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**Stabilisation Policy in a Model of Consumption,  
Housing Collateral and Bank Lending**

Jagjit S. Chadha, Germana Corrado and Luisa Corrado

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# STABILISATION POLICY IN A MODEL OF CONSUMPTION, HOUSING COLLATERAL AND BANK LENDING\*

Jagjit S. Chadha<sup>†</sup>      Germana Corrado<sup>‡</sup>      Luisa Corrado<sup>§</sup>

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## Abstract

We decompose aggregate consumption by modelling both savers and their links to collateral constrained borrowers through a bank which prices credit risk. Savers own both firms and the commercial bank while borrowers require loans from the commercial bank to effect their consumption plans. The bank lends at a premium over the interest rate on central bank money in proportion to the riskiness of assets, the demand for loans, the asset price and the quantity of housing collateral. We show that even though house price do not represent wealth, aggregate consumption is not independent of movements in house prices. We consider the case for employing macro-prudential policy jointly with monetary and fiscal policy in order to minimise losses for a representative household.

JEL Classification: E31; E40; E51.

Keywords: Credit constrained households, housing collateral, asset prices, bank lending, default risk, macro-prudential, fiscal and monetary policy.

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# 1 Introduction

The interplay between household consumption and housing wealth seems to have become a dominant force in driving recent business cycle fluctuations but both the mechanism and its implications for policy remain rather opaque. We pursue a line of enquiry suggested by a number of recent studies that have employed collateral constrained models (see, for example, Almeida *et al.*, 2006 and Ortalo-Magné and Rady, 2006) to understand better this interplay. We explore the collateral channel of housing demand - a variant of the ‘financial accelerator’ model developed by Bernanke *et al.* (1999) and Kiyotaki and Moore (1997) - in order to disentangle the role of house prices in households’ consumption decisions within the framework of a micro-founded macroeconomic model (see Aoki *et al.*, 2004; Iacoviello, 2005). We emphasize the role of financial intermediation in our model by having the links between savers and borrowers mediated by a bank that allows us to endogenize the external finance premium, defined as the difference between the cost of providing external funding by banks and the opportunity cost of internal funds. An advantage of our approach is that, as well as standard questions of monetary policy, we can then also consider the scope for using macro-prudential instruments to help stabilize welfare in our economy.

In this paper, we unbundle the representative agent assumption and consider two household types, savers and borrowers. Saver households maximize their lifetime expected utility subject to a standard cash-in-advance constraint (CIA), but are otherwise asset rich as they own the housing stock, financial intermediaries, firms and government bonds and behave as standard intertemporal optimizing consumers. Borrowers face the same CIA constraint but obtain loans from banks up to the expected loan to collateral value of houses. Banks intermediate between savers’ deposits and loans to borrowers on the basis of house prices, for which we derive an explicit demand function. We are thus able to analyze the interaction between both types of households, banks and assess the role of various policies in maximizing household welfare.

This work thus fills another gap in the literature. It has been forcefully argued that micro-founded macroeconomic models do not adequately model monetary imbalances or financial frictions. Standard models used for policy analysis have, by construction, no banks, borrowing constraints or any risk of default and so the risk free short-term interest rate suffices to model the monetary side of the economy (on this point, see Chadha *et al.*, 2013). As a consequence, money or credit aggregates and asset prices play no substantive role in explaining economic fluctuations. In comparison we stress that a link between credit and house prices may arise via collateral effects on credit and via the repercussions arising from credit supply fluctuations on house prices. We emphasize that even in a life-cycle model of household consumption, changes in house prices lead to changes in household spending and borrowing when homeowners try to smooth consumption over the life cycle.

There has been increasing interest in introducing a banking sector within micro-founded macroeconomic models in order to analyze economies where different financial assets are available to agents (see, for example, Canzoneri *et al.*, 2008 and Goodhart and Hofmann, 2008). We have framed a banking sector where the risk of households’ default on residential loans is explicitly modelled. This element of uncertainty might explain why the anticipation of potential defaults leads to contractions of credit and deleveraging, even without the necessity of formal default events. Therefore, in our model, the accelerator effect from increasing asset prices operates through the ‘collateral’ channel of housing, and an attenuator operates via the lending rate which reflects the probability of default on residential loans. Our work confirms that a strong shocks amplification and propagation mechanism originates from the External Finance Premium (Goodfriend and McCallum, 2007) and from fluctuations in asset (housing) prices, which determines what we might wish to term a collateral channel, for propagating

real and monetary shocks.

The paper contributes on three dimensions. The model captures the salient features of aggregate consumption dynamics and their apparent relationship to house prices, as it delivers strongly procyclical house prices with no wealth effects (see, for example, Attanasio *et al.*, 2009). We show that house prices, which are forward-looking, are closely linked to the path of borrowers' consumption, loan to value ratios, inflation and the lending rate. Secondly, consumption dynamics are shown to follow a higher order process when there are two types of households. Saver households have considerable volatility injected into their consumption titling plans by movements in real deposit rates. Borrower households need to generate sufficient collateral to allow credit to flow to them in the form of loans and these loans then suppress consumption in future periods and lead to a cycle in aggregate consumption. There are also spillovers in this economy from one type of consumer to the other, as changes in the expected price of durable goods affect borrower consumption via bank lending; the complementarity between consumption of the two types originates from the policy rate which, in our model, effects the deposit rate and savers' consumption decision. Finally, we also consider the appropriate role of monetary and macro-prudential policies in stabilizing this economy, specifically we can use the model to understand the scope instrumental control on financial activities and understand the steady-state implications of policy induced changes in the reserve requirement, risk-weighting of assets and the loan to deposit ratio.

The paper is organized as follows. Section 2 describes how the model fits to the existing literature. Section 3 presents the model comprising a household sector with two types of agents - savers and borrowers - a banking sector, real and monetary sectors and both monetary and fiscal policy. Section 4 describes the steady-state of the model alongside the solution method employed. Section 5 illustrates the response of key variables in our model to real and financial shocks, reports the main results, and considers the appropriate role of stabilization policy in this class of model, noting that in a traditional representative agent framework active interest rates tend to be sufficient to obtain a welfare allocation close to optimal levels under commitment. Section 6 concludes and offers a tentative normative conclusion.

## 2 Background

The role of collateral constraints has been mainly assessed in a closed economy setting, where agents are constrained in the amount of funds they can borrow by the value of collateral they can pledge as a guarantee to the lenders. For example, in the presence of durable goods, Kiyotaki and Moore (1997) consider the case of collateral constraints with heterogeneous agents. Their analysis shows that the collateral constraint plays an important role in transmitting the effects of various shocks to other sectors through the 'financial accelerator mechanism'. The benchmark model linking the macroeconomy to financial markets is Bernanke *et al.*'s model (1999) which Bernanke and Gertler (2001) exploit to analyze the supply-side effects of asset-price fluctuations and assess the implications of an explicit monetary-policy response to stock prices.

Empirical work has also focussed on the relationship between consumption and house price. Attanasio *et al.* (2009) stress that over the past 25 years, house price and consumption growth have been highly correlated. Three main hypotheses for this have been proposed: increases in house prices raise household wealth and so their consumption; house price growth relaxes borrowing constraints by increasing the collateral available to households; and house prices and consumption are together influenced by common factors. Using microeconomic data from the Family Expenditure Survey (FES)

for UK, they find that the relationship between house prices and consumption is stronger for younger -typically borrowing constrained- than older households, contradicting the wealth channel. Using data from the British Panel Household Survey (BHPS) for the years 1991-2008 Table 1 shows that there is a positive correlation between consumption<sup>1</sup> and house prices<sup>2</sup> (both in growth and in levels) for the median borrower<sup>3</sup> validating the house price effect hypothesis on consumption for credit constrained agents. Changes in house prices, in fact, affect consumption by changing the degree to which credit constraints are binding.<sup>4</sup> The median saver,<sup>5</sup> instead, displays either a negative correlation (in levels) or no correlation (growth) between house prices and consumption. Therefore, an increase in house prices which raises household wealth does not affect consumption, contradicting the wealth channel hypothesis. Hence, homeowners who are not facing credit constraints seem to be more hedged against fluctuations in house prices; these fluctuations have no effect on their real wealth and do not affect their consumption choices.

Our work relates to different strands of literature. First, it is strictly related to some recent DSGE models with heterogeneous agents<sup>6</sup> and durables (housing). Such as Iacoviello's study (2005), where he introduces a borrowing constraint tied to housing values both for impatient households and for entrepreneurs; in this framework a rise in asset prices increases the borrowing capacity of the debtors (both households and firms), allowing them to consume and invest more. Hence collateral effects can significantly strengthen the response of the real economy to demand shocks, including those hitting house prices.

Our paper also relates to the literature on optimal monetary policy with heterogeneous consumers and collateral constraints. Recently, Kannan *et al.* (2012) examine the potential role of monetary policy in mitigating the effects of asset price booms and study the role of macroprudential policies in a New Keynesian model with a banking sector and financial accelerator effects. The main feature of this model is the presence of financial intermediaries. In fact, the analysis assumes that savers cannot lend to borrowers directly, whereas banks take deposits from savers and lend them to borrowers, charging a spread that depends on the net worth of borrower. They find that having monetary policy which responds to credit conditions or introducing a loan-to-value rule for borrowers helps to reduce the volatility of the output gap and credit aggregates when the economy is hit by financial or housing demand shocks; however, here the functional form for the determination of the spread is assumed rather than derived from a profit maximization problem. Whereas in our model, savers and borrowers face not only different degrees of impatience but also different interest rates; in fact the wedge between the deposit rate and lending rate generates sources for banks' profits and credit frictions. Moreover, we assume that the interest rate wedge is not constant, but varies with expected durable goods prices (i.e., the collateral value), and the amount of granted loans. Since durable goods are secured for loans, changes in the expected price of durable goods will affect lending rate, borrowers' credit availability and consumption.

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<sup>1</sup> Consumption is defined as difference between real household monthly income and real savings (base year 2005).

<sup>2</sup> We consider the real house value defined as the ratio between the house value and the consumer price index (base year 2005).

<sup>3</sup> In line with the borrowing constraint hypothesised in our model borrowers are homeowners facing a mortgage or loan repayment against a house purchase.

<sup>4</sup> This is also in line with the findings by Aoki *et al.* (2004) who pointed out that a rise in house prices increases the collateral available to homeowners encouraging them to borrow more and to finance higher consumption.

<sup>5</sup> Savers are defined as those who own their house outright without mortgage or loans repayment.

<sup>6</sup> Iacoviello (2005) assumes that the heterogeneity among agents is in the discount rates. Aoki *et al.* (2004) assume instead that a certain fraction of households have accumulated enough wealth so that their consumption decisions are well approximated by the permanent income hypothesis. The other households do not have enough wealth to smooth consumption and they face borrowing constraints.

### 3 The Model

In this section we illustrate the main features of our model summarized in Figure 1. The economy operates over an infinite-time horizon and comprises a continuum of households in the interval  $\mathbb{R} \in [0, 1]$ . Households who consume, work and demand housing and financial assets, are divided into two groups, which we refer to as saver (creditor) households and borrower households.<sup>7</sup> Saver households maximize their lifetime expected utility facing a cash-in-advance constraint, while borrower (leveraged) households, whose ability of borrowing is endogenously linked to the market value of their housing wealth, face both liquidity and collateral (borrowing) constraints. The latter ‘doubly-constrained’ collateralize their debt repayment in order to borrow from financial intermediaries and use these additional credit lines and money to finance their current consumption. The dichotomy between savers, who are essentially standard optimizing consumers and borrowers who are liquidity constrained is key to this paper.

The banking sector collects money in the form of time deposits from savers and lends against housing equity to households who are borrowing constrained. Saver households purchase a positive amount of bonds and time deposits and do not borrow from banks, while leveraged households borrow a positive amount of money from banks and have no deposits. We assume that the savers are also the owners of monopolistic firms in the production sector and of financial intermediaries in the banking sector. Saver households derive utility from consumption of non-durable goods (consumption goods) while leveraged households derive utility from consumption of both non-durable goods and durable goods (housing services). Leveraged households supply labor to firms. Entrepreneurs produce differentiated intermediate goods using leveraged households’ labor. They sell the differentiated goods at a price which includes a markup over the purchasing cost and is subject to adjustment costs. Finally, the monetary authorities set the policy interest rate endogenously, in response to inflation and output, and fiscal policy can be set passively or actively to foster stabilization.

#### 3.1 Households’ Utility Maximization

##### 3.1.1 Saver Households

The preferences for this type of household can be expressed as:

$$\max U = E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t + \chi_B \log B_t) \quad (1)$$

where  $\beta \in (0, 1)$  is the discount or time-preference factor that measures how patient people are,  $C_t$  denotes household’s real consumption of non-durable goods and  $B_t$  real government bond holdings with a weight coefficient  $\chi_B > 0$ .

The representative saver household maximizes the above utility function subject to a sequence of constraints expressed in real terms (i.e., in units of consumption):

(i) Resource constraint:

$$C_t + (\omega m_{t+1} + d_{t+1} + B_{t+1}) \pi_{t+1} = \omega m_t + R_t^D d_t + R_t^B B_t + q_t(H_t - H_{t-1}) + \Theta_t + \Pi_t - Tax_t \quad (2)$$

(ii) Cash-in-Advance (CIA) constraint:

$$C_t \leq \omega m_t \quad (3)$$

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<sup>7</sup>We use saver and creditor, as well as borrower and leveraged as interchangeable terms.

Households enter each period  $t$  with real saving deposits at the bank,  $d_t$ , and real money balances,  $m_t$ , carried over from period  $t-1$  to period  $t$  with  $\omega$  denoting the fraction of real money balances held by savers. Saving deposits pay a gross nominal interest rate,  $R_t^D$ , at the end of the period,  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross inflation rate. In our model, saving deposits (or saving accounts) like a risk-free financial assets provide a store of value and no liquidity services to households, since we assume that they cannot be withdrawn from the bank before the beginning of the next period. Following Canzoneri *et al.* (2008) we put government bonds into household utility to reflect their value in providing liquidity services in addition to being store of value; the liquidity services are represented as direct contributions to household's utility and therefore they command a liquidity premium, for this reason the bond rate will be lower than the saving deposit rate. We also assume that savers enters each period with an endowment of a fixed credit good, therefore it does not enter savers' preferences. We might think of the credit good as a house,  $H_t$ , which savers sell at time  $t$  to the borrowers who demand and consume housing services in the same period. Hence, the term  $q_t(H_t - H_{t-1})$  stands for net real housing holdings with  $q_t$  denoting the relative price of residential goods expressed in terms of non-durable goods,  $q_t \equiv \frac{Q_t}{P_t}$  where  $Q_t$  is the nominal house price. The terms  $\Theta_t$  and  $\Pi_t$  denote real profits (dividends)<sup>8</sup> respectively from firms and banks owned by saver households; and the term  $Tax_t$  stands for taxation. According to our timing convention,  $d_{t+1}$ ,  $m_{t+1}$  and  $B_{t+1}$  are respectively bank saving accounts, real cash balances and short-term government bonds accumulated in period  $t$  and carried over into period  $t+1$ .

Relationship (3) is the familiar CIA requirement that introduces an extreme transactions-technology in which money (cash) is not simply used as a mean to economize on transactions, but is also essential for carrying them out (Lucas and Stokey, 1987). We assume that the CIA constraint is binding  $C_t = m_t$  and that the return on money is no greater than the return on financial savings,  $\frac{P_t}{P_{t-1}} < \min(R_t^D, R_t^B)$  which implies that in a neighborhood of the steady state  $\min(R^D, R^B) > 1$ . Using the CIA constraint we can rewrite the budget constraint (2) as follows:

$$[C_{t+1} + d_{t+1} + B_{t+1}] \pi_{t+1} = R_t^D d_t + R_t^B B_t + q_t(H_t - H_{t-1}) + \Pi_t + \Theta_t - Tax_t \quad (4)$$

Using (4) the maximization problem reads as:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \log \left[ \frac{1}{\pi_t} (R_{t-1}^D d_{t-1} + R_{t-1}^B B_{t-1} + q_{t-1}(H_{t-1} - H_{t-2}) + \Pi_{t-1} + \Theta_{t-1} - Tax_t) - d_t - B_t \right] \\ + \chi_B \log B_t \end{array} \right\} \quad (5)$$

By differentiating (5) with respect  $d_t$  and  $B_t$  we get the efficiency conditions:

*Euler Equation:*

$$\frac{1}{C_t} = \frac{\beta R_t^D}{\pi_{t+1} C_{t+1}} \quad (6)$$

*Bond Demand:*

$$\frac{\chi_B}{B_t} = \left( \frac{1}{C_t} - \frac{\beta R_t^B}{\pi_{t+1} C_{t+1}} \right) \quad (7)$$

Equation (6) is the relation between the marginal utility of current period consumption, next period consumption and the real interest rate. With respect to the standard consumption Euler

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<sup>8</sup>Profits rebated to saver households by the real and banking sectors are respectively  $\Theta_t = \int_0^1 \Theta_{z,t} dz$  and  $\Pi_t = \int_0^1 \Pi_{j,t} dj$ .



condition, in (6) there is an additional cost defined as the opportunity cost of holding positive money balances, which is given by the foregone one-period deposit interest rate,  $R_t^D$ . (7) defines the demand for bonds,  $B_t$ , which depends positively on the bond rate,  $R_t^B$ , and inversely on the the difference of marginal utility of consumption in two consecutive periods.

### 3.1.2 Borrower Households

Each household is allowed to acquire housing services by owning a house. Owning a house in our model serves a dual purpose; it provides the household housing services, and also allows household to own equity. Housing enters in this model both as a good but also as an asset which can be used as collateral to get loans in the credit market. This group of households is facing an additional borrowing constraint that limits the amount they can borrow to the expected market value of their housing holdings; this sort of home equity release scheme allows households to access their housing wealth for financing consumption and housing.

The representative borrower household's maximization problem then reads as:

$$\max U^b = E_0 \sum_{t=0}^{\infty} \check{\beta}^t \left( \log C_t^b + \chi \log H_t - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) \quad (8)$$

where the discount factor is  $\check{\beta} \in (0, 1)$  and  $\check{\beta} < \beta$  indicating that borrowers are also more impatient than savers.  $H$  denotes services from the fixed stock of residential goods (housing services), with a weight coefficient  $\chi > 0$ ;  $N$  denotes labour supplied by leveraged households to the goods sector, and  $\varsigma > 0$  is the labour disutility parameter and it is equal to the inverse of the (Frisch) elasticity of labour supply with respect to the real wage.<sup>9</sup> The superscript  $b$  denotes borrower (leveraged) consumers who are subject to both liquidity and borrowing constraints.

This household maximizes the above utility function subject to the following constraints expressed in real terms (units of consumption):

(i) a resource constraint:

$$C_t^b + q_t(H_t - H_{t-1}) + (1 - \omega) m_{t+1} \pi_{t+1} + \frac{R_{t-1}^L l_{t-1}}{\pi_t} = l_t + (1 - \omega) m_t + w_t N_t - \tau_h q_t H_t \quad (9)$$

(ii) a CIA constraint:

$$C_t^b \leq (1 - \omega) m_t + w_t N_t \quad (10)$$

(iii) and a borrowing constraint:

$$l_t \leq \kappa_t q_{t+1} \pi_{t+1} H_t \quad (11)$$

In (9) the term  $q_t(H_t - H_{t-1})$  denotes net real housing holdings. Among the resources there are wage earnings  $w_t N_t$  from supplying labor to the goods sector with  $w_t$  denoting real wage and loans from the banking sector expressed in real terms,  $l_t$ , with  $R_{t-1}^L$  denoting the nominal interest rate on previous period borrowing. Finally, the last term on the right hand side of the resource constraint denotes tax payments to the government in the form of a property tax,  $\tau_h q_t H_t$ , levied on the value

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<sup>9</sup>The Frisch elasticity of labour supply is defined as the elasticity of the labour supply with respect to wages holding the marginal utility of consumption constant. Empirical estimates on the Frisch elasticity of labour supply are numerous. Prescott (2005) estimates that the Frisch elasticity of labour supply is 3 in the United States, so  $\varsigma = 0.3$ . With  $\varsigma$  approaching 1, the utility function becomes linear in leisure.

of the housing assets where  $\tau_h$  is the house-tax rate. Again according to our timing convention,  $m_t$  is the real quantity of money accumulated in period  $t - 1$  and carried over into period  $t$ , so that  $m_t$  are real money balances at the start of period  $t$ .

Borrower households also face two constraints: a cash-in-advance constraint according to which household's real holdings of money have to be used for cash purchases at time  $t$ , and a borrowing constraint according to which household's borrowing capacity is constrained by the future value of her collateral, that is her housing assets.

The CIA constraint (10) explains why households hold money, that is they must enter the period with enough liquidity to pay for (nondurables) consumption. Since we assume that the CIA constraint is binding then the return on money is no greater than the return on housing equity, that is  $\frac{P_{t+1}}{P_t} < \frac{q_{t+1}H_t}{l_t}$  with  $\frac{q_{t+1}H_t}{l_t} = \frac{1}{\kappa_t}$ . So in a neighborhood of the steady state the following condition holds  $\frac{1}{\kappa} > 1$ .

Following the 'collateral' channel of housing, our work aims at disentangling the important role of housing wealth in the households' decisions of consumption over the life-cycle. The above collateral constraint (11) implies that in each period borrower households cannot borrow from banks more than a fraction,  $\kappa_t$ ,<sup>10</sup> of the expected value of today's stock of housing which in real terms is equal to  $q_{t+1}\pi_{t+1}H_t$ . The stochastic term  $\kappa_t$  can be then interpreted as a shock to the loan-to-value ratio (i.e., the real value of collateralizing housing assets) and represents an indirect measure of the flexibility of the credit market.

This approach is a variant of the 'financial accelerator' model developed by Bernanke *et al.* (1999) where borrowing is procyclical with respect to the underlying business cycles which affect asset prices and therefore the value of the collateral. The collateral channel can work either by relaxing a liquidity constraint directly, by rising the loan to value ratio, or by providing equity that can be extracted at some point in the future, affecting individuals' consumption decisions. Among other things, the collateral channel can also amplify the effects of monetary policy in the economy (see Goodfriend and McCallum, 2007; Chadha *et al.*, 2013; Aoki *et al.*, 2004). As house prices affect the collateral value of houses, then real house price fluctuations have a considerable role in determining the access to credit lines (11).

By incorporating the binding constraint (10) into the budget constraint (9) we get:

$$q_t(H_t - H_{t-1}) + C_{t+1}^b \pi_{t+1} + \frac{R_{t-1}^L l_{t-1}}{\pi_t} = l_t + w_{t+1} N_{t+1} \pi_{t+1} - \tau_h q_t H_t \quad (12)$$

Equation (12) gives the level of consumption for borrowers and we use it to substitute for  $C_t$  into the utility function. Therefore, the maximization problem reads:

$$\max U = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \log \left[ \frac{1}{\pi_t} \left( l_{t-1} - q_{t-1}(H_{t-1} - H_{t-2}) - \tau_h q_{t-1} H_{t-1} - \frac{R_{t-2}^L l_{t-2}}{\pi_{t-1}} \right) + w_t N_t \right] + \right. \\ \left. + \chi \log H_t - \frac{(N_t)^{1+\varsigma}}{1+\varsigma} \right\} \quad (13)$$

By differentiating (13) subject to (11) with respect to  $l_t$ ,  $H_t$  and  $N_t$  the efficiency conditions for leveraged consumer are:

<sup>10</sup>We assume that the LTV shock evolves as follows:

$$\log \left( \frac{\kappa_t}{\kappa} \right) = \phi_\kappa \log \left( \frac{\kappa_{t-1}}{\kappa} \right) + u_{\kappa,t}$$

where  $\kappa$  is the calibrated steady state,  $\phi_\kappa$  is the persistence of the LTV innovation, and the error term is i.i.d., with mean zero and variance  $\sigma_\kappa$ .

*Euler Condition:*

$$\frac{\check{\beta}}{C_{t+1}^b \pi_{t+1}} - \frac{\check{\beta}^2 R_t^L}{C_{t+2}^b \pi_{t+2} \pi_{t+1}} = \nu_t \quad (14)$$

where  $\nu_t$  is the Lagrange multiplier associated to the borrowing constraint (11). Equations (14) and (6) imply that the borrowing constraint is binding in a neighborhood of the steady state, that is  $\nu > 0$  if and only if  $(1 - \check{\beta}R^L) > 0$  or equivalently if  $\beta R^D > \check{\beta}R^L$  which holds in our calibration exercise.

A binding collateral constraint, implying  $\nu_t > 0$ , has two main effects on household's decisions: (i) it prevents a consumption smoothing behavior (14); (ii) increases the marginal value of housing as it is also used as collateral (see (16)).

*Residential Good Demand:*

$$-\frac{\check{\beta}q_t}{C_{t+1}^b \pi_{t+1}} + \frac{\check{\beta}^2 q_{t+1}}{C_{t+2}^b \pi_{t+2}} - \frac{\check{\beta}\tau_h q_t}{C_{t+1}^b \pi_{t+1}} + \nu_t \kappa_t q_{t+1} \pi_{t+1} + \frac{\chi}{H_t} = 0 \quad (15)$$

Given (14) the above relationship can be also rewritten as follows:

$$\frac{\chi}{\check{\beta}H_t} = \frac{q_t}{C_{t+1}^b \pi_{t+1}} (1 + \tau_h) - q_{t+1} J(\bullet) \quad (16)$$

where  $J(\bullet) = \kappa_t \left( \frac{1}{C_{t+1}^b} - \frac{\check{\beta}R_t^L}{C_{t+2}^b \pi_{t+2}} \right) + \frac{\check{\beta}}{C_{t+2}^b \pi_{t+2}}$ .

*Labor Supply:*

$$N_t^s C_t^b = w_t \quad (17)$$

The usual Euler condition (14) states that the utility foregone in sacrificing a unit of current consumption is equal to the expected marginal benefit of future additional consumption appropriately discounted. But because of the cash-in-advance constraint, households who wish to consume more of nondurable goods face the opportunity cost given by foregoing returns on interest-bearing assets - as they must hold higher positive money balances which do not pay a return. In addition, the collateral constraint implies that because the borrowing capacity, and therefore the availability of loans, is strictly tied to the real value of housing holdings we are also expecting a higher demand of housing.

Equation (16) can be interpreted as a modified intertemporal Euler condition for residential goods. It states that the purchase of durable goods (housing) is partly an investment. In fact, (16) shows that the path of housing consumption is optimal when the marginal cost of acquiring one unit of housing which comprises a housing tax,  $\tau_h$ , (the first term on the RHS) is equal to its marginal utility (the last term on the RHS). The latter depends on (i) the direct utility gain of each additional unit of real estate; (ii) on the value of housing used as collateral to borrow funds from the credit market; (iii) from the expected utility coming from next period net resale value of each unit of housing purchased in the previous period. Consumption for borrower households is determined by their flow of funds (12).

In (16) the last term  $q_{t+1}J(\bullet)$  on the right-hand side is linked to the shadow value of the collateral constraint (11) which depends on several model variables. The first is the loan-to-value ratio,  $\kappa_t$ , which is a measure of the flexibility of the credit market. The second variable is represented by the real expected house prices,  $q_{t+1}$ , which directly affects the ability of households to get loans by relaxing their collateral constraint. Therefore, when house prices rise, especially in case of bubbles and overconfidence in future house prices, households can borrow and spend more. This implies that with a cheaper and easier access to home equity lines of credit, a rise in house prices will allow for

additional borrowing to finance consumption. Another variable is the interest rate on loans,  $R_t^L$ , which is negatively related to housing demand, since the amount of debt to be repaid is increasing with the interest rate charged by banking intermediaries on the new borrowing against housing collateral.

In the compound term  $J(\bullet)$  the difference between the marginal utility of consumption of two consecutive periods can be interpreted as a modified version of a standard Euler equation for consumption. It states that there is the possibility of expanding consumption by means of purchasing a unit of housing and increasing borrowing via a relaxation of the collateral constraint. As recalled above, these results occur because the borrowing constraints affect both intertemporal and within-period households' choices of lifetime consumption. And the housing demand of doubly-constrained households may increase over time as the shadow price of lifting the collateral (borrowing) constraint exceeds the marginal utility of consumption; therefore  $J(\bullet)$  measures the collateral constraint effects.

Given that housing supply is assumed to be fixed and equal to 1, we can derive the real house price from (16):

$$q_t = \frac{C_{t+1}^b \pi_{t+1}}{(1 + \tau_h)} \left( \frac{\chi}{\beta} + q_{t+1} J(\bullet) \right) \quad (18)$$

and we can thus study directly how asset prices interact with the consumption plans of borrowers.

## 3.2 Banking Sector and Macroprudential Policy

Banks collect deposits from savers and make loans to borrowers under monopolistic competition; this market power allows each individual bank to set its own interest rates on loans and deposits to maximize profits. In this section we outline the optimal lending and deposit rates and point to three parameters, the loan default rate,  $\lambda$ , the fraction of seizable collateral,  $\delta$ , and the loan-to-value ratio,  $\hat{\kappa}_t$ , that might be set to influence bank policy as part of a macroprudential framework. In our analysis of this model we will set these parameters to either a lax or restrictive level in order to understand the implications for monetary and fiscal policy.

### 3.2.1 Bank Profit Maximization

The representative bank  $j$  seeks to maximize profits:

$$\max E_t \sum_{s=0}^{\infty} \Lambda_{t+s} \Pi_{j,t+s} \quad (19)$$

where  $\Pi_{j,t+s}$  denotes real profits and the nominal discount rate  $\Lambda_{t+s} = \beta^s \left( \frac{C_t}{C_{t+s}} \right)$  comes from the saver households' maximization problem. The coincidence of discount factors comes from the assumption that households (saver households) are the ultimate owner of banks and their profits.

Bank's profits,  $\Pi_j$ , expressed in real terms read:

$$\Pi_{j,t} = \int_0^{s(\bar{\kappa}_t)} \delta \kappa_t q_t H_t \phi(\kappa_t) d\kappa_t + \int_{s(\bar{\kappa}_t)}^{\infty} l_{j,t} R_{j,t}^L \phi(\kappa_t) d\kappa_t - R_{j,t}^D d_{j,t} + R_t^M r r_{j,t} \quad (20)$$

The representative bank maximizes the expected flow of profits subject to the ex-ante real budget constraint:

$$r r_{j,t} + l_{j,t} = d_{j,t} \quad (21)$$

where  $rr_{j,t}$  denotes high powered money (reserves) on which the Central Bank pays an interest rate equal to the policy rate,  $R_t^M$ , and since we assume a fractional reserves system then  $rr_{j,t} = rrd_{j,t}$  where  $rr$  is the reserve requirement coefficient.

In the profit function  $\phi(\kappa_t)$  is the probability density function of the idiosyncratic shock on the LTV or, equivalently, on house prices;  $\delta$  is the fraction of collateral  $\kappa_t q_t H_t$  seized by the bank in case of borrower's insolvency; therefore it can be interpreted as the collateral value net of monitoring cost faced by banks to assess and seize the collateral connected to the original loan, and this foreclosure cost is assumed to be constant (Bernanke *et al.*, 1999). Borrowers that default on their loan lose their housing holdings. Finally, let be  $\bar{\kappa}_t$  the threshold value of the idiosyncratic shock for which borrowers are still willing to repay the loan, then the shortfall on loan repayment  $s(\bar{\kappa}_t)$  reads as:

$$s(\bar{\kappa}_t) = l_{j,t} - \delta \kappa_t q_t H_t = 0 \quad (22)$$

Therefore, with high realizations of the idiosyncratic shock on house price  $\kappa_t \in [\bar{\kappa}_t, \infty]$  loans are repaid, while loans with low realizations of the shock  $\kappa_t \in [0, \bar{\kappa}_t)$  are defaulted on.

### 3.2.2 Optimal Loan Rate

Deposit and loan contracts bought by households are a composite basket of slightly differentiated products, loans and deposits, each supplied by a branch of a bank  $j$  with elasticities of substitution equal to  $\mu_L$  and  $\mu_D$  respectively.<sup>11</sup>

Given the assumption that banking intermediaries operate in a regime of monopolistic competition, each bank faces an upward sloping demand curve for deposits and a downward sloping demand for loans, as we will show below. This market power allows each individual bank to set its own interest rates on loans and deposits to maximize profits. As in Hüelsewig *et al.* (2006), we assume that the individual bank  $j$  that operates in an environment that is characterized by banker-customer relationships faces the following demand for lending from households:

$$l_{j,t} = \left( \frac{R_{j,t}^L}{R_t^L} \right)^{-\mu_L} l_t \quad (23)$$

where  $\mu_L > 1$  represents the interest rate elasticity of loan demand,  $R_{j,t}^L$  is the interest rate on the loan  $l_{j,t}$  provided by bank  $j$ , and  $l_t$  is the aggregate demand for loans. According to (23) we assume that banks provide differentiated loans as they act under monopolistic competition. Following Carletti *et al.* (2007), we interpret the parameter  $\mu_L$  as the household's willingness to modify the customer relationship with the bank in the event of a change in loan rates. The higher is  $\mu_L$  the weaker become the ties between the bank  $j$  and the customers, that is the market power measured by  $1/\mu_L$  decreases; and for values of  $\mu_L$  approaching infinity the loan market resembles perfect competition.

By replacing  $rr_{j,t}$  using the resource constraint (21) bank's profits can be rewritten as follows:

$$\Pi_{j,t} = \int_0^{s(\bar{\kappa}_t)} \delta \kappa_t q_t H_t \phi(\kappa_t) d\kappa_t + \int_{s(\bar{\kappa}_t)}^{\infty} l_{j,t} R_{j,t}^L \phi(\kappa_t) d\kappa_t - R_{j,t}^D d_{j,t} + R_t^M (d_{j,t} - l_{j,t}) \quad (24)$$

An equivalent formulation of bank's profits reads as:

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<sup>11</sup>Thus as in a standard Dixit-Stiglitz framework for goods markets, agents have to purchase loan (deposit) contracts by each banking intermediary in order to borrow (save) one unit of resources. This assumption allows to capture the existence of market power in the banking industry. In fact, leveraged households would allocate their borrowing among different banks so as to minimize the due total repayment. Saver households would allocate their savings in form of deposits among different banks so as to maximise the revenues.

$$\Pi_{j,t} = l_{j,t} \left[ \int_{s(\bar{\kappa}_t)}^{\infty} R_{j,t}^L \phi(\kappa_t) d\kappa_t - R_t^M \right] + \int_0^{s(\bar{\kappa}_t)} \delta \kappa_t q_t H_t \phi(\kappa_t) d\kappa_t - d_{j,t} (R_{j,t}^D - R_t^M) \quad (25)$$

By maximizing the expected flow of profits (25) subject to (23) we get the optimal loan rate:

$$R_{j,t}^L = \frac{\mu_L}{(\mu_L - 1)} \frac{1}{\int_{s(\bar{\kappa}_t)}^{\infty} \phi(\kappa_t) d\kappa_t} R_t^M \quad (26)$$

If we assume that the probability function for  $\kappa_t$  has an exponential distribution<sup>12</sup>  $\phi(\kappa_t) = \lambda e^{-\lambda \kappa_t}$  the cumulative function reads as  $\Phi(\kappa_t) = \int_0^{s(\kappa_t)} \lambda e^{-\lambda s(\kappa_t)} d\kappa_t = 1 - e^{-\lambda s(\kappa_t)}$  so the probability of repayment which is at the denominator of (26) is simply its complement  $1 - \Phi(\bullet) = \int_{s(\kappa_t)}^{\infty} \lambda e^{-\lambda s(\kappa_t)} d\kappa_t = e^{-\lambda s(\kappa_t)}$  where  $\lambda$  is the default rate.

We can write (26) in a more compact form as follows:

$$R_{j,t}^L = X_{\mu_L} e^{\lambda s(\kappa_t)} R_t^M \quad (27)$$

Note that the optimal loan rate,  $R_{j,t}^L$ , is given by a constant mark-up  $X_{\mu_L} = \frac{\mu_L}{(\mu_L - 1)}$  over the policy rate  $R_t^M$  plus a risk premium  $e^{\lambda s(\kappa_t)}$  where  $\lambda$  is the default rate.

After imposing a symmetric equilibrium and log-linearizing, the optimal loan rate (27) reads:

$$\hat{R}_t^L = \hat{R}_t^M + \frac{\lambda}{1 - \delta} \left[ \hat{l}_t - \delta (\hat{\kappa}_t + \hat{q}_t + \hat{H}_t) \right] \quad (28)$$

According to (28) a fall in leveraging, that is a decrease in the level of households' liability to asset ratio, leads to an increase in the probability of repayment thus reducing the lending rate; and such a fall depends on the coefficient  $\lambda$  which can be interpreted as the default rate that is  $\lambda = \frac{\phi(\cdot)}{\Phi(\cdot)} = \frac{\lambda e^{-\lambda}}{e^{-\lambda}}$ . We can note immediately that this default rate,  $\lambda$ , the fraction of seizable collateral,  $\delta$ , and the loan-to-value ratio,  $\hat{\kappa}_t$ , will each impact on the elasticity of the loan rate to the state of the economy.

The result in (28) introduces a 'financial accelerator' in monetary policy as lending will expand when the collateral value increases. In nominal terms, an increase in the house price as well as an increase in the fraction of the residential good that can be used as a collateral, raises the value of households' collateralized net worth relative to their stock of outstanding loans. In other words, in real terms the value of their outstanding loans falls relative to that of their collateral following an increase in house prices. The implication is that banks are willing to accept a lower risk premium, thus reducing the lending rate. The collateralized wealth could also act as a strict quantity constraint on bank borrowing, as for instance in the model of Kiyotaki and Moore (1997) and its variants (see, for instance, Krishnamurthy, 2003) where shocks to credit-constrained firms would then be amplified through changes in collateral values and transmitted to output.

### 3.2.3 Optimal Deposit Rate

In a similar way followed by Hüelsewig *et al.* (2006), we also assume that the bank  $j$  faces the following demand for deposits:

$$d_{j,t} = \left( \frac{R_{j,t}^D}{R_t^D} \right)^{\mu_D} d_t \quad (29)$$

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<sup>12</sup>So higher expected housing prices are reducing the failure rate of banks.

where  $d_{j,t}$  is the demand for bank  $j$  deposits,  $d_t$  is the economy-wide demand for deposits,  $R_t^D$  is the average deposit interest rate prevailing in the market, taken as given by the single bank when solving the problem and  $\mu_D$  is elasticity of substitution among deposit varieties. Banks exploit their market power to lower their marginal cost (deposit interest rates) in order to increase profits and  $\mu_D$  is a measure of the existing competition in the banking sector; the degree of competition in the banking sector is measured by the inverse of  $\mu_D$ .

Therefore, by maximizing the flow of profits (25) with respect  $R^D$  subject to (29) we get the optimal deposit rate:

$$R_{j,t}^D = X_{\mu_D} R_t^M \quad (30)$$

where  $X_{\mu_D} = \frac{(1+\mu_D)}{\mu_D}$ . Therefore, with fully flexible deposit rates, the cost of deposits depends on the elasticity of substitution among deposit varieties,  $\mu_D$ , and the optimal deposit rate  $R_t^D$  would be determined as a mark-down,  $\frac{1}{X_{\mu_D}}$ , over the policy rate,  $R_t^M$ . Conversely, the policy rate is simply given by a constant mark-up over the deposit rate, that is  $R_t^M = (X_{\mu_D})^{-1} R_{j,t}^D$ . This implies that the bank views households' deposits and reserves as perfect substitutes at the margin so the spread between the policy rate and the cost of deposits only depends on the elasticity of substitution among deposit varieties. The latter condition implies that money market credits and deposits are assumed to be perfect substitutes so that the deposit rate is then assumed to equal the policy rate (at least in log-linear form) and are therefore exogenous for the bank.

### 3.3 Real Sector

#### 3.3.1 Final Good Producers

In a perfectly competitive market, each firm producing final good uses a continuum of intermediate goods indexed by  $z \in [0, 1]$  according to the following CES technology:

$$Y_t = \left( \int_0^1 Y_{z,t}^{\frac{\psi-1}{\psi}} dz \right)^{\frac{\psi}{\psi-1}} \quad (31)$$

where  $Y_{z,t}$  is the demand by the final good producer of the intermediated good  $z$ , and  $\psi > 1$  is the elasticity of substitution between differentiated varieties of intermediate goods.

Profit maximization implies a downward sloping demand function for the typical intermediate good  $z$ :

$$Y_{z,t}^d = \left( \frac{P_{z,t}}{P_t} \right)^{-\psi} Y_t \quad (32)$$

where  $P_{z,t}$  denotes the price of the intermediate good,  $Y_{z,t}$ , and  $P_t$  is the price index of final consumption goods which is equal to:

$$P_t = \left( \int_0^1 P_{z,t}^{1-\psi} dz \right)^{\frac{1}{1-\psi}} \quad (33)$$

where the price index (33) is consistent with the maximization problem<sup>13</sup> of the final good producer earning zero profits and subject to the production function (31).

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<sup>13</sup>Hence the problem of the final good producer is:  $\max P_{i,t} Y_{i,t} - \int_0^1 P_{i,t}(z) Y_{i,t}(z)$  subject to the demand function (32).

### 3.3.2 Intermediate Goods Producers

There is a continuum of firms producing intermediate goods. Each firm has a monopolistic power in the production of its own good variety and therefore has a leverage in setting prices. The representative monopolistic firm,  $z$ , will choose a sequence of prices and labour inputs  $\{N_{z,t}, P_{z,t}\}$  to maximize expected discounted profits:

$$\max E_t \sum_{s=0}^{\infty} \Lambda_{t+s} \Theta_{z,t+s} \quad (34)$$

where  $\Lambda_{t,s} = \beta^s \left( \frac{C_{t+s}}{C_t} \right)^{-1} \left( \frac{P_t}{P_{t+s}} \right)$  is the relevant creditor household discount factor and  $\Theta$  denotes profits.

A Cobb-Douglas-type production function is adopted with decreasing return on labour which is assumed to be the only input:

$$Y_{z,t} = A_t (N_{z,t})^{1-\gamma} \quad (35)$$

where  $0 < \gamma < 1$  is a measure of decreasing returns,  $N_{z,t}$  denote firm's  $z$  demand of labor, and  $A_t$  is the productivity shock that is assumed to be common to all firms and evolves exogenously over time.<sup>14</sup>

The intratemporal demand function for good  $z$  is given by:

$$Y_{z,t}^d = \left( \frac{P_{z,t}}{P_t} \right)^{-\psi} Y_t \quad (36)$$

where  $\psi$  is the price elasticity of demand for individual goods faced by each monopolist and  $P$  is the general price level. Therefore,  $\frac{P_{z,t}}{P_t}$  is the relative price of good variety  $z$ .

Given the consumer' demand schedule (36) and taking wages as given, the cost minimization implies the following demand for labor:

$$\frac{W_t}{P_t} = \frac{MPL_t}{X_{\psi,t}} = \frac{1}{X_{\psi,t}} \frac{(1-\gamma) Y_t}{N_t} \quad (37)$$

where  $X_{\psi,t}$  is the markup (or the inverse of the real marginal cost,  $MC_t = 1/X_{\psi,t}$ ) which in steady state is  $X_{\psi} \equiv \frac{\psi}{\psi-1}$  and  $MPL_t \equiv (1-\gamma) A_t N_t^{-\gamma}$  is the marginal product of labor.

**Price Setting.** As it is standard in the New Keynesian literature, we assume Calvo staggered nominal price adjustment. We assume that intermediate firms set nominal prices in a staggered fashion, according to a stochastic time dependent rule. The sale price can be changed in every period only with probability  $1 - \theta$ , independently of the time elapsed since the last adjustment. Thus, in each period a measure  $1 - \theta$  of producers reset their prices, while a fraction  $\theta \in [0; 1]$  of firms keep their prices unchanged with an implied average price duration of  $1/(1 - \theta)$ . A firm  $z$  resetting its price in period  $t$  will choose the price  $P_t^*$  to maximize

$$\max_{P_t^*} \sum_{s=0}^{\infty} \theta^s E_t \{ \Lambda_{t,s} Y_{t,t+s} (P_t^* - P_{t+s} MC_{t,t+s}) \} \quad (38)$$

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<sup>14</sup>We assume that the productivity shock evolves exogenously as follows:

$$\log A_t = \phi_a \log A_{t-1} + u_{a,t}$$

where  $\phi_a$  is the persistence of the productivity innovation, and the error term is i.i.d., with mean zero and variance  $\sigma_a$ .



for  $s = 1, 2, 3, \dots$  and subject to the sequence of demand constraints for its product variety  $Y_{t,t+s} = \left(\frac{P_{z,t}^*}{P_{t+s}}\right)^{-\psi} Y_{t+s}$  where  $P_t^*$  is the price set in period  $t$  by all firms re-optimizing their price in that period,  $Y_{t,t+s}$  and  $MC_{t,t+s}$  denote respectively output and real marginal cost in period  $t+s$  for a firm that has reset its price in period  $t$ .

The first order condition associated with the above problem is given by:<sup>15</sup>

$$\sum_{s=0}^{\infty} \theta^s E_t \left[ \Lambda_{t,s} Y_{t,t+s} \left( \frac{P_t^*}{P_{t-1}} - X_\psi MC_{t,t+s} \pi_{t-1,t+s} \right) \right] = 0 \quad (39)$$

where  $MC = (1/X_\psi)$  and  $\pi_{t-1,t+s} \equiv P_{t+s}/P_{t-1}$  and in the zero inflation steady state  $\pi_{t,t+s} = 1$ . This relationship states that the optimal price  $P_t^*$  equates the expected discounted marginal revenues to the expected discounted marginal costs as required in this setup where monopolistic competition in the intermediate goods market is assumed.<sup>16</sup>

Finally, the equation describing the dynamics for the aggregate price level is given by (see Galí, 2008) :

$$P_t = \left[ \theta P_{t-1}^{1-\psi} + (1-\theta) (P_t^*)^{(1-\psi)} \right]^{1/(1-\psi)} \quad (40)$$

**Inflation Dynamics.** Combining (39) and (40) and log-linearizing as shown in the Technical Appendix we get a forward-looking Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \eta \widehat{MC}_t + \xi_{mc,t} \quad (41)$$

where  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$  which is strictly decreasing in the price stickiness parameter  $\theta$ , in the demand elasticity  $\psi$  and in the measure of decreasing returns on labor  $1-\gamma$ ; and  $\xi_{mc,t}$  denotes a cost-push shock.<sup>17</sup> Relationship (41) states that inflation depends positively both on expected inflation and on the real marginal costs which according to (37) and ignoring the constant terms read as:

$$\widehat{MC}_t = \hat{w}_t - A_t \quad (42)$$

where  $\hat{w}_t$  is the real wage and  $A_t = \hat{Y}_t - \hat{N}_t$  is the technology (productivity) variable.

### 3.4 The Fiscal Rule

The government's budget constraint expressed in real terms is

$$B_t = \frac{R_{t-1}^B B_{t-1}}{\pi_t} + (G_t - T_t) - \Delta m_t \quad (43)$$

<sup>15</sup>All firms choose the same price because they face an identical problem so we can drop  $z$  in the notation.

<sup>16</sup>Given that  $MC_{t+s} = (X_{\psi,t+s})^{-1}$  then we can rewrite the relationship (39) as follows:

$$\sum_{s=0}^{\infty} \theta^s E_t \left\{ \Lambda_{t,s} Y_{z,t+s} \left( \frac{P_t^*}{P_{t+s}} - \frac{X_\psi}{X_{\psi,t+s}} \right) \right\} = 0$$

<sup>17</sup>We assume that the mark-up shock evolves exogenously as follows:

$$\log \xi_{mc,t} = \phi_{\xi_{mc}} \log \xi_{mc,t-1} + u_{\xi_{mc},t}$$

where  $\phi_{\xi_{mc}}$  is the persistence of the shock, and the error term is i.i.d., with mean zero and variance  $\sigma_{\xi_{mc}}$ .

where  $G_t$  is real net government spending (i.e., net of lump-sum taxes),  $B_t$  is the real value of one-period government liabilities issued at the end of period  $t - 1$  and with maturity in  $t$ ,  $\frac{R_{t-1}^B B_{t-1}}{\pi_t}$  denotes real debt service on existing government debt,  $T_t \equiv \tau_h q_t H_t + Tax_t$  are total tax revenues from (i) real property taxes,  $\tau_h q_t H_t$ , with  $q_t H_t$  denoting the real value of existing housing stock held by borrower households where  $\tau_h$  is the tax rate; and (ii) from savers  $Tax_t = \tau_y Y_t$ . Finally, the term  $\Delta m_t \equiv \frac{M_t - M_{t-1}}{P_t} \equiv m_t - \frac{m_{t-1}}{\pi_t}$  is the seigniorage revenue in real terms.

The log-linearization of (43) in per-capita terms reads as:

$$\frac{B}{Y} \hat{B}_t = \frac{B}{Y} R^B \left( \hat{R}_{t-1}^B + \hat{B}_{t-1} - \hat{\pi}_t \right) + \frac{G}{Y} \hat{G}_t - \frac{T}{Y} \hat{T}_t - \frac{m}{Y} (\Delta \hat{m}_t + \hat{\pi}_t) \quad (44)$$

where

$$\frac{T}{Y} \hat{T}_t = \tau_h \frac{qH}{Y} \left( \hat{q}_t + \hat{H}_t \right) + \tau_y \hat{Y}_t \quad (45)$$

We also assume a feedback rule on government spending:

$$\hat{G}_t = -f_y \hat{Y}_t + f_T \hat{T}_t + f_m \Delta \hat{m}_t + \xi_{g,t} \quad (46)$$

where  $f_T$  is a government spending feedback parameter<sup>18</sup> from tax revenues,  $f_m = \frac{R^M}{1+R^M}$  and  $f_y$  are respectively the fiscal policy parameters on the seigniorage revenue and output respectively. We are considering some feedback rules for fiscal policy which apply to government spending over the cycle  $G_t = F(f_T, f_y)$ , which act on tax receipts both from housing sales and consumption through the cash-in-advance constraints. And the term  $\xi_{g,t}$  is a fiscal policy shock.<sup>19</sup>

### 3.5 Monetary Policy Rule

The model is closed by the Central Bank's reaction function. The Central bank is assumed to set the nominal interest rate according to a simple linear interest rate rule:

$$\hat{R}_t^M = \rho \hat{R}_{t-1}^M + (1 - \rho) \left( \alpha_\pi \hat{\pi}_t + \alpha_y \hat{Y}_t \right) + \xi_{m,t} \quad (47)$$

where  $R^M$  is the (net) policy rate and  $\rho$  captures the degree of interest rate smoothing, while  $\alpha_\pi$  and  $\alpha_y$  are the central bank's reaction coefficients with respect to expected consumer price index (CPI) and the output gap while  $\xi_{m,t}$  denotes the monetary policy shock.<sup>20</sup>

We also consider an alternative specification of the Taylor rule with a monetary policy that also responds to house prices.<sup>21</sup> The augmented Taylor rule specification is:

<sup>18</sup>The parameters  $f_T$ ,  $f_m$ ,  $f_y$  are set exogenously in the impulse response analysis. We then choose optimally these parameter values when performing the welfare analysis.

<sup>19</sup>We assume that the government spending shock evolves exogenously as follows:

$$\log \xi_{g,t} = \phi_{\xi_g} \log \xi_{g,t-1} + u_{\xi_g,t}$$

where  $\phi_{\xi_g}$  is the persistence of the shock, and the error term is i.i.d., with mean zero and variance  $\sigma_{\xi_g}$ .

<sup>20</sup>The monetary policy shock evolves as follows:

$$\log \xi_{m,t} = \phi_{\xi_m} \log \xi_{m,t-1} + u_{\xi_m,t}$$

where  $\phi_{\xi_m}$  is the persistence of the monetary policy innovation, and the error term is i.i.d., with mean zero and variance  $\sigma_{\xi_m}$ .

<sup>21</sup>Darracq-Pariès and Notarpietro (2008) use a similar specification for the augmented Taylor rule where both consumption goods and house prices appear.

$$\hat{R}_t^M = \rho \hat{R}_{t-1}^M + (1 - \rho) \left( \alpha_\pi \hat{\pi}_t + \alpha_y \hat{Y}_t \right) + \alpha_q \hat{q}_t + \xi_{m,t} \quad (48)$$

where  $\alpha_q$  is the asset price target coefficient.

### 3.6 Welfare Analysis

The aggregate welfare function depends on households' preferences over consumption, housing, labor, bonds<sup>22</sup> and public spending; the argument  $G_t$  represents government spending and is determined by the government in each period so that the representative consumer takes it as exogenously given. Thus social welfare is given by:<sup>23</sup>

$$U_t = \frac{C}{Y} (\log C_t + \chi_B \log B_t) + \frac{C^b}{Y} \left( \log C_t^b + \chi \log H_t - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) + \frac{G}{Y} (\log G_t) \quad (49)$$

where we attach weight coefficients equal to the steady state value of consumption over output,  $\frac{C}{Y}$ , and government spending over output,  $\frac{C^b}{Y}$ .

A second-order approximation of the welfare function yields:<sup>24</sup>

$$(U_t - U) \simeq \frac{1}{2} \left[ \left( 1 - \frac{1+\varsigma}{1-\gamma} \right) \hat{Y}_t^2 - \left( \frac{C}{Y} \hat{C}_t^2 + \frac{C^b}{Y} (\hat{C}_t^b)^2 + \frac{G}{Y} \hat{G}_t^2 \right) - \frac{C}{Y} \chi_B \hat{B}_t^2 - \frac{C^b}{Y} \chi \hat{H}_t^2 - \frac{\psi}{\eta} \hat{\pi}^2 \right] \\ + t.i.p. + \mathcal{O}(\|a\|^3) \quad (50)$$

where the hatted variable is the log-deviation from steady state,  $\mathcal{O}(\|a\|^3)$  collects all the terms of third order or higher, and “*t.i.p.*” denotes terms independent of policy.

As shown in the Appendix the above welfare function can be also expressed in terms of aggregate welfare losses using the following purely quadratic loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t (U_t - U) = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + \mathcal{O}(\|a\|^3) \quad (51)$$

with  $L_t = \varphi_Y \sigma_Y^2 + \varphi_C \sigma_C^2 + \varphi_{C^b} \sigma_{C^b}^2 + \varphi_G \sigma_G^2 + \varphi_B \sigma_B^2 + \varphi_H \sigma_H^2 + \varphi_\pi \sigma_\pi^2$

where the weight coefficients are given by  $\varphi_Y \equiv \frac{\varsigma+\gamma}{1-\gamma}$ ,  $\varphi_C \equiv \frac{C}{Y}$ ,  $\varphi_{C^b} \equiv \frac{C^b}{Y}$ ,  $\varphi_B \equiv \frac{C}{Y} \chi_B$ ,  $\varphi_H \equiv \frac{C^b}{Y} \chi$ ,

$\varphi_G \equiv \frac{G}{Y}$  and  $\varphi_\pi \equiv \frac{\psi}{\eta}$  with  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$  and the welfare loss is given by a linear combination of the variances of output, inflation, consumption levels, housing and government bond holdings.

## 4 Model Solution

### 4.1 Steady States

Table 2 reports the parameter values used to calibrate this model and Table 3 reports the steady-state values. The saver households' intertemporal discount factor is set at the conventional (quarterly) value

<sup>22</sup>We assume that households' utility function is separable in consumption, labor, housing and bond holdings, which implies that all the cross-derivatives are zero.

<sup>23</sup>Here we introduce a welfare function which is fairly standard in the literature (Woodford, 2003). The main difference in our framework stems from the introduction of government consumption.

<sup>24</sup>Derivation of the second-order approximation to the welfare function is reported in the Appendix.

of  $\beta = 0.99$ . This implies a steady state real interest rate on (time) deposits of 4% in annual terms, as  $(1/\beta)^4 = 1.04$ . Lawrance (1991) estimates intertemporal discount factors for poor households between 0.95 and 0.98 so we set the borrower intertemporal discount factor at  $\beta = 0.97$ . Our calibration implies that borrowing constrained households using housing as collateral derive a higher per unit utility from housing services, so we set the housing parameter in the utility function  $\chi = 0.4$ . This value implies a steady-state ratio of total housing wealth to annualized GDP of 1.62. The corresponding ratio in U.S. data has ranged between 1.2 and 2.3 over the period 1952 to 2008, hence our steady state ratio is approximately an average of these values.<sup>25</sup>

The elasticity of substitution between varieties of nondurables is set to 6 which yields a steady-state mark-up over the nominal costs of 20% in the real sector. We set the stickiness parameter for consumer prices,  $\theta$ , equal to the standard value 0.75 which implies a mean duration of price contracts of one year. We also assume that house prices are flexible.

We calibrate the loan-to-value ratio (LTV),  $\kappa$ , to 0.6 which is a value close to the euro area average. In fact, existing empirical studies show the presence of a substantial degree of heterogeneity within the euro area in terms of mortgage markets flexibility, with some countries as the Netherlands being close to the US where the value of the LTV ratio exceeds 0.9 for the period 1973-2006, and others, like for example Germany, France and Italy, displaying a much smaller degree of flexibility (IMF, 2008). The proposed calibration of  $\kappa$  thus provides an approximated average across European countries (Darracq-Pariès *et al.*, 2008).

The inverse of the elasticity of labor supply,  $\varsigma$ , is set equal to 0.3 as in Prescott (2005); and we set the parameter associated with the elasticity of output with respect to labor at 0.6 implying that  $\gamma$  is 0.4. The loan demand elasticity,  $\mu_L$ , is assumed to be equal to 3.5 implying a mark-up of the banking industry over the nominal costs of 40% (see Huelsewig *et al.*, 2006). Finally, the central bank's mark-up over the nominal deposit rate is assumed to be equal to 10%, this implies a deposit demand elasticity,  $\mu_D$ , equal to 10; this mark-up represents the refinancing costs for the banking sector, thus by varying this mark-up the central bank might help stabilizing the system. Finally, the policy parameters are chosen as follows: we set the interest rate smoothing parameter,  $\rho$ , to 0.85, the size of the response to inflation,  $\alpha_\pi$ , to 1.5, the response to output,  $\alpha_y$ , to 0.5 and to relative house price,  $\alpha_q$ , to 0.2 (see Darracq-Pariès and Notarpietro, 2008).

The steady-state<sup>26</sup> values of the endogenous variables as well as the value of the complex parameters are reported in Table 3. Starting with the interest rates, note that the steady state is computed at zero inflation so that we can interpret all the interest rates as real interest rates. Calibrations give a policy rate,  $R^M$ , at 4.4%, a saving deposit rate,  $R^D$ , at 4% and a loan rate,  $R^L$ , at 7.6% and a bond rate,<sup>27</sup>  $R^B$ , at 1.2% per annum. For borrowing constrained households the consumption to output ratio,  $C^b/Y$ , is 0.46 (against a value for  $C/Y$  of 0.38 for savers); also their level of housing wealth,  $qH^b/Y$ , is 1.62 times total output, while their level of indebtedness over consumption,  $l/Y$ , is 2.11. Finally, we set the steady state expected value of repayment of the leveraged households at 0.8 implying a value of  $\lambda$ , that is the default rate, of 1.25.

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<sup>25</sup>Data in the National Income and Product Accounts (NIPA) for U.S. are provided by the Bureau of Economic Analysis.

<sup>26</sup>For details on the derivation of the steady-states see the Technical Appendix, available on request.

<sup>27</sup>We assume that government bonds are used by saver households in managing their liquidity. This implies that there is a spread between the consumption capital asset pricing model rate (i.e. the Euler for consumption) and the government bond rate, since government bonds command a liquidity premium.

## 4.2 Solution

For the impulse response analysis and simulation exercise we consider the real and financial shocks described in Table 3, which reports the volatility and persistence parameters chosen for the calibration and simulation exercise. These are standard parameters in the literature. The model is then solved using the solution methods of King and Watson (1998) to derive the impulse responses of the endogenous variables to different shocks, to obtain asymptotic variance and covariances of the variables and to simulate the data. This system of linear difference equations can be expressed as a singular dynamic system of the following form:

$$\check{\mathbf{A}}E_t\mathbf{y}_{t+1} = \check{\mathbf{B}}\mathbf{y}_t + \check{\mathbf{C}}\boldsymbol{\varepsilon}_t \quad \forall t \geq 0, \quad (52)$$

where  $\mathbf{y}_t$  is the vector of endogenous variables comprising both predetermined and non-predetermined variables including policy rules for the nominal interest rate,  $\boldsymbol{\varepsilon}_t$  is a vector of exogenous forcing variables, and  $\check{\mathbf{A}}$ ,  $\check{\mathbf{B}}$  and  $\check{\mathbf{C}}$  are matrices of fixed, time-invariant, coefficients.  $E_t$  is the expectations operator conditional on information available at time  $t$ . King and Watson (1998) demonstrate that if a solution exists and is unique then we may write any such solution in state-space form as follows,

$$\begin{aligned} \mathbf{y}_t &= \mathbf{\Pi}\mathbf{s}_t \\ \mathbf{s}_t &= \mathbf{M}\mathbf{s}_{t-1} + \mathbf{G}\mathbf{e}_t, \end{aligned} \quad (53)$$

where the  $\mathbf{s}_t$  matrix includes the state variables of the model (predetermined variables along with exogenous variables),  $\mathbf{e}_t$  is a vector of shocks to the state variables and  $\mathbf{\Pi}$ ,  $\mathbf{M}$  and  $\mathbf{G}$  are coefficient matrices. There are six shocks in  $\mathbf{G}$  and the variance-covariance as well as the autocorrelation matrices associated with these shocks are described in Table 3. The impulse responses of this system are given by:

$$\begin{aligned} E_t\mathbf{y}_{t+k} - E_{t-1}\mathbf{y}_{t+k} &= \mathbf{\Pi}(E_t\mathbf{s}_{t+k} - E_{t-1}\mathbf{s}_{t+k}) \\ &= \mathbf{M}^k\mathbf{G}\mathbf{e}_t. \end{aligned} \quad (54)$$

And the variance of the states,  $\mathbf{V}_{ss}$ , is given by:

$$vec(\mathbf{V}_{ss}) = (\mathbf{I} - \mathbf{M} \otimes \mathbf{M})^{-1} vec(\mathbf{G}\mathbf{V}_{ee}\mathbf{G}^T). \quad (55)$$

For the analysis of the optimal rule in the next section we minimize the variance of the arguments in the representative household's loss function,  $L$ :

$$L = \varphi_Y\sigma_Y^2 + \varphi_C\sigma_C^2 + \varphi_{Cb}\sigma_{Cb}^2 + \varphi_G\sigma_G^2 + \varphi_B\sigma_B^2 + \varphi_H\sigma_H^2 + \varphi_\pi\sigma_\pi^2 \quad (56)$$

In the appendix, this expression is shown to be an approximation of welfare loss and it is minimized subject to policy choice on monetary and/or fiscal policy such that  $[f_y, f_T, f_m, \alpha_q] = \arg \min L$ .

## 5 Dynamic Model Results

In this section we report the results of specific exercises designed to understand this model's properties.<sup>28</sup> First, we examine the moments of the simulated model in Table 4. Then in Figures 2 to 7, we show the impulse responses of the benchmark calibration model to a set of forcing variables used to drive this model: a shock in the loan to value ratio offered by commercial banks, a goods productivity shock, a canonical monetary policy shock to the interest rate rule, a cost-push shock, an unanticipated shock to the house price and a government spending shock. The impulse response are drawn for the benchmark case and also the cases of restrictive and lax regimes, which are set when the probability of default is 0% ( $\lambda = 1$ ), the steady-state LTV is 90% ( $\kappa = 0.9$ ) and when up to 90% of capital can be seized ( $\delta = 0.9$ ) in the former case and when the probability of default is 20% ( $\lambda = 5$ ), steady state LTV is 0.4 ( $\kappa = 0.4$ ) and seizable capital is only 0.2 ( $\delta = 0.2$ ). Finally, we undertake a welfare analysis of the model under restrictive and lax regimes.

### 5.1 Simulated model moments

The model can be solved for its moments to understand the basic relationships listed in the Appendix. Table 4 gives the relative standard deviation of key endogenous variables and their correlations with output for the benchmark simulation corresponding to the calibrated parameters in Tables 2-3. Both categories of consumption are volatile as well as aggregate consumption. Recall there is no investment *per se* in this model but trade in durable consumption of the housing stock drives up the standard deviation of consumption somewhat. Government consumption is much less volatile as consumption and acyclical. Lending is more volatile than aggregate consumption type and is strongly procyclical. The deposit rate is relatively sticky and less volatile than aggregate consumption and it is considerably less procyclical. But the EFP is both volatile and procyclical, reflecting the dominance of demand shocks in the loans market. House prices and wages are procyclical in this model. House prices have a correlation of 0.45 with output.

The central two columns of Table 4 show the same moments but for the case of an interest rate rule that also targets asset (house) prices. Note in this case the standard deviation of aggregate consumption does not change. But wages and the deposit rate each become somewhat destabilized as asset price targeting introduces some losses as compared to standard monetary policy stabilization, which does not target asset prices explicitly. The final two columns show the moments for the model with an optimal fiscal rule, with choices of the feedback from taxation receipts, output and money growth, set by parametric choice of minimization of the loss function. Broadly speaking and certainly relative to an interest rate rule targeting asset prices, there seems to be some overall welfare gain to the representative household from more active fiscal policy. We return to this point in detail below. But note that fiscal policy acts on the tax and spends from the receipts of house purchases, and these purchases are the counterpart of bank lending in this model - recall that loans are used exclusively to finance borrower household consumption. Whereas monetary policy acts on the creditor household first by impacting on the deposit rate and so when the policy maker targets asset prices, she is acting on the consumption plans of savers whose optimal plans are directly affected by interest rates. To that extent monetary policy targets for asset prices may be somewhat misdirected.

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<sup>28</sup>The list of log-linearized model equations is reported in the Appendix.

## 5.2 Impulse responses

Figures 2 to 7 plot the responses of some key variables in this model to the forcing variables in each of three cases. For the benchmark model, which is indicated with straight lines, and for both the restrictive regime with crosses and the lax regime as a dotted line. Figure 2 shows a 1% shock to the loan to value ratio, a so-called collateral shock. The higher is the loan to value ratio the higher is the marginal amount of new lending that can be accorded for a given value of the collateral (housing). The quantity of loans to borrower households then jumps up as housing demand rises driving up the spread between the lending rate and the policy rate. Borrowers' consumption increases as a result of increased leverage and drives a temporary boom. But in this calibration the lending boom is short lived and in later periods, borrowers are paying back their loans and gradually reducing the level of loans, which depresses output. The savers in the benchmark scenario reduce current consumption following the increase in the saving deposit rate. Note that under the restrictive regime the quantity of lending is considerably less and the lending rate considerably higher and house price actually fall as the higher LTV means that there is less incentive to use houses as collateral.

Figure 3 plots the response of this economy to a 1% shock in goods productivity. Wages and output rise and borrowers' consumption jumps up and remains persistent as loans are offered against a falling EFP following an increase in the policy rate. This boom in borrowers' consumption is financed by a reduction in savers' consumption, as deposit rates rise to attract funds. Figure 4 plots the responses to a 1% shock in the policy rate. This shock leads to a temporary recession in saver households who reduce consumption in response to higher deposit rates. Borrower, or leveraged, households are insulated temporarily as they benefit from the reduction in the cost of external finance, but in the following periods there is a longer recession for these borrowers as they repay back their loans. In response to a 1% cost-push shock rising wages (Figure 5) both consumption rise, to some degree, as policy and deposit rates rise as well as wages. Figure 6 shows that an unanticipated 1% shock to house prices brings about a delayed boom as deposit rates fall and lending stimulates consumption. Note that initially the EFP reduces as the increase in the value of households' collateralized net worth relative to the stock of outstanding loans drives down the loan rate since the risk premium asked by lenders decreases. The reduction in the interest rate spread is weak and short-termed and then it starts increasing as demand for loans outstrips supply. In fact, the increase in house prices relaxes the credit constraint of the leveraged households, allowing them to borrow and consume more.

Finally, when government spending increases, see Figure 7, it acts, as we would expect, like a demand shock raising output and inflation temporarily. The increase in the policy and deposit rates reduces savers' consumption whereas the increase in borrowers' consumption drives up the demand for additional loans and so the EFP rises.

Figure 8 shows the middle segment, as an illustration, from a simulation of 10,000 data points, discarding the first 500 observations, of the benchmark model. The simulated data are HP filtered ( $\lambda = 1600$ ). Borrower's consumption covaries positively with house prices and it is negatively correlated with saver's consumption. This replicates the empirical evidence from the BHPS data for UK where a much stronger house-price effect was found for credit constrained households as well as a negative correlation between consumption of borrowers and savers.

## 5.3 Fiscal, Monetary and Macro-prudential Policy

Using the welfare criterion, we can analyze, as an illustrative calculation, the implications of varying policy parameters for fiscal, monetary and macroprudential policy. Table 5 illustrates the impact on optimal monetary and fiscal rules from the perceived default rate of the commercial bank. Along

with an increase in the perceived default rate, there seems to be a case for strong countercyclical fiscal policy and also for more monetary policy feedback from asset prices. In this sense monetary and fiscal policy seem to be trying to offset more restrictive behaviour of banks when their external finance premium becomes more sensitive to the state of the economy by injecting more demand into the economy and acting to stabilize changes in house prices. We do not want to introduce a too normative conclusion here but this would seem to imply that the correct parametric response of stabilization policy is not independent of the lending behaviour of banks.

In Table 6 we investigate the welfare losses of households for different loan default rates but also from different loan to value ratios and different levels of seizable collateral. We also examine losses under a hypothetical lax and restrictive regime. What we seem to find is that losses are in general less when the seizable collateral, lending and perceived default rate correspond to more restrictive regimes. This is because the bank profit maximization might not match exactly the welfare of saver and borrower consumption paths - the loans offered through our collateral channel do not necessarily offer a perfect hedge for both types of households in the presence of a typical sequence of macroeconomic shocks.

## 6 Conclusions

In this paper we model two types of households, ones that are savers and ones that face both a liquidity and a borrowing constraint. The second type of households take on debt to finance their consumption. In this model banks have a specific role in pricing the risk of these leveraged households by setting the margin for lending rates as a spread over the deposit rates. We find that compared to a standard representative agent model, consumption displays considerably more volatility in this set-up, as access to credit pushes consumption up and the requirement for debt repayments pulls consumption back again with countercyclical lending spreads. A complementary cycle also emerges for savers. So a deeper and more persistent business cycle emerges endogenously from this model.

Households' welfare is shown not necessarily to be maximized under a standard interest rate rule. In fact, stabilizing inflation and output with a jointly determined monetary and fiscal policy responses seems preferable. Furthermore we analyze the aggregate welfare of households when some form of macroprudential policy also operates, which limits the lending, capital returns and perceived default rate of loans, and these produce lower losses for the representative household. Therefore, some augmentation of policy to include an additional role for macroprudential policy may improve welfare. Two tentative normative points emerge from this analysis. First, there may be more scope for countercyclical fiscal and macroprudential policy to stabilize the economy than we thought when models were unable to speak about intermediation. Secondly, when banks do not price risk in a substantive manner the policy frontier deteriorates and this would imply there is an ongoing need to calibrate macroprudential policies, particularly in a world where lending is driven by home ownership.



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## Appendix

### A List of Steady-State Relationships

Assuming that the steady state gross inflation rate  $\pi \equiv \frac{P_t}{P_{t-1}}$  is equal to 1, that is inflation is zero in the steady state and that housing supply which is fixed and equal to 1,  $\bar{H} = 1$ , then the steady state equilibrium will be described by:

$$R^D = \frac{1}{\beta} \quad (57)$$

$$R^M = X_{\mu_D} R^D \quad (58)$$

$$R^L = X_{\mu_L} \frac{1}{1 - \Phi} R^M \quad (59)$$

$$R^B = \frac{1}{\beta} \left( 1 - \chi_B \frac{C}{B} \right) \quad (60)$$

$$\frac{qH}{Y} = \frac{\frac{\chi}{\beta} \frac{C^b}{Y} - \left[ \frac{G}{Y} - \frac{B}{Y} (1 - R^B) - \frac{Tax}{Y} \right]}{(1 - \kappa - \Omega)} \quad (61)$$

$$\frac{C^b}{qH} = \frac{\check{\beta} Z}{\chi} \quad (62)$$

$$\frac{C^b}{Y} = 1 - \frac{C}{Y} - \frac{G}{Y} \quad (63)$$

$$\frac{l}{C^b} = \frac{\kappa \chi}{\check{\beta} Z} \quad (64)$$

$$\frac{m}{Y} = \frac{C}{Y} + \frac{C^b}{Y} + \frac{WN}{Y} \quad (65)$$

$$l = (1 - rr)d \quad (66)$$

$$\tau_h = \left[ \frac{G}{Y} - \frac{B}{Y} (1 - R^B) - \frac{Tax}{Y} \right] \left( \frac{qH}{Y} \right)^{-1} \quad (67)$$

$$\tau_m = \frac{R^M}{1 + R^M} \quad (68)$$

where  $X_{\mu_L} \equiv \frac{\mu_L}{(\mu_L - 1)}$  and  $X_{\mu_D} \equiv \frac{1 + \mu_D}{\mu_D}$  are respectively loan and deposit interest rate markup and the compound terms are  $\Omega \equiv \check{\beta} (1 - \kappa R^L)$ , and  $Z \equiv (1 + \tau_h - \kappa - \Omega)$ .

## B The Log-Linearized Model

The model can be reduced to the following log-linearized system in which all the hatted variables denote percent changes from the steady state, and those without subscript denote steady-state values:<sup>29</sup>

*Saver Household*

- Consumption Demand:

$$\hat{C}_{t+1} - \hat{C}_t = \left( \hat{R}_t^D - \hat{\pi}_{t+1} \right) \quad (69)$$

- Government Bonds Demand:

$$\hat{B}_t = \chi_B^{-1} \left( \frac{B}{C} \right) \left[ \hat{C}_t + \beta R^B \left( \hat{R}_t^B - \hat{C}_{t+1} - \hat{\pi}_{t+1} \right) \right] \quad (70)$$

*Borrower Household*

- Consumption Demand:

Using the labor demand relationship  $w_t N_t = \frac{1}{X_{\psi,t}} (1 - \gamma) Y_t$  (37) and knowing that the real marginal costs  $MC_t = 1/X_{\psi,t}$ , the borrowers' budget constraint (9) written in log-linearized form reads as:

$$\begin{aligned} \hat{C}_{t+1}^b + \hat{\pi}_{t+1} &= -\frac{qH}{C^b} (\hat{H}_t - \hat{H}_{t-1}) - \frac{R^L l}{C^b} \left( \hat{R}_{t-1}^L + \hat{l}_{t-1} - \hat{\pi}_t \right) + \frac{l}{C^b} \hat{l}_t \\ &+ \frac{(1-\gamma) Y}{X_{\psi}} \frac{1}{C^b} \left( \hat{Y}_t + \widehat{MC}_t \right) - \frac{\tau_h qH}{C^b} \left( \hat{q}_t + \hat{H}_t \right) \end{aligned} \quad (71)$$

- Residential Goods Demand:

$$\hat{H}_t = \frac{\check{\beta} qH}{\chi C^b} \left\{ \begin{aligned} &(1 + \tau_h) (\hat{\pi}_{t+1} - \hat{q}_t) + (1 + \tau_h - \kappa) \hat{C}_{t+1}^b + (\Omega + \kappa) \hat{q}_{t+1} \\ &- \Omega (\hat{C}_{t+2}^b + \hat{\pi}_{t+2}) + \hat{\kappa}_t \kappa \left( 1 - \check{\beta} R^L \right) - \check{\beta} \kappa R^L \hat{R}_t^L \end{aligned} \right\} \quad (72)$$

where  $\Omega = \check{\beta} (1 - \kappa R^L)$

- Real House Price:

Given the market clearing condition  $\hat{H}_t = 0$  under a fixed supply of housing and using the residential goods demand relationship, the log-linearized form of (18) reads:

$$\hat{q}_t = \hat{\pi}_{t+1} + \frac{1}{1 + \tau_h} \left[ \begin{aligned} &(1 + \tau_h - \kappa) \hat{C}_{t+1}^b - \Omega (\hat{C}_{t+2}^b + \hat{\pi}_{t+2}) + \\ &+ (\Omega + \kappa) \hat{q}_{t+1} + \hat{\kappa}_t \kappa \left( 1 - \check{\beta} R^L \right) - \check{\beta} \kappa R^L \hat{R}_t^L \end{aligned} \right] + \xi_{q,t} \quad (73)$$

where  $\xi_{q,t}$  denotes a non-fundamental shock to house prices.<sup>30</sup>

<sup>29</sup>For details on the derivation of the log-linearised equations and steady-states see Technical Appendix, available on request.

<sup>30</sup>We assume that the house price shock evolves exogenously as follows:

$$\log \xi_{q,t} = \phi_{\xi_q} \log \xi_{q,t-1} + u_{\xi_q,t}$$

where  $\phi_{\xi_q}$  is the persistence of the shock, and the error term is i.i.d., with mean zero and variance  $\sigma_{\xi_q}$ .

- Borrowing Constraint:

$$\hat{l}_t = \hat{k}_t + \hat{q}_{t+1} + \hat{\pi}_{t+1} + \hat{H}_t \quad (74)$$

- Labor Supply:

$$\hat{w}_t = \hat{C}_t^b + \varsigma \hat{N}_t \quad (75)$$

*Banking Sector*

- Loan Rate:

$$\hat{R}_t^L = \hat{R}_t^M + \frac{\lambda}{1-\delta} \left[ \hat{l}_t - \delta (\hat{k}_t + \hat{q}_t + \hat{H}_t) \right] \quad (76)$$

- Deposit Rate:

$$\hat{R}_t^D = \hat{R}_t^M \quad (77)$$

*Real Sector*

- Phillips Curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \eta \widehat{MC}_t + \xi_{mc,t} \quad (78)$$

where  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$ .

- Aggregate Supply:

$$\hat{Y}_t = A_t + (1-\gamma) \hat{N}_t \quad (79)$$

- Labor Demand:

$$\hat{w}_t = (\hat{Y}_t - \hat{N}_t) + \widehat{MC}_t \quad (80)$$

*Monetary Policy*

$$\hat{R}_t^M = \rho \hat{R}_{t-1}^M + (1-\rho) \left( \alpha_\pi \hat{\pi}_t + \alpha_y \hat{Y}_t \right) + \alpha_q \hat{q}_t + \xi_{m,t} \quad (81)$$

*Fiscal Policy*

$$\frac{B}{Y} \hat{B}_t = \frac{B}{Y} R^B \left( \hat{R}_{t-1}^B + \hat{B}_{t-1} - \hat{\pi}_t \right) + \frac{G}{Y} \hat{G}_t - \frac{T}{Y} \hat{T}_t - \frac{m}{Y} (\Delta \hat{m}_t + \hat{\pi}_t) \quad (82)$$

The feedback rule on government spending is:

$$\hat{G}_t = -f_y \hat{Y}_t + f_T \hat{T}_t + f_m \Delta \hat{m}_t + \xi_{g,t} \quad (83)$$

where  $\xi_{g,t}$  is a fiscal policy shock and taxation is:

$$\frac{T}{Y} \hat{T}_t = \tau_h \frac{qH}{Y} \left( \hat{q}_t + \hat{H}_t \right) + \tau_y \hat{Y}_t \quad (84)$$

*Market Clearing*

- Banking Sector: <sup>31</sup>

$$\hat{d}_t = \hat{l}_t \quad (85)$$

- Real Sector:

$$\hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{C^b}{Y} \hat{C}_t^b + \frac{G}{Y} \hat{G}_t \quad (86)$$

- Housing:<sup>32</sup>

$$H \hat{H}_t = 0 \quad (87)$$

- Monetary Market:

$$m \hat{m}_t = C \hat{C}_t + C^b \hat{C}_t^b + WN (\hat{w}_t + \hat{N}_t) \quad (88)$$

## C Welfare Analysis

The aggregate welfare function depends on households' preferences over consumption, housing, labor, bonds and public spending; the argument  $G_t$  represents government spending and is determined by the government in each period so that the representative consumer takes it as exogenously given. Thus social welfare is given by:

$$U_t = \frac{C}{Y} (\log C_t + \chi_B \log B_t) + \frac{C^b}{Y} \left( \log C_t^b + \chi \log H_t - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) + \frac{G}{Y} (\log G_t) \quad (89)$$

where we attach weight coefficients equal to the steady state value of consumption over output and government spending over output. All derivations are calculated at the steady state values  $C, C^b, N, G$  and the variables signed with a “ $\sim$ ” denote second-order approximations in terms of log-deviations:

$$\tilde{X} = X_t - X \simeq X \left( \hat{X}_t + \frac{1}{2} \hat{X}_t^2 \right) \quad (90)$$

where  $\hat{X}$  is the log-deviation from steady state for a generic variable  $X_t$ .

### C.1 Deriving the second order approximation

Since the utility is additively separable between consumption, labor, housing then we can consider the second-order approximations to each term in (89) separately.

- The second-order approximation to saver's consumption is given by:

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<sup>31</sup>According to the bank's resource constraint (21)  $rr_t + l_t = d_t$  and assuming a fractional reserves system that is  $rr_t = rr d_t$  where  $rr$  is the reserve requirement coefficient then  $l_t = (1 - rr)d_t$ .

<sup>32</sup>We assume that the supply of housing is fixed and normalised to 1,  $\bar{H} \equiv 1$ .

$$\begin{aligned}
\log C_t &\approx U_C \tilde{C}_t + \frac{U_{CC} \tilde{C}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \frac{1}{C} C \left( \hat{C}_t + \frac{1}{2} \hat{C}_t^2 \right) - \frac{1}{C^2} \frac{C^2 \hat{C}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \hat{C}_t + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{91}$$

where  $\mathcal{O}(\|a\|^3)$  collects all the terms of third order or higher, in the bound  $\|a\|$  on the amplitude of the relevant shocks.

- The second-order approximation to bond holdings is given by:

$$\begin{aligned}
\chi_B \log B_t &= \chi_B U_B \tilde{B}_t + \chi_B U_{BB} \frac{\tilde{B}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&= \frac{\chi_B}{B} B \left( \hat{B}_t + \frac{1}{2} \hat{B}_t^2 \right) - \omega \frac{\chi_B}{B^2} \frac{B^2 \hat{B}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&= \chi_B \hat{B}_t + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{92}$$

- The second-order approximation to borrower's consumption is given by:

$$\begin{aligned}
\log C_t^b &\approx U_{C^b} \tilde{C}_t^b + \frac{U_{C^b C^b} (\tilde{C}_t^b)^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \frac{1}{C^b} C^b \left( \hat{C}_t^b + \frac{1}{2} (\hat{C}_t^b)^2 \right) - \frac{1}{(C^b)^2} \frac{(C^b)^2 (\hat{C}_t^b)^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \hat{C}_t^b + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{93}$$

- The second-order approximation to real estate holdings is given by:

$$\begin{aligned}
\chi \log H_t &\approx \chi U_H \tilde{H}_t + \frac{\chi U_{HH} \tilde{H}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \frac{\chi}{H} H \left( \hat{H}_t + \frac{1}{2} \hat{H}_t^2 \right) - \frac{\chi}{H^2} \frac{H^2 \hat{H}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \chi \hat{H}_t + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{94}$$

- The second-order approximation to labor is given by:

$$\frac{N_t^{1+\varsigma}}{1+\varsigma} \approx U_N \tilde{N}_t + U_{NN} \frac{\tilde{N}_t^2}{2} + \mathcal{O}(\|a\|^3) \tag{95}$$

Given that  $U_{NN} = \varsigma N U_N$  with  $U_N = N^\varsigma$  we can re-write the above relationship as follows:

$$\begin{aligned}
\frac{N_t^{1+\varsigma}}{1+\varsigma} &\approx U_N N \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \mathcal{O}(\|a\|^3) \\
&\approx N^{\varsigma+1} \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{96}$$

Finally using the labor market clearing when the economy remains in a neighborhood of an efficient steady state yields  $N^\varsigma C^b = (1-\gamma) \frac{Y}{N}$  from which we get  $N^{\varsigma+1} = \frac{(1-\gamma)Y}{C^b}$ . Substituting this into the above relationship gives:

$$\frac{N_t^{1+\varsigma}}{1+\varsigma} = \frac{(1-\gamma)Y}{C^b} \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \mathcal{O}(\|a\|^3) \tag{97}$$

- The second-order approximation to government spending is given by:

$$\begin{aligned}
\log G_t &\approx U_G \tilde{G}_t + \frac{U_{GG} \tilde{G}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \frac{1}{G} G \left( \hat{G}_t + \frac{1}{2} \hat{G}_t^2 \right) - \frac{1}{G^2} \frac{\hat{G}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \hat{G}_t + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{98}$$

## C.2 Simplifying the Welfare Function

We now add equations (91), (93), (94), (97) in order to get a second-order approximation to the welfare function (89):

$$U_t - U \simeq \frac{C}{Y} \left( \hat{C}_t + \chi_B \hat{B}_t \right) + \frac{C^b}{Y} \left( \hat{C}_t^b + \chi \hat{H}_t \right) - (1-\gamma) \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \frac{G}{Y} \hat{G}_t + \mathcal{O}(\|a\|^3) \tag{99}$$

(i) From the second order approximation of the housing market clearing condition we have  $H \left( \hat{H}_t + \frac{1}{2} \hat{H}_t^2 \right) = 0$  which implies that  $\hat{H}_t = -\frac{1}{2} \hat{H}_t^2$  so that (99) can be rewritten as follows:

$$U_t - U \simeq \frac{C}{Y} \left( \hat{C}_t + \chi_B \hat{B}_t \right) + \frac{C^b}{Y} \hat{C}_t^b - \frac{C^b}{Y} \chi \frac{1}{2} \hat{H}_t^2 - (1-\gamma) \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \frac{G}{Y} \hat{G}_t + \mathcal{O}(\|a\|^3) \tag{100}$$

- (ii) From the second-order approximation of the aggregate resource constraint:

$$\left( \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right) = \frac{C}{Y} \left( \hat{C}_t + \frac{1}{2} \hat{C}_t^2 \right) + \frac{C^b}{Y} \left( \hat{C}_t^b + \frac{1}{2} (\hat{C}_t^b)^2 \right) + \frac{G}{Y} \left( \hat{G}_t + \frac{1}{2} \hat{G}_t^2 \right) + \mathcal{O}(\|a\|^3) \tag{101}$$

which implies

$$\frac{C}{Y} \hat{C}_t + \frac{C^b}{Y} \hat{C}_t^b + \frac{G}{Y} \hat{G}_t = \left( \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right) - \frac{1}{2} \left( \frac{C}{Y} \hat{C}_t^2 + \frac{C^b}{Y} (\hat{C}_t^b)^2 + \frac{G}{Y} \hat{G}_t^2 \right) + \mathcal{O}(\|a\|^3) \tag{102}$$



(iii) To eliminate  $\hat{N}_t$  which allows us to express the welfare function in terms of output, we use both the production function (35) and the aggregate intratemporal demand function (36) to rewrite  $\hat{N}_t$  in terms of output:

$$N_t = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\gamma}} \int_0^1 \left( \frac{P_{z,t}}{P_t} \right)^{-\frac{\psi}{1-\gamma}} dz \quad (103)$$

which in log-linear form can be rewritten as follows

$$(1-\gamma)\hat{N}_t = \hat{Y}_t - A_t + \widehat{dp}_t \quad (104)$$

where  $\widehat{dp}_t \equiv (1-\gamma) \log \left[ \int_0^1 \left( \frac{P_{z,t}}{P_t} \right)^{-\frac{\psi}{1-\gamma}} dz \right]$  is a measure of consumption goods price dispersion in the intermediate sector with  $z$  denoting the price of good variety  $z$ .

**Lemma 1** *In a neighborhood of a symmetric steady state and up to a second-order approximation  $\widehat{dp}_t$  is proportional to the cross-sectional variance of relative prices,  $\widehat{dp}_t \equiv \frac{1}{2} \frac{\psi}{\varrho} \text{var} \{P_{z,t}\} + \mathcal{O}(\|a\|^3)$  where  $\varrho \equiv \frac{1-\gamma}{1+\gamma(\psi-1)}$*

**Proof.** See Galì (2008), Chapter 4. ■

**Lemma 2**  $\sum_{t=0}^{\infty} \beta^t \text{var} \{P_{z,t}\} = \frac{\theta}{(1-\theta)(1-\beta\theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$

**Proof.** See Woodford (2003), Chapter 6. ■

iv) In a stationary equilibrium money grows at a constant rate  $\Delta m = 0$  which implies zero inflation in steady state and the primary fiscal position is balanced  $PD_t = G_t - T_t = 0$ . Therefore, the second order approximation to the per-capita government budget constraint reads as:

$$\begin{aligned} \frac{B}{Y} \left( \hat{B}_t + \frac{1}{2} \hat{B}_t^2 \right) &= \frac{BR^B}{Y} \left( \hat{B}_{t-1} + \frac{1}{2} \hat{B}_{t-1}^2 \right) + \frac{BR^B}{Y} \left( \hat{R}_{t-1}^B + \frac{1}{2} \left( \hat{R}_{t-1}^B \right)^2 \right) + \mathcal{O}(\|a\|^3) \\ \hat{B}_t &= -\frac{1}{2} \hat{B}_t^2 + \mathcal{O}(\|a\|^3) + t.i.p. \end{aligned} \quad (105)$$

where “*t.i.p.*” denotes terms independent of policy.

Substituting (102) and (104) and (105) we can re-write the welfare function (100) as follows:

$$\begin{aligned} U_t - U &\simeq \left( \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right) - \frac{1}{2} \left( \frac{C}{Y} \hat{C}_t^2 + \frac{C^b}{Y} \left( \hat{C}_t^b \right)^2 + \frac{G}{Y} \hat{G}_t^2 \right) - \frac{1}{2} \frac{C^b}{Y} \chi \hat{H}_t^2 + \\ &+ \frac{C}{Y} \chi_B \left( -\frac{1}{2} \hat{B}_t^2 \right) - \left[ \left( \hat{Y}_t - A_t + \widehat{dp}_t \right) + \frac{1}{2} \left( \frac{1+\varsigma}{1-\gamma} \right) \left( \hat{Y}_t - A_t + \widehat{dp}_t \right)^2 \right] + \mathcal{O}(\|a\|^3) \end{aligned} \quad (106)$$

By simplifying and using Lemma 1 and 2 in order to rewrite the terms involving the price dispersion as a function of inflation and knowing that  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$  and collecting “*t.i.p.*” terms the above relationship can be rewritten as follows:

$$(U_t - U) \simeq \frac{1}{2} \left[ \left( 1 - \frac{1+\varsigma}{1-\gamma} \right) \hat{Y}_t^2 - \left( \frac{C}{Y} \hat{C}_t^2 + \frac{C^b}{Y} \left( \hat{C}_t^b \right)^2 + \frac{G}{Y} \hat{G}_t^2 \right) - \frac{C}{Y} \chi_B \hat{B}_t^2 - \frac{C^b}{Y} \chi \hat{H}_t^2 - \frac{\psi}{\eta} \hat{\pi}_t^2 \right] \quad (107)$$

$$+ t.i.p. + \mathcal{O}(\|a\|^3) \quad (108)$$

The welfare function (107) can be also expressed in terms of aggregate welfare losses using the following purely quadratic loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t (U_t - U) = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t + t.i.p + \mathcal{O}(\|a\|^3) \quad (109)$$

$$\text{with } L_t = \varphi_Y \sigma_Y^2 + \varphi_C \sigma_C^2 + \varphi_{C^b} \sigma_{C^b}^2 + \varphi_G \sigma_G^2 + \varphi_B \sigma_B^2 + \varphi_H \sigma_H^2 + \varphi_\pi \sigma_\pi^2$$

where the weight coefficients are given by  $\varphi_Y \equiv \frac{s+\gamma}{1-\gamma}$ ,  $\varphi_C \equiv \frac{C}{Y}$ ,  $\varphi_{C^b} \equiv \frac{C^b}{Y}$ ,  $\varphi_B \equiv \frac{C}{Y} \chi_B$ ,  $\varphi_H \equiv \frac{C^b}{Y} \chi$ ,  $\varphi_G \equiv \frac{G}{Y}$  and  $\varphi_\pi \equiv \frac{\psi}{\eta}$  with  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$ .

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TABLE 1. CORRELATION BETWEEN  
CONSUMPTION AND HOUSE VALUE

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	Borrower	Saver	All
Levels:*	0.709	-0.452	0.602
Growth:*	0.453	0.080	0.391

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Source: BHPS 1991-2008. Data are detrended (HP-filtered).

\* Values refer respectively to Median Saver, Median Borrower  
and Median Consumer (All Sample).

TABLE 2. CALIBRATED PARAMETERS		
Parameter	Description	Value
<i>Preference Parameters and Collateral</i>		
$\beta$	Intertemporal discount rate for savers	0.99
$\check{\beta}$	Intertemporal discount rate for borrowers	0.97
$\chi_B$	Financial assets weight for savers	0.01
$\chi$	Housing weight for borrowers	0.4
$\varsigma$	Labour supply aversion	0.3
$\kappa$	Loan-to-Value	0.6
$\delta$	Fraction of seizable collateral	0.5
<i>Sticky Prices</i>		
$\psi$	Elasticity of substitution between goods	6
$X_\psi$	Price markup	1.2
$\theta$	Index of price stickiness	0.75
<i>Technology</i>		
$\gamma$	Production function parameter	0.4
<i>Loans, Deposits and Fractional Reserves</i>		
$X_{\mu_D}$	Markup on deposit rate	1.1
$X_{\mu_L}$	Markup on loan rate	1.4
$rr$	Fractional reserve coefficient	0.005
$\lambda$	Default rate	1.25
$1 - \Phi$	Loan repayment probability	0.8
<i>Fiscal Policy</i>		
$G/Y$	Government spending to output	0.16
$B/Y$	Public debt to output	0.56
$T/Y$	Taxation to output	0.3
$f_T$	Fiscal feedback parameter on housing tax	0.01
$f_y$	Fiscal feedback parameter on output	0.01
$f_m$	Fiscal feedback parameter on seignorage	0.01
$\tau_y$	Lump-sum tax	0.01
<i>Monetary Policy</i>		
$\rho$	Degree of interest rate smoothing	0.85
$\alpha_\pi$	Central Bank reaction to expected inflation	1.5
$\alpha_y$	Central Bank reaction to output gap	0.5
$\alpha_q$	Central Bank reaction to asset prices	0.2

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TABLE 2 (CONTINUED) IMPLIED STEADY STATES

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	Description	Value
<i>Banking Sector</i>		
$R^D-1$	Deposit rate	0.010
$R^M-1$	Policy rate	0.011
$R^L-1$	Loan rate	0.019
$l/Y$	Loan-to-output ratio	0.97
$l/d$	Loan-to-deposit ratio	0.995
<i>Public Sector</i>		
$\tau_h$	Housing tax	0.09
$m/Y$	Money to output ratio	1.34
$R^B$	Bond rate (gross)	1.003
<i>Savers</i>		
$C/Y$	Consumption to output ratio	0.38
<i>Borrowers</i>		
$C^b/Y$	Consumption to output ratio	0.46
$l/C^b$	Loan to consumption ratio	2.11
$qH/Y$	Housing wealth to output ratio	1.62

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TABLE 3. BASELINE MODEL SHOCKS\*

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	Description	Value
$\rho_\kappa$	Autocorrelation of the collateral shock	0.90
$\rho_{\xi_q}$	Autocorrelation of house price shock	0.50
$\rho_a$	Autocorrelation of productivity shock	0.70
$\rho_{\xi_m}$	Autocorrelation of monetary policy shock	0.01
$\rho_{\xi_g}$	Autocorrelation of fiscal policy shock	0.90
$\rho_{\xi_{mc}}$	Autocorrelation of the cost-push shock	0.50
$\sigma_\kappa$	Standard deviation of the loan to value shock	0.01
$\sigma_{\xi_q}$	Standard deviation of house price shock	0.01
$\sigma_a$	Standard deviation of the productivity shock	0.01
$\sigma_{\xi_m}$	Standard deviation of the monetary policy shock	0.01
$\sigma_{\xi_g}$	Standard deviation of the fiscal policy shock	0.01
$\sigma_{\xi_{mc}}$	Standard deviation of the cost-push shock	0.01

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\*Source: Chadha *et al.* (2009)

TABLE 4. MOMENTS						
	Rel stdev ( $Y$ )	Corr ( $Y$ )	Rel stdev ( $Y$ )	Corr ( $Y$ )	Rel stdev ( $Y$ )	Corr ( $Y$ )
	Benchmark Policy		Optimal Monetary Policy		Optimal Fiscal Policy	
	Base Regime					
$C^*$	1.18	0.99	1.18	0.99	1.26	0.98
$C$	2.50	-0.61	2.68	-0.66	2.51	-0.60
$C^b$	3.81	0.90	4.00	0.91	3.89	0.91
$G$	0.09	0.07	0.09	0.06	1.05	-0.31
$l$	1.91	0.40	1.87	0.39	1.94	0.41
$R^L$	0.71	0.43	0.69	0.46	0.73	0.44
$R^M, R^D$	0.59	0.24	0.63	0.27	0.60	0.27
$EFP$	0.69	0.24	0.66	0.21	0.70	0.23
$q$	2.05	0.45	2.15	0.45	2.11	0.49
$\pi$	1.16	0.01	1.17	-0.01	1.16	0.06
$w$	4.10	0.90	4.29	0.90	4.20	0.90

Note:  $C^*$  denotes aggregate consumption and  $EFP = R^L - R^M$ . We solve the model for the asymptotic moments and present the HP filtered results with  $\lambda^{HP} = 1,600$ . For optimal monetary policy we use the benchmark policy parameters plus a optimal weight on asset prices in the interest rate rule  $\alpha_q = 0.2$ . For optimal fiscal we use the feedback parameters  $f_y, f_T$  and  $f_m$  [1.024, 1.027, 0.097]

TABLE 5. OPTIMAL POLICY PARAMETERS					
Default Rate		Optimal Policy Parameters			
		$f_y$	$f_T$	$f_m$	$\alpha_q$
$\lambda = 1$	Fiscal Policy	-1.02	1.03	0.10	-
	Monetary Policy	-	-	-	0.1
$\lambda = 1.25$	Fiscal Policy	-1.024	1.027	0.097	-
	Monetary Policy	-	-	-	0.2
$\lambda = 5$	Fiscal Policy	-1.06	1.27	0.10	-
	Monetary Policy	-	-	-	0.3

Note:  $\lambda = 1$  corresponds to a loan repayment probability of 100%  
 $\lambda = 5$  corresponds to a loan repayment probability of 20%  
For the optimal monetary policy we set  $\alpha_\pi = 1.5$  and  $\alpha_y = 0.1$   
and optimize over  $\alpha_q$ .

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**TABLE 6. WELFARE LOSSES**


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		Loss			
$\lambda = 1.25$					
$\kappa = 0.6$	$\delta = 0.2$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.9$	
Benchmark Policy	0.0561	<b>0.0552</b>	0.0575	0.0608	
Monetary Policy	0.0543	<b>0.0518</b>	0.0522	0.0532	
Fiscal Policy	0.0580	<b>0.0559</b>	0.0571	0.0600	
$\delta = 0.5$	$\kappa = 0.25$	$\kappa = 0.4$	$\kappa = 0.7$	$\kappa = 0.9$	
Benchmark Policy	0.0540	0.0545	0.0552	0.0546	
Monetary Policy	0.0500	0.0506	0.0520	0.0521	
Fiscal Policy	0.0534	0.0545	0.0560	0.0555	
$\lambda = 1$					
$\kappa = 0.6$	$\delta = 0.2$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.9$	
Benchmark Policy	0.0574	0.0568	0.0593	0.0626	
Monetary Policy	0.0562	0.0547	0.0559	0.0577	
Fiscal Policy	0.0595	0.0576	0.0592	0.0625	
$\delta = 0.5$	$\kappa = 0.25$	$\kappa = 0.4$	$\kappa = 0.7$	$\kappa = 0.9$	
Benchmark Policy	0.0546	0.0555	0.0570	0.0564	
Monetary Policy	0.0522	0.0531	0.0550	0.0548	
Fiscal Policy	0.0538	0.0553	0.0579	0.0575	
$\lambda = 5$					
$\kappa = 0.6$	$\delta = 0.2$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.9$	
Benchmark Policy	0.0521	0.0510	0.0517	0.0554	
Monetary Policy	0.0514	0.0488	0.0478	0.0484	
Fiscal Policy	0.0531	0.0515	0.0513	0.0527	
$\delta = 0.5$	$\kappa = 0.25$	$\kappa = 0.4$	$\kappa = 0.7$	$\kappa = 0.9$	
Benchmark Policy	0.0505	0.0506	0.0511	0.0513	
Monetary Policy	0.0477	0.0483	0.0489	0.0490	
Fiscal Policy	0.0510	0.0514	0.0515	0.0515	
Lax Regime	$\lambda = 1, \delta = 0.9, \kappa = 0.9$				
Benchmark Policy	0.0628				
Monetary Policy	0.0586				
Fiscal Policy	0.0631				
Restrictive Regime	$\lambda = 5, \delta = 0.2, \kappa = 0.4$				
Benchmark Policy	0.0531				
Monetary Policy	0.0529				
Fiscal Policy	0.0543				

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Note:  $\lambda = 1$  corresponds to a loan repayment probability of 100%  $\lambda = 5$  corresponds to a loan repayment probability of 20%. For optimal monetary policy we use the benchmark policy parameters plus a optimal weight on asset prices in the interest rate rule  $\alpha_q = 0.2$  For optimal fiscal policy we optimise over  $f_y$   $f_T$  and  $f_m$ . The optimal value for both policies are shown in Table 5. Numbers in bold denote the losses associated to the base parameters  $\lambda = 1.25, \delta = 0.5$  and  $\kappa = 0.6$ .

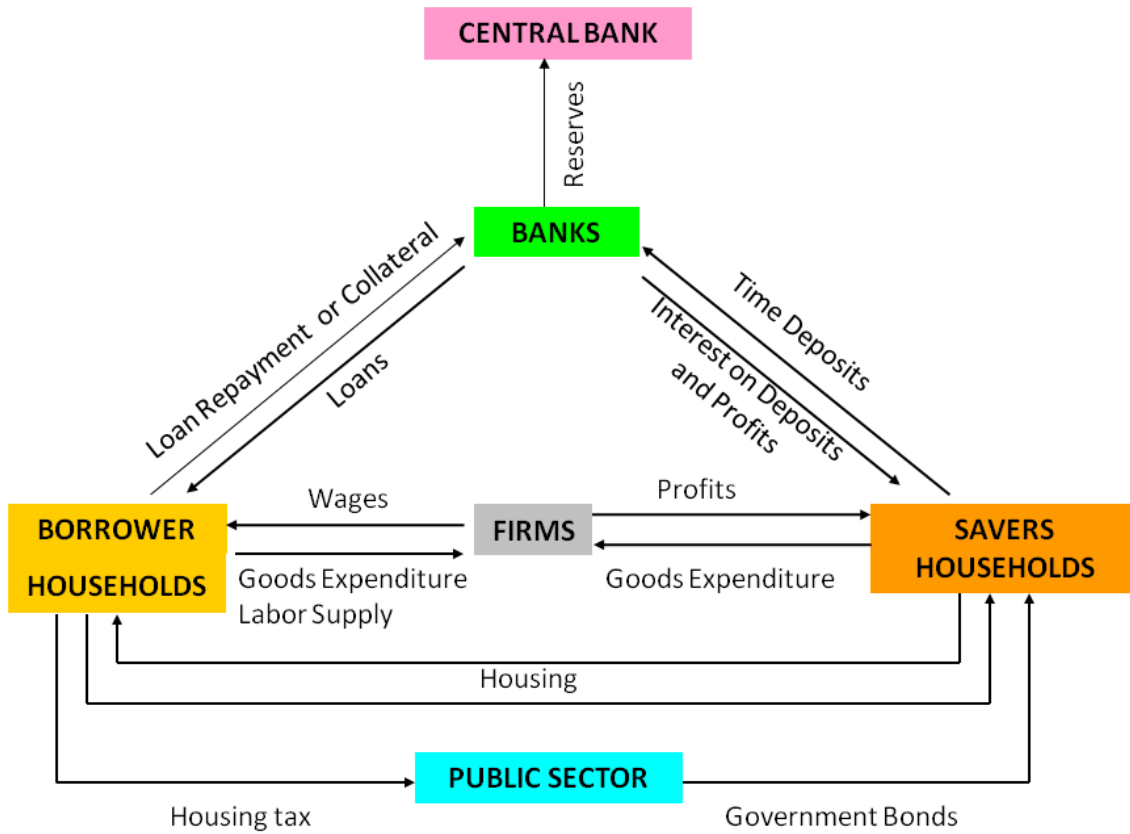


Figure 1: The Model



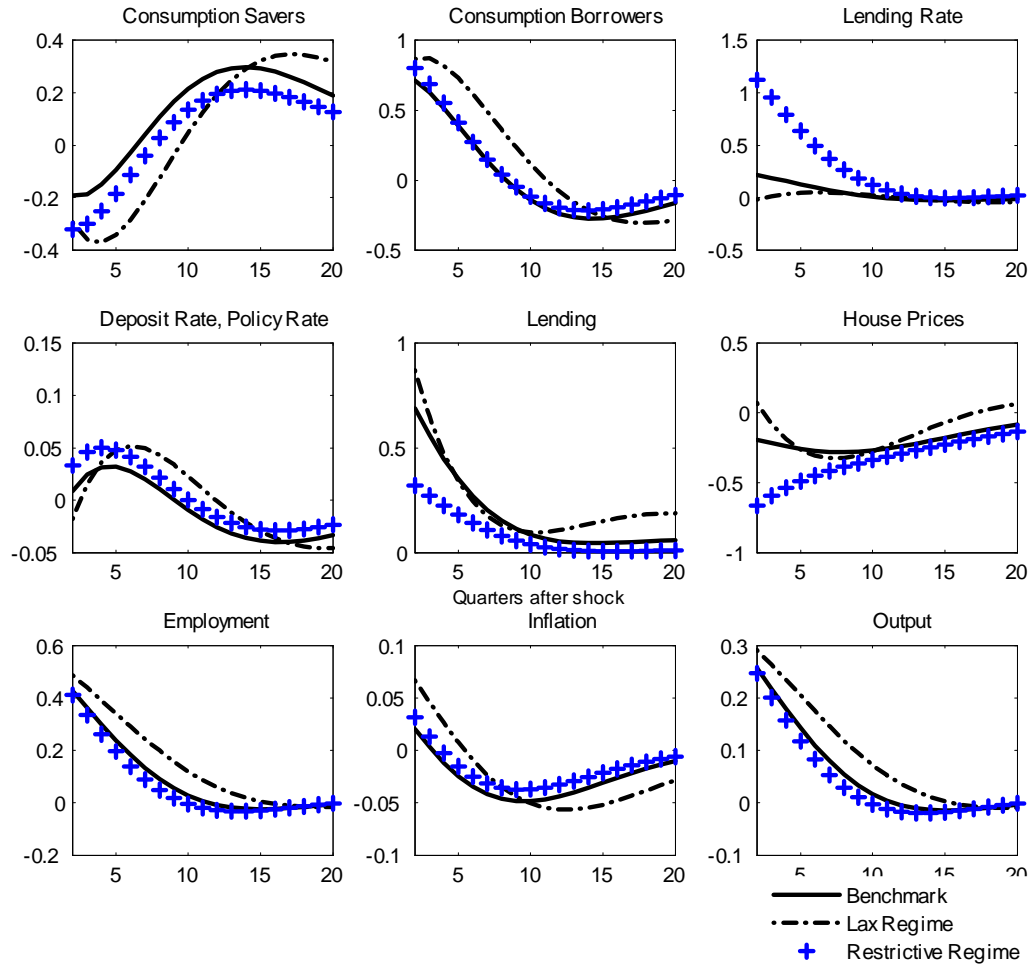


Figure 2: Impulse Response Analysis to a Loan to Value Shock. Base Regime:  $\delta = 0.5$ ;  $\kappa = 0.6$ ;  $\lambda = 1.25$   
 Restrictive Regime:  $\delta = 0.2$ ;  $\kappa = 0.4$ ;  $\lambda = 5$ . Lax Regime:  $\delta = 0.9$ ;  $\kappa = 0.9$ ;  $\lambda = 1$

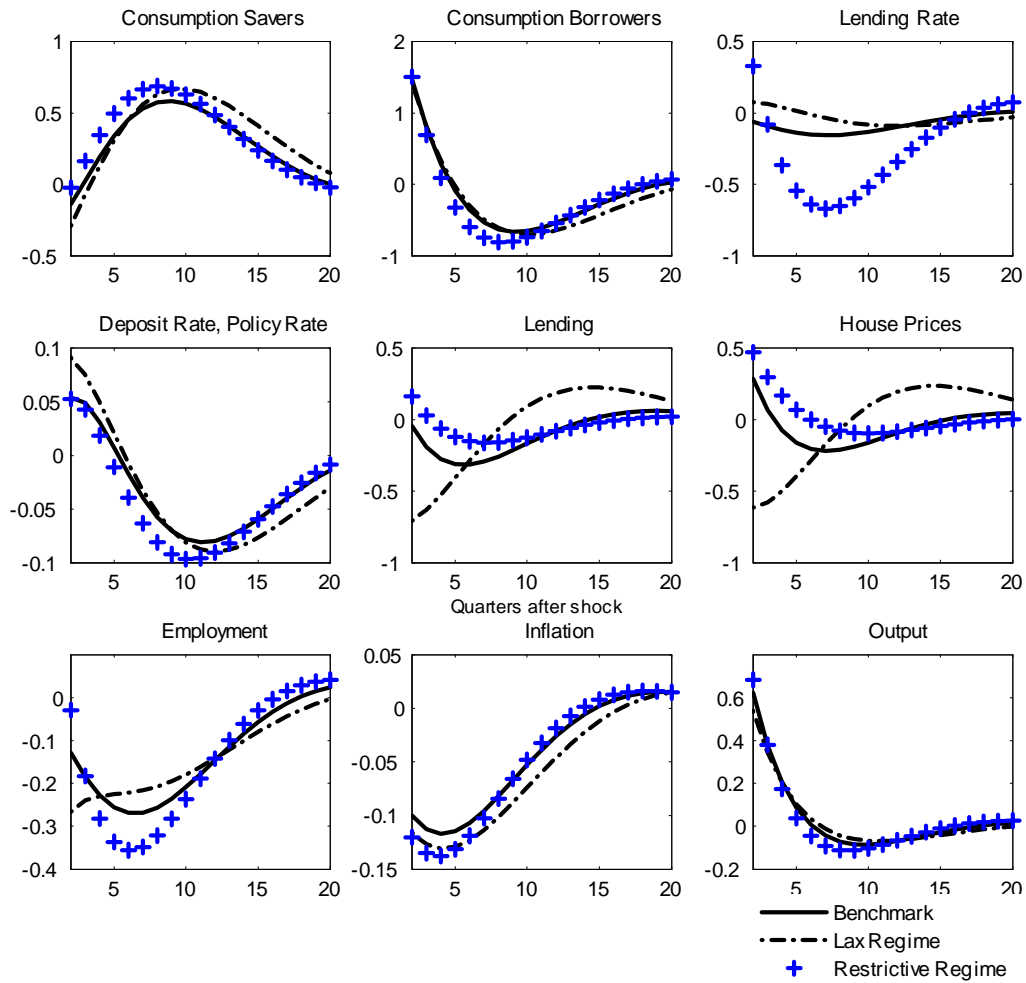


Figure 3: Impulse Response Analysis to a Productivity Shock. Base Regime:  $\delta = 0.5$ ;  $\kappa = 0.6$ ;  $\lambda = 1.25$   
 Restrictive Regime:  $\delta = 0.2$ ;  $\kappa = 0.4$ ;  $\lambda = 5$ . Lax Regime:  $\delta = 0.9$ ;  $\kappa = 0.9$ ;  $\lambda = 1$

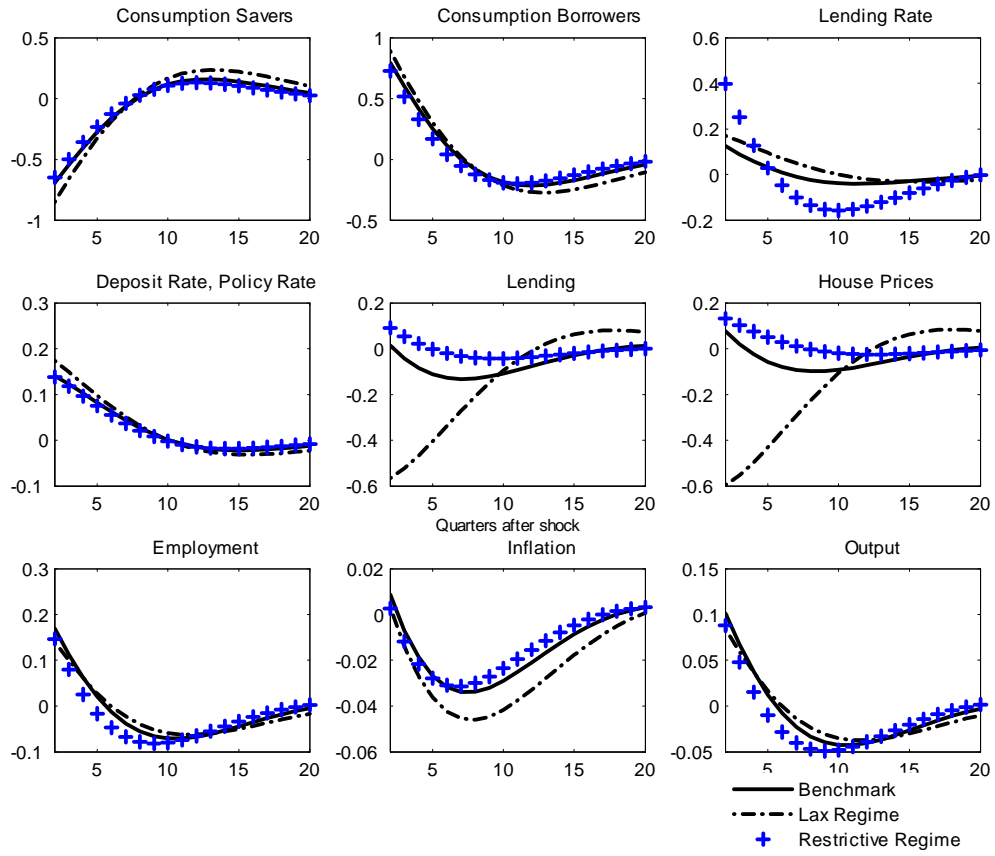


Figure 4: Impulse Response Analysis to a Monetary Policy Shock. Base Regime:  $\delta = 0.5$ ;  $\kappa = 0.6$ ;  $\lambda = 1.25$   
 Restrictive Regime:  $\delta = 0.2$ ;  $\kappa = 0.4$ ;  $\lambda = 5$ . Lax Regime:  $\delta = 0.9$ ;  $\kappa = 0.9$ ;  $\lambda = 1$

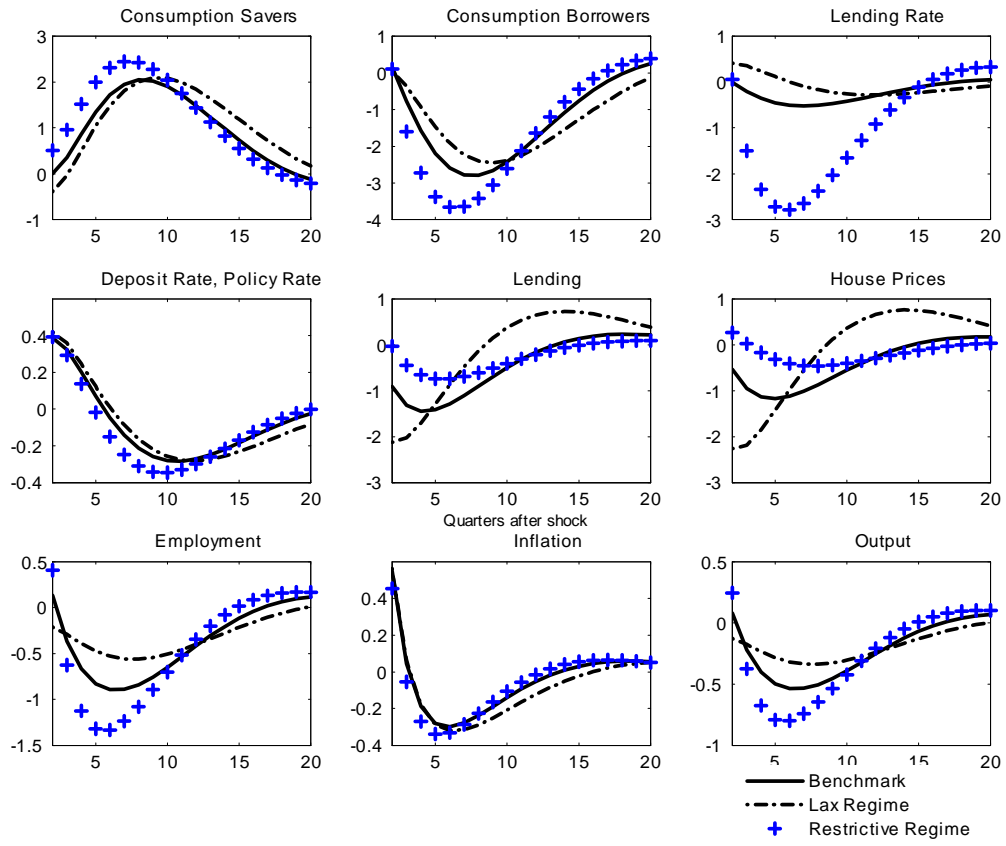


Figure 5: Impulse Response Analysis to a Cost-Push Shock. Base Regime:  $\delta = 0.5$ ;  $\kappa = 0.6$ ;  $\lambda = 1.25$   
 Restrictive Regime:  $\delta = 0.2$ ;  $\kappa = 0.4$ ;  $\lambda = 5$ . Lax Regime:  $\delta = 0.9$ ;  $\kappa = 0.9$ ;  $\lambda = 1$

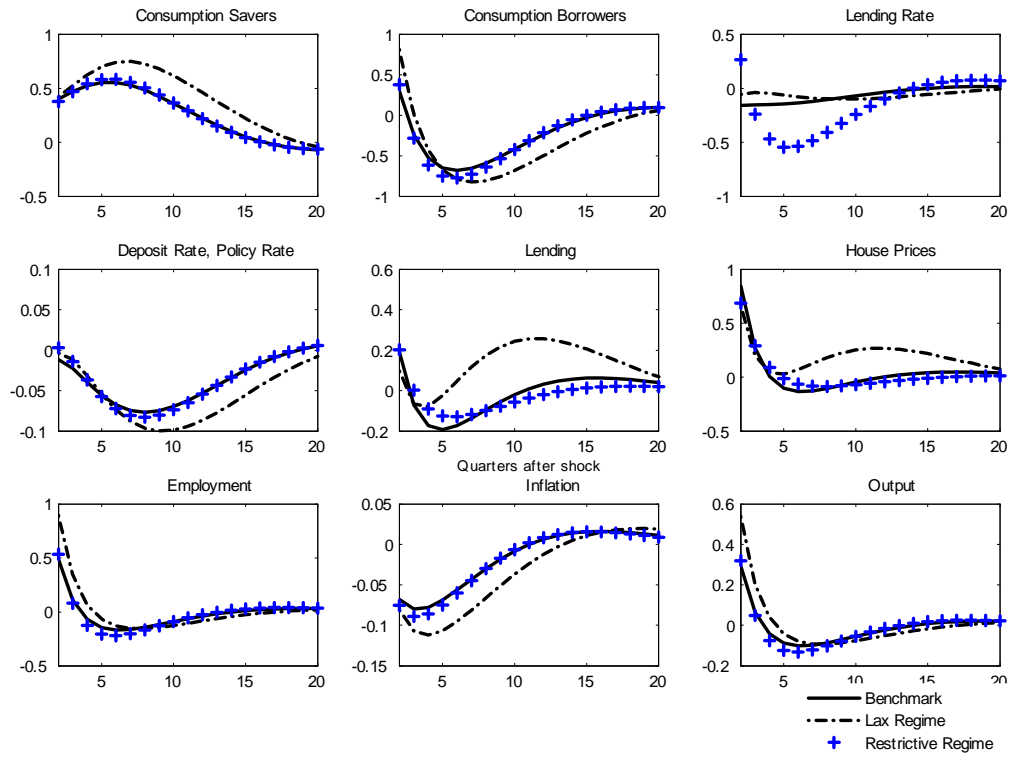


Figure 6: Impulse Response Analysis to a House Price Shock. Base Regime:  $\delta = 0.5$ ;  $\kappa = 0.6$ ;  $\lambda = 1.25$   
Restrictive Regime:  $\delta = 0.2$ ;  $\kappa = 0.4$ ;  $\lambda = 5$ . Lax Regime:  $\delta = 0.9$ ;  $\kappa = 0.9$ ;  $\lambda = 1$

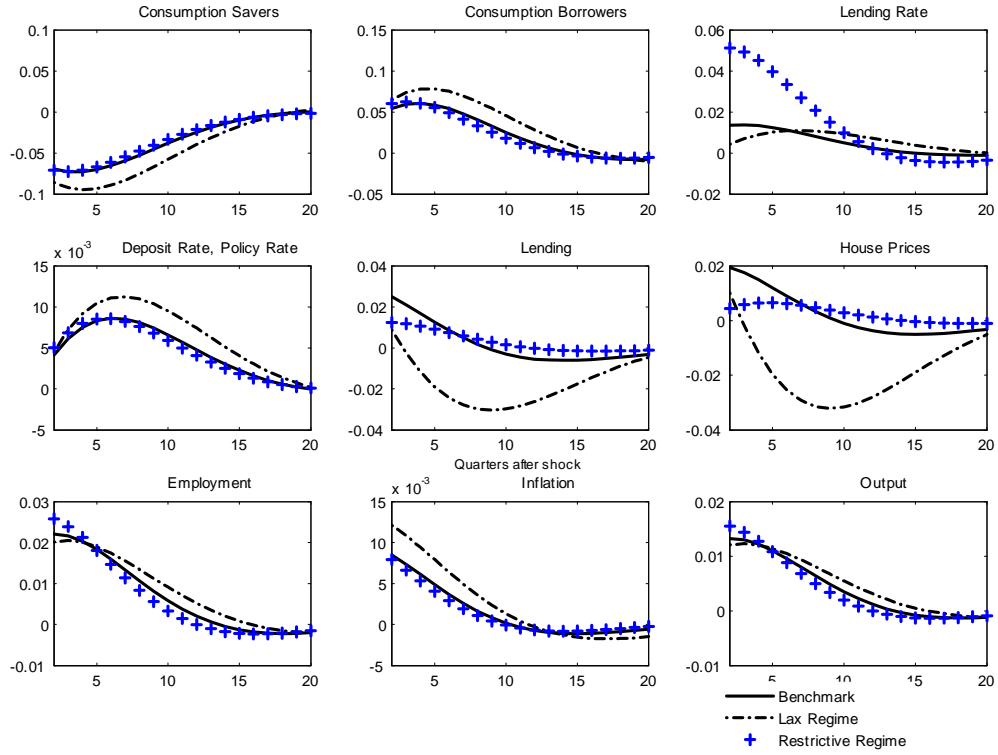


Figure 7: Impulse Response Analysis to a Government Spending Shock. Base Regime:  $\delta = 0.5$ ;  $\kappa = 0.6$ ;  $\lambda = 1.25$  Restrictive Regime:  $\delta = 0.2$ ;  $\kappa = 0.4$ ;  $\lambda = 5$ . Lax Regime:  $\delta = 0.9$ ;  $\kappa = 0.9$ ;  $\lambda = 1$

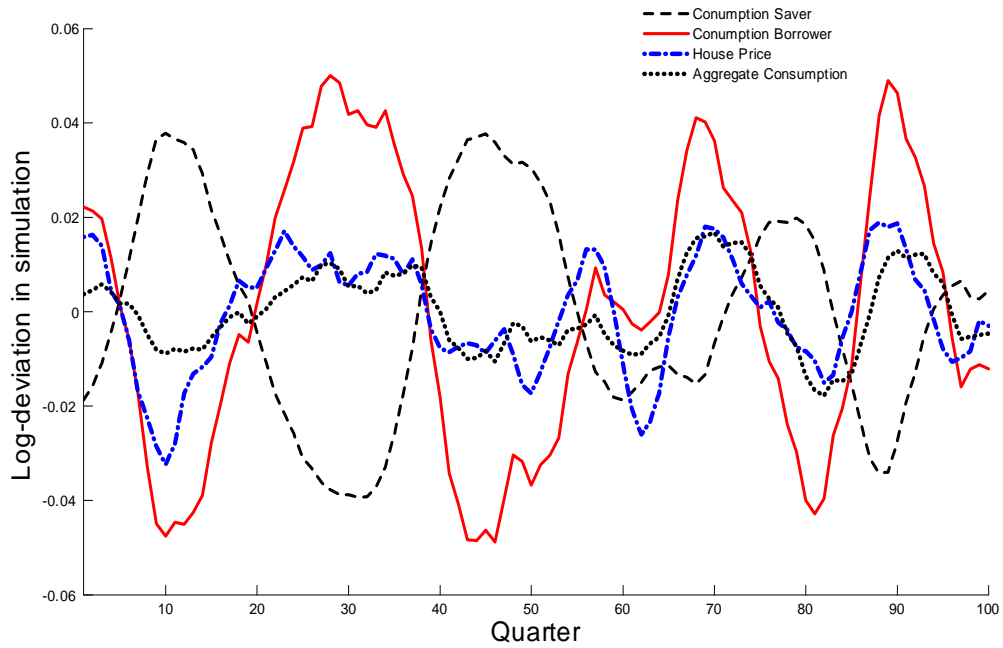


Figure 8: Model Simulation