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Instability, stationary utilization and effective demand: A synthesis of Harrodian and Kaleckian growth theory

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Christian Schoder[1]

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Within a Kaleckian framework, Harrodian instability and a constant long-run utilization rate are reconciled with the principle of effective demand by endogenizing the capacity output-capital ratio. Its change over time is argued to be a positive function of the utilization rate. As stabilizing forces, distribution and debt dynamics are considered. We argue that, with plausible non-linearities in the investment function, limit cycles consistent with empirical observations for the US can be generated by our model with reasonable parameter values and functional forms. With an endogenous capacity-capital ratio, the paradox of thrift as well as the paradox of cost may hold despite a constant long-run utilization rate.

Keywords: Kaleckian growth model, Harrodian instability, stationary utilization, effective demand, endogenous capital productivity, endogenous cycles

JEL Classication: E12, E16, E22, E32

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Instability, stationary utilization and effective demand: A synthesis of Harrodian and Kaleckian growth theory.*

Christian Schoder†
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Within a Kaleckian framework, Harrodian instability and a constant long-run utilization rate are reconciled with the principle of effective demand by endogenizing the capacity output-capital ratio. Its change over time is argued to be a positive function of the utilization rate. As stabilizing forces, distribution and debt dynamics are considered. We argue that, with plausible non-linearities in the investment function, limit cycles consistent with empirical observations for the US can be generated by our model with reasonable parameter values and functional forms. With an endogenous capacity-capital ratio, the paradox of thrift as well as the paradox of cost may hold despite a constant long-run utilization rate.

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1 Introduction

The Kaleckian growth model popularized by Rowthorn (1981), Dutt (1984) and Taylor (1985) faces two serious challenges as summarized by Skott (2012): First, given persistent demand shocks, the principle of effective demand predicts for the canonical Kaleckian model a non-stationary rate of capacity utilization. Yet, as documented by Skott (2012) and Schoder (2012c) and noted by Taylor (2012), the utilization rate seems to be stationary empirically even though highly persistent.

Second, stability of the Kaleckian model requires the savings effect of a change in capacity utilization to be stronger than the investment effect. Yet, the long-run effect of utilization on investment is typically strong likely exceeding its effect on savings and indicating Harrodian instability (Skott 2012).

Moreover, even though some contributors such as Taylor (2004) emphasize the importance of endogenous cycles in Kaleckian growth theory, much of the theoretical and almost all of the empirical work in the Kaleckian tradition relies on static models.[1] This is surprising since endogenous growth cycles seem to be an uncontested stylized fact among heterodox economists. In particular, there seems to exist strong evidence for Goodwin type of cycles with the wage share following utilization as observed by Barbosa-Filho and Taylor (2006) and Zipperer and Skott (2010) for the US and by Flaschel (2009) for European economies. This is because high rates of utilization implying low unemployment and strong trade union tend to cause profit margins to go up rather than down (cf. Steindl 1979, Kurz 1994). The existence of similar cycles between utilization and debt, with the latter following the former, has been implied by the work of Minsky (1976) and Adrian and Shin (2010).

Alternatives to the baseline Kaleckian model have been proposed in the literature to account for these two shortcomings. Yet, none of them is fully convincing from a Kaleckian perspective emphasizing equilibrating adjustments of quantities rather than prices and the principle of effective demand, i.e. growth effects of demand shocks. Taylor (2012) puts forward various aggregative growth models one of which features de-stabilizing Harrodian investment dynamics as well as a constant steady-state utilization rate. Yet, the economy in this model is supply-determined in the long run and the possibility of the paradoxes of thrift and cost disappear.

The growth models proposed by Skott (1989a,b, 2010), Flaschel and Skott (2006) include Harrodian investment dynamics and are built around a constant steady-state utilization rate. Among these, only the Kaldorian model for the *mature* economy also predicts Goodwin-type of cycles between utilization and distribution. It is characterized by instanteneous price adjustment and sluggish output adjustment as well as a constrained labor supply with the labor market condition affecting the desired output growth rate: With low employment, output expands fast causing utilization and, therefore, investment (through the investment function) and the profit share (through the endogenous change of distribution to align savings to investment) to go up. This tightens the labor market which, overall, impairs the business

[1] See, for instance, the influential theoretical contributions of Bhaduri and Marglin (1990), Lavoie (1992) and Hein (2007). For empirical studies relying on static models, see Naastepad and Storm (2006-07), Stockhammer and Onaran (2004) and Hein and Schoder (2011).

climate and causes the expansion of output to slow down. The principle of effective demand can be introduced by assuming the growth rate of the labor supply to depend on employment. Kaleckians may articulate three objections: First, the assumption of an instantaneous price adjustment may be questioned due to evidence of considerable price rigidies (cf. Blinder et al. 1998, Klenow and Kryvtsov 2008, Nakamura and Steinsson 2008). Second, the notion of a pre-determined output may be seen as too strong an assumption in light of widespread *just-in-time* production, delivery lags instead of production lags as well as the existence of considerable inventories. Third, the argument that the adjustment costs for a given output expansion or investment increase with the level of employment is not fully convincing (cf. Hein et al. 2012).[2]

The present paper seeks to reconcile Harrodian instability, a constant long-run utilization rate and the principle of effective demand within a Kaleckian rather than Kaldorian framework, i.e. with equilibrating adjustments of quantities rather than prices, while being able to generate counter-clockwise utilization-wage share and utilization-debt cycles, respectively, as observed empirically.

This is achieved by endogenizing the capacity output-capital ratio. Its change over time is argued to be a positive function of the utilization rate. Hence, a demand shock has long-run impacts on steady state growth through changes in capital productivity.

A pro-cyclical capacity-capital ratio is the core implication of Schoder's (2012a, 2012b) empirical analyses. The former contribution studies the long-run relationships between capacity output, output and demand expectations for the US manufacturing sector. There seems to exist some evidence for a small but significant long-run adjustment of capacity to realized output. For various US industrial sectors since the late 1940s, Schoder (2012b) studies the capacity-capital ratio as a function of the business cycle. As the core result, a significant and positive response of the cyclical element of the growth rate of the capacity-capital ratio to a change in the cyclcal element of utilization is estimated.

The remainder of the paper proceeds as follows: Section 2 presents the canonical Kaleckian growth model, reviews its shortcomings and summarizes the features required to reproduce the stylized facts. Section 3 introduces these long-run features to the model and derives the dynamics implied. Leaving out the principle of effective demand by assuming a constant capacity-capital ratio, the dynamics between utilization and distribution, utilization and debt as well as utilization, distribution and debt are studied. This section is related to and extends Taylor (2012). Section 4 introduces the principle of effective demand by endogenizing the capacity-capital ratio and discusses various demand shocks to the system. Section 5 concludes the paper.

[2]Even if such a relationship exists, the model neglects the impact of labor market conditions on distribution, which is purely an accomodating variable, despite the emphasis on conflict.

2 A canonical Kaleckian growth model

2.1 A benchmark short-run model

We consider a closed economy comprising a representative household, firm and bank. Production is characterized by Leontief-type of production function in capital and labor with a fully elastic labor supply and a, in the long run, endogenous capital-coefficient as discussed below. Tables 1 and 2 report the stocks and flows, respectively, normalized by the capital stock of the simplified economy considered here. Since prices and price expectations are irrelevant for the purpose of the present paper, we assume price expectations to be correct and prices to be equal to unity.[3] As shown in Table 1, the household's assets are money and deposits held at the bank, μ, which equal the household's net worth, ω_H. The firm's capital stock is matched by the sum of its debt-capital ratio, λ, and its net worth, ω_F. The bank's assets is the outstanding debt, λ, and its liabilities are money, μ, and net worth ω_B.[4]

Table 1: Balance sheets

Households		Firms		Banks	
μ	ω_H	1	λ	λ	μ
			ω_F		ω_B

Notes: μ is money over capital, λ debt over capital and ω_i is net worth over capital for sector i.

The sources and uses of the corresponding flows are reported in Table 2. Let $u = Y/Y^C$ and $\sigma = Y^c/K$ denote the rate of capacity utilization and the capacity-capital ratio, respectively, with K, Y and Y^c being capital, output and full-capacity output, respectively. Output equals household consumption, c, and investment, g^i. The production of output generates wage income (normalized by capital), $\psi u\sigma$, and profit income, $(1-\psi)u\sigma$, where ψ is the wage share. Apart from wage income, households also earn interest income, $i\mu$. The part of this income saved, g_H^s, is used for acquiring new deposits, $\dot{M}/K$. A part of the firm's profit income is distributed to the bank, $i\lambda$, and the rest is saved. The firm's savings plus the funds acquired from new loans, $\dot{L}/K$, are used for investment. The bank's savings is the the part of the interest income not used for interest payments on deposits. The newly granted loans, $\dot{L}/K$, are matched by these savings plus new money, $\dot{M}/K$.

In the short run, our economy is fully characterized by the above relations of stocks and flows and two behavioral assumptions: First, for the sake of simplicity, we assume investment to be exogenous in the short run.[5] Hence, we suppose

$$g^i = \rho + \eta + \nu, \tag{1}$$

[3]For Keynesian macro-dynamics derived from price expectations, see Taylor (2012).

[4]We abstract from equity finance since it complicates the algebra while not contributing to the argument made or changing the results.

[5]As common in the Kaleckian literature, one may also suppose an investment function in u and ψ. See, for instance, Rowthorn (1981), Dutt (1984), Amadeo (1986) and Bhaduri and Marglin (1990). Since the following algebra would get rather cumbersome, we stick with exogenous investment in the short run.

Table 2: Social accounting matrix

	Current expenditures					Changes in claims		
	Output cost	Households	Firms	Banks	Capital formation	Bank assets	Bank liabilities	Totals
Output		c			g^i			y
Incomes								
Households	$\psi u\sigma$			$i\mu$				y_H
Firms	$(1-\psi)u\sigma$							y_F
Banks			$i\lambda$					y_B
Flow of funds								
Households		g_H^s					$-\dot{M}/K$	0
Firms			g_F^s		$-g^i$	$\dot{L}/K$		0
Banks				g_B^s		$-\dot{L}/K$	$\dot{M}/K$	0
Total	y	y_H	y_F	y_B	0	0	0	

Notes: $u = Y/Y^C$ and $\sigma = Y^c/K$ denote the rate of capacity utilization and the capacity-capital ratio, respectively, with K, Y and Y^c being capital, output and full-capacity output, respectively. ψ is the wage share, y_i and g_i^s the income and savings, respectively, for sector i over K, c consumption over K, g^i the accumulation rate of capital, i the interest rate, λ debt, L over K and μ money, M over K. $\dot{X}$ indicates the change of X.

where ρ, η and ν are parameters to be endogenized below in the long-run analysis.[6] Second, we assume a constant saving rate of the household, s, leading to an aggregate saving function of the form

$$g^s = s(\psi, u, \sigma)u\sigma, \tag{2}$$

with

$$s(\psi, u, \sigma) = 1 - (1-s)\psi - (1-s)\frac{i\mu}{u\sigma} \tag{3}$$

then following from Table 2. The macroeconomic balance condition,

$$g^i = g^s, \tag{4}$$

then implies a short-run equilibrium of utilization, $u^*(\psi, \sigma)$, as a function of the wage share and the capacity-capital ratio with $\partial u^*(\cdot)/\partial\psi > 0$, i.e wage-led demand, and $\partial u^*(\cdot)/\partial\sigma < 0$. Keynesian stability holds since $\partial(s(\cdot)\sigma)/\partial u > 0$. Note that $s(\cdot)$ is independent of λ since a change in λ is just a redistribution between firm and bank profits as banks are allowed to have net worth (cf. Taylor 2012).

Variants and extensions of this baseline model are the work horse of theoretical and empirical analysis of economic growth in the Kaleckian tradition.[7] All have in common that the principle of effective demand holds in the long run: For instance, a shift in ρ in the investment function, has a permanent effect on u^* and g^*.

[6]For the long run, we will suppose that the changes in ρ, η and ν are driven by utilization, distribution and debt, respectively.

[7]Several features of market economies have been studied within this framework: First, the effects of distri-

2.2 Shortcomings of the benchmark Kaleckian model

The model outlined above as well as most of its cousins, however, have some severe theoretical and empirical shortcomings (cf. Skott 2012). First, they are typically not fully adjusted, i.e. the target utilization rate of the firm is, in general, inconsistent with the equilibrium rate. The representative firm cannot be expected to settle if its target is incongruent with the realization (Committeri 1986, Auerbach and Skott 1988). To defend the Kaleckian model against this criticism, Lavoie (1995b, 1996) and Dutt (1997, 2009) introduced an endogenous adjustment of the target rate through hysteresis effects, thus maintaining the principle of effective demand in the long run. Even though there seems to exist some evidence for hysteresis in the target utilization rate as discussed by Schoder (2012c), it cannot solve the second, empirical problem of the canonical Kaleckian growth model.

The second shortcoming is related to the predicted non-stationarity of the utilization rate which is not tackled by the endogenous target rate. The principle of effective demand in the Kaleckian framework typically implies for the equilibrium utilization rate to follow a random walk given random permanent demand shocks. Yet, this is inconsistent with the empirical observation of a highly persistent but still stationary utilization rate as reported by Skott and Zipperer (2010), Skott (2012) and Schoder (2012c).

Third, stability of the Kaleckian model requires the sensitivity of investment to changes in utilization to be lower than the utilization sensitivity of saving, also in the long run. Yet, simulation and empirical studies such as Dallery (forthcoming), Skott and Zipperer (2010) and Skott (2012), respectively, suggest that the stability condition is hardly met in the long run due to a strong accelerator effect on investment. Hence, there seems to exist convincing support for a Harrodian long-run specification of investment according to which the accumulation rate is self-enforcing which is absent from the Kaleckian model (cf. Skott 2012).[8]

The long-run extension of the short-run model presented here is an attempt to take these empirical stylized facts into account without dropping the principle of effective demand. In particular, the model outlined below with opportune but not implausible parameter values and functional forms features (a) Harrodian investment dynamics, (b) a stationary utilization rate (and, therefore, full adjustment in the steady state), (c) cycles that are consistent with empirical observations, i.e. counter-clockwise in the (u, ψ)-space and (u, λ)-space, respectively, and (d) the principle of effective demand in the long run allowing for the possibility of the paradox of thrift and paradox of cost.[9]

bution on capacity utilization and growth by, among others, Naastepad and Storm (2006-07), Stockhammer et al. (2009) and Hein and Vogel (2008). Second, open economy issues have been discussed, among others, by Blecker (1989). Third, endogenous labor productivity and distribution have been added by Taylor (2004, ch. 9) to study the dynamic interaction of distribution and growth. Fourth, the phenomenon of *financialization* has been recently discussed by Lavoie (1995a) and Hein (2007) and Hein and Schoder (2011).

[8]For a survey of the debate on stability from a Kaleckian perspective, see Hein et al. (2012, 2011).

[9]Note the following: First, within a Kaleckian framework of quantity adjustment, (b) and (d) can only hold simultaneously if the capacity-capital ratio is endogenous, i.e. its change is rising with the utilization gap. Second, once Harrodian investment dynamics prevail, additional forces need to be introduced which keep the system globally stable. We shall consider two possible stabilizing mechanisms: distribution dynamics

3 Dynamics of utilization, distribution and debt

The aim is to close the Kaleckian model outlined above for the long run such that the features (a) to (c) are included. In the next section we will add feature (d), i.e. the principle of effective demand.

3.1 Introducing long-run features

Harrodian investment. We first introduce long-run investment by endogenizing ρ, η and ν in (1). Some preliminary empirical evidence for the suggested dynamics is provided below.

An Harrodian element of instability is introduced by stating that

$$\dot{\rho} = \phi_\rho(u - \bar{u}) \tag{5}$$

with $\partial\phi_\rho(\cdot)/\partial u > 0$ and $\partial^2\phi_\rho(\cdot)/(\partial u)^2 < 0$. If the representative firm faces an inconsistency of the realized utilization rate with the target, it will keep adjusting the rate of accumulation until the gap disappears.

The negative second-order derivative will turn out to be important for generating the non-linearities required for limit cycles. It can be motivated by increasing adjustment costs of investment. The cost of changing the speed of accumulation is increasing with the desired change which may be proportional to the utilization gap.[10]

As can easily be verified, the economy characterized by (1) to (5) is unstable even if Keynesian stability holds. To obtain global stability, additional forces have to be introduced which over-compensate Harrodian instability. We will consider distribution and debt dynamics and accordingly endogenize η and ν, respectively.

Cost effect on investment. One way to achieve global stability despite the de-stabilizing Harrodian investment dynamics is to introduce a negative cost effect on investment which dominates the Harrodian effect at extreme values of utilization. Hence, we suppose the following: With a high profit rate, r, the firm can be expected to seek growing faster facilitated by better availability of external funds from incomplete financial markets due to higher creditworthiness. With a low profit rate, the reverse may hold. The firm might have little access to external funds and seek to reduce growth and save costs instead. Therefore, a profit rate, $\bar{r}$, may exist which is associated with a constant accumulation rate. Since $r = (1 - \psi)u\sigma$ with $u = \bar{u}$ and σ changing slowly in the long run, there may be a wage share $\psi = \bar{\psi}$ such that the profit rate is consistent with a constant accumulation rate.[11] Hence, we suppose

$$\dot{\eta} = -\phi_\eta(\psi - \bar{\psi}) \tag{6}$$

and debt dynamics.

[10] This is similar to the specification of the production expansion frontier put forward by Skott (1989a) which is also subject to convex adjustment costs and assumed to be bounded.

[11] If σ is constant, there is a proportional relationship between $\bar{r}$ and $\bar{\psi}$, given $\bar{u}$. If σ is changing, $\bar{\psi}$ has to move in the same direction to leave $\bar{r}$ constant. Yet, for simplicity, we assume $\bar{r}$ to change accordingly such that $\bar{\psi}$ is constant.

with $\partial\phi_\eta(\cdot)/\partial\psi > 0$ and, again, $\partial^2\phi_\eta(\cdot)/(\partial\psi)^2 < 0$. Again, rising adjustment costs of investment may justify the negative partial derivative.

Debt effect on investment. Another stabilizing mechanism can be obtained by introducing a negative debt effect on investment. Loosely following Minsky (1976), the firm may have a target debt-capital ratio which it tries to achieve by adjusting the accumulation rate accordingly. On the one hand, a high debt-capital ratio bears the risk of bankruptcy and the firm may attempt to de-leverage by slowing down accumulation. On the other hand, with a low debt-capital ratio additional profits outweigh the risk of default associated with an acceleration of accumulation. Hence, we assume

$$\dot{\nu} = -\phi_\nu(\lambda - \bar{\lambda}) \tag{7}$$

with $\partial\phi_\nu(\cdot)/\partial\psi > 0$ and, again, $\partial^2\phi_\nu(\cdot)/(\partial\psi)^2 < 0$.

Since the long-run investment dynamics specified in (5), (6) and (7) are crucial to our argument let us briefly discuss their empirical relevance. As a preliminary look at the data, we estimate

$$\Delta g_{i,t} = \alpha + \phi_\rho(u - \bar{u})_{i,t} + \phi_\eta(\psi - \bar{\psi})_{i,t} + \phi_\nu(\lambda - \bar{\lambda})_{i,t} + \mu_i + \varepsilon_{i,t}$$

for 18 US manufacturing industries from 1984Q1 to 2007Q4 where α is a constant and μ_i are group-specific fixed effects.[12] Note that the ϕ's are assumed to be parameters. Using a fixed effects estimator, we obtain $\hat{\phi}_\rho = 0.010(0.004)$, $\hat{\phi}_\eta = -0.016(0.009)$ and $\hat{\phi}_\nu = -0.001(0.003)$ where the numbers in parenthesis are the standard errors.

We find a significant, positive Harrodian effect of $u - \bar{u}$ on the change in g which is consistent with Hein and Schoder (2011), Skott (2012) and Schoder (2012c). As expected, we also obtain a negative effect of $\psi - \bar{\psi}$ on the change in g which is significant at the 10% level. The point estimate of the effect of $\lambda - \bar{\lambda}$ on the change in g is negative but not significant. This may be related to the fact that we approximate λ by the debt-assets ratio which includes a lot of noise due to revaluations of debt and assets which we have abstracted from in the theoretical model. Other studies such as Ndikumana (1999), Orhangazi (2008) and Schoder (forthcoming) find strong anti-cyclical debt effects. Overall, there seems to exist some evidence for our investment specification for the long run.

Distribution. Distributional dynamics can be analyzed in terms of the nominal wage rate, w, the price level, p and labor productivity, ξ, since $\hat{\psi} = \hat{w} - \hat{p} - \hat{\xi}$ (cf. Taylor 2004, ch. 7). A rise in utilization can be expected to accelerate wage inflation due to tighter labor markets

[12] $x - \bar{x}$ is approximated by the cyclical element of x obtained from applying the HP filter to the series. Data on u has been taken from the Fed. Similar to Schoder (2012c), sectorally consistent data on g, ψ and λ has been computed by aggregating (and seasonally adjusting) firm-level data on US corporations provided by S&P's Compustat database. g is quarterly investment expenses over the capital stock at the beginning of the period. ψ is approximated by 1 minus operational income before depreciation over sales. λ is the debt-assets ratio. We limit our sample to the end of 2007 in order to exclude the period of economic disruptions caused by the financial crisis.

as well as price inflation due to higher demand on goods markets. A rise in the wage share may provoke a stronger resistance of the firm to wage increases and a stronger incentive to find ways of price increases. Labor productivity growth can be expected to speed up with utilization trough Okun's Law and Kaldor-Verdoorn effects as well as with a rising wage share due to induced technical change. Overall, we assume the utilization effect on wages to be higher than on prices and labor productivity together. This gives rise to a radical distribution mechanism with the change in the wage share being determined by

$$\dot{\psi} = \psi\phi_\psi(u - \bar{u}, \psi - \bar{\psi}) \tag{8}$$

with $\partial\phi_\psi(\cdot)/\partial u > 0$ and $\partial\phi_\psi(\cdot)/\partial\psi < 0$. Note that the steady state values $\bar{u}$ and $\bar{\psi}$ in the distribution equation are the same as the steady state values in the investment equation. In our model framework, there is no particular reason why this should be the case.[13] Yet, this assumption facilitates the succeeding mathematical analysis enormously since the steady states of both differential equations are consistent with each other.[14]

Monetary policy rule. Since it makes stable counter-clockwise limit cycles in debt and utilization more likely, we follow Duménil and Lévy (1999) and suppose the monetary authority to follow an interest rate rule,

$$i = \bar{i} + \phi_i(u - \bar{u}), \tag{9}$$

with $\partial\phi_i(\cdot)/\partial u < 0$ and, for simplicity, $\partial^2\phi_i(\cdot)/(\partial u)^2 = 0$. Note that here the choice of $\bar{u}$ as the reference value is not too strong an assumption since the specification of the dynamics of distribution implies that inflation is stable at $u = \bar{u}$ (and $\psi = \bar{\psi}$).[15]

[13] Let $\bar{u}^i$ and $\bar{u}^d$ denote the steady state utilization rates in the investment function and distribution function, respectively. Let $\bar{\psi}^i$ and $\bar{\psi}^d$ be equivalently defined. In the present paper, we merely assume $\bar{u}^i = \bar{u}^d = \bar{u}$ and $\bar{\psi}^i = \bar{\psi}^d = \bar{\psi}$. In an extension of the present analysis, the equalities may hold if $\bar{u}^d$ and $\bar{\psi}^d$ are endogenized through changes in the degree of monopoly. For instance, if $\bar{u}^i > \bar{u}^d$ ($\bar{u}^i < \bar{u}^d$) and firms attempt to realize $\bar{u} = \bar{u}^i$, the wage share will keep rising (falling). Over the very long run, this will force unprofitable firms to leave the market (attract rival firms into the market) and increase (decrease) the degree of monopoly. Thus, $\bar{u}^d$ may be pushed up (down) to $\bar{u}$ since, with more relaxed (fierce) competition, unions may become weaker (stronger) at a given utilization rate. With these forces at work, we will have $\bar{u}^i = \bar{u}^d = \bar{u}$. An equivalent argument can be made for $\bar{\psi}^d$. If $\bar{\psi}^i < \bar{\psi}^d$ ($\bar{\psi}^i > \bar{\psi}^d$), the wage share will keep rising (falling), increasing (decreasing) the degree of monopoly. Similar to before, $\bar{\psi}^d$ may be pushed down (up) to $\bar{\psi}$ and we will have $\bar{\psi}^i = \bar{\psi}^d = \bar{\psi}$.

[14] Since Taylor (2012) does not specify the distributional dynamics in terms of deviations form the steady state values, his dynamical analysis is not entirely correct. In fact, the dynamical system in utilization and distribution characterized by his equations (8) and (12) does not have stable nullclines since the left-hand sides of both equations include state variables in growth rates. To see this point, note that the steady state value for utilization in his model is obviously $\bar{u}$ and his equation (8) will then be zero only if $\hat{\psi}$ is also zero, i.e. only if distribution is in the steady state. Hence, evaluated at the steady state where $\dot{\psi} = 0$, the u-nullcline will be independent of ψ and, hence, vertical in the (u, ψ)plain and not downward sloping as in his figure 1 or upward sloping as in his figure 2. Around the steady state, the system will, therefore, be unstable. Even if this problem was accounted for by substituting (12) into (8), there still will not exist a steady state since, for the steady state value of ψ, there is no reason why equation (12) should be zero for $u = \bar{u}$ which is the steady stae for u.

[15] Moreover, since (9) is a static equation, a certain i can be achieved by an arbitrary reference value of u which is $\bar{u}$ in (9) as long as $\bar{i}$ and $\phi_i(\cdot)$ are conformative.

Endogenous capacity-capital ratio. The core feature of the present model which gives rise to the principle of effective demand in the long run is an endogenous capacity-capital ratio, σ, which is modeled as

$$\dot{\sigma} = \phi_\sigma(u - \bar{u}) \tag{10}$$

with $\partial\phi_\sigma(\cdot)/\partial u > 0$. Since it is the crucial extension of the Kaleckian framework in the present paper, we shall briefly discuss it here. An in-depth discussion can be found in Schoder (2012b).

In heterodox growth models, the capacity-capital ratio is typically assumed to be constant (cf. Taylor 2004, Skott 2012, Hein et al. 2012). Regarding the trend of the variable, a large body of literature analyzes if technical change is labor saving or augmenting in the long run, i.e. if the capacity-capital ratio tends to decrease or increase. In industrialized countries technical change has been found to be slightly labor saving in the long run (cf. Foley and Michl 1999, pp. 37-41 and Duménil and Lévy 2004). Since we are primarily interested in the cyclical behavior of the capacity-capital ratio, we assume its trend to be zero.

Regarding the cyclical pattern, there are good reasons for a pro-cyclical capacity-capital ratio as pointed out in detail by Schoder (2012b). First, as argued by Nikiforos (2011) full-capacity output as reported by the Fed does not measure the technically feasible capacity but the highest level of output that can be produced under *normal* conditions and maintained sustainably. Indivisibilities in the production process such as shift work may then cause capacity output to change endogenously. Any number of shifts is associated with a certain full-capacity output beyond which profits may become negative and production cannot be sustained unless another shift is introduced. Running another shift is associated with additional fixed costs but reduces unit variable costs since over-time labor can be saved. In a boom with high demand expectations some firms may introduce additional shifts and, hence, raise their full-capacity output even though no capital investment need to have taken place. Capacity and, therefore, the capacity-capital ratio will be endogenous.

Second, investment induced technical change may affect the capacity-capital ratio pro-cyclically (cf. Schoder 2012b). If a deviation of the utilization rate from the desired rate implies some form of costs arising from an inefficient use of resources, on the one hand, and a lack of flexibility in accommodating demand required to deter market entry of potential competitors, on the other, then a firm will seek to invest in capital which helps realigning the utilization rate to the desired rate. For instance, if utilization is too high, a firm will choose structures and equipment which raise the productivity of capital, since this increases capacity output and, therefore, reduces utilization for a given demand to be accommodated. Hence, the capacity output-capital ratio will move pro-cyclically.

Two empirical studies support this view. First, Schoder (2012b) estimates a linear variant of (10) with the cyclical element of the growth rate of σ on the left-hand side for a panel of several US industrial sectors from 1948 to 2011.[16] A small but significantly positive response

[16]For each sector, the utilization gap has been approximated by the cyclical element of the HP-filtered rate of capacity utilization as provided by the Fed. Data on industrial capacity can also be obtained from this source while capital stock data is available at the Bureau of Economic Analysis.

coefficient robust across different samples is estimated. Second, Schoder (2012a) estimate a Cointegrated VAR model for the US manufacturing sector from 1955Q1 to 2012Q2 in output, capacity and a composite leading indicator and provide evidence that production capacities adjust slowly to current output in the long run.[17]

Dynamics. Having specified the long-run behavioral dynamics of investment, distribution and the capacity-capital ratio as well as the monetary policy rule, we can now put the parts together and analyze the dynamics of the system.

Let us first, derive the dynamics of the debt-capital ratio, λ. Note that investment can be financed internally, $(1-\psi)u\sigma - i\lambda$ and by new debt, $\lambda\hat{L}$.[18] Hence,

$$g = (1-\psi)u\sigma - i\lambda + \lambda\hat{L}. \tag{11}$$

Using $\hat{L} = \hat{\lambda} + g$, (11) implies together with the short-run equilibrium characterized by (1) to (4) and the interest rate rule in (9) that

$$\begin{aligned}\dot{\lambda} = (1-\lambda)\Big(\big(1-(1-s)\psi\big)u\sigma - (1-s)\big(\bar{i} + \phi_i(u-\bar{u})\big)\mu\Big) \\ + \big(\bar{i} + \phi_i(u-\bar{u})\big)\lambda - (1-\psi)u\sigma.\end{aligned} \tag{12}$$

Note that (12) evaluated at $\bar{u}$, $\bar{\psi}$ and $\bar{i}$ implies for the steady-state value of λ to be a function of the steady-state value of σ, i.e.

$$\bar{\lambda}(\sigma^{**}) = \frac{g^{**} - (1-\psi)\bar{u}\sigma^{**}}{g^{**} - \bar{i}} \tag{13}$$

with $g^{**} = s(\bar{\psi}, \bar{u}, \sigma^{**})\bar{u}\sigma^{**}$. Hence, we assume implicitly the target level of the debt-capital ratio to be endogenous.

To obtain the dynamics of u, we log-differentiate (2), use (1) differentiated with respect to time, (5) to (10) as well as (1) to (4) characterizing the short-run equilibrium growth rate and re-arrange which leads to

$$\begin{aligned}\dot{u} = \frac{\phi_\rho(u-\bar{u}) - \phi_\eta(\psi-\bar{\psi}) - \phi_\nu(\lambda-\bar{\lambda}) - \phi_\sigma(u-\bar{u})\big(1-(1-s)\psi\big)u}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial\phi_i(u-\bar{u})}{\partial u}} \\ + \frac{(1-s)\psi\phi_\psi(u-\bar{u}, \psi-\bar{\psi})u\sigma}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial\phi_i(u-\bar{u})}{\partial u}}.\end{aligned} \tag{14}$$

[17] In particular, two cointegrating relationships have been identified between the three variables: the first one between output, capacity and a constant; the second one between output and the composite leading indicator. Output is found to be fully characterized by the composite leading indicator, since output is error-correcting to the output relation while it is not error-correcting to the utilization relation. Moreover, capacity seems to be error-correcting to the utilization relation, i.e. to be endogenously adjusting. The composite leading indicator has been found to be weakly exogenous. Note that even though the variables are not normalized by the capital stock the observed endogenous capacity is not the ordinary capacity effect of investment present in any heterodox model. This is because the effect of output on capacity is a long-run and not a short-run phenomenon.

[18] Recall that we ignore equity finance.

Equations (8), (10), (12), (13) and (14) characterize a four-dimensional dynamical system in u, ψ, λ and σ. The corresponding Jacobian Matrix can be found in Appendix A. With reasonable parameter values its elements can be expected to have the following signs:

$$J = \begin{bmatrix} + & + & + & + \\ - & - & + & 0 \\ - & 0 & - & 0 \\ -/+ & 0 & -/+ & 0 \end{bmatrix} \tag{15}$$

In the remainder of this section, we impose restrictions on parameters and functional forms in order to reduce the dimensions and make the dynamics of the system tractable.

3.2 Utilization and distribution cycles

As observed for the US by Barbosa-Filho and Taylor (2006) and Zipperer and Skott (2010) and, for selected European economies, by Flaschel (2009), there seem to exist counter-clockwise cycles in the (u, ψ)-plain. The late upswing and the early downturn are typically associated with a rising wage share, whereas the late downturn and the early recovery tend to feature a falling wage share. As we will argue here, the model outlined above is able to generate these cycles.

The following restrictions are imposed: First, we exclude λ from the system since we are interested in the interaction between utilization and distribution. Hence, we assume that the debt-capital ratio does not affect the investment decision, i.e. $\phi_\nu(\cdot) = 0$. Second, we take the capacity-capital ratio as constant, i.e. $\phi_\sigma(\cdot) = 0$, an assumption to be relaxed in the next section in order to introduce the principle of effective demand.

The system collapses to two dimensions in u and ψ. With the restrictions imposed, the Jacobian reported in Appendix A exhibits the following signs in the neighborhood of the steady state:

$$J = \begin{bmatrix} + & + \\ - & - \end{bmatrix} \tag{16}$$

Because of the Harrodian element in the investment function and the positive effect of utilization on the growth of saving, $\partial\dot{u}/\partial u > 0$. Further, $\partial\dot{u}/\partial\psi < 0$ due to a strong cost effect on investment. Obviously, $\partial\dot{\psi}/\partial u > 0$ and $\partial\dot{\psi}/\partial\psi < 0$ as specified in (8).

Counter-clockwise spiral-like trajectories then require additionally that $\det(J) > \mathrm{tr}(J)^2/4$. If, evaluated at the steady state, $\mathrm{tr}(J) > 0$, i.e the de-stabilizing own-effects of u are larger then the stabilizing own effects of ψ, then the long-run equilibrium will be a source. However, the assumption of non-linear adjustment costs of investment represented by $\partial^2\phi_\rho(\cdot)/(\partial u)^2 < 0$ and $\partial^2\phi_\nu(\cdot)/(\partial u)^2 < 0$ allow for global stability since $\partial^2\phi_\psi(\cdot)/(\partial\psi)^2 \geq 0$. In this case the trajectories will produce limit cycles.[19]

A parameter constellation favorable to counter-clockwise limit cycles implies a strong cost effect on investment, a fast adjustment of accumulation to the utilization gap, a high

[19] In Taylor's (2012) Harrodian specification, the signs of the elements of the corresponding Jacobian are

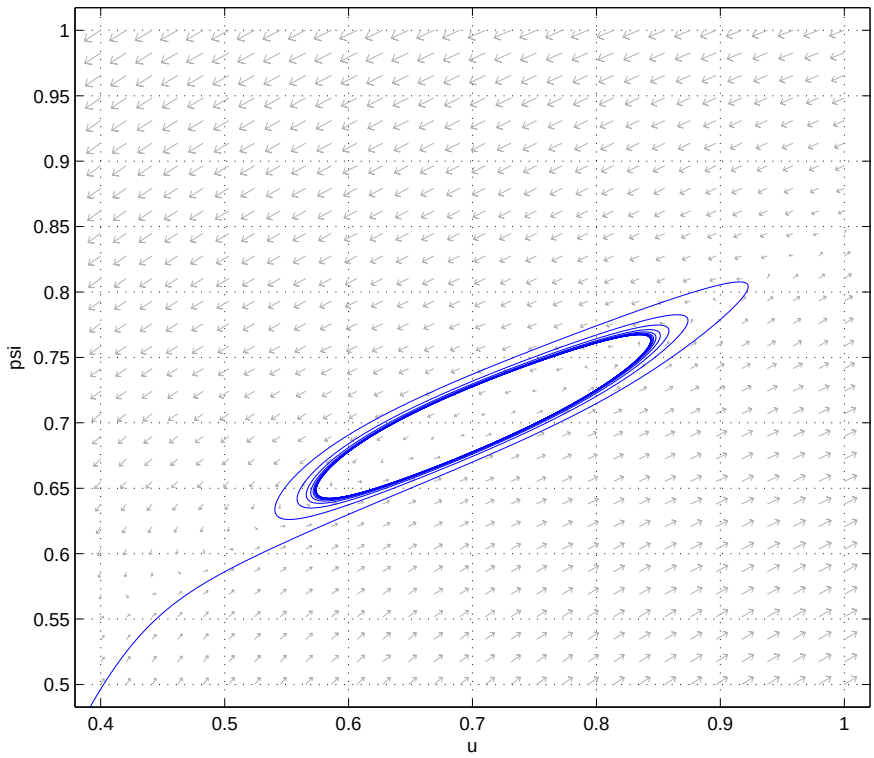

Figure 1: Endogenous limit cycles of utilization and distribution

sensitivity of the trade unions' wage bargaining power to changes in utilization relative to the firms' price setting power, $\partial\hat{w}/\partial u$ with w being the real wage, a low Kaldor-Verdoorn effect, a low sensitivity of the firms' resistance to wage pressures with respect to changes in the wage share, and low induced technical change.

Figure 1 shows the vector field and a particular solution of such a dynamical system calibrated to be roughly consistent with the US economy.[20] Close to the steady state Harrodian instability dominates the stabilizing effects of the wage share on its change causing the economy to divert from the equilibrium. Yet, due to rising adjustment costs of investment, the Harrodian forces become weaker the further the economy moves away from the steady state. At some point destabilizing and stabilizing forces perfectly compensate each other

$J = \begin{bmatrix} + & - \\ + & - \end{bmatrix}$ generating a clockwise cyclical adjustment in the (u, ψ)-plain. This result originates in two behavioral assumptions we slightly deviate from: First, compared to Taylor, we assume a weak Kaldor-Verdoorn effect of growth on labor productivity since we separate out the effect on the capacity-capital coefficient. With a strong Kaldor-Verdoorn effect, an increasing u may accelerate growth such that the following rise in $\hat{\xi}$ overcompensates the rise in $\hat{w}$ with $\hat{p}$ assumed to be constant. Second, Taylor excludes the cost effect on investment. Hence, the result that $\partial\dot{u}/\partial\psi > 0$ reflects the negative effect of an increase in the wage share on the saving rate. In the case considered here, a strong cost effect is required to stabilize the system.

[20] Details on the functional relationships are reported in Appendix B.

causing the economy to move along a counter-clockwise limit cycle.

In the upswing a Harrodian investment boom is initiated leading to ever-increasing utilization which strengthens the bargaining power of trade unions and cause the wage share to increase. At some point the rising wage share will cause a profit squeeze and the dynamics of utilization reverse while the wage share still rises. A falling utilization rate first dominated by high labor costs and then driven by Harrodian dynamics precedes a fall in the wage share which is the seeds of a new recovery.

As long as parameter values and functional relationships are such that the system generates counter-clockwise limit cycles, changes in parameter values will have the following effects: First, the stronger the cost effect on investment compared to the Harrodian effect, the lower the amplitude of the wage share. Second, the higher the elasticities of distribution compared to the elasticities of investment, the smaller the cycles are. Third, the higher the elasticity of the interest rate, the bigger the cycles. Note that the reverse might hold, if there was a direct effect of the interest rate on investment which we do not consider here. Note, further, that the system may also produce limit cycles if the interest rate is constant, even if it is less likely.

Cycles driven by distribution alone may be a plausible characterization of the post-war period with incomplete financial markets, high employment and strong labor unions. Yet, it seems implausible to explain the downturns since the 1980s by profit squeezes given the persistently high unemployment rates and weak labor market institutions. Moreover, the cost effect on investment is typically lower than it would be required to generate a trend reversal. Therefore, other stabilizing forces have to be added to the model such as debt dynamics to be considered next.

3.3 Utilization and debt cycles

Since the 1980s, labor market liberalization, globalization of production and capital market deregulation have deteriorated the bargaining power of trade unions. Hence, distribution being the primary driving force of the business cycle may well be doubted. Because of financial market deregulation, debt dynamics may have gained an important role in determining the turning points of the cycle. As reported by Adrian and Shin (2010) for the US, financial leverage tends to behave pro-cyclically, following utilization with a delay, similar to the wage share.

To analyze the dynamics of utilization and debt within our model, let us impose the following restrictions on the benchmark long-run model: First, we assume distribution to be exogenous, i.e. $\phi_\psi(\cdot) = 0$. Second, we also suppose $\phi_\eta(\cdot) = 0$. Third, we maintain that $\phi_\sigma(\cdot) = 0$. Note that in contrast to the previous sub-section, the assumption of an endogenous interest rate following the rule specified in (9) is a necessity for the model to generate limit cycles with plausible parameter values.[21]

[21] Note that Taylor (2012) ensures the possibility of cycles not by assuming a pro-cyclical interest rate rule but by large and permanent share buybacks. While we assume away equity finance, Taylor assumes new equity issues normalized by the capital stock to be inversely proportional to the accumulation rate. In this case $\partial\dot{\lambda}/\partial u > 0$ may hold in the extended variant of the model. Yet, the assumption of a permanent share

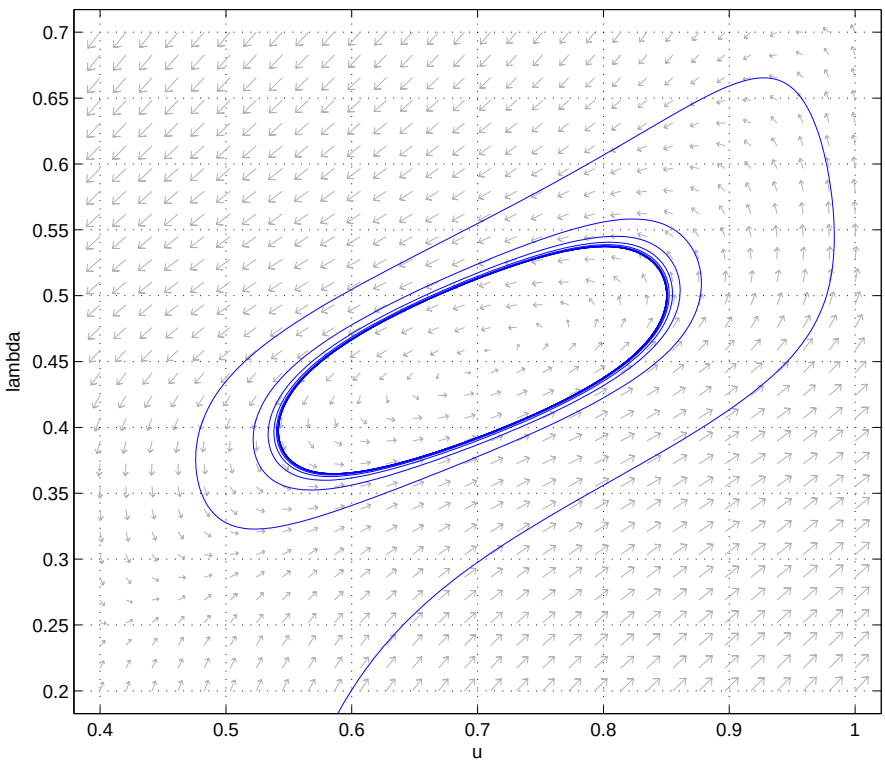

Figure 2: Endogenous limit cycles of utilization and debt

With ψ and σ being exogenous, we obtain from (12) to (14) a two-dimensional system in u and λ with the elements of the Jacobian, again reported in Appendix A, exhibiting the following signs:

$$J = \begin{bmatrix} + & + \\ - & - \end{bmatrix} \tag{17}$$

As in the previous model, $\partial\dot{u}/\partial u > 0$. Obviously, $\partial\dot{u}/\partial\lambda < 0$ since increasing debt dampens investment. For not too high levels of debt, it may well hold that $\partial\dot{\lambda}/\partial u > 0$, i.e. debt accelerates with rising investment. If the solvency condition holds, i.e. $g > j$, then $\partial\dot{\lambda}/\partial\lambda < 0$.

The non-linearities introduced in the investment function allow for global stability while the steady state is locally unstable. Assuming plausible parameter values for the US economy, the system may well produce counter-clockwise limit cycles in the (u, λ)-plain.

Counter-clockwise cycles are likely if investment reacts sensitively to changes in utilization as well as debt, and if the monetary policy response to the utilization gap is strong.

The vector field and a solution of the system are plotted in Figure 2. Mathematically, both the nullclines and the trajectories are equivalent to the previous model. Yet, the

buyback cannot be valid in the steady state since, at some point, all equity will have been bought back.

interpretation has changed. In the upswing, an expansion of utilization induces firms to speed up accumulation with the share of new capital financed by debt increasing. Rising debt-capital ratios negatively feed back into investment causing a trend reversal at the peak of the cycle. While utilization starts decreasing, debt overshoots since firms still try to expand capacity by investing financed partly by debt. Yet, soon a process of falling investment and de-leveraging sets in reducing both the debt-capital ratio and utilization. At low levels of debt, investment picks up driving utilization up again.

Again, one can analyze how the cycles behave in response to changes in parameter values. First, the stronger the response of the interest rate to the utilization gap, the higher the amplitude of λ. In fact, the cycles are very sensitive to the utilization elasticity of the interest rate.

3.4 Utilization, distribution and debt cycles

The assumption of an a-cyclical wage share may overshoot the point that the cost effect of distribution on investment does not drive the business cycle. Even though there may not be a strong cost effect on investment, distribution may still be affected by utilization through the labor market and, therefore, affect utilization through the overall propensity to save out of income (in a de-stabilizing way). With primarily debt-dynamics driving the business cycle and a radical distribution mechanism, the cost effect on investment contributes to stabilizing the system whereas the effect on saving destabilizes.

To analyze these dynamics between utilization, distribution and debt we maintain only the assumption of a constant capacity-capital ratio, $\phi_\sigma(\cdot) = 0$.

This assumption together with (8) and (12) to (14) characterize a three-dimensional system in u, ψ and λ. The corresponding Jacobian is reported in Appendix A. With a plausible parameterization, the signs of its elements are simply the Jacobians of the previous two sub-sections combined, i.e.

$$J = \begin{bmatrix} + & + & + \\ - & - & + \\ - & 0 & - \end{bmatrix} \tag{18}$$

Again, the assumption of non-linear adjustment costs of investment allow the system to generate limit-cycles with plausible parameterization. Compared to section 3.2, the cost effect on investment is assumed to be weak in the neighborhood of the steady state.

Figure 3 plots a particular solution for the parameter values and functional forms reported in Appendix B. The top-left panel depicts the solution in a 3-dimensional plot while the other panels show the solution in the three subsets. The model calibration generates counter-clockwise cycles in the (u, ψ)- and (u, λ)-plain, respectively. Since the amplitudes of λ are more pronounced then the amplitudes of ψ, the also predicts narrow cycles in the (ψ, λ)-plain.

Since there is no direct interest cost effect on investment, the monetary policy rule contributes to de-stabilization as the change in debt increases faster with utilization. Since the change in debt is also an increasing function of the wage share, the monetary policy rule

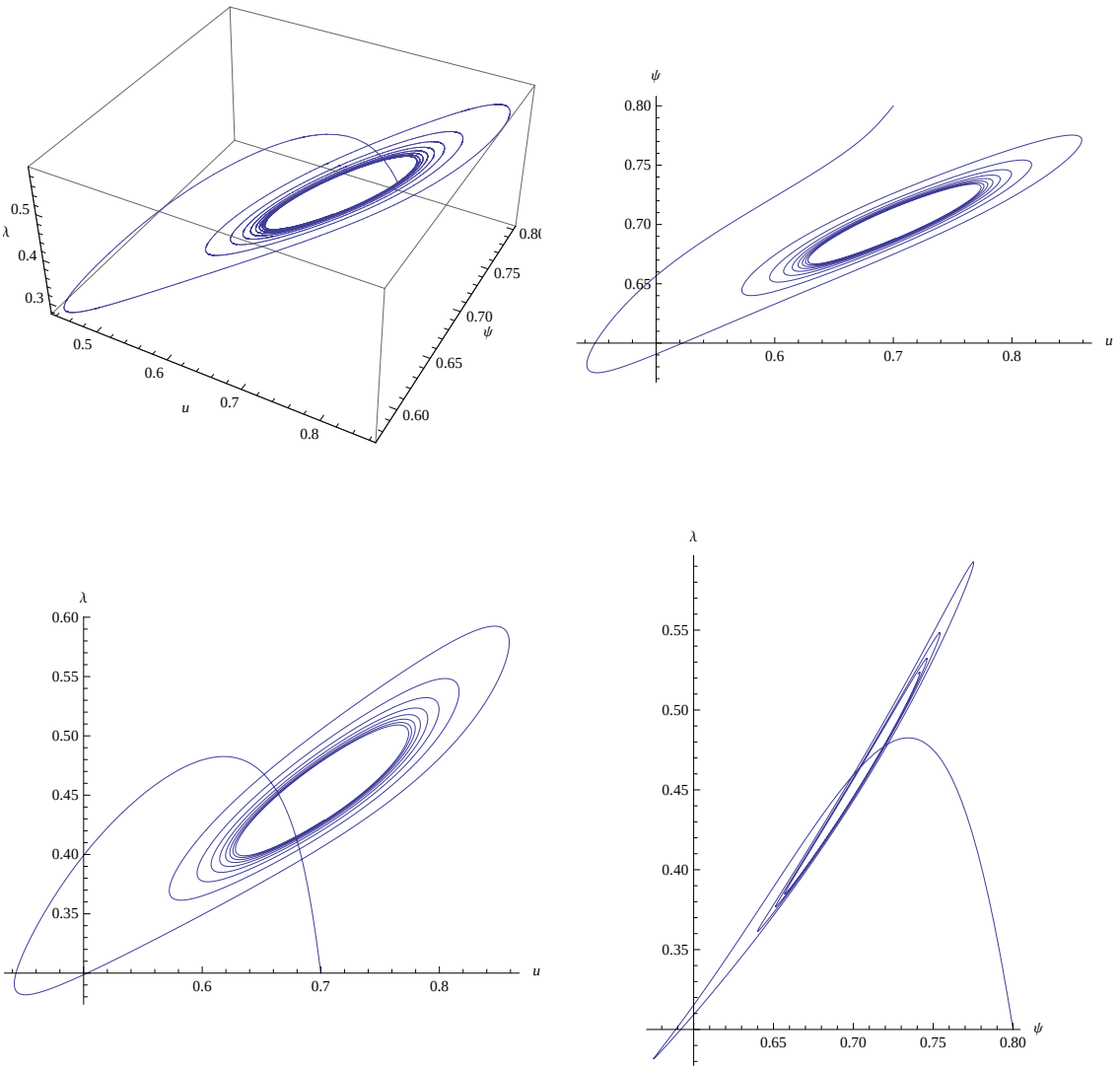

Figure 3: Endogenous limit cycles of utilization, distribution and debt

would not be required anymore to generate cycles. Note that, with both endogenous changes in distribution and debt affecting the system, the trajectories are less sensitive to changes in the parameter values specifying the marginal impacts of utilization, distribution and debt, respectively on investment in the neighborhood of the steady state.

In this section, we studied cycles around an exogenous steady-state. Hence, the economy considered so far is not demand-driven in the long run. In the following, we will introduce the principle of effective demand in our model by endogenizing the capacity-capital ratio.

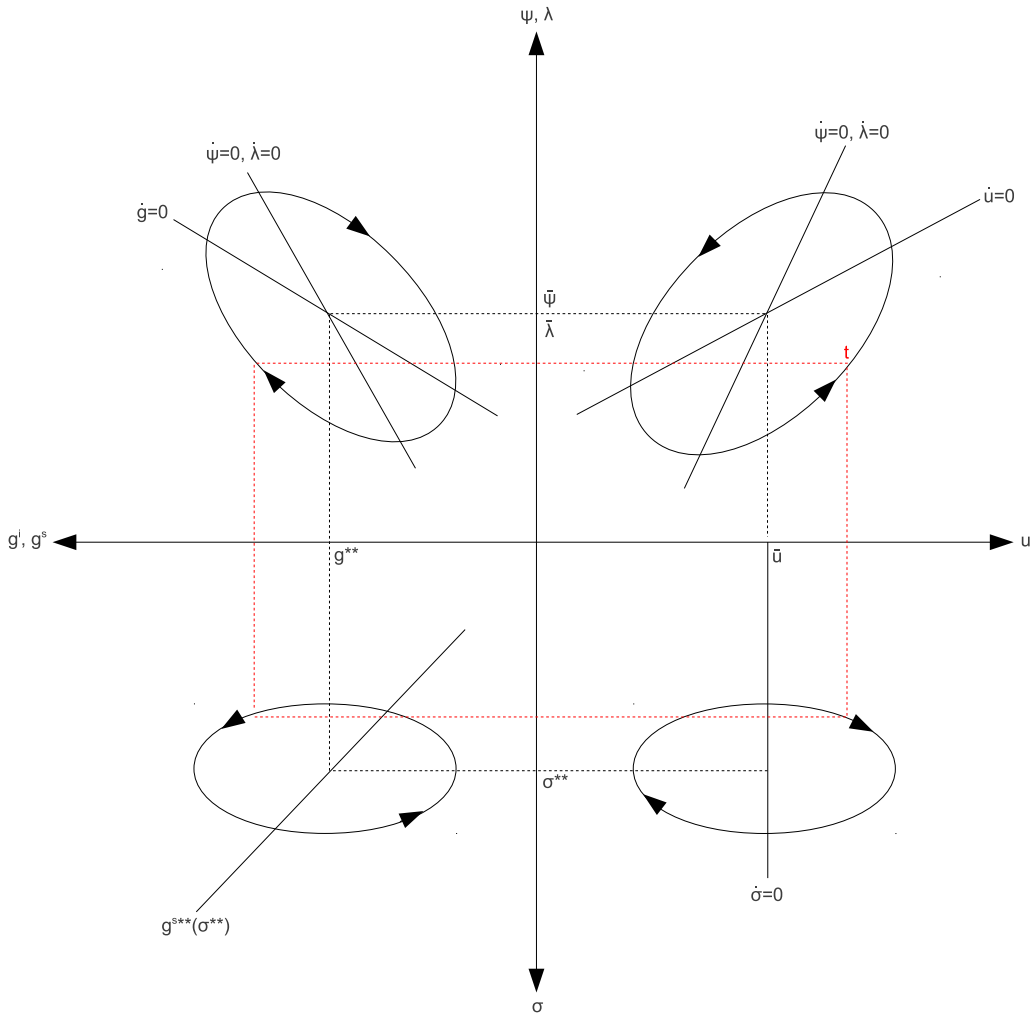

Figure 4: Cycles with endogenous capacity-capital ratio

4 Endogenizing the capacity-capital ratio

In this section, we loosen the restriction on σ of the previous section, $\phi_\sigma(\cdot) = 0$, and assume a pro-cyclical capacity-capital ratio. We study how this change affects the cycles previously discussed as well as how demand shocks affect the dynamics and steady states. As it turns out, the principle of effective demand will prevail if the capacity-capital ratio is endogenous. The paradoxes of thrift and cost may hold depending on parameter values.

4.1 Cycles with an endogenous capacity-capital ratio

Endogenizing the capacity-capital ratio, σ, implies adding another state variable to the system. To avoid the cumbersome analysis of a 4×4 dynamical system, we consider utilization-

distribution cycles and utilization-debt cycles separately. Moreover, we assume parameters and non-linear functional forms such that limit-cycles as illustrated in the previous section exist. Note from the forth term in the numerator in equation (14) that an endogenous σ contributes to stabilizing the system by reducing the overall Harrodian instability in the system. Hence, in both the (ψ, u) and (λ, u)-plain, the limit cycles will be smaller as compared to the cycles generated by the systems with an exogenous σ.

Such a limit cycle around the steady state is illustrated in the north-east quadrant of Figure 4. Since the dynamical systems of utilization and distribution as well as utilization and debt are mathematically equivalent, the ordinate and the relevant nullcline have two interpretations. The u-nullcline and the ψ-nullcline and λ-nullcline, respectively, intersect at the steady state, $(\bar{u}, \bar{\psi})$ and $(\bar{u}, \bar{\lambda})$, respectively. At the steady state, the utilization gap is zero and the capacity-capital ratio is constant at σ^{**} as illustrated in the south-east quadrant. From (2) to (4), $\bar{u}$, $\bar{\psi}$ and σ^{**} imply the steady-state value of the accumulation rate, g^{**} (south-west quadrant), which is consistent with the steady-state accumulation rate implied by the accumulation-distribution and accumulation-debt dynamical system (north-west quadrant). For given ψ, λ and σ as well as model parameters, u and g are monotonically related through the macroeconomic balance condition.

All quadrants in Figure 4 illustrate the same limit cycle from different perspectives. The swings in u implied by the counter-clockwise cycle in the (ψ, u) and (λ, u)-plain, respectively (north-east quadrant), generate swings in σ through (10) (south-east quadrant). The cyclical behavior of u, ψ and σ imply a counter-clockwise cycle in the $(-g, -\sigma)$-plain (south-west quadrant). Given the cycles in the north-east quadrant, the monotonicity between g and u implies cycles as illustrated in the north-west quadrant.

Suppose the economy is at point t in the north-east quadrant. Both utilization and the wage share and the debt-capital ratio, respectively, are increasing. Since the utilization gap is positive, the capacity-capital ratio is increasing as well. With utilization, the wage share and the capacity-capital ratio rising, saving goes up, and so does investment. Within this graphical framework, shocks to the system can now be analyzed.

4.2 The effect of a demand shock on long-run growth

Suppose the economy moving along the limit cycle is hit by a permanent positive demand shock during the downturn. This case is illustrated in Figure 5. For instance, the model above could be extended to include a government sector, or autonomous investment could be added to the investment function. An autonomous expansion of government expenditures or investment increases the short-run equilibrium of the utilization rate, given distribution and debt, respectively. Hence, the economy leaves the limit cycle and moves eastwards in the north-eastern quadrant. From there, the long-run forces, i.e. the pressures on wages and debt, respectively, as well as on utilization, let the economy spiral outwards until it reaches the limit cycle again. The positive demand shock during the downturn reduces the negative amplitude of the cycle in terms of utilization. This has long-run effects on the capacity-capital ratio. As indicated in the south-east quadrant, the reduction of the depth and duration of the recession implies a smaller loss of capital productivity following from (10) characterizing

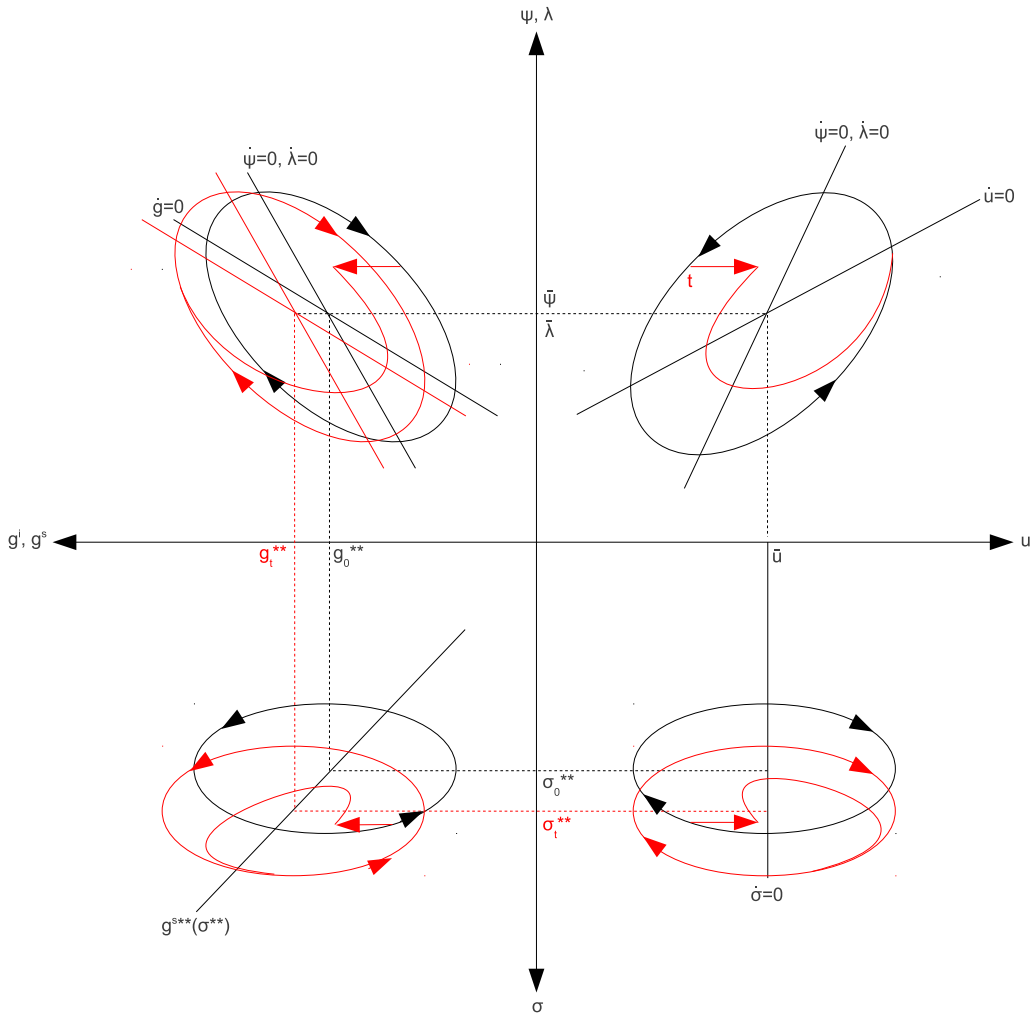

Figure 5: The effects of a permanent demand shock on the system

the change in σ as a function of the utilization gap. After the demand shock, σ will oscillate around a higher steady-state level. A permanently increased σ^{**} shifts the saving cycle in the south-west quadrant south-westwards. Because of the increased capacity-capital ratio, the nullclines in the north-western quadrant will shift outwards such that the new steady growth rate is consistent with the steady saving rate. Overall, an expansionary permanent demand shock has a permanent growth effect due to an increased average capacity-capital ratio. The principle of effective demand holds in the long run.

Note that the effect of a demand shock on the system depends on its timing. An expansionary shock is more effective in the downturn rather than in the early recovery. However, it may also be effective in the late upswing by extending the amplitude and duration of the boom.

Further, the centrifugal forces and centripetal forces, respectively, influence the long-run effectiveness of a demand shock. For instance, if the forces are strong and the economy quickly returns to the limit cycle, the change in the long-run growth rate will be moderate. The same holds for very week forces with the economy returning to the orbit after several cycles.

4.3 The paradox of thrift in the long run

Provided parameter values and functional relationships generate the type of limit cycles considered here, the paradox of thrift may hold or not in the long run. Since $g^{**} = s(\bar{\psi}, \bar{u}, \sigma^{**})\bar{u}\sigma^{**}$, the response of the growth rate to a change in the personal saving rate, s, depends on the latter's effects on σ^{**} and $s(\cdot)$. Basically, it will only hold if the steady-state capacity-capital ratio increases more than the overall economy's propensity to save out of income at the steady state decreases in a response to a drop in s. Again, these relative effects depend on parameter values, centrifugal and centripetal forces as well as the timing of the shock.

Figure 6 illustrates a constellation of the dynamics of ψ, u and σ for which the paradox of thrift holds with a very elastic σ and a very inelastic $s(\cdot)$. A reduction of the personal saving rate increases demand and utilization moving the economy eastwards in the north-east quadrant. With a lower s the u-nullcline will be steeper implying a larger limit cycle. Because of the prolonged and intensified boom in terms of u, the σ-cycle moves downwards around a higher steady state value. In the demand shock with a constant saving rate previously considered, a higher σ necessitates higher saving. Yet, an expansion due to a drop in the saving rate is also associated with a downward rotation of the saving curve, $g^{s**}(\sigma^{**})$. Hence the expansionary effect of a higher σ on g is reduced (and may be overcompensated) by a reduction of the overall propensity to save out of income. In the case illustrated in Figure 6, the drop in s has positive growth effects. The paradox of thrift holds.

For the sake of completeness, Figure 7 depicts the equivalent case for the dynamics of λ, u and σ. The only difference to the previous case worth to note is that a drop in s now causes the u-nullcline to shift downwards while the λ-nullcline moves and rotates westwards. Note that the endogenous steady-state value of λ decreases.

4.4 The paradox of cost in the long run

Figure 7 also describes a potential behavior of the system after a permanent rise in the wage share. In the short-run, our economy is wage-led. Hence, a redistribution of income from profits to wages increases utilization and moves the economy eastwards in the northeast quadrant. Both the u and λ-nullclines shift downwards with a new steady state for λ. The extension of the boom lets σ go up in the steady state. The saving curve rotates counter-clockwise due to a higher wage share. Nevertheless, with a relatively strong increase in σ, the steady-state accumulation rate may go up. Hence, the paradox of cost holds in the economy after the shock described in Figure 7.

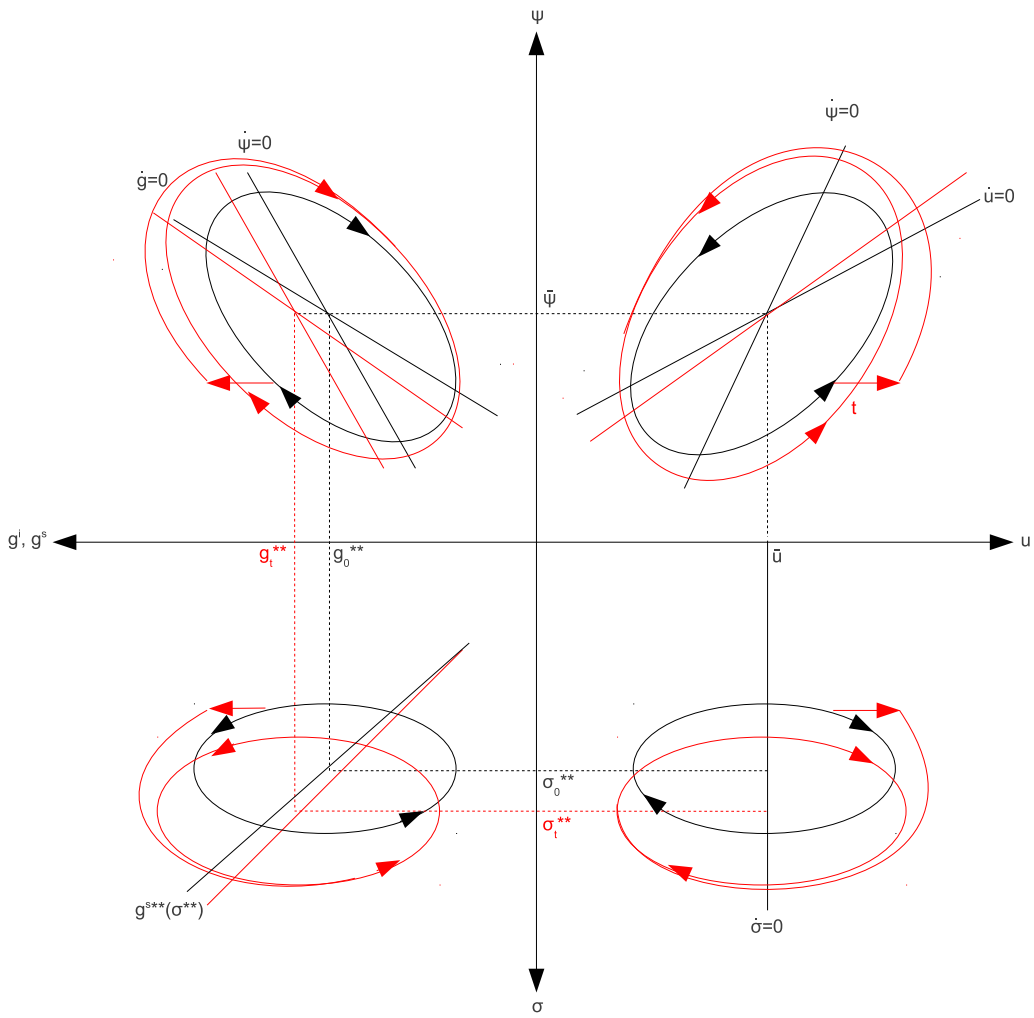

Figure 6: Drop in the saving rate with utilization-distribution cycles

5 Concluding remarks

The paper has sought to contribute to reconciling Harrodian and Kaleckian growth theory. Different specifications of a Kaleckian growth model have been presented featuring Harrodian investment dynamics, a fully adjusted steady state, an exogenous long-run utilization rate, cycles consistent with empirical observation, i.e. counter-clockwise in the utilization-wage share-space and utilization-debt-space, respectively as well as the principle of effective demand in the long run possibly including the paradox of thrift and paradox of cost.

The crucial innovation allowing the model being consistent with all these stylized facts while preserving the principle of effective demand is a pro-cyclical capacity-capital ratio as suggested theoretically and empirically by Schoder (2012a,b).

Starting from a Kaleckian model for the short run comprising an investment function

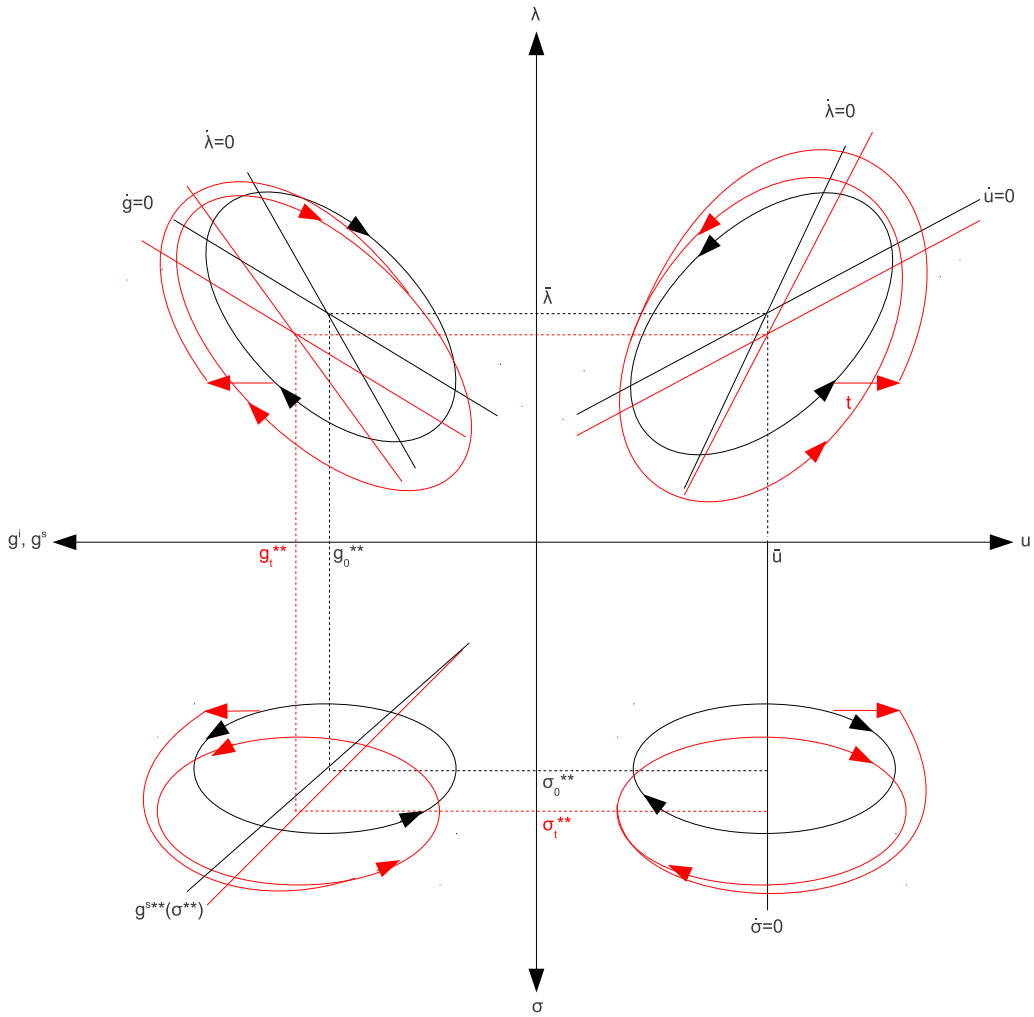

Figure 7: Drop in the saving rate/rise in the wage share with utilization-debt cycles

and a saving function, the following long-run features have been included: (a) destabilizing Harrodian investment dynamics, (b) a stabilizing labor-cost effect on investment, (c) a stabilizing debt effect on investment, (d) a radical distribution mechanism, (e) a monetary policy rule and (f) an endogenous capacity-capital ratio.

Imposing restrictions on the resulting 4-dimensional system, we have studied the dynamics between utilization and distribution, utilization and debt as well as utilization, distribution and debt. With plausible parameter values and partly non-linear functional forms empirically observed counter-clockwise limit cycles can be generated despite the economy being wage-led in the short run and including Harrodian investment.

Demand effects on steady-state growth has then been studied by endogenizing the capacity-capital ratio. Since opportune demand shocks, shocks to the saving rate and distributional

shocks may shorten the downturn or prolong the upswing, permanent growth effects can be realized through permanent changes in the capacity-capital ratio even though the long-run utilization rate is constant.

References

Adrian, T., Shin, H. S. (2010): Liquidity and leverage, *Journal of Financial Intermediation*, 19(3), pp. 418–437.

Amadeo, E. J. (1986): Notes on capacity utilisation, distribution and accumulation, *Contributions to Political Economy*, 5, pp. 83–94.

Auerbach, P., Skott, P. (1988): Concentration, competition and distribution, *International Review of Applied Economics*, 2(1), pp. 42–61.

Barbosa-Filho, N. H., Taylor, L. (2006): Distributive and demand cycles in the US economy-A structuralist Goodwin model, *Metroeconomica*, 57(3), pp. 389–411.

Bhaduri, A., Marglin, S. (1990): Unemployment and the real wage: the economic basis for contesting political ideologies, *Cambridge Journal of Economics*, 14, pp. 375–93.

Blecker, R. A. (1989): International competition, income distribution and economic growth, *Cambridge Journal of Economics*, 13, pp. 395–412.

Blinder, A. S., Canetti, E. R. D., Lebow, D. E., Rudd, J. B. (1998): *Asking About Prices*, Russell Sage Foundation, New York.

Committeri, M. (1986): Some comments on recent contributions on capital accumulation, income distribution and capacity utilization, *Political Economy*, 2(2), pp. 161–86.

Dallery, T. (forthcoming): Kaleckian models of growth and distribution revisited: Evaluating their relevance through simulation, *Intervention. European Journal of Economics and Economic Policies*.

Duménil, G., Lévy, D. (1999): Being Keynesian in the short term and classical in the long term: The traverse to classical long-term equilibrium, *The Manchester School*, 67(6), pp. 684–716.

Duménil, G., Lévy, D. (2004): *Capital Resurgent: Roots of the Neoliberal Revolution.*, Harvard University Press, Cambridge, MA.

Dutt, A. K. (1984): Stagnation, income distribution and monopoly power, *Cambridge Journal of Economics*, 8(1), pp. 25–40.

Dutt, A. K. (1997): Equilibrium, path dependence and hysteresis in post-Keynesian models, in: Arestis, P., Palma, G., Sawyer, M., eds., *Markets, Unemployment and Economic Policy: Essays in Honour of Geoff Harcourt*, Routledge, London, vol. 2, pp. 238–53.

Dutt, A. K. (2009): Path dependence, equilibrium and economic growth, in: Arestis, P., Sawyer, M., eds., *Path Dependency in Macroeconomics. International Papers in Political Economy*, Macmillan Palgrave, Houndmills, New York, pp. 119–61.

Flaschel, P. (2009): *The Macrodynamics of Capitalism: Elements for a Synthesis of Marx, Keynes, and Schumpeter*, Springer, Berlin.

Flaschel, P., Skott, P. (2006): Steindlian Models Of Growth And Stagnation, *Metroeconomica*, 57(3), pp. 303–338.

Foley, D. K., Michl, T. R. (1999): *Growth and Distribution*, Harvard University Press, Cambridge, London.

Hein, E. (2007): Interest rate, debt, distribution and capital accumulation in a post-Kaleckian model, *Metroeconomica*, 58(2), pp. 310–39.

Hein, E., Lavoie, M., van Treeck, T. (2011): Some instability puzzles in Kaleckian models of growth and distribution: A critical survey, *Cambridge Journal of Economics*, 35(3), pp. 587–612.

Hein, E., Lavoie, M., van Treeck, T. (2012): Harrodian instability and the 'normal rate' of capacity utilisation in Kaleckian models of distribution and growth - a survey, *Metroeconomica*, 63(1), pp. 139–169.

Hein, E., Schoder, C. (2011): Interest rates, distribution and capital accumulation – A post-Kaleckian perspective on the US and Germany, *International Review of Applied Economics*, 25(6), pp. 693–723.

Hein, E., Vogel, L. (2008): Distribution and growth reconsidered - empirical results for six OECD countries, *Cambridge Journal of Economics*, 32(3), pp. 479–511.

Klenow, P. J., Kryvtsov, O. (2008): State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?, *The Quarterly Journal of Economics*, 123(3), pp. 863–904.

Kurz, H. D. (1994): Growth and distribution, *Review of Political Economy*, 6(4), pp. 393–420.

Lavoie, M. (1992): *Foundations of Post-Keynesian Economic Analysis*, Edward Elgar, Aldershot.

Lavoie, M. (1995a): Interest rates in post-Keynesian models of growth and distribution, *Metroeconomica*, 46, pp. 146–77.

Lavoie, M. (1995b): The Kaleckian Model of Growth and Distribution and Its Neo-Ricardian and Neo-Marxian Critiques, *Cambridge Journal of Economics*, 19(6), pp. 789–818.

Lavoie, M. (1996): Traverse, hysteresis, and normal rates of capacity utilization in Kaleckian models of grwoth and distribution, *Review of Radical Political Economics*, 28(4), pp. 113–47.

Minsky, H. P. (1976): *John Maynard Keynes*, Macmillan, London.

Naastepad, C. W. M., Storm, S. (2006-07): OECD demand regimes (1960-2000), *Journal of Post Keynesian Economics*, 29(2), pp. 211–46.

Nakamura, E., Steinsson, J. (2008): Five Facts about Prices: A Reevaluation of Menu Cost Models, *The Quarterly Journal of Economics*, 123(4), pp. 1415–1464.

Ndikumana, L. (1999): Debt service, financing constraints, and fixed investment: evidence from panel data, *Journal of Post Keynesian Economics*, 21(3), pp. 455–78.

Nikiforos, M. (2011): On the Desired Rate of Capacity Utilization, Working Paper 16-2011, The New School, New York.

Orhangazi, O. (2008): Financialisation and capital accumulation in the non-financial corporate sector, *Cambridge Journal of Economics*, 32(6), pp. 863–86.

Rowthorn, R. E. (1981): Demand, real wages and economic growth, *Thames Papers in Political Economy*, Autumn, pp. 1–39.

Schoder, C. (2012a): Effective demand, exogenous normal utilization and endogenous capacity in the long run. Evidence from a CVAR analysis for the US, Working Paper 103, Macroeconomic Policy Institute (IMK), Düsseldorf, Germany.

Schoder, C. (2012b): Endogenous capital productivity in the Kaleckian growth model. Theory and Evidence, Working Paper 102, Macroeconomic Policy Institute (IMK), Düsseldorf, Germany.

Schoder, C. (2012c): Hysteresis in the Kaleckian growth model: A Bayesian Analysis for the US manufacturing sector from 1984 to 2007, *Metroeconomica*, 63(3), pp. 542–568.

Schoder, C. (forthcoming): Demand, q, financial constraints and shareholder value revisited: an econometric micro-analysis of US fixed investment, *International Journal of Economics and Business Research*.

Skott, P. (1989a): *Conflict and Effective Demand in Economic Growth*, Cambridge University Press, Cambridge.

Skott, P. (1989b): Effective Demand, Class Struggle and Cyclical Growth, *International Economic Review*, 30(1), pp. 231–47.

Skott, P. (2010): Growth, instability and cycles: Harrodian and Kaleckian models of accumulation and income distribution, in: Setterfield, M., ed., *Handbook of Alternative Theories of Economic Growth*, Edward Elgar, pp. 108–32.

Skott, P. (2012): Theoretical and empirical shortcomings of the Kaleckian investment function, *Metroeconomica*, 63(1), pp. 109–138.

Skott, P., Zipperer, B. (2010): An empirical evaluation of three post Keynesian models, Working Papers 2010-08, University of Massachusetts Amherst, Department of Economics.

Steindl, J. (1979): Stagnation Theory and Stagnation Policy, *Cambridge Journal of Economics*, 3(1), pp. 1–14.

Stockhammer, E., Onaran, Ö. (2004): Accumulation, distribution and employment: a structural VAR approach to a post-Keynesian macro model, *Structural Change and Economic Dynamics*, 15, pp. 421–47.

Stockhammer, E., Onaran, Ö., Ederer, S. (2009): Functional income distribution and aggregate demand in the Euro area, *Cambridge Journal of Economics*, 33(1), pp. 19–40.

Taylor, L. (1985): A stagnationist model of economic growth, *Cambridge Journal of Economics*, 9(4), pp. 381–403.

Taylor, L. (2004): *Reconstructing Macroeconomics: Structuralist Proposals and Critiques of the Mainstream*, Harvard University Press, Cambridge.

Taylor, L. (2012): Growth, cycles, asset prices and finance, *Metroeconomica*, 63(1), pp. 40–63.

Zipperer, B., Skott, P. (2010): Cyclical patterns of employment, utilization and profitability, Working Papers 2010-02, University of Massachusetts Amherst, Department of Economics.

A Jacobians of the dynamical systems considered

A.1 The unrestricted system

The elements of the Jacobian of the unrestricted system in u, ψ, λ and σ characterized by (8), (10) and (12) to (14) are as follows:

$$J_{11} = \frac{\frac{\partial \phi_\rho(\cdot)}{\partial u} - \big(1-(1-s)\psi\big)\big(\phi_\sigma(\cdot) + u\frac{\partial \phi_\sigma(\cdot)}{\partial u}\big) + (1-s)\psi\sigma\big(\phi_\psi(\cdot) + u\frac{\partial \phi_\psi(\cdot)}{\partial u}\big)}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}}$$

$$J_{21} = \frac{-\frac{\partial \phi_\eta(\cdot)}{\partial \psi} + \phi_\sigma(\cdot)(1-s)u + (1-s)u\sigma\big(\sigma_\psi(\cdot) + \psi\frac{\partial \phi_\psi(\cdot)}{\partial \psi}\big)}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}} +$$

$$+ \frac{\Big(\phi_\rho(\cdot) - \phi_\eta(\cdot) - \phi_\nu(\cdot) - \phi_\sigma(\cdot)\big(1-(1-s)\psi\big)u + (1-s)\psi\phi_\psi(\cdot)u\sigma\Big)(1-s)\sigma}{\Big(\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}\Big)^2}$$

$$J_{31} = \frac{-\frac{\partial \phi_\nu(\cdot)}{\partial \lambda}}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}}$$

$$J_{41} = \frac{(1-s)\psi\sigma_\psi(\cdot)u}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}} -$$

$$- \frac{\Big(\phi_\rho(\cdot) - \phi_\eta(\cdot) - \phi_\nu(\cdot) - \phi_\sigma(\cdot)\big(1-(1-s)\psi\big)u + (1-s)\psi\phi_\psi(\cdot)u\sigma\Big)\big(1-(1-s)\psi\big)}{\Big(\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}\Big)^2}$$

$$J_{12} = \psi\frac{\partial \phi_\psi(\cdot)}{\partial u}$$

$$J_{22} = \phi_\psi(\cdot) + \psi\frac{\partial \phi_\psi(\cdot)}{\partial \psi}$$

$$J_{32} = 0$$

$$J_{42} = 0$$

$$J_{13} = (1-\lambda)\Big(\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}\mu\Big) + \frac{\partial \phi_i(\cdot)}{\partial u}\lambda - (1-\psi)\sigma$$

$$J_{23} = -(1-\lambda)(1-s)u\sigma + u\sigma$$

$$J_{33} = -\big(1-(1-s)\psi\big)u\sigma + (1-s)\big(\bar{i} + \phi_i(\cdot)\big)\mu + \big(\bar{i} + \phi_i(\cdot)\big)$$

$$J_{43} = (1-\lambda)\big(1-(1-s)\psi\big)u - (1-\psi)u$$

$$J_{14} = \frac{\partial \phi_\sigma(\cdot)}{\partial u}$$

$$J_{24} = 0$$

$$J_{34} = 0$$

$$J_{44} = 0$$

A.2 The system restricted to utilization and distribution

The elements of the Jacobian of the system in u and ψ characterized by (8) and (14) as well as $\phi_\nu(\cdot) = 0$, $\phi_i(\cdot) = 0$ and $\phi_\sigma(\cdot) = 0$ are as follows:

$$
\begin{aligned}
J_{11} &= \frac{\frac{\partial \phi_\rho(\cdot)}{\partial u} + (1-s)\psi\sigma\big(\phi_\psi(\cdot) + u\frac{\partial \phi_\psi(\cdot)}{\partial u}\big)}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}} \\
J_{21} &= \frac{-\frac{\partial \phi_\eta(\cdot)}{\partial \psi} + (1-s)u\sigma\big(\sigma_\psi(\cdot) + \psi\frac{\partial \phi_\psi(\cdot)}{\partial \psi}\big)}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}} + \\
&+ \frac{\Big(\phi_\rho(\cdot) - \phi_\eta(\cdot) + (1-s)\psi\phi_\psi(\cdot)u\sigma\Big)(1-s)\sigma}{\Big(\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}\Big)^2} \\
J_{12} &= \psi\frac{\partial \phi_\psi(\cdot)}{\partial u} \\
J_{22} &= \phi_\psi(\cdot) + \psi\frac{\partial \phi_\psi(\cdot)}{\partial \psi}
\end{aligned}
$$

A.3 The system restricted to utilization and debt

The elements of the Jacobian of the system in u and λ characterized by (12) to (14) as well as $\phi_\psi(\cdot) = 0$, $\phi_\eta(\cdot) = 0$ and $\phi_\sigma(\cdot) = 0$ are as follows:

$$
\begin{aligned}
J_{11} &= \frac{\frac{\partial \phi_\rho(\cdot)}{\partial u}}{\big(1-(1-s)\psi\big)\sigma} \\
J_{21} &= \frac{-\frac{\partial \phi_\nu(\cdot)}{\partial \lambda}}{\big(1-(1-s)\psi\big)\sigma} \\
J_{12} &= (1-\lambda)\Big(\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}\mu\Big) + \frac{\partial \phi_i(\cdot)}{\partial u}\lambda - (1-\psi)\sigma \\
J_{22} &= -\big(1-(1-s)\psi\big)u\sigma + (1-s)\big(\bar{i} + \phi_i(\cdot)\big)\mu + \big(\bar{i} + \phi_i(\cdot)\big)
\end{aligned}
$$

A.4 The system restricted to utilization, distribution and debt

The elements of the Jacobian of the system in u, ψ and λ characterized by (8) and (12) to (14) as well as $\phi_\sigma(\cdot) = 0$ are as follows:

$$
\begin{aligned}
J_{11} &= \frac{\frac{\partial \phi_\rho(\cdot)}{\partial u} + (1-s)\psi\sigma\big(\phi_\psi(\cdot) + u\frac{\partial \phi_\psi(\cdot)}{\partial u}\big)}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}} \\
J_{21} &= \frac{-\frac{\partial \phi_\eta(\cdot)}{\partial \psi} + (1-s)u\sigma\big(\sigma_\psi(\cdot) + \psi\frac{\partial \phi_\psi(\cdot)}{\partial \psi}\big)}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}} + \\
&+ \frac{\Big(\phi_\rho(\cdot) - \phi_\eta(\cdot) - \phi_\nu(\cdot) + (1-s)\psi\phi_\psi(\cdot)u\sigma\Big)(1-s)\sigma}{\Big(\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}\Big)^2} \\
J_{31} &= \frac{-\frac{\partial \phi_\nu(\cdot)}{\partial \lambda}}{\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}} \\
J_{12} &= \psi\frac{\partial \phi_\psi(\cdot)}{\partial u} \\
J_{22} &= \phi_\psi(\cdot) + \psi\frac{\partial \phi_\psi(\cdot)}{\partial \psi} \\
J_{32} &= 0 \\
J_{13} &= (1-\lambda)\Big(\big(1-(1-s)\psi\big)\sigma - (1-s)\frac{\partial \phi_i(\cdot)}{\partial u}\mu\Big) + \frac{\partial \phi_i(\cdot)}{\partial u}\lambda - (1-\psi)\sigma \\
J_{23} &= -(1-\lambda)(1-s)u\sigma + u\sigma \\
J_{33} &= -\big(1-(1-s)\psi\big)u\sigma + (1-s)\big(\bar{i} + \phi_i(\cdot)\big)\mu + \big(\bar{i} + \phi_i(\cdot)\big)
\end{aligned}
$$

B Calibration

B.1 for utilization-distribution dynamics

$\phi_\psi(\cdot) = 0.05(u - \bar{u}) - 0.1(\psi - \bar{\psi})$
$\phi_\rho(\cdot) = -0.002 + \frac{0.004}{1+e^{10(-(u-\bar{u}))}}$
$\phi_\eta(\cdot) = -0.002 + \frac{0.004}{1+e^{30(-(\psi-\bar{\psi}))}}$
$\phi_i(\cdot) = \bar{i} + 0.04(u - \bar{u})$
$s = 0.3$
$\sigma = 0.6$
$\bar{u} = 0.7$
$\bar{\psi} = 0.7$

B.2 for utilization-debt dynamics

$\phi_\rho(\cdot) = -0.001 + \frac{0.002}{1+e^{10(-(u-\bar{u}))}}$
$\phi_\nu(\cdot) = -0.001 + \frac{0.002}{1+e^{30(-(\lambda-\bar{\lambda}))}}$

$\phi_i(\cdot) = \bar{i} + 0.4(u - \bar{u})$
$s = 0.3$
$\sigma = 0.6$
$\psi = 0.7$
$\mu = 0.1$
$\bar{i} = 0.02$
$\bar{u} = 0.7$

B.3 for utilization-distribution-debt dynamics

$\phi_\psi(\cdot) = 0.05(u - \bar{u}) - 0.1(\psi - \bar{\psi})$
$\phi_\rho(\cdot) = -0.001 + \frac{0.002}{1+e^{10(-(u-\bar{u}))}}$
$\phi_\eta(\cdot) = -0.001 + \frac{0.002}{1+e^{1(-(\psi-\bar{\psi}))}}$
$\phi_\nu(\cdot) = -0.001 + \frac{0.002}{1+e^{20(-(\lambda-\bar{\lambda}))}}$
$\phi_i(\cdot) = \bar{i} + 0.1(u - \bar{u})$
$s = 0.3$
$\sigma = 0.6$
$\mu = 0.1$
$\bar{i} = 0.02$
$\bar{u} = 0.7$
$\bar{\psi} = 0.7$

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