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## The Mechanics of Dualistic Models: "Comparable" Structures and Comparative Statics Results \*

By Ana Paula MARTINS †

**Abstract.** In this paper we present and confront the main comparative static results - effects of labor supply, demand and institutional wage movements - of the (a) basic two-sector model in six different scenarios: free market; partial (one-sector) coverage with perfect intersector mobility; partial (one-sector) coverage with imperfect mobility (Harris-Todaro); multiple (two-sector) coverage with imperfect mobility (Bhagwati-Hamada); partial (one-sector) coverage with affiliation restrictions in the covered sector; partial (one-sector) coverage with limited employment generation ability in the traditional uncovered sector.

**Keywords.** Migration, Mobility, Minimum Wages, Segmented Labor Markets, Regional Labor Markets.

**JEL.** O15, O18, R23, J42, J61, J62, F22.

### 1. Introduction

This paper affiliates to the [Todaro \(1969\)](#) and [Harris-Todaro \(1970\)](#) model of a dualistic economy<sup>i</sup>. This model has had several applications in labor, international and development economics; it sheds light on issues as migration - international or rural-urban labor flows-, inter-sector mobility, dualistic and segmented (labor) markets<sup>ii</sup>, and even implications of minimum wage<sup>iii</sup> laws with partial coverage. The principle has had wide applications in issues concerning regional economics, and even in subjects such as language choice<sup>iv</sup>. International economics literature<sup>v</sup> dealt with the problems of welfare implications and optimal policies to achieve efficiency.

The aim of this research is to distinguish the three scenarios that have been advanced in the literature and allow for two other frameworks – the latter being the free market benchmark solution and the existence of labor force restrictions in one of the sectors. The scenarios are reduced to a common and simple algebraic format, allowing us to proceed to the derivation and comparison of several comparative static results.

Firstly, we distinguish perfect from no (short-run) mobility across the two regions or sectors in models with only one covered sector by the institutional wage rules. Secondly, we allow for institutional arrangements to force wage determination in both of the sectors or regions. In this setting, adjustments are

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forced through supply<sup>vi</sup> – these arrangements may be found in [Bhagwati and Hamada \(1974\)](#).

Finally, we use the two versions of the model with unemployment and introduce size restrictions in one of the sectors. On the one hand, we allow for the possibility of covered sector's affiliation to be restricted; the reasons for such an occurrence may rest with "insider-outsider" arguments, with urban congestion, or even existence of migration quotas. On the other, we consider the possibility of a limited employment (and output) generation ability of the secondary (traditional) region or sector; in this scenario, there will also be "rural" unemployment.

The approach followed is essentially valid for long or medium-run assessment. We ignore inputs other than labor that may affect economic outcomes - wage bill effects are, thus, indistinguishable from total output effects. Labor may be seen as homogeneous and we interpret the two sectors as regions; or we may interpret that there are two types of labor - as in [Bhagwati and Hamada \(1974\)](#) - that (can or) cannot immediately switch from one sector to the other. We ignore search issues<sup>vii</sup>.

In each particular framework, we establish the basic hypothesis, and analyze changes in the labor market implied by:

- change in the institutional wage(s)
  - sector/region or global demand fluctuations<sup>viii</sup>
  - alterations in total labor supply - always considered inelastic.
- and compare the effects in the different scenarios.

A distinction is made between the impact on absolute and relative measures of unemployment: total or global unemployment, total unemployment rate and local or sector unemployment rates.

We focus on issues that are relevant for labor (and regional) economics and business cycle analysis rather than for international trade. The research results may be applicable to interpret local problems or in small countries for which output demand is perfectly elastic - output price is exogenous.

We depart from the benchmark case - free market with perfect mobility across regions or sectors -, outlined in section II. In section III, partial coverage with perfect mobility - i.e., people not employed in the primary sector can immediately get a job in the secondary sector and wait there for an opportunity to switch, and thus, there is no unemployment generation - is introduced. In section IV, a version of the Harris-Todaro model - with imperfect mobility and institutionally fixed wage in one of the sectors - is presented. In section V, the Bhagwati-Hamada economy is forwarded; comparative statics are analyzed for two cases: multiple coverage (wage is set institutionally in both sectors or regions but may differ from sector to sector) and total or complete coverage (wage is equal in both sectors). Section VI deals with frameworks where there are size restrictions: in the primary sector size - the counterpart of the H.-T. model; and in the employment generation capacity of the secondary sector - the "dual" case of the B.-H. model. Section VII provides the comparison of the exogenous demand or supply shocks on some main macroeconomic aggregates across the four standard dualistic scenarios. The exposition ends with a brief summary in section VIII.

## 2. Dualistic Scenario - Free Market Solution

### 2.1. The Model

In this section, we exposit the basic dualistic scenario, common to most dualistic models, and highlight the general free market outcome and the essential comparative statics results within that model that may interest labor economists and migration/mobility and business cycle analysts.

Consider the following scenario:

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1. There are two sectors in the economy with specific local/industry labor demands which determine local or sector employment,  $L_i$ :

$$L_i = L^i(W_i), \quad i = 1, 2 \quad (\text{II.1})$$

or in inverse form

$$W_i = W^i(L_i), \quad i = 1, 2 \quad (\text{II.2})$$

which depend negatively on the wage paid to local or industry employees. There are no cross effects, i.e.,  $\partial L_i / \partial W_j = 0$  for  $i \neq j$ . Capital and trade are ignored: both regions produce an homogeneous good with distinct technologies and no capital.

$L^i(W_i)'$  is the slope of the demand in sector  $i$ ;  $W^i(L_i)'$  is, of course, its inverse – both usually assumed negative. We will denote the elasticity of demand in sector  $i$  by  $\square^i = L^i(W_i)' W_i / L^i(W_i) = W^i(L_i)' / [W^i(L_i)' L_i]$ . As is well-known, at an elastic labor demand,  $|\square^i| > 1$ , a rise in the wage decreases the wage bill and vice-versa: a large number of the conclusions in dualistic frameworks will depend on which is the case, others on relative size of demand elasticities in the two sectors.

2. Total supply is perfectly inelastic and fixed or exogenous,  $\bar{L}$ . People affiliate to (or locate in) either of the sectors, with:

$$\bar{L} = \bar{L}_1 + \bar{L}_2 \quad (\text{II.3})$$

3. Individuals are risk neutral and maximize expected income.

These three assumptions are common to most of the dualistic models. The free market outcome will arise if also:

4. Wage in first sector is determined by market conditions, i.e.,  $L_1 = \bar{L}_1$ .

5. Wage in second sector is determined by market conditions, i.e.,  $L_2 = \bar{L}_2$ .

6. Job rotation is completely free, i.e., individuals in one sector can costlessly and immediately switch to an employment in the other sector.

Then,

$$\bar{L}_i = L_i = L^i(W_i), \quad i = 1, 2 \quad (\text{II.4})$$

The equilibrium in dualistic models results from individuals' maximizing expected wage. In the present scenario, people will move from one to the other sector's employment till equalization of wages. That is, wage  $W$  will adjust till  $W^*$  that solves:

$$L^1(W^*) + L^2(W^*) = \bar{L} \quad (\text{II.5})$$

In Fig. 1 we depicted the equilibrium solution; in the graph, of horizontal size  $\bar{L}$ , demands are depicted with respect to each of the vertical axis where the wages corresponding to each of the sectors are read.

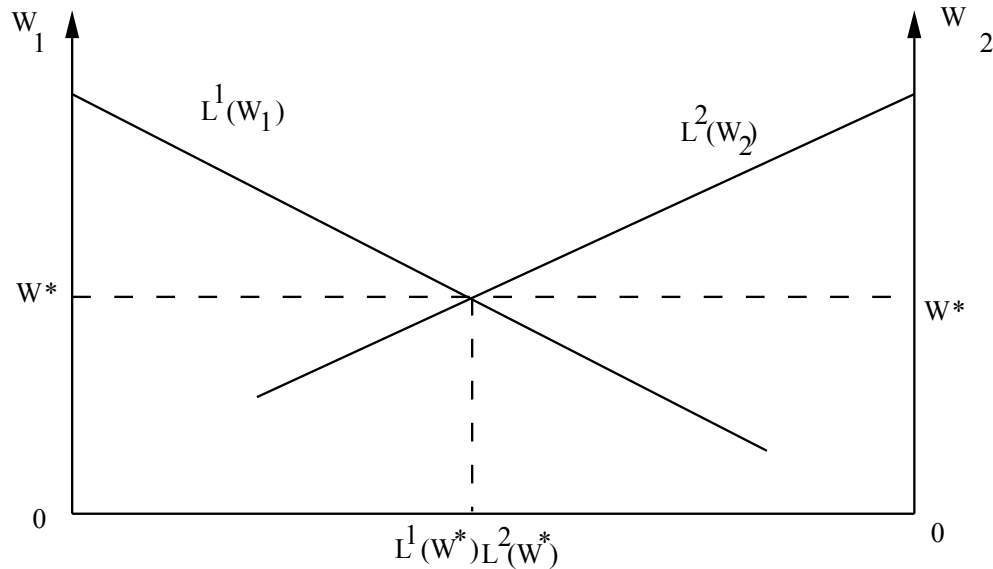


Fig. 1

In equilibrium, there will be no unemployment; the average wage in the economy,  $(W_1 L_1 + W_2 L_2) / \bar{L}$ , and the expected wage in each region,  $W_i L_i / \bar{L}_i$ ,  $i = 1, 2$ , are equal to  $W^*$ .

### 2.2. Comparative Statics: Change in Labor Supply

Consider a change in  $\bar{L}$ . We get that, from (II.5):

$$\frac{\partial W^*}{\partial \bar{L}} = 1 / [L^1(W^*)' + L^2(W^*)'] < 0 \quad (\text{II.6})$$

The wage will decrease to accommodate employment of new labor supply, which will be distributed between the two sectors. The increase in employment will be larger in the region where the slope of the demand,  $L^i(W^*)'$ , is larger in absolute value.

The wage bill in either sector will increase iff elasticity of demand in that sector is larger than 1 in absolute value. The total wage bill will increase iff:

$$L^1(W^*) (1 + \epsilon^1) + L^2(W^*) (1 + \epsilon^2) < 0 \quad (\text{II.7})$$

### 2.3. Comparative Statics: Shift in Labor Demands

1. Take a change in labor demand that does not alter the slope - a parallel shift outwards of the demand in one of the sectors, say sector 1. Let, then, demand in sector 1 be represented by  $L_1 = L^1(W_1) + a$ , or, alternatively,  $W_1 = W^1(L_1 - a)$ , where  $a$  is the shift parameter<sup>ix</sup>. Then (II.5) becomes:

$$L^1(W^*) + a + L^2(W^*) = \bar{L} \quad (\text{II.8})$$

A positive change in  $a$  will imply an increase in the equilibrium wage:

$$\frac{\partial W^*}{\partial a} = -1 / [L^1(W^*)' + L^2(W^*)'] > 0 \quad (\text{II.9})$$

Employment will decrease with  $a$  in region 2 (where labor demand does not shift, being negatively sloped); consequently, it will increase in region 1, once  $\bar{L}$  is

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fixed. Thus, the wage bill will always increase in region 1. It will increase in region 2 iff elasticity of demand in that sector is smaller than 1 in absolute value. Total wage bill will always increase, because total employment is, after all, fixed (equal

to  $\bar{L}$ ) and  $W^*$  rises.

2. Consider a general increase in demand. That is, let demand in sector  $i$  be represented by  $L^i(W_i) + c$ , or  $W_i = W^i(L_i - c)$ ,  $i = 1, 2$ , and consider changes in  $c$ , which affect both sectors simultaneously. Then, the equilibrium condition becomes:

$$L^1(W^*) + 2c + L^2(W^*) = \bar{L} \quad (\text{II.10})$$

A rise in  $c$  will imply an increase in the equilibrium wage:

$$\partial W^* / \partial c = -2 / [L^1(W^*)' + L^2(W^*)'] > 0 \quad (\text{II.11})$$

Again, total wage bill will always increase, because total employment is equal to  $\bar{L}$  and  $W^*$  rises with  $c$ .

Employment in region  $i$  will move according to:

$$\partial [L^i(W^*) + c] / \partial c = [L^j(W^*)' - L^i(W^*)'] / [L^1(W^*)' + L^2(W^*)'] \quad (\text{II.12})$$

Consequently, employment will increase in (population will move towards) the region of lower demand slope in absolute value. The wage bill in region  $i$  will move according to:

$$\partial \{W^* [L^i(W^*) + c]\} / \partial c = \quad (\text{II.13})$$

$$= - \{2 [L^i(W^*) + c] + W^* [L^i(W^*)' - L^j(W^*)']\} / [L^1(W^*)' + L^2(W^*)']$$

Wage bill in region  $i$  will increase iff

$$[L^i(W^*) + c] (2 + \square^i) > [L^j(W^*) + c] \square^j \quad (\text{II.14})$$

This will occur provided that

$$L^i(W^*)' > L^j(W^*)' \quad (\text{II.15})$$

The demand slope is higher in absolute value in sector  $j$ ; or (another sufficient but not necessary condition) that the elasticity of demand in sector  $i$  is lower than 2 in absolute value.

Summarizing:

### Proposition 1:

**1.1.** Under free market, the usual dualistic model will result in equalization of wages across regions or sectors and there will be no unemployment.

**1.2.** An increase in exogenous labor supply will:

- decrease the equilibrium wage to accommodate the new total supply
- increase the wage bill in a sector iff elasticity of demand in that sector is larger than 1 (in absolute value).

**1.3.** An increase of either or both demands will always increase the equilibrium wage and the total wage bill.

**1.4.** If the (outward) demand shift is unilateral, it will rise the wage bill in the sector where it occurred. Employment will decrease in the other sector.

**1.5.** If we have a general (outward) demand increase, employment will rise in the region of lower demand slope (in absolute value).

**3. Partial Coverage - The Perfect Mobility Case**

*3.1. The Model*

Consider we maintain assumptions 1. to 3. of section II and:

4. Wage in one of the sectors, sector 1, is institutionally determined.

5. Wage in second sector is determined by market conditions, i.e.,  $L_2 = \bar{L}_2$ .

6. People in one sector can immediately get a job in the second sector.

Then

$$L_2 = \bar{L} - L^1(W_1) \tag{III.1}$$

If a person does not get a job in the first sector, where he can get the higher wage  $W_1$ , he can immediately go to the second sector, where he can wait for an eventual sector 1 job offer at the same time he works. Therefore, equilibrium will yield:

$$L^1(W_1) + L^2(W_2) = \bar{L} \tag{III.2}$$

or

$$W_2[\bar{L} - L^1(W_1)] = W_2 \tag{III.3}$$

Graphically, we can see the result in Fig. 2: the wage in the second sector will absorb all the supply not employed in the first sector.

Notice that

$$W_1 > W^* > W_2 \tag{III.4}$$

otherwise minimum wage is not binding. That is,  $W_1$  must be higher than the competitive wage  $W^*$ ; the wage in the other sector,  $W_2$ , will decrease to accommodate all excess demand generated in the first sector.

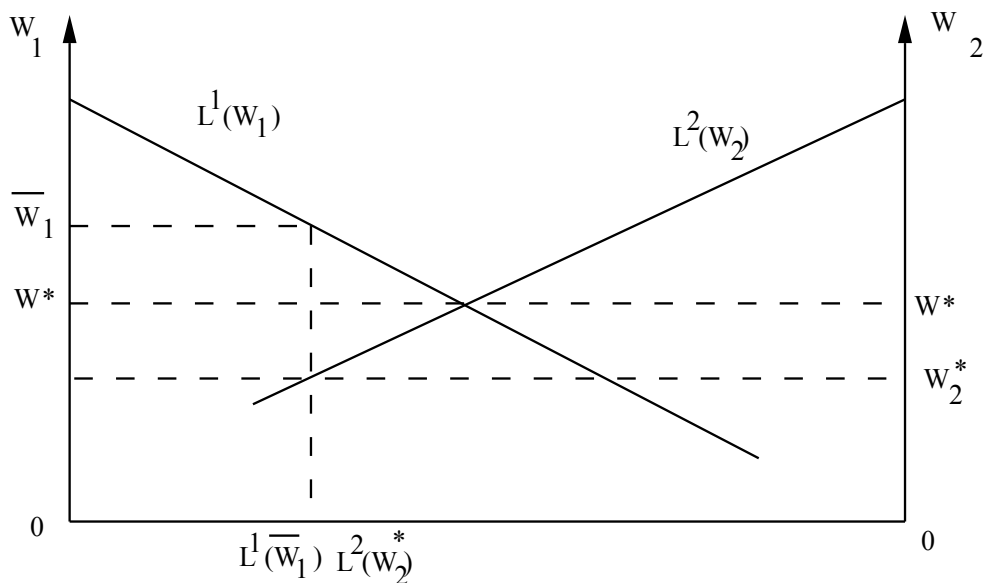


Fig. 2

In equilibrium, the expected wage in each region,  $\bar{W}_i L_i / \bar{L}_i$ ,  $i = 1, 2$ , is always equal to  $\bar{W}_i$ .

**3.2. Comparative Statics: Change in Covered Sector Wage.**

Consider a change in the institutional wage. In this environment:

$$\partial W_2 / \partial W_1 = -L^1(W_1)' / L^2(W_2)' < 0 \quad (\text{III.5})$$

Then, an increase in  $W_1$  will imply:

- a decrease in 1's employment
- an increase in 2's employment and a fall in  $W_2$
- an increase in 1's wage bill iff  $\square^1$  is smaller than 1 in absolute value.
- an increase in 2's wage bill iff  $\square^2$  is larger than 1 in absolute value.
- an increase in total wage bill iff

$$W_1 (1 + \square^1) / \square^1 < W_2 (1 + \square^2) / \square^2 \quad (\text{III.6})$$

**3.3. Comparative Statics: Change in Labor Supply**

An increase in  $\bar{L}$  will be absorbed in sector 2, with no particular innovation with respect to its consequences relative to a one-sector standard model. Employment in the second sector will increase and  $W_2$  will decrease. The wage bill will increase iff the elasticity of demand in the second sector is larger than 1 in absolute value. The average wage in the economy will decrease with supply:

$$\partial[(W_1 L_1 + W_2 L_2) / \bar{L}] / \partial \bar{L} = [\bar{L} L_2 / L^2 - (W_1 - W_2) L_1] / \bar{L}^2 < 0 \quad (\text{III.7})$$

**3.4. Comparative Statics: Shift in Labor Demands**

The consequences of a shift in demand(s) has opposite effects on  $W_2$  to a similar change in  $\bar{L}$ , once we can write the equilibrium solution as:

$$L^1(W_1) + a + L^2(W_2) + b = \bar{L} \quad (\text{III.8})$$

or

$$W_2 [\bar{L} - a - b - L^1(W_1)] = W_2 \quad (\text{III.9})$$

Therefore,  $W_2$  always increases with either a or b. The wage bill in either case moves according to the change we are considering:

1.  $b = 0$ . Then, we have a change in the institutional sector demand. Consider, thus, an increase in a. The wage bill in the first sector will rise. The wage bill in the second sector will rise iff elasticity of demand in that sector is smaller than 1 in absolute value. Total wage bill will move according to:

$$\partial\{W_1 [L^1(W_1) + a] + W_2 L^2(W_2)\} / \partial a = W_1 - (1 + 1 / \square^2) W_2 \quad (\text{III.10})$$

Therefore, it will increase with a iff:

$$W_1 > (1 + 1 / \square^2) W_2 \quad (\text{III.11})$$



This always occurs, once in the equilibrium solution  $W_1 > W_2 > (1-1/|\sigma^2|) W_2$ .

2.  $a = 0$ . Then, we have a change in the uncovered sector demand. Employment will not change in either sector. The wage bill in the first sector will stay constant. The wage bill in the second sector, and thus total wage bill, will always rise:

$$\partial\{W_2 [L^2(W_2) + b]\}/\partial b = -W_2^2[\bar{L} - b - L^1(W_1)]' [L^2(W_2) + b] > 0 \quad (\text{III.12})$$

3.  $a = b = c$ . Then, we have a general demand increase. The wage bill in the first sector will rise, as well as its employment. The wage bill in the second sector will move according to:

$$\partial\{W_2 [L^2(W_2) + c]\}/\partial c = \quad (\text{III.13})$$

$$\begin{aligned} &= -2 W_2^2[\bar{L} - 2c - L^1(W_1)]' [L^2(W_2) + c] - W_2 = \\ &= -W_2 (1 + 2/|\sigma^2|) = -W_2 (1 - 2/|\sigma^2|) \end{aligned}$$

Therefore, it will increase with demand iff demand elasticity in the uncovered sector is smaller than 2 (in absolute value). Total wage bill will increase iff

$$W_1 > W_2 (1 - 2/|\sigma^2|) \quad (\text{III.14})$$

This always occurs, once in the equilibrium solution  $W_1 > W_2 > (1-2/|\sigma^2|) W_2$ .

Summarizing:

**Proposition 2:**

**2.1.** In a dualistic model with perfect mobility and institutional wage fixed in one of the sectors, the equilibrium wage in the second sector is lower than the free market wage. There will be no unemployment.

**2.2.** An increase in the institutional wage will:

- decrease employment in the covered sector
- increase employment and imply a fall in wage of the uncovered sector
- increase the wage bill in the covered sector iff demand elasticity in that sector is smaller than 1 in absolute value.
- increase the wage bill in the uncovered sector iff demand elasticity in that sector is larger than 1 in absolute value.

**2.3.** An increase in total supply will

- decrease the wage in the uncovered sector, where the additional supply is accommodated by local demand
- increase the wage bill in the uncovered sector iff demand elasticity in that sector is larger than 1 in absolute value.

**2.4.** An increase of either or both demands will always increase the equilibrium wage in the uncovered sector and the total wage bill. It will always rise the wage bill in the sector where the shift occurred.

**2.5.** If the (outward) demand shift is unilateral, employment will decrease in the uncovered sector if the shift occurred in the covered sector. The employment distribution will not change if the shift occurred in the uncovered sector.

**2.6.** If we have a general (outward) demand increase, employment of the covered sector will rise and the employment in the other will decrease.

#### 4. Partial Coverage - The Harris-Todaro Model

##### 4.1. The Model

In this section, we exposit the Harris-Todaro model and the essential comparative statics results within that model that may interest labor economists and migration/mobility and business cycle analysts.

Let us start by the three previous assumptions of the dualistic context. In Harris-Todaro, they are complemented by:

4. Wage in one of the sectors, sector 1, is institutionally determined.

5. Wage in the second sector is determined by market conditions, i.e.,  $L_2 = \bar{L}_2$ .

6. Job rotation is only locally accomplished, i.e., individuals in one sector cannot get an employment in the other sector.

Then

$$\bar{L}_2 = L_2 = L^2(W_2) \quad (IV.1)$$

In region 2 there is no unemployment and expected wage is  $W_2$ . In region 1, there is unemployment and expected wage is

$$W_1 \times \text{Probability of Employment in region 1} \quad (IV.2)$$

Probability of employment in region 1 is

$$L^1(W_1) / \bar{L}_1 \quad (IV.3)$$

As long as  $W_1 \times \text{Probability of Employment in region 1} > W_2$ , people flow to region 1. Therefore, employment will distribute among the two sectors or regions in such a way that, as long as  $W_1 > W_2$ :

$$W_1 \times \text{Probability of Employment in region 1} = W_2 \quad (IV.4)$$

That is, in the long run we expect that:

$$W_1 L^1(W_1) / \bar{L}_1 = W_2 \{L^1(W_1) / [\bar{L} - L^2(W_2)]\} = W_2 \quad (IV.5)$$

or<sup>x</sup>:

$$W_1 L^1(W_1) = W_2 [\bar{L} - L^2(W_2)] = W_2 \bar{L} - W_2 L^2(W_2) \quad (IV.6)$$

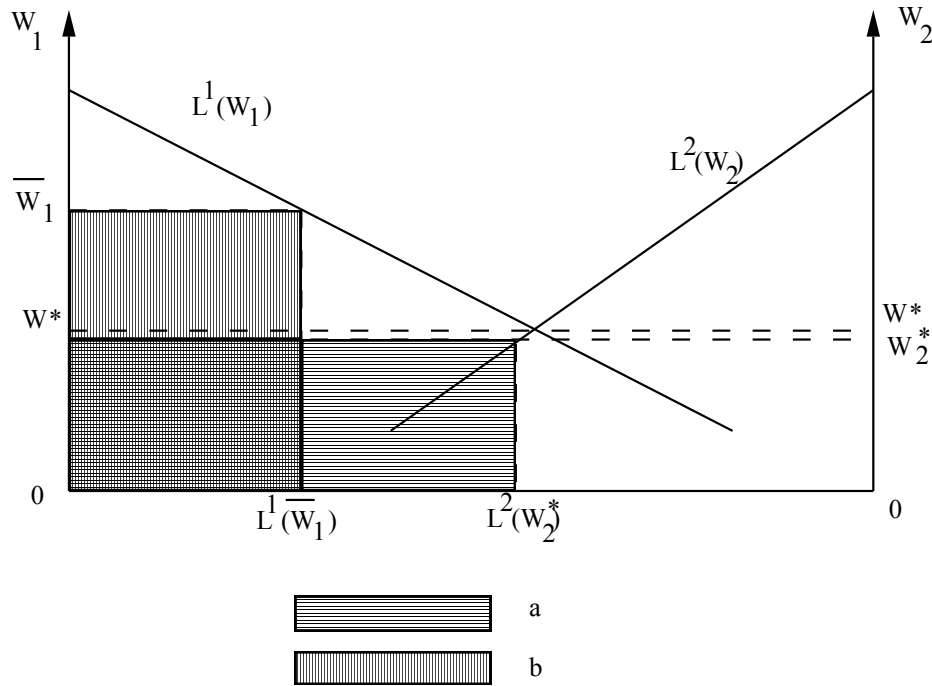


Fig. 3

Graphically - see Fig.3, where a box of base dimension  $\bar{L}$  is depicted along with the two sector demands<sup>xi</sup> -, (IV.5) implies that wage in sector 2 will go down till the point where the area a equalizes the area below the covered sector wage, b. (IV.6) establishes that the area under  $W_2^*$  is equal to the equilibrium total wage bill,

i.e., total wage bill will equal  $W_2^* \bar{L}$ .

Equilibrium wage in the uncovered sector may be higher or lower than the competitive wage  $W^*$ .

In equilibrium, expected wage in each sector as well as average wage in the economy - due to (IV.6) - are equal to  $W_2^*$ .

#### 4.2. Comparative Statics: Change in Covered Sector Wage

1. Consider a change in wage in sector 1. Then

$$\partial W_2 / \partial W_1 = L^1(W_1) (1 + \square^1) / [\bar{L} - L^2(W_2) - W_2 L^2(W_2)] \quad (IV.7)$$

Recall that if  $\square^1 = L^1(W_1)' W_1 / L^1(W_1) > -1$ , an increase in  $W_1$  will increase  $L_1 W_1$ .

#### Proposition 3:

**3.1.** Consider a dualistic model with no mobility. The equilibrium wage in the second sector may be higher or lower than the free market equilibrium, in which there will be unemployment in the institutional sector or urban region.

**3.2.** If wage-elasticity of demand of the covered sector is smaller than 1 (in absolute value), that is, if an increase in the wage increases the wage bill in the

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covered sector, a rise in the covered sector wage will also increase the wage in the uncovered sector.

**3.3.** Therefore, if  $W_1$  is close to (but always larger than) the latter, the wage in the second sector will also be larger iff elasticity of demand in the covered sector is smaller than 1 in absolute value.

The implications for the wage of the uncovered sector of a rise of the covered sector wage are derived geometrically in [Corden and Findlay \(1975:62\)](#).

Consider the effect on the wage bill in sector 2:

$$\begin{aligned} \partial[L^2(W_2) W_2] / \partial W_1 &= \{\partial[L^2(W_2) W_2] / \partial W_2\} \partial W_2 / \partial W_1 = & (IV.8) \\ &= L^2(W_2) (1 + \square^2) (\partial W_2 / \partial W_1) = \\ &= L^1(W_1) L^2(W_2) (1 + \square^1) (1 + \square^2) / [\bar{L} - L^2(W_2) - W_2 L^2(W_2)] \end{aligned}$$

As for total wage bill:

$$\begin{aligned} \partial[L^1(W_1) W_1 + L^2(W_2) W_2] / \partial W_1 &= & (IV.9) \\ &= \bar{L} L^1(W_1) (1 + \square^1) / [\bar{L} - L^2(W_2) - W_2 L^2(W_2)] = \\ &= \bar{L} \partial W_2 / \partial W_1 \end{aligned}$$

(Last equality comes from looking at (IV.6)).

Let us see what happens to unemployment. Within this framework, we may be interested in what happens to:

- total unemployment,  $U = \bar{L} - L^1(W_1) - L^2(W_2)$
- global unemployment rate,  $u = U / \bar{L}$ ;
- local or sector unemployment rate,  $u_1 = U / [U + L^1] = U / [\bar{L} - L^2(W_2)]$ .

Notice that  $u_1 = 1 - \text{Probability of Employment in region 1}$ .

Unemployment,  $U = \bar{L} - L^1(W_1) - L^2(W_2)$  - and the unemployment rate, given

that  $\bar{L}$  is fixed -, will vary with the wage in the covered sector in such a way that:

$$\begin{aligned} \partial U / \partial W_1 &= - [L^1(W_1)' + L^2(W_2)' \partial W_2 / \partial W_1] = & (IV.10) \\ &= - \{(W_1 - W_2) L^1(W_1)' L^2(W_2)' + L^1(W_1)' [\bar{L} - L^2(W_2)] + L^2(W_2)' L^1(W_1)\} / \\ & \quad / [\bar{L} - L^2(W_2) - W_2 L^2(W_2)] \end{aligned}$$

Even if  $\partial W_2 / \partial W_1 < 0$ , which occurs if  $\square^1 < -1$ , unemployment may still increase with the wage in the covered sector. If  $\partial W_2 / \partial W_1 > 0$ , unemployment will necessarily increase with  $W_1$ . If  $W_2$  is close to  $W_1$ , i.e., if  $W_1$  is close to free market wage, unemployment will increase with the institutional wage.

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Let us consider what happens with the local unemployment rate,  $u_1$ , in region 1. One can show that:

$$\frac{\partial \{1 - L^1(W_1) / [\bar{L} - L^2(W_2)]\} / \partial W_1 = -L^1 [(\bar{L} - L^2) \square^1 + L^2 \square^2] / (IV.11)}{\{W_1 [\bar{L} - L^2(W_2) - W_2 L^2(W_2)] [\bar{L} - L^2(W_2)]\} > 0}$$

**Proposition 4:** An increase in the wage of the covered sector will increase the

**4.1.** wage bill in the uncovered sector iff either:

- both labor demands have wage-elasticities smaller than one in absolute value  
or

- both labor demands have wage-elasticities larger than one in absolute value.

**4.2.** total wage bill iff wage-elasticity of demand of the covered sector is smaller than 1 (in absolute value).

**4.3.** total unemployment (and total unemployment rate) if wage-elasticity of demand of the covered sector is smaller than 1 (in absolute value); or if  $W_2$  is close to  $W_1$  (i.e., around free market solution).

**4.4.** local (sector) unemployment rate.

### 4.3. Comparative Statics: Change in Labor Supply

Consider a change in  $\bar{L}$ . We get that, from (IV.6):

$$\frac{\partial W_2}{\partial \bar{L}} = -W_2 / [\bar{L} - L^2(W_2) - W_2 L^2(W_2)] < 0 \quad (IV.12)$$

Therefore employment in sector 2 increases. Total unemployment will increase:

$$\begin{aligned} \frac{\partial U}{\partial \bar{L}} &= 1 - \frac{\partial L_2}{\partial \bar{L}} = 1 - L^2(W_2)' \frac{\partial W_2}{\partial \bar{L}} = \\ &= [\bar{L} - L^2(W_2)] / [\bar{L} - L^2(W_2) - W_2 L^2(W_2)] > 0 \end{aligned} \quad (IV.13)$$

As for the global unemployment rate,  $u = U / \bar{L}$ :

$$\frac{\partial (U/\bar{L})}{\partial \bar{L}} = [(\bar{L} - L^2) (L^1 + L^2) + W_2 L^2' (\bar{L} - L^1 - L^2)] / \quad (IV.14)$$

$$[(\bar{L} - L^2 - W_2 L^2)' \bar{L}^2]$$

It will be positive iff:

$$\{\bar{L} [(L^1 + L^2) / L^2] - L^1 - L^2\} / (\bar{L} - L^1 - L^2) > -\square^2 = |\square^2| \quad (IV.15)$$

The left hand-side of the expression is larger than 1. Therefore, even if  $|\square^2| > 1$ , the global unemployment rate may still increase when  $\bar{L}$  increases. If  $|\square^2| < 1$ , the unemployment rate will necessarily increase with  $\bar{L}$ , once the inequality (IV.15) will always hold.

Local unemployment rate,  $u_1 = 1 - L^1(W_1) / [\bar{L} - L^2(W_2)]$ , will vary in the same direction as  $[\bar{L} - L^2(W_2)]$ , and, because  $L^1(W_1)$  is fixed, as  $\bar{L}$  does; therefore, it will increase with  $\bar{L}$ . (Alternatively,  $W_1$  is fixed and  $W_2$  decreases; so, the probability of employment in region 1 must decrease as  $\bar{L}$  rises for equilibrium to hold.)

Total wage bill will equal  $\bar{L} W_2$ . Therefore:

$$\partial(\bar{L} W_2) / \partial \bar{L} = W_2 + \bar{L} \partial W_2 / \partial \bar{L} \quad (IV.16)$$

$$= -W_2 L^2(W_2) (1 + \epsilon^2) / [\bar{L} - L^2(W_2) - W_2 L^2(W_2)']$$

The wage bill will increase iff demand in sector 2 has absolute wage-elasticity larger than 1. As the average wage in the economy,  $[W_1 L^1(W_1) + W_2 L^2(W_2)] / \bar{L}$ , will equal  $W_2$  in equilibrium, it will move in the same direction as the wage in the uncovered sector.

**Proposition 5:** An increase in labor supply will decrease the wage in sector 2. It will increase total wage bill iff elasticity of demand in sector 2 is larger than 1 in absolute value. It will increase total unemployment and local unemployment rate in region 1; the global unemployment rate will increase if elasticity of demand of the uncovered sector is smaller than 1 (in absolute value).

#### 4.4. Comparative Statics: Shift in Labor Demands

This section, in some instances, reproduces the findings of [Corden and Findlay \(1975\)](#) and [McDonald and Solow \(1985\)](#). These last authors analyze the effects of business cycle fluctuations on a segmented labor market economy with a primary sector where wage determination is subject to collective bargaining and a secondary competitive sector. [Corden and Findlay \(1975\)](#) also consider the subject of economic expansion without - the H.-T. framework - and in the presence of capital mobility between the two sectors.

1. Take a change in labor demand that does not alter the slope - a parallel shift outwards of the demand in sector 1. Let, then, demand in sector 1 be represented by  $L^1(W_1) + a$ , where  $a$  is the shift parameter<sup>xii</sup>. Then (IV.6) becomes:

$$W_1 [L^1(W_1) + a] = W_2 \bar{L} - W_2 L^2(W_2) \quad (IV.17)$$

A change in  $a$  will imply a change in sector 2's wage in the following way:

$$\partial W_2 / \partial a = W_1 / [\bar{L} - L^2(W_2) - W_2 L^2(W_2)'] > 0 \quad (IV.18)$$

Therefore employment in sector 2 decreases with  $a$ : some people leave sector or region 2 for the other where the positive shift occurred. Total unemployment,  $\bar{L} -$

$(L^1 + a) - L^2$  - and global unemployment rate - will likely decrease but by less than 1's employment increases:

$$\frac{\partial[\bar{L} - (L^1 + a) - L^2]/\partial a = - \{(W_1 - W_2) L^2(W_2)' + [\bar{L} - L^2(W_2)]\}}{[\bar{L} - L^2(W_2) - W_2 L^2(W_2)']} \quad (IV.19)$$

This will be negative if  $W_1 = W_2$ , i.e., around the free market equilibrium; or if

$$|\square^2| < [L^1(W_1) + a] / L^2(W_2) \quad (IV.20)$$

but neither are necessary conditions.

Given that  $W_2$  increases with  $a$ , and  $W_1$  is fixed, the probability of employment in sector or region 1 must rise with  $a$  in equilibrium - therefore, the local unemployment rate must decrease with  $a$ .

The total wage bill will increase, once  $\bar{L} \partial W_2 / \partial a > 0$ .

Notice that this occurs without any change in 1's wage.

2. Consider a parametric change in demand of sector 2. Write it as  $L^2(W_2) + b$ , which means that (IV.6) becomes:

$$W_1 \times L^1(W_1) = W_2 \bar{L} - W_2 [L^2(W_2) + b] \quad (IV.21)$$

A change in  $b$  will imply a change in sector 2's wage in the following way:

$$\partial W_2 / \partial b = W_2 / [\bar{L} - L^2(W_2) - b - W_2 L^2(W_2)'] > 0 \quad (IV.22)$$

and

$$\partial(L^2(W_2) + b) / \partial b = L^2(W_2)' \partial W_2 / \partial b + 1 = \quad (IV.23)$$

$$= [\bar{L} - L^2(W_2) - b] / [\bar{L} - L^2(W_2) - b - W_2 L^2(W_2)'] > 0$$

The increase in 2's employment is smaller than the shift. In this case:

$$\partial[\bar{L} - L^1 - (L^2(W_2) + b)] / \partial b = - \partial(L^2(W_2) + b) / \partial b < 0 \quad (IV.24)$$

Global unemployment - and unemployment rate - decreases with  $b$ . As  $L^1$  is fixed, also local unemployment rate in region 1 will decrease with  $b$ .

The total wage bill will increase, once  $\bar{L} \partial W_2 / \partial b > 0$ .

3. Consider now a simultaneous shift in demand. We have the equilibrium condition:

$$W_1 \times [L^1(W_1) + c] = W_2 \bar{L} - W_2 [L^2(W_2) + c] \quad (IV.25)$$

Then:

$$\partial W_2 / \partial c = (W_1 + W_2) / [\bar{L} - L^2(W_2) - c - W_2 L^2(W_2)'] > 0 \quad (IV.26)$$

$$\partial(L^2(W_2) + c) / \partial c = L^2(W_2)' \partial W_2 / \partial c + 1 = \quad (IV.27)$$

$$= [\bar{L} - L^2(W_2) - c + W_1 L^2(W_2)] / [\bar{L} - L^2(W_2) - c - W_2 L^2(W_2)']$$

This will be positive iff:

$$|\square^2| < (L^1(W_1) + c) / (L^2(W_2) + c) \quad (IV.28)$$

i.e., the elasticity of demand in the uncovered sector is smaller than the relative employment size of the other region's employment.

The impact  $\partial(L^2(W_2) + c) / \partial c$  is smaller than 1 - because  $W_1 L^2(W_2)' < -W_2 L^2(W_2)'$ , the impact on 1's employment. This could be seen as reproducing the [McDonald and Solow \(1985\)](#) theoretical result, consistent with empirical evidence: over the business cycle, fluctuations of primary sector employment will be higher than those of the secondary sector. We also expect the total unemployment - and global unemployment rate - to decrease with  $c$  because:

$$\partial U / \partial c = -\partial(L^2(W_2) + c) / \partial c - 1 = \quad (IV.29)$$

$$= -[2(\bar{L} - L^2(W_2) - c) + (W_1 - W_2) L^2(W_2)] / [\bar{L} - L^2(W_2) - c - W_2 L^2(W_2)']$$

This will be negative if  $W_1 = W_2$ , i.e., around the unrestricted equilibrium; or if

$$|\square^2| < 2(L^1(W_1) + c) / (L^2(W_2) + c) \quad (IV.30)$$

but neither are necessary conditions.

As  $W_2$  increases, because  $W_1$  is fixed, the probability of employment in sector 1 must increase, and therefore the local unemployment rate in sector 1 will decrease with  $c$ .

The total wage bill will increase. Wage bill in sector 2 will move according to:

$$\partial\{W_2 [L^2(W_2) + c]\} / \partial c = \quad (IV.31)$$

$$= \{W_2 \bar{L} + W_1 [L^2(W_2) + c + W_2 L^2(W_2)']\} / [\bar{L} - L^2(W_2) - c - W_2 L^2(W_2)']$$

This will be positive iff

$$\{\bar{L} [L^1(W_1) + c]\} / \{[\bar{L} - L^2(W_2) - c] [L^2(W_2) + c]\} + 1 > |\square^2| \quad (IV.32)$$

This will hold if  $|\square^2| < 1$ , not a necessary condition.

**Proposition 6:**

**6.1.** An increase in the demand of either or both sectors will increase the wage of the uncovered sector and the total wage bill. It will likely decrease total unemployment; it will always decrease local unemployment rate in region 1.

**6.2.** Response movements will be higher if the shift of one of the demands occurs in the covered sector. If the shift occurs only on the covered sector, it will have an anti-cyclic effect on employment of the uncovered sector.

**6.3.** A general increase in demand will imply higher response movements in the employment of the covered sector.

The implications for wage of the uncovered sector and unemployment rate in the covered sector of economic expansion working as a demand shift are derived geometrically in [Corden and Findlay \(1975:62\)](#).



## 5. Multiple and Global Coverage in the Presence of Mobility Costs - The Bhagwati-Hamada Model

### 5.1. The Model

In this section we want to quantify the effects of several changes in a two-sector model with institutional rules in both sectors. One can see this same (technically speaking) scenario in, for example, [Bhagwati and Hamada \(1974\)](#). The authors model a two-sector economy where both sector wages are institutionally fixed - each sector is interpreted as employing individuals of different skills - and examine the issue of emigration (or immigration...) of skilled population out of (into) the economy. Instead, we focus on the internal migration of an equally skilled population - or mobility process between two sectors of people with "horizontally but not vertically" different skills - between two areas induced by several exogenous changes. Our interpretation would also apply to the consequences of those changes in the presence of integrated labor markets when there are high mobility costs.

We distinguish two environments:

- multiple coverage. By multiple coverage we mean that the wage is fixed institutionally in both sectors, but may differ from one sector to another.

- total coverage. By total coverage we mean a model where the same wage applies to both sectors or regions.

In a setting where wages are exogenously fixed, adjustments are made through labor supply movements.

We consider the common assumptions 1. to 3., assumption 4. of the previous sector and:

5. Wage in second sector is also institutionally determined.

6. Job rotation is only locally accomplished, i.e., individuals in one sector cannot get an employment in the other sector.

Then, employment, being demand determined:

$$L_i = L^i(W_i) \quad (V.1)$$

Let  $U_i$  be the local unemployment in region  $i$ , i.e.:

$$U_i = \bar{L}_i - L_i \quad (V.2)$$

where  $L_i$  is employment in sector  $i$  and demand determined, i.e.,  $L_i = L^i(W_i)$ .

Denote by  $u_i$  the unemployment rate in sector/region  $i$ . Define:

$$u_i = U_i / \bar{L}_i = 1 - L_i / \bar{L}_i \quad (V.3)$$

Notice that these definitions are consistent with the ones of the previous section - just consider that  $U_2 = 0$  for the H.-T. case. One can see that the previous section results would apply in the case where the wage restriction is non-binding in one of the sectors - the lower wage one.

Consider first the case of multiple coverage. The equilibrium condition will establish equalization of expected income in both sectors:

$$(1 - u_1) W_1 = (1 - u_2) W_2 \quad (V.4)$$

that is, equilibrium is defined by:

$$W_1 L^1(W_1) / \bar{L}_1 = W_2 L^2(W_2) / \bar{L}_2 \quad (V.5)$$

and, of course, (II.3). Local unemployment rate will be higher in the region of higher wage.

In equilibrium, the average wage in the economy,  $[W_1 L^1(W_1) / \bar{L}_1] \bar{L}_1 / \bar{L} + [W_2 L^2(W_2) / \bar{L}_2] \bar{L}_2 / \bar{L}$ , is equal to the expected wage in each sector,  $W_i L^i(W_i) / \bar{L}_i$ .

Graphically, areas c and d, the wage bills in each sector - Fig. 4 - are fixed; the population will be distributed in such a way as to equalize expected wages.

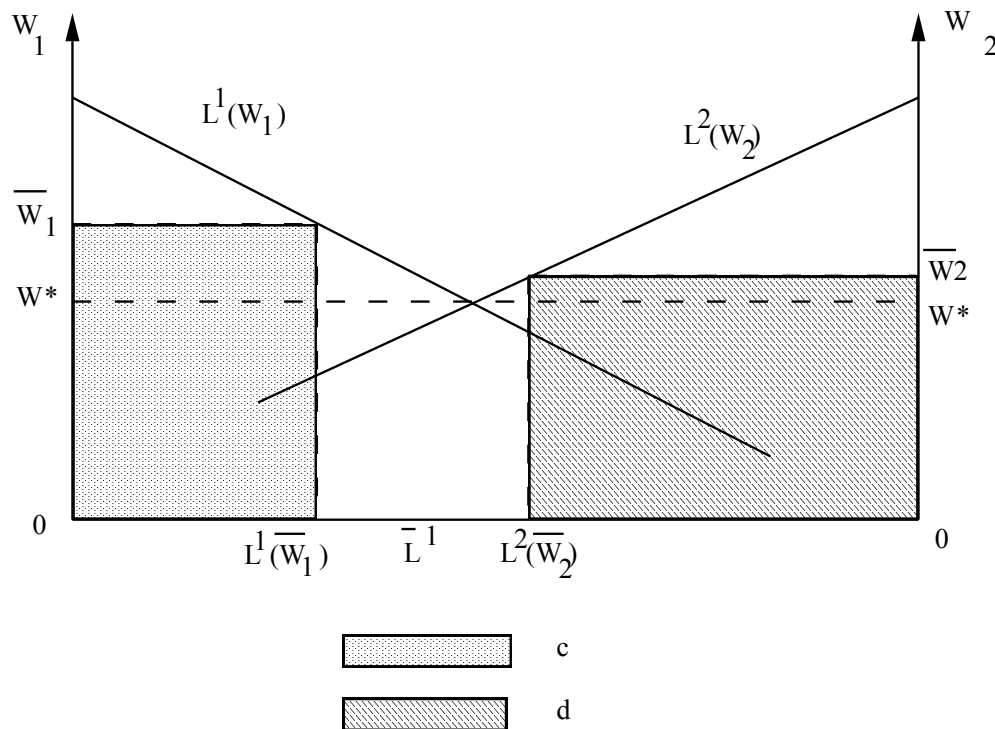


Fig. 4

### 5.2. Comparative Statics: Change of the Sector Wage

Take a change in the wage of sector/region 1. As usual the wage bill in this sector (and total wage-bill) will increase iff the wage-elasticity of demand is smaller than one (in absolute value). We have that:

$$\frac{\partial \bar{L}_1}{\partial W_1} = -\frac{\partial \bar{L}_2}{\partial W_1} = \frac{\bar{L}_2 L^1(W_1) (1 + \epsilon^1)}{[W_1 L^1(W_1) + W_2 L^2(W_2)]} \quad (V.6)$$

This condition is stated in Bhagwati and Hamada (1974).

Iff the wage elasticity of demand in sector 1 is smaller than one, then supply in sector 1 increases and in sector 2 decreases. As  $W_2 L^2(W_2)$  is fixed, this also implies that equilibrium expected wage  $W_2 L^2(W_2) / \bar{L}_2 = W_1 L^1(W_1) / \bar{L}_1$  will increase.

If such elasticity is smaller than 1, local unemployment in region 2 will decrease

- because  $L^2(W_2)$  is fixed and  $\bar{L}_2$  decreases. The local unemployment rate in region 1 will always increase:

$$\frac{\partial [L^1(W_1)/\bar{L}_1]}{\partial W_1} = [L^1(W_1)' W_1 L^1(W_1) \bar{L}_1 - \bar{L}_2 L^1(W_1)^2] / \quad (V.7)$$

$$/ \{ \bar{L}_1^2 [W_1 L^1(W_1) + W_2 L^2(W_2)] \} < 0$$

See, again, [Bhagwati and Hamada \(1974\)](#).

Local unemployment in region 1 will move according to:

$$\frac{\partial U_1}{\partial W_1} = \frac{\partial [\bar{L}_1 - L^1(W_1)]}{\partial W_1} = \frac{\partial \bar{L}_1}{\partial W_1} - L^1(W_1)' = \quad (V.8)$$

$$= \{ \bar{L}_2 L^1(W_1) - \bar{L}_2 L^1(W_1) | \square^1 | + | \square^1 | L^1(W_1) \bar{L} [L^1(W_1)/\bar{L}_1] \} /$$

$$/ [W_1 L^1(W_1) + W_2 L^2(W_2)]$$

This will be positive iff:

$$| \square^1 | \{ 1 - (\bar{L}/\bar{L}_2) [L^1(W_1)/\bar{L}_1] \} < 1 \quad (V.9)$$

It is a sufficient condition for (V.9) to hold that either  $| \square^1 | < 1$  or:

$$\bar{L}_2/\bar{L} < L^1(W_1)/\bar{L}_1 \quad (V.10)$$

**Proposition 7:** With multiple coverage, the increase in one sector's wage:

- increases the wage bill
- increases supply in that sector (and decreases the supply in the other sector)
- increases the equilibrium expected wage iff the wage-elasticity of labor demand of the sector in which the wage increase was registered is smaller than 1 (in absolute value). The reverse would happen if such elasticity was larger than 1. Total unemployment and the local unemployment rate in the region where the wage increase took place will necessarily increase.

### 5.3. Comparative Statics: Change in Labor Supply

Consider a change in  $\bar{L}$ . Wages and employment are fixed - therefore, also the wage bill. We get that <sup>xiii</sup>:

$$\frac{\partial \bar{L}_i}{\partial \bar{L}} = W_i L^i(W_i) / [W_1 L^1(W_1) + W_2 L^2(W_2)] > 0 \quad (V.11)$$

**Proposition 8:** An increase in total supply will be distributed between the two sectors proportionally to the corresponding share in the total wage bill. It will increase (all measures of) unemployment and decrease the expected equilibrium wage.

### 5.4. Comparative Statics: Shift in Labor Demands

1. The equilibrium condition (V.5) can be seen as:

$$W_1 [L^1(W_1) + a] / \bar{L}_1 = W_2 L^2(W_2) / \bar{L}_2 \quad (V.12)$$

Consider then a change in a.

$$\frac{\partial \bar{L}_1}{\partial a} = - \frac{\partial \bar{L}_2}{\partial a} = W_1 \bar{L}_2 / \{W_1 [L^1(W_1)+a] + W_2 L^2(W_2)\} > 0 \quad (V.13)$$

We have symmetric effects in both sectors: with an increase in the demand of sector 1 there is a labor force shift from sector 2 to sector 1. The flow of people will be larger if the shift occurs in the sector of lower employment.

The sector or local employment rates will increase in both sectors:

$$\frac{\partial [L^2(W_2) / \bar{L}_2]}{\partial a} = L^2(W_2) (- \frac{\partial \bar{L}_2}{\partial a}) / \bar{L}_2^2 = \quad (V.14)$$

$$= W_1 [L^2(W_2) / \bar{L}_2] / \{W_1 [L^1(W_1)+a] + W_2 L^2(W_2)\} = (W_1 / W_2) / \bar{L}$$

and

$$\frac{\partial \{[L^1(W_1)+a] / \bar{L}_1\}}{\partial a} = (W_2 / W_1) \frac{\partial [L^2(W_2) / \bar{L}_2]}{\partial a} = \quad (V.15)$$

$$= W_2 [L^2(W_2) / \bar{L}_2] / \{W_1 [L^1(W_1)+a] + W_2 L^2(W_2)\} = 1 / \bar{L}$$

The increase in the (local) employment rate (decrease in local unemployment rate) will be larger in the sector of lower wage, regardless of the sector where the demand shift occurred. The effect on the employment rate of the sector where the shift occurred will be larger, the smaller is total supply.

Unemployment will decrease in region 2. In region 1:

$$\frac{\partial U_1}{\partial a} = \frac{\partial \bar{L}_1}{\partial a} - 1 = \{W_1 \bar{L}_2 - W_1 [L^1(W_1)+a] - W_2 L^2(W_2)\} / \{W_1 [L^1(W_1)+a] + W_2 L^2(W_2)\} \quad (V.16)$$

This will be negative iff:

$$[L^1(W_1)+a] / \bar{L}_1 > \bar{L}_2 / \bar{L} \quad (V.17)$$

2. Suppose we want to consider a simultaneous shift in the demand of both sectors. The equilibrium can be stated as:

$$W_1 [L^1(W_1) + c] / \bar{L}_1 = W_2 [L^2(W_2) + c] / \bar{L}_2 \quad (V.18)$$

and a change in c originates:

$$\frac{\partial \bar{L}_1}{\partial c} = - \frac{\partial \bar{L}_2}{\partial c} = \quad (V.19)$$

$$= (W_1 \bar{L}_2 - W_2 \bar{L}_1) / \{W_1 [L^1(W_1)+c] + W_2 [L^2(W_2)+c]\}$$

Supply in sector 1 will increase with an increase in demand iff

$$W_1 / W_2 > \bar{L}_1 / \bar{L}_2 \quad (V.20)$$

Using the equilibrium condition (V.18), this means:

$$L^2(W_2) + c > L^1(W_1) + c \quad (V.21)$$

i.e., local supply increases in the sector of lower (absolute) employment.

Looking at the employment rate:

$$\begin{aligned} \partial\{[L^1(W_1) + c] / \bar{L}_1\} / \partial c &= \{\bar{L}_1 - [L^1(W_1) + c] \partial \bar{L}_1 / \partial c\} / \bar{L}_1^2 = \quad (V.22) \\ &= W_2 [L^1(W_1) / \bar{L}_1 + L^2(W_2) / \bar{L}_2] / \{W_1 [L^1(W_1) + c] + W_2 [L^2(W_2) + c]\} = \\ &= (W_2 / W_1 + 1) / \bar{L} \end{aligned}$$

and

$$\begin{aligned} \partial\{[L^2(W_2) + c] / \bar{L}_2\} / \partial c &= \quad (V.23) \\ &= W_1 [L^1(W_1) / \bar{L}_1 + L^2(W_2) / \bar{L}_2] / \{W_1 [L^1(W_1) + c] + W_2 [L^2(W_2) + c]\} = \\ &= (W_1 / W_2 + 1) / \bar{L} \end{aligned}$$

Therefore, the impact on the local unemployment rate will be negative and larger (in absolute value) in the sector of lower wage (lower local unemployment rate).

The expected wage (in both sectors) increases with  $c$  according to:

$$\begin{aligned} \partial\{W_2 [L^2(W_2) + c] / \bar{L}_2\} / \partial c &= \partial\{W_1 [L^1(W_1) + c] / \bar{L}_1\} / \partial c = \quad (V.24) \\ &= W_1 W_2 [L^1(W_1) / \bar{L}_1 + L^2(W_2) / \bar{L}_2] / \{W_1 [L^1(W_1) + c] + W_2 [L^2(W_2) + c]\} = \\ &= (W_1 + W_2) / \bar{L} \end{aligned}$$

As for local unemployment:

$$\begin{aligned} \partial U_1 / \partial c &= \partial \bar{L}_1 / \partial c - 1 = \{W_1 \bar{L}_2 - W_2 \bar{L}_1 - W_1 [L^1(W_1) + c] - W_2 [L^2(W_2) + c]\} / \quad (V.25) \\ &\quad / \{W_1 [L^1(W_1) + c] + W_2 [L^2(W_2) + c]\} \end{aligned}$$

This will be negative iff:

$$[L^2(W_2) + c] / \bar{L} < [L^1(W_1) + c] / \bar{L} + \{[L^1(W_1) + c] / \bar{L}_1\} \{[L^2(W_2) + c] / \bar{L}_2\} \quad (V.26)$$

or

$$\bar{L}_2 / \bar{L} < \{[L^1(W_1) + c] / [L^2(W_2) + c]\} \bar{L}_2 / \bar{L} + [L^1(W_1) + c] / \bar{L}_1 \quad (V.27)$$

A sufficient condition for this to hold is that:

$$\bar{L}_2 / \bar{L} < [L^1(W_1) + c] / \bar{L}_1 \quad (V.28)$$

**Proposition 9:**

**9.1.** An increase in the demand of one of the two sectors will increase the supply in that sector and decrease the supply of the other sector. Equilibrium expected wage will increase. The flow of people will be larger if the shift occurs in the sector of lower employment. The effect on the employment rate of the sector where the shift occurred will be larger, the smaller is total supply.

**9.2.** An increase in demand in both sectors increases the supply of the sector where employment is lower.

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**9.3.** (Local) unemployment rate fluctuations will be higher in the low wage (low local unemployment rate) region/sector.

### 5.5. Global Coverage

1. Take now the case where  $W_1 = W_2 = W$ , i.e., we have global, total or complete coverage. Equilibrium condition (V.5) becomes:

$$L^1(W) / \bar{L}_1 = L^2(W) / \bar{L}_2 \quad (V.29)$$

(V.29) establishes equality between employment - hence, also local unemployment - rates of the two sectors or regions.

Notice that a change in  $W$  implies a double wage increase: in sector 1 and in sector 2. A change in the wage rate will imply:

$$\begin{aligned} \frac{\partial \bar{L}_1}{\partial W} = - \frac{\partial \bar{L}_2}{\partial W} = [L^1(W)' \bar{L}_2 - L^2(W)' \bar{L}_1] / \\ / [L^1(W) + L^2(W)] \end{aligned} \quad (V.30)$$

Then, the labor force will flow to region 1 with a rise in  $W$  iff:

$$L^1(W)' / \bar{L}_1 > L^2(W)' / \bar{L}_2 \quad (V.31)$$

or

$$\square^1 (1 - u_1) > \square^2 (1 - u_2) \quad (V.32)$$

As  $u_1 = u_2$ , this means:

$$\square^1 > \square^2 \text{ or } |\square^1| < |\square^2| \quad (V.33)$$

Unemployment rate in region 1 will move according to:

$$\begin{aligned} \frac{\partial U_1}{\partial W} = - [L^1(W)' \{L^1(W) - [\bar{L}_2 - L^2(W)]\} + L^2(W)' \bar{L}_1] / \\ / [L^1(W) + L^2(W)] \end{aligned} \quad (V.34)$$

It is a sufficient condition for this to be positive that:

$$L^1(W) > U_2 \quad (V.35)$$

or

$$L^1(W) + L^2(W) > \bar{L}_2 \quad (V.36)$$

Of course, the wage bill will increase in region  $i$  iff elasticity of demand in that region is smaller than 1 in absolute value. The total wage bill will increase iff:

$$L_1 (1 + \square^1) + L_2 (1 + \square^2) > 0 \quad (V.37)$$

The employment rate will decrease (the local unemployment rate will increase) in region 1 (i.e., in both regions...):

$$\begin{aligned} \frac{\partial [L^1(W) / \bar{L}_1]}{\partial W} = L^1(W) [L^1(W)' + L^2(W)'] / \\ / \{ \bar{L}_1 [L^1(W) + L^2(W)] \} < 0 \end{aligned} \quad (V.38)$$

The expected wage will increase iff:

$$L^1(W) / \bar{L}_1 > - W \frac{\partial [L^1(W) / \bar{L}_1]}{\partial W} \quad (V.39)$$

Developing this condition, we can arrive at:

$$(\square^1 + 1) L_1 + (\square^2 + 1) L_2 > 0 \quad (\text{V.40})$$

That, is the same condition as for total wage bill to increase.

**Proposition 10:** With total coverage, the increase of the wage:

- increases the unemployment rates in both sectors

- increases supply in the sector for which  $\square^i$  is lower in absolute value. As usual, it will increase the wage bill in each sector iff the elasticity of labor demand of the sector is smaller than 1 (in absolute value). It will increase the expected wage in the economy (and total wage bill) if both elasticities of labor demand are smaller than 1 (in absolute value).

2. Parametric changes in supply or demand will yield very similar results as the ones seen for multiple coverage:

Consider first a change in labor supply. Then (V.11) becomes:

$$\frac{\partial \bar{L}_1}{\partial L} = L^i(W) / [L^1(W) + L^2(W)] > 0 \quad (\text{V.41})$$

With respect to shifts in demand, (V.13) turns into:

$$\frac{\partial \bar{L}_1}{\partial a} = - \frac{\partial \bar{L}_2}{\partial a} = \bar{L}_2 / \{ [L^1(W)+a] + L^2(W) \} > 0 \quad (\text{V.42})$$

and a general demand increase implies, from (V.19) and (V.20):

$$\frac{\partial \bar{L}_1}{\partial c} = - \frac{\partial \bar{L}_2}{\partial c} = \quad (\text{V.43})$$

$$= (\bar{L}_2 - \bar{L}_1) / \{ [L^1(W) + c] + [L^2(W) + c] \}$$

Supply in sector 1 will increase with an increase in demand iff

$$\bar{L}_2 > \bar{L}_1 \quad \text{or} \quad L_2 > L_1 \quad (\text{V.44})$$

that is, supply increases in the sector with smaller local supply, therefore, smaller employment.

## 6. Size Restrictions in the Sectors

In this section we want to quantify the effects of several changes in the two-sector model with institutional wage fixed in sector 1 but with size restriction in the areas.

We consider the common assumptions 1. to 4. of the previous sections and 6.:

6. Job rotation is only locally accomplished, i.e., individuals in one sector cannot get an employment in the second sector.

We distinguish two cases:

### Model A:

Region 1 has a limited housing capacity, or there are barriers to membership or affiliation in region 1 (say, "insiders" limit entry). We add assumption:

5. Wage in second sector is market determined and entry location restrictions in region 1 place an upper bound of  $\bar{L}_1^*$  on the amount of people that can actually live there.

If the restriction is binding in equilibrium, supply in the second sector will be:

$$\bar{L}_2 = \bar{L} - \bar{L}_1^* \quad (\text{VI.1})$$

and the wage in the second sector:

$$W_2 = W^2(\bar{L} - \bar{L}_1^*) \quad (VI.2)$$

It must be the case that  $\bar{L}_1^*$  is smaller than the equilibrium local supply in the institutional sector generated in the Harris-Todaro framework. As long as that condition holds, dynamics of this scenario have some of the same properties of the partial coverage - perfect mobility case.

It is straightforward to show that:

**Proposition 11:**

**11.1.** In a dualistic model with housing or membership restrictions and institutional wage fixed in one of the sectors, the equilibrium wage in the uncovered sector is higher than the perfect mobility case. There will be unemployment but less than in the H.-T. framework.

**11.2.** An increase in the size of the covered sector will:

- decrease employment and increase the wage of the uncovered sector
- increase the wage bill in the uncovered sector iff demand elasticity in that sector is smaller than 1 in absolute value.
- have no effect on covered sector's employment or wage bill, but it will rise local and total unemployment and unemployment rate.

**11.3.** An increase of the wage of the covered sector will have no impact on employment of the other sector and will decrease employment in the first sector. It will rise the wage bill iff demand elasticity in the covered sector is smaller than 1 in absolute value.

**11.4.** An increase in total supply will have the same effects as in the perfect mobility case.

**11.5.** An increase of either or both demands will always increase the total wage bill. It will always rise the wage bill in the sector where the shift occurred.

**11.6.** If the (outward) demand shift is unilateral, employment will increase in the covered sector if the shift occurred in that sector. Employment will not change if the shift occurred in the uncovered sector, where the wage bill will rise.

**11.7.** If we have a general (outward) demand increase, employment of the covered sector will rise and the employment in the other will not change. The wage in the uncovered sector will rise.

Notice that we could consider that the limitations occurred in the second sector,

say, locals could not exceed  $\bar{L}_2^*$ , being  $\bar{L}_2^*$  larger than employment in the uncovered sector in the H.-T. solution. All the above implications would be valid apart from 11.2., which would hold for a "decrease in the size of the uncovered sector".

**Model B:**

The traditional sector has a limited ability of employment generation, say land (and land productivity) is fixed or limited; or there are employment quotas in the region. We consider assumption 5:

5. Wage in second sector is demand determined but there is a(n exogenous) limit of access to employment in the sector given by  $L_2^*$ .

Then, the wage in the second sector is determined by:

$$L_2^* = L^2(W_2) \quad \text{or} \quad W_2 = W^2(L_2^*) \quad (VI.3)$$



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In this setting, there will be unemployment in both regions and everything works as in the Bhagwati-Hamada model. Changes in  $L_2^*$  have similar effect as changes in  $W_2$  multiplied by the slope of the demand in the second sector - and, thus, symmetric to the impact of a change in  $W_2$ :

$$dL_2^* = L_2^2(W_2)' dW_2 \quad \text{or} \quad dW_2 = W_2^2(L_2^*)' dL_2^* \quad (\text{VI.4})$$

If employment in sector 2 is fixed, we have the situation where the wage bill is also fixed in that sector. Therefore shifts of the wage of the covered sector, of total supply or of the demand of the first sector will have the same effects as in B.-H.

Shifts in demand of the second sector will have different implications according to the way we introduce them.

If we consider that a shift in the demand of the second sector implies a corresponding increase in the employment of the second sector but no change in productivity, i.e., the equilibrium condition can be stated as:

$$W_1 [L^1(W_1) + a] / \bar{L}_1 = (L_2^* + b) W_2^2(L_2^*) / \bar{L}_2 \quad (\text{VI.5})$$

we have the same conclusions as previously in the B.-H. model.

If we consider that the changes affect productivity in the second sector but not the size capacity or employment limitations of that sector, we can write the equilibrium condition as:

$$W_1 [L^1(W_1) + a] / \bar{L}_1 = W_2^2(L_2^* - b) L_2^* / \bar{L}_2 \quad (\text{VI.6})$$

1. Consider then that  $a$  is 0 and a change in  $b$ . The wage in the second sector increases with  $b$ . As for local supplies:

$$\begin{aligned} \partial \bar{L}_2 / \partial b &= - \partial \bar{L}_1 / \partial b = - L_2^* W_2^2(L_2^* - b)' \bar{L}_1 / \\ &/ \{ W_1 [L^1(W_1) + W_2^2(L_2^* - b) L_2^*] \} > 0 \end{aligned} \quad (\text{VI.7})$$

The wage bill increases with  $b$ , as well as the expected wage in the economy.

The sector or local employment rate will increase in sector 1 because  $W_1$  is fixed and expected wage increases; it will decrease in sector 2, because employment is fixed and  $\bar{L}_2$  increases with  $b$ .

2. Suppose we want to consider a simultaneous shift in the demand of both sectors. The equilibrium can be stated as:

$$W_1 [L^1(W_1) + c] / \bar{L}_1 = W_2^2(L_2^* - c) L_2^* / \bar{L}_2 \quad (\text{VI.8})$$

and a change in  $c$  originates:

$$\begin{aligned} \partial \bar{L}_1 / \partial c &= - \partial \bar{L}_2 / \partial c = [W_1 \bar{L}_2 + W_2^2(L_2^* - c) L_2^* \bar{L}_1] / \\ &/ \{ W_1 [L^1(W_1) + c] + W_2^2(L_2^* - c) L_2^* \} \end{aligned} \quad (\text{VI.9})$$

Supply in sector 1 will increase with an increase in demand iff

$$W_1 / [-W_2^2(L_2^* - c)' L_2^*] > \bar{L}_1 / \bar{L}_2 \quad (\text{VI.10})$$

Using the equilibrium condition (VI.8), this means:

$$W_2^2(L_2^* - c) / [-W_2^2(L_2^* - c)' L_2^*] = |\square|^2 > [L^1(W_1) + c] / L_2^* \quad (\text{VI.11})$$

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i.e., local supply increases in sector 1 iff elasticity of demand in that sector is larger than the employment share of the other sector relative to the second sector.

Wage in the uncovered sector, expected wage in both regions (and total wage bill) will always increase with  $c$ . Therefore, as  $W_1$  is fixed, the unemployment rate in the covered sector will decrease with  $c$ . As for sector 2:

$$\partial(L_2^*/\bar{L}_2)/\partial c = -L(\partial\bar{L}_2/\partial c)/\bar{L}_2^2 \quad (\text{VI.12})$$

The unemployment rate in sector 2 will move in the same direction of local supply, as well as total unemployment in region 2. As for unemployment in region 1:

$$\begin{aligned} \partial U_1/\partial c = \partial\bar{L}_1/\partial c - 1 = \{W_1\bar{L}_2 + W^2(L_2^* - c)L_2^*/\bar{L}_1 - \\ - W_1[L^1(W_1+c) - W^2(L_2^* - c)L_2^*] / \\ / \{W_1[L^1(W_1+c) + W^2(L_2^* - c)L_2^*]\} \end{aligned} \quad (\text{VI.13})$$

This will be negative iff:

$$|\square^2| \{1 - (\bar{L}/\bar{L}_2)[L^1(W_1+c)/\bar{L}_1]\} < [L^1(W_1+c)/L^2(W_2+c)] \quad (\text{VI.14})$$

It is a sufficient condition for the expression to hold that:

$$\bar{L}_2/\bar{L} < [L^1(W_1+c)/\bar{L}_1] \quad (\text{VI.15})$$

### Proposition 12:

**12.1.** With this scenario, the increase in the covered sector's wage, of labor supply or of the demand of the covered sector has the same effects as in the B.-H. framework. The same occurs if the demand shifter in the uncovered sector works through employment but not productivity and we consider the different demand shift combinations.

**12.2.** The increase in the employment restriction in sector 2

- increases the wage bill
- increases supply in that sector (and decreases the supply in the other sector)
- increases the equilibrium expected wage iff the wage-elasticity of labor demand of the sector in which the wage increase was registered is larger than 1 (in absolute value). The reverse would happen if such elasticity was smaller than 1. Local unemployment rate in the second region will necessarily decrease.

**12.3.** If we consider that the demand shift in the uncovered sector affects productivity but not employment:

- An increase in the demand of the uncovered or of the two sectors will increase the wage in the uncovered sector, total wage bill and the expected wage in the economy and decrease the unemployment rate in the covered sector.
- An increase in demand in both sectors increases the supply of the covered sector, and thus, the corresponding local unemployment and unemployment rate iff elasticity of demand in that sector is higher (in absolute value) of the ration of sector 1 to sector 2's employment.
- An unilateral increase in the demand of the uncovered sector will increase local supply in the uncovered sector, where unemployment rate will increase.

## 7. Further Applications: Comparison of Effects Across Different Scenarios

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. A possible application of the complete analysis is the comparison of the comparative static results across different scenarios.

1. Consider the effect on the expected wage - and, in this framework, on output and per capita output - of the economy of an exogenous increase in demand - change in  $a, b, c$ . Let:

$$\partial W^e / \partial j^{F.-M.} = (a) \text{ (Free market)}$$

$$\partial W^e / \partial j^{P.-M.} = (b) \text{ (Perfect mobility)}$$

$$\partial W^e / \partial j^{H.-T.} = (c) \text{ (Harris-Todaro)}$$

$$\partial W^e / \partial j^{B.-H.} = (d) \text{ (Bhagwati-Hamada)}$$

$j = a, b, c$ . Then one can show that around the free market solution, i.e.,  $W_1 = W_2, u_1 = u_2 = 0$ :

. If  $|\square^2| > |\square^1| > 1$ , then  $(b) < (a) < (c) < (d)$ . The effects of the business cycle increases as institutional arrangements or mobility restrictions are imposed; free market would exhibit a low depth of the cycle.

. If  $|\square^2| < |\square^1| < 1$ , then  $(d) < (c) < (a) < (b)$ . The effects of the business cycle decreases as institutional arrangements or mobility restrictions are imposed; free market would exhibit a low depth of the cycle.

. If  $|\square^1| < 1 < |\square^2|$ , then: If  $|\square^1| L_1 / \bar{L}_1 + |\square^2| L_2 / \bar{L}_2 < 1$ ,  $(b) < (c) < (d) < (a)$ ; If  $|\square^1| L_1 / \bar{L}_1 + |\square^2| L_2 / \bar{L}_2 > 1$ ,  $(b) < (c) < (a) < (d)$ . The depth of the cycle is enlarged as institutional arrangements or mobility restrictions are imposed; but free market would exhibit a high degree depth of the cycle too.

. If  $|\square^1| > 1 > |\square^2|$ , then: If  $|\square^1| L_1 / \bar{L}_1 + |\square^2| L_2 / \bar{L}_2 < 1$ ,  $(d) < (a) < (c) < (b)$ ; If  $|\square^1| L_1 / \bar{L}_1 + |\square^2| L_2 / \bar{L}_2 > 1$ ,  $(a) < (d) < (c) < (b)$ . The depth of the cycle decreases as institutional arrangements or mobility restrictions are imposed; but free market would exhibit also a low depth of the cycle too.

The same conclusions would hold for the effect (in absolute value) of an exogenous increase of the total population,  $\bar{L}$  on the overall expected wage and on per capita output.

One can see that, consistently, the (increasing or decreasing, depending on demand elasticities) trend always comes from (b) - one institutional sector with perfect mobility - to (c) - one institutional sector with no mobility - to (d) - both sectors are subject to institutional rules. However, free market results do not necessarily obey to a systematic comparison with the others.

2. Also, with respect to the unemployment rate  $u$ , one can show that, around the free market solution it is always the case that:

$$\partial u / \partial \bar{L}^{H.-T.} < \partial u / \partial \bar{L}^{B.-H.}$$

$$|\partial u / \partial j^{H.-T.}| < |\partial u / \partial j^{B.-H.}|, \quad j = a, b, c$$

That is the unemployment rate will suffer larger fluctuations when institutional arrangements are more widely spread.

**Proposition 13:**

**13.1.** If the demand elasticity of the secondary sector is larger than 1 in absolute value, the output cycle is enlarged as we move from the scenario of one institutional sector with perfect mobility, to the model with one institutional sector with no mobility and, finally, to the case where both sectors are subject to institutional rules; if demand elasticity of the secondary sector is smaller than 1 in absolute value, the output cycle is dampened as we move in that direction. Free market results do not necessarily obey to a systematic comparison with the others.

**13.2.** The same pattern is observed for induced movements of per capita output in response to changes in the exogenous labor supply.

**13.3.** The overall unemployment rate will always show stronger fluctuations when both sectors are institutionally covered.

### 8. Summary and Conclusions

1. It is clear that migration/mobility decisions are based in differential wages in the several regions or sectors where wage setting is not competitive. The main contribution of this research is a unified formal treatment of the two-sector economy under different mobility and institutional conditions and the investigation of sensitivity to several shifts.

The analysis starts from an exposition of results implied in a two sector model under free market. Then it follows for a two-sector model with one-covered sector but perfect mobility across sectors - which generates no unemployment. Next, it considers the Harris-Todaro model where one sector is competitive and imperfect mobility across regions is imposed; then goes to the two-sector institutionally ruled framework of Bhagwati-Hamada; and finally describes a (two) possible equilibrium (equilibria) where there are size restrictions, either in the region or affiliation possibilities (for example, migration quotas), or in the employment generation capacity of the traditional sector.

2. Several conclusions can be drawn with the simple models used - the results are summarized in Tables 1 to 6. The most interesting ones are on the direction of the migration/mobility flow - local or sector supply, or people that rotate in a sector - and also on average (and/or expected) wage outcomes in response to a change in the wage, exogenous supply or demands.

The sign effects of supply and unilateral demand shifts on each macroeconomic aggregate seem invariant to the dualistic scenario considered. All other changes in exogenous conditions may have different sign effects on some of the aggregates according to the framework chosen.

Some conclusions depend upon the size of demand elasticities; others on relative expected wage in the two regions; and still others on the employment size of the two regions. In general,

- a low demand elasticity of a region where a wage increase takes place favors both regions average wage.

- a shift in supply will rise (more) local unemployment in the region of higher wage bill - or higher employment.

3. With respect to local or sector unemployment rate fluctuations induced by demand shifts (the business cycle), they seem to be higher in the low(er) wage sector (or lower unemployment rate sector) when we are in presence of multiple coverage (and or global coverage).

4. The output cycle is enlarged or dampened (depending on demand elasticities of the secondary sector being large or small) as we move from the scenario of one institutional sector with perfect mobility, to the model with one institutional sector with no mobility and, finally, to the case where both sectors are subject to institutional rules. Free market results do not necessarily obey to a systematic

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comparison with the others. The same is observed for induced movements of per capita output in response to changes in the exogenous labor supply.

The overall unemployment rate will always show stronger fluctuations when both sectors are institutionally covered.

5. Size restrictions or local population quotas may work as policy devices to counteract the effects of (other) institutional arrangements. In general, affiliation restrictions in the institutional sector of a Harris-Todaro framework originate macroeconomic behavior patterns similar to perfect mobility with partial (or one-sector) coverage. Employment sluggishness of the traditional sector originates comparative statics results which are similar to those of the Bhagwati-Hamada model.

| TABLE 1. Institutional Wage Increases (or of Wage in Sector 1) |   |  |   |   |                       |   |
|--|---|--|---|---|-----------------------|---|
| Variable   | Model                                     |  |   |   |                       |   |
|  | P.-M.                                     | H.-T.                                  | B.-H.   | B.-H.TC                                       | A                     | B   |
| $L_1$  | -   | -                                      | -   | -   | -                     | -   |
| $W_1 L_1$  | + iff $ \square  < 1$                     | + iff $ \square  < 1$                  | + iff $ \square  < 1$                                 | + iff $ \square  < 1$                         | + iff $ \square  < 1$ | + iff $ \square  < 1$                                 |
| $L_1$  | -   | + iff $ \square  < 1$                  | + iff $ \square  < 1$                                 | + iff $ \square  < \square^2$                 | ..                    | + iff $ \square  < 1$                                 |
| $W_1 L_1 / L_1$  | +   | + iff $ \square  < 1$                  | + iff $ \square  < 1$                                 | + iff $L_1(1+\square) + L_2(1+\square^2) > 0$ | + iff $ \square  < 1$ | + iff $ \square  < 1$                                 |
| $U_1$  | .   | + if $ \square  < 1$<br>or $W_2 = W_1$ | + if $ \square  < 1$ or<br>- - -<br>$L_2/L < L_1/L_1$ | + if $L_1 > U_2$                              | +                     | + if $ \square  < 1$ or<br>- - -<br>$L_2/L < L_1/L_1$ |
| $u_1$  | .   | +                                      | +   | +   | +                     | +   |
| $W_2$  | -   | + iff $ \square  < 1$                  | ..  | +   | 0                     | 0   |
| $L_2$  | +   | - iff $ \square  < 1$                  | 0   | -   | 0                     | ..  |
| $W_2 L_2$  | + iff $ \square  > 1$                     | + iff $ \square  - 1$<br>of same sign  | 0   | + iff $ \square  < 1$                         | 0                     | 0   |
| $L_2$  | +   | - iff $ \square  < 1$                  | - iff $ \square  < 1$                                 | + iff $ \square  < \square^2$                 | 0                     | - iff $ \square  < 1$                                 |
| $W_2 L_2 / L_2$  | -   | + iff $ \square  < 1$                  | + iff $ \square  < 1$                                 | + iff $L_1(1+\square) + L_2(1+\square^2) > 0$ | 0                     | + iff $ \square  < 1$                                 |
| $U_2$  | .   | .                                      | - iff $ \square  < 1$                                 | + if $L_2 > U_1$                              | .                     | - iff $ \square  < 1$                                 |
| $u_2$  | .   | .                                      | - iff $ \square  < 1$                                 | +   | .                     | - iff $ \square  < 1$                                 |
| $W_1 L_1 + W_2 L_2$  | + iff $W_1(1+\square) < W_2(1+\square^2)$ | + iff $ \square  < 1$                  | + iff $ \square  < 1$                                 | + iff $L_1(1+\square) + L_2(1+\square^2) > 0$ | + iff $ \square  < 1$ | + iff $ \square  < 1$                                 |
| $U$  | .   | + if $ \square  < 1$<br>or $W_2 = W_1$ | +   | +   | +                     | +   |

| TABLE 2. Size Restrictions |                        |  |
|----------------------------|------------------------|--|
| Variable                   | Model                  |  |
|                            | A                      | B  |
| Effects on Sector 1        |                        |  |
| $L_1$                      | 0                      | 0  |
| $W_1 L_1$                  | 0                      | 0  |
| $-L_1$                     | +                      | + iff $ \sigma^2  < 1$                       |
| $-W_1 L_1 / L_1$           | -                      | + iff $ \sigma^2  > 1$                       |
| $U_1$                      | +                      | + iff $ \sigma^2  < 1$                       |
| $u_1$                      | +                      | + iff $ \sigma^2  < 1$                       |
| Effects on Sector 2        |                        |  |
| $W_2$                      | +                      | -  |
| $L_2$                      | -                      | +  |
| $W_2 L_2$                  | + iff $ \sigma^2  < 1$ | + iff $ \sigma^2  > 1$                       |
| $-L_2$                     | -                      | + iff $ \sigma^2  > 1$                       |
| $-W_2 L_2 / L_2$           | +                      | + iff $ \sigma^2  > 1$                       |
| $U_2$                      | .                      | - if $ \sigma^2  < 1$ or $L_1/L_2 < L_2/L_2$ |
| $u_2$                      | .                      | -  |
| Total Effects              |                        |  |
| $W_1 L_1 + W_2 L_2$        | + iff $ \sigma^2  < 1$ | + iff $ \sigma^2  > 1$                       |
| $U$                        | +                      | -  |

**TABLE 3. Supply Shifts**

| Variable                   | Model  |                            |                            |       |                            |    |
|----------------------------|--|----------------------------|----------------------------|-------|----------------------------|----|
|                            | F.-M.  | P.-M.                      | H.-T.                      | B.-H. | A                          | B  |
| $W_1$                      | -  | ..                         | ..                         | ..    | ..                         | .. |
| $L_1$                      | +  | 0                          | 0                          | 0     | 0                          | 0  |
| $W_1 L_1$                  | + iff<br>$ \square^1  > 1$                     | 0                          | 0                          | 0     | 0                          | 0  |
| $-L_1$                     | +  | 0                          | +                          | +     | ..                         | +  |
| $-W_1 L_1 / L_1$           | -  | 0                          | 0                          | -     | 0                          | -  |
| $U_1$                      | .  | .                          | +                          | +     | 0                          | +  |
| $u_1$                      | .  | .                          | +                          | +     | 0                          | +  |
| $W_2$                      | -  | -                          | -                          | ..    | -                          | 0  |
| $L_2$                      | +  | +                          | +                          | 0     | +                          | .. |
| $W_2 L_2$                  | + iff<br>$ \square^2  > 1$                     | + iff<br>$ \square^2  > 1$ | + iff<br>$ \square^2  > 1$ | 0     | + iff<br>$ \square^2  > 1$ | 0  |
| $-L_2$                     | +  | +                          | +                          | +     | +                          | +  |
| $-W_2 L_2 / L_2$           | -  | -                          | -                          | -     | -                          | -  |
| $U_2$                      | .  | .                          | .                          | +     | .                          | +  |
| $u_2$                      | .  | .                          | .                          | +     | .                          | +  |
| $W_1 L_1 + W_2 L_2$        | + iff<br>$L_1(1+\square^1)+L_2(1+\square^2)<0$ | + iff<br>$ \square^2  > 1$ | + iff<br>$ \square^2  > 1$ | 0     | + iff<br>$ \square^2  > 1$ | 0  |
| $-(W_1 L_1 + W_2 L_2) / L$ | -  | -                          | -                          | -     | -                          | -  |
| $U$                        | .  | .                          | +                          | +     | 0                          | +  |
| $u$                        | .  | .                          | + if<br>$ \square^2  < 1$  | +     | 0                          | +  |



**TABLE 4.** Demand Shifts in Covered Sector or Sector 1

| Variable               | Model                     |                           |  |                            |    |                            |
|------------------------|---------------------------|---------------------------|--|----------------------------|----|----------------------------|
|                        | F.-M.                     | P.-M.                     | H.-T.  | B.-H.                      | A  | B                          |
| $W_1$                  | +                         | ..                        | ..   | ..                         | .. | ..                         |
| $L_1$                  | +                         | +                         | +  | +                          | +  | +                          |
| $W_1 L_1$              | +                         | +                         | +  | +                          | +  | +                          |
| $-$<br>$L_1$           | +                         | +                         | +  | +                          | .. | +                          |
| $-$<br>$W_1 L_1 / L_1$ | +                         | 0                         | +  | +                          | +  | +                          |
| $U_1$                  | .                         | .                         | - if<br>$\sigma^2 < L_1/L_2$<br>or $W_2 = W_1$ | - iff<br>$L_2/L < L_1/L_1$ | -  | - iff<br>$L_2/L < L_1/L_1$ |
| $u_1$                  | .                         | .                         | -  | -                          | -  | -                          |
| $W_2$                  | +                         | +                         | +  | ..                         | 0  | 0                          |
| $L_2$                  | -                         | -                         | -  | 0                          | 0  | ..                         |
| $W_2 L_2$              | + iff<br>$ \sigma^2  < 1$ | + iff<br>$ \sigma^2  < 1$ | + iff<br>$ \sigma^2  < 1$                      | 0                          | 0  | 0                          |
| $-$<br>$L_2$           | -                         | -                         | -  | -                          | 0  | -                          |
| $-$<br>$W_2 L_2 / L_2$ | +                         | +                         | +  | +                          | 0  | +                          |
| $U_2$                  | .                         | .                         | .  | -                          | .  | -                          |
| $u_2$                  | .                         | .                         | .  | -                          | .  | -                          |
| $W_1 L_1 + W_2 L_2$    | +                         | +                         | +  | +                          | +  | +                          |
| $U$                    | .                         | .                         | - if<br>$\sigma^2 < L_1/L_2$<br>or $W_2 = W_1$ | -                          | -  | -                          |

**TABLE 5.** Demand Shifts in Uncovered Sector or Sector 2

| Variable            | Model                      |       |       |                          |    |                          |    |
|---------------------|----------------------------|-------|-------|--------------------------|----|--------------------------|----|
|                     | F.-M.                      | P.-M. | H.-T. | B.-H.                    | A  | B1                       | B2 |
| $W_1$               | +                          | ..    | ..    | ..                       | .. | ..                       | .. |
| $L_1$               | -                          | 0     | 0     | 0                        | 0  | 0                        | 0  |
| $W_1 L_1$           | + iff<br>$ \square^1  < 1$ | 0     | 0     | 0                        | 0  | 0                        | 0  |
| $-L_1$              | -                          | 0     | -     | -                        | .. | -                        | -  |
| $-W_1 L_1 / L_1$    | +                          | 0     | +     | +                        | 0  | +                        | +  |
| $U_1$               | .                          | .     | -     | -                        | 0  | -                        | -  |
| $u_1$               | .                          | .     | -     | -                        | 0  | -                        | -  |
| $W_2$               | +                          | +     | +     | ..                       | +  | ..                       | +  |
| $L_2$               | +                          | 0     | +     | +                        | 0  | +                        | .. |
| $W_2 L_2$           | +                          | +     | +     | +                        | +  | +                        | +  |
| $-L_2$              | +                          | 0     | +     | +                        | 0  | +                        | +  |
| $-W_2 L_2 / L_2$    | +                          | +     | +     | +                        | +  | +                        | +  |
| $U_2$               | .                          | .     | .     | - iff<br>$1/L < L_2/L_2$ | .  | - iff<br>$1/L < L_2/L_2$ | +  |
| $u_2$               | .                          | .     | .     | -                        | .  | -                        | +  |
| $W_1 L_1 + W_2 L_2$ | +                          | +     | +     | +                        | +  | +                        | +  |
| $U$                 | .                          | .     | -     | -                        | 0  | -                        | 0  |

**TABLE 6.** Demand Shifts in Both Sectors

| Variable             | Model                  |                         |   |                            |    |                            |                             |
|----------------------|------------------------|-------------------------|---|----------------------------|----|----------------------------|-----------------------------|
|                      | F.-M.                  | P.-M.                   | H.-T.                                       | B.-H.                      | A  | B1                         | B2                          |
| $W_1$                | +                      | ..                      | ..  | ..                         | .. | ..                         | ..                          |
| $L_1$                | + iff $L_2^1 < L_1^1$  | +                       | +   | +                          | +  | +                          | +                           |
| $W_1 L_1$            | + if $ \square_1  < 2$ | +                       | +   | +                          | +  | +                          | +                           |
| -<br>$L_1$           | + iff $L_2^1 < L_1^1$  | +                       | + iff $\square_2 > L_1/L_2$                 | + iff $L_1 < L_2$          | .. | + iff $L_1 < L_2$          | + iff $\square_2 > L_1/L_2$ |
| -<br>$W_1 L_1 / L_1$ | +                      | 0                       | +   | +                          | +  | +                          | +                           |
| $U_1$                | .                      | .                       | - if $\square_2 < 2 L_1/L_2$<br>$W_2 = W_1$ | - if $\square_2 < L_1/L_1$ | -  | - if $\square_2 < L_1/L_1$ | - if $\square_2 < L_1/L_1$  |
| $u_1$                | .                      | .                       | -   | -                          | -  | -                          | -                           |
| $W_2$                | +                      | +                       | +   | ..                         | +  | ..                         | +                           |
| $L_2$                | + iff $L_1^1 < L_2^1$  | -                       | + iff $\square_2 < L_1/L_2$                 | +                          | 0  | +                          | ..                          |
| $W_2 L_2$            | + if $ \square_2  < 2$ | + iff $ \square_2  < 2$ | + if $ \square_2  < 1$                      | +                          | +  | +                          | +                           |
| -<br>$L_2$           | + iff $L_1^1 < L_2^1$  | -                       | + iff $\square_2 < L_1/L_2$                 | + iff $L_2 < L_1$          | 0  | + iff $L_2 < L_1$          | + iff $\square_2 < L_1/L_2$ |
| -<br>$W_2 L_2 / L_2$ | +                      | +                       | +   | +                          | +  | +                          | +                           |
| $U_2$                | .                      | .                       | .   | - if $\square_1 < L_2/L_2$ | .  | - if $\square_1 < L_2/L_2$ | + iff $\square_2 < L_1/L_2$ |
| $u_2$                | .                      | .                       | .   | -                          | .  | -                          | + iff $\square_2 < L_1/L_2$ |
| $W_1 L_1 + W_2 L_2$  | +                      | +                       | +   | +                          | +  | +                          | +                           |
| $U$                  | .                      | .                       | - if $\square_2 < 2 L_1/L_2$ or $W_2 = W_1$ | -                          | -  | -                          | -                           |

## Notes

<sup>i</sup> A good survey of theoretical literature can be found in [Bhattacharya \(1993\)](#).

<sup>ii</sup> See [McNabb and Ryan \(1990\)](#) for a recent survey. Literature on segmented labor markets seems to have followed an independent path of international economics. See also Saint-Paul (1996) for applications of the theory with microfoundations for several dualistic structures.

<sup>iii</sup> See for example, [Mincer \(1976\)](#) and [McDonald and Solow \(1985\)](#).

<sup>iv</sup> See [Robinson \(1988\)](#).

<sup>v</sup> See, for example, [Bhagwati and Srinivasan \(1974\)](#), [Srinivasan and Bhagwati \(1975\)](#), [Todaro \(1976\)](#).

<sup>vi</sup> The recent rise of Institutional Economics, specially in Europe, may be associated to fact that in the Old Continent market forces seem to work in a different, more rigid, context. Nevertheless, those forces (may) still work...

<sup>vii</sup> These were considered in enlarged versions of Harris-Todaro type by [Fields \(1989\)](#), for example.

<sup>viii</sup> [McDonald and Solow \(1985\)](#) also analyze this issue in a two-sector economy where primary wages are formed under collective bargaining and the secondary market is competitive. We discard the problem of how the primary sector jobs are determined...

<sup>ix</sup> See [Fields \(1989\)](#), for example, for a similar definition of demand shifter, but interpreted there as government induced.

<sup>x</sup> Equation (2) in [Corden and Findlay \(1975\)](#).

<sup>xi</sup> See, for example, [Corden and Findlay \(1975\)](#) or [Krueger \(1983\)](#) for a similar graph.

<sup>xii</sup> See [Fields \(1989\)](#), for example, for a similar definition of demand shifter, but interpreted there as government induced.

<sup>xiii</sup> We reproduce the same - and not the symmetric - effect of emigration of skilled labor found in [Hamada and Bhagwati \(1983\)](#). This is due to different assumptions.

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