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**Working Paper**

## Currency Runs, International Reserves Management and Optimal Monetary Policy Rules

IMK Working Paper, No. 8/2009

**Provided in Cooperation with:**

Macroeconomic Policy Institute (IMK) at the Hans Boeckler Foundation

*Suggested Citation:* Kato, Mika; Proaño, Christian R.; Semmler, Willi (2009) : Currency Runs, International Reserves Management and Optimal Monetary Policy Rules, IMK Working Paper, No. 8/2009, Hans-Böckler-Stiftung, Institut für Makroökonomie und Konjunkturforschung (IMK), Düsseldorf,  
<https://nbn-resolving.de/urn:nbn:de:101:1-20091104844>

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## Currency Runs, International Reserves Management and Optimal Monetary Policy Rules

Mika Kato, Christian R. Proaño, Willi Semmler

September 2009

# Currency Runs, International Reserves Management and Optimal Monetary Policy Rules

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September 24, 2009

## Abstract

This paper studies the design of optimal monetary policy rules for emerging economies confronted to sharp capital outflows and speculative attacks. We extend Taylor type monetary policy rules by allowing the central bank to give some weight to the level of precautionary foreign reserve balances as one of its targets. We show that a currency crisis scenario can easily occur when the weight is zero, and that it can be avoided when the weight is positive. The impacts of the central bank's monetary control on the output level, the inflation rate, the exchange rate, and the foreign reserve level are investigated as well. By applying both the Hamiltonian as well as the Hamilton-Jacobi-Bellman (HJB) equation (the latter leading to a dynamic programming formulation of the problem), we can explore safe domains of attractions in a variety of complicated model variants. Given the uncertainties the central banks faces, we also show of how central banks can enlarge safe domains of attraction.

**Keywords:** Currency Crises, Capital Outflows, Monetary Policy Rules

**JEL CLASSIFICATION SYSTEM:** E5, F3

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## 1 Introduction

As pointed out for example in the recent IMF World Economic Outlook (IMF, 2009, p.79ff) and by Aslund (2009), numerous Central and East European (CEE) countries have been particularly affected by the global economic crisis originated in the U.S. market for subprime housing loans. As it was for example the case in the 1997/98 East Asian crisis, in recent times many Central and Eastern European countries have suffered from significant rises in their country risk premia, sharp capital outflows and an increased nominal exchange rate volatility which have made once again particularly clear the fragility of emerging market economies to external shocks.

The potential macroeconomic consequences of such developments are well-known from the numerous currency and financial crises which have occurred in the last decades: As discussed e.g. by Radelet and Sachs (2000), and Frankel (2005), the occurrence of large currency depreciations in countries with a large share of unhedged foreign currency-denominated liabilities (such as the majority of East Asian crisis before the 1997/98 crisis and as it is currently also the case of many of the Central and East European economies) involved disastrous consequences for the macroeconomic performance of those countries due to the sharp deterioration of the balance sheets of the domestic sectors, the activation of credit constraints by the foreign and domestic financial sectors and the subsequent credit crunch and the partial collapse of aggregate investment.<sup>1</sup>

Against the background of the sharp capital outflows and increase exchange rate volatility experienced by CEE countries in the recent months, the effectiveness of large foreign currency reserves held by the domestic monetary authorities as a buffer instrument against potential currency runs has become once again particularly relevant: Indeed, as discussed for example by Aizenman and Marion (2003), Aizenman and Lee (2007) and Mendoza (2007), following especially the 1997/98 East Asian crisis, many emerging economies engaged in a remarkable buildup of precautionary international reserves, despite of the significant costs that such large liquidity buffers represent (Rodrik, 2006) – see Levy-Yeyati (2008) for an alternative view in this respect –. As pointed out e.g. by Feldstein (1999, p.93), such a strategy seems rational since liquidity “is the key to self-protection: A country that has substantial international liquidity – large foreign exchange reserves and a ready source of foreign

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<sup>1</sup>See Krugman (2000), Aghion, Bacchetta and Banerjee (2001, 2004) and Flaschel and Semmler (2006) for theoretical models that relate the Bernanke and Gertler (1995) balance sheet effect with the liability dollarization phenomenon.

currency loans – is less likely to be the object of a currency attack. Substantial liquidity also permits a country that is attacked from within or without to defend itself better and to make more orderly adjustments.” Indeed, given that an IMF bailout cannot be taken for certain and that the concerned economies might be shut out from additional lines of credit during episodes of currency runs, a sufficiently large precautionary stock of foreign currency reserves might be vital for a successful defense of the domestic currency.<sup>2</sup>

Besides from the large body of literature based on the seminal papers by seminal papers by Krugman (1979) and Flood and Garber (1984), in recent times new studies have delivered important additional insights on the rationale a buildup of precautionary foreign currency reserves. Aizenman and Turnovsky (2002) for example show that sufficient international reserves of a country that is an extensive international borrower can serve as a collateral for creditors who want to secure international loans. Thereafter, the increase of reserves in heavily borrowing countries reduces default risk and may raise the welfare of both the high income lending countries as well as the emerging market economies. Aizenman and Marion (2004) in turn investigate the role of political factors in the demand for international reserves and the level of external borrowing. Arifovic and Maschek (2006), building on Arifovic and Masson (2004), investigate the role of minimum reserves holdings for the occurrence of currency crises in a model with heterogeneous beliefs and learning. In contrast, Caballero and Panageas (2007) compare the efficiency of the accumulation of international with other hedging strategies in a global equilibrium model of sudden stops, Jeanne and Rancière (2006, 2009), derive optimal levels of precautionary reserves using a representative agent model where the agent has the possibility to enter into a “reserve insurance contract” with foreign investors.

In contrast, the contribution of this paper to the literature relies in the analysis of optimal monetary policy rules within a stylized macroeconomic framework of a

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<sup>2</sup>An alternative view concerning the significant accumulation of foreign currency reserves by emerging economies is that such a development reflects a mercantilist behavior of such economies. Aizenman and Lee (2007) compare empirically the importance of precautionary versus mercantilist motivations for reserve holding using data for a sample of 49 industrialized and developing countries during the period 1980 – 2000, finding that a) variables associated with mercantilist motivations (e.g. export growth, trade openness, etc.) are statistically significant but they account for a very small part of total reserve accumulation, b) that reserve holding is strengthened significantly by more liberal capital accounts, and c) that variables related to precautionary motivations (e.g. capital account openness, crisis dummies, etc.) are both statistically and economically significant in explaining reserve holding trends.

small open emerging economy prone to speculative attacks and sudden capital outflows. As it will be discussed below, in our model the sole anticipation of a domestic currency depreciation can lead to a sharp capital outflow (which in turn decreases the foreign reserves of the domestic central bank). This, in turn, may trigger a run on the domestic currency caused by the market participant's fear of a termination of the currency conversion. Since a sharp depreciation of the domestic currency gives further incentives to foreign investors to pull their funds out of the country as quickly as possible, their expectations become self-fulfilling and a currency crisis occurs, as discussed in Krugman (1979). By incorporating a reserves target into an otherwise standard linear-quadratic monetary control problem, we study the ability of the so-designed monetary policy to diminish the country's vulnerability to speculative attacks and sharp capital outflows which arises when the monetary authorities acknowledge and incorporate the external financial fragility of their economy in their decisions.

In solving our model variants we apply both the Hamiltonian as well as the Hamilton-Jacobi-Bellman (HJB) equation leading to a dynamic programming algorithm. With these methods we can explore the question what policies may allow an increase of the safe domains of attraction so that the economy is less vulnerable to currency shocks. As we will show, the resulting build-up of large stockpiles of foreign currency reserves is an advantageous result of such a monetary policy strategy, since these might forestall – or at least weather – runs on the domestic currency. In this line of reasoning, adequate monetary and foreign currency reserve policies might indeed lower the likeliness of a currency run due to self-fulfilling expectations.

The remainder of the paper is organized as follows. In section 2 the theoretical framework is discussed. In section 3 we explore by means of numerical simulations the stabilization effects of monetary policy when the domestic central bank includes a term of reserves targeting in its loss function. Section 4 draws some concluding remarks from this study.

## 2 The Theoretical Framework

The small open economy prone to speculative attacks is assumed to be represented by the following macroeconomic relationships:

$$y = c_o - \alpha_{yr}(i - \pi^e) + X(e^e); \quad X'(e^e) > 0 \quad (1)$$

$$\pi = \beta_{\pi y}(y - y_n) + \pi^e \quad (2)$$

$$\dot{e}^e = i - i^f - \psi(i - i^f, R); \quad \partial\psi/\partial(i - i^f) > 0, \quad \partial\psi/\partial R < 0, \quad (3)$$

where  $e^e$  is the log expected exchange rate of domestic for foreign currency. An increase in  $e^e$  means an anticipation of depreciation of the domestic currency. Eq.(1) represents a standard IS relationship where the real interest rate  $i - \pi^e$  affects negatively the firm's investment decisions,  $c_o$  and  $\alpha_{yr}$  being positive constants.<sup>3</sup> With  $X(e^e)$  we characterize the net exports which are assumed to react positively to the depreciation of the expected exchange rate. Eq.(2) is a standard Phillips curve according to which the domestic inflation rate  $\pi$  is explained by the deviation of  $y$  from the natural output (or NAIRU) level  $y_n$  and an expected inflation term  $\pi^e$  (which in our most simplest variant will be presumed to be constant).<sup>4</sup> For simplicity, we assume that the policy maker sets  $y^* = y_n$ .

Finally, eq.(3) represents an extended interest rate parity condition, which is assumed to consist of two factors: the standard Uncovered Interest Rate Parity (where  $i$  is the domestic riskless rate and  $i^f$  is the foreign riskless rate), and  $\psi$ , a risk premium term assumed to comprise the exchange rate- as well as the country risk.<sup>5</sup> Concerning the former, since high interest rate differentials are usually associated with a high exchange rate volatility and therefore with a high exchange rate risk, we use  $i - i^f$  as a proxy for the exchange rate risk. Concerning the latter, because the domestic central bank's foreign reserve holdings are commonly acknowledged as one of the key factors that affects market's perception of country risk, precisely  $R$  is the second term in the risk premium term  $\psi$ .

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<sup>3</sup>In our IS equation (2) we have neglected the negative exchange rate effect on investment incorporated in Krugman (2000), Flaschel and Semmler (2006) and Proaño, Flaschel and Semmler (2008) in presence of the liability dollarization phenomenon. In order to simplify matters, in our context, we revert back to a traditional IS equation, defining output by a static and not by a dynamic equation.

<sup>4</sup>Note that in this current version of the Phillips-curve we neglect an open economy component and therefore possible direct exchange rate pass-through effects as done for example in Ball (1999).

<sup>5</sup>For details of such risk premium, see Evans and Lyons (2005).

The currency reserves dynamics in our model are described by

$$\dot{R} = X(e^e) + F(e^e, R) \quad \partial F / \partial e^e < 0, \quad \partial F / \partial R > 0 \quad (4)$$

where

$$F(e^e, R) = n(e^e) - z(e^e, R) \quad \partial z / \partial e^e > 0, \quad \partial z / \partial R < 0 \quad (5)$$

As formulated in eq.(4), there are two sources of foreign currency reserves: the net export of goods and services (represented by  $X(e^e)$ ) and the net inflow of financial funds, represented by  $F(e^e, R)$ . Eq.(5) shows that the net inflow of financial funds is equal to the inflow  $n$  minus the outflow  $z$ .<sup>6</sup> While  $n$  is a function of  $e^e$  only,  $z$  is a function of two factors:  $e^e$  and  $R$ . The reason why investors give weight to foreign reserves in their decision of pulling their funds out is somewhat related to investors' psychology. Since they understand that a serious shortage of foreign reserves may bring about a panic caused by the fear of a rapid depreciation of the currency, foreign investors will show a more sensitive reaction, in case of lower reserves, to an anticipated depreciation of the domestic currency. In short, eq.(5) tells us that investors' reaction is state-dependent.

Concerning monetary policy, we assume that the domestic central bank has three target variables: the inflation rate, the output level and a precautionary level of foreign currency reserves. As previously discussed, by building up the precautionary international reserve holdings, central banks keep the ability to defend their currency value and avoid a currency crisis scenario (we assume that these variables can be indirectly affected by the central bank through controlling the short-term nominal interest rate).

The central bank's objective is to minimize its loss function, which is defined as:

$$\mathcal{L} = \int_0^\infty e^{-rt} \left[ \frac{\lambda_1}{2} (\pi - \pi^*)^2 + \frac{\lambda_2}{2} (y - y^*)^2 + \frac{\lambda_3}{2} (R - R^*)^2 \right] dt \quad (6)$$

where  $\pi$  represents the inflation rate,  $y$  the output level,  $R$ , the stock of foreign currency reserves and  $\pi^*$ ,  $y^*$  and  $R^*$  the respective target levels. The weights  $\lambda_1$  and  $\lambda_2$  and  $\lambda_3$  measure the relative importance of the three target variables in the central bank's objective function.

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<sup>6</sup>In the short-run, the outflow and the inflow of the foreign currency due to financial investments depends on the difference of the domestic and the rest of the world's interest rates and the expected exchange rate. Here we neglect the difference in the interest rates and assume the inflow to be constant. Those simple assumptions allow us to highlight the mechanism of the currency crises resulting from the investors speculative behavior.



By inserting eqs. (1) and (2) in eq.(6), the problem for the domestic monetary policy authority can be rewritten as:

$$\begin{aligned} \min_i \mathcal{L} = & \int_0^\infty e^{-rt} \left[ \frac{\lambda_1}{2} (\pi^e + \beta_{\pi y}(c_o - \alpha_{yr}(i - \pi^e) + X(e^e) - y^*) - \pi^*)^2 \right. \\ & \left. + \frac{\lambda_2}{2} (c_o - \alpha_{yr}(i - \pi^e) + X(e^e) - y^*)^2 + \frac{\lambda_3}{2} (R - R^*)^2 \right] dt \end{aligned} \quad (7)$$

subject to eqs. (3) and (4) and the boundary conditions for  $e^e$  and  $R$ .

For the sake of deriving more specific results, we use the following explicit formulation of eq.(3):

$$\psi(R) = \frac{\sigma_1}{R} + \frac{\sigma_2}{2}(i - i^f)^2 \quad (8)$$

where the first term is country risk associated with a currency run and the second term is exchange rate risk driven by the volatility of the interest rate differentials. The terms  $\sigma_1$  and  $\sigma_2$  are constant coefficients.

In the same sense, we define the net exports function as

$$X(e^e) = \varepsilon e^e - \bar{m} \quad (9)$$

where  $\varepsilon$  represents the elasticity of exports to a change in the exchange rate, and  $\bar{m}$  represents the autonomous import level. Respecting the net inflow of financial funds, we assume the following functional form:

$$F(R, e^e) = n(e^e) - z(e^e, R) = \frac{\nu}{e^e} - \frac{\mu}{R} e^{e^2} \quad (10)$$

where  $\nu$  and  $\mu$  are constant coefficients.

By introducing these explicit expressions (8)-(10) into eq.(7), it follows for the current value Hamiltonian

$$\begin{aligned} H = & \frac{\lambda_1}{2} \{ \pi^e + \beta_{\pi y}(c_o - \alpha_{yr}(i - \pi^e) + \varepsilon e^e - \bar{m} - y^*) - \pi^* \}^2 \\ & + \frac{\lambda_2}{2} (c_o - \alpha_{yr}(i - \pi^e) + \varepsilon e^e - \bar{m} - y^*)^2 + \frac{\lambda_3}{2} (R - R^*)^2 \\ & + q_1 \left[ i - i^f - \left( \frac{\sigma_1}{R} + \frac{\sigma_2}{2}(i - i^f)^2 \right) \right] + q_2 \left( \varepsilon e^e - \bar{m} + \frac{\nu}{e^e} - \frac{\mu}{R} e^{e^2} \right), \end{aligned} \quad (11)$$

with the corresponding first-order conditions

$$\begin{aligned} \frac{\partial H}{\partial i} = & -\lambda_1 \alpha_{yr} \beta_{\pi y} (\pi^e - \pi^*) - \alpha_{yr} (\lambda_1 \beta_{\pi y}^2 + \lambda_2) (c_o - \alpha_{yr} (i - \pi^e) + \varepsilon e^e - \bar{m} - y^*) \\ & + q_1 (1 - \sigma_2 (i - i^f)) = 0 \end{aligned} \quad (12)$$

$$\begin{aligned} \dot{q}_1 = & r q_1 - \left( \epsilon + \frac{\nu}{e^{e2}} + 2\mu \frac{e^e}{R} \right) q_2 - \lambda_1 \beta_{\pi y} \epsilon (\pi^e - \pi^*) \\ & - (\lambda_1 \beta_{\pi y} + \lambda_2) (\alpha - \alpha_{yr} (i - \pi^e) + \varepsilon e^e - \bar{m} - y^*) \end{aligned} \quad (13)$$

$$\dot{q}_2 = \left[ r - \mu \left( \frac{e^e}{R} \right)^2 \right] q_2 - \lambda_3 (R - R^*) - \frac{\sigma_1}{R^2} q_1 \quad (14)$$

$$\dot{e}^e = i - i^f - \frac{\sigma_1}{R} - \frac{\sigma_2}{2} (i - i^f)^2 \quad (15)$$

$$\dot{R} = \varepsilon e^e - \bar{m} + \frac{\nu}{e^e} - \frac{\mu}{R} e^{e2}. \quad (16)$$

From eq.(12), we obtain

$$q_1 = \frac{\lambda_1 \alpha_{yr} \beta_{\pi y} (\pi^e - \pi^*) + \alpha_{yr} (\lambda_1 \beta_{\pi y}^2 + \lambda_2) (c_o - \alpha_{yr} (i - \pi^e) + \varepsilon e^e - \bar{m} - y^*)}{1 - \sigma_2 (i - i^f)}. \quad (17)$$

Incorporating (17) into (13), (15), (16), and (14) and solving  $\dot{q}_1 = \dot{q}_2 = \dot{e}^e = \dot{R} = 0$  for  $i$ ,  $e^e$ ,  $R$ , and  $q_2$  gives a new set of steady states.

Next, we discuss some numerical results from our theoretical model.

### 3 Numerical Analysis

#### 3.1 Monetary Policy without Foreign Reserves Control

In order to highlight the importance of the targeting of the foreign currency reserves for dynamics of the analyzed small open economy, we begin by presuming that the domestic monetary authority does not attempt to control the foreign reserves but only aims at controlling the inflation rate and output through steering the interest rate. In the next numerical example we set thus  $\lambda_3 = 0$  (the relative weight of the reserves targeting term in the loss function of the domestic central bank).

Note that in this first example (Table 1 shows the corresponding parameter values) we set  $\sigma_1 = 0$  and  $\sigma_2 = 0$  (and therefore  $\psi = 0$ ), meaning that there is no risk premium for the asset holding in the domestic currency. Yet, we have introduced with  $\nu > 0$ ,  $\mu > 0$ , an effect on foreign exchange reserves triggered by the level of exchange rates and the level of reserves. Note that we here also presume the most

simplest variant where, as in the output determination of eq.(1) and the Phillips-curve of eq.(2) the expected inflation rate,  $\pi^e$ , is a constant. We for convenience also take  $\bar{m} = 0$ .

Table 1: Parameter values: No Foreign Reserves Targeting,  $\psi = 0$

$\lambda_1$	$\lambda_2$	$\lambda_3$	$\pi^*$	$\pi^e$	$y^*$	$R^*$	$i^f$	$r$
1	1	0	.03	.03	10	.00	.05	.02
$\alpha$	$\alpha_{yr}$	$\beta_{\pi y}$	$\sigma_1$	$\sigma_2$	$\varepsilon$	$\bar{m}$	$\nu$	$\mu$
5	1	.01	.00	.00	1	.00	100	10

Under the parameters of Table 1 there exists a unique steady state, which values for the reserves  $R$ , interest rate,  $i$ , the exchange rate,  $e$ , output  $y$ , and the inflation rate,  $\pi$ , are shown in Table 2.<sup>7</sup>

Table 2: Steady State Values: No Foreign Reserves Targeting,  $\psi = 0$

Steady State	$R$	$i$	$e$	$y$	$\pi$
1	10.104	0.05	5.02	10	0.03

The dynamics about the steady state of the state variables  $R$  and  $e$  are computed using a dynamic programming algorithm as applied to dynamic economic problems in Grüne and Semmler (2004). Figure 1 shows the value function about the unique steady state.

Figure 1 shows that there is a deep valley of the value function about the steady state demonstrating that the value function indeed achieves a minimum about the steady state values in the vicinity of the steady state  $R^* = 10.104$  and  $e^* = 5.02$ . The vector field for the solution of the dynamic programming problem described by eqs.(12) – (14), represented by the state equations (15) and (16), illustrates the dynamics about the steady state. As can be observed from the trajectories all trajectories move toward the unique steady state  $\bar{R}, \bar{e}$  in the vicinity of the steady state. Thus, all currency reserves, displaced by a shock from the steady – but with initial conditions in the region between zero and the steady state  $\bar{R}$  in the deep valley – will

<sup>7</sup>We computed the steady state for the above parameter values, using eqs. (13)–(17), employing the software package Mathematica. The eigenvalues obtained from the 4D system in  $q_1, q_2, e, R$  at this unique steady state are  $\{2.46829, -2.44829, 1.0, -0.989899\}$ . The steady state is thus a saddle point in terms of state and co-state variables. It is, however, a general mathematical result, that a saddle point in the state-costate space corresponds to a stable point in the state space dynamics of a HJB-equation.

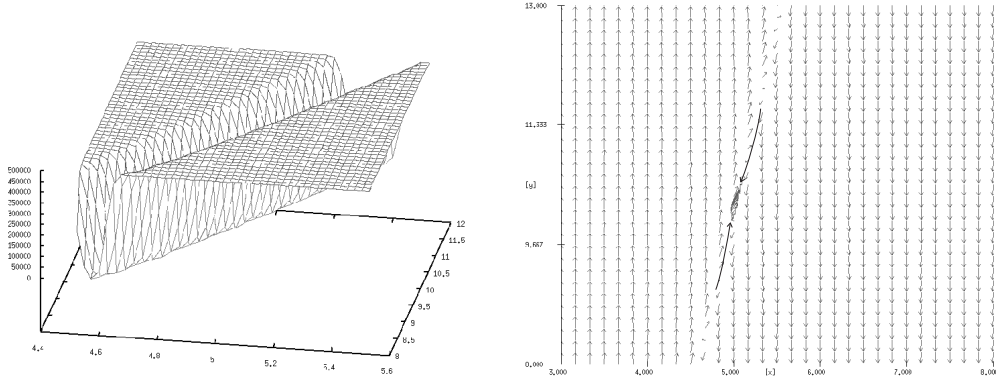


Figure 1: Value function about the unique steady state and vector field, Case 1

safely move back to the steady state  $\bar{R}$ . Yet, shocks that move the initial conditions beyond that region will, as Figure 1 shows, lead to a divergence of the currency reserves,  $R$ , and the exchange rate  $e$ .<sup>8</sup>

We now set  $\sigma_1 = 0.3$ , thus permitting the function  $\psi > 0$  due to a risk premium arising from low currency reserves (we thus again refer to the simple variant just discussed). The other parameters remain the same as in Table 1, and we still keep  $\sigma_2 = 0$ .

As Table 3 makes clear, due to the introduction of a risk premium  $\sigma_1 > 0$ , multiple steady state equilibria arise.<sup>9</sup>

Table 3: Multiple steady states: No Foreign Reserves Targeting,  $\sigma_1 = 0.3$ ,  $\sigma_2 = 0$ ,  $\psi > 0$

Steady State	$R$	$i$	$e$	$y$	$\pi$
1	10.257	0.079	5.049	10.000	0.030
2	0.519	0.627	1.7497	6.150	-0.008
3	-0.051	-5.773	-0.803	10.000	0.030
4	-0.519	-0.527	-1.749	3.800	-0.031

<sup>8</sup>Numerical Simulations using the actual inflation rate  $\pi$  instead of  $\pi^e = \text{constant}$  delivered quite similar results, see Kato and Semmler (2005). Since neither the shape of the value function nor the dynamics significantly changed, we subsequently work with this simpler variant.

<sup>9</sup>The eigenvalues obtained from the 4D system in  $q_1, q_2, e, R$  at each steady state (from SS1 to SS4) are:  $\{-2.43215, 2.3969, 1.02606, -0.960707\}$ ,  $\{-114.343, 112.387, 0.992782 + 2.75418 i, 0.992782 - 2.75418 i\}$ ,  $\{-114.246, 112.286, 4.84355, -2.85307\}$ ,  $\{-2453.37, 2409.97, 22.8586, 20.574\}$ . Note that SS(1) is stable, SS(2) is unstable, SS(3) is stable and SS(4) again unstable.

Note that the highest steady state is almost the same as in Table 2 for the first example (with  $\sigma_1 = 0$ ,  $\sigma_2 = 0$ ), yet in this second analyzed case there are three additional steady states arising, with of them featuring a negative  $R$  (this case should be interpreted as a situation where the country may have to obtain a credit line on external reserves for example from the IMF in order to serve its liabilities). Overall, due to country risk ( $\sigma_1 > 0$ ) the domain of attraction of the steady state  $\bar{R} = 10.257$  has slightly decreased, since a new unstable positive steady state has emerged at  $\bar{R} = 0.519$ . Now, due to the fact that there is a state dependent risk premium,  $\psi > 0$ , shocks may produce a more vulnerable situation for the country's currency reserves.<sup>10</sup>

In the following case we permit both  $\sigma_1 > 0$ ,  $\sigma_2 > 0$ , employing still the parameters as shown in Table 4.

Table 4: Parameters values: No Foreign Reserves Targeting,  $\psi > 0$  ( $\sigma_1 > 0$ ,  $\sigma_2 > 0$ )

$\lambda_1$	$\lambda_2$	$\lambda_3$	$\pi^*$	$\pi^e$	$y^*$	$R^*$	$i^f$	$r$
1	1	10	.03	.03	10	.00	.05	.02
$\alpha$	$\alpha_{yr}$	$\beta_{\pi y}$	$\sigma_1$	$\sigma_2$	$\varepsilon$	$\bar{m}$	$\nu$	$\mu$
5	1	.01	2	2	1	.00	100	10

As in the previous case, we also obtain in this numerical example (where the risk premium rises due to both exchange and the country risk effects) multiple steady states. We have left aside the detailed exploration of the number of steady state equilibria but explore instead, using dynamic programming, in what direction the largest steady state moves.

As Figure 2 demonstrates, the required equilibrium reserves in this case are upward shifting when the home country's risk premium is rising due to  $\sigma_1 > 0$  and  $\sigma_2 > 0$ . This shows that with small risk premia the required reserves will be small, but the region of stability is also small. Yet, with larger risk premia, the required foreign currency reserves rise (although the region of a stable domain may rise too). But as we will show below, the region of a stable domain will rise even much more if currency reserve targeting is directly included in the objective function of the central bank.

<sup>10</sup>Note that a large negative  $\psi$ , due to a negative  $R$ , an external credit line of a country, might not be very reasonable, so one may disregard the steady with a negative  $R$  or constrain the  $\psi$  by a lower bound.

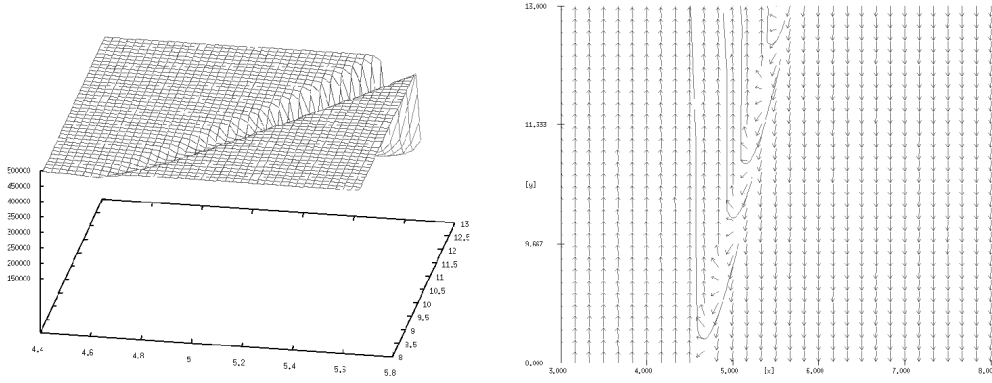


Figure 2: Minimum of the value function moves up (equilibrium  $R$  is rising) and vector field pointing upward

### 3.2 Monetary Policy with Foreign Reserves Control

As Table 5, we now assume that the domestic monetary authorities pursue also an exogenously determined foreign currency reserves target  $R^*$  (as originally formulated in eq.(6)) besides the standard inflation and output gap targets.

Table 5: Parameters Values: Foreign Reserves Targeting,  $\psi = 0$

$\lambda_1$	$\lambda_2$	$\lambda_3$	$\pi^*$	$\pi^e$	$y^*$	$R^*$	$i^f$	$r$
1	1	10	.03	.03	10	100	.05	.02
$\alpha$	$\alpha_{yr}$	$\beta_{\pi y}$	$\sigma_1$	$\sigma_2$	$\varepsilon$	$\bar{m}$	$\nu$	$\mu$
5	1	.01	.00	.00	1	.00	100	10

As Table 6 makes clear, the introduction of  $\lambda_3 > 0$  (and therefore  $R^*$ ) into the linear quadratic monetary control problem given by eq.(7) does not prevent the arise of multiple steady states.<sup>11</sup>

Figure 3 depicts the value function about the high steady state characterized by  $R = 100.042$  and  $e = 14.659$ .

Figure 3 illustrates some important insights: In the first place, compared with

<sup>11</sup>The eigenvalues obtained from the 4D system in  $q_1, q_2, e, R$  at each steady state (from SS1 to SS3) are:  $\{3.27181, -3.25073, 0.00 + 3.14646 i, 0.00 - 3.14646 i\}$ ,  $\{3.83562 \times 10^6, -3.83562 \times 10^6, 8.79169, -8.78159\}$ ,  $\{3.46176 \times 10^6, -3.46176 \times 10^6, 0.00 + 8.7866 i, 0.00 - 8.7866 i\}$ . Note that no reference on the stability properties of the equilibria can be made when the real parts of the eigenvalues are zero. Dynamic programming has to be used instead.

Table 6: Steady State Values, Foreign Reserves Targeting,  $\psi = 0$

Steady State	$R$	$i$	$e$	$y$	$\pi$
1	100.042	.050	14.659	19.639	0.126
2	0.0002	.0500	0.127	5.107	-0.018
3	-0.0002	.050	-0.130	4.849	-0.021

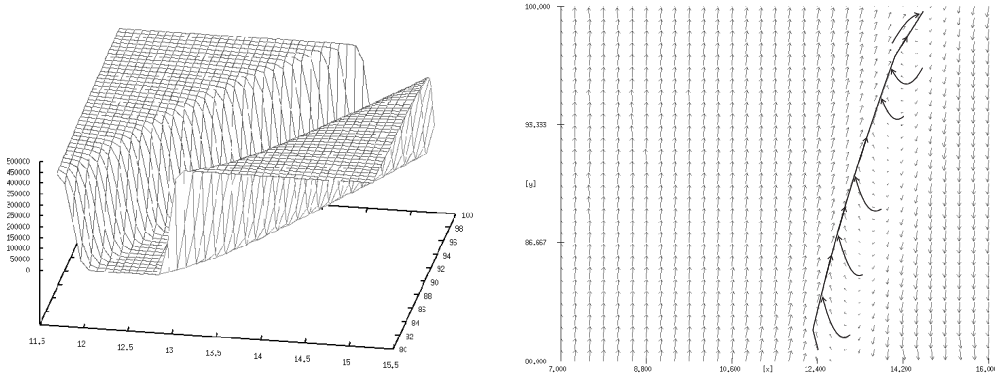


Figure 3: Value function and vector field about high steady state

the case illustrated in Figure 2, in the reserves targeting case depicted in Figure 3 the minimum of the value function lies again in a deep valley, what clearly shows the stronger attraction of the high steady state. This is the case despite that the required reserves level  $R^*$  is assumed to be very high and that the introduced  $\lambda_3 = 10$  implies a strong reaction of the monetary authority to the deviation of foreign reserves to its target. This is purposely undertaken in order to obtain distinct and clearly separated steady states. This figure shows there are converging dynamics toward the high steady state which is indicated by the arrows, thus the highest steady state SS(1) is an attractor. Thus, as we can observe from Table 6 and Figure 3, when the central bank targets a currency reserve level, the distance between the lower (unstable) steady of  $R$  (roughly zero) and the high steady state of  $R$  (roughly 100) increases significantly, what means that the domain of attraction is enlarged to an important extent. We can also observe from Table 6 that the steady state output for the high level steady state of  $R$  is much higher than for the lower one. In other words, as compared to the case when the central bank does not target the currency reserves, in the case above when the central bank does target the reserves a much larger domain of attraction emerges.

As next, we employ again the parameters reported in Table 5 for the previous example, but assume additionally  $\sigma_1 = 0.3$ , and  $\sigma_2 = 0$ . There is now a risk premium term  $\psi > 0$  responding to low currency reserves.

Table 7: Steady State Values, Foreign Reserves Targeting,  $\sigma_1 = 0.3$ ,  $\sigma_2 = 0$ ,  $\psi > 0$

Steady State	$R$	$i$	$e$	$y$	$\pi$
1	100.042	0.053	14.659	19.636	0.126
2	-0.032	-9.229	-0.687	13.572	0.065

As shown in Table 7 this time due to the introduction of the risk premium, arising from  $\sigma_1 = 0.3$ , the unstable steady state has moved down into the negative region and only two steady states are detectable.<sup>12</sup> However, the upper steady state (SS1) remains the same as in the previous case. We thus do not expect, when employing the dynamic programming algorithm, different results in the local stability properties and the local behavior of the value function. We thus can by-pass a detailed dynamic programming study for this variant. Overall, even in this case of the existence of a state dependent risk premium, a large stable domain of attraction exists and the economy is more likely to escape a currency crisis scenario when currency shocks occur.

## 4 Concluding Remarks

In this paper we have investigated, by means of an extended linear quadratic monetary policy control problem, the rationale of a foreign currency reserves target in a macroeconomic environment where the central bank is confronted to the occurrence of sharp capital outflows, speculative attacks on its domestic currency and foreign investors' state-dependent reactions and self-fulfilling expectations. We have focused on two factors:  $\lambda_3$ , the weight that the central bank gives to the level of precautionary foreign reserve balances, and  $\psi$ , the risk premium term in the extended interest rate parity condition, which represented the foreign investors' state-dependent

<sup>12</sup>The eigenvalues obtained from the 4D system in  $q_1, q_2, e, R$  at each steady state (from SS1 to SS2) are:  $\{3.27182, -3.25075, 0.00 + 3.14645 i, 0.00 - 3.14645 i\}$ ,  $\{-4561.26, 4480.39, 40.4486 + 35.3196 i, 40.4486 - 35.3196 i\}$  The latter case shows instability, but again no inference can be made for the case of zero real roots. Yet the dynamic programming result showed, as for the case of Table 6 (depicted in Figure 3) that the upper equilibrium is stable.



reactions which is likely to fuel currency runs.

In the standard monetary policy model firstly analyzed, the central bank pursued only two targets, the output level and the inflation rate. With a state dependent risk premium term ( $\psi > 0$ ), multiple steady states arose, two domains of attraction appeared in a positive state space and the safe domain of attraction shrank. While the upper steady state (a saddle point) was close to the unique steady state obtained when  $\psi = 0$ , the lower steady state (an unstable focus) occurred at a low reserve level, indicating that the lower steady state was unstable in the state space. Therefore, the lower steady state divided the positive state space into two domains: an upper stable and a lower unstable domain, where the trajectories moved downward, reducing the stable domain of attraction. Since the lower steady state level of  $R$  moved up when the risk premium factor was high, it suggests that a higher  $\psi$  enlarges the domain of attraction of the lower steady state. In economic terms, such a situation may be reflected by continuously falling levels of foreign currency reserves, persistently high interest rates and the eventual occurrence of a currency crisis which may (jointly with the high interest rates) lead the economy to a economic recession and a period of deflation (see e.g. Christiano, Gust and Roldos (2004) and Proaño et al. (2008) for a study of such monetary policy effects in currency crisis episodes).

In the alternative scenario, the central bank targeted also the level of precautionary foreign reserves. In this case, when the risk premium factor was zero, multiple steady states arose and thus two domains of attraction appeared in a positive state space. Since the lower steady state was close to zero and the upper steady state of  $R$  was very high and close to the target reserve level, there was a low probability of a currency crisis. The lower steady state moved even closer to zero when the central bank gave higher weight to the level of precautionary foreign reserve balances, which means that the domain of attraction of the upper steady state enlarges as  $\lambda_3$  increases. The central bank's actual reserve level tends to be as large as the central bank's desired target level. Such a strategy could avoid the currency crisis scenario even when the risk premium factor  $\psi > 0$  was introduced, since there was a unique steady state in a positive state space, and the steady state of  $R$  remained close to the central bank's target level. Thus, in this second scenario, as compared to the first one where the central bank does not target the reserve level, a much larger safe region of attraction between the upper and lower steady states emerges.

Overall, we may conclude that if the central bank takes the level of precautionary foreign reserve balances into account as one of its targets, it is possible to avoid the economy flipping into a currency crisis scenario. Given the usual uncertainty that

central banks face in terms of the data of the country, the underlying model and its parameters, the domains of bad outcomes and the size of shocks, this appears as a very useful strategy for central banks to follow in order to increase such domains of attraction decrease the probability of a currency run on their domestic currencies.

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**Publisher:** Hans-Böckler-Stiftung, Hans-Böckler-Str. 39, 40476 Düsseldorf, Germany  
**Phone:** +49-211-7778-331, [IMK@boeckler.de](mailto:IMK@boeckler.de), <http://www.imk-boeckler.de>

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**ISSN:** 1861-2199

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