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Non-Homothetic Growth Models for the Environmental Kuznets Curve

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Technical Note on:

Non-Homothetic Growth Models for the Environmental Kuznets Curve*

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Abstract

This technical note shows the detailed derivation of the Slutsky decomposition of Models I and II in Fraser and Shibayama (2012) "Non-Homothetic Growth Models for the Environmental Kuznets Curve".

KEYWORDS: Environmental Kuznets Curve, Economic Growth, Non-Homothetic Preferences, Generalized Isoelastic Preferences.

JEL CLASSIFICATION: O13, Q56

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<http://www.kent.ac.uk/economics/research/papers/index.html>

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1 Technical Appendeces

1.1 Exponential (DES) Utility with Linear Production in Details

In this subsection, we show the detailed derivation of Model I. Note first that, in the model, two W appears; one as the shadow price of R , which is equal to the marginal product of X in equilibrium and the other as wealth, which is the environmental endowment 1 times its shadow price. In this appendix, we let W be the shadow price and \tilde{W} be the wealth (see 1b), which we set to be the same at the end of the derivation; $W = \tilde{W}$. We first show the results of the direct optimization (which is equivalent to the planner's optimization) since this is rather standard solution concept, and then show the household's cost minimization problem to obtain the Slutsky decomposition.

1.1.1 Direct Optimization

◇ Optimization ($\tilde{W} \neq W$):

$$\max_{C,R} \quad -e^{-\alpha_C C} - \phi e^{-\alpha_R R} \quad (1a)$$

$$\text{s.t.} \quad C + WR = \tilde{W} \quad (1b)$$

◇ FOCs ($\tilde{W} \neq W$):

$$\phi \alpha_R e^{-\alpha_R R} = \lambda W \quad \text{and} \quad \alpha_C e^{-\alpha_C C} = \lambda$$

These two FOCs imply that the shadow price P_R of R is W , which is defined as the ratio of two marginal utilities; $P_R = (\partial U / \partial R) / (\partial U / \partial C)$. From the constraint and FOCs,¹ we obtain the equilibrium as a

¹Perhpas, a matrix notation is easier to see the problem.

$$\begin{pmatrix} \ln \phi \alpha_R / W \\ \ln \alpha_C \\ \tilde{W} \end{pmatrix} = \begin{bmatrix} \alpha_R & 0 & 1 \\ 0 & \alpha_C & 1 \\ W & 1 & 0 \end{bmatrix} \begin{pmatrix} R \\ C \\ \ln \lambda \end{pmatrix}$$

$$\begin{pmatrix} R \\ C \\ \ln \lambda \end{pmatrix} = \frac{1}{\alpha_R + \alpha_C W} \begin{bmatrix} 1 & -1 & \alpha_C \\ -W & W & \alpha_R \\ \alpha_C W & \alpha_R & -\alpha_R \alpha_C \end{bmatrix} \begin{pmatrix} \ln \phi \alpha_R / W \\ \ln \alpha_C \\ \tilde{W} \end{pmatrix}$$

function of W and \tilde{W} .

$$\begin{aligned} R &= \frac{\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C \tilde{W}}{\alpha_R + \alpha_C W} \\ C &= \frac{-W \ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_R \tilde{W}}{\alpha_R + \alpha_C W} \\ \ln \lambda &= \frac{\alpha_C W \ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_R \ln \alpha_C - \alpha_R \alpha_C \tilde{W}}{\alpha_R + \alpha_C W} \\ P_R &= W \left(= \frac{\partial U / \partial R}{\partial U / \partial C} \right) \end{aligned}$$

Confirm that, if $W = \tilde{W}$, $C = W(1 - R)$. Hence, if $W = \tilde{W}$,

$$\frac{dR}{dW} = \frac{\alpha_C - \frac{1}{W}}{\alpha_R + \alpha_C W} - \frac{\alpha_C \left(\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C W \right)}{(\alpha_R + \alpha_C W)^2} = \frac{\alpha_C (1 - R) - \frac{1}{W}}{\alpha_R + \alpha_C W} = \frac{\alpha_C C - 1}{W(\alpha_R + \alpha_C W)} \quad (2)$$

$$\begin{aligned} \frac{dC}{dW} &= \frac{\alpha_R + 1 - \ln \frac{\phi \alpha_R}{\alpha_C W}}{\alpha_R + \alpha_C W} - \frac{\alpha_C \left(-W \ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_R W \right)}{(\alpha_R + \alpha_C W)^2} = \frac{\alpha_R + 1 - \ln \frac{\phi \alpha_R}{\alpha_C W} - \alpha_C C}{\alpha_R + \alpha_C W} \\ &= \frac{C}{W} - \frac{\alpha_C C - 1}{\alpha_R + \alpha_C W} = \frac{\alpha_R (1 - R) + 1}{\alpha_R + \alpha_C W} = (1 - R) - W \frac{dR}{dW} \end{aligned} \quad (3)$$

◇ Elasticity of Substitution ($\tilde{W} = W$): Note that in our definition η is defined as the ratio of observed changes in quantities and prices; hence, we do not discriminate \tilde{W} from W .

$$\begin{aligned} \eta &= -\frac{d(R/C)}{dW} \frac{W}{(R/C)} = \frac{dC}{dW} \frac{W}{C} - \frac{dR}{dW} \frac{W}{R} \\ &= 1 - \frac{\alpha_C C - 1}{\alpha_R + \alpha_C W} \frac{1}{1 - R} - \frac{\alpha_C C - 1}{\alpha_R + \alpha_C W} \frac{1}{R} = 1 - \frac{dR}{dW} \left(\frac{W}{R} + \frac{W}{1 - R} \right) \\ &= 1 - \left(\frac{-W \ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_R W}{\alpha_R + \alpha_C W} - 1 \right) \left(\frac{1}{\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C W} + \frac{1}{\alpha_R - \ln \frac{\phi \alpha_R}{\alpha_C W}} \right) \end{aligned}$$

Obviously, $\eta > 1$ implies that $dR/dW < 0$.

1.1.2 Household's Cost Minimization for Slutsky Decomposition

◇ Dual cost minimization problem ($\tilde{W} \neq W$):

$$\begin{aligned} &\min C + WR \\ &\text{s.t. } (e^{-\alpha_C C} + \phi e^{-\alpha_R R}) < \bar{U} \quad (\bar{U} > 0 \text{ is the negative of utility}) \end{aligned}$$

◇ FOCs ($\tilde{W} \neq W$):

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R} &: W = \phi \alpha_R \lambda e^{-\alpha_R R} \\
\frac{\partial \mathcal{L}}{\partial C} &: 1 = \alpha_C \lambda e^{-\alpha_C C} \\
\lambda &= \frac{1/\alpha_C + W/\alpha_R}{e^{-\alpha_C C} + \phi e^{-\alpha_R R}} = \frac{1/\alpha_C + W/\alpha_R}{\bar{U}} \\
\frac{\phi e^{-\alpha_R R}}{e^{-\alpha_C C} + \phi e^{-\alpha_R R}} &= \frac{W/\alpha_R}{1/\alpha_C + W/\alpha_R}, \quad e^{-\alpha_R R} = \frac{\alpha_C W/\phi}{\alpha_R + \alpha_C W} \bar{U} \\
\frac{e^{-\alpha_C C}}{e^{-\alpha_C C} + \phi e^{-\alpha_R R}} &= \frac{1/\alpha_C}{1/\alpha_C + W/\alpha_R}, \quad e^{-\alpha_C C} = \frac{\alpha_R}{\alpha_R + \alpha_C W} \bar{U}
\end{aligned}$$

◇ Hicksian demand functions ($\tilde{W} \neq W$):

$$\begin{aligned}
R_h &= -\frac{\ln(\alpha_C W/\phi)}{\alpha_R} + \frac{\ln(\alpha_R + \alpha_C W) - \ln \bar{U}}{\alpha_R} \\
C_h &= -\frac{\ln(\alpha_R)}{\alpha_C} + \frac{\ln(\alpha_R + \alpha_C W) - \ln \bar{U}}{\alpha_C}
\end{aligned}$$

◇ Slutsky Decomposition ($\tilde{W} \neq W$):

$$\begin{aligned}
\frac{dR}{dW} &= \frac{dR_h}{dW} - \frac{dR}{d\tilde{W}} R \\
&= \frac{-1}{\alpha_R} \left(\frac{1}{W} - \frac{\alpha_C}{\alpha_R + \alpha_C W} \right) - \frac{\alpha_C R}{\alpha_R + \alpha_C W} = \frac{-1/W}{\alpha_R + \alpha_C W} - \frac{\alpha_C R}{\alpha_R + \alpha_C W} \\
&= \frac{1}{\alpha_R + \alpha_C W} \left(\frac{-1}{W} - \alpha_C \frac{\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C \tilde{W}}{\alpha_R + \alpha_C W} \right) \\
\frac{dC}{dW} &= \frac{dC_h}{dW} - \frac{dC}{d\tilde{W}} R \\
&= \frac{1}{\alpha_R + \alpha_C W} - \frac{\alpha_R R}{\alpha_R + \alpha_C W} = \frac{1}{\alpha_R + \alpha_C W} \left(1 - \alpha_R \frac{\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C \tilde{W}}{\alpha_R + \alpha_C W} \right) \\
\frac{dR/R}{dW/W} &= \frac{W}{\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C \tilde{W}} \left(\frac{-1}{W} - \alpha_C \frac{\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C \tilde{W}}{\alpha_R + \alpha_C W} \right) = \frac{-1/R}{\alpha_R + \alpha_C W} - \frac{\alpha_C W}{\alpha_R + \alpha_C W} \\
\frac{dC/C}{dW/W} &= \frac{W}{-W \ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_R \tilde{W}} \left(1 - \alpha_R \frac{\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C \tilde{W}}{\alpha_R + \alpha_C W} \right)
\end{aligned}$$

◇ Hicksian compensated substitution effects ($\tilde{W} \neq W$):

$$\begin{aligned}\frac{dR_h}{dW} &= \frac{-1/W}{\alpha_R + \alpha_C W} < 0 \quad , \quad \frac{dR_h}{dW} \frac{W}{R_h} = \frac{-1}{\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_C \tilde{W}} < 0 \\ \frac{dC_h}{dW} &= \frac{1}{\alpha_R + \alpha_C W} > 0 \quad , \quad \frac{dC_h}{dW} \frac{W}{C_h} = \frac{1}{-\ln \frac{\phi \alpha_R}{\alpha_C W} + \alpha_R \tilde{W}/W}\end{aligned}$$

◇ Hicksian income effects through CPI change ($\tilde{W} \neq W$):

$$\begin{aligned}-\frac{dR}{d\tilde{W}} R &= -\frac{\alpha_C \alpha_R}{\alpha_R + \alpha_C W} \frac{R}{\alpha_R} < 0 \quad , \quad -\frac{dR}{d\tilde{W}} R \frac{W}{R} = -\frac{\alpha_C \alpha_R}{\alpha_R + \alpha_C W} \frac{WR}{\alpha_R R} < 0 \\ -\frac{dC}{d\tilde{W}} R &= -\frac{\alpha_C \alpha_R}{\alpha_R + \alpha_C W} \frac{R}{\alpha_C} < 0 \quad , \quad -\frac{dC}{d\tilde{W}} R \frac{W}{C} = -\frac{\alpha_C \alpha_R}{\alpha_R + \alpha_C W} \frac{WR}{\alpha_C C} < 0\end{aligned}$$

◇ Direct income effects due to a change in \tilde{W} ($\tilde{W} \neq W$):

$$\begin{aligned}\frac{dR}{d\tilde{W}} &= \frac{\alpha_C}{\alpha_R + \alpha_C W} > 0 \quad , \quad \frac{dR/R}{d\tilde{W}/\tilde{W}} = \frac{\alpha_C \tilde{W}}{\ln \phi \alpha_R / W \alpha_C + \alpha_C \tilde{W}} = \frac{\alpha_C \alpha_R}{\alpha_R + \alpha_C W} \frac{\tilde{W}/\alpha_R}{R} \\ \frac{dC}{d\tilde{W}} &= \frac{\alpha_R}{\alpha_R + \alpha_C W} > 0 \quad , \quad \frac{dC/C}{d\tilde{W}/\tilde{W}} = \frac{\alpha_R \tilde{W}}{-W \ln \phi \alpha_R / W \alpha_C + \alpha_R \tilde{W}} = \frac{\alpha_C \alpha_R}{\alpha_R + \alpha_C W} \frac{\tilde{W}/\alpha_C}{C}\end{aligned}$$

◇ Total effects ($W = \tilde{W}$):

$$\begin{aligned}\left. \frac{dR}{dW} + \frac{dR}{d\tilde{W}} \right|_{\tilde{W}=W} &= \frac{-1/W}{\alpha_R + \alpha_C W} - \frac{\alpha_C R}{\alpha_R + \alpha_C W} + \frac{\alpha_C}{\alpha_R + \alpha_C W} = \frac{\alpha_C (1 - R) - 1/W}{\alpha_R + \alpha_C W} \\ \left. \frac{dC}{dW} + \frac{dC}{d\tilde{W}} \right|_{\tilde{W}=W} &= \frac{1}{\alpha_R + \alpha_C W} - \frac{\alpha_R R}{\alpha_R + \alpha_C W} + \frac{\alpha_R}{\alpha_R + \alpha_C W} = \frac{\alpha_R (1 - R) + 1}{\alpha_R + \alpha_C W}\end{aligned}$$

where the first, second and third terms are Hicks substitution, Hicks income and the direct income effects, respectively. Confirm that these results are indeed identical to (2) and (3).

1.2 Power (CES) Utility with Linear Production in Detail

We show the same things as in Appendix 1.1 for CRRA utility function. Here, we treat η as a parameter (not endogenous variable) for the sake of simplicity.

1.2.1 Planner's Optimization

◇ Optimization:

$$\max_{C,R} \quad \frac{C^{1-1/\eta}}{1-1/\eta} + \phi \frac{R^{1-1/\eta}}{1-1/\eta} \quad (4a)$$

$$\text{s.t.} \quad C + WR = \tilde{W} \quad (4b)$$

◇ FOCs:

$$\frac{\partial \mathcal{L}}{\partial R} : \phi R^{-1/\eta} = \lambda W \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial C} : C^{-1/\eta} = \lambda$$

Note that these two FOCs imply that the shadow price P_R of R is W . Hence, we find $\lambda = \frac{C^{1-1/\eta} + \phi R^{1-1/\eta}}{WR + C}$

and

$$R = \frac{\tilde{W}}{(W/\phi)^\eta + W} \quad \text{and} \quad C = \frac{\tilde{W}}{(W/\phi)^\eta + W} (W/\phi)^\eta$$

◇ R and C as functions of W : If $\tilde{W} = W$,

$$\begin{aligned} \frac{dR}{dW} &= \frac{-1}{W^{\eta-1}\phi^{-\eta} + 1} \frac{(\eta-1)W^{\eta-2}\phi^{-\eta}}{W^{\eta-1}\phi^{-\eta} + 1} = (1-\eta) \frac{R}{W} \frac{C}{W} \\ \frac{dC}{dW} &= \frac{1}{W(W/\phi)^{-\eta} + 1} - \frac{W}{W(W/\phi)^{-\eta} + 1} \frac{(1-\eta)(W/\phi)^{-\eta}}{W(W/\phi)^{-\eta} + 1} = (1 - (1-\eta)R) \frac{C}{W} \\ \frac{dR}{dW} \frac{W}{R} &= (1-\eta) \frac{W^{\eta-1}\phi^{-\eta}}{W^{\eta-1}\phi^{-\eta} + 1} = (1-\eta)(1-R) \\ \frac{dC}{dW} \frac{W}{C} &= 1 - (1-\eta) \frac{W(W/\phi)^{-\eta}}{W(W/\phi)^{-\eta} + 1} = 1 - (1-\eta) \left(1 - \frac{C}{W}\right) = 1 - (1-\eta)R \end{aligned}$$

◇ Elasticity of Substitution:

$$-\frac{d(R/C)}{dW} \frac{W}{(R/C)} = \eta$$

1.2.2 Household's Cost Minimization for Slutsky Decomposition

◇ Dual cost minimization problem ($\tilde{W} \neq W$):

$$\begin{aligned} \min \quad & C + WR \\ \text{s.t.} \quad & \left(\frac{C^{1-1/\eta}}{1-1/\eta} + \phi \frac{R^{1-1/\eta}}{1-1/\eta} \right) > \bar{U} \end{aligned}$$

◇ FOCs:

$$W = \lambda \phi R^{-1/\eta} \quad \text{and} \quad 1 = \lambda C^{-1/\eta}$$

Also, we find $\lambda = \frac{C+WR}{C^{1-1/\eta} + \phi R^{1-1/\eta}}$ and $R/C = (W/\phi)^{-\eta}$.

$$\begin{aligned} \frac{\phi R^{1-1/\eta}}{C^{1-1/\eta} + \phi R^{1-1/\eta}} &= \frac{WR}{WR + C} \quad , \quad \frac{\phi R^{1-1/\eta} WR + C}{1 - 1/\eta} = \frac{\phi R^{1-1/\eta}}{1 - 1/\eta} \left(1 + \frac{(W/\phi)^\eta}{W}\right) = \bar{U} \\ \frac{C^{1-1/\eta}}{C^{1-1/\eta} + \phi R^{1-1/\eta}} &= \frac{C}{WR + C} \quad , \quad \frac{C^{1-1/\eta} WR + C}{1 - 1/\eta} = \frac{C^{1-1/\eta}}{1 - 1/\eta} \left(1 + \frac{W}{(W/\phi)^\eta}\right) = \bar{U} \quad , \end{aligned}$$

◇ Hicksian demand functions:

$$\begin{aligned} R_h &= \left(\frac{W/\phi}{(W/\phi)^\eta + W} (1 - 1/\eta) \bar{U} \right)^{\frac{1}{1-1/\eta}} = \left(\frac{1/\phi}{\phi^{-\eta} W^{\eta-1} + 1} (1 - 1/\eta) \bar{U} \right)^{\frac{1}{1-1/\eta}} \\ C_h &= \left(\frac{(W/\phi)^\eta}{(W/\phi)^\eta + W} (1 - 1/\eta) \bar{U} \right)^{\frac{1}{1-1/\eta}} = \left(\frac{1}{\phi^\eta W^{1-\eta} + 1} (1 - 1/\eta) \bar{U} \right)^{\frac{1}{1-1/\eta}} \end{aligned}$$

◇ Slutsky Decomposition ($\tilde{W} \neq W$):

$$\begin{aligned} \frac{dR}{dW} &= \frac{dR_h}{dW} - \frac{dR}{d\tilde{W}} R \\ &= \frac{\eta}{\eta - 1} R_h \frac{(1 - \eta) \phi^{-\eta} W^{\eta-2}}{\phi^{-\eta} W^{\eta-1} + 1} - \frac{R}{(W/\phi)^\eta + W} = -\eta \frac{R_h}{\tilde{W}} \frac{C}{W} - R \frac{R}{\tilde{W}} \\ \frac{dC}{dW} &= \frac{dC_h}{dW} - \frac{dC}{d\tilde{W}} R \\ &= \frac{\eta}{\eta - 1} C_h \frac{(\eta - 1) \phi^\eta W^{-\eta}}{\phi^\eta W^{1-\eta} + 1} - \frac{R}{W (W/\phi)^{-\eta} + 1} = \eta \frac{C_h}{\tilde{W}} R - C \frac{R}{\tilde{W}} \end{aligned}$$

Note that the above results show that $\frac{dC}{dW} \frac{W}{C} - \frac{dR}{dW} \frac{W}{R} = \eta$ at $R_h = R$ and $C_h = C$. This means that, if we define $\tilde{\eta}$ as an elasticity of substitution between R and C keeping the wealth at a certain level, we find $\tilde{\eta} = -\frac{d(R/C)}{dW} \frac{W}{(R/C)} \Big|_{\tilde{W} \text{ constant}} = \eta$. This is because, under CES utility, the income effects through R and C are identical, and they offset each other in the calculation of η .

◇ Total effects ($\tilde{W} = W$): Hence, at $R_h = R$, $C_h = C$ and $\tilde{W} = W$, we can decompose the total

effect as follows.

$$\begin{aligned}\left. \frac{dR/R}{dW/W} + \frac{dR/R}{d\tilde{W}/\tilde{W}} \right|_{\tilde{W}=W} &= -\eta(1-R) - R + 1 = (1-\eta)(1-R) \\ \left. \frac{dC/C}{dW/W} + \frac{dC/C}{d\tilde{W}/\tilde{W}} \right|_{\tilde{W}=W} &= \eta R - R + 1 = 1 - (1-\eta)R\end{aligned}$$

Confirm $-\frac{d(R/C)}{dW} \frac{W}{R/C} = \frac{dC}{dW} \frac{W}{C} - \frac{dR}{dW} \frac{W}{R} = \eta$. Here, Hick substitution effects for R and C are $-\eta(1-R) < 0$ and $\eta R > 0$, respectively. For both R and C , Hicks income effect and direct income effect are $-R$ and 1 , respectively.