of the high-quality firm, which may also lower its product quality to the level of the minimum quality standard.

Consumers benefit from compliance with a minimum quality standard. The group of consumers as a whole loses due to non-compliance. But subgroups of consumers are affected differently by non-complying behavior. Three subgroups can be identified: The first group consists of consumers, who buy high-quality products under compliance and buy high-quality products under non-compliance. They receive a lower quality product, but pay a higher price. In the second group, consumers buy low-quality products under compliance and buy high-quality products under non-compliance. They obtain the same quality level ($s_{CL}^C = S = s_{NC}^{HC}$) but pay a higher price. The third group buys low-quality products under compliance and still buys low-quality products under non-compliance. They obtain a lower quality ($s_{CL}^C < s_{NC}^{HC}$), but also pay a lower price. This is the only group of consumers that might gain under non-compliance, provided that the low quality does not imply damaging effects for consumers.

Proposition 5 summarizes the main results.

**Proposition 5** If both firms comply with the minimum quality standard, its introduction i) lowers profits for both firms, ii) increases consumer surplus, if $S < S^*$, and decreases consumer surplus, if $S > S^*$, iii) increases the regulator’s payoff, and iv) increases (decreases) total welfare, if $\alpha$ is sufficiently high. Non-compliance of the low quality firm i) increases profits of the high quality firm and the low quality firm, iii) reduces consumer surplus and iv) reduces (increases) the regulator’s, if the market size is sufficiently small (if the market size and the detection probability are sufficiently high), and increases (decreases) welfare, if the market size $b$ is sufficiently small (large) and $\alpha$ is sufficiently low (high) and $\phi$ and $S$ are sufficiently high (low).

**Proof.** See Appendix.

5 Conclusion

In this paper, we have studied the effect of non-compliance with a minimum quality standard on prices and quality levels in a vertical differentiation model. We have assumed a duopolistic market structure with a high-quality firm and a low-quality firm. Non-complying behavior is an endogenous result of our model. Since in many markets non-compliance with quality standards
frequently occurs, our results offer some insight in the effectiveness of standard-setting and/or increasing the detection probability.

Non-compliance by the low-quality firm reduces both quality levels. It shifts demand from the low-quality to the high-quality firm. An increase in the minimum quality standard increases the quality difference, increases the price difference, and shifts demand from the high-quality to the low-quality firm. Non-compliance by the low quality firm also increases profits of the high quality firm, affects the quality level of the high-quality product negatively, reduces consumer surplus, and increases or decreases welfare, depending on the market size, the effect of quality levels of the externality, the detection probability and the minimum quality level. Consumers are not affected equally by non-compliance. While on average consumers lose due to non-compliance, a subgroup of consumers might benefit.

Under non-compliance, an enhancement of the minimum quality standard induces an increase of the quality level of low-quality products, increases the average quality level but also increases the quality difference. If the government is interested in raising the average quality of available products, this strategy may be preferable.

A higher level of government enforcement, however, lowers the quality of the low quality firm, but shifts demand from the low-quality firm to the high-quality firm. If consumers buying the low-quality products are at risk or pollute the environment (and would be also at risk or pollute the environment under an enhanced minimum quality standard under non-compliance), while there is no such risk or pollution effect associated with high-quality products, increasing ϕ might be preferable, because it shifts demand from the low-quality firm to the high quality firm.

The main idea of the “Product Safety and Market Surveillance Package” proposed by the European Commission is to increase the probability of detecting non-compliant products. Our results, however, show that a higher level of government enforcement has a negative effect on the quality level of low-quality products. It also increases the probability, that non-complying behavior is welfare-increasing. Therefore maybe it should be accompanied by an additional program that raises the awareness of consumer to problems of non-compliant products.
References


[14] and international effects, Nota di lavoro 20.93, Fondazione ENI Enrico Mattei, Milan.


Appendix

No Regulation

\[ s_H = \frac{4b+1}{8}, \quad s_L = \frac{4b-5}{8} \]
\[ p_H = \frac{25+8b+16b^2}{64}, \quad p_L = \frac{49-40b+16b^2}{64} \]
\[ \Delta s = s_H - s_L = \frac{3}{2} \]
\[ \Delta p = p_H - p_L = \frac{9b+3}{8} \]
\[ q_H = q_L = \frac{1}{2} \]

Minimum Quality Standard and Compliance

Introduction of a Minimum Standard

\[ S \geq s_H = \frac{4b+1}{8} \]
\[ S \leq S_{\text{max}} = \frac{b+1}{2} \]
\[ s_H^C = \frac{1+b-S}{3}, \quad s_L^C = S \]
\[ \Delta s^C = s_H^C - s_L^C = \frac{b+1-2S}{3} \]
\[ p_H^C = \frac{5(8b+2)}{27} \]
\[ p_L^C = (b+1)(7-2b)+2S(4b-5)+19S^2 \]
\[ q_H^C - p_H = \frac{(8S-4b+5)(88S+28b-71)}{1728} > 0, \text{ if } S > S_{\text{min}} \text{ and } b > b_{\text{min}}. \]
\[ q_L^C - p_L = \frac{(8S-4b+5)(152S+140b-175)}{1728} > 0, \text{ if } S > S_{\text{min}} \text{ and } b > b_{\text{min}}. \]
\[ \Delta p^C = p_H^C - p_L^C = \frac{(b+1-2S)(4S+17b-2)}{27} \]
\[ q_H^C = \frac{2(b+1)-4S}{9}, \quad q_L^C = \frac{7-2b+4S}{9} \]
\[ q_H^C - q_L^C = -\frac{8S-4b+5}{18} < 0, \text{ if } S > S_{\text{min}}. \]
\[ q_H^C = q_L^C = \frac{8S-4b+5}{18} > 0, \text{ if } S > S_{\text{min}}. \]

Increase in Minimum Standard

\[ \frac{\partial s^C}{\partial S} = \frac{1}{4} > 0 \]
\[ \frac{\partial p_H^C}{\partial S} = \frac{2(11S-b-1)}{27} > 0, \text{ if } S > S_{\text{min}} \]
\[ \frac{\partial p_L^C}{\partial S} = \frac{2(19S+4b-5)}{27} > 0 \]
\[ \frac{\partial (q_H^C - q_L^C)}{\partial S} = \frac{-2(8S+5b-4)}{27} < 0, \text{ if } S > S_{\text{min}} \text{ and } b > b_{\text{min}} \]
\[ \frac{\partial q_H^C}{\partial S} = -\frac{4}{9} < 0 \]
\[ \frac{\partial q_L^C}{\partial S} = \frac{4}{9} > 0 \]

Minimum Quality Standard and Non-Compliance

Compliance vs. Non-Compliance

\[ s_H^{NC} = S, \quad s_L^{NC} = \frac{4S-\Omega - (1-\phi)(2-b)}{6}, \]
with \( \Omega = \sqrt{(1-\phi)^2 (b-2)^2 - 4S (1 - \phi) (b - 2) + 4S^2 (3\phi + 1)}. \)
\[
\frac{d\xi}{dS} = 4S(3\phi+1) - 2(1-\phi)(b-2) > 0, \text{ if } S > S_{\text{min}} \\
\text{for } S_{\text{min}}, \Omega = \Omega_{\text{min}} = \frac{1}{4} \sqrt{3(27-47\phi) + 48b\phi(b+2) + 16\phi^2(b-2)^2 + 2}, \text{ if } b > \alpha_{\text{min}} \\
\text{for } S_{\text{max}}, \Omega = \Omega_{\text{max}} = \sqrt{9(1-\phi) + 3b\phi(b+4) + \phi^2(b-2)^2 + 2}, \text{ if } b > \alpha_{\text{min}} \\
\Delta_{S_{\text{NC}}} - \Delta_{S_{C}} = \frac{(6S + \Omega - 3b + \phi(b-2))}{6} > 0 \\
P_{H}^{NC} = \left(1-\phi\right)(b+5) \Omega + 4S(1-\phi)(b+4) + 2S^2(23-15\phi) - (1-\phi)^2(b+5)(b-2) \\
P_{H}^{N} = \frac{5(1-\phi)^2(2-b)^2 - 5(1-\phi)(b-2)\Omega - 8S\Omega - 2S(1-\phi)(b-2) + 2S^2(19-3\phi)}{54(1-\phi)} \\
\Delta_{1} = (1-\phi)(\phi(2b+5)(b-2) - 3b(4b+7)) \\
\Delta_{2} = 2S(3(1-\phi)(2b+1) + 4S(3-\phi)) \\
\Delta_{3} = \Omega((1-\phi)(2b+5) - 4S) \\
\Delta_{2} = 2S(3(1-\phi)(2b+1) + 4S(3-\phi)) > 0, \text{ min}(\Delta_{2}) \text{ for } S_{\text{min}}, \text{ max}(\Delta_{2}) \text{ for } S_{\text{max}} \\
\text{min}(\Delta_{3}) \text{ for } S_{\text{min}} \wedge S < S_{*} = \frac{(1-\phi)(2b+5)}{4}, \text{ max}(\Delta_{3}) \text{ for } S_{\text{max}} \wedge S < S_{*} = \frac{(1-\phi)(2b+5)}{4} \\
\text{I. } \text{min}(\Delta_{2}) \text{ and } \text{min}(\Delta_{3}) \text{ for } S_{\text{min}} \\
\Delta_{1} + \Delta_{2} + \Delta_{3} \\
\text{II. } \text{max}(\Delta_{2}) \text{ and } \text{max}(\Delta_{3}) \text{ for } S_{\text{max}} \\
\text{if } \phi > \phi^{*} = \frac{3(6-40b+736b^2+384b^3-665) + 15\sqrt{44806+4928b^2+768b^4+104b^6}}{8(b+5)(-70b+48b^2+32b^3-65)} > 0, b > \frac{3}{8}, \\
\text{if } S > S^{*} = \frac{(1-\phi)(2b+5)}{4}, \text{ if } S < \tilde{S} \\
\text{If } b = 3, \phi = 0.5, \tilde{S} \approx 1.6352. \\
\text{For } S_{\text{max}}, \text{ min}(\Delta_{1}) \text{ for } S_{\text{max}} \\
\text{if } b = 3, \phi = 0.5, \tilde{S} \approx 1.6352. \\
\text{For } S_{\text{max}}, \text{ min}(\Delta_{1}) \text{ for } S_{\text{max}} \\
\text{if } b = 3, \phi = 0.5, \tilde{S} \approx 1.6352. \\
\text{For } S_{\text{max}}, \text{ min}(\Delta_{1}) \text{ for } S_{\text{max}}
\[
\begin{align*}
\min(\Delta_2) \text{ for } S_{\min}^{\text{min}} & \iff S > S^* = \frac{3(1-\phi)(3b-4)}{16\phi}, \\
\max(\Delta_2) \text{ for } S_{\min}^{\max} & \iff S > S^* = \frac{3(1-\phi)(3b-4)}{16\phi}, \\
S^* - S_{\min}^{\text{max}} & = \frac{3(3b-4) - \phi(17b-4)}{16\phi} > 0 \text{ if } \phi < \frac{3(3b-4)}{17b-4} \\
\Delta_3 & = (1-\phi)\left(3(2-10b+3b^2) - 5\phi(b-2)^2\right) > 0, \\
\text{if } b > b^* & = \frac{5(3-2\phi)+3\sqrt{19-16\phi}}{9-5\phi}.
\end{align*}
\]

I. \(\min(\Delta_1)\) and \(\min(\Delta_2)\) for \(S_{\min}\)
\[
\Delta_1 - \Delta_2 - \Delta_3 = \frac{9(3b-4)-3\phi(35b+4b^2-38)-20\phi(2b-2)\phi^2+(9(3b-2)-5\phi(b-2))\Omega_{\min}}{4} > 0,
\]
if \(\phi < 1\) and \(b \geq b_{\min}\).

II. \(\max(\Delta_1)\) and \(\max(\Delta_2)\) for \(S_{\max}\)
\[
\Delta_1 - \Delta_2 - \Delta_3 = 9(3b-2) - 3\phi(2b^2+10b) - 5\phi(2b-2)^2 + (3b-2)-5\phi(b-2))\Omega_{\max} > 0,
\]
if \(\phi < 1\) and \(b \geq b_{\min}\).

\[
\begin{align*}
p^N_C & < p^L_C, \\
qu^N_C & = \frac{(1-\phi)(4b+1)-8\phi+2\Omega}{9(1-\phi)}, \\
qu^L_C & = \frac{8\phi - 2\Omega - 4(1-\phi)(b-2)}{9(1-\phi)}, \\
qu^N_C - q^L_C & = \frac{\Delta_1 - \Delta_2 - \Delta_3}{9(1-\phi)},
\end{align*}
\]
with \(\Delta_1 = (1-\phi)(2b-1), \Delta_2 = 4S(\phi+1), \Delta_3 = 2\Omega.
\]
\[
\begin{align*}
\Delta_1 & = (1-\phi)(2b-1) > 0, \\
\Delta_2 & = 4S(\phi+1) > 0, \min(\Delta_2) \text{ for } S_{\min}, \max(\Delta_2) \text{ for } S_{\max} \\
\Delta_3 & = 2\Omega > 0, \min(\Delta_3) \text{ for } S_{\min}, \max(\Delta_3) \text{ for } S_{\max} \\
\text{I. } & \min(\Delta_2) \text{ and } \min(\Delta_3) \text{ for } S_{\min} \\
\Delta_1 - \Delta_2 + \Delta_3 & = 4\Omega_{\min} - \phi - 8b\phi - 3 > 0, \text{ if } \phi < 1 \\
\text{II. } & \max(\Delta_2) \text{ and } \max(\Delta_3) \text{ for } S_{\max} \\
\Delta_1 - \Delta_2 + \Delta_3 & = 2\Omega - 4b\phi - \phi - 3 > 0, \text{ if } \phi < 1 \\
qu^N_C - q^L_C & = \frac{\Delta_1 - \Delta_2 + \Delta_3}{9(1-\phi)}, \\
\text{cf. } q^N_C & > q^L_C, \\
qu^N_C - q^L_C & < q^C_C,
\end{align*}
\]
\[
\text{with } \Delta_1 = (1-\phi)(2b-1), \Delta_2 = 4S(\phi+1), \Delta_3 = 2\Omega.
\]
\[
\text{Increase in Minimum Standard under Non-Compliance}
\]
\[
\frac{\partial S_{\min}^{N}}{\partial b} = \frac{\Delta_1 - \Delta_2 + \Delta_3}{3\Omega},
\]
with \(\Delta_1 = (1-\phi)(b-2), \Delta_2 = 2S(\phi+1), \Delta_3 = 2\Omega.\)
\[ \Delta_1 = (1 - \phi)(b - 2) > 0, \text{if } b > 2, \]
\[ \Delta_2 = 2S(3\phi + 1) > 0, \text{min}(\Delta_2) \text{ for } S_{\text{min}}, \text{max}(\Delta_2) \text{ for } S_{\text{max}}, \]
\[ \Delta_3 = 2\Omega > 0, \text{min}(\Delta_3) \text{ for } S_{\text{min}}, \text{max}(\Delta_3) \text{ for } S_{\text{max}}. \]

I. \text{min}(\Delta_2) \text{ and } \text{min}(\Delta_3) \text{ for } S_{\text{min}}
\[ \Delta_1 - \Delta_2 + \Delta_3 = \frac{3M_{\text{min}} - 9 + 5\phi - 10b\phi}{4} > 0 \text{ if } \phi < 1. \]

II. \text{max}(\Delta_2) \text{ and } \text{max}(\Delta_3) \text{ for } S_{\text{max}}
\[ \frac{\partial S_{\text{NC}}}{\partial S} > 0 \]
\[ \frac{\partial (S - S_{\text{NC}})}{\partial S} = \Delta_1 - \Delta_2 + \Delta_3, \]
with \[ \Delta_1 = \Omega, \Delta_2 = (1 - \phi)(b - 2), \Delta_3 = 2S(3\phi + 1). \]
\[ \Delta_1 = \Omega > 0, \text{min}(\Delta_1) \text{ for } S_{\text{min}}, \text{max}(\Delta_1) \text{ for } S_{\text{max}}, \]
\[ \Delta_2 = (1 - \phi)(b - 2) > 0, \]
\[ \Delta_3 = 2S(3\phi + 1) > 0, \text{min}(\Delta_3) \text{ for } S_{\text{min}}, \text{max}(\Delta_3) \text{ for } S_{\text{max}} \]
\[ \text{min}(\Delta_1) \text{ and } \text{min}(\Delta_3) \text{ for } S_{\text{min}} \]
\[ \text{min}(\Delta_1) \text{ and } \text{min}(\Delta_3) \text{ for } S_{\text{max}} \]
\[ \frac{\partial S_{\text{NC}}}{\partial S} = \frac{\Delta_1 - \Delta_2 - \Delta_3}{2(1 - \phi)\Omega}, \text{ with } \]
\[ \Delta_1 = \Omega(2S(23 - 15\phi) + (1 - \phi)(4b + 1)), \]
\[ \Delta_2 = (1 - \phi)^2(4b + 1)(b - 2), \]
\[ \Delta_3 = 2S(8S(3\phi + 1) - (1 - \phi)(8b - 7 + 3\phi(2b + 5))) \]
\[ \Delta_1 = \Omega(2S(23 - 15\phi) + (1 - \phi)(4b + 1)) > 0, \]
\[ \text{min}(\Delta_1) \text{ for } S_{\text{min}}, \text{max}(\Delta_1) \text{ for } S_{\text{max}}, \]
\[ \Delta_2 = (1 - \phi)^2(4b + 1)(b - 2) > 0, \text{if } b > 2, \]
\[ \Delta_3 = 2S(8S(3\phi + 1) - (1 - \phi)(8b - 7 + 3\phi(2b + 5))) > 0, \]
\[ \text{if } S > S^* = \frac{(1 - \phi)(8b - 7 + 3\phi(2b + 5))}{8(3\phi + 1)}, \]
\[ S^* - S_{\text{max}} = \frac{4b - 11 - 2\phi(7b - 5) - 3\phi^2(2b + 5)}{8(3\phi + 1)} > 0, \]
\[ \text{if } \phi < \frac{\sqrt{(73b + 70)(b - 2) - 7b + 5}}{4(2b + 5)}, \]
\[ \text{min}(\Delta_3) \text{ for } S_{\text{min}}, \text{max}(\Delta_3) \text{ for } S_{\text{max}} \]

I. \text{min}(\Delta_1) \text{ and } \text{min}(\Delta_3) \text{ for } S_{\text{min}}
\[ \Delta_1 - \Delta_2 - \Delta_3 = \frac{(4b + 1)(4M_{\text{min}}(27 - 19\phi) - 12\phi(2b - 1) - 4\phi^2(10b + 7))}{16} > 0, \]
\[ \text{if } \phi < 1 \]

II. \text{max}(\Delta_1) \text{ and } \text{max}(\Delta_3) \text{ for } S_{\text{max}}
\[ \Delta_1 - \Delta_2 - \Delta_3 = (27b + 24 - \phi(19b + 16))\Omega_{\text{max}} - 3(3 - 2\phi) - 6b\phi(b + 3) - \phi^2(14b + 10b^2 + 13) > 0, \]
\[ \text{if } \phi < 1 \]
\[ \frac{\partial S_{\text{NC}}}{\partial S} > 0 \]
\[
\frac{\partial \rho^{NC}}{\partial S} = \Delta_1 - \Delta_2 + \Delta_3, \text{ with } \\
\Delta_1 = (1 - \phi)^2 (b - 2)^2, \\
\Delta_2 = 2S (16S (3\phi + 1) - (7 - 15\phi) (1 - \phi) (b - 2)), \\
\Delta_3 = \Omega (2S (19 - 3\phi) - (1 - \phi) (b - 2)). \\
\] 

\[
\Delta_1 = (1 - \phi)^2 (b - 2)^2 > 0, \\
\Delta_2 = 2S (16S (3\phi + 1) - (7 - 15\phi) (1 - \phi) (b - 2)) > 0, \\
\text{if } S > S^* = \frac{(7 - 15\phi)(1 - \phi)(b - 2)}{16(3\phi + 1)}, \\
S^* - S^\max = \frac{b + 2\phi (23b - 10) - 15\phi^2 (b - 2)}{16(3\phi + 1)} < 0, \\
\min(\Delta_2) \text{ for } S^\min, \max(\Delta_2) \text{ for } S^\max. \\
\Delta_3 = \Omega (2S (19 - 3\phi) - (1 - \phi) (b - 2)) > 0 \text{ if } S > S^* = \frac{(1 - \phi)(b - 2)}{2(19 - 3\phi)}, \\
S^* - S^\max = \frac{-21 + 18\phi - 9(2b + 5)}{2(19 - 3\phi)} < 0, \\
\min(\Delta_3) \text{ for } S^\min, \max(\Delta_3) \text{ for } S^\max. \\
I. \ \min(\Delta_2) \text{ and } \min(\Delta_3) \text{ for } S^\min, \\
\Delta_1 - \Delta_2 + \Delta_3 \\
= \left( \frac{3(6b + 7) - \phi (2b + 5)}{16} \right) \Omega^\max - 9(3b + 2) - 6\phi (3b + 8b^2 - 2) + \phi^2 (16b + 13) (b - 2) > 0, \\
\text{if } \phi < 1. \\
II. \ \max(\Delta_2) \text{ and } \max(\Delta_3) \text{ for } S^\max, \\
\Delta_1 - \Delta_2 + \Delta_3 \\
= \left( 3 \left[ (6b + 7) - \phi (2b + 5) \right] \Omega^\max - 9(3b + 2) - 6\phi (3b + 8b^2 - 2) + \phi^2 (16b + 13) (b - 2) \right) > 0, \\
\text{if } \phi < 1, \\
\frac{\partial \rho^{NC}}{\partial S} > 0, \\
\frac{\partial (\rho^{NC} - \rho^{NC})}{\partial S} = \frac{\Delta_1 - \Delta_2 + \Delta_3}{2(1-\phi)^2}, \text{ with } \\
\Delta_1 = \Omega (8S (1 - 3\phi) + (1 - \phi) (5b - 1)), \\
\Delta_2 = (1 - \phi)^2 (5b - 1) (b - 2), \\
\Delta_3 = 2S (8S (3\phi + 1) + 7 + 2\phi (10b - 11) - 3\phi^2 (7b - 5)), \\
\Delta_1 = \Omega (8S (1 - 3\phi) + (1 - \phi) (5b - 1)) > 0, \\
\min(\Delta_1) \text{ for } S^\min, \max(\Delta_1) \text{ for } S^\max, \\
\Delta_2 = (1 - \phi)^2 (5b - 1) (b - 2) > 0, \\
\Delta_3 = 2S (8S (3\phi + 1) + 7 + 2\phi (10b - 11) - 3\phi^2 (7b - 5)) > 0, \\
\min(\Delta_3) \text{ for } S^\min, \max(\Delta_3) \text{ for } S^\max. \\
\text{min}(\Delta_1) \text{ and min}(\Delta_3) \text{ for } S^\min, \\
\Delta_1 - \Delta_3 + \Delta_3 \\
= \left( \frac{81b + 3\phi (8b^2 - 2) - \phi^2 (13b + 1) (8b - 7) + 4\Omega^\min (8b - \phi (17b + 2))}{4} \right) > 0, \\
\text{if } b \geq \frac{\rho^\min}{\partial (\rho^{NC} - \rho^{NC})} > 0, \\
\frac{\partial \rho^{NC}}{\partial S} = \frac{\Delta_1 - \Delta_2 + \Delta_3}{9(1 - \phi)^2}, \\
\text{with } \Delta_1 = (1 - \phi) (b - 2), \Delta_2 = 2S (3\phi + 1), \Delta_3 = 2\Omega.
\[
\frac{\partial s_{NC}}{\partial S} > 0
\]
\[
\frac{\partial s_{NC}}{\partial \phi} < 0
\]
\[
\frac{\partial s_{NC}}{\partial S} = \frac{4 \Delta_1 - \Delta_2 + \Delta_3}{9(1-\phi)^2},
\]
with \(\Delta_1 = (1-\phi)(b-2), \Delta_2 = 2S(3\phi + 1), \Delta_3 = 2\Omega\)
\[
\frac{\partial s_{NC}}{\partial \phi} > 0
\]

**Government Enforcement**

\[
\frac{\partial s_{NC}}{\partial \phi} = -\Delta_1 + \Delta_2 - \Delta_3
\]
with \(\Delta_1 = 2S(3S + b - 2), \Delta_2 = \Omega(b - 2), \Delta_3 = (b - 2)^2 (1 - \phi)\).
\[
\Delta_1 = 2S(3S + b - 2) > 0, \text{ if } S > S^{\min}, \text{ min}(\Delta_1) \text{ for } S^{\min}, \text{ max}(\Delta_1) \text{ for } S^{\max},
\]
\[
\Delta_2 = \Omega(b - 2) > 0, \text{ min}(\Delta_2) \text{ for } S^{\min}, \text{ max}(\Delta_2) \text{ for } S^{\max},
\]
\[
\Delta_3 = (b - 2)^2 (1 - \phi) > 0,
\]
\[
\text{min}(\Delta_1) \text{ and min}(\Delta_2) \text{ for } S^{\min}
\]
\[
\Delta_1 + \Delta_2 - \Delta_3 = \frac{966-141-128(\phi(b-1)+16\phi^2(2b+3)+32(b-2)\Omega^{\min}}{32} > 0,
\]
if \(\phi < 1 \land b \geq 6\phi^{\min}\),
\[
\frac{\partial s_{NC}}{\partial \phi} < 0
\]
\[
\frac{\partial s_{NC}}{\partial \phi} = \frac{\Delta_1 + \Delta_2 + \Delta_3 - \Delta_4}{64(1-\phi)^2}, \text{ with } \Delta_4 = (1 - \phi)^2 (2b + 5)(b - 2)(2S - (1 - \phi)(b - 2)), \Delta_2 = \Omega\left((1-\phi)^2 (2b + 5)(b - 2) + 16S^2\right), \Delta_3 = 2S^2(1 - \phi)(10b + 7 - 3\phi(2b + 5)), \Delta_4 = 8S^3(3\phi + 5).
\]
\[
\Delta_1 = (1 - \phi)^2 (2b + 5)(b - 2)(2S - (1 - \phi)(b - 2)) > 0 \text{ if } S > S^{\min}, \text{ min}(\Delta_1) \text{ for } S^{\min}, \text{ max}(\Delta_1)
\]
for \(S^{\max}\)
\[
\Delta_2 = \Omega\left((1-\phi)^2 (2b + 5)(b - 2) + 16S^2\right) > 0, \text{ min}(\Delta_2) \text{ for } S^{\min}, \text{ max}(\Delta_2) \text{ for } S^{\max}
\]
\[
\Delta_3 = 2S^2(1 - \phi)(10b + 7 - 3\phi(2b + 5)) > 0, \text{ if } \phi < \phi^{\min} = \frac{10b + 7}{66 + 115}, \text{ min}(\Delta_3) \text{ for } S^{\min}, \text{ max}(\Delta_3)
\]
for \(S^{\max}\)
\[
\Delta_4 = 8S^3(3\phi + 5) > 0, \text{ min}(\Delta_4) \text{ for } S^{\min}, \text{ max}(\Delta_4) \text{ for } S^{\max}
\]
\[\text{I. } \text{min}(\Delta_1), \text{ min}(\Delta_2), \text{ min}(\Delta_3), \text{ and } \text{min}(\Delta_4) \text{ for } S^{\min}
\]
\[
\Delta_1 + \Delta_2 + \Delta_3 - \Delta_4 = 4\sqrt{3(27-47\phi+48\phi^2(b+2)+16\phi^2(2b+5))\left(3(4b+8S^2-13)-4\phi(2b+5)(b-2)(2-\phi)\right)}
\]
\[
+ \frac{27(8b+16b^2-53)-9\phi(10b+208b^2+64b^3-457)}{64}
\]
\[
+ \frac{25^2(2b+5)(352b-16b^2-397)+64\phi^2(2b+5)(b-2)^2}{64} > 0
\]
\[\text{II. } \text{max}(\Delta_1), \text{ max}(\Delta_2), \text{ max}(\Delta_3), \text{ and } \text{max}(\Delta_4) \text{ for } S^{\max}
\]
\[
\Delta_1 + \Delta_2 + \Delta_3 - \Delta_4 = \frac{2\sqrt{9(1-\phi)+3b(2b+4)+\phi^2(2b+5)(b-2)^2(2-\phi)}}{2}
\]
\[
+ \frac{9(b^2-7)-6\phi(19b+17b^2+3b^3-22)}{2}
\]
\[
- \frac{-\phi^2(2b+5)(-28b+b^2+25)+2\phi^2(2b+5)(b-2)^2}{2} > 0
\]
\[
\frac{\partial s_{NC}}{\partial \phi} > 0
\]
\[ \frac{\partial \rho^{NC}}{\partial \phi} = -\frac{\Delta_1 - \Delta_2 + \Delta_3 + \Delta_4}{\mu_4 \phi (\phi - 1)^2 \Omega}, \text{ with } \Delta_1 = (1 - \phi)^2 (b - 2)^2 (2S - (1 - \phi) (b - 2)), \]
\[ \Delta_2 = \Omega \left( 32S^2 - 5 (1 - \phi)^2 (b - 2)^2 \right), \Delta_3 = 2S^2 (7 - 15\phi) (1 - \phi) (b - 2), \Delta_4 = 16S^2 (3\phi + 5) \]
\[ \Delta_1 = (1 - \phi)^2 (b - 2)^2 (2S - (1 - \phi) (b - 2)) > 0 \text{ if } S > S_{\text{min}}, \min(\Delta_1) \text{ for } S_{\text{min}}, \max(\Delta_1) \text{ for } S_{\text{max}} \]

for \( S_{\text{max}} \)
\[ \Delta_2 = \Omega \left( 32S^2 - 5 (1 - \phi)^2 (b - 2)^2 \right) > 0, \min(\Delta_2) \text{ for } S_{\text{min}}, \max(\Delta_2) \text{ for } S_{\text{max}} \]
\[ \Delta_3 = 2S^2 (7 - 15\phi) (1 - \phi) (b - 2) > 0, \min(\Delta_3) \text{ for } S_{\text{min}}, \max(\Delta_3) \text{ for } S_{\text{max}} \]
\[ \Delta_4 = 16S^2 (3\phi + 5) > 0, \min(\Delta_4) \text{ for } S_{\text{min}}, \max(\Delta_4) \text{ for } S_{\text{max}} \]
I. \( \min(\Delta_1), \min(\Delta_2), \min(\Delta_3), \text{ and } \min(\Delta_4) \text{ for } S_{\text{min}} \)
\[ \Delta_1 - \Delta_2 + \Delta_3 + \Delta_4 = \frac{9(37b-16b^2-48b^3-31)+\phi(2b+5)(122b-64b^5-157)+\phi^2(b-2)(448b+176b^5-385)}{2} + \frac{32\phi^2(b-2)^2(2\Omega - (1 - \phi)(b - 2))^2}{3} > 0 \text{ if } b \geq 1.7 \]

II. \( \max(\Delta_1), \max(\Delta_2), \max(\Delta_3), \text{ and } \max(\Delta_4) \text{ for } S_{\text{max}} \)
\[ \Delta_1 - \Delta_2 + \Delta_3 + \Delta_4 = \frac{9(5b+22b^5+9b^6+10)-2(\phi(b+4)-(2b+4b^5+1)+\phi^2(b-2)(52b+11b^5-13)}{2} + \frac{2\phi^2(b-2)^2(2\Omega - (1 - \phi)(b - 2))^2}{3} > 0 \text{ if } b \geq 1.5 \]
\[ \frac{\partial \rho^{NC}}{\partial \phi} < 0 \text{ if } b \geq 1.7 \]
\[ \frac{\partial \rho^{NC}}{\partial \phi} = -4S \frac{(3\phi + 5) - 2\phi - (1 - \phi)(b - 2)}{9(\phi - 1)^2 \Omega} < 0 \]
\[ \frac{\partial \rho^{NC}}{\partial \phi} = 4S \frac{(3\phi + 5) - 2\phi - (1 - \phi)(b - 2)}{9(\phi - 1)^2 \Omega} > 0 \]
\[ \frac{\partial \rho^{NC}}{\partial \phi} = 4S \frac{\Delta_1 - \Delta_2 - \Delta_3}{9(\phi - 1)^2 \Omega}, \]

with \( \Delta_1 = S(3\phi + 5), \Delta_2 = 2\Omega, \Delta_3 = (1 - \phi)(b - 2) \)
\[ \Delta_1 = S(3\phi + 5) > 0, \min(\Delta_1) \text{ for } S_{\text{min}}, \max(\Delta_1) \text{ for } S_{\text{max}}, \]
\[ \Delta_2 = 22\Omega > 0, \min(\Delta_2) \text{ for } S_{\text{min}}, \max(\Delta_2) \text{ for } S_{\text{max}}, \]
\[ \Delta_3 = (1 - \phi)(b - 2) > 0, \text{ if } b > 2 \]
I. \( \min(\Delta_1) \text{ and } \min(\Delta_2) \text{ for } S_{\text{min}} \)
\[ \Delta_1 - \Delta_2 - \Delta_3 = \frac{2(4b+7)+\phi(20b-13)-16\Omega_{\text{min}}}{8} > 0, \]
if \( \phi < 1 \text{ and } b \geq b_{\text{min}} \).

II. \( \max(\Delta_1) \text{ and } \max(\Delta_2) \text{ for } S_{\text{max}} \)
\[ \Delta_1 - \Delta_2 - \Delta_3 = \frac{2(5b+3)+\phi(5b-1)-4\Omega_{\text{max}}}{2} > 0 \]
if \( b \geq b_{\text{min}} \)
\[ \frac{\partial \rho^{NC}}{\partial \phi} > 0 \]
\[ \frac{\partial \rho^{NC}}{\partial \phi} = -4S \frac{\Delta_1 - \Delta_2 - \Delta_3}{9(\phi - 1)^2 \Omega}, \]

with \( \Delta_1 = S(3\phi + 5), \Delta_2 = 2\Omega, \Delta_3 = (1 - \phi)(b - 2) \)
\[ \text{cf. } \frac{\partial \rho^{NC}}{\partial \phi}, \frac{\partial \rho^{NC}}{\partial \phi} < 0 \]
Welfare

No Regulation

\[ \pi_H = \pi_L = \frac{3}{16} \]

\[ CS = \int_0^b (\theta s_H - p_H) \, d\theta + \int_a^b (\theta s_L - p_L) \, d\theta \]

\[ = \frac{16b^2 - 16b - 23}{64} \]

\[ R = \alpha \left( \frac{2b-1}{4} \right) - \eta \]

\[ W = \pi_H + \pi_L + CS + R \]

\[ = \frac{117b^2 - 16b}{64} + \alpha \left( \frac{2b-1}{4} \right) - \eta \]

Minimum Quality Standard and Compliance

\[ \pi^C_H = \frac{4(b-2S+1)^2}{243} \]

\[ \pi^C_L = \frac{4(b-2S+1)^3}{243} \]

\[ \pi^C_H - \pi^C_L = \frac{(8S-4b+5)(8S(8S-8b-17)+68b+16b^2+1332)}{3888} < 0 \]

if \( S > S^\min \)

\[ \frac{\partial \pi^C_H}{\partial S} = -\frac{8(b-2S+1)^2}{81} < 0 \]

if \( S > S^\min \)

\[ \frac{\partial \pi^C_L}{\partial S} = \frac{(b-2S+1)(4S-2b+7)^2}{81} \]

\[ \pi^C_L - \pi^C_L = -\frac{(8S-4b+5)(8S(8S-8b+19)-76b+16b^2-11)}{3888} < 0 \]

if \( S > \frac{4b+1}{8} \)

\[ \frac{\partial \pi^C_L}{\partial S} = \frac{2(4S-2b+1)(4S-2b+7)}{81} < 0 \]

if \( S > \frac{4b+1}{8} \)

\[ CS^C = \frac{2(b+1)(2b+2b^2-61)+S(3(96b-29-8b^2)+2S(24b-147-16bS))}{15552} \]

\[ CS^C - CS = \frac{(8S-4b+5)(54Sb-325b^2+337-8S(16S-16b^2+137))}{15552} > 0, \]

if \( S < S^* \)

\[ \frac{\partial CS^C}{\partial S} = -\frac{4S(8S-8b+40)+8b^2+29-98b}{162} < 0, \]

if \( S > S^\min \)

\[ WC^C = \frac{4(5b-4)(b+1)^2+S(110b-40b^2-73)+10S(24b-39-16bS)}{15552} \]

\[ WC^C - W = -\left( 7008S^2-3504b+3840bS^2-7680S^2-12480S^2+12480b^2+5120S^2+3120b^2-640b^2+28943 \right) < 0, \]

if \( S > S^\min \)

\[ \frac{\partial WC^C}{\partial S} = -\frac{20S(8S-8b+13)-130b+40b^2+73}{162} < 0, \]

if \( S > S^\min \)

\[ RC^C = \frac{S(8S-8b+19)+2(b+1)^2}{27} - \eta \]

\[ \frac{\partial RC^C}{\partial S} = \frac{16S(8b-19)}{45} > 0 \text{ if } S > S^\min \]

\[ \frac{\partial RC^C}{\partial b} = \frac{4(4b-5)(2b-7)-4S(8b-8S+49)}{108b} > 0 \text{ if } S > S^\min \]

\[ WC^C = \frac{4(5b-4)(b+1)^2+S(110b-40b^2-73)+10S(24b-39-16bS)}{15552} + \alpha \frac{S(8S-8b+19)+2(b+1)^2}{27} - \eta \]
\[ W^C - W = \frac{(7008S - 3504b + 3840S^2 - 7680S^2b - 12480Sb + 12480S^2 + 5120S^3 + 3120b^2 - 640b^3 + 28943)}{108} \]

\[ \delta W^C \frac{dS}{dS} = -\frac{20S(8S - 8b + 13) - 130b + 40b^2 + 73}{102} > 0 \text{ if } \alpha > \frac{108(32S^2S - 3900b - 240S^2 + 160S^2 + 219) + 3120b^2 - 3504b - 640b^3 + 28943}{15552(4S - 2b + 17)(1S - 4b + 5)} \]

**Minimum Quality Standard and Non-Compliance**

\[ \pi^N_C = \frac{(8S - 2b - (1 - \phi)(4b + 1))((1 - \phi)^2(2b + 5)(b - 2) + 2S(4S(1 - 3\phi) - (1 - \phi)(4b + 1)) + \Omega(4S - (1 - \phi)(2b + 5)))}{486(1 - \phi)^2} \]

\[ \pi^C_H - \pi^C_H = \frac{64S^3(2 - 5\phi + \phi^2) - 24S^2(1 - \phi^2)(8b + 5) + 16S(3\phi + 1) + 6S(1 - \phi)^2(20b + 16b^2 - 5) - 8S\Omega((1 - \phi)(2b + 5) + \Omega)}{486(1 - \phi)^2} \]

Numerical examples are used to illustrate the effect: Assuming \( b = 2, b = 5, b = 10, b = 15, \) or \( b = 20 \) yields the result \( \pi^H_C - \pi^C_H > 0. \)

\( b = 2, b = 5, b = 10, b = 20: \) \( \pi^H_C - \pi^C_H > 0 \)

\[ \pi^H_C - \pi^C_L = \frac{6S(1 - \phi)^2(1 - \phi)(2b - 5) + 4S(1 - \phi)^2((1 - \phi)(b - 2) + 1)) + 4(180b + 4S^2 + 8S^2 + 17)}{243(1 - \phi)^2} \]

\[ \pi^L_C = \frac{6S(1 - \phi)^2(2b + 5) + 4S(1 - \phi)^2((1 - \phi)(b - 2) + 1)) + 4(4b + 16b^2 + 30b + 1)}{243(1 - \phi)^2} \]

\[ \pi^H_C - \pi^C_L = \frac{6S(1 - \phi)^2((1 - \phi)^2(b^2 - 2) + 4S(1 - \phi)(b^2 - 2) + 1)) + 4(180b + 4S^2 + 8S^2 + 17)}{243(1 - \phi)^2} \]

\[ CS^N C = \frac{(1 - \phi)^2(b^2 - 13 + 2b - 2) + 4S((1 - \phi)(b^2 - 13 + 2b - 2) + 1)) + 4(180b + 4S^2 + 8S^2 + 17)}{243(1 - \phi)^2} \]

\[ CS^N C = \frac{6S(1 - \phi)^2((1 - \phi)^2(b^2 - 13 + 2b - 2) + 1)) + 4S((1 - \phi)(b^2 - 13 + 2b - 2) + 1)}{243(1 - \phi)^2} \]

Numerical examples are used to illustrate the overall effect: Assuming \( b = 2, b = 5, b = 10, b = 15, \) or \( b = 20 \) yields the result: \( CS^N C - CS^C < 0. \)

\[ R^N C = \frac{2(1 - \phi)(1 - \phi)^2\Omega(2S - (2b - 1)(1 - \phi) + 3b + 11\Omega + 8S)}{27(1 - \phi)} \]

\[ R^N C - R^C = \frac{2(1 - \phi)(1 - \phi)^2\Omega(2b - 1)(1 - \phi) - 8S((1 - \phi)(b - 2)(1 - \phi))(1 - \phi) - 2S(1 - \phi)(b - 2)(1 - \phi))}{27(1 - \phi)} \]

For \( b = 2: \) \( R^N C - R^C < 0 \)
Figure 1: $R^{NC} - R^C$, $b = 5$

Figure 2: $R^{NC} - R^C$, $b = 10$
\[ W_{NC}^{\phi} = 2\Omega((\phi-1)^2(b-2)(5b-1-2\phi(b-2))+2S(2S(3\phi+1)(5-2\phi)-(1-\phi)(10b-11+4\phi(b-2))))-2(1-\phi)^2(b-2)^2(5b-1-2\phi(b-2)) \\
+ 8S(\phi-1)^2\left(106b+20\phi^2-49-8\phi(b-2)^2\right)+2S\left(8S(1-\phi)(5-2\phi)-3(1-\phi)(20b+53+19b-68\phi+24\phi^2(b-2))\right) \\
+ \alpha \frac{-2(b-2)^2(1-\phi)-2\Phi(2S-\Omega-(b-2)(1-\phi))+S(8b+11)(1-\phi)-8S}{27(1-\phi)} - \eta \\
\]

\[ W_{NC}^{\rho} - W_{C}^{\rho} = \frac{\Omega(1-\phi)^2(b-2)(5b-1-2\phi(b-2)) - (1-\phi)^2\left(186-98^2+15b^2-12-\phi(b-2)^2(76-5+2\phi(b-2))\right)}{243(1-\phi)^2} \\
+ \frac{8S^2(1-\phi)^2(3\phi+1)(5-2\phi) - 2S\left(120b+766\phi-240b\phi^2-339\phi^2+120b\phi^2-235\right) - 2S(1-\phi)(10b-11+4\phi(b-2))}{243(1-\phi)^2} \\
+ \frac{8S^2(3\phi+1)(5-2\phi)-S^2\left(120b+766\phi-240b\phi^2-339\phi^2+120b\phi^2-235\right) - 2S(1-\phi)(10b-11+4\phi(b-2))}{243(1-\phi)^2} \\
+ \alpha \frac{2(1-\phi)(\phi(b-2)^2-(2\phi^2+2b+5)) - 8S(2S(2\phi)-2b-1)(1-\phi)-9(2S-\Omega-(b-2)(1-\phi))}{27(1-\phi)} \]
Figure 4: $W^{NC} - W^C, b = 2, \alpha = 1$

Figure 5: $W^{NC} - W^C, b = 2, \alpha = 2$

$b = 5, b = 10, b = 15 \ W^{NC} - W^C > 0$

**Incentive to non-comply**

$$
\pi_{L}^{NC} - \pi_{L}^{C} = \frac{64S^3(1-\phi)-48S^2(1-\phi)(2-\phi)(b-2)+4S\Omega(3\phi+1)+6S(1-\phi)(-32\phi+8\phi^2+23-4\phi(b-2)^2)}{243(1-\phi)}
+ \frac{2\Omega(1-\phi)(b-2)((1-\phi)(b-2)+\Omega)-4S\Omega(2(1-\phi)(b-2)+\Omega)-(1-\phi)(606-48\phi^2+8\phi^3+17+4\phi(b-2)^3(\phi-2))}{243(1-\phi)}
$$
Figure 6: $\pi^{NC}_L - \pi^C_L$, $b = 2$

Figure 6 shows the incentive for the low quality firm to comply or to not comply with the minimum quality standard depending on combinations of values of $S$ and $\phi$ for a given maximum willingness to pay for quality $b = 2$. In the area $I$ the firm has an incentive for non-compliance, as $\pi^{NC}_L - \pi^C_L > 0$. In the area $NI$ the firm has no incentive for non-compliance, as $\pi^{NC}_L - \pi^C_L < 0$. Given the market size of $b = 2$, a higher level of $S$ requires a higher detection probability $\phi$ in order to avoid non-compliance behavior of the low quality firm.
Figure 7 is similar. The market size is now set to $b = 5$. Irrespective of the detection probability a higher level of the minimum quality standard is needed to trigger non-compliance.

For a market size of $b = 10$, the picture becomes more complex (see Figure 8). For most combinations of $\phi$ and $S$ there is an incentive for non-compliance. Compared to lower levels of $b$ a higher minimum quality level $S$ is needed to trigger non-compliance. Only for very modest
values of \( \phi \) and for very strict levels of government enforcement combinations of \( \phi \) and \( S \) exist, where compliance is the best strategy for the low-quality firm.

For a market size of \( b = 15 \), again a higher level of \( S \) is needed to trigger non-compliance. But once this level is reached, non-compliance is the dominant strategy for the low-quality firm for most combinations of \( \phi \) and \( S \). The NI-area is smaller than in Figure 8.