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Working Paper
The Housing Wealth Effect on Consumption Reconsidered

IMK Working Paper, No. 115

Provided in Cooperation with:
Macroeconomic Policy Institute (IMK) at the Hans Boeckler Foundation


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Fabian Lindner

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June 2013

Abstract

Much of the literature on the effect of housing wealth on consumption has been embedded in a simple life-cycle model in which housing price changes work as a „wealth effect“. In such models, windfall gains in housing always lead to positive changes in consumption. However, this might be a fallacy of composition. Such models ignore that changes in housing wealth have distributional consequences between those planning to sell their house and those planning to buy a house. Further, since most housing is not simply financed out of current cash holdings but by mortgages, the institutions on mortgage markets have to be considered when looking at the „wealth effect“ of housing. In this paper, a model is presented from which the classic Ando-Modigliani consumption function augmented by housing wealth can be deduced. It is shown that the deeper structural model from which this equation is deduced implies that changes in housing wealth are not necessarily positively correlated with consumption. It will be argued that changes both in demographics (the composition of the age groups in the population) as well in mortgage markets have led to a structural break in the effect of housing wealth on consumption in the mid-1980s in the US. In the empirical part of the paper, a vector auto-regressive model is estimated and impulse-response functions are computed that show that housing wealth changes did negatively affect consumption before the mid-1980s and positively only afterward.

JEL ref.: C32, E21, E44, R31.

Keywords: Housing, Consumption, Wealth Effect, Demographics, Mortgage Markets.

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The Housing Wealth Effect on Consumption Reconsidered

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October 2, 2013, corrected version

Abstract

Much of the literature on the effect of housing wealth on consumption has been embedded in a simple life-cycle model in which housing price changes work as a "wealth effect". In such models, windfall gains in housing always lead to positive changes in consumption. However, this might be a fallacy of composition. Such models ignore that changes in housing wealth have distributional consequences between those planning to sell their house and those planning to buy a house. Further, since most housing is not simply financed out of current cash holdings but by mortgages, the institutions on mortgage markets have to be considered when looking at the "wealth effect" of housing. In this paper, a model is presented from which the classic Ando-Modigliani consumption function augmented by housing wealth can be deduced. It is shown that the deeper structural model from which this equation is deduced implies that changes in housing wealth are not necessarily positively correlated with consumption. It will be argued that changes both in demographics (the composition of the age groups in the population) as well in mortgage markets have led to a structural break in the effect of housing wealth on consumption in the mid-1980s in the US. In the empirical part of the paper, a vector auto-regressive model is estimated and impulse-response functions are computed that show that housing wealth changes did negatively affect consumption before the mid-1980s and positively only afterward.

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Introduction

The study of the influence of housing wealth on consumption has gained much interest since the steady rise in housing prices since the mid-1990s - and even more so since the fall in housing prices that led to the global financial crisis [Duca et al. 2011]. Especially the paper by Case et al. (2005) gained much prominence and did start a new interest in the housing wealth effect on consumption. The authors find significant positive effects of housing wealth on consumption. Many other authors have reached similar results (Benjamin et al., 2004; Carroll et al., 2006; Kundan, 2007).

Most of the research on the effect of housing wealth on consumption is conducted in the framework of the theory of the “wealth effect”, going back to the classic life cycle hypothesis (LCH) formulated by Ando and Modigliani (1963). The LCH states that wealth is accumulated by households to maintain a relatively constant level of consumption in the face of varying income over the life cycle. The “wealth effect” is one corollary of that hypothesis, namely that households consume out of wealth and that changes in the prices of their accumulated assets may influence consumption.

However, it is not clear that a change in asset prices always has beneficial effects on consumption. Asset price changes do not necessarily make all consumers better off since they have distributional consequences. Economic agents that own the asset gain by an increase in asset prices while those planning to purchase the asset are worse off (Attanasio et al., 2011; Attanasio and Weber, 1995; Li and Yao, 2007). Those distributional effects are likely to be larger for housing than for financial assets since the demand for housing is less elastic. Owner-occupied housing is not only an asset but also a durable consumption good that provides essential housing services (Fernández-Villaverde and Krueger, 2011). Financial assets do not provide consumption services so that the price elasticity of demand of financial assets is likely to be higher.

Also, since one can buy small units of financial assets, no credit financing is necessary for acquiring those assets. However, most housing is financed by a mortgage loan, especially by first-time buyers. Thus, while house price increases benefit home owners who plan to sell their house, they might lead first-time buyers to save more for their down payment, thus possibly depressing their consumption. The net effect of housing price changes is thus not clear ex ante (Bajari et al., 2005; Muellbauer and Lattimore, 1995).

The theoretical literature on the distributional consequences on housing price changes is still rather small. Li and Yao (2007) have developed and simulated a life-cycle model with distributional consequences of housing price changes and their consequences for
household welfare. They found large welfare losses due to housing price increases in the USA for renters and young owners who plan to upgrade their housing stock over the life cycle. Old homeowners with high housing equity gain and middle aged owners are hardly affected since they neither plan to upgrade nor to downgrade their housing stock.

Kiyotaki et al. (2011) develop and simulate a general equilibrium model and estimate welfare changes between net sellers and net buyers of housing wealth for Japan. They find that net buyers have large welfare costs if housing prices increase.

Both papers focus on welfare costs, i.e. changes in utility due to housing price changes and less on the classic macroeconomic question of the effect of housing value changes on consumption. However, this is what Attanasio et al. (2011) do. They calibrate aggregate and age-specific consumption of households with different housing equity and simulate income and housing price shocks for the UK. They are mostly interested in the effect of housing prices on homeowners and less on the effects on households that plan to increase their housing stock. The authors consider the endogenous effect of housing price changes on homeowner rates with a given credit constraint so that some households planning to buy housing cannot afford it and thus abstain from buying.

However, Attanasio et al. do not consider the effect of “target-saving” of household, i.e. targeted saving for the downpayment and a potential inelasticity of housing demand by such savers. This motive can be very important for US household saving (Browning and Lusardi, 1996).

For instance, Sheiner (1995) finds that US potential first time buyers are more likely to increase their saving with higher housing prices and do not abstain from buying. She estimates that downpayment saving accounts for a quarter of household saving. Engelhardt (1996) also finds such saving to be very important. This is in contrast to other countries. For Japan, Yoshikawa and Ohtake (1989) find that households tend to abstain from buying their house when housing prices increase. It might be the case that the behavior of potential first-time buyers also depends on cultural influences.

This paper focuses on down payment saving and develops a simple, partial-equilibrium overlapping generations model. In the model, young households save for their down payment based on the expected value of a house they plan to buy. When they are in their middle age, they buy their house from the current old generation and use their accumulated down payment saving. When they are old, they sell their house to the current middle aged. A positive housing price shock has distributional consequences between the current members of the different generations. Old homeowners who trade down their housing unambiguously gain from higher housing prices. However, since housing price increases change the actual down payment relative to the expected down payment for
the middle aged, this generation is forced to save more and consume less if their demand for housing is not elastic.

The sign of the effect of housing price changes on consumption is not clear ex ante. In the model, the overall effect depends on demographics and financial market institutions. More liberalized financial markets will lead to lower down payments for first-time buyers (Ortalo-Magné and Rady, 1999). The lower the required minimum down payment is, the lower additional saving will have to be with a given unexpected housing price shock. This mitigates the negative effect of higher housing prices on consumption. Demographics change the ratio of middle-aged to old households. With a given housing stock, the existence of more old households relative to young households in the economy will lead to a more positive relation between housing and consumption. Then, more households profit from the realization of capital gains than lose.

While the model uses a borrowing constraint for housing, it does not allow households to use their house as a collateral for non-housing borrowing, an aspect of housing that has been widely discussed in the literature (Aoki et al., 2004; Iacoviello, 2004). The decision to leave that aspect out of the model has a theoretical and an empirical reason.

The theoretical reason is that collateralized borrowing for consumption does not constitute a wealth effect. The wealth effect measures changes in housing net worth, i.e. housing assets minus mortgage debt. If households increase their mortgage debt in line with the increased value of the housing asset, housing net worth would not increase. Then, one would observe varying housing values, mortgages and consumption but not varying housing equity. However, this is not what can be observed in the US. Figure 1 shows that housing equity changes and that most of the change is caused by changes in housing values which are not counteracted by mortgage credit.

Also, empirical studies have shown that mortgage equity extraction does not seem to finance higher paths of aggregate consumption. Studies found that mortgage equity extraction might finance consumption in the case of an adverse income shock, e.g. due to an unemployment shock, but not an increase of overall consumption (Cooper, 2009; Hurst and Stafford, 2004; Klyuev and Mills, 2007). Thus, equity extraction works as an insurance against income shocks (Lustig and Van Nieuwerburgh, 2005). Also, taking out a mortgage secured by a house might not only be used to finance consumption, but also in order to improve the housing asset. This use of mortgages has been extensive (Canner et al., 2002; Greenspan and Kennedy, 2008).

The implications of the model for aggregate US consumption will be tested using a structural vector autoregressive model and by computing impulse-response functions. This is done to show how shocks of housing value, labor income and financial wealth

The lack of a co-integration relationship between the variables is likely to reflect the changes in demography and financial market institutions that are argued to influence the relation between housing and consumption. The estimation shows that there was a negative influence of changes in housing wealth on consumption before the mid-1980s and a positive influence only after the mid-1980s. Thus, the housing wealth effect is highly context specific.

The paper is structured as follows. In the next part, the overlapping generation model with three generations will be presented. From this model an aggregate consumption function augmented by net housing wealth will be deduced. It will be shown that its deeper structure implies that unanticipated housing wealth changes are not unambiguously positively correlated with consumption. In the second part, a vector autoregressive model will be estimated and impulse-response functions will be computed. A final part will conclude.
1 An overlapping generations model of the housing market

Here, an overlapping generation model with three generations is developed with housing and a composite consumption good. In the model, young households rent and save for their house, middle aged households buy their house and pay off their mortgage, and old households sell their house and realize possible capital gains. Households maximize both non-housing and housing consumption over the life-cycle. The middle aged households buy the complete housing stock from the old and rent out housing to both the young and the old households. Changes in housing prices affect both middle aged households who buy their house and old households who realize capital gains.

The interesting dynamic of the model comes from young households’ saving decision. They form expectations about the housing price that will be realized once they enter their middle-age and save accordingly. However, if housing prices increase once households want to buy their house, they are worse off and have to reduce their consumption in order to pay the higher down payment.

The strength of the effect depends on the down payment ratio, i.e. the own funds relative to the housing value they have to come up with. The lower the down payment ratio, the less they will have to save additionally in the period and the less will housing price increases affect their consumption. On the other hand, housing price increases will always have a positive effect on consumption for the old who realize their capital gains.

Another factor that influences the aggregate effect is the demographic situation. A higher proportion of old homeowners to middle aged first-time buyers will increase the positive correlation between housing wealth and consumption; a lower proportion will tend to decrease it.

The basic set-up is similar to the models developed by Brueckner and Pereira (1994, 1997). However, those authors neither derive an aggregate consumption function nor look at the distributional consequences of housing price changes between generations; they only look at two generations and do not model the mortgage market and especially liquidity constraints explicitly.

The structure of the model is exogenously imposed and not endogenously derived. Other models explicitly model the housing choice of different households given their income, preference of housing relative to consumption and borrowing constraints (Attanasio et al., 2009, 2011; Li and Yao, 2007). This is not done here in order to focus on the effect of a non-elastic housing demand given saving for down payments.

This somewhat inflexible approach is justified on empirical grounds. First-time buyers in the US do not buy smaller houses or abstain from buying but save more and
consume less when housing prices have increased (Engelhardt, 1996; Sheiner, 1995). For the middle-aged US consumers, housing does not seem to play an important role for their consumption because they are less likely to sell their house (Skinner, 1989).

As far as the old generation is concerned, Lehnert (2004) finds that the housing wealth effect for households shortly before retirement (52 to 62 years) was highest among different age groups he looked at, confirming that those who plan to trade down their stock of housing benefit most from a housing price increase. The model uses the insights of those micro-econometric studies and draws the implications for aggregate consumption.

Empirically, there is a clear life-cycle pattern present for owning homes in the US. Figure 2 shows the rate of ownership by age of the households’ heads in 1982, 1998 and 2005. Data before 1982 is not available. One can see that the basic age pattern of housing ownership has hardly changed since the early 1980s. The biggest difference is that in 2005 a higher proportion of older households did own their housing. On first sight, the model’s assumption that older households sell their house is not documented by the data.

However, the data only shows whether households own or not, not the value, i.e. either the size or price of their house. Figure 3 shows the mean value of primary
residences weighted by the percentage of families who own their house from 1989 - the earliest available data - until 2007. The data is taken from the Federal Reserve’s Survey of Consumer Finance. The data captures the total housing value, not just the homeowner rate, for each age group in the economy. A clear hump shaped life-cycle pattern is evident: older households reduce their housing so that they own houses with lower values. The different levels of value of houses in the period reflect the increase in housing prices, especially from 1995 until 2007. According to this date, the assumption of a hump shaped life-cycle behavior in the model makes sense.

An additional bequest motive in the model could be used to model the high ownership of old households ([Attanasio et al., 2011](https://www.jstor.org/stable/2477061)). However, such a motive would not mitigate the distributional consequences between credit constrained first-time buyers and old sellers. It would mitigate the strength of the effect: Old households would not gain since they would not sell their house and heirs would not have to save for their housing. Since according to the data presented in figure 3, downsizing takes place to a significant degree and somebody has to buy the houses that are sold, I stick with the assumption that households sell their house when they are old and leave no bequests.
The model

The three generations have a simple logarithmic, time-separable additive utility function. The utility function of the young is written:

\[ U^y_t = \ln(c^y_t) + \beta \ln(h^y_t) + (1 + \rho)^{-1}E_t[\ln(c^m_{t+1}) + \beta \ln(h^m_{t+1})] + (1 + \rho)^{-2}E_t[\ln(c^o_{t+2}) + \beta \ln(h^o_{t+2})] \]

(1)

Where the indices \( y \), \( m \), and \( o \) denote households’ consumption when young, when in the generation of the middle aged and when old, respectively. The subjective discount factor is \( \rho \) and the term \( \beta \) is a parameter for consumers’ tastes for housing relative to non-housing consumption. It is assumed that both parameters are the same irrespective of age. Consumers maximize both non-housing consumption \( c \) and housing services \( h \) throughout their lives.

Middle aged consumers maximize the same kind of utility function at time \( t \):

\[ U^m_t = \ln(c^m_t) + \beta \ln(h^m_t) + (1 + \rho)^{-1}E_t[\ln(c^o_{t+1}) + \beta \ln(h^o_{t+1})] \]

(2)

The old maximize:

\[ U^o_t = \ln(c^o_t) + \beta \ln(h^o_t) \]

(3)

There is a fixed housing stock which has to be shared by the young, the middle-aged and the old and which is normalized to 1 in order to ease the exposition:

\[ H = h^y_t + h^m_t + h^o_t = 1 \]

(4)

For each episode in their life, households face a period budget constraint. Young households receive labor income, \( y^y_t \), they rent housing, \( h^y_t \) at a rental rate \( R \), save in the form of financial assets, \( s^f_t \), and they save for the discounted expected down payment for their house, \( H \frac{\phi E_t(p_{t+1})}{1+r} \). Here, \( \phi \) is the percentage of the house value households have to put up in order to buy the house in period \( t + 1 \), the so called down payment ratio. Thus, the young’s period budget constraint reads:

\[ y^y_t - \frac{\phi E_t(p_{t+1})}{1+r} - s^f_t - c^y_t - R_t h^y_t = 0 \]

(5)
Note that the term $H$ has been skipped in the budget constraint because it is normalized to one. Total saving of young households, $s_t$, is equal to their financial saving and saving for the downpayment, so that:

$$s_t = \frac{\phi E_t(p_{t+1})}{1+r} + s_t^f = y_t^Y - c_t^Y - R_t h_t^Y$$  \hspace{1cm} (6)$$

At the beginning of their middle age, households buy a house. It is assumed that they buy the entire housing stock from the old even if prices change. That means that the change in the housing price is assumed never to be as high as to lead households to buy a smaller house or to abstain from buying.

Households in their middle age have to meet their mortgage payments with a mortgage rate, $r_m$, so that their mortgage payment is equal to the amount they pay for their house net of the down payment, i.e $r_m(1 - \phi)p_t$. Since the middle aged rent out part of their housing to the old and the young, they have both a rental income, $R_t$, and opportunity costs for living themselves in the house, $-R_t h_t^m$. Since, by (4) the housing used by the young, the middle aged and the old sum to one, one can write $R(1 - h_t^m) = R_t(h_t^y + h_t^o)$. Then, the period budget constraint of the middle-aged generation at time $t$ reads:

$$y_t^m + R_t(1 - h_t^m) + (1 + r)s_{t-1} - s_t - c_t^m - r_m(1 - \phi)p_t + \phi p_t = 0$$  \hspace{1cm} (7)$$

To pay the down payment, $\phi p_t$, middle aged households draw on their savings from the previous period, given by (6). Combining (6) and (7) and rearranging yields the period budget constraint of the middle-aged generation at time $t$:

$$y_t^m + R_t(1 - h_t^m) + (1 + r)s_t^f - s_t - c_t^m - r_m(1 - \phi)p_t + \phi(E_{t-1}(p_t) - p_t) = 0$$  \hspace{1cm} (8)$$

One can see that, if the young’s expectations from the previous period about housing prices differ from the actual price they have to pay - once they are in the middle age - the term $\phi(E_{t-1}(p_t) - p_t)$ is different from zero. If actual prices were higher, households will have to pay more for their house than expected; if it is lower, they will have to pay less than expected.

Finally, when households are old, they sell their house and pay off their mortgage. It is assumed that they live off their financial savings and do not receive other income:

$$p_t - (1 - \phi)p_{t-1} + (1 + r)s_t^f - c_t^o - R_t h_t^o = 0$$  \hspace{1cm} (9)$$
(1 − \phi)p_{t−1} is the amount of outstanding mortgage debt.

From the different period constraints, the life-cycle budget constraint for the young and the middle-aged can be computed. For the old, the life-cycle budget constraint is equal to their period budget constraint since they die at the end of the period and consume all their wealth.

For the young the life-cycle budget constraint is derived by combining the different period budget constraints (5), (8) and (9):

\[
\begin{align*}
   c^y_t + R_t h^y_t + \frac{c^m_{t+1} + R_{t+1} h^m_{t+1}}{(1 + r)} + \frac{c^o_{t+2} + R_{t+2} h^o_{t+2}}{(1 + r)^2} &= y^y_{t+1} + \frac{y^m_{t+1} + R_{t+1}}{(1 + r)} - \frac{E_t(p_{t+1})(1 - \phi)(r_m + (1 + r)^{-1})}{(1 + r)} + \frac{E_t(p_{t+2})}{(1 + r)^2} \\
y^y_t + \frac{y^m_{t+1} + R_{t+1}}{(1 + r)} - E_t(p_{t+1})(1 - \phi)(r_m + (1 + r)^{-1}) + E_t(p_{t+2})}{(1 + r)^2}
\end{align*}
\]  

The term \( E_t(p_{t+1})(1 - \phi)(r_m + (1 + r)^{-1}) \) is the expected total debt service that households will have to pay over their life-cycle and \( E_t(p_{t+2}) \) is the price of their house that they expect to get when they sell their house at the beginning of their old age.

Equivalently, the life-cycle budget constraint of the middle-aged is a substitution of (9) into (8):

\[
\begin{align*}
   c^m_t + R_t h^m_t + \frac{c^o_{t+1} + R_{t+1} h^o_{t+1}}{(1 + r)} &= y^m_{t} + R_t + (1 + r)s^{a}_{t-1} + \\
\phi(E_{t-1}(p_t) - p_t) - p_t(1 - \phi)(r_m + (1 + r)^{-1}) + \frac{E_t(p_{t+2})}{(1 + r)}
\end{align*}
\]  

After having bought their house, middle aged households have locked in an actual mortgage service and expect the selling value of their houses.

In order to derive the aggregate consumption function, the utility functions (1), (2) and (3) have to be maximized under the constraints (9), (10) and (11) (see the appendix for the derivation). Further, overall consumption depends on the share of each generation \( G \) in the whole population, \( pop \):

\[
\begin{align*}
1 = \frac{G^y + G^m + G^o}{pop} = g^y + g^m + g^o
\end{align*}
\]
Total per capita consumption at time $t$, $C_t/pop = c_t$, is the sum of consumption of all three generations at time $t$, weighted by their share in the population:

$$c_t = g^y c^y_t + g^m c^m_t + g^o c^o_t =$$

$$g^y mpc^y \left( y^y_t + \frac{y^m_{t+1} + R_{t+1}}{(1 + r)} - E_t(p_{t+1})d + E_t(p_{t+2}) \right) +$$

$$g^m mpc^m \left( y^m_t + R_t + (1 + r)s_{t-1}^{fa} + \phi(E_{t-1}(p_t) - p_t) - p_t d + \frac{E_t(p_{t+1})}{(1 + r)} \right) +$$

$$g^o mpc^o \left( p_t - (1 - \phi)p_{t-1} + (1 + r)s_{t-1}^{fa} \right)$$

(13)

The term $d$ stands for debt service:

$$d \equiv (1 - \phi)(r_m + (1 + r)^{-1})$$

(14)

The term $mpc$ stands for the marginal propensities to consume for the old, the middle generation and the young. They have been derived from the first order conditions (see appendix):

(15)  

$$mpc^o = (1 + \beta)^{-1}$$

(16)  

$$mpc^m = ((1 + \beta)(1 + (1 + \rho)^{-1}))^{-1}$$

(17)  

$$mpc^y = ((1 + \beta)(1 + (1 + \rho)^{-1} + (1 + \rho)^{-2}))^{-1}$$

Since the generations are assumed to be homogenous so that both $\beta$ and $\rho$ are equal for all generations, it is clear that $mpc^o > mpc^m > mpc^y$. The old will spend all of their income before they die while the young and the middle generation discount their future income at their subjective discount rate. The marginal propensity changes with the subjective discount rate $\rho$. An increase in $\rho$ will lead to a higher $mpc$ because households will discount their life-time income at a higher rate, i.e. they are less patient.

To close the model, a no-arbitrage condition has to be introduced that determines the price of housing. Under perfect competition, arbitrage should lead to the state in which costs (the debt service) and revenues (rent and the sale price) from the housing asset are the same so that, for the determination of $p_t$:

$$p_t = \left( R_t + (1 + r)^{-1}E_t(p_{t+1}) \right) d^{-1}$$

(18)
Assuming equilibrium \((p_t = E_{t-1}(p_t))\) and substituting the no-arbitrage condition \((18)\) into \((13)\) yields:

\[
\begin{align*}
    c_t &= g^y c_t^y + g^m c_t^m + g^o c_t^o = \\
    &\quad g^y mpc^y \left( y_t^y + \frac{y_{t+1}^y}{(1+r)} \right) + \\
    &\quad g^m mpc^m \left( y_t^m + R_t + (1 + r)(s_{t-1}^a) \right) + \\
    &\quad g^o mpc^o \left( p_t - (1 - \phi)p_{t-1} + (1 + r)s_{t-1}^a \right)
\end{align*}
\]

(19)

In equation \((19)\), the no-arbitrage condition leads to an elimination of the housing market terms for the young and the middle generation. The term \(\phi (E_{t-1}(p_t) - p_t)\) does not appear in \((19)\) because in equilibrium, expectations are fulfilled so that there is no difference between the actual price \(p_t\) and its expected value one period before, \(E_{t-1}(p_t)\). Housing prices are determined under the no arbitrage conditions \((18)\) so that a change in housing prices could only come about by changes in one of the variables of this condition.

Thus, if one assumes equilibrium in the system, housing prices would only play a role for the old. Note also that it is housing net wealth which matters, i.e. \(p_t - (1 - \phi)p_{t-1}\). The term \(p_t\) is the housing asset that old households hold and \((1 - \phi)p_{t-1}\) is the mortgage stock they have to pay back.

Now, it is straightforward to deduce the classic life-cycle consumption function from equation \((19)\). For simplicity, assume \(y_t^y\), \(y_t^m\) and \(y_t^o\) to be the same and equal to \(Y\) (they could also be expressed as multiples of each other, see Ando and Modigliani (1963)), then the classical life-cycle function can be written, augmented by housing net worth:

\[
C_t = \alpha_1 Y_t + \alpha_2 A_t + \alpha_3 (p_t - m_t)
\]

(20)

Where:

\[
\begin{align*}
    \alpha_1 &= g^y \left( mpc^y + \frac{mpc^y}{(1+r)} \right) + g^m mpc^m \\
    \alpha_2 &= (g^m mpc^m + g^o mpc^o)(1 + r) \\
    \alpha_3 &= g^o mpc^o \\
    A_t &= s_{t-1}^a \\
    m_t &= (1 - \phi)p_{t-1}
\end{align*}
\]
\( A_t \) is the stock of financial wealth at the beginning of the period that households have saved in the previous period. The term \( m_t \) captures the outstanding mortgage debt so that the third term in \( 20 \) is the housing net wealth of the household sector out of which they can consume.

On first inspection of the equation it seems that a change in housing prices at time \( t \) would only have an effect on the consumption of the old. But this is not the case because a surprise change in housing prices due e.g. to a change in expected housing prices also affects the middle generation through the term \( \phi(\mathbb{E}_{t-1}(p_t) - p_t) \) in equation \( 13 \). Higher or lower prices would lead to changes of what they have to pay for their house relative to what they have saved when they were young.

Where could an unexpected exogenous change in prices come from? Attanasio et al. (2011) and Li and Yao (2007) simply assume exogenous shocks but do not motivate them. In the model presented here, a housing price shock could be caused by any of the variables in the no-arbitrage condition, i.e. current rent, expected future house prices or the debt service. Since current rents are determined in the model, they are not exogenous. Only changes in future expected house prices or the debt service could thus be used as an exogenous shock.

To see how such a shock influences non-housing consumption, substitute the no-arbitrage condition into \( 13 \) but now without setting \( \mathbb{E}_t(p_t) \) equal to current actual housing prices:

\[
C_t = g^\nu m p c^\nu \left( y_t + \frac{y_{t+1}}{1+r} \right) +
\]

\[
g^m m p c^m \left( y_t^m + (1+r)s_{t-1}^{fa} + \phi \left( \mathbb{E}_{t-1}(p_t) - \left( R_t + (1+r)^{-1} E_t(p_{t+1}) d^{-1} \right) \right) \right) +
\]

\[
g^o m p c^o \left( (R_t + (1+r)^{-1} E_t(p_{t+1})) d^{-1} - (1-\phi)p_{t-1} + (1+r)s_{t-1}^{fa} \right)
\]

Differentiating aggregate consumption with respect to expected prices yields:

\[
\frac{\partial C_t}{\partial \mathbb{E}_t(p_{t+1})} = (-g^m m p c^m \phi + g^o m p c^o) (1+r)^{-1}
\]

Differentiating aggregate consumption with respect to the debt service yields:

\[
\frac{\partial C_t}{\partial d} = -(g^m m p c^m \phi + g^o m p c^o) (R_t + (1+r)^{-1} E_t(p_{t+1})) d^{-2}
\]

1Note that \( s_{t-1}^{fa} \) is the sum of both the financial savings of the middle aged and the old.
In both cases, the sign of the effect depends on the term \(-g^m \text{mpc}^m \phi + g^o \text{mpc}^o\) and thus on demographics, \(g^m\) and \(g^o\), and the down payment ratio, \(\phi\). Given the marginal propensity to consume, the higher the proportion of young households to old households and the higher the down payment ratio, \(\phi\), the more will the positive influence of housing price changes on consumption be mitigated. With a high down payment ratio and a young population the effect of housing price changes on consumption are likely to be negative.

Figure 4: Ratio of households younger than 35 to households older than 65

The evidence on demographics and down payment ratios shows that one can distinguish between two phases in the relation between housing prices and consumption, one before the mid-1980s where the correlation of the two variables is likely to be low because many young potential first-time buyers plan to buy houses in an economy with high down payment ratios. After the mid-1980s there are both more older households who already own a house and lower down payment requirements due to financial deregulation. Figure 4 shows the demographic situation. One can observe an ever decreasing share of young potential first-time buyers (households younger than 35 years old) to older households (older than 65). In 1960, there were 6 times as much young households than older households while in 2008, there were only 3.7 times more younger households than older households. In terms of the model, this would mean that the aggregate negative effect of a house price change on first-time buyers should have decreased in time.
Figure 5 shows that loan-to-value ratio for first-time buyers which is the inverted down-payment ratio. Data has been computed by Duca et al. (2011). Unfortunately, this data is only available since 1979. However, one can see a clear upward pattern and thus ever decreasing down payments for young first time buyers, from roughly 15 % in 1990 to 5 % in 2005. That means that credit market liberalization did have an influence on credit restriction thus making it easier for first-time buyers to buy a house and be less affected by increases in housing prices. Combined with the data, the model makes the prediction that the housing wealth effect is likely to have been smaller or even negative before the mid-1980s than afterward. This will be tested by estimating a wealth effect for time series data.

2 Estimation of the Housing Wealth Effect

The implications of the model will be tested for aggregate time series. It will be tested whether and to what degree the housing wealth effect before the mid-1980s and afterward differed. A Vector Autoregression model (VAR) will be estimated and impulse-response functions will be estimated. This is done to capture the effects of unexpected shocks of housing wealth on consumption. Before estimating the model, the data will be presented in detail.
2.1 The Data

The implications of the model are tested using four time series: non-housing consumption, labor income, financial wealth and housing wealth. The use of the data of each of the variables will be explained in turn.

The consumption data used measures consumption expenditure less the services from housing and durable consumption goods. It has become standard to exclude durable consumption in studies of the wealth effect and of the life-cycle model. In their classic study of the life-cycle hypothesis, Ando and Modigliani (1963) consider current outlays for non-durable goods and services plus the rental value of the stock of service-yielding consumer durable goods. Hall (1978), on the other hand, excludes the services from durables and only examines non-durables and services.

This has become the standard procedure in the literature on consumption. Hall’s argument is mainly practical: he does not want to discuss the sensitivity of his findings to the method of imputation of services from durables which are not part of the official National Income and Product Accounts (NIPA) statistics. A problem with using durables in estimations of consumption models is that their introduction in the consumption measure is likely to lead to some form of serial correlation in the estimation of a consumption function (Mankiw, 1982).

In the estimation a measure of consumption will be used that only includes non-durables and services minus housing services. Housing prices are likely to be correlated with housing services, since a rise in rents would ceteris paribus also lead to a rise in housing prices. If housing services were included in the consumption function, there would be both a problem of endogeneity (if changes in housing services caused changes in housing wealth) and a problem of testing whether housing wealth changes have an effect on consumption since housing wealth increases would automatically lead to higher consumption if housing services and housing wealth were correlated. This problem might lead to serious problems in the estimates by Lettau and Ludvigson (2001; 2004) since they do not control for this correlation in their estimates of the wealth effect.

The next variable to consider is labor income. Disposable personal income cannot be used as labor income in a wealth effect model because it also contains property and capital income. Both the effect of property and capital income should be gauged by the wealth measures which are theoretically present values of future expected property and capital income. This is why one has to isolate disposable labor income from disposable personal income. Blinder and Deaton (1985) have proposed a measure that is now stan-
Palumbo et al. (2006) and Campbell and Mankiw (1990). They use the disaggregated income data by the BEA and compute labor income thus:

\[
\text{Labor income} = \text{wages and salaries + transfer payments} - \text{social security contributions} - \text{labor taxes}
\]

The problem with labor taxes is that the NIPA only registers overall income taxes and not whether those income taxes are applicable to labor or capital income. The part of taxes paid by labor is then computed as the part of wages and salaries as a share of all labor and capital income:

\[
\text{Labor taxes} = \frac{\text{taxes}}{\text{wages and salaries, interest, dividend and rental income}} \times \text{wages and salaries}
\]

Next, wealth data is discussed. As Rudd and Whelan (2006) argue, wealth data and consumption data have to be consistent. If only a consumption measure without durable goods is used, the stock of durables also has to be excluded from aggregate wealth since consumption would then measure additions to the stock of wealth. Thus, although the Federal Reserve Flow of Funds also contains data on the stock of durables as part of overall wealth, these items are excluded here.

The wealth data used for the estimation consist of financial wealth and net housing wealth. Net housing wealth is the value of the stock of residential housing minus mortgage debt. While consumer debt could also be deducted from wealth, this is not done here. Since most of consumer debt is used to finance durable consumption and durable consumption is excluded from the model, consequently, consumer debt is also excluded.

Palumbo et al. (2006) stress that the deflator has also to be consistent with the data used. The deflator has to take into account which data is used and which is excluded. They show that this is very important for statistical tests since some tests show cointegration between variables only because different deflators are used. This might be the case because the price level for different items changes and the composition of overall consumption changes. If one uses the deflator of all personal consumption expenditures one would also have included the prices of items that were explicitly excluded beforehand,
thereby possibly biasing one’s results. This is why a special deflator has been constructed here, which is computed in the following way:

\[ p_c^d = \frac{(c_{pce} - c^d - c^{hs})}{(c_{pce} - c^d - c^{hs})} \]

All \( p \)'s are price indices and \( c \)'s are nominal expenditures. Then, \( pce \) stands for total personal consumption expenditure, \( c \) for durables and \( hs \) for housing services. All variables are expressed in per capita terms\(^2\) in order to account for the movements in population, consistent with the model. That the use of different deflators can be crucial is shown in figure 6. The figure shows the ratio of the chosen consumption measure deflated by two different deflators, the deflator for personal consumption expenditure from the NIPA and the consumption deflator as it has been computed here:

\[ \frac{c_{pce} - c^d - c^{hs}}{p_{pce}^d} = p^{pce}/p^d \]

This ratio has a clear upward trend so that the deflation by the NIPA deflator for personal consumption expenditure would possibly bias results downward.

Figure 6: Ratio of pce deflator and computed consumption deflator

Source: Bureau of Economic Analysis, own calculations

\(^2\)The population series are the official NIPA population series that the BEA uses to compute per capita values.
2.2 Estimation

In the following a vector autogression will be estimated and the wealth effect deduced in the theoretical model will be tested using an impulse-response function. The impulse-response function is used because it captures the effect of a shock of one variable on another variable. Since theoretically a wealth effect can only occur after a shock, this is the best method available to test for a shock of housing wealth on consumption. Similar methods, but only for overall wealth, have been used by Lettau et al. (2001; 2004). Kundan did also look at housing wealth (2007). However, these authors estimated co-integrated vector autoregressive models. This will not be done here since tests do reject co-integration in the consumption function. Gali (1990), Palumbo et al. (2006), Rudd et al. (2006) do not find that the consumption, income and wealth are co-integrated. However, they do not distinguish between housing and financial wealth. Distinguishing between housing and financial wealth, Benjamin et al. (2004) also do not find a co-integration relationship between the variables.

Before turning to a test for co-integration, the first step is to test whether the time series under consideration have a unit root. I test for the presence of a unit root using the ADF test. Since the series in levels have a clear upward trend, a deterministic trend is
included in the test equation. The length of the lags is determined by the Hannan-Quinn criterion.

Table 1: Unit Root Tests

<table>
<thead>
<tr>
<th>Sample</th>
<th>Series</th>
<th>Trend</th>
<th>Lag Length</th>
<th>t-statistics</th>
<th>95% Critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959q1-2012q2</td>
<td>C</td>
<td>Yes</td>
<td>2</td>
<td>-1.19</td>
<td>-3.43</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>Yes</td>
<td>0</td>
<td>-1.78</td>
<td>-3.43</td>
</tr>
<tr>
<td></td>
<td>FW</td>
<td>Yes</td>
<td>1</td>
<td>-2.22</td>
<td>-3.43</td>
</tr>
<tr>
<td></td>
<td>HW</td>
<td>Yes</td>
<td>2</td>
<td>-2.16</td>
<td>-3.43</td>
</tr>
</tbody>
</table>

The test results are reported in table 1. They show that the hypothesis of the presence of a unit root cannot be rejected. Thus, the variables are integrated of order one. If the variables were co-integrated, a VECM should be estimated. In order to test for co-integration, I perform the Johansen procedure. The Johansen procedure tests for different co-integration vectors between the variables in a multiple-equation system. In order to use the Johansen procedure, one has to choose a lag length for the whole system that is tested. The information criteria show a lag length of 3 for the whole sample.

Further, critical values are affected if a constant and a trend are taken into the relationship. A deterministic trend is used since one can clearly discern from the data that they are trended. In the Johansen procedure, two tests can be conducted. The first test, the trace tests, tests the null hypothesis that there are no co-integration relationships between the variables higher than the rank. For each rank - that means for each possible co-integration relationship - the null hypothesis is that there is no co-integration of the order of the rank or higher. With the second test, the Lmax test, the exact order of co-integration can be tested. The null is that there are no co-integration relationships equal to the rank.

Table 2: Johansen test procedure, with linear deterministic trend

<table>
<thead>
<tr>
<th>Sample</th>
<th>Rank</th>
<th>Eigenvalue</th>
<th>Trace-test</th>
<th>p-value</th>
<th>Lmax-Test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959q1-2012q2</td>
<td>0</td>
<td>0.09</td>
<td>40.37</td>
<td>0.21</td>
<td>20.69</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.05</td>
<td>19.68</td>
<td>0.44</td>
<td>10.04</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.04</td>
<td>9.64</td>
<td>0.31</td>
<td>8.41</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.01</td>
<td>1.23</td>
<td>0.27</td>
<td>1.23</td>
<td>0.27</td>
</tr>
</tbody>
</table>

As can be seen in table 2, the null of no co-integration relationship cannot be rejected for all combinations of possible co-integration relationships. This means that the whole system cannot be estimated using an error-correction model as many authors have done (Case et al., 2005; Davis and Palumbo, 2001; Klyuev and Mills, 2007).
The hypothesis brought forward in the theoretical part has claimed that housing prices are likely to have different effects given different demographic as well as financial market regimes. This is why it is likely that the relationship between housing and non-housing consumption is not stable throughout time.

The stability of the relationship will be tested using a Chow-test applied by Candelon et al. (2001) to vector autoregressive models. Because in VAR models all variables are endogenous, the number of coefficients to be estimated is the square of a simple multivariate regression with only one variable exogenous and the rest endogenous. This is why the degrees of freedom in a VAR are much smaller than in a simple multivariate regression. The authors circumvent this problem by using bootstrap methods to estimate the standard errors and to derive the test statistics. Not using bootstrapping methods would bias the tests towards accepting structural breaks too easily. Two tests have been conducted: First, a simple Chow breakpoint test and second, a Chow forecasting test. Both tests are conducted in the dataset for each data point between the first quarter of 1963 until the second quarter of 2012 (see figure 8).

With the Chow test, the sample is cut into two sub-samples. The breakpoint test compares the sum of squared residuals that are obtained by fitting a single equation to the entire sample with the sum of squared residuals obtained when separate equations are fit to each subsample of the data. The tested hypothesis is whether the sub-samples are the same. A rejection of the null means that there is likely to be a breakpoint. With a Chow forecast test on the other hand, two equations are estimated, one using the full sample and the other only one sub-sample. The degrees of freedom are higher for the forecast test than for the breakpoint test.

Figure 8 shows that the two tests lead to relatively equal results until 1980 but diverge afterwards. The breakpoint test would establish the breakpoint early in the 1980s while the forecast test would establish it late in the 1980s or early in the 1990s. I decided to use the fourth quarter of 1984 as a breakpoint. This has the advantage that the data is almost exactly cut in half, thereby having the same degrees of freedom for both sub-samples. Further, different estimations (not reported) have shown that the results are hardly different when choosing other cutoff points, either in the early or in the late 1980s.

Since I want to estimate two models, one before and one after 1984, I test for separate co-integration relationships in the two sub-samples. In table 3, the tests for co-integration in both periods are shown. No co-integration relationship can be detected in either of the two periods.

---

3Tests have been conducted with the freely available econometric program JMullTi 4.24.
Figure 8: Chow breakpoint and forecast test

Table 3: Johansen test procedure, with linear deterministic trend

<table>
<thead>
<tr>
<th>Time</th>
<th>Rank</th>
<th>Eigenvalue</th>
<th>Trace-test</th>
<th>p-value</th>
<th>Lmax-Test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959q4-1984q4</td>
<td>0</td>
<td>0.14</td>
<td>34.92</td>
<td>0.45</td>
<td>14.61</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.11</td>
<td>20.31</td>
<td>0.4</td>
<td>11.48</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.08</td>
<td>8.84</td>
<td>0.38</td>
<td>7.88</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.01</td>
<td>0.95</td>
<td>0.33</td>
<td>0.95</td>
<td>0.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Rank</th>
<th>Eigenvalue</th>
<th>Trace-test</th>
<th>p-value</th>
<th>Lmax-Test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985q1-2012q2</td>
<td>0</td>
<td>0.17</td>
<td>43.2</td>
<td>0.13</td>
<td>20.13</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.12</td>
<td>23.07</td>
<td>0.24</td>
<td>14.41</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.05</td>
<td>8.66</td>
<td>0.4</td>
<td>5.96</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.02</td>
<td>2.7</td>
<td>0.1</td>
<td>2.7</td>
<td>0.1</td>
</tr>
</tbody>
</table>

I will estimate a VAR model in levels and then compute impulse-response functions. For both periods, three lags are used and a deterministic trend. Lags have been chosen to minimise autocorrelation and heteroskedasticity. Lag exclusion tests show that all lags have significant explanatory power. In the first period, a dummy has been introduced for the second quarter of 1975 to control for the high temporary tax rebate of this quarter which leads to an outlier (Campbell and Mankiw, 1990). The use of the dummy does not affect the value of the coefficients but leads to normality of the residuals.

Table 4 presents tests for autocorrelation, normality and heteroskedasticity in both periods for which the VAR has been estimated. In both periods, no autocorrelation is present and the residuals are normal. However, while residuals are heteroskedastic in the
Table 4: System tests

<table>
<thead>
<tr>
<th>Time</th>
<th>Autocorrelation (1-5) LM test</th>
<th>Normality Jarque Bera statistics</th>
<th>Heteroskedasticity Chi-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959q1-1984q4</td>
<td>14.90 (0.53)</td>
<td>12.9 (0.11)</td>
<td>236.29 (0.99)</td>
</tr>
<tr>
<td>1985q1-2012q2</td>
<td>13.63 (0.62)</td>
<td>4.0 (0.86)</td>
<td>344.68 (0.00)</td>
</tr>
</tbody>
</table>

first period, they are not in the second period. This is why the model has been estimated with HC1 consistent error terms.

In order to grasp their effects on each other, impulse-response functions are computed. In order to compute such functions, the system has to be identified. A standard Cholesky-decomposition is chosen with the following ordering:

**Financial wealth → Housing wealth → Labor income → Consumption**

Financial and housing wealth are both pre-determined values since they are stocks at the beginning of the period that cannot be influenced by the flow variables income and consumption of the same period. However, both can influence income and consumption. The effect of all variables on consumption is theoretically established by the model. The effects of the two wealth variables on labor income are likely to be indirect, for instance via the influence of wealth on overall economic activity and thus wages and employment.
If one compares the two periods, one can see that the difference between shocks to consumption by changes in housing values are significant. In the first period, housing wealth does not significantly affect consumption until the 8th quarter and than has a significantly negative effect on consumption.

In the second period, it has a significantly positive effect on consumption until the 8th quarter after the shock. As such, the theoretical model’s implications for aggregate housing wealth on consumption seem not to be rejected.

The impulse-response functions for the other variables also seem sensible. Financial wealth affects consumption positively in both periods and with comparable intensity. Labor income affects consumption more strongly in the first than in the second period. This is consistent with the implications of the theoretical model. In an economy with many young households, labor income is a more important source of income so that changes in labor income have a more important role for the economy in the first period in which there was a predominance of young households relative to older households.

**Conclusion**

In much of the literature on the US, the wealth effect of housing has been analyzed as if it was identical to financial wealth. However, there are important differences. On the one hand, housing is a necessary good. Everybody has to live somewhere. This is why
housing price changes are likely to have distributional consequences. Further, housing is mostly financed via mortgages and not bought out of current savings as is the case for financial assets. This has an influence on the distributional consequences of housing price changes. This is why most empirical time series studies on the US have not looked at the possible instability of the relationship between housing and consumption in time and are thus likely to be mis-specified.

This study, on the other hand, has developed an explicit model of the housing market in which demographics and the features of the credit market determine the aggregate effect of housing wealth changes on non-housing consumption. The model’s analytical result is that the higher the proportion of young first-time buyers is with respect to old sellers, the higher are the negative distributional consequences of surprise housing price changes on aggregate consumption. This is modified by the financial market regime. The higher the required down payment, the higher is the negative effect of housing price changes on aggregate consumption.

Analysis of demographic and mortgage market data has shown that it is likely that both less younger households and lower down payment requirements since the 1980s are likely to have reduced the negative impact of housing prices changes on consumption. This has led to a domination of the positive impact on consumption for those who realize their capital gains when they sell their house. The vector autoregressive model which has been estimated in the last part does not refute the theory. It finds that housing price shocks before the mid-1980s led even to a negative effect of housing wealth changes on consumption. Only after the 1980s did a housing wealth shock lead to a positive influence on consumption.
Appendix

The first order conditions for the maximization of the young, the middle aged and the old are given. The old maximize

$$ U_t^o = \ln(c_t^o) + \beta \ln(h_t^o) $$

(28)

s.t.

$$ c_t^o + R_t h_t^o - p_t + (1 - \phi)p_{t-1} - (1 + r)s_{t-1}^{fa} = 0 $$

(28a)

The first order conditions for non-housing consumption and housing consumption are ($\lambda$ being the Lagrange multiplier):

$$ \frac{1}{c_t^o} - \lambda = 0 $$

(29a)

$$ \frac{\beta}{h_t^o} - \lambda R_t = 0 $$

(29b)

$$ c_t^o + R_t h_t^o - p_t + (1 - \phi)p_{t-1} - (1 + r)s_{t-1}^{fa} = 0 $$

(29c)

Then, equations (29a) and (29b) are combined, so that:

$$ h_t^o = \frac{\beta c_t^o}{R_t} $$

(30)

This is plugged into (29c) and solved for consumption $c_t^o$. This yields the olds’ consumption function:

$$ c_t^o = (1 + \beta)^{-1}(p_t - (1 - \phi)p_{t-1} + (1 + r)s_{t-1}^{fa}) $$

(31)

The olds’ marginal propensity to consume out of their life-cycle income is:

$$ mpc^o = (1 + \beta)^{-1} $$

The middle aged maximize:

$$ U_t^m = \ln(c_t^m) + \beta \ln(h_t^m) + (1 + \rho)^{-1}E_t[\ln(c_t^{m+1}) + \beta \ln(h_t^{m+1})] $$

s.t.

$$ c_t^m + R_t h_t^m + \frac{c_{t+1}^m + R_{t+1} h_{t+1}^m}{1 + r} - y_t^m - R_t - (1 + r)s_{t-1}^{fa} - \phi(E_t(p_t) - p_t) + p_t(1 - \phi)(r_m + (1 + r)^{-1}) - \frac{E_t(p_{t+2})}{1 + r} = 0 $$

(32)
The first order conditions for the middle aged are:

\[(33a) \quad \frac{1}{c_{t}^m} - \lambda = 0\]
\[(33b) \quad \beta \frac{h_{t}^m}{h_{t}^m} - \lambda R_t = 0\]
\[(33c) \quad \frac{1}{c_{t+1}^m(1 + \rho)} - \frac{\lambda}{(1 + r)} = 0\]
\[(33d) \quad \frac{\beta}{h_{t+1}^m(1 + \rho)} - \frac{\lambda R_{t+1}}{(1 + r)} = 0\]
\[(33e) \quad c_{t}^m + R_t h_{t}^m + \frac{c_{t+1}^m + R_{t+1} h_{t+1}^o}{(1 + r)} - y_{t}^m - R_t - (1 + r)s_{t-1}^o - \phi(E_{t-1}(p_t) - p_t) + p_t(1 - \phi)(r_m + (1 + r)^{-1}) - \frac{E_t(p_{t+2})}{(1 + r)} = 0\]

Combining (33a) with (33b), (33c), and (33d) yields:

\[(34a) \quad h_{t}^m = \frac{\beta c_{t}^m}{R_t}\]
\[(34b) \quad c_{c+1}^o = \frac{c_{t}^m(1 + r)}{(1 + \rho)}\]
\[(34c) \quad h_{t+1}^o = \frac{\beta c_{t}(1 + r)}{R_{t+1}(1 - \rho)}\]

Substituting (34a)–(34c) into (33e) and solving for $c_{t}^m$ yields the consumption function of the middle aged:

\[(35) \quad c_{t}^m = (1 + \beta(1 + (1 + \rho)^{-1}))^{-1}\left( y_{t}^m + R_t + (1 + r)s_{t-1}^o + \phi(E_{t-1}(p_t) - p_t) - p_t(1 - \phi)(r_m + (1 + r)^{-1}) + \frac{E_t(p_{t+2})}{(1 + r)} \right)\]

The marginal propensity to consume of the middle aged is thus

\[mpc^m = (1 + \beta(1 + (1 + \rho)^{-1}))^{-1}\]

28
Finally, the young maximize:

\[ U_t^y = \ln(c_t^y) + \beta \ln(h_t^y) + (1 + \rho)^{-1} E_t[ \ln(c_{t+1}^m) + \beta \ln(h_{t+1}^m)] + (1 + \rho)^{-2} E_t[ \ln(c_{t+2}^o) + \beta \ln(h_{t+2}^o)] \]

subject to

\[
\begin{align*}
    & c_t^y + R_t h_t^y + \frac{c_{t+1}^m + R_{t+1} h_{t+1}^m}{1 + r} +
    \frac{c_{t+2}^o + R_{t+2} h_{t+2}^o}{(1 + r)^2} - y_t^y - \frac{y_{t+1}^m + R_{t+1}}{(1 + r)} +
    \frac{E_t(p_{t+1})(1 - \phi)(r_m + (1 + r)^{-1}) - E_t(p_{t+2})}{(1 + r)^2} = 0
\end{align*}
\]

The youngs’ first order conditions are:

\[
\begin{align*}
    (37a) & \quad \frac{1}{c_t^y} - \lambda = 0 \\
    (37b) & \quad \frac{\beta}{h_t^y} - \lambda R_t = 0 \\
    (37c) & \quad \frac{1}{c_{t+1}^y(1 + \rho)} - \frac{\lambda}{(1 + r)} = 0 \\
    (37d) & \quad \frac{\beta}{h_{t+1}^y(1 + \rho)} - \frac{\lambda R_{t+1}}{(1 + r)} = 0 \\
    (37e) & \quad \frac{1}{c_{t+2}(1 + \rho)^2} - \frac{\lambda}{(1 + r)^2} = 0 \\
    (37f) & \quad \frac{\beta}{h_{t+1}(1 + \rho)^2} - \frac{\beta R_{t+2}}{(1 + r)^2} = 0 \\
    (37g) & \quad c_t^y + R_t h_t^y + \frac{c_{t+1}^m + R_{t+1} h_{t+1}^m}{1 + r} +
    \frac{c_{t+2}^o + R_{t+1} h_{t+2}^o}{(1 + r)^2} - y_t^y - \frac{y_{t+1}^m + R_{t+1}}{(1 + r)} +
    \frac{E_t(p_{t+1})(1 - \phi)(r_m + (1 + r)^{-1}) - E_t(p_{t+2})}{(1 + r)^2} = 0
\end{align*}
\]

Substituting (37a)-(37f) into (37g) and solving for \(c_t^y\) yields the youngs’ consumption function:

\[
\begin{align*}
    c_t^y = & \quad ((1 + \beta)(1 + (1 + \rho)^{-1} + (1 + \rho)^{-2}))^{-1} \left( y_t^y + \frac{y_{t+1}^m - R_{t+1}}{(1 + r)} -
    \frac{E_t(p_{t+1})(1 - \phi)(r_m + (1 + r)^{-1})}{(1 + r)} + \frac{E_t(p_{t+2})}{(1 + r)^2} \right)
\end{align*}
\]
The youngs’ marginal propensity to consume is

$$mpc^y = ((1 + \beta)(1 + (1 + \rho)^{-1} + (1 + \rho)^{-2}))^{-1}$$
References


Hurst, Erik and Frank Stafford, “Home is Where the Equity is: Mortgage Refinancing and Household Consumption,” Journal of Money, Credit and Banking, 2004, 36 (6), 985–1014.


